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Maria Vanina Martínez
Universidad de Buenos Aires and CONICET, Argentina

Ivan Varzinczak
CRIL, Univ. Artois & CNRS, France

Inductive Reasoning with Difference-making Conditionals

Meliha Sezgin[1] , **Gabriele Kern-Isberner**[1] , **Hans Rott**[2]
[1]Department of Computer Science, TU Dortmund University, Germany
[2]Department of Philosophy, University of Regensburg, Germany
meliha.sezgin@tu-dortmund.de, gabriele.kern-isberner@cs.uni-dortmund.de, hans.rott@ur.de

Abstract

In belief revision theory, conditionals are often interpreted via the Ramsey test. However, the classical Ramsey Test fails to take into account a fundamental feature of conditionals as used in natural language: typically, the antecedent is relevant to the consequent. Rott has extended the Ramsey Test by introducing so-called difference-making conditionals that encode a notion of relevance. This paper explores difference-making conditionals in the framework of Spohn's ranking functions. We show that they can be expressed by standard conditionals together with might conditionals. We prove that this reformulation is fully compatible with the logic of difference-making conditionals, as introduced by Rott. Moreover, using c-representations, we propose a method for inductive reasoning with sets of difference-making conditionals and also provide a method for revising ranking functions by a set of difference-making conditionals.

1 Introduction

On most accounts of conditionals, a conditional of the form 'If A then B' is true or accepted if (but not only if) B is true or accepted and A does not undermine B's truth or acceptance. On the suppositional account, for instance, if you believe B and the supposition that A is true does not remove B, you may (and must!) accept 'If A, then B'. On this account, there is no need that A furthers B or supports B or is evidence or a reason for B. This does not square well with the way we use conditionals in natural language. Skovgaard-Olsen et al. (2019) have conducted an empirical study and concluded that the positive relevance reading (reason-relation reading) of indicative conditionals is a conventional aspect of their meaning which cannot be cancelled 'without contradiction'. This, of course, is helpful only if the notion of contradiction is clear, but we aim to flesh out the positive relevance reading in an intuitive and yet precise way. The *difference-making conditionals* studied in this paper aim at capturing the relevance reading that is conveyed semantically or pragmatically by the utterance of conditionals in natural language. (Unfortunately, use of the term 'relevance conditionals' has been preempted by a completely different use in linguistics). Let us begin by giving an example that illustrates what we mean by the term 'relevance':

Example 1. *An agent wanted to escape the hustle and bustle of the city and decided to move into an old farm house in the countryside. Unfortunately, the weather quickly changed and it became cold (c). Due to the low temperatures one of the rather old pipes in the house broke (b) and the agent had to call a plumber (p) to get the damage fixed.*

In this example, it is clear that the cold temperatures are the reason for the broken pipe. Yet, this is not well reflected if we use a standard conditional 'If it is cold then the pipe will break'. We would rather say that the pipe broke *because* it was cold. The notion of relevance featuring here is encoded in the *Relevant Ramsey Test* which governs difference-making conditionals first introduced under a different name by Rott (1986) and then studied in Rott (2019). Except for a very recent paper by Raidl (2020), the logic of difference-making conditionals has been explored only in a purely qualitative framework. We characterize difference-making conditionals in the framework of Spohn's (1988) ranking functions and provide a simple and elegant semantics which we can use to define an inductive representation, that is, to build up an epistemic state from a (conditional) knowledge base, as well as a revision method for difference-making conditionals. Our main contributions in this paper are the following:

- We transfer Rott's notion of difference-making conditionals to the framework of ordinal conditional functions and reformulate the relevant Ramsey Test in this framework.
- We define an inductive representation for a set of difference-making conditionals in the framework of ranking functions.
- We set up a method for revising a ranking function by a set of difference-making conditionals, and we elaborate this general method for revising by a single difference-making conditional in the ranking functions framework, based on the c-revisions introduced by Kern-Isberner (2001).
- We compare the notion of evidence or support captured by difference-making conditionals to the one offered in related approaches like the 'evidential conditionals' of Crupi and Iacona (2019a) or Spohn's (2012) notion of 'reason'.

The rest of this paper is organized as follows: In section 2, we define the formal preliminaries and notations used throughout the paper. Section 3 summarizes concepts and results from Rott's (2019) work on difference-making conditionals. Then, in section 4, we define a ranking semantics for difference-making conditionals via an OCF-version of the Relevant Ramsey Test and prove the basic principles using a reformulation of a difference-making conditional as a pair of more standard conditionals. In section 5, we construct an inductive representation for sets of difference-making conditionals using c-representations. Section 6 introduces a method for revising by difference-making conditionals based on c-revisions in the framework of ranking functions. In section 7, we discuss alternative approaches to incorporating relevance in conditionals. The concluding section 8 sums up our findings.

2 Formal Preliminaries

Let $\mathcal{L}$ be a finitely generated propositional language over an alphabet Σ with atoms $a, b, c, \ldots$ and with formulas $A, B, C, \ldots$. For conciseness of notation, we will omit the logical and-connector, writing AB instead of $A \wedge B$, and overlining formulas will indicate negation, i.e., $\overline{A}$ means $\neg A$. The set of all propositional interpretations over Σ is denoted by Ω_Σ. As the signature will be fixed throughout the paper, we will usually omit the subscript and simply write Ω. $\omega \vDash A$ means that the propositional formula $A \in \mathcal{L}$ holds in the possible world $\omega \in \Omega$; then ω is called a *model* of A, and the set of all models of A is denoted by $Mod(A)$. For propositions $A, B \in \mathcal{L}$, $A \vDash B$ holds iff $Mod(A) \subseteq Mod(B)$, as usual. By slight abuse of notation, we will use ω both for the model and the corresponding conjunction of all positive or negated atoms. This will allow us to ease notation a lot. Since $\omega \vDash A$ means the same for both readings of ω, no confusion will arise. The set of classical consequences of a set of formulas $\mathcal{A} \subseteq \mathcal{L}$ is $Cn(\mathcal{A}) = \{B \mid \mathcal{A} \models B\}$. The deductively closed set of formulas which has exactly a subset $\mathcal{W} \subseteq \Omega$ as a model is called the *formal theory of* $\mathcal{W}$ and defined as $Th(\mathcal{W}) = \{A \in \mathcal{L} \mid \omega \models A \text{ for all } \omega \in \mathcal{W}\}$.

We extend $\mathcal{L}$ to a conditional language $(\mathcal{L}|\mathcal{L})$ by introducing a conditional operator $(\cdot|\cdot)$, so that $(\mathcal{L}|\mathcal{L}) = \{(B|A) \mid A, B \in \mathcal{L}\}$. $(\mathcal{L}|\mathcal{L})$ is a flat conditional language, no nesting of conditionals is allowed. A is called the antecedent of $(B|A)$, and B is its consequent. $(B|A)$ expresses *'If A, then (plausibly) B'*. In the following, conditionals $(B|A) \in (\mathcal{L}|\mathcal{L})$ are referred to as *standard conditionals* or, if there is no danger of confusion, simply *conditionals*.

We further extend our framework of conditionals to a language with *might conditionals* $\langle\!|\mathcal{L}|\mathcal{L}|\!\rangle$ by introducing a might conditional operator $\langle\!|\cdot|\cdot|\!\rangle$ (the angle brackets are supposed to remind the reader of a split diamond operator). For a might conditional $\langle\!|D|C|\!\rangle$, we call C the antecedent and D the consequent. As for standard conditionals, $\langle\!|\mathcal{L}|\mathcal{L}|\!\rangle$ is a flat conditional language, and $\langle\!|D|C|\!\rangle$ expresses *'If C, then D might be the case'*. In a way, the might conditional $\langle\!|D|C|\!\rangle$ is the negation of the standard conditional $(\overline{D}|C)$ (Lewis 1973). The former is accepted iff the latter isn't.

