5-2007

Consumption, Time Preference, and the Life Cycle

Michael Charles Bailey
Utah State University

Consumption, Time Preference, and the Life Cycle

by

Michael Charles Bailey

Thesis submitted in partial fulfillment
of the requirements for the degree

of

HONORS IN UNIVERSITY STUDIES
WITH DEPARTMENTAL HONORS

in

Economics
in the Department of Economics

Approved:

Thesis/Project Advisor
Dr. L. Dwight Israelsen

Departmental Honors Advisor
Dr. Clifford Skousen

Director of Honors Program
Dr. Christie Fox

UTAH STATE UNIVERSITY
Logan, UT

Spring 2007

Consumption, Time Preference, and the Life Cycle

Michael C. Bailey
Utah State University

March 2007

Abstract

This paper presents two life-cycle models of consumption implementing novel assumptions about time preference and subjective time. The goal of this paper is to investigate implications of the existence of subjective time to consumption decisions over the life cycle. The first model is a model of 'systematic impatience' and implements the assumption of increasing subjective time by specifying a time dependent rate of time preference upon which the rational consumers in this model maximize lifetime utility. The second model investigates consumer behavior in subjective time, or the subjective sense of the actual passage of time. Consumers in this model maximize lifetime utility in subjective time. The optimal subjective consumption and saving functions are then mapped into real time. Both models are then compared to empirical findings on consumption theory. [1]

[1] I would like to thank my mentor and friend Dr. L. Dwight Israelsen whose comments inspired the topic of this paper and who has provided invaluable help and encouragement during my undergraduate career. This paper is the result of a discussion about a seminar presentation by Frank Caliendo (2006) at Utah State Univeristy . I would also like to thank Shantanu Bagchi who provided invaluable assistance for the mathematical programming.

1 Introduction

A child claims that the summer drags forever. His grandfather regrets how fast the summer flew by. – Paul Samuelson

This statement was penned nearly thirty years ago in a seminal article by Samuleson (1977), but economists have just barely begun researching the relationship between people's perception of time and how this affects their economic behavior. Behavioral economics is a nascent field blending economic theory and psychology and is becoming a mainstream field in economics. Given that much of economic theory is devoted to the study of human and consumer behavior it is surprising that many developments in psychology have yet to be integrated into mainstream economic theory.

This paper implements novel assumptions about people's intertemporal preferences and perception of time into the life-cycle theory of consumption. In the first model presented in Section 4, younger consumers are more impatient than older ones, i.e. the rate of time preference (RTP) changes systematically through time.

It is an empirical fact that younger consumers consume a larger proportion of their income than older ones, this phenomenon is attributed to the 'smoothing' of consumption, or the attempt by consumers to maintain a consistent consumption profile (this is intimately related to risk aversion and in fact it can be shown that the elasticity of intertemporal substitution is the inverse of the rate of relative risk aversion see Mas-Colell 2002) and to the presence of budget constraints. This 'smoothing' phenomenon could also be the result of a consumer's perception of time. If time seems to move slower for younger consumers, then it is reasonable to assume that younger consumers need more consumption to attain a given amount of utility, given that utility is a function of the *flow* of consumption. The first model which I call 'systematic impatience', presented in section 4, implements this assumption using a systematic change in the RTP, i.e. future consumption per unit of real time is discounted at a higher and higher rate because time subjectively seems to be moving faster.

The second model which I call 'subjective mapping' builds upon findings by Samuelson (1977) on subjective time, or the sense of the speed at which time passes. A life-cycle model is built where consumers maximize lifetime utility in subjective time and the optimal consumption path in subjective time is found. This consumption path is then mapped into real time. Four simple functional forms proposed by Samuelson linking real and subjective time are used to map the standard optimal consumption and saving functions from the standard life-cycle model into real time. The critical assumption of this paper is that consumers base their consumption decisions on subjective time and try to smooth their consumption in subjective time or alternatively they consume so that the marginal utility of money is constant (per unit of subjective time).

One of the major puzzles in the macroeconomics literature is that though the standard life-cycle model of consumption predicts a monotonic consumption path, empirical studies have consistently found a robust hump-shaped consumption path peaking near the age of 50. This paper shows that with certain values of the parameters not varying too greatly from those in the literature, a model of 'systematic impatience' can produce hump-shaped consumption profiles. The goal of this paper is not to show that the hump is the result of these kind of preferences, but rather that the existence of subjective time could have interesting implications for consumption decisions.

Section 2, Intertemporal Allocation of Consumption, introduces the theory of the intertemporal allocation of consumption over the life cycle upon which the models of this paper rely. This section provides evidence that consumers consume so that the marginal utility of money remain constant over the life cycle, a key assumption made in many seminal papers on life-cycle consumption theory.

Section 3, Time Preference, builds upon the theory of intertemporal allocation of consumption adding the important element of time preference. This section presents two key parameters used in modeling intertemporal decisions, the RTP and the elasticity of intertemporal substitution (EIS). This section shows that consumers try to smooth consumption over the life-cycle and that the life-cycle model of consumption predicts a monotonic path of consumption over the life cycle.

