UNSTEADY NEWTONIAN AND NON-NEWTONIAN FLUID FLOWS IN THE CIRCULAR TUBE IN THE PRESENCE OF MAGNETIC FIELD USING CAPUTO-FABRIZIO DERIVATIVE

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ABSTRACT

This thesis investigates analytically the magnetohydrodynamics (MHD) transport of Newtonian and non-Newtonian fluids flows inside a circular channel. The flow was subjected to an external electric field for the Newtonian model and a uniform transverse magnetic field for all models. Pressure gradient or oscillating boundary condition was employed to drive the flow. In the first model Newtonian fluid flow without stenotic porous tube was considered and in the second model stenotic porous tube was taken into account. The third model is concerned with the temperature distribution and Nusselt number. The fourth model investigates the non-Newtonian second grade fluid velocity affected by the heat distribution and oscillating walls. Last model study the velocity, acceleration and flow rate of third grade non-Newtonian fluid flow in the porous tube. The non-linear governing equations were solved using the Caputo-Fabrizio time fractional order model without singular kernel. The analytical solutions were obtained using Laplace transform, finite Hankel transforms and Robotnov and Hartley's functions. The velocity profiles obtained from various physiological parameters were graphically analyzed using Mathematica. Results were compared with those reported in the previous studies and good agreement were found. Fractional derivative and electric field are in direct relation whereas magnetic field and porosity are in inverse relation with respect to the velocity profile in Newtonian flow case. Meanwhile, fractional derivative and Womersely number are in direct relation whereas magnetic field, third grade parameter, frequency ratio and porosity are in inverse relation in third grade non-Newtonian flow case. In the case of second grade fluid, Prandtl number, fractional derivative and Grashof number are in direct relation whereas second grade parameter and magnetic field are in inverse relation. The fluid flow model can be regulated by applying a sufficiently strong magnetic field.

ABSTRAK

Tesis ini mengkaji pengangkutan magnetohidrodinamik (MHD) bagi aliran bendalir Newtonan dan bukan Newtonan di dalam saluran membulat secara analitik. Kesemua model tertakluk kepada medan magnet merentas lintang yang seragam, sementara khusus bagi model Newtonan pula, aliran tersebut juga tertakluk kepada medan elektrik luaran. Kecerunan tekanan atau syarat sempadan yang berayun dikenakan supaya aliran berlaku. Pada model pertama, aliran bendalir Newtonan tanpa tiub berliang yang tersumbat telah dikaji dan model kedua pula mengambilkira tiub berliang yang tersumbat. Model ketiga mengambilkira taburan suhu dan nombor Nusselt. Model keempat mengkaji halaju bendalir bukan Newtonan gred kedua dengan kesan taburan haba dan dinding yang berayun. Model terakhir mengkaji halaju, pecutan dan kadar aliran bendalir bukan Newtonan gred ketiga di dalam tiub berliang. Persamaan menakluk tak-linear telah diselesaikan menggunakan model peringkat pecahan masa Caputo-Fabrizio tanpa inti singular. Penyelesaian analitik diperoleh menggunakan jelmaan Laplace, jelmaan Hankel terhingga, dan fungsi Robotnov dan Hartley. Profail halaju diperoleh dari pelbagai parameter fizikal telah dianalisis secara graf menggunakan Mathematica. Keputusan yang didapati telah dibandingkan dengan kajian terdahulu dengan hasil yang memuaskan. Terbitan pecahan dan medan elektrik berada dalam bentuk hubungan terus, sementara medan magnet dan sifat keliangan pula berkadar songsang terhadap profail halaju pada model aliran Newtonan. Untuk model aliran bukan Newtonan gred ketiga pula, terbitan pecahan dan nombor Womersely mempunyai sifat hubungan terus terhadap profail halaju, sementara medan magnet, parameter gred ketiga, kadar frekuensi dan keliangan pula berkadaran songsang. Bagi model bendalir gred kedua, nombor Prandtl, terbitan pecahan dan nombor Grashof berkadar terus terhadap profail halaju, sementara parameter bendalir gred kedua dan medan magnet berkadar songsang. Model aliran bendalir boleh dikawal dengan mengenakan kekuatan medan magnet yang secukupnya.

