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A Multi-stage Approach for Empty Container Repositioning under Coordination among Liner Carriers

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Abstract This paper studies the empty container repositioning problem considering the exchange of slots and empty containers among liner shipping companies. It is common that an individual shipping company seeks for the optimal solution of empty container repositioning and cargo routing to maximize its own benefits. To achieve cooperation among shipping companies, a multi-stage solution strategy is proposed. With the inverse optimization technique, the guide leasing prices of slots and empty containers among shipping companies are derived considering the schedule of vessels and cargo routing. Based on the guide leasing price, a cooperative model is formulated to minimize the total cost, which includes the transportation cost for laden containers, the inventory holding cost, the container leasing cost, and the repositioning cost. All the involved shipping companies are expected to follow the best solution of empty container repositioning and cargo routing to achieve the cooperative and stable optimum. A real-world shipping network operated by three liner shipping companies is used as a case study with promising numerical results.

Keywords Empty container repositioning; Guide leasing price; Inverse optimization; Multi-stage optimization

1 Introduction

Container, as a fundamental and vital component, plays an important role in global freight distribution. According to [1], the container penetration in global trade rose from 22% in 1980 to 67% in 2012. Due to the disequilibrium of global trade in different regions, imbalanced containerized flows between import and export activities are inevitable, which results in that some ports have a surplus of empty containers while others lack of empty containers. Taking the Trans-Pacific trade lane as an example, the unbalanced flow was 10 million twenty-foot equivalent unit (TEU) of containers in 2016, which caused a large amount of empty containers accumulated in North America while a serious shortage of empty containers in East Asia [2]. Although empty container repositioning (ECR) is an effective way to address the imbalance problem, it usually generates high repositioning cost, which has increased from \$11 billion to \$16 billion between 2003 and 2012 [3]. It is evident that cooperation among shipping companies is a feasible strategy to reduce the high repositioning cost because it offers higher flexibility and efficiency of ECR operation by allowing exchange of empty containers and slots among different shipping companies [4].

In this paper, a variant of the ECR problem with explicit consideration of both slots and empty containers exchangeable among shipping companies, named cooperative empty container repositioning (CECR) problem, is addressed by a multi-stage approach. All the involved shipping companies are expected to follow the derived CECR solution to achieve a collaborative optimum. Multiple practical considerations are included in our models, like the schedule of vessels, cargo routing, devanning process, and leasing activities from both lessors and shipping companies. Naturally, individual shipping company strives to minimize its own operational cost and to maximize its profit. In order to lead shipping companies to a stable cooperation, proper pricing strategy is derived to help shipping companies to make decision on empty containers and slots leasing from other shipping companies. More importantly, to reflect the dynamics of market, we further consider the guide leasing prices as time-varying rather than fixed.

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The contribution of this work is threefold. First, we introduce a CECR model with cooperation among shipping companies and multiple practical considerations, like the schedule of vessels, cargo routing, devanning process, and leasing activity from container leasing company. Second, a multi-stage approach based on inverse optimization technique is applied to deriving the time-varying guide leasing prices, which are useful for decision making on leasing activities among shipping companies. Finally, in contrast to the system optimal solution with potential sacrifice of individual shipping company's benefits, our proposed solution shows similar total system cost, but none individual shipping company would suffer from increased total cost under cooperation.

The rest of this paper unfolds as follows. The subsequent section reviews relevant literature. Then an ECR model and an inverse optimization model are developed to determine the collaborative optimum. After that, computational experiments are conducted and the corresponding results are presented and discussed. The last section concludes the paper with a discussion of future work.

2 Literature review

Over the past decade, there have been extensive investigations on the ECR problem and its variants. The ECR problem is often combined with other problems, such as shipping network design [5; 6], ship deployment [7; 8], cargo routing [9], inventory control [10], and leasing activities [11]. This paper focuses on the ECR problem considering cargo routing, and the relevant literature studies are reviewed as follows.

Crainic et al. [12] introduced the characteristics and the structure of an inland ECR problem, which provided a general framework to formulate this class of problems. Cheung and Chen [13] developed a two-stage stochastic programming model to investigate a dynamic empty container allocation problem, where the uncertain demands, supplies and ship capacities for empty containers were determined at the beginning of the second stage. Song [14] and Shi and Xu [15] applied the Markov decision process to determining the optimal policy of empty container inventory control in a two-port system. Later, Song and Dong [16] extended the previous studies to the cycling shipping route and compared it with three other heuristic policies. Considering stochastic parameters, Di Francesco et al. [17] proposed multi-scenario policies to improve shipping companies' ability of responding to uncertain demand for empty containers. Song and Dong [9] combined the laden container routing problem and the ECR problem under fixed vessel schedules and a given shipping service network. Song and Dong [18] developed an three stage solution procedure to tackle the joint problem of long-haul route design, ship deployment and empty container repositioning, where the direction of empty container flow was specified based on the container flow pattern and the route topologic structure. Hu et al. [19] derived a guide leasing price to help make decision on repositioning or leasing empty containers to minimize total operational cost for individual linear carrier. Furthermore, some researchers explored the usage of foldable containers and found that foldable containers could save repositioning cost compared to the standard ones [20]. Numerical experiments demonstrated that using foldable containers would reduce 1.06% of total repositioning cost when the purchasing price of a foldable container was 50% higher than that of a standard one [21].

Prior research on the CECR problem with the consideration of cooperation among shipping companies is limited in number. Theofanis and Boile [22] discussed the CECR problem from a qualitative point of view, and they claimed that a liner carrier could lease out empty containers to other liner carriers if the inflow of containers exceeded the outflow, otherwise they might consider leasing in from other liner carriers. Zheng et al. [23] developed a two-stage empty container exchange model among liner carriers, where only the quantity of empty containers from one node to another was specified. Cargo routing was not taken into account, therefore the travel time for empty container delivery and the capacity of vessel were neglected. Perceived price was proposed to determine the exchange price of empty containers from other liner carriers, but the exchange price was fixed and identical for different liner carriers without consideration of marketing dynamics and heterogeneous prices for different liner carriers. Moreover, the leasing activity from container leasing company was not included, which actually had impacts on the exchange activities among liner carriers. This study is motivated by the issues

raised heretofore that reflect a practical need: how to design a dynamic pricing strategy to maintain the CEQR solution under cooperation among shipping companies, in the meanwhile to include multiple practical considerations in operation.

3 Problem description and formulations

In this study, we consider multiple shipping companies that provide weekly shipping service among various ports according to the predetermined services routes and timetable. Each shipping company operates a container fleet to satisfy the customers' demand for empty containers at each port covered by its service routes. When the owned containers cannot satisfy the demands, leased containers will be used, which can be leased from the vendors or other shipping companies. The leased containers refer to the short-term leased containers, whose leasing period starts when they are collected by customers at the origin port and ends when they are returned at the destination.

To simplify model formulation, relevant assumptions are given as below:

- i. All containers and demands are measured by TEU. The forty-foot equivalent unit can be treated as two TEUs, and the other types of containers like reefer containers are not considered in this study;
- ii. No changes happen on the liner shipping network, service schedules and deployed vessels in the planning horizon;
- iii. The weekly laden container with consignments accumulated between two adjacent round trips must be fulfilled by the latter round trip;
- iv. The lead times of returning empty containers and empty containers becoming laden containers are given.

Assumption i and Assumption ii are commonly used assumptions in the literature. In practice, shipping companies usually design their service routes several months in advance. Assumption iii guarantees the accumulated consignments to be transported to destination ports on time. Assumption iv describes that once laden containers are unloaded from the vessels at the destination, they become empty and can be reused after a certain time period [9].