Section 4, Systematic Impatience, builds the first of two models of this paper in which the RTP is time dependent. This differs from the approach taken by Uzawa (1968) and Epstein (1983) in which the RTP varies based upon the level of wealth or consumption. The RTP in this model varies systematically throughout time and is independent of wealth and consumption. The model is then compared to empirical findings in the literature.

Section 5, Subjective Time and Life-cycle Consumption, builds the second model which is based upon Samuelson's (1977) findings on subjective time. This model uses the findings of the standard life cycle model of consumption and assumes that the consumer is maximizing in subjective time. Various functions are presented liking real and subjective time and the optimal real consumption and saving functions are found from by mapping the subjective functions into real time. The model is then compared to empirical findings in the literature.

2 Intertemporal Allocation of Consumption

Some of the most important models used in public finance, macroeconomics, and consumption theory are optimizing models of the intertemporal allocation of consumption. The benchmark or standard model is the life-cycle model suggested by Modigliani and Brumberg (1954), Tobin (1967), Yaari (1964) and others and which a modern example can be found in Butler (2001).

We begin by assuming that a consumer has a given utility function $U(x)$ describing their preferences over various goods (for a similar treatment to the following see Silberberg (2001) or Mas-Colell (2002)). Given a bundle or vector of the amounts of n goods which the consumer would choose to consume, say $\vec{x} = (x_1, x_2, ..., x_n)^T$, and the relative prices of these goods, $\vec{p} = (p_1, p_2, ..., p_n)^T$, the consumer will choose the amount of each good to maximize their utility function given total income I, i.e. they will choose to:

$$\text{Maximize } U(\vec{x}) \text{ subject to } p^T \bullet \vec{x} = I$$

Differentiating the Lagrangian $\Psi(\vec{x}) = U(\vec{x}) - \lambda \bullet (p^T \bullet \vec{x} - I)$ with respect to $\vec{x}$ and λ yields the following first order conditions (assuming an interior solution):

$$\frac{\partial U(\vec{x})}{\partial x_i} - \lambda p_i = 0 \text{ for } 1 \le i \le n \qquad (1)$$

$$p^T \bullet \vec{x} = I \qquad (2)$$

and upon rearranging (1):

$$\frac{\frac{\partial U(\vec{x})}{\partial x_i}}{\frac{\partial U(\vec{x})}{\partial x_j}} = \frac{p_i}{p_j} \qquad (3)$$

equation (3) shows that the ratio of the marginal utilities of the goods must equal the ratio of their prices.

Geometrically, the optimum is found where the budget set is tangent to the indifference curve. Now assume that there are different periods over which the consumer can consume the different goods. Let $\vec{x}_j = (x_{j1}, x_{j2}, ..., x_{jn})$ denote the goods available to the consumer in period j. Then the previous analysis applies with $\vec{x} = (x_{11}, x_{12}, ..., x_{21}, x_{22}, ...)$, a re-labeling of each good according to the period in which the good is consumed. A consumer deciding between good 1 and good 2 will follow the same rule as deciding between good 1 in period 1 and good 1 in period 2, the ratio of the marginal utility must be equal to the ratio of the *intertemporal* price. In this framework the consumer as deciding between many goods, where a good available in a different period is treated as a different good.

The standard analysis needs revision when one considers that if the consumer saves say X dollars of his income in one period, he can earn $(1+r)\bullet x$ dollars of income for the next period, given an interest rate of r (the continuous case follows analogously) i.e. the consumer can forgo present consumption for future consumption (see Romer (2006)). Hence, if intertemporal prices remain constant, an amount q of a good in the second period is worth $\frac{q}{1+r}$ of the good in the first period since this is the saving required at an interest rate r to obtain q of the good. This leads to the concept known as discounting.

If the consumer earns an income of Y_i in period i and consumes c_i worth of goods in period i then the intertemporal budget constraint that present value of consumption over the life cycle be less than the present value of income over the life-cycle can be expressed as:

$$\sum_{i=0}^{i=n}\frac{c_i}{(1+r)^i}\leq\sum_{i=0}^{i=n}\frac{Y_i}{(1+r)^i} \tag{4}$$

If the consumer has a utility function $u(C_i)$ for period i (where period 0 is the current period) and lifetime utility $U(c_0,\dots,c_n)=\sum_{i=0}^{i=n}u(c_i)$, such that $u'(c_i)>0$ and $u''(c_i)<0$, then the consumer will choose to:

$$\text{Maximize } U(c_0,\dots,c_n)=\sum_{i=0}^{i=n}u(c_i) \text{ subject to } \sum_{i=0}^{i=n}\frac{c_i}{(1+r)^i}\leq\sum_{i=0}^{i=n}\frac{Y_i}{(1+r)^i}$$

Since the marginal utility of consumption is positive, the budget constraint will be satisfied with equality and the Lagrangian and first order conditions for this problem are:

$$\Psi(c_0,\dots,c_n)=U(c_0,\dots,c_n)-\lambda\bullet\left(\sum_{i=0}^{i=n}\frac{c_i}{(1+r)^i}-\sum_{i=0}^{i=n}\frac{Y_i}{(1+r)^i}\right) \tag{5}$$

$$u'(c_i)=\frac{\lambda}{(1+r)^i} \quad \text{for } 0\leq i\leq n \tag{6}$$

$$\sum_{i=0}^{i=n}\frac{c_i}{(1+r)^i}=\sum_{i=0}^{i=n}\frac{Y_i}{(1+r)^i} \tag{7}$$