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LIST OF ABBREVIATIONS

HPM	-	Homotopy perturbation method
OHAM	-	Optimal homotopy asymptotic method
MHD	-	Magnetohydrodynamic flow
HAM	-	Homotopy analysis method
BVP	-	Boundary value problem
IVP	-	Initial value problem
ADM	-	Adomian decomposition method
DTM	-	Differential transformation method
UFD_t	-	Caputo derivative notation in the literature
NFD_t	-	Caputo-Fabrizio derivative notation in the literatre
CF	-	Caputo-Fabrizio derivative notation in the thesis

NOMENCLATURE

Roman Letters

A_0	-	steady part of the pressure fluctuation
A_1	-	amplitude of the pressure fluctuation
A_g	-	acceleration amplitude
B_0	-	magnetic field in the radial direction
$\overrightarrow{B}$	-	external magnetic field
C	-	volume fraction density of the particle
C_f	-	skin friction coefficient
C_p	-	pressure heat capacity
c	-	stenosis shape parameter
$D_t^{(\alpha)}$	-	Caputo-Fabrizio fractional order derivative
d	-	stenosis distance
$\frac{du}{dt}$	-	material time derivative
$\overrightarrow{E}$	-	external electric field
Ec	-	Eckert number
E_z	-	electric field in the axial direction
e_0	-	electronic charge
$F_a(-b,\, t)$	-	Robotnov function
$\overrightarrow{F}_{em}$	-	electro-magnetic field force
$\overrightarrow{F}_{uv}$	-	relative motion between velocities
$\overrightarrow{f}$	-	Body force per unit mass
f_p	-	frequency of the pulse rate
g(t)	-	body acceleration
g_z	-	vertically downward gravity componenet
G	-	mass parameter of magnetic particles

Gr	-	Grashof number
Ha	-	Hartmann number
$H^1(a,\, b)$	-	class of all integrable functions on [a, b]
$\mathcal{H}_{n,i}$	-	Hankel transform
I	-	identity tensor
$\vec{J}$	-	current density
J_0, J_1	-	Bessel functions of first kind with zero and first order
K	-	non-dimensional electrokinetic width
k_B	-	Boltzmann constant
k_p	-	porosity parameter
k_t	-	thermal conductivity
$\vec{k}$	-	unit vector in z-direction
L_0	-	stenosis length
$\mathcal{L}$	-	Laplace transform
$M(\alpha)$	-	normalization function
m	-	average mass of the magnetic particles
N	-	magnetic particles per unit volume
Nu	-	Nusselt number
$\tilde{N}_b$	-	brownian motion
$\tilde{N}_t$	-	thermophoresis
n	-	velocity power index parameter
n_0	-	ionic concentration
Pr	-	Prandtl number
Δp	-	pressure drop
p	-	pressure N/m^2
Q	-	stokes constant
R	-	particles concentration parameter
R_1 & R_2	-	first and second Rivlin-Ericksen kinematic tensors
Re	-	Reynolds number
$R_{a,\,c}(-b,\, t)$	-	Hartley function

R_0	-	artery radius without stenosis
R_z	-	stenosis function
r	-	radial component
r_n	-	positive roots of the bessel function
S_j	-	tensors
T & T_ω	-	fluid temperature
T_a	-	absolute temperature
T_s	-	stress tensor
T_r	-	transpose of a matrix
T_∞	-	ambient temperature
t	-	dimensionless time
tr	-	trace of a square matrix
$U(t)$	-	unit step size
$u(r,t)$	-	blood flow velocity
∇u	-	viscous dissipation
u_r	-	radial component of velocity
u_0	-	characteristic velocity
u_θ	-	angular component of velocity
u_z	-	axial component of velocity
$\overrightarrow{V}$	-	Velocity field
V_0	-	characteristic velocity
$v(r,t)$	-	magnetic particles velocity
y	-	thermophoretic parameter
z	-	axial component
$\bar{z}$	-	location of stenosis

Greek Letters

α	-	order of the fractional differential operator
ζ^2	-	dimension less Womersley number
α_1, α_2	-	visco-elasticity and cross-viscosity
δ	-	maximum height of the stenosis
ε	-	dielectric constant
κ	-	Debye-Huckel parameter
κ^{-1}	-	thickness of electric double layer
$\wp_n$	-	positive roots of the bessel function
μ	-	fluid viscosity
μ_0	-	magnetic permeability
ζ	-	Womersley number
ρ	-	fluid density
σ	-	electrical conductivity
σ_e	-	net charge density
ψ	-	Stream function
τ	-	cauchy stress tensor
ν	-	kinematic viscosity
ϑ	-	body acceleration and pressure gradient angle
Φ	-	third grade non-Newtonian fluid parameters
γ	-	steady and unsteady pressure gradient ratio
ψ	-	second grade non-Newtonian fluid parameter
ω_p	-	frequency of the heart pressure
ω_g	-	frequency