Relevant parameters and variables used for the model formulation are shown as following:

Parameter

- A The set of shipping companies, $a \in A$;
- K The planning horizon, $k \in K$;
- I^a The set of ports of shipping company a , $i \in I^a$;
- $I^{a,r}$ The set of ports with service route r of shipping company a ;
- R^a The set of service routes of shipping company a , $r \in R^a$;
- R_i^a The set of service routes of shipping company a covering port i ;
- V^a The set of vessels belonging to shipping company a , $v \in V^a$;
- $V^{a,r}$ The set of vessels deployed in service routes r by shipping company a ;
- $Path_{ij}$ The set of paths connecting port i to port j , $p_{ij} \in Path_{ij}$;
- $r^{a,v}$ The route where vessel v of shipping company a deployed in;
- $Cap^{a,v}$ The capacity of vessel v of shipping company a ;
- $d_{ij,n}^a$ The customer demands from port i to port j of shipping company a at n th week, $i \neq j$;
- $Inventory_i^a$ The initial inventory of empty containers at port a of shipping company i ;
- $\tau_{ij}^{a,r}$ The travel time from port i to port j along route r of shipping company a ;
- $\tau_{p_{ij}^n}^a$ The travel time of n th segment p_{ij}^n on path p_{ij} of shipping company a ;
- $h_{imj}^{a,rr'}$ Waiting time of shipping company a at transit port m between port i to port j with two different routes r and r' , $i \neq j, j \neq m, r \neq r'$;
- $C_i^{hold,a}$ Unit empty container holding cost per period at port i of shipping company a ;
- $C_{ij}^{del,a,r}$ Unit transportation cost per empty container from port i to port j along route r of shipping company a , including transportation cost and lift-on/off cost, $i \neq j$;

- $C_{p_{ij}}^{laden,a}$ Unit transportation cost per laden container from port i to port j on path p_{ij} of shipping company a , including the transportation cost, lift-on/off cost and waiting cost at transit ports, $i \neq j$;
 $C_i^{long,a}$ Unit long-term leasing cost per container at port i for shipping company a ;
 $C_{p_{ij}}^{short,a}$ Unit short-term leasing cost per container for shipping company a from port i to port j on path p_{ij} , $i \neq j$;
 $C_{ij,k}^{slot,ab,r}$ Unit cost for shipping company a leasing in one slot from shipping company b from port i to port j along route r at period k , $i \neq j$, $a \neq b$, $r \in R_i^b$;
 $C_{p_{ij},k}^{empty,ab}$ Unit cost for shipping company a leasing in one container from shipping company b from port i to port j on path p_{ij} at period k , $i \neq j$, $a \neq b$;
 $W_{ij,k}^{slot,ab,r}$ Unit profit for shipping company a leasing out one slot to shipping company b from port i to j along route r at period k , $i \neq j$, $a \neq b$, $r \in R_i^a$;
 $W_{p_{ij},k}^{empty,ab}$ Unit profit for shipping company a leasing out one container to shipping company b from port i to port j on path p_{ij} at period k , $i \neq j$, $a \neq b$;
 ξ_1^a Turn-around time of returning empty container from customers to shipping company a ;
 ξ_2^a Lead time of empty container becoming laden container to shipping company a ;
 $\beta_{ij,k}^{a,v}$ Takes 1 if vessel v of shipping company a arrives at port i at period k and has not called port j at this trip, but will do it later; 0 otherwise, $i \neq j$;
 $\mu_{p_{ij},k}^{a,v}$ Takes 1 if vessel v of shipping company a starts p_{ij}^n , the n th segment of path p_{ij} at the period k ; 0 otherwise, $i \neq j$;
 $\varphi_{p_{ij},k}^a$ Takes 1 if a vessel of shipping company a starts path p_{ij} at period k ; 0 otherwise, $i \neq j$.

Variable

- $s_{i,k}^a$ The inventory of empty containers at port i at period k of shipping company a ;
 $Cap_k^{a,v}$ The remained capacity of vessel v at period k of shipping company a ;
 $x_{ij,k}^{a,r}$ The number of empty containers repositioned by shipping company a from port i to port j along route r at period k , $i \neq j$;
 $y_{p_{ij},k}^a$ The number of laden containers transported from port i to port j through path p_{ij} at period k of shipping company a , $i \neq j$;
 $l_{p_{ij},k}^{short,a}$ The number of empty containers short-term leased in at port i by shipping company a to port j on path p_{ij} at period k , $i \neq j$;
 $e_{ij,k}^{slot,ab,r}$ The number of slots at port i at period k leased in by shipping company a from shipping company b to port j , $i \neq j$, $a \neq b$;
 $e_{p_{ij},k}^{empty,ab}$ The number of empty containers leased in by shipping company a from shipping company b on path p_{ij} from port i to port j at period k , $i \neq j$, $a \neq b$.

In this study, path p_{ij} is defined as a sequence of service routes and transshipment points connecting two ports, and $Path_{ij}$ is the set of feasible paths from port i to port j , $p_{ij} \in Path_{ij}$, $i \neq j$. In this research, laden containers are allowed to be transshipped twice at maximum. Thus, $Path_{ij}$ consists of three types of paths depending on the number of transshipment ports involved: direct service $p_{ij} = (i \xrightarrow{r} j)$ denotes a direct path from port i to port j on route r without transshipment; transshipment once $p_{ij} = (i \xrightarrow{r} m \xrightarrow{r'} j)$ means two routes r and r' are adopted to connect port i with transshipment point m , and point m with port j respectively; and transshipment twice $p_{ij} = (i \xrightarrow{r} m \xrightarrow{r'} m' \xrightarrow{r''} j)$ includes two transshipment points m and m' and three corresponding routes r , r' and r'' in sequence to connect port i and j . Particularly, path with transshipment can be split into a sequence of segments according to the number of transshipment points, and each segment represents a direct route. Let p_{ij}^n denote the n th segment of path p_{ij} and P_{ij}^n represent the segments from the first segment to the n th segment of path p_{ij} , thus we have $P_{ij}^n = p_{ij}^1 + \dots + p_{ij}^n$, $1 \leq n \leq 3$.

Since the schedules and routes of vessels are fixed, and regular services are provided by shipping companies in the planning horizon, the corresponding travel time on each path can be calculated. Taking a path p_{ij} with two transshipment points m and m' and three corresponding routes r , r' and

r'' as instance, the corresponding cumulative travel time covering different segments can be calculated. Let $T_{P_{ij}^1}^a$ be the cumulative travel time of the first segment P_{ij}^1 on path p_{ij} of shipping company a , thus

$$T_{P_{ij}^1}^a = \tau_{im}^{a,r} \quad (1)$$

Similarly, let $T_{P_{ij}^2}^a$ be the cumulative travel time covering the first two segments P_{ij}^2 on path p_{ij} of shipping company a , then

$$T_{P_{ij}^2}^a = \tau_{im}^{a,r} + h_{imm'}^{a,rr'} + \tau_{mm'}^{a,r'} \quad (2)$$

and $T_{P_{ij}^3}^a$ be the cumulative travel time covering all the three segments P_{ij}^3 on path p_{ij} of shipping company a , such that

$$T_{P_{ij}^3}^a = \tau_{im}^{a,r} + h_{imm'}^{a,rr'} + \tau_{mm'}^{a,r'} + h_{mm'j}^{a,r'r''} + \tau_{m'j}^{a,r''} \quad (3)$$