Equation (6) has an important economic interpretation. Because λ is the Lagrange multiplier for the budget constraint, it is the value to the consumer of an additional dollar of income, therefore it is the marginal utility of money. Equation (6) implies that the marginal utility of money must equal the marginal utility of consumption in period 0,

$u'(c_0)=\lambda$, and that the marginal value of consumption in subsequent time periods be equal to the discounted value of the marginal utility of money, i.e. the present value of the marginal utility of money. To see why this must be true assume that the consumer decreases his consumption in period j and increases his consumption by (1+r) the decrease in period j+1, if the lifetime utility is at a maximum then this marginal change should have no effect on total utility (Romer 2006). The marginal utilities in periods j and (j+1) from equation (6) are $\frac{\lambda}{(1+r)^j}$ and $\frac{\lambda}{(1+r)^{j+1}}$ and therefore:

$$\frac{\lambda}{(1+r)^j}=(1+r)\bullet\frac{\lambda}{(1+r)^{j+1}}=\frac{\lambda}{(1+r)^j} \tag{8}$$

Hence it is satisfied for periods $0\le i\le n$. This shows that *the present value of the marginal utility of money must be constant throughout the life-cycle.*

Without specifying the form of the underlying utility function, three special cases emerge:

1.) $r=0$: In this case the marginal utility of consumption is constant throughout the life cycle and this implies that consumption is constant throughout the life cycle, $c_0=c_1=...=c_n$. From equation (7) we can solve for c_i:

$$c_i=\frac{1}{n}\bullet\sum_{i=0}^{i=n}Y_i \tag{9}$$

2.) $r>0$: this implies that $u'(c_0)>u'(c_1)>...>u'(c_n)$ and since by assumption $u'(c_i)>0$, we conclude that $c_0<c_1<...<c_n$. This result is expected, since the consumer derives the same utility from a given amount of consumption in any time period, it would make sense to save income and earn a positive interest and consume in the final period. Since $u''(c_i)<0$, it is not optimal to consume all in the last period, but the amount of consumption increases each period.

3.) $r<0$: Using a similar argument to 2.), $u'(c_0)<u'(c_1)<...<u'(c_n)$ and $c_0>c_1>...>c_n$.

These 3 cases provide evidence of a result to be shown in the following section that consumers try to smooth their consumption over the life-cycle and in the standard framework consumption is either monotonically increasing or decreasing.

3 Time Preference

In this section we introduce the concept of time preference. It is obvious that people prefer to have wealth now as opposed to later, say in the form of goods or money (assuming it can be costlessly stored), because if one has it now one can always save it for later and having it now would thus increase one's opportunities (Silberberg 2001). It is not surprising therefore that empirical studies show that people are impatient (Chung 1967). Empirical studies indicate if a person is given the choice between 50$ now and 100$ a year from now they would take the $50. This implies more than opportunities, but preferences.

To represent these preferences, we would discount not only consumption based upon the interest rate but we would also discount utility based upon a rate of time preference, ρ. A lifetime utility function might have the form of

$$U(c_0,\ldots,c_n)=\sum_{i=0}^{i=n}\frac{u(c_i)}{(1+\rho)^i} \tag{10}$$

if the rate of time preference is constant over time, or if the rate of time preference differs for each period it might take the form of

$$U(c_0,\ldots,c_n)=\sum_{i=0}^{i=n}\frac{u(c_i)}{\prod_{j=0}^{j=i}(1+\rho_i)} \tag{11}$$

where ρ_i is the rate of time preference for period i. Using (4) as our budget constraint we can see the consumer will try to maximize (10) or (11) subject to (4). For (10) this yields (5) as its Lagrangian and a first order condition of (7) and:

$$u'(c_i)=\lambda\bullet\frac{(1+\rho)^i}{(1+r)^i}\quad\text{for } 0\le i\le n \tag{12}$$

For consecutive time periods i and (i+1):

$$\frac{u'(c_i)}{u'(c_{i+1})}=\frac{\lambda\bullet\dfrac{(1+\rho)^i}{(1+r)^i}}{\lambda\bullet\dfrac{(1+\rho)^{i+1}}{(1+r)^{i+1}}}=\frac{(1+r)}{(1+\rho)} \tag{13}$$

Equation (13) gives the relationship between the market interest rate and the rate of time preference. If the rate of time preference is greater than the market interest rate than consumption will be shifted towards the current period. If the interest rate exceeds the rate of time preference, than consumption will be shifted towards later periods. We see that the growth rate in consumption is constant, i.e. consumption increases, decreases, or stays the same monotonically. The standard life cycle model in continuous time leads to the same conclusion (see appendix 1).

In the case where $\rho = r$, we notice that $c_0 = c_1 = ... = c_n$. This along with (9) shows that *consumers will try to smooth their consumption over the life cycle.* Smoothing does not necessarily mean that consumers will try to maintain constant consumption, but as already stated that they will try to maintain a constant marginal utility of money, which means with well-behaved utility functions, the consumption path will also be well-behaved (Brumberg). We would only see a dramatic shift in consumption in two adjacent time periods if the market interest rate differs dramatically from the rate of time preference, otherwise the consumer would not be a utility maximizer since a smoothing of consumption would increase utility.