A (conditional) *knowledge base* is a finite set of conditionals $\Delta = \{(B_1|A_1), \ldots, (B_n|A_n)\} \cup \{\langle\!|B_{n+1}|A_{n+1}|\!\rangle, \ldots, \langle\!|B_m|A_m|\!\rangle\}$. To give an appropriate semantics to (standard resp. might) conditionals and knowledge bases, we need richer semantic structures like epistemic states in the sense of Halpern (2003), most commonly represented as probability distributions, possibility distributions (Dubois and Prade 2006) or ordinal conditional functions (Spohn 1988, 2012). A knowledge base is *consistent* if and only if there is (a representation of) an epistemic state that accepts the knowledge base, i.e., all conditionals in Δ.

Ordinal conditional functions (OCFs, also called *ranking functions*) $\kappa : \Omega \to \mathbb{N} \cup \{\infty\}$, with $\kappa^{-1}(0) \neq \emptyset$, assign to each world ω an implausibility rank $\kappa(\omega)$. OCFs were first introduced by Spohn (1988). The higher $\kappa(\omega)$, the less plausible ω is, and the normalization constraint requires that there are worlds having maximal plausibility. Then one puts $\kappa(A) := \min\{\kappa(\omega) \mid \omega \vDash A\}$ and $\kappa(\emptyset) = \infty$. Due to $\kappa^{-1}(0) \neq \emptyset$, at least one of $\kappa(A)$ and $\kappa(\overline{A})$ must be 0. A proposition A is believed if $\kappa(\overline{A}) > 0$, and the belief set of a ranking function κ is defined as $Bel(\kappa) = Th(\kappa^{-1}\{0\})$.

Definition 1. *A (standard) conditional $(B|A)$ is accepted in an epistemic state represented by an OCF κ, written as $\kappa \models (B|A)$, iff $\kappa(AB) < \kappa(A\overline{B})$ or $\kappa(A) = \infty$.*

That is, the verification of $(B|A)$ is more plausible than its falsification or the premise of the conditional is always false.

Definition 2. *A might conditionals $\langle\!|D|C|\!\rangle$ is accepted in an epistemic state represented by an OCF κ, written as $\kappa \models \langle\!|D|C|\!\rangle$, if and only if $\kappa \not\models (\overline{D}|C)$ or $\kappa(C) = \infty$, i.e., $\kappa(CD) \leq \kappa(C\overline{D})$ or $\kappa(C) = \infty$.*

Note that accepting a might conditional is not equivalent to the acceptance of the conditional with negated consequent ($\kappa \models (\overline{D}|C)$) but weaker since it allows for indifference between CD and $C\overline{D}$. In this case both $(D|C)$ and $(\overline{D}|C)$ fail to be accepted.

3 The Ramsey Test, the Relevant Ramsey Test and difference-making conditionals

In the following, let Ψ be an epistemic state of any general format, and let Bel be an operator on belief states that assigns to Ψ the set of beliefs held in Ψ. Let $*$ be a revision operator on epistemic states, and let $(B|A)$ be a conditional. The *Ramsey Test* (so-called after a footnote in Ramsey 1931) was made popular by Stalnaker (1968). According to it, 'If A then B' is accepted in a belief state just in case B is an element of the belief set $Bel(\Psi * A)$ that results from a revision of the belief state Ψ by the sentence A. Formally:

(RT) $\Psi \models (B|A)$ iff $B \in Bel(\Psi * A)$.

If belief states are identified with ranking functions, the Ramsey Test reads as follows: $\kappa \models (B|A)$ iff $B \in Bel(\kappa * A)$; this, taken together with Definition 1 implies a constraint on $\kappa * A$. The condition $B \in Bel(\Psi * A)$ can be reformulated using some basic properties of ranking functions:

$$\begin{aligned} & B \in Bel(\kappa * A) = Th((\kappa * A)^{-1}\{0\}) \\ \Leftrightarrow\quad & \forall\, \omega \in \min(Mod(\kappa * A)) \text{ it holds that } \omega \models B \\ \Leftrightarrow\quad & (\kappa * A)(B) < (\kappa * A)(\overline{B}) \\ \Leftrightarrow\quad & (\kappa * A)(\overline{B}) > 0 \quad \Leftrightarrow \quad \kappa * A \models B. \end{aligned}$$

We can also define a *Ramsey Test for might conditionals*: $\Psi \models \langle\!\langle B|A\rangle\!\rangle$ iff $\overline{B} \notin Bel(\Psi * A)$, that is, iff $\Psi \not\models (\overline{B}|A)$. Or more specifically, in terms of ranking functions: $\kappa \models \langle\!\langle B|A\rangle\!\rangle$ iff $\overline{B} \notin Bel(\kappa * A)$, that is, iff $\kappa \not\models (\overline{B}|A)$, which follows from Definition 2. The condition $\overline{B} \notin Bel(\Psi * A)$ can again be reformulated using some properties of ranking functions:

$$\begin{aligned} & \overline{B} \notin Bel(\kappa * A) = Th((\kappa * A)^{-1}\{0\}) \\ \Leftrightarrow \quad & \exists\, \omega \in \min(Mod(\kappa * A)) \text{ such that } \omega \models B \\ \Leftrightarrow \quad & (\kappa * A)(B) = 0 \quad \Leftrightarrow \quad \kappa * A \not\models \overline{B}. \end{aligned}$$

Given assumptions on belief revision in the tradition of Alchourrón, Gärdenfors and Makinson (1985), Ramsey Test conditionals are known to satisfy, among other things, the following principles of And, Right Weakening, Cautious Monotonicity, Cut and Or:

(And) If $(B|A)$ and $(C|A)$, then $(BC|A)$.
(RW) If $(B|A)$ and $C \in Cn(B)$, then $(C|A)$.
(CM) If $(B|A)$ and $(C|A)$, then $(C|AB)$.
(Cut) If $(B|A)$ and $(C|AB)$, then $(C|A)$.
(Or) If $(C|A)$ and $(C|B)$, then $(C|A \vee B)$.

All of these principles are to be read as quantified over all belief states Ψ: '$(B|A)$' is short for '$\Psi \models (B|A)$'. Roughly, a principle of the form 'If Δ, then $(B|A)$' is *valid* iff *for every belief state* Ψ, if the conditionals mentioned in Δ are all accepted in Ψ, then $(B|A)$ is accepted in Ψ, too.

The Ramsey Test falls squarely within the paradigm of the suppositional account mentioned above. Assume that an agent happens to believe B. Assume further that her beliefs are consistent with A (or that she actually already believes that A). Then, given a widely endorsed condition of belief preservation, the Ramsey Test rules that the agent is committed to accepting the conditional $(B|A)$. There need not be any relation of relevance or support between A and B. In particular, if you happen to believe A and B, this is sufficient to require acceptance of $(B|A)$.

How can the Ramsey Test be adapted to capture the idea that the antecedent should be relevant to the consequent? One straightforward way is to interpret conditionals as being contrastive: The antecedent should *make a difference* to the consequent. In order to implement this idea without introducing a dependence on the actual belief status of the antecedent, Rott (2019) suggests the following *Relevant Ramsey Test*:

(RRT) $\Psi \models A \gg B$ iff $B \in Bel(\Psi * A)$ and $B \notin Bel(\Psi * \overline{A})$.

We call conditionals that are governed by (RRT) *difference-making conditionals*, and we have changed the notation here from $(B|A)$ to $A \gg B$ in order to mark our transition from standard would conditionals to difference-making conditionals. $A \gg B$ can be read as *'If A, then (relevantly) B.'* Here the consequent is accepted if we revise the belief state by the antecedent, but the consequent fails to be accepted if we revise by the negation of the antecedent. Rott's idea was to liken conditionals to the natural-language connectives *'because'* and *'since'* that are widely taken to express the contrast that a cause or a reason is making to its effect. Thus Rott took $\gg$ to be an intrinsically contrastive connective. It is important to note, however, that unlike *'B because A'* and *'Since A, B'*, which can only be accepted if A is believed to be true, the acceptance of $A \gg B$ neither entails nor is entailed by a particular belief status of A. (RRT) provides a clear and simple doxastic semantics for relevance-encoding conditionals with antecedents and consequents that may be arbitrary compounds of propositional sentences.