Moreover, the time duration of leasing containers from port i to port j using path p_{ij} of shipping company a can be calculated as below:

$$t_{p_{ij}}^{lease,a} = T_{P_{ij}^3}^a + \xi_1^a + \xi_2^a \quad (4)$$

3.1 Inflow and outflow of empty containers

$In_{i,k}^a$ represents the number of empty containers that flow in port i at period k of shipping company a , which includes: (1) the empty containers repositioned in by self and other shipping companies; (2) the empty containers returned from other shipping companies and customers; and (3) the empty containers leased in from lessors and other shipping companies. Let $N_{i,k}^{in,a}$ be the number of repositioned empty containers arriving at port i at period k of shipping company a , and then we have

$$N_{i,k}^{in,a} = \sum_{r \in R_i^a} \sum_{j \in I^{a,r}} \sum_{v \in V^{a,r}} \beta_{ji,(k-\tau_{ji}^{a,r})}^{a,v} x_{ji,(k-\tau_{ji}^{a,r})}^{a,r} + \sum_{b \in A \setminus a} \sum_{r \in R_i^b} \sum_{j \in I^{b,r}} \sum_{v \in V^{b,r}} \beta_{ji,(k-\tau_{ji}^{b,r})}^{b,v} e_{ji,(k-\tau_{ji}^{b,r})}^{slot,ab,r} \quad (5)$$

wherein the first and second terms represent the empty containers repositioned in by self and other shipping companies, respectively. Similarly, let $N_{i,k}^{out,a}$ be the number of empty containers leaving port i at period k of shipping company a repositioned by self (first term in (6)) and other shipping companies (second term in (6)), then we have:

$$N_{i,k}^{out,a} = \sum_{r \in R_i^a} \sum_{j \in I^{a,r}} \sum_{v \in V^{a,r}} \beta_{ij,k}^{a,v} x_{ij,k}^{a,r} + \sum_{b \in A \setminus a} \sum_{r \in R_i^b} \sum_{j \in I^{b,r}} \sum_{v \in V^{b,r}} \beta_{ij,k}^{b,v} e_{ij,k}^{slot,ab,r} \quad (6)$$

Let $E_{p_{ij},k}^{empty,a}$ define the total number of leased-in empty containers, which should be transported from port i to port j along path p_{ij} at period k of shipping company a . According to Assumption iv, empty containers are only needed at a certain time before they are transported. Therefore, we get

$$E_{p_{ij},k}^{empty,a} = \varphi_{p_{ij},(k+\xi_2^a)}^a (I_{p_{ij},k}^{short,a} + \sum_{b \in A \setminus a} e_{p_{ij},k}^{empty,ab}) \quad (7)$$

and furthermore,

$$\begin{aligned} In_{i,k}^a &= \sum_{j \in I^a} \left(\sum_{b \in A \setminus a} \sum_{p_{ji} \in Path_{ji}} \varphi_{p_{ji},(k-T_{P_{ji}^3}^b - \xi_1^b)}^b e_{p_{ji},(k-t_{p_{ji}}^{lease,b})}^{empty,ba} \right) + \sum_{p_{ij} \in Path_{ij}} E_{p_{ij},k}^{empty,a} + N_{i,k}^{in,a} \\ &+ \sum_{j \in I^a} \sum_{p_{ji} \in Path_{ji}} \varphi_{p_{ji},(k-T_{P_{ji}^3}^a - \xi_1^a)}^a y_{p_{ji},(k-T_{P_{ji}^3}^a - \xi_1^a)}^a \end{aligned} \quad (8)$$

Similarly, let $Out_{i,k}^a$ be the number of empty containers that flow out of port i at period k of shipping company a , which consists of: (1) the empty containers returned to shipping companies and leasing companies; (2) the empty containers leased out to other shipping companies; (3) the customers' demand; and (4) the empty containers repositioned out by self and other shipping companies. Hence, $Out_{i,k}^a$ can be calculated as below:

$$Out_{i,k}^a = \sum_{j \in I^a} \left(\sum_{b \in A \setminus a} \sum_{p_{ij} \in Path_{ij}} \varphi_{p_{ij},(k+\xi_2^b)}^b e_{p_{ij},k}^{empty,ba} + \sum_{p_{ji} \in Path_{ji}} E_{p_{ji},(k-t_{p_{ji}}^{lease,a})}^{empty,a} \right) + N_{i,k}^{out,a} + \sum_{j \in I^a} \sum_{p_{ij} \in Path_{ij}} \varphi_{p_{ij},(k+\xi_2^a)}^a y_{p_{ij},(k+\xi_2^a)}^a \quad (9)$$

Let $load_k^{a,v}$ denote the number of laden and empty containers loaded to vessel v of shipping company a at period k , then we have

$$load_k^{a,v} = \sum_{i,j \in P^{a,r^a,v}} \beta_{ij,k}^{a,v} (x_{ij,k}^{a,r^a,v} + \sum_{b \in A \setminus a} e_{ij,k}^{slot,ba,r^a,v}) + \sum_{i \in I^a} \sum_{j \in I^a} \sum_{p_{ij} \in Path_{ij}} \sum_{n \leq 3} \mu_{p_{ij}^n,k}^{a,v} y_{p_{ij},(k-T_{P_{ij}^n}^a + \tau_{P_{ij}^n}^a)}^a \quad (10)$$

wherein the first and second terms represent the empty and laden containers loaded on ship v , respectively. The empty containers include the owned empty containers and the slots leased to other shipping companies, and the laden containers contain the original and the transshipped containers. Let $unload_k^{a,v}$ represent the number of containers unloaded from vessel v of shipping company a at period k , which includes the empty containers repositioned to this port (first term in (11)) and the arriving laden containers (second term in (11)). Therefore, equation (11) can be given by

$$unload_k^{a,v} = \sum_{i,j \in P^{a,r^a,v}} \beta_{ij,(k-\tau_{ij}^{a,r^a,v})}^{a,v} (x_{ij,(k-\tau_{ij}^{a,r^a,v})}^{a,r^a,v} + \sum_{b \in A \setminus a} e_{ij,(k-\tau_{ij}^{a,r^a,v})}^{slot,ba,r^a,v}) + \sum_{i,j \in I^a} \sum_{p_{ij} \in Path_{ij}} \sum_{n \leq 3} \mu_{p_{ij}^n,(k-T_{P_{ij}^n}^a)}^{a,v} y_{p_{ij},(k-T_{P_{ij}^n}^a)}^a \quad (11)$$

3.2 A multi-stage modelling framework

The framework of the multi-stage model is shown in [Figure 1](#). In the first stage, the model focuses on ECR, cargo routing, and deriving the guide leasing price of each shipping company without consideration of cooperation. The model for ECR and cargo routing with cooperation is proposed first, named **M1**. In **M1**, the number of exchanged slots and empty containers is assumed to be zero, then ECR and cargo routing plan without cooperation for each shipping company can be obtained. Based on the dual of **M1**, a corresponding inverse optimization model (**M2**) is formulated based on the inverse optimization technique. By solving **M2**, the guide leasing prices of slot and empty container among shipping companies can be derived. The second stage aims to determine the CECR and cargo routing plan of each shipping company under cooperation. Based on the guide leasing prices derived in the first stage, some constraints are incorporated to guarantee that such leasing activity under cooperation will not increase individual cost. Combined with **M1**, the model for CECR and cargo routing under cooperation is formulated as **M3**, and then the CECR and cargo routing plan under cooperation can be derived.