There is one other parameter that is important in analyzing intertemporal consumption decisions, the Elasticity of Intertemporal Substitution. While the RTP determines how much the consumer discounts future utility relative to current utility, the EIS determines how willing the consumer is to take advantage of differences in the RTP and the interest rate.

Technically, the EIS between periods i and j is the percentage change in the relative consumption between those two periods for a one percentage change in the ratio of marginal utilities. Since the marginal utilities depend upon the level of consumption, one computes the EIS by finding the inverse of the percentage change in the ratio of the marginal utilities of two periods for a one-percentage change in relative consumption between those two periods. Formally this can be written:

$$\frac{\dfrac{d(c_i/c_j)}{c_i/c_j}}{\dfrac{d(u(c_i)/u(c_j))}{u(c_i)/u(c_j)}} = \frac{d\ln(c_i/c_j)}{d\ln(u(c_i)/u(c_j))} \qquad (14)$$

It can be shown that the inverse of the EIS is the Arrow-Pratt measure of relative risk aversion, $-c \bullet \frac{u''(c)}{u'(c)}$ (See Mas-Collel 2002). A common utility function used in the literature is the constant-relative-risk-aversion instantaneous utility defined as:

$$u(c(t)) = \frac{c(t)^{1-\theta}}{1-\theta} \text{ for } \theta > 0 \qquad (15)$$

where θ is the relative risk aversion and is constant, and the EIS is $\frac{1}{\theta} = \varepsilon$. This utility function has many nice properties and in the limit as $\theta \to 0$ this function converges to the simple function $u(c) = \ln(c(t))$, which has been shown to match empirical observations very well (Attanasio 1999). Using this form of utility and equation (13) we can derive:

$$\frac{u'(c_i)}{u'(c_{i+1})} = \frac{c(t)^{-\theta}}{c(t+1)^{-\theta}} = \frac{(1+r)}{(1+\rho)} \Rightarrow \frac{c(t+1)}{c(t)} = \left(\frac{(1+r)}{(1+\rho)}\right)^{\frac{1}{\theta}} = \left(\frac{(1+r)}{(1+\rho)}\right)^{\varepsilon} \qquad (16)$$

Equation (16) leads to the same implications as (13). Studies have shown that consumption growth responds relatively little to changes in the real interest rate, which implies that the EIS is low (Romer 2006).

4 Systematic Impatience

The first model is that of systematic impatience. The consumers in this model are impatient in the sense that the RTP is a function of time and varies systematically over the life cycle. This form of the RTP is an implementation of the assumption that the subjective speed of time is increasing over time. If people view time as passing more quickly as they get older, and if they are trying to keep the marginal utility of money per unit of real time constant, then they would systematically discount future utility at a higher and higher rate.

It can be shown that for the varying rate of time preference case, using the same procedure in the derivation of (13) that:

$$\frac{u'(c_i)}{u'(c_{i+1})} = \frac{\lambda \bullet \dfrac{\prod_{j=0}^{j=i}(1+\rho_i)}{(1+r)^i}}{\lambda \bullet \dfrac{\prod_{j=0}^{j=i+1}(1+\rho_{i+1})}{(1+r)^{i+1}}} = \frac{(1+r)}{(1+\rho_{i+1})} \tag{17}$$

Depending on the rates of time preference in each period consumption could be growing or shrinking. Consider the case of CRRA utility where $\theta \to 0$ and hence $u(c) = \ln(c(t))$. As mentioned previously this form matches 'conventional and empirical evidence' (Caliendo 2007). In this case (17) becomes;

$$\frac{u'(c_i)}{u'(c_{i+1})} = \frac{c(i+1)}{c(i)} = \frac{(1+r)}{(1+\rho_{i+1})} \tag{18}$$

The model of systematic impatience shows that consumption increases from period i to period i +1 if the consumption in period i + 1 is discounted greater than the interest rate. Using (18) and the budget constraint (7) we can solve for the optimal consumption path c*(t) given $\rho(t)$.

We would expect $\rho(t)$ to be related to the mapping between subjective and real time $\theta(t)$, perhaps by some transformation $\rho(t) = g(\theta(t))$ for the reason that as time is 'speeding up' subjectively the consumer discounts that interval of time at a higher rate. In section 5 we present various forms of $\theta(t)$ introduced originally by Samuelson (1977) including linear, quadratic, logarithmic, exponential, and square root forms.

We now assume the consumer consumes over 40 years (ages 20 – 60) and assume an interest rate equal to 0.05. Figures 1-3 plot relative consumption (consumption divided by initial consumption at age 20) for the five mentioned functional forms for the rate of time preference. Figures 1 plots for $\rho(20) = .03$ and $\rho(60) = .07$ and Figure 2 plots for $\rho(20) = .00$ and $\rho(60) = .20$. In the life-cycle literature, empirical results find that the relative consumption peak is between 1.1 and 1.5 the initial consumption value peaking about age 45 to 50. In figure 5 we plot each function for different $\rho(20)$ and $\rho(60)$ so as to obtain consumption paths that approximate these values.