Since (RRT) is more complex than (RT), it is hardly surprising that difference-making conditionals don't satisfy some of the usual principles for standard conditionals such as CM, Cut and Or. Rott discusses some examples showing how CM, Cut and OR can fail with difference-making conditionals. The most striking fact, however, is that difference-making conditionals do not even validate Right Weakening which has long seemed entirely innocuous to conditional logicians. Rott even called the invalidity of RW *the hallmark of difference-making conditionals* and indeed *of the relevance relation*. Another notable property of difference-making conditionals is that $B \in Cn(A)$ does not imply that $A \gg B$ is accepted. If B is accepted "anyway" (like for instance a logical truth B is), then A cannot be relevant to B, even if it implies B.

That many of the familiar principles for standard conditionals become invalid for difference-making conditionals does not mean that there is no logic to the latter. Here are the *basic principles of difference-making conditional operators* that Rott (2019) shows to be complete with respect to the basic AGM postulates for belief revision (actually Rott uses a slight weakening of the basic AGM postulates that allows that revisions by non-contradictions may result in inconsistent belief sets):

($\gg$0) $\bot \gg \bot$.
($\gg$1) If $A \gg BC$, then $A \gg B$ or $A \gg C$.
($\gg$2a) $A \gg C$ iff ($A \gg AC$ and $A \gg A \vee C$).
($\gg$2b) $A \gg AC$ iff (not $\overline{A} \gg \overline{A} \vee C$ and $A \gg A$).
($\gg$3–4) $\bot \gg A \vee C$ iff ($\bot \gg A$ and $A \gg A \vee C$).
($\gg$5) $A \vee B \gg \bot$ iff ($A \gg \bot$ and $B \gg \bot$).
($\gg$6) If $Cn(A) = Cn(B)$ and $Cn(C) = Cn(D)$, then: $A \gg C$ iff $B \gg D$.

All of these principles are to be read as quantified over all belief states Ψ: '$A \gg C$' is short for '$\Psi \models A \gg C$' and 'not $A \gg C$' is short for '$\Psi \not\models A \gg C$'. Roughly, a principle of the form 'If Δ, then Γ' is *valid* iff *for every belief state* Ψ, if the (possibly negated) conditionals mentioned in Δ are all accepted in Ψ, then the (possibly negated) conditionals mentioned in Γ are accepted in Ψ, too.

It follows from principles ($\gg$0) – ($\gg$6) that (And) is also valid for difference-making conditionals. ($\gg$1) is dual to the well-known principle of Disjunctive Rationality; it is called *Conjunctive Rationality* in Rott (2020). Like its dual, Conjunctive Rationality is a non-Horn condition. Another non-Horn condition is the right-to-left direction of ($\gg$2b). The presence of non-Horn conditions means that reasoning with difference-making conditionals is not trivial. In order to determine what may be inferred from a knowledge base containing difference-making conditionals, we cannot simply use the axioms as closure operators. This is analogous to the problem of rational consequence relations in the sense of Lehmann and Magidor (1992) that have made it necessary to invent special inference methods like rational closure/system Z and c-representations. In the following, we will use the method of c-representations to deal with difference-making conditionals. A major part of our task ahead may be described as doing for c-representations what Booth and Paris (1998) achieved for rational closure.

4 Ranking semantics for difference-making conditionals

In this section, we define a semantics for difference-making conditionals in the framework of Spohn's ranking functions. We make use of standard conditionals and might conditionals in order to express that the antecedent of the conditional is relevant to the consequent. We justify our definition of difference-making conditionals by showing that the Relevant Ramsey Test holds and we show that the Basic principles are satisfied.

Definition 3 (Relevant Ramsey Test for OCFs)**.** *Let κ be an OCF, $A \gg B$ be a difference-making conditional and $*$ a revision operator for OCFs. We define the* ***Relevant Ramsey Test for OCFs*** *as follows:*

(RRT^{ocf}) $\kappa \models A \gg B$ *iff* $B \in Bel(\kappa * A)$ *and* $B \notin Bel(\kappa * \overline{A})$.

Using some basic properties of ranking functions, we can reformulate (RRT$^{\text{ocf}}$) :

$$\kappa \models A \gg B \text{ iff } \kappa * A \models B \text{ and } \kappa * \overline{A} \not\models B. \quad (1)$$

From (1), we obtain for A with $\kappa(A), \kappa(\overline{A}) < \infty$:

$$\kappa \models A \gg B \text{ iff } \kappa \models \{(B|A), \langle\!\langle \overline{B}|\overline{A} \rangle\!\rangle\} \quad (2)$$

iff both of the following two conditions hold:

$$\kappa(AB) < \kappa(A\overline{B}) \text{ and} \quad (3)$$

$$\kappa(\overline{A}\overline{B}) \leq \kappa(\overline{A}B). \quad (4)$$

Difference-making conditionals defined by (RRT$^{\text{ocf}}$) can be expressed by pairs of conditionals. The first conditional $(B|A)$ corresponds to the first part of the (RRT$^{\text{ocf}}$) , $B \in Bel(\kappa * A)$, using basically the standard Ramsey Test. The clause for (RRT$^{\text{ocf}}$) implies the clause for the standard Ramsey Test. The second conditional $\langle\!\langle \overline{B}|\overline{A} \rangle\!\rangle$ corresponds to the second part of the (RRT$^{\text{ocf}}$) , namely $B \notin Bel(\kappa * \overline{A})$. We now continue with Example 1 in order to elucidate our reformulation in (2).

Example 2 (Continue Example 1)**.** *The agent's pipe broke because the temperatures were too low, and therefore she had to call a plumber to have the pipe fixed. These connections can be expressed using difference-making conditionals $c \gg b$ and $b \gg p$. Applying* (2)*, we can reformulate $\Delta^{\gg} = \{c \gg b, b \gg p\} = \{(b|c), \langle\!\langle \overline{b}|\overline{c} \rangle\!\rangle, (p|b), \langle\!\langle \overline{p}|\overline{b} \rangle\!\rangle\}$. The standard conditionals express that if it is cold, then the pipe will break, and if the pipe breaks, then the agent will call a plumber. But the reason relation would get neglected if we only used standard conditionals. The might conditionals express that if it is not cold, then the pipe might not break, and if the pipe does not break, we might not call the plumber. Here the might conditionals formulated in natural language perhaps sound a bit odd, but together with the standard conditionals they express the reason relations introduced by the difference-making conditionals.*

Next, we turn to the basic principles for difference-making conditionals. Note that when checking the principles of Rott, instead of a general epistemic state Ψ, we use a ranking function κ.

Theorem 1. *Let κ be a ranking function and let $\kappa \models A \gg B$ be as defined in* (2)*. Then $\cdot \gg \cdot$ satisfies the basic principles of difference-making conditionals.*

Proof. ($\gg$0): We show that $\kappa \models \bot \gg \bot$, i.e., $\kappa * \bot \models \bot$ and $\kappa * \top \not\models \bot$. These are true by the success and consistency conditions for revisions, respectively.

($\gg$1): Let $\kappa \models A \gg BC$. We have to show that $\kappa \models A \gg B$ or $\kappa \models A \gg C$. Via (2) it follows that we have to show that $\kappa(ABC) < \kappa(A(\overline{B} \vee \overline{C}))$ and $\kappa(\overline{A}(\overline{B} \vee \overline{C})) \leq \kappa(\overline{A}BC)$ implies $\kappa(AB) < \kappa(A\overline{B})$, $\kappa(\overline{A}\overline{B}) \leq \kappa(\overline{A}B)$, or $\kappa(AC) < \kappa(A\overline{C})$, $\kappa(\overline{A}\overline{C}) \leq \kappa(\overline{A}C)$.
From $\kappa(ABC) < \kappa(A(\overline{B} \vee \overline{C}))$, we derive $\kappa(ABC) < \kappa(A\overline{B} \vee A\overline{C}) = \min\{\kappa(A\overline{B}), \kappa(A\overline{C})\}$, and hence both $\kappa(ABC) < \kappa(A\overline{B})$ and $\kappa(ABC) < \kappa(A\overline{C})$. Since $ABC \models AB, AC$ we obtain $\kappa(AB) < \kappa(A\overline{B})$ and $\kappa(AC) < \kappa(A\overline{C})$.
Moreover, from $\kappa(\overline{A}(\overline{B} \vee \overline{C})) \leq \kappa(\overline{A}BC)$, we derive that either $\kappa(\overline{A}\overline{B}) \leq \kappa(\overline{A}BC)$ or $\kappa(\overline{A}\overline{C}) \leq \kappa(\overline{A}BC)$. Since $\overline{A}BC \models \overline{A}B, \overline{A}C$ we obtain that either $\kappa(\overline{A}\overline{B}) \leq \kappa(\overline{A}B)$ or $\kappa(\overline{A}\overline{C}) \leq \kappa(\overline{A}C)$.