3.2.1 Model without cooperation

The first model (defined as **M1**) aims to determine the weekly number of laden containers and empty containers delivered from one port to another by each shipping company without consideration of leasing slots and empty containers among shipping companies, which is subject to the following constraints:

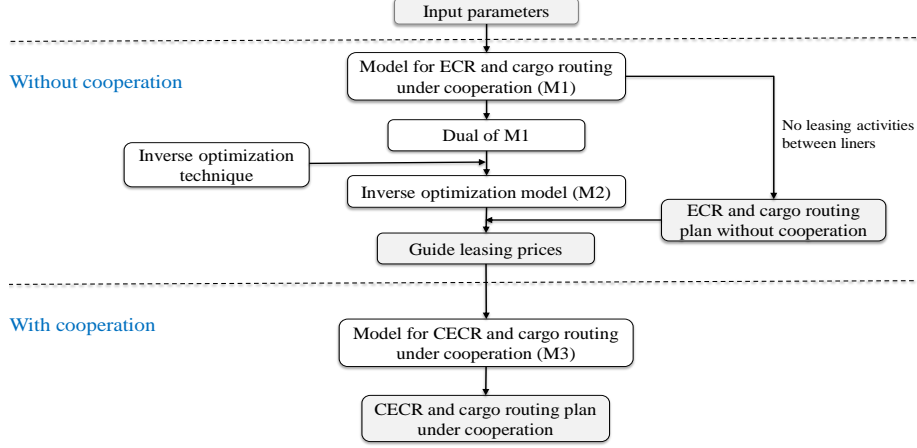


Figure 1. A multi-stage modelling framework

Constraint 1. The number of short-term leased-in empty containers is no more than the laden ones transported on the same path.

$$y_{p_{ij},k}^a - E_{p_{ij},(k-\xi_2^a)}^{empty,a} \geq 0 \quad (12)$$

Constraint 2. The number of laden containers transported within a week is equal to the weekly demand, and $\tilde{k} = \lceil k/7 \rceil$.

$$\sum_{p_{ij} \in Path_{ij}} \sum_{k=7\tilde{k}-6}^{7\tilde{k}} \varphi_{p_{ij},k}^a y_{p_{ij},k}^a = d_{ij,\tilde{k}}^a \quad (13)$$

Constraint 3. The total number of empty containers repositioned out and leased to other shipping companies does not exceed the inventory in the last period, and $s_{i,0}^a = Inventory_i^a$.

$$s_{i,(k-1)}^a - \sum_{b \in A \setminus a} \sum_{j \in I^b} \sum_{p_{ij} \in Path_{ij}} \varphi_{p_{ij},(k+\xi_2^b)}^b e_{p_{ij},k}^{empty,ab} - N_{i,k}^{out,a} \geq 0 \quad (14)$$

Constraint 4. The flow balance of empty and laden containers on vessels, and vessel's capacity is considered, and $Cap_0^{a,v} = Cap^{a,v}$.

$$Cap_k^{a,v} = Cap_{(k-1)}^{a,v} - load_k^{a,v} + unload_k^{a,v} \quad (15)$$

Constraint 5. The inventory change of empty containers at a certain port in a certain period is calculated as below, where $s_{i,0}^a = l_i^{long,a}$.

$$s_{i,k}^a = s_{i,(k-1)}^a + In_{i,k}^a - Out_{i,k}^a \quad (16)$$

Constraint 6. Non-negative variables include:

$$x_{ij,k}^{a,r}, y_{p_{ij},k}^a, l_{p_{ij},k}^{short,a}, e_{ij,k}^{slot,ab,r}, e_{p_{ij},k}^{empty,ab}, s_{i,k}^a, Cap_k^{a,v} \geq 0 \quad (17)$$

The objective of **M1** is to minimize the total cost J^a of individual shipping company in the whole horizon, which is the sum of the laden container transportation cost J_1^a , the empty container holding cost J_2^a , the empty container repositioning cost J_3^a , and the empty container leasing-in cost J_4^a , minus the revenue generated by leasing slots and empty containers to other shipping companies J_5^a . Each

component is formulated as follows.

$$J_1^a = \sum_k \sum_{i \in I^a} \sum_{j \in I^a} \sum_{p_{ij} \in Path_{ij}} C_{p_{ij}}^{laden,a} y_{p_{ij},k}^a \quad (18)$$

$$J_2^a = \sum_k \sum_{i \in I^a} \sum_{j \in I^a} C_i^{hold,a} s_{i,k}^a \quad (19)$$

$$J_3^a = \sum_k \sum_{r \in R^a} \sum_{i \in I^{a,r}} \sum_{j \in I^{a,r}} x_{ij,k}^{a,r} C_{ij}^{del,a,r} + \sum_k \sum_{b \in A \setminus a} \sum_{r \in R^b} \sum_{i \in I^{b,r}} \sum_{j \in I^{b,r}} e_{ij,k}^{slot,ab,r} C_{ij,k}^{slot,ab,r} \quad (20)$$

$$J_4^a = \sum_k \sum_{i \in I^a} \sum_{j \in I^a} \sum_{p_{ij} \in Path_{ij}} (l_{p_{ij},k}^{short,a} C_{p_{ij}}^{short,a} + \sum_{b \in A \setminus a} e_{p_{ij},k}^{empty,ab} C_{p_{ij},k}^{empty,ab}) \quad (21)$$

$$J_5^a = \sum_k \sum_{b \in A \setminus a} (\sum_{i \in I^b} \sum_{j \in I^b} \sum_{p_{ij} \in Path_{ij}} e_{p_{ij},k}^{empty,ba} W_{p_{ij},k}^{empty,ab} + \sum_{r \in R^a} \sum_{i \in I^{a,r}} \sum_{j \in I^{a,r}} e_{ij,k}^{slot,ba,r} W_{ij,k}^{slot,a,r}) \quad (22)$$

To sum up, the objective function is given as below:

$$\min J^a = J_1^a + J_2^a + J_3^a + J_4^a - J_5^a \quad (23)$$

Combining the constraints (12)-(17) and the objective function (18)-(23), we can get the complete model formulation of **M1**. By solving **M1**, we can obtain the optimal ECR solution and the corresponding cargo routing plan, with only consideration of leasing activity from container leasing company. Moreover, the dual of model **M1** can be derived, which formulates the key component of the inverse optimization model of **M1** in the subsequent section.

3.2.2 Inverse optimization model of M1

An Inverse Optimization (IO) technique proposed by Ahuja and Orlin [24] is adopted to determine the slot and empty container leasing prices. The inverse optimization aims to adjust a set of parameters to make a feasible solution optimal. Let $\lambda_{p_{ij},k}$, $\theta_{ij,\bar{k}}$, $\eta_{i,k}$, ψ_k^v , $\omega_{i,k}$ denote the dual variables associated with the constraints (12)-(16), respectively, $\pi_{M1} = (x_{ij,k}^{a,r}, y_{p_{ij},k}^a, l_{p_{ij},k}^{short,a}, e_{ij,k}^{slot,ab,r}, e_{p_{ij},k}^{empty,ab}, s_{i,k}^a, Cap_k^{a,v})$ be the optimal solution to **M1**, and $\pi_{M1-dual} = (\lambda_{p_{ij},k}, \theta_{ij,\bar{k}}, \eta_{i,k}, \psi_k^v, \omega_{i,k})$ be the optimal solution to **M1-dual**. According to the primal-dual complementary slackness conditions, additional sets can be defined as below:

- (1) $Z_{ik}^1 = \{(i, k) | s_{i,k}^a > 0, \forall i \in I^a, k \in K\}$
- (2) $Z_{ijkr}^2 = \{(i, j, k, r) | x_{ij,k}^{a,r} > 0, \forall r \in R^a, \forall i, j \in r, i \neq j, k \in K\}$
- (3) $Z_{p_{ij}k}^3 = \{(p_{ij}, k) | l_{p_{ij},k}^{short,a} > 0, \forall i, j \in I^a, i \neq j, \forall p_{ij} \in Path_{ij}, k \in K\}$
- (4) $Z_{vk}^4 = \{(v, k) | Cap_k^{a,v} > 0, \forall v \in V^a, k \in K\}$
- (5) $Z_{p_{ij}k}^5 = \{(p_{ij}, k) | y_{p_{ij},k}^a > 0, \forall i, j \in I^a, j \neq i, p_{ij} \in Path_{ij}, k \in K\}$
- (6) $Z_{p_{ij}k}^6 = \{(p_{ij}, k) | y_{p_{ij},k}^a - l_{p_{ij},(k-\xi_2^a)}^{short,a} > 0, \forall i, j \in I^a, j \neq i, p_{ij} \in Path_{ij}, k > \xi_2^a\}$
- (7) $Z_{ik}^7 = \{(i, k) | s_{i,(k-1)}^a - \sum_{r \in R_i^a} \sum_{j \in I^{a,r}} x_{ij,k}^{a,r} > 0, \forall i \in I^a, k \in K\}$