As the figures show, with values of $\rho(20) < r$ and $\rho(60) > r$ there is a consumption 'hump' as our model would predict. The purpose of these graphs is not to show that a model of systematic impatience is the primary reason for the consumption hump, but that a varying rate of time preference due to the presence of subjective time function might greatly affect the shape of the optimal consumption profile. If such a subjective time function does exist it could be an important aspect in consumption decisions over the life cycle. The next section discusses the subjective time function. The existence of a time dependent rate of time preference could imply the existence of subjective time if consumers do consume so as to keep the marginal utility of money constant per unit of subjective time.

Figure 1: Relative Consumption Paths with $\rho(20) = .02$ and $\rho(60) = .07$ for five functional forms of $\rho(t)$

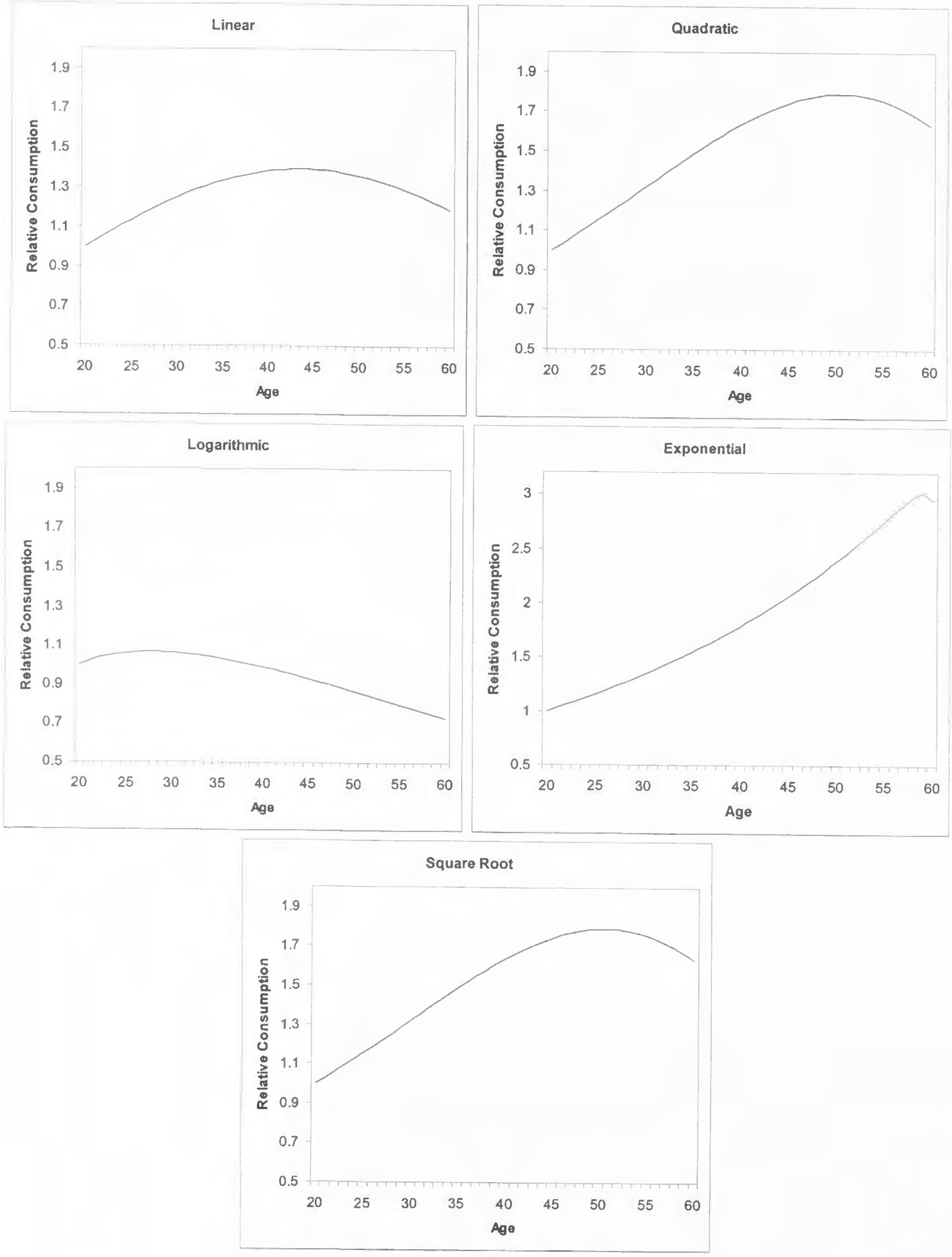


Figure 2: Relative Consumption Paths with $\rho(20) = .00$ and $\rho(60) = .20$ for five functional forms of $\rho(t)$