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CHAPTER 1

INTRODUCTION

1.1 Research background

According to ordinary calculus, a function can be differentiated to the rst or second order. On the basis of results, some potential applications or meanings may be identified. In the 17th century, sir Isaac Newton and Wilhelm Leibnitz independently discovered their own calculus. Over three hundred years, the question raised by Leibnitz about fractional-order derivative was a prevailing topic. It has long been regarded as a pure mathematical domain with no practical applications. Nevertheless, recently, this matter has changed due to improvement in the fractional calculus Sengar, Sharma and Trivedi (2015). The most significant advantage of modeling with fractional-order derivative is its non-local property (where we take non-integer order derivative like half order derivative), which differentiates it from the local model (where we take integer-order derivative like first order derivative, second order derivative etc). The local model only describes the current stage of the system whereas, the non-local model describes the historical stage of the system. According to Devendra, Singh and Kumar (2015) non-local property of the fractional differential equations differentiates it from the other models which predict the next stage of a system based on the historical background and doesn't rely on the current state of the system. According to Caputo (2008) and Riesz (2016) fractional calculus is concerned with derivatives and integrals of arbitrary (real or complex) orders in applied mathematics. Nowadays it gained importance and popularity, normally due to the established applications in science and engineering. It includes problem modeling in fluid flow, electric networks, propagation of seismic waves, rheology, oscillation, anomalous and reaction-diffusion, turbulence, polymer and chemical physics, electrochemistry, relaxation and dynamical processes and many other physical phenomena in the complex systems.

A fluid can be called a continuum because its particles have an identical topological relationship between each other. Ockendon and Tayler (1983) states that flow velocity, pressure, density, and temperature describes the state of fluid. Most of the organic and inorganic liquids being tiny molecular weight like gases, inorganic salts, and solution of liquefied metals with salts exhibit Newtonian flow characteristics. In these types of matter, shear stress is proportional to the shear rate at constant temperature and pressure, in simple shear, as by Chhabra (2010) dynamic viscosity is the constant of proportionality. In the past decades, most of the works of literature practice the Navier-Stokes equation to model the Newtonian fluid. However, Newtonian models are less applicable generally. In fact, according to Chen, Lai and Chen (2010) many complex types of fluids like blood, soaps, oils and greases, suspensions, clay coatings, and many emulsions are non-Newtonian fluids. Fluid having flow curve (shear stress versus shear rate) being nonlinear and deviated from the origin can be categorized as a non-Newtonian fluid. Furthermore, viscosity and shear stress does not vary at the specific pressure and temperature. Flow conditions like flow geometry and shear rate and even kinematic history also affect the non-Newtonian fluid characteristics. In the past, there were many mathematical models, focused on the flow parameters (like magnetic field, electricity, wall porosity and stenosis) affecting the non-Newtonian flow of a fluid inside by using various numerical approaches (Homotopy perturbation method (HPM), Homotopy analysis method (HAM), Adomian decomposition method (ADM), Perturbation method, Variation parameter method (VPM), Variation iteration method (VIM) and Caputo fractional derivative (UFD_t) in Calculus. Recently, there is an advancement towards fractional calculus, because it describes the sub-diffusion process between tissues and sub-cellular space during the fluid flow.

Unsteady flow refers to the state where the fluid properties at a point in the system change over time. In other words, time-dependent flow is known as unsteady flow. Many examples can be given from everyday life like water flow out of a tap which has just been opened. This flow is unsteady to start with, but with time does become steady. Some flows, though unsteady, become steady under certain frames of reference. These are called pseudo steady flows. On the other hand, a flow such as a wake behind a bluff body is always unsteady. Unsteady flows are undoubtedly difficult to calculate while with steady flows, we have one degree less complexity. Whether a particular flow is steady or unsteady, it depends on the chosen frame of reference. For instance, laminar flow over a sphere is steady in the frame of reference that is stationary with respect to the sphere. In a frame of reference that is stationary with respect to a background flow, the flow is unsteady. Turbulent flows are unsteady by definition. Steady flows are often more tractable than otherwise similar unsteady flows. The governing equations of a steady problem have one dimension fewer (time) than the governing equations of the same problem without taking advantage of the steadiness of flow field.

Fluid or gas flow through pipes is common in the distribution networks of fluids. The fluid is normally powered by a pump via a flow section in such applications. Friction resists flow-through pipes which are directly linked to the drop in pressure and heat loss. The pressure drop is then used to determine the pumping power requirement. Most of the fluids, especially liquids, are transported in circular pipes. This is because pipes with a circular cross-section can withstand large pressure differences between the inside and the outside without undergoing significant distortion. The fluid velocity in a pipe changes from zero at the surface because of the no-slip condition to a maximum at the pipe center. The region around a magnet that exerts a magnetic force is called a magnetic field. It is generated by the movement of electric charges. The presence and strength of a magnetic field are denoted by magnetic flux lines. These lines also indicate the direction of the magnetic field. The flux lines are clearly visible when iron particles are positioned on a magnet. Magnetic fields also generate power in particles that come in contact with it as shown in the Figure 1.1. Electrical fields are formed around particles that carry an electrical charge. This attracts positive charges, while negative charges are repelled. A moving charge always has both a magnetic and an electric field, and that's precisely the reason they are associated with each other. They are two different fields with nearly the same characteristics. Therefore, they are interrelated in a field called the electromagnetic field. In this field, the electric field and the magnetic field move at right angles to each other. However, they are not dependent on each other. They may also exist independently. Without the electric field, the magnetic field exists in permanent magnets and electric fields exist in the form of static electricity, in absence of the magnetic field.