Let $\delta = (e_{ij,k}^{slot,ab,r}, e_{p_{ij},k}^{empty,ab})$ and $\sigma = (C_{p_{ij},k}^{empty,ab}, C_{ij,k}^{slot,ab,r}, W_{ij,k}^{slot,ab,r}, W_{p_{ij},k}^{empty,ab})$ when vector π is the optimal solution. Obviously, when the leasing activities of slots and empty containers among shipping companies are not taken into consideration, we can still get the feasible solution $\tilde{\pi}(\delta = 0)$. Let $\varepsilon_{p_{ij},k1}^{empty,ab}$ and $\gamma_{p_{ij},k1}^{empty,ab}$ be the adjustments of $C_{p_{ij},k}^{empty,ab}$, and $\varepsilon_{p_{ij},k2}^{empty,ab}$ and $\gamma_{p_{ij},k2}^{empty,ab}$ be the adjustments of $W_{p_{ij},k}^{empty,ab}$, respectively, then we have:

$$\tilde{C}_{p_{ij},k}^{empty,ab} = C_{p_{ij},k}^{empty,ab} + \varepsilon_{p_{ij},k1}^{empty,ab} - \gamma_{p_{ij},k1}^{empty,ab} \quad (24)$$

$$\tilde{W}_{p_{ij},k}^{empty,ab} = W_{p_{ij},k}^{empty,ab} + \varepsilon_{p_{ij},k2}^{empty,ab} - \gamma_{p_{ij},k2}^{empty,ab} \quad (25)$$

where $\tilde{C}_{p_{ij},k}^{empty,ab}$ and $\tilde{W}_{p_{ij},k}^{empty,ab}$ represent the adjusted variables of $C_{p_{ij},k}^{empty,ab}$ and $W_{p_{ij},k}^{empty,ab}$, respectively. Obviously, we have $\tilde{C}_{p_{ij},k}^{empty,ab} \geq C_{p_{ij},k}^{empty,ab}$ and $\tilde{W}_{p_{ij},k}^{empty,ab} \leq W_{p_{ij},k}^{empty,ab}$ when π and $\tilde{\pi}(\delta = 0)$ are optimal solutions to the original model and the adjusted model, respectively. Moreover, the inverse optimization technique aims to minimize the adjustment of parameters, thus we can have $\gamma_{p_{ij},k1}^{empty,ab}, \varepsilon_{p_{ij},k2}^{empty,ab} = 0$. Zheng et al. [25] derived the perceived price of leasing empty containers by minimizing the difference between leasing-in price and leasing-out price. However, in reality, the leasing-in price and leasing-out price can be different because shipping companies can use different paths even between the same origin-destination pair. Moreover, they concluded that the profit of leasing out an empty container was equal to the cost of leasing in an empty container at a certain port, which neglected different paths aforementioned, and when it was leased in. In this study, we consider different paths used by different shipping companies, and the leasing price is assumed to be time-varying. Therefore, the total time consumed from port i to port j varies for different shipping companies due to different paths used, and the leasing-in and the leasing-out prices of a specific shipping company can be different. Similarly, we can define the adjustments of slot leasing price as $\varepsilon_{ij,k1}^{slot,ab,r}, \gamma_{ij,k1}^{slot,ab,r}, \varepsilon_{ij,k2}^{slot,ab,r}$, and $\gamma_{ij,k2}^{slot,ab,r}$. The inverse optimization technique aims to determine the parameters' values to make the feasible solution optimal with the minimal adjustment. In the case of norm l_1 , the inverse optimization model based on **M1-dual** can be formulated as follows, defined as **M2**:

$$\begin{aligned} \min \quad & \sum_{b \in A \setminus a} \left(\sum_{i,j \in I^b} \sum_{p_{ij} \in Path_{ij}} \gamma_{p_{ij},k2}^{empty,ab} + \sum_{r \in R^a} \sum_{i,j \in I^{a,r}} \gamma_{ij,k2}^{slot,ab,r} \right. \\ & \left. + \sum_{r \in R^b} \sum_{i,j \in I^{b,r}} \varepsilon_{ij,k1}^{slot,ab,r} + \sum_{i,j \in I^a} \sum_{p_{ij} \in Path_{ij}} \varepsilon_{p_{ij},k1}^{empty,ab} \right) \end{aligned} \quad (26)$$

s.t.

$$\eta_{i,(k+1)} + \omega_{i,(k+1)} - \omega_{i,k} \leq C_i^{hold,a}, \forall (i,k) \notin Z_{ik}^1 \quad (27)$$

$$\sum_{v \in V^{a,r}} \beta_{ij,k}^{a,v} (\omega_{j,(k+\tau_{ij}^{a,r})} - \eta_{i,k} - \omega_{i,k} - \psi_k^v + \psi_{(k+\tau_{ij}^{a,r})}^v) \leq C_{ij}^{del,a,r}, \forall (i,j,k,r) \notin Z_{ijk}^2 \quad (28)$$

$$\varphi_{p_{ij},(k+\xi_2^a)}^a (\omega_{i,k} - \lambda_{p_{ij},(k+\xi_2^a)} - \omega_{j,(k+\tau_{p_{ij}}^{lease,a})}) \leq C_{p_{ij}}^{short,a}, \forall (p_{ij},k) \notin Z_{p_{ij}k}^3 \quad (29)$$

$$\psi_{(k+1)}^v - \psi_k^v \leq 0, \forall (v,k) \notin Z_{vk}^4 \quad (30)$$

$$\begin{aligned} \sum_{v \in V^a} \sum_{n \leq 3} \mu_{p_{ij},(k+T_{P_{ij}}^a - \tau_{p_{ij}}^a)}^{a,v} (\psi_{(k+T_{P_{ij}}^a)}^v - \psi_{(k+T_{P_{ij}}^a - \tau_{p_{ij}}^a)}^v) \\ + \lambda_{p_{ij},k} + \varphi_{p_{ij},k}^a (\theta_{ij,k} - \omega_{i,(k-\xi_2^a)} + \omega_{j,(k+T_{P_{ij}}^a + \xi_1^a)}) \leq C_{p_{ij}}^{laden,a}, \forall (p_{ij},k) \notin Z_{p_{ij}k}^5 \end{aligned} \quad (31)$$

$$\lambda_{p_{ij},k} = 0, \forall (p_{ij},k) \in Z_{p_{ij}k}^6 \quad (32)$$

$$\eta_{i,k} = 0, \forall (i,k) \in Z_{ik}^7 \quad (33)$$

$$\eta_{i,(k+1)} + \omega_{i,(k+1)} - \omega_{i,k} = C_i^{hold,a}, \forall (i,k) \in Z_{ik}^1 \quad (34)$$

$$\sum_{v \in V^{a,r}} \beta_{ij,k}^{a,v} (\omega_{j,(k+\tau_{ij}^{a,r})} - \eta_{i,k} - \omega_{i,k} - \psi_k^v + \psi_{(k+\tau_{ij}^{a,r})}^v) = C_{ij}^{del,a,r}, \forall (i,j,k,r) \in Z_{ijk}^2 \quad (35)$$

$$\varphi_{p_{ij},(k+\xi_2^a)}^a(\omega_{i,k} - \lambda_{p_{ij},(k+\xi_2^a)} - \omega_{j,(k+t_{p_{ij}}^{lease,a})}) = C_{p_{ij}}^{short,a}, \forall (p_{ij}, k) \in Z_{p_{ij}k}^3 \quad (36)$$