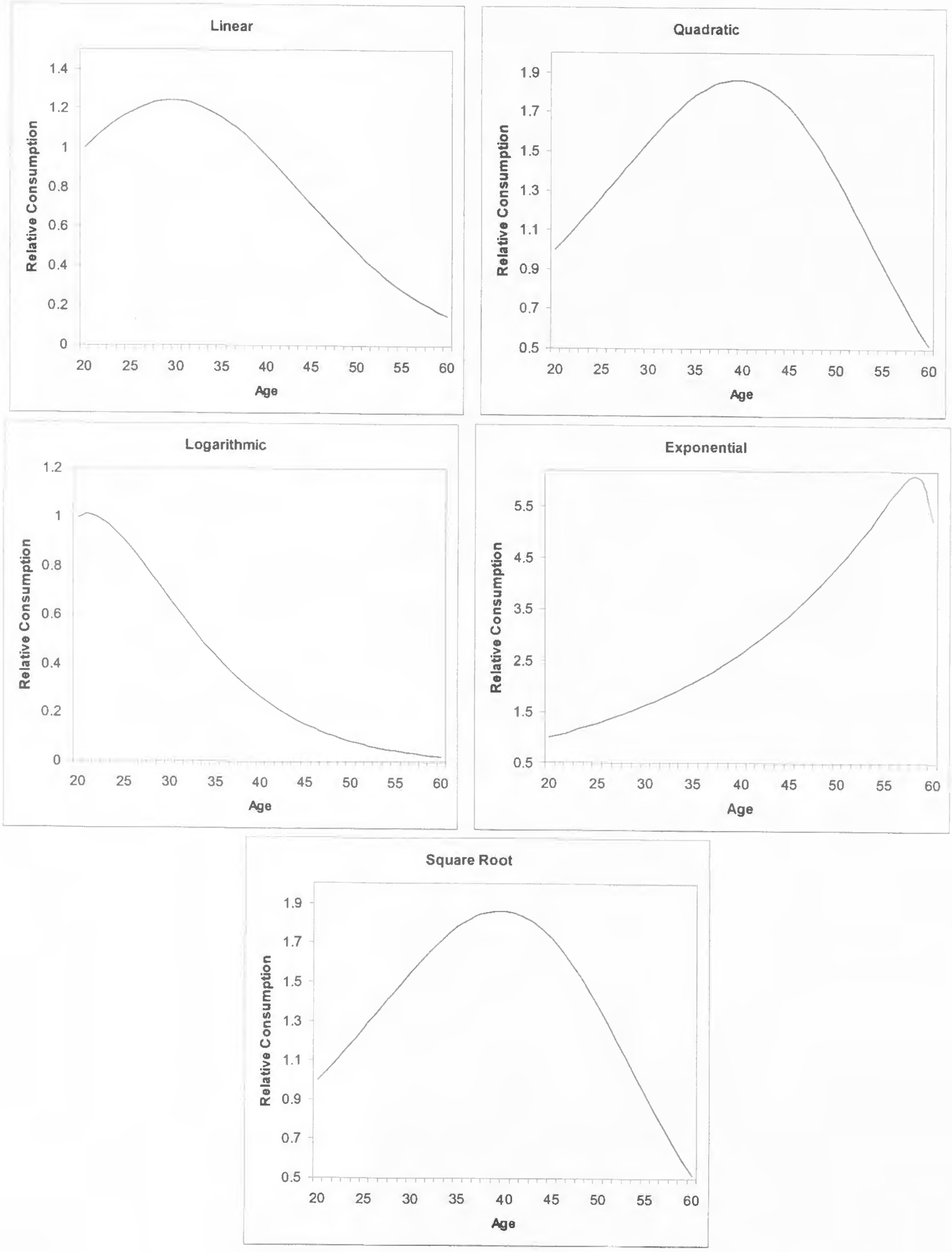
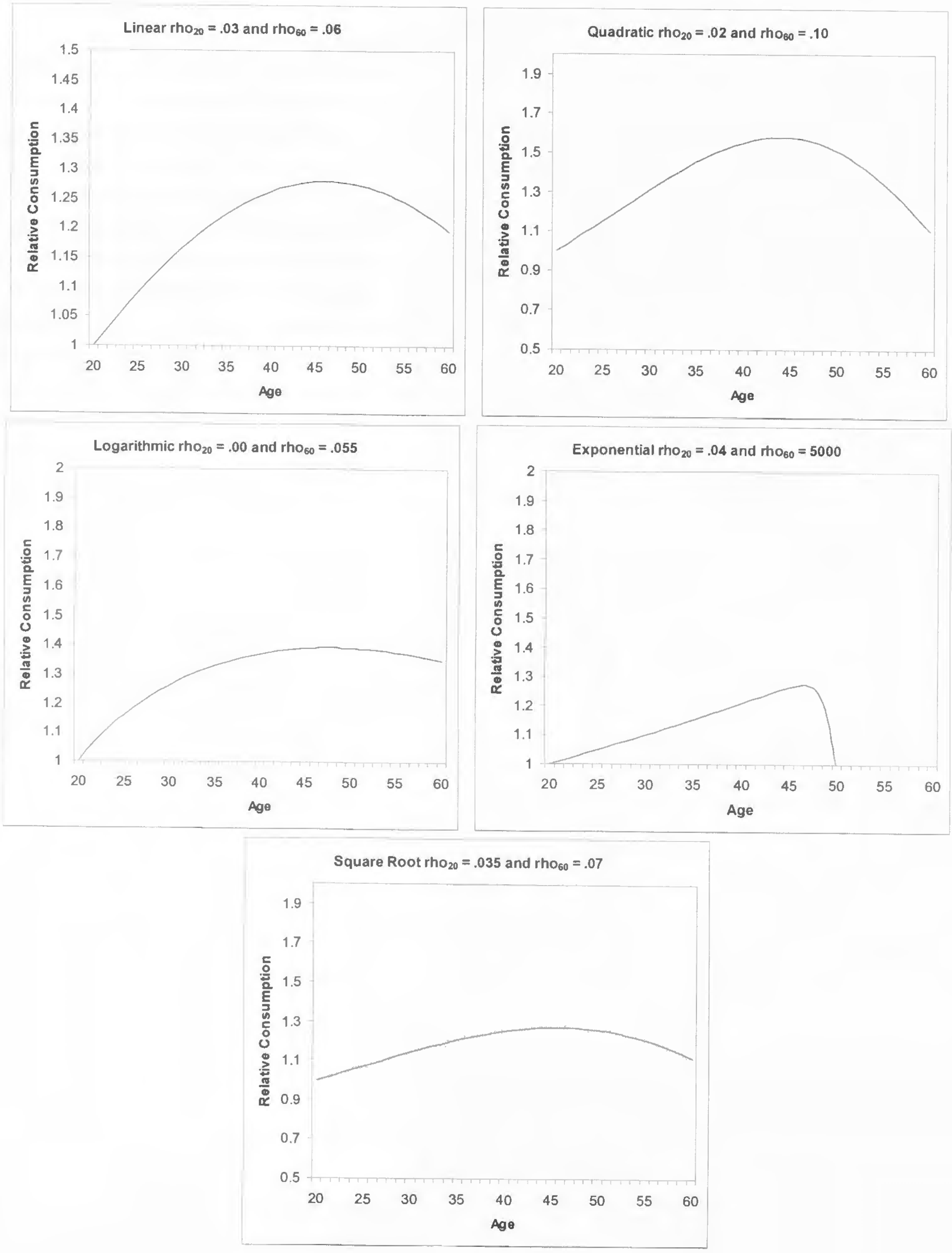