($\gg$2a): We have to show that $\kappa \models A \gg C$ iff ($\kappa \models A \gg AC$ and $\kappa \models A \gg A \vee C$). Via (2) it follows that we have to show that $\kappa(AC) < \kappa(A\overline{C})$ and $\kappa(\overline{A}\overline{C}) \leq \kappa(\overline{A}C)$ iff ($\kappa(AC) < \kappa(A\overline{C})$ and $\kappa(\overline{A}) \leq \kappa(\bot)$) and ($\kappa(A) < \kappa(\bot)$ and $\kappa(\overline{A}\overline{C}) \leq \kappa(\overline{A}C)$). This holds trivially.

($\gg$2b): We have to show that $\kappa \models A \gg AC$ iff (not $\kappa \models \overline{A} \gg \overline{A} \vee C$ and $\kappa \models A \gg A$). Via (2) it follows that we have to show that $\kappa(AC) < \kappa(A\overline{C})$, $\kappa(\overline{A}) \leq \kappa(\bot)$ iff ($\kappa(\overline{A}) \geq \kappa(\bot)$ or $\kappa(AC) < \kappa(A\overline{C})$) and $\kappa(A) < \kappa(\bot)$ and $\kappa(\overline{A}) \leq \kappa(\bot)$. This holds trivially.

($\gg$3–4): We have to show that $\kappa \models \bot \gg A \vee C$ iff ($\kappa \models \bot \gg A$ and $\kappa \models A \gg A \vee C$). Via (2) it follows that we have to show that $\kappa(\overline{A}\overline{C}) \leq \kappa(A \vee C)$ iff $\kappa(\overline{A}) \leq \kappa(A)$ and $\kappa(\overline{A}\overline{C}) \leq \kappa(\overline{A}C)$. The direction from left to right is immediate. For the converse direction, note that $\kappa(\overline{A}\overline{C}) \leq \kappa(\overline{A}C)$ implies that $\kappa(\overline{A}\overline{C}) = \kappa(\overline{A})$. So we get from $\kappa(\overline{A}) \leq \kappa(A)$ and $\kappa(\overline{A}\overline{C}) \leq \kappa(\overline{A}C)$ that $\kappa(\overline{A}\overline{C}) \leq \min\{\kappa(A), \kappa(\overline{A}C)\} = \kappa(A \vee C)$, as desired.

($\gg$5): We have to show that $\kappa \models A \vee B \gg \bot$ iff ($\kappa \models A \gg \bot$ and $\kappa \models B \gg \bot$). But conditionals with impossible consequents are accepted iff the antecedents are impossible, i.e., have κ-rank ∞. So the claim follows from the fact that $\kappa(A \vee B) = \min\{\kappa(A), \kappa(B)\}$.

$\bot \gg \bot$	$\kappa * \top \not\models \bot$	$Bel(\kappa)$ is consistent
$A \gg \bot$	$\kappa * A \models \bot$	A is a doxastic impossibility
$\overline{A} \gg \bot$	$\kappa * \overline{A} \models \bot$	A is a doxastic necessity
$\bot \gg A$	$\kappa * \top \not\models A$	A is a non-belief
$A \gg A$	$\kappa * \overline{A} \not\models A$	A is contingent
$A \gg AC$	$\kappa * A \models C$ and $\kappa * \overline{A} \not\models \bot$	C is in $Bel(\kappa * A)$ and A is contingent
$A \gg A \vee C$	$\kappa * \overline{A} \not\models C$	C is not in $Bel(\kappa * \overline{A})$
not $\overline{A} \gg \overline{A} \vee C$	$\kappa * A \models C$	C is in $Bel(\kappa * A)$

Table 1: The meanings of some basic difference-making conditionals.

($\gg$6): If $Cn(A) = Cn(B)$ and $Cn(C) = Cn(D)$, then $A \gg C$ iff $B \gg D$. This follows trivially since structurally analogous compounds of logically equivalent sentences are logically equivalent and thus get the same κ-ranks. □

The basic principles explore the logic of conditionals governed by (RRT). The reformulation in Definition 3 shows that the notion of the Relevant Ramsey Test can be transferred to the OCF framework. The relevance of the antecedent to the consequent can be expressed by splitting the two directions within the (RRT^{ocf}) into two conditionals, one might and one standard conditional. In Theorem 1, we have shown that this reformulation serves the logic behind difference-making conditionals. Theorem 1 should be compared with the results of Raidl (2020).

Rott (2019) explained the meanings of some basic difference-making conditionals, and the explanations still work within the OCF framework. They are collected in table 1. Note that the meanings also reflect the idea of the basic principles. For example, ($\gg$2a) says that C is in the revision of $\kappa * A$ and not in the revision $\kappa * \overline{A}$ iff $A \gg AC$ and $A \gg A \vee C$, which is exactly the meaning of these two basic difference-making conditionals. Also for ($\gg$2b) the meanings of the difference-making conditionals of both sides of 'iff' are exactly the same.

5 Inductive representation of difference-making conditionals

In this section, we define an inductive representation of sets of difference-making conditionals $\Delta^{\gg}$ by setting up epistemic states in form of OCFs that are admissible with respect to $\Delta^{\gg}$. We use the approach of c-representations firstly introduced by Kern-Isberner (2001). C-representation are not only capable of setting up epistemic states that represent sets of standard conditionals but were extended to might conditionals (see Eichhorn, Kern-Isberner and Ragni 2018). By combining the representation of standard and might conditionals, we get a c-representation of sets of difference-making conditionals.

First, we will turn to the application of the technique of c-representations to sets of standard and might conditionals.

Proposition 2 (C-representation of sets of standard and might conditionals)**.** *Let* $\Delta = \{(B_i|A_i)\}_{i=1,\ldots,n} \cup \{\langle\!\langle B_i|A_i \rangle\!\rangle\}_{i=n+1,\ldots,m}$ *be a set of standard and might conditionals. A c-representation of* Δ *is given by an OCF of the form*

$$\kappa^c_\Delta(\omega) = \sum_{\omega \models A_i\overline{B}_i} \kappa_i^- \tag{5}$$

with non-negative impact factors κ_i^- *for each conditional* $(B_i|A_i) \in \Delta$ *resp.* $\langle\!\langle B_i|A_i \rangle\!\rangle \in \Delta$ *satisfying*

$$\kappa_i^- \underset{(-)}{>} \min_{\omega \models A_iB_i} \{ \sum_{\substack{\omega \models A_k\overline{B}_k \\ i \neq k}} \kappa_k^- \} - \min_{\omega \models A_i\overline{B}_i} \{ \sum_{\substack{\omega \models A_k\overline{B}_k \\ i \neq k}} \kappa_k^- \} \tag{6}$$

for all $1 \leq i \leq m$*. If* $i \in \{1, \ldots, n\}$*, i.e. the impact factor stands for a standard conditional, then we need strict inequalities '$>$'. If* $i \in \{n+1, \ldots, m\}$*, i.e. the impact factor stands for a might conditional, then we do not need strict inequalities and '$\geq$' is sufficient.*