$$\psi_{(k+1)}^v - \psi_k^v = 0, \forall (v, k) \in Z_{vk}^4 \quad (37)$$

$$\begin{aligned} & \sum_{v \in V^a} \sum_{n \leq 3} \mu_{p_{ij},(k+T_{P_{ij}^n}^a - \tau_{p_{ij}^n}^a)}^{a,v} (\psi_{(k+T_{P_{ij}^n}^a)}^v - \psi_{(k+T_{P_{ij}^n}^a - \tau_{p_{ij}^n}^a)}^v) \\ & + \lambda_{p_{ij},k} + \varphi_{p_{ij},k}^a(\theta_{ij,\tilde{k}} - \omega_{i,(k-\xi_2^a)} + \omega_{j,(k+T_{P_{ij}^3}^a + \xi_1^a)}) = C_{p_{ij}}^{laden,a}, \forall (p_{ij}, k) \in Z_{p_{ij}k}^5 \end{aligned} \quad (38)$$

$$\sum_{v \in V^{b,r}} \beta_{ij,k}^{b,v} (\omega_{j,(k+\tau_{ij}^{b,r})} - \eta_{i,k} - \omega_{i,k}) - \varepsilon_{ij,k1}^{slot,ab,r} \leq C_{ij,k}^{slot,ab,r} \quad (39)$$

$$\varphi_{p_{ij},(k+\xi_2^a)}^a(\omega_{i,k} - \lambda_{p_{ij},(k+\xi_2^a)} - \omega_{j,(k+t_{p_{ij}}^{lease,a})}) - \varepsilon_{p_{ij},k1}^{empty,ab} \leq C_{p_{ij},k}^{empty,ab} \quad (40)$$

$$\sum_{v \in V^{a,r}} \beta_{ij,k}^{a,v} (\psi_{(k+\tau_{ij}^{a,r})}^v - \psi_k^v) - \gamma_{ij,k2}^{slot,ab,r} \leq -W_{ij,k}^{slot,ab,r} + C_{ij,k}^{del,a,r} \quad (41)$$

$$\varphi_{p_{ij},(k+\xi_2^b)}^a(\omega_{j,(k+t_{p_{ij}}^{lease,b})} - \eta_{i,k} - \omega_{i,k}) - \gamma_{p_{ij},k2}^{empty,ab} \leq -W_{p_{ij},k}^{empty,ab} \quad (42)$$

$$\gamma_{ij,k2}^{slot,ab,r}, \gamma_{p_{ij},k2}^{empty,ab}, \varepsilon_{ij,k1}^{slot,ab,r}, \varepsilon_{p_{ij},k1}^{empty,ab}, \lambda_{p_{ij},k}, \eta_{i,k} \geq 0 \quad (43)$$

By solving **M2**, we can derive the guide leasing prices of slots and empty containers for each shipping company, which includes both the leasing-in and the leasing-out prices. Such leasing prices will be input into the cooperative model, in the subsequent section, to derive the cooperative optimum.

3.2.3 Cooperative model

Since each shipping company tends to pursue its own maximal profits, multiple constraints are incorporated in the CECR model under cooperation among shipping companies (defined as **M3**) to avoid the leasing activities of slots and empty containers among shipping companies if such activities will increase the shipping company's cost.

$$(1 - \rho_{p_{ij},k}^{empty,ab}) e_{p_{ij},k}^{empty,ab} = 0 \quad (44)$$

$$(1 - \rho_{ij,k}^{slot,ab,r}) e_{ij,k}^{slot,ab,r} = 0 \quad (45)$$

where $\rho_{p_{ij},k}^{empty,ab} = 1$ when $C_{p_{ij},k}^{empty,ab} \geq W_{p_{ij},k}^{empty,ba}$, and 0 otherwise. Similarly, we have $\rho_{ij,k}^{slot,ab,r} = 1$ when $C_{ij,k}^{slot,ab,r} \geq W_{ij,k}^{slot,ba,r}$, and 0 otherwise.

The mechanism of leasing activity among shipping companies under cooperation is described as follows. If the guide leasing-in price of empty container of shipping company a is no less than the leasing-out price of shipping company b , $C_{p_{ij},k}^{empty,ab} \geq W_{p_{ij},k}^{empty,ba}$, then shipping company a should lease in empty containers from b , $\rho_{p_{ij},k}^{empty,ab} = 1$. In this way, shipping company a 's leasing-in cost is lower than its expectation (guide leasing-in price), while shipping company b will get its expected profit (guide leasing-out price). Hence, no partner will increase its cost under cooperation. On the contrary, leasing activity should not occur if $C_{p_{ij},k}^{empty,ab} \leq W_{p_{ij},k}^{empty,ba}$. The remaining constraints of **M3** are as same as those in **M1**, and its objective function is given as below:

$$\min J = \sum_{a \in A} J_a \quad (46)$$

In this paper, **M1** and **M3** are formulated as integer programming (IP) models, and **M2** is a linear

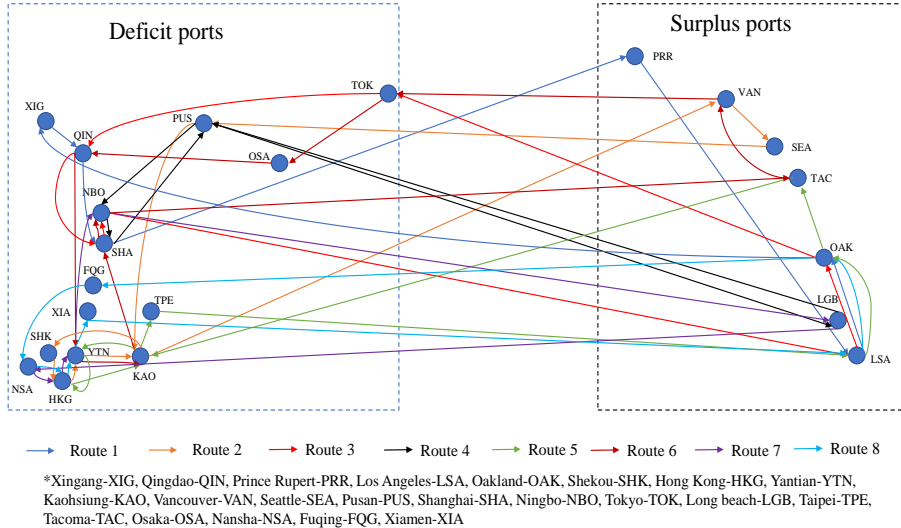


Figure 2. The service routes of shipping company 1

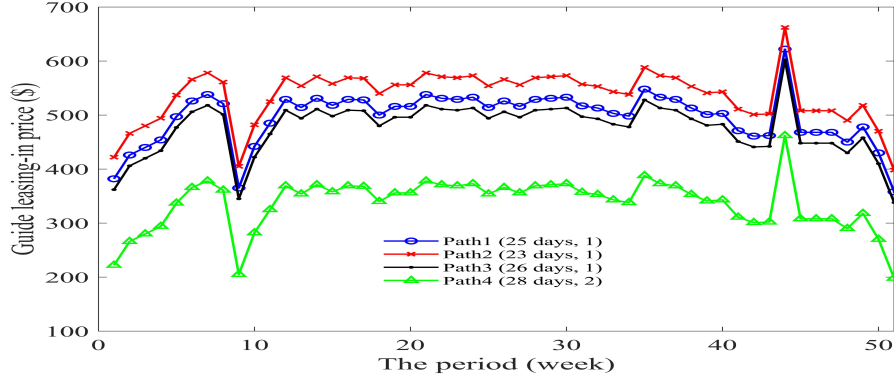
programming (LP) model, which can be solved efficiently by various existing solvers. The experiments are performed on a 64-bit Windows 10 platform with an Intel i7 2.2GHz CPU and 16GB RAM. The free toolbox Yalmip together with Matlab R2016 is adopted to model the problems. The commercial optimization solver Gurobi is used as an external solver of Yalmip to solve the proposed IP and LP models. In average, **M1** and **M2** can be solved in half an hour, and **M3** can be solved in an hour.