Figure 3: Relative Consumption Paths with various values of $\rho(20)$ and $\rho(60)$ for five functional forms of $\rho(t)$



5 Subjective Time and Life-cycle Consumption

Everyone has experienced periods where time seems to pass more quickly or slowly. Samuelson (1977) defines subjective time as the 'sense of the speed at which, subjectively, actual time passes.' Samuelson presents two explanations for the existence of a subjective time, the backward look hypothesis and the forward look hypothesis.

The backward look hypothesis focuses on the duration from birth to the current age. Suppose that subjective time is a function of the percentage of marginal experience gained compared to total experience received over a lifetime. As one accumulates life experience the marginal increase in experience is small in comparison to the total and hence leads to a smaller and smaller percentage increase in total experience. Since people report a speeding up of subjective time we might suppose that subjective time is a function of the percentage increase in experience, that somehow subjective time seems to move faster for those who are experiencing a larger percentage increase in life experience, i.e. are having more 'new' experiences.

Another way to look at the backward model is to say that subjective time is a function of receiving 'new' experience. Suppose that one receives experiences in life at a constant rate taken from a set of possible experiences. We can treat the experiences we receive as a random variable with a probability distribution for example it is much more probable that one will receive the experience, say, 'late for work', than, say, 'win the lottery'. We can treat subjective time as discrete function dependent on the number of 'new' received experiences, or experiences never received before. As one accumulates experience it becomes less and less likely that one will receive new experiences and thus subjective time moves faster. This could be modeled as a Poisson process or each time interval could be modeled using a hypergeometric distribution.

Samuelson shows that a subjective time function following the rule that the same actual time intervals will have the same subjective time measurement if and only if there has been the same percentage increase in accumulated experience will look like:

$$\theta = \ln(t / t_0) \qquad (19)$$

where t_0 is the first age of experience accumulation. Hence the subjective time will follow a logarithmic rule. It can be tested if a person did experience subjective time following a logarithmic rule, in fact it can be tested if a person followed any subjective time

$$\theta = \frac{t^\alpha}{\alpha}, 0 < \alpha \leq 1 \qquad (20)$$

where $\theta \to \ln(t)$ as $\alpha \to 0$ (Samuelson 1977). We could also test for the curvature or any functional form of θ to see if subjective time speeds up or slows down over the life-cycle.

The forward look hypothesis focuses on the duration from the current age to death. The age of death is unknown but follows a probability distribution such that the probability of death increases with age. Suppose that the subjective time is a function of remaining years of life. At age 20 one might have an expected remaining life of 58 years and hence there is (58/78) = 78% of ones lifetime remaining and the next year is only about 2% of ones remaining lifetime. However, at age 70 one might expect to only life 8 more years and the next year would be about 12% of that persons remaining lifetime. Hence the subjective speeding up of time could be attributed to the increasing percentage of one's remaining lifetime per unit of time as one get's older, an increasing 'recognition of life as fleeting.'

A case of the forward look model can be found in the following passage written about Evariste Galois, the mathematician who died in a dual at age 21. Eric Bell (1937) writes:

> *(Galois was) writing against time to glean a few of the great things in his teeming mind before the death which he foresaw would overtake him. Time after time he broke off to scribble in the margin, "I have not the time; I have not the time." And passed on to the next frantically scrawled outline.*

For Galois, the speeding up of time is apparent as is the speeding up of my own subjective time in finishing this paper before its deadline. In the case where death is known at age T, Samuelson proposes the following functional form for subjective time:

$$\theta(t) = b \int_{t=0}^{t=T} (T - t)dt = b(T * t - (1/2)t^2), 0 < t < T \qquad (21)$$

And another simpler form that he proposes is:

$$\theta(t) = \int_{u=0}^{u=t} e^{-u} du = 1 - e^{-t} \qquad (22)$$

Samuelson shows that the forward look and backward look models have the same implications.