To calculate a c-representation of a set of conditionals Δ we need to solve a system of inequalities, given by formula (6) for each $i = 1, \ldots, m$, which ensure $\kappa^c_\Delta \models \Delta$. More precisely, with the ranks of formulas, and formula (5) the constraint $\kappa^c_\Delta(A_iB_i) < \kappa^c_\Delta(A_i\overline{B}_i)$ for $1 \leq i \leq n$ resp. $\kappa^c(A_iB_i) \leq \kappa^c(A_i\overline{B}_i)$ for $n+1 \leq i \leq m$ expands to

$$\underbrace{\min_{\omega \models A_iB_i} \{ \sum_{\omega \models A_k\overline{B}_k} \kappa_k^- \}}_{(7a)} \underset{(-)}{<} \underbrace{\min_{\omega \models A_i\overline{B}_i} \{ \sum_{\omega \models A_k\overline{B}_k} \kappa_k^- \}}_{(7b)} \tag{7}$$

The left minimum ranges over the models A_iB_i, so the conditional $(B_i|A_i)$ resp. $\langle\!\langle B_i|A_i \rangle\!\rangle$ is not falsified by any considered world and thus κ_i^- is no element of any sum (7a). As opposed to this, the right minimum ranges over the models of $A_i\overline{B}_i$, so the conditional $(B_i|A_i)$ resp. $\langle\!\langle B_i|A_i \rangle\!\rangle$ is falsified by every considered world and thus κ_i^- is an element of every sum (7b). With these deliberations, we can rewrite the inequalities to

$$\min_{\omega \models A_iB_i} \{ \sum_{\substack{\omega \models A_k\overline{B}_k \\ i \neq k}} \kappa_k^- \} \underset{(-)}{<} \kappa_i^- + \min_{\omega \models A_i\overline{B}_i} \{ \sum_{\substack{\omega \models A_k\overline{B}_k \\ i \neq k}} \kappa_k^- \} \tag{8}$$

and therefore

$$\kappa_i^- \underset{(-)}{>} \min_{\omega \models A_iB_i} \{ \sum_{\substack{\omega \models A_k\overline{B}_k \\ i \neq k}} \kappa_k^- \} - \min_{\omega \models A_i\overline{B}_i} \{ \sum_{\substack{\omega \models A_k\overline{B}_k \\ i \neq k}} \kappa_k^- \} \tag{9}$$

for all $1 \leq i \leq m$. As we have seen, save for the strictness the inequalities defining impact factors for standard and might conditionals are the same and therefore can be expressed using '$\underset{(-)}{>}$'. Also note that c-representations are not unique since the solution of the system of inequalities is not unique. If the system of inequalities in (6) has a solution then Δ is consistent and (5) is a model of Δ. For the converse, Kern-Isberner (2001, p. 69, 2004, p. 26) has shown that every finite consistent knowledge base consisting solely of standard conditionals has a c-representation; but it is still an open question whether this result extends to knowledge bases including might conditionals.

According to (2), a difference-making conditional $A \gg B$ can be reformulated as a set of a standard and a might conditional $\{(B|A), \langle\!\langle \overline{B}|\overline{A} \rangle\!\rangle\}$. So, for a set of difference-making conditionals $\Delta^{\gg} = \{A_i \gg B_i \mid i = 1, \ldots, n\}$, $\Delta^{\gg}$ can be implemented via $\{(B_k|A_k) \mid k = 1, \ldots, n\} \cup \{\langle\!\langle \overline{B}_l|\overline{A}_l \rangle\!\rangle \mid l = 1, \ldots, n\}$. In this way, we can get an inductive representation of $\Delta^{\gg}$ by a c-representation as follows:

Definition 4 (C-representation for sets of difference-making conditionals). *Let $\Delta^{\gg} = \{A_i \gg B_i \mid i = 1, \ldots, n\}$ be a set of difference-making conditionals. An OCF κ is a c-representation of $\Delta^{\gg}$ iff*

$$\kappa^c_{\Delta^{\gg}}(\omega) = \sum_{\substack{\omega \models A_k\overline{B}_k \\ k=1}}^{n} \kappa^-_k + \sum_{\substack{\omega \models \overline{A}_l B_l \\ l=1}}^{n} \lambda^-_l . \qquad (10)$$

with non-negative impact factors κ^-_k resp. λ^-_l for each conditional $(B_k|A_k) \in \Delta^{\gg}$ resp. $\langle\!\langle \overline{B}_j|\overline{A}_j \rangle\!\rangle \in \Delta^{\gg}$ satisfying

$$\begin{aligned} \kappa^-_k > \min_{\omega \models A_k B_k} \{ \sum_{\substack{\omega \models A_j \overline{B}_j \\ k \neq j}} \kappa^-_j + \sum_{\omega \models \overline{A}_l B_l} \lambda^-_l \} \\ - \min_{\omega \models A_k \overline{B}_k} \{ \sum_{\substack{\omega \models A_j \overline{B}_j \\ k \neq j}} \kappa^-_j + \sum_{\omega \models \overline{A}_l B_l} \lambda^-_l \} \end{aligned} \qquad (11)$$

$$\begin{aligned} \lambda^-_l \geq \min_{\omega \models \overline{A}_l \overline{B}_l} \{ \sum_{\substack{\omega \models \overline{A}_j B_j \\ l \neq j}} \lambda^-_j + \sum_{\omega \models A_k \overline{B}_k} \kappa^-_k \} \\ - \min_{\omega \models \overline{A}_l B_l} \{ \sum_{\substack{\omega \models \overline{A}_j B_j \\ l \neq j}} \lambda^-_j + \sum_{\omega \models A_k \overline{B}_k} \kappa^-_k \} \end{aligned} \qquad (12)$$

Equations (11) and (12) ensure that the impact factors are chosen such that $\kappa^c_{\Delta^{\gg}} \models \Delta^{\gg}$. Just like in (6), (11) resp. (12) follows from the success condition in (3) resp. (4). Since we chose different impact factors κ^- resp. λ^- for the standard resp. the might conditionals, the terms in the minima look more complex even though they can be derived from (6). Also we replaced the general form of might conditionals $\langle\!\langle B|A \rangle\!\rangle$ by the more specific might conditional $\langle\!\langle \overline{B}_i|\overline{A}_i \rangle\!\rangle$, taking advantage of the special structure of difference-making conditionals. C-representations of difference-making conditionals exist iff all inequalities (11) and (12) are solvable.

Sets of difference-making conditionals, can be inductively represented by a c-representation. The crucial part is the reformulation of difference-making conditionals as sets of one standard and one might conditional in (2). Due to the high adaptability of the approach of c-representations, it is possible to deal with such a set of mixed conditionals.

In order to illustrate c-representations of difference-making conditionals, we now turn to the special case when the set $\Delta^{\gg}$ consists just of one difference-making conditional $\Delta^{\gg} = \{A \gg B\}$. In this case, the system of inequalities always has a solution and we can define the $\kappa^c_{\Delta^{\gg}}$ as follows:

Theorem 3. *Let $A \gg B$ be a difference-making conditional. $\kappa^c_{A \gg B}$ is a c-representation of $A \gg B$ iff there are integers $\kappa^-_{st}, \kappa^-_w$, such that, for $\omega \in \Omega$*

$$\kappa^c_{A \gg B}(\omega) = \begin{cases} \kappa^-_{st}, & \omega \models A\overline{B} \\ \kappa^-_w, & \omega \models \overline{A}B \text{ , for all } \omega \in \Omega \\ 0, & else \end{cases} \qquad (13)$$

and

$$\kappa^-_{st} > 0 \text{ and } \kappa^-_w \geq 0 \qquad (14)$$

Proof. Let $\Delta^{\gg} = \{A \gg B\}$. Since $A_i\overline{B}_i\overline{A}_iB_i \equiv \bot$, (13) follows immediately from (10). $\kappa^-_{st} > 0$ follows from (11) and $\kappa^-_w \geq 0$ follows from (12), since there are no other difference-making conditional to interact with. □

Let us now continue with our example concerning the agent's broken pipe:

Example 3 (Continue Example 2). *Using the representation of the set of difference-making conditionals $\Delta^{\gg} = \{c \gg b, b \gg p\}$ as pairs of standard and weak conditionals from Example 2, we can construct a c-representation $\kappa^c_{\Delta^{\gg}}$ using Definition 4. First we have to solve the system of inequalities defining the impact factors. Let κ^-_1 and λ^-_1 correspond to the standard and the might conditional representations of $c \gg b$, and let κ^-_2 and λ^-_2 apply similarly to $b \gg p$:*

$$\begin{aligned} \kappa^-_1 &> \min\{0, \kappa^-_2\} - \min\{0, \lambda^-_2\} = 0, \\ \lambda^-_1 &\geq \min\{0, \lambda^-_2\} - \min\{0, \kappa^-_2\} = 0, \\ \kappa^-_2 &> \min\{0, \lambda^-_1\} - \min\{0, \lambda^-_1\} = 0, \\ \lambda^-_2 &\geq \min\{0, \kappa^-_1\} - \min\{0, \kappa^-_1\} = 0. \end{aligned}$$

The minima on the left-hand side range over worlds verifying the corresponding (standard resp. might) conditional and the minima on the right-hand side range over worlds falsifying these. We take the minimum of the summed up impact factors indicating that other conditionals are falsified. Since the impact factors are non-negative the minima equal zero. We choose $\kappa^-_1 = \kappa^-_2 = 1$ and $\lambda^-_1 = \lambda^-_2 = 0$ and get the c-representation presented in table 2. It is easy to verify that $\kappa^c_{\Delta^{\gg}} \models c \gg p$, so in this example the difference-making conditionals satisfy transitivity. Note, however, that transitivity is only 'valid by default', that is, it can easily be undercut by the addition of another premise. For instance, it is possible to consistently add $c \gg \overline{p}$ as a third premise to $\Delta^{\gg}$. The extended knowledge base has a c-representation (based on $\kappa^-_1 = \kappa^-_3 = 2$, $\kappa^-_2 = 1$ and $\lambda^-_1 = \lambda^-_2 = \lambda^-_3 = 0$) that does not satisfy $c \gg p$ because it does not even satisfy $(p|c)$.

6 Revision by difference-making conditionals

In this section we discuss a revision method for epistemic states represented by an OCF with one difference-making conditional. Therefore, we make use of the characterisation

ω	$\kappa^c_{\Delta\gg}(\omega)$	ω	$\kappa^c_{\Delta\gg}(\omega)$
cbp	0	$\overline{c}bp$	$\lambda_1^- = 0$
$cb\overline{p}$	$\kappa_2^- = 1$	$\overline{c}b\overline{p}$	$\kappa_2^- + \lambda_1^- = 1$
$c\overline{b}p$	$\kappa_1^- + \lambda_2^- = 1$	$\overline{c}\overline{b}p$	$\lambda_2^- = 0$
$c\overline{b}\overline{p}$	$\kappa_1^- = 1$	$\overline{c}\overline{b}\overline{p}$	0

Table 2: The ranking function $\kappa^c_{\Delta\gg}$ of Example 3.

of a difference-making conditional as a set of one standard conditional and one might conditional in (2) and provide a method for simultaneously revising an epistemic state with a standard and a might conditional.

C-revisions, introduced by Kern-Isberner (2001), provide a highly general framework for revising epistemic states by sets of conditionals. In the framework of ranking functions, c-revisions are capable of revising an OCF by a set of conditionals with respect to conditional interaction within the new information, while preserving conditional beliefs in the former belief state. This is all depicted in the *principle of conditional preservation*, which implies the Darwiche-Pearl postulates for revising epistemic states (Kern-Isberner 2001, 2004). We will now introduce a simplified version of c-revisions for sets of standard and might conditionals.

Proposition 4 (C-revisions by sets of standard and might conditionals)**.** *Let κ be an OCF specifying a prior epistemic state and let $\Delta = \{(B_i|A_i) \mid i = 1,\dots,n\} \cup \{\langle\!\langle B_i|A_i \rangle\!\rangle \mid i = n+1,\dots,m\}$ be a set of standard and might conditionals which represent the new information. Then a c-revision of κ by Δ is given by an OCF of the form*

$$\kappa * \Delta(\omega) = \kappa^*_\Delta(\omega) = \kappa_0 + \kappa(\omega) + \sum_{\omega \models A_i\overline{B}_i} \kappa_i^- \qquad (15)$$

with non-negative impact factors κ_i^- for each conditional $(B_i|A_i) \in \Delta$ resp. $\langle\!\langle B_i|A_i \rangle\!\rangle \in \Delta$ satisfying

$$\kappa_i^- \underset{(}{\geq}{}_{)} \min_{\omega \models A_i B_i} \{\kappa(\omega) + \sum_{\substack{\omega \models A_k\overline{B}_k \\ i \neq k}} \kappa_k^-\} - \min_{\omega \models A_i\overline{B}_i} \{\kappa(\omega) + \sum_{\substack{\omega \models A_k\overline{B}_k \\ i \neq k}} \kappa_k^-\} \qquad (16)$$

κ_0 is a normalization factor to ensure that κ^*_Δ is an OCF. The κ_i^- can be considered as impact factors of the single conditional $(B_i|A_i) \in \Delta$ resp. $\langle\!\langle B_i|A_i \rangle\!\rangle \in \Delta$ for falsifying the conditionals in Δ which have to be chosen so as to ensure success $\kappa^*_\Delta \models \Delta$ by (16). As before, we use '$\underset{(}{\geq}{}_{)}$' as a dummy operator which is replaced by the strict inequality symbol $>$ for standard conditionals, while for might conditionals it is replaced by the inequality symbol $\geq$. From the success condition $\kappa^*_\Delta(A_iB_i) \underset{(}{\leq}{}_{)} \kappa^*_\Delta(A_i\overline{B}_i)$ and the ranks of formulas, it holds that

$$\min_{\omega \models A_i B_i} \{\kappa_0 + \kappa(\omega) + \sum_{\omega \models A_k\overline{B}_k} \kappa_k^-\} \underset{(}{\leq}{}_{)} \min_{\omega \models A_i\overline{B}_i} \{\kappa_0 + \kappa(\omega) + \sum_{\omega \models A_k\overline{B}_k} \kappa_k^-\}.$$

Since κ_0 is a constant factor, it can be removed from the inequality. As in c-representation, the factor κ_i^- is no element of the left sum, whereas the right sum ranges over worlds falsifying $(B_i|A_i)$ resp. $\langle\!\langle B_i|A_i \rangle\!\rangle$ and therefore the factor κ_i^- is an element of every sum. With these deliberations we can rewrite the inequalities to (16) for all $1 \leq i \leq m$. Note that the impact factors defining c-revisions are not unique because there are multiple solutions of the system of inequalities in (16). The question as to which choice of the impact factors is 'best' is part of our ongoing work.

Now we turn to the revision of an epistemic state by a single difference-making conditional in the framework of OCFs. In (2) we showed that the revision by a difference-making conditional is equivalent with revising a ranking function by a special set of conditionals, since $A \gg B$ corresponds to $\{(B|A), \langle\!\langle \overline{B}|\overline{A} \rangle\!\rangle\}$. Thus, we need a revision method which is capable of dealing with a mixed set of conditionals. As we have seen before, c-revisions are an adaptable revision method for sets of conditionals, both for standard and for might conditionals. Following the general schema of c-revisions, we get:

Definition 5 (C-revision by a difference-making conditional)**.** *Let κ be an OCF specifying a prior epistemic state and let $A \gg B = \{(B|A), \langle\!\langle \overline{B}|\overline{A} \rangle\!\rangle\}$ be a difference-making conditional which represents the new information. Then a c-revision of κ by $A \gg B$ is given by an OCF of the form*

$$\kappa * A \gg B(\omega) = \kappa^*_{\Delta\gg}(\omega) = \kappa_0 + \kappa(\omega) + \begin{cases} \kappa_{st}^-, & \omega \models A\overline{B} \\ \kappa_w^-, & \omega \models \overline{A}B \end{cases} \qquad (17)$$

with

$$\kappa_{st}^- > \kappa(AB) - \kappa(A\overline{B}) \qquad (18)$$

$$\kappa_w^- \geq \kappa(\overline{A}\overline{B}) - \kappa(\overline{A}B). \qquad (19)$$