4 Numerical experiments

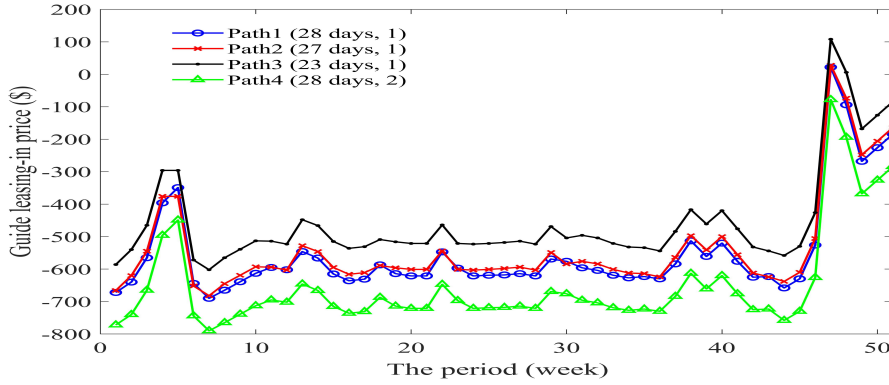
4.1 Experiment setting

A real-world shipping network connecting Asia and the West Coast of North America with three different shipping companies is employed to investigate the ECR problem under cooperation. The optimal ECR solution under cooperation for each shipping company is derived. For illustration purpose, we take shipping company 1 as an example to demonstrate the guide leasing-in and leasing-out activities between shipping company 1 and the other two companies, 2 and 3. Using the shipping network in [26], the service routes operated by shipping company 1 are shown in Figure 2. A route segment refers to a portion of a path, and a path consists of several route segments. For example, laden containers can be transported from Fuqing to Vancouver through a path consisting of three route segments: Fuqing to Hongkong along Route 8, Hongkong to Kaohsiung along Route 5, and Kaohsiung to Vancouver along Route 2. In the numerical experiments, the planning horizon is set as 52 weeks with given pathways and the corresponding travel times, and the demand beyond the planning horizon will be ignored. According to [8], the imbalance ratio of laden container flow for Asia-North America trade route is set around 1.99 (i.e., Eastbound/Westbound). Based on the ratio, the weekly laden container transportation consignments are randomly generated for each selected shipping service route. The transportation consignments from an Asian port to a North American port follow the uniform distribution $U(600, 800)$, and the ones on the inverse direction follow $U(300, 400)$. Here, we assume that the imbalance ratio of laden container flow in the same region is 1.00 and the transportation consignments follow the distribution $U(100, 150)$. According to the imbalance ratio of laden container flow, we can identify some ports in Asia (like Ningbo and Shanghai) as deficit ports and some in North America (like Oakland and Seattle) as surplus ports. For all the routes, there are no transportation consignments within the same country.

All vessels have the capacity between 6000 TEUs and 6800 TEUs, and they make round trips to provide a weekly service. All the holding costs and the transportation costs for both empty and laden containers are the same at each port: $C_i^{hold} = \$42/\text{week}$, $C_{ij}^{del,r} = \$140/\text{week}$. The short-term leasing



(a) Ningbo port to Oakland port



(b) Oakland port to Ningbo port

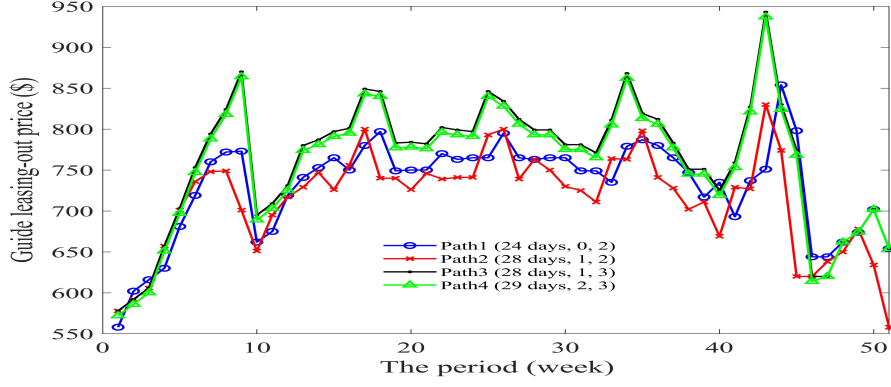
Figure. 3. The time-varying guide leasing-in price of empty containers of shipping company 1

cost is charged based on the travel time between the origin and the destination ports. According to [27], the unit short-term leasing cost is set as \$170/week. For example, if the travel time between an origin port and a destination port is two weeks and the time of devanning and loading for customers is two weeks, then the short-term leasing cost is \$680.

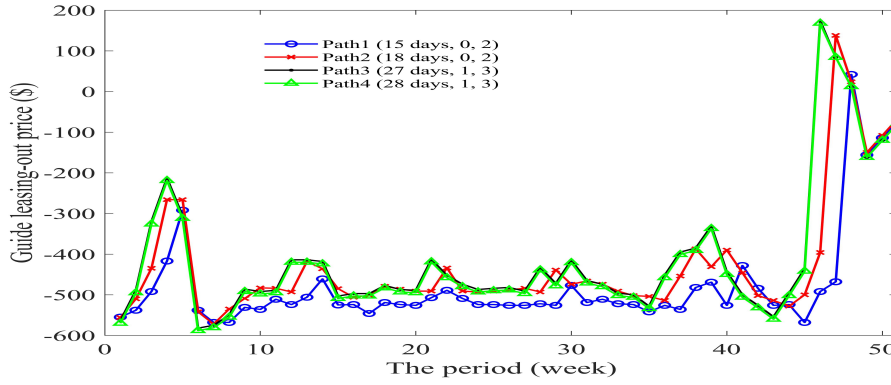
4.2 Result analysis

4.2.1 Guide leasing-in price of empty containers

In this subsection, the leasing-in activities of empty containers from shipping company 2 and 3 to shipping company 1 are analysed. Under cooperation, Figure 3(a) and Figure 3(b) show the time-varying guide leasing-in prices of empty container for shipping company 1 at Ningbo Port (deficit port) and Oakland port (surplus port), respectively. The legend in Figure 3 shows each path's detailed information, including the travel time and the number of transshipments. For example, Path1 (25 days,1) represents the containers along Path 1 will travel 25 days and be transshipped once. For the shipping services from Ningbo to Oakland, the guide leasing-in price of empty containers on each path is always above \$198. The positive price indicates that leasing in empty containers at Ningbo port could avoid empty containers accumulated at Oakland port, which reduces the storage and repositioning cost. On the contrary, the guide leasing-in prices along the paths from Oakland to Ningbo are negative in most periods, which indicates that leasing in empty containers is not an economical option for shipping company 1 at Oakland port since it is a surplus port. It is shown that the guide leasing-in price of empty containers at both Ningbo and Oakland ports rises firstly and then drops sharply at the beginning of the planning horizon. At both ports, leasing empty containers would result in more storage costs since a certain number of empty containers is stored at each port initially. As time elapses, the storage cost of empty containers caused by leasing-in activities decreases and thus the guide leasing-in price



(a) Ningbo port to Oakland port



(b) Oakland port to Ningbo port

Figure. 4. The time-varying guide leasing-out price of empty containers of shipping company 1

risers. After a certain period, empty containers initially accumulated at deficit ports arrive at Ningbo port, and lots of empty containers are returned to Oakland port by customers. Thus, the guide leasing prices at both ports drop. It is noticeable that the guide leasing prices show an opposite trend at the end of the planning horizon due to the assumption of no demand beyond the planning horizon. It can also be observed that the paths with shorter travel time show higher guide leasing-in prices in Figure 3, which indicates that the guide leasing-in price of empty containers is correlated with the path used.