Every numerical approach present and portrait a comparative study of the nonlinear velocity profile, wall shear stress, flow rate and pressure gradient in the presence of certain external flow parameters. The fluid flow model in the study is defined in Caputo-Fabrizio fractional-order derivative (CF) approach without a singular kernel.

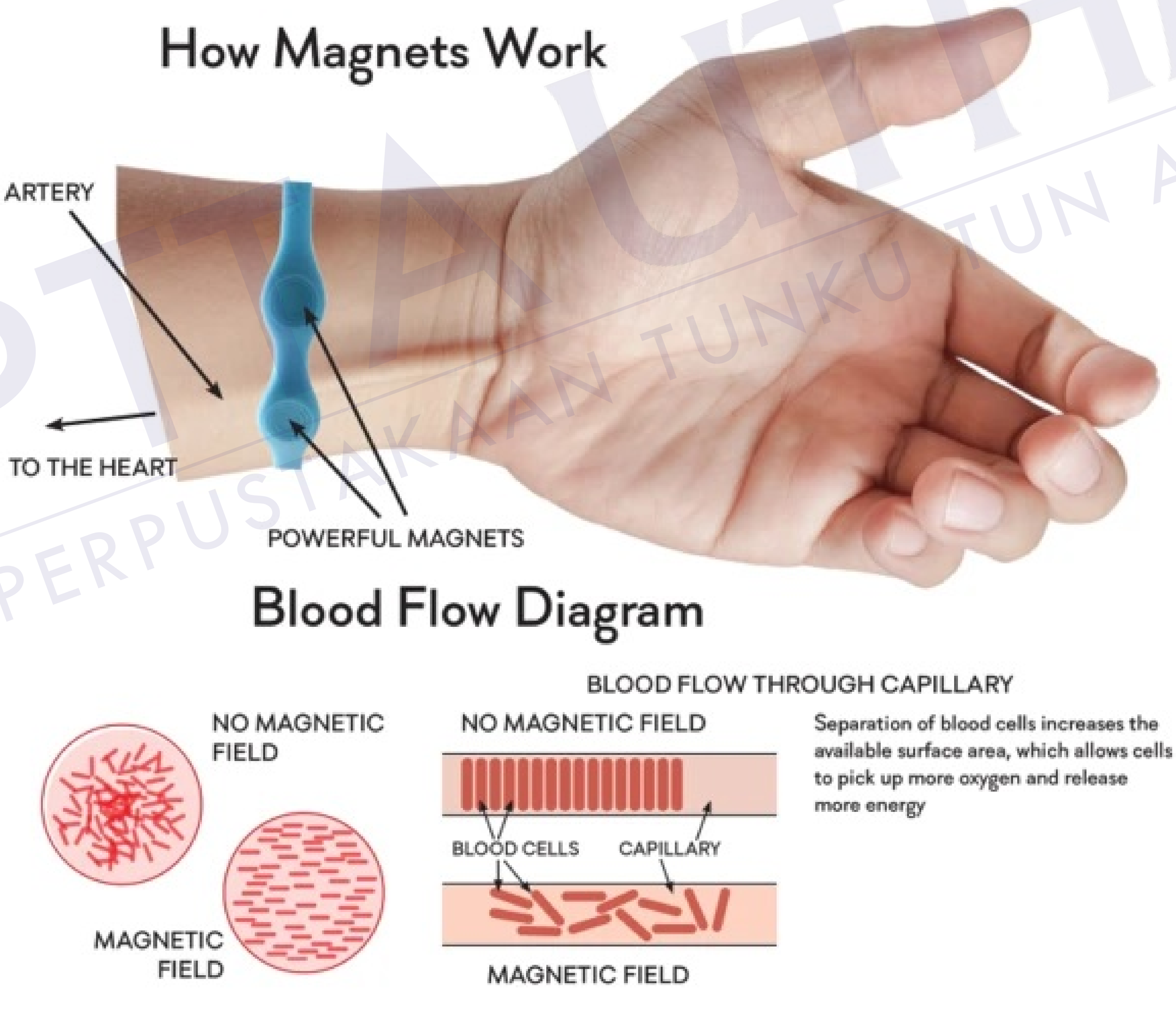

Figure 1.1: Magnetic Therapy as mentioned in Lusk (2018)

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