As before, κ_0 is a normalization factor. The premises of the standard and the might conditional defining the difference-making conditional $A \gg B$ are exclusive, so the set $A \gg B = \{(B|A), \langle\!\langle \overline{B}|\overline{A} \rangle\!\rangle\}$ is consistent and $\kappa^*_{\Delta\gg}$ always exists. The form of $\kappa^*_{\Delta\gg}$ in (17) follows from (15):

$$\kappa * (A \gg B)(\omega) = \kappa * \{(B|A), \langle\!\langle \overline{B}|\overline{A} \rangle\!\rangle\}(\omega) = \kappa_0 + \kappa(\omega) + \sum_{\omega \models A_i\overline{B}_i} \kappa_{st}^- + \sum_{\omega \models \overline{A}_i B_i} \kappa_w^-$$

Since $A\overline{B}$ and $\overline{A}B$ are exclusive and we revise with just a single difference-making conditional, we get (17). The success condition for the standard conditional $(B|A)$ in (3) and the success condition for might conditional $\langle\!\langle \overline{B}|\overline{A} \rangle\!\rangle$ in (4) lead to inequalities defining impact factors κ_{st}^- resp. κ_w^-. For κ_{st}^- it holds that (18) follows immediately from (16):

$$\kappa_{st}^- > \min_{\omega \models AB}\{\kappa(\omega) + \sum_{\omega \models A_k\overline{B}_k} \kappa_k^-\} - \min_{\omega \models A\overline{B}}\{\kappa(\omega) + \sum_{\omega \models A_k\overline{B}_k} \kappa_k^-\}.$$

The minimal range over worlds AB resp. $A\overline{B}$, so the might conditional $\langle\!\langle \overline{B}|\overline{A} \rangle\!\rangle$ are not falsified by any considered world

ω	$\kappa^*(\omega)$	ω	$\kappa^*(\omega)$
$cbpd$	0	$\bar{c}bpd$	0
$cbp\bar{d}$	$0+\kappa_w^-=0$	$\bar{c}bp\bar{d}$	$0+\kappa_w^-=0$
$cb\bar{p}d$	1	$\bar{c}b\bar{p}d$	1
$cb\bar{p}\bar{d}$	$1+\kappa_w^-=1$	$\bar{c}b\bar{p}\bar{d}$	$1+\kappa_w^-=1$
$c\bar{b}pd$	$1+\kappa_{st}^-=2$	$\bar{c}\bar{b}pd$	$0+\kappa_{st}^-=1$
$c\bar{b}p\bar{d}$	1	$\bar{c}\bar{b}p\bar{d}$	0
$c\bar{b}\bar{p}d$	$1+\kappa_{st}^-=2$	$\bar{c}\bar{b}\bar{p}d$	$0+\kappa_{st}^-=1$
$c\bar{b}\bar{p}\bar{d}$	1	$\bar{c}\bar{b}\bar{p}\bar{d}$	0

Table 3: Schematic c-revised ranking function $\kappa^* = \kappa^c_{\Delta\gg} * (d \gg b)$ of Example 4. Note that $\kappa_0 = 0$ which is why it is not represented in this table.

and thus the sums are empty and we get (18). Analogously, (19) follows from (16):

$$\kappa_w^- \geq \min_{\omega \models \overline{A}\,\overline{B}} \{\kappa(\omega) + \sum_{\omega \models A_l \overline{B}_l} \kappa_l^-\} - \min_{\omega \models \overline{A}B} \{\kappa(\omega) + \sum_{\omega \models A_l \overline{B}_l} \kappa_l^-\}.$$

The sums in the minima are empty because the strong conditional is not falsified for any world satisfying $\overline{A}\overline{B}$ resp. $\overline{A}B$. Conditions (18) and (19) ensure the success condition $\kappa^*_{\Delta^w} \models \Delta^w$.

As we have seen, c-revision provides a revision method for OCFs which can handle sets of standard and might conditionals. The admissible impact factors allow for a combination of standard and might conditionals in the revision. Together with the special structure of difference-making conditionals, we obtain a revision method for epistemic states which take a difference-making conditional as input and therefore ensure that the premise of the conditional is relevant for the antecedent.

Now we give an example of a c-revision by a single difference-making conditional:

Example 4 (continue Example 3)**.** *The plumber arrives at the agent's house and tells her that another common reason for broken pipes are deposits in the pipe (d). Since the house is pretty old, the pipe could have also broken because of these deposits. The agent revises her belief state $\kappa^c_{\Delta\gg}$ with the new information $d \gg b = \{(b|d), \langle\!\langle \bar{b}|\bar{d} \rangle\!\rangle\}$. Note that $\kappa^c_{\Delta\gg}(\dot{c}\dot{b}\dot{p}) = \kappa^c_{\Delta\gg}(\dot{c}\dot{b}\dot{p}\dot{d})$ with $\dot{a} = \{a, \overline{a}\}$ for any boolean variable a. Using* (18) *and* (19)*, we calculate $\kappa_{st}^- > \kappa^c_{\Delta\gg}(bd) - \kappa^c_{\Delta\gg}(\bar{b}d) = 0$ and $\kappa_w^- \geq \kappa^c_{\Delta\gg}(\bar{b}\bar{d}) - \kappa^c_{\Delta\gg}(b\bar{d}) = 0$, and choose $\kappa_{st}^- = 1$ and $\kappa_w^- = 0$. Using Definition 4 we get $\kappa^c_{\Delta\gg} * (d \gg b) = \kappa^*$ which is depicted in table 3. Note that in κ^* still the difference-making conditional $c \gg b$ holds, so the new reason-relation between the deposits and the broken pipe does not overwrite the connection between cold temperatures and the broken pipe.*

7 Related Work

Difference-making conditionals establish a notion of relevance for conditionals, namely that the antecedent A of a conditional 'If A, then B' is relevant for its consequent B. The idea of incorporating relevance into the analysis of conditionals has been around for a long time, and several attempts to implement this kind of connective have been made. In this section, we explore and compare some of these ideas.

The earliest work establishing a tight connection between conditionals and belief revision was Gärdenfors (1979). In a similar vein Fariñas and Herzig (1996) uncover a strong link between belief contraction (which is known to be dual to belief revision) and *dependence*. Their idea is close to the idea of relevance introduced in Rott (1986) and their work is cited by Rott (2019). This is what Fariñas and Herzig understand by the phrase 'B depends on A':

FHD $\Psi \models A \rightsquigarrow B$ iff $B \in Bel(\Psi)$ and $B \notin Bel(\Psi \dot{-} A)$.

So B depends on A if and only if B is believed in the current belief state Ψ and B is no longer believed if A is withdrawn from the belief set of Ψ. There are some notable differences to the Relevant Ramsey Test. The most striking one is that the domain of Fariñas and Herzig's dependency relation is restricted to the agent's current belief set, since $\Psi \models A \rightsquigarrow B$ implies that $A, B \in Bel(\Psi)$. It fails to acknowledge dependencies between non-beliefs, i.e., propositions that the agent either believes to be false or suspends judgement on, like the propositions featuring in counterfactuals which typically are non-beliefs.

A second strand of research to compare with the present one is the study of conditionals incorporating relevance in a probabilistic framework that was begun by Douven (2016) and Crupi and Iacona (2019b). Crupi and Iacona (2019a) suggested a non-probabilistic possible-worlds semantics for the 'evidential conditional' that can be defined as follows:

CPC $A \rhd B$ iff $(B|A)$ and $(\overline{A}|\overline{B})$.

Let us call such conditionals *contraposing conditionals*. Crupi and Iacona call a rule essentially identical to (CPC) the 'Chrysippus Test' (Crupi and Iacona 2019a) and say that it characterizes the evidential interpretation of conditionals according to which 'a conditional is true just in case its antecedent provides evidence [or support] for its consequent.' Raidl (2019) provided the first completeness proof for the 'evidential conditional' which has been improved in Raidl, Crupi and Iacona (2020). Independently, Booth and Chandler (2020, Proposition 12) hit upon the same concept of contraposing conditionals and have started investigating it.