4.2.2 Guide leasing-out price of empty containers

In this subsection, we demonstrate the leasing-out activities of empty containers from shipping company 1 to shipping companies 2 and 3. Figure 4 depicts the time-varying guide leasing-out prices of empty containers along different paths between Ningbo and Oakland ports. The legend in Figure 4 shows each path's detailed information. For example, Path1 (24 days,0,2) represents Path1 with travel time of 24 days and 0 transshipment, which belongs to shipping company 2. As shown in Figure 4, shipping company 1 is willing to lease out empty containers to other shipping companies at high price at Ningbo port, while the guide leasing-out prices at Oakland port are always negative. In most periods, Ningbo port, as a deficit port, lacks of empty containers, and leasing out empty containers at Ningbo port will worsen the situation and cause additional repositioning of empty containers. Therefore, the corresponding guide leasing-out prices are at a high level to prevent the occurrence of leasing-out activity. As a surplus port, Oakland port holds a great number of empty containers. Hence, leasing out empty containers at Oakland port would help to reduce the storage cost and reposition the surplus empty containers to Ningbo port simultaneously. It can be found that the guide leasing-out price of empty containers at Ningbo port rises firstly and then drops significantly at the beginning of the planning horizon. As a deficit port, Ningbo holds a certain number of initial stored empty containers to satisfy customers' demand. As time elapses, the storage cost decreases and the guide leasing-out

Table 1. Comparison of costs among shipping companies (million \$)

	Shipping company	Leasing cost			Repositioning cost			Total cost		
		1	2	3	1	2	3	1	2	3
Scenario 2	[1, 2]	6.7	47.3	-	0	0	-	1408.4	883.5	-
	[1, 3]	173.7	-	282.7	96.9	-	137.1	1456.8	-	1420.8
	[2, 3]	-	11.0	55.3	-	0	0	-	895.0	1369.1
	[1, 2, 3]	0	0	5.8	0	0	0	963.5	795.1	1222.1
Scenario 3	[1, 2]	8.9	42.3	-	0	0	-	1490.1	800.8	-
	[1, 3]	181.8	-	277.5	94.8	-	136.2	1455.4	-	1420.4
	[2, 3]	-	8.5	51.7	-	0	0	-	865.5	1397.6
	[1, 2, 3]	0	0	0	0	0	0	832.0	936.3	1182.9
Scenario 1	254.0	167.4	208.9	131.0	87.9	124.8	1477.4	1080.4	1430.0	

price rises. Then, empty containers initially accumulated at deficit ports arrive at Ningbo port, which causes a significant reduction of the guide leasing-out price. Similarly, the guide leasing-out price rises firstly at Oakland port as well. Due to the turn-around time, the number of empty containers at Oakland port would not increase, however, more and more empty containers are returned to liner 1 and accumulated at Oakland port. Thus, the guide leasing-out price drops, and shipping company 1 is more willing to leasing out empty containers to reposition surplus empty containers and reduce storage cost at Oakland. It is assumed that there is no demand beyond the planning horizon, hence repositioning empty containers is not considered at the end of the planning horizon. Consequently, the corresponding guide leasing-out price of empty container rises at the end of the planning horizon. It is noticeable that although the travel time and leasing time on path 2 and path 3 are identical in [Figure 4\(a\)](#), their guide leasing-out prices differ, which indicates that the guide leasing-out price not only depends on the travel time and leasing time, but also is related to the vessel schedule.

4.2.3 Comparison between cooperation and non-cooperation scenarios

In this subsection, we compare individual shipping company's various costs before and after considering cooperation. Three scenarios, (1) ECR without cooperation, (2) CECR under cooperation and following guide leasing prices, and (3) CECR under cooperation but ignoring guide leasing prices, are investigated. Scenario 1 considers no cooperative activity among shipping companies, but they can lease in empty containers from leasing companies. Scenario 2 represents the solution to model **M3**, and the leasing activities among shipping companies only happen when no individual shipping company's total cost is increased, which is a win-win solution to all shipping companies. Scenario 3 considers cooperative activities among shipping companies, but its objective function is to minimize the total system cost, which means the total cost of individual shipping company may rise.

As shown in [Table 2](#), under cooperation in Scenario 2, the total cost of shipping companies 1, 2 and 3 can be reduced by up to \$513.9 million, \$285.3 million and \$207.9 million, respectively; and the total system cost can be reduced by \$1007.1 million when cooperation among all the shipping companies happens. It is noticeable that the repositioning cost becomes zero in several cases under cooperation since shipping companies would lease out empty containers to other liners at surplus ports instead of repositioning. However, each liner's storage cost would increase since lots of owned empty containers would not be used. Compared to Scenario 2, the total system cost can be further reduced in Scenario 3. However, the total cost of shipping company 1 is increased by \$12.7 million when only shipping company 1 and 2 cooperate, which means the optimal solution in Scenario 3 is not beneficial for shipping company 1. Consequently, Scenario 3 may result in unstable cooperation among these three shipping companies. On the contrary, although with a slightly higher total system cost, Scenario

Table 2. Comparison between cooperation and non-cooperation scenarios (thousand TEUs)

	Liner	Empty containers repositioned		Empty containers leased	
		by self	by other liners	from lessors	from other liners
Cooperation	1	55.08	48.10	0	1643.15
	2	86.13	0	0	1825.73
Non-cooperation	1	537.7	0	201.4	0
	2	382.8	0	137.6	0

2 maintains a stable optimum for all participatory shipping companies.

The number of empty containers repositioned and leased-in under cooperation and non-cooperation is compared in Table 3. The number of empty containers repositioned drops while leasing activities between liners increase significantly. Under cooperation, the liners can better utilize the owned empty containers and reduce empty container movement by leasing out surplus empty containers to cooperators. Although this would result in higher storage cost, the total cost drops, and the storage cost can be reduced by optimizing the container fleet of each liner.

5 Conclusion and future work

In this paper, the problem of CECR in shipping network was studied considering leasing activities of slots and empty containers among shipping companies, and multiple practical conditions, like routing of laden containers, traveling time, and the devanning process. In order to maintain the cooperation among shipping companies stable, the time-varying guide leasing prices of slots and empty containers of each shipping company were derived by the inverse optimization technique. Shipping companies were guided to follow the optimal solution to CECR. By comparing the guide leasing prices of empty containers and slots along different paths between a certain pair of deficit port and surplus port, we found that the guide leasing prices were related to the schedule of vessels and they changed over time. Compared with the total cost under no cooperation, shipping companies could benefit from cooperation with other shipping companies. In contrast to the system optimal solution with potential sacrifice of individual benefit (increasing total cost of individual shipping company), the proposed solution shows similar total system cost, but the total cost of individual shipping company would not increase under cooperation. In the future work, the proposed model can be extended by incorporating cargo routing under stochastic demand, the impacts of devanning process on inventory control, and multi-type containers such as reefer containers. The CECR model will be further explored for application to other allocation or repositioning problems in car sharing and bike sharing systems [28; 29; 30]. Moreover, the competition between different shipping companies was ignored in this paper. Cooperation usually causes reduced competition with potential negative effects. Therefore, the trade-off between cooperation and competition, and proper strategies to balance cooperation and competition between different shipping companies are worthy of further investigation.

6 Author contributions

The authors confirm contribution to the paper as follows: study conception and design: B. Du, H. Hu; data collection: H. Hu; analysis and interpretation of results: B. Du, H. Hu, J. Zhang, M. Meng; draft manuscript preparation: B. Du, H. Hu, J. Zhang, M. Meng. All authors reviewed the results and approved the final version of the manuscript.

7 Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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