Rott (2020) raises doubts as to whether contraposition really captures the idea of evidence or support. It is true that contraposing conditionals do violate RW, and this violation was called the hallmark of relevance by Rott. Except for that, contraposing conditionals are formally very well-behaved as they validate, for example, Or, Cautious Monotony, Negation Rationality and Disjunctive Rationality. These principles are all violated by difference-making conditionals. However, Rott argues that the contrastive notion of difference-making is better motivated as an explication of evidence and support than contraposition. The Relevant Ramsey Test—which can be found, under the name 'Strong Ramsey Test', already in Rott (1986)—has ancestors in Gärdenfors' (1980) notion of explanation and in Spohn's (1983) notion of reason which both encode the idea that the explanans (or the reason) should raise the doxastic status of the explanandum (or of what the reason is a reason for).

If we define a ranking semantics for contraposing conditionals using the framework of Spohn's ranking functions, we can compare these two notions of relevance from technical point of view. Let κ be a ranking function and $A \rhd B$ be a contraposing conditional with contingent A and B. Then

(CPC^{ocf}) $\quad \kappa \models A \rhd B$

iff $\kappa \models (B|A)$ and $\kappa \models (\overline{A}|\overline{B})$

iff both of the following two conditions hold:

$$\kappa(AB) < \kappa(A\overline{B}) \text{ and} \tag{20}$$

$$\kappa(\overline{A}\overline{B}) < \kappa(A\overline{B}). \tag{21}$$

Difference-making and contraposing conditionals both require the acceptance of the standard conditional $(B|A)$, but they differ in the case when the antecedent is denied. Compare (20) and (21) with (3) and (4). Difference-making conditionals require the $\overline{A}\overline{B}$-worlds to be more or equally plausible as the $\overline{A}B$-worlds stressing that the denial of the antecedent should not lead to acceptance of the consequent. For contraposing conditionals, the denial of the consequent leads to denial of the antecedent, so some $\overline{A}\overline{B}$-worlds are required to be strictly more plausible than all the $A\overline{B}$-worlds. Difference-making conditionals place inequality constraints on all possible worlds in $\Omega_{\{A,B\}}$, whereas contraposing conditionals do not deal with the position of $\overline{A}B$-worlds at all.

To give a feel for the contrast between difference-making conditionals and contraposing conditionals, we present an example from Rott (2020) and transfer it to the framework of ranking functions. Suppose an infectious disease breaks out with millions of cases, and consider the following two scenarios concerning a treatment:

> **Scenario 1:** Almost all of the people infected were administered a medicine and almost all of them have recovered. However, only few of the persons who did not receive the medicine have recovered.
>
> **Scenario 2:** Only very few of the people infected were administered the medicine. But fortunately, most people end up recovering anyway. It turns out that within the group of people who got the medicine slightly less people have recovered than within the group who did not get it.

We compare these two scenarios and imagine an agent who has contracted the disease, but of whom it is not know whether she got the medicine. In Scenario 1, the fact that the agent received the medicine would clearly support the fact that she recovered, as it would clearly make the recovery more likely. So we are justified in accepting the conditional 'If the agent received the medicine, she has recovered'. However, in scenario 2 it does not make sense to apply this conditional. It is likely that the agent has recovered, but having received the medicine would not be evidence for the recovery. We depict these two scenarios using ranking functions. Let m stand for 'the agent received the medicine' and r for 'the agent recovered'. The ranking function κ_1 with $\kappa_1(mr) = 0$, $\kappa_1(m\overline{r}) = 1$, $\kappa_1(\overline{m}\,\overline{r}) = 2$ and $\kappa_1(\overline{m}r) = 3$ captures scenario 1 and κ_2 with $\kappa_2(\overline{m}r) = 0$, $\kappa_2(\overline{m}\,\overline{r}) = 1$, $\kappa_2(mr) = 2$ and $\kappa_2(m\overline{r}) = 3$ captures scenario 2.As we can see $\kappa_1 \models m \gg r$, since $\kappa_1(mr) = 0 < 1 = \kappa(m\overline{r})$ and $\kappa_1(\overline{m}\,\overline{r}) = 2 \leq 3 = \kappa_1(\overline{m}r)$, but $\kappa_1 \not\models m \rhd r$, since $\kappa_1(\overline{m}\,\overline{r}) = 2 > 1 = \kappa_1(m\overline{r})$. For the second scenario, it holds $\kappa_2 \models m \rhd r$ but $\kappa_2 \not\models m \gg r$. If we compare this with our intuition towards the relation between medicine and recovery, we find that the difference-making conditional gets the example right.

Another argument for the notion of relevance encoded by difference-making conditionals is that they comply with Spohn's work who defines causation as follows:

> A is a cause of B iff A and B obtain, A precedes B, and A raises the metaphysical or epistemic status of B given the obtaining circumstances. (Spohn 2012, p. 352)

As we can see, this is a compound of facts, times, obtaining circumstances and a reason relation. We do not deal with the first three components, but we can compare difference-making conditionals with Spohn's concept of reason. In terms of ranking functions, A is a reason for B if the following inequality holds for a ranking functions κ:

$$\kappa(\overline{B}|A) - \kappa(B|A) > \kappa(\overline{B}|\overline{A}) - \kappa(B|\overline{A}). \tag{22}$$

Compare Spohn (2012, p. 105, using the definition of two-sided ranks $\tau(B|A) = \kappa(\overline{B}|A) - \kappa(B|A)$). Inequality (22) expresses that the conditional $(B|A)$ is stronger than $(B|\overline{A})$. Thus, A is a direct[!] cause of B in Spohn's sense just in case A and B are true, the event represented by A precedes the event represented by B and Spohn's inequality (22) holds, given the obtaining circumstances. For $\kappa \models A \gg B$, equations (3) and (4) hold. Via the definition of ranks for conditionals we first elaborate on (22):

$$\begin{aligned}(22) \;\Leftrightarrow\;\; & \kappa(A\overline{B}) - \kappa(A) - (\kappa(AB) - \kappa(A)) \\ & > \kappa(\overline{A}\overline{B}) - \kappa(\overline{A}) - (\kappa(\overline{A}B) - \kappa(\overline{A})) \\ \Leftrightarrow\;\; & \kappa(A\overline{B}) - \kappa(AB) > \kappa(\overline{A}\overline{B}) - \kappa(\overline{A}B).\end{aligned}$$

Now if $\kappa \models A \gg B$, then the left-hand side is positive, due to (3), whereas the right-hand side is not, due to (4). So, the inequality expressing the notion of reason defined by Spohn follows immediately from the definition of difference-making conditionals as a set of standard and might conditionals. As was pointed out by Eric Raidl (2020, p. 17), $A \gg C$ expresses that A is a 'sufficient reason' for C in the terminology of Spohn (2012, pp. 107–108).

8 Conclusion

Difference-making conditionals aim at capturing the intuition that the antecedent A of a conditional is relevant to its consequent B, that A supports B or is a reason or evidence for it. The Relevant Ramsey Test encodes this idea, ruling that revising by the antecedent should lead to acceptance of the consequent, which is the standard Ramsey Test, but also ruling that revising by the negation of the antecedent should *not* lead to the acceptance of the consequent. Rott (2019) defined the Relevant Ramsey Test and difference-making conditionals in a purely qualitative framework. In the present paper we extended his approach to ranking functions by first transferring the Relevant Ramsey Test to the framework of OCFs. We defined difference-making conditionals as a pair consisting of a standard and a might conditional, which is in full compliance with the basic principles that Rott identified for difference-making conditionals. Using this transformation we benefitted from the flexible approach of c-representations and c-revisions, defining an inductive representation and a revision method for conditionals incorporating relevance. To the best of our knowledge, there is no other revision method capable of dealing with not only sets of conditionals but also sets of conditionals of different types, namely standard and might-conditionals. Finally, drawing on the ranking semantics for difference-making conditionals, we compared different approaches to relevance or evidence in conditionals. We showed that difference-making conditionals express something very close to Spohn's concept of reason in the context of ranking functions, but that they are fundamentally different from the evidential (or contraposing) conditionals studied by Crupi, Iacona and Raidl.

For future work we plan on elaborating on the inductive representation of mixed sets of conditionals. Moreover, we will continue working on the incorporation of relevance in different kinds of epistemic states and examine different revision methods for conditionals incorporating relevance.

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