A third explanation for the existence of subjective time could involve the degree of mental concentration. In my personal experience, I have noticed that time seems to pass faster when I am at a greater degree of mental concentration, for example when completing homework assignments or trying something new, and seems to pass slowest when my mental concentration is low, for example during soporific meetings and lengthy seminars. A common experience amongst students is that during exams one does not even recognize the passage of time because subjective time is passing so quickly. Dr. L. Dwight Israelsen proposed in a letter to Samuelson that the passage of subjective time might be dependent on the activity one is engaged in; physical concentration (brief moments of intense physical activity) is related to a slowing of subjective time and mental concentration results in a speeding of subjective time.

> *I have experienced brief moments of maximum exertion and concentration when time seemed to slow tremendously, and activity seemed to be proceeding in slow motion...witness Ted William's claim that he could see the rotation of the seams on the baseball, and his demonstrated ability to see the exact spot on the baseball struck by his bat. ... Contrast this to Bell's account of Galois' speeding up of time during the night before his death.*

We might posit then that subjective time is speeding up because as people get older their degree of physical activity decreases significantly compared to their mental activity. The account by some elderly that time seems to pass slowly may attributed to their greatly decreased degree of mental concentration.

In this paper I will not discuss the existence of some tangible or coherent subjective time or a notion of subjective time that is 'well-behaved' nor will I discuss the conditions for which one can test for the existence of subjective time, both of these matters are discussed at length in Samuelson in which he derives axioms that a subjective time function must obey (1977). The previous discussion demonstrates the plausibility that people observe some form of subjective time. I will now discuss what implications the existence of subjective time would have for consumption behavior.

Suppose that consumers did observe some form of subjective time where subjective time passes faster with age. The critical assumption of this paper is that consumers will try to keep the marginal utility of money per unit of subjective time constant, i.e. consumers are maximizers in subjective time and not in actual time.

Consider the standard result of the life cycle model derived in appendix 1, that the growth rate of consumption is constant (given the EIS, RTP is an exponential function, and the interest rate is constant).

$$\frac{\dot{c}(t)}{c(t)} = \varepsilon\{r - \rho\} \tag{a13}$$

Suppose we rewrite this as $\frac{\dot{c}(\theta)}{c(\theta)} = \varepsilon\{r - \rho\}$, i.e. that the consumer was really maximizing with respect to subjective time and not actual time. To solve for the optimal subjective consumption path we rewrite this differential equation:

$$\dot{c}(\theta) - c(\theta)(\varepsilon\{r - \rho\}) = 0 \tag{23}$$

which has as a set of solutions

$$c(\theta(t)) = ae^{(\varepsilon\{r-\rho\})\theta} = ae^{q\theta} \tag{24}$$

where the parameter a is chosen so that $c(\theta)$ satisfies the initial conditions and $q = \varepsilon\{r - \rho\}$ a constant.

Let $\theta(t)$ be defined as in (19)-(22), substituting these values into (24) we obtain a mapping of $f : c(\theta) \to c(t)$ and we get (25) – (28):

$$c(\theta(t)) = a(\frac{t}{t_0})^q \Rightarrow \frac{\dot{c}(t)}{c(t)} = \frac{q}{t} \tag{25}$$

$$c(\theta(t)) = ae^{\frac{q}{\alpha}t^{\alpha}} \Rightarrow \frac{\dot{c}(t)}{c(t)} = qt^{\alpha-1} \; 0 < \alpha \leq 1 \tag{26}$$

$$c(\theta(t)) = ae^{qb(Tt-(1/2)t^2)} \Rightarrow \frac{\dot{c}(t)}{c(t)} = qb(T-t) \; 0 \leq t \leq T \tag{27}$$

$$c(\theta(t)) = ae^{q(1-e^{-t})} \Rightarrow \frac{\dot{c}(t)}{c(t)} = qe^{-t} \tag{28}$$

Equations (25)-(28) reveal that the consumption growth in real time depends upon the consumption growth in subjective time, q. In all cases constant growth in subjective time becomes a time dependent consumption growth in real time. For example in the case where $\theta = \ln(t/t_0)$, consumption growth is now a function of the inverse of time, if consumption was growing at a constant rate q in subjective time, it's growth now decreases in proportion with the inverse of time.

What is interesting is that the growth of consumption in real time has approximately the same functional form as the subjective time function. The four functional forms of the growth of consumption are vastly different, (26) implies growth at some power where as (28) implies that growth is exponentially damped.

(26) – (28) show that the existence of a subjective time function could have significant impacts on consumer behavior if they do indeed maximize and make decisions based upon some sort of subjective time. We might conclude that since subjective time is what consumers observe that it is logical that they do base decisions upon subjective time. However we have to remember that while people observe subjective time, they live in real time and the passage of real time is how many decisions are based. If consumers consume 'per-day' or 'per-month' and not per unit of subjective time than the existence of a subjective time function may not have any implications for consumer behavior.

As previously indicated the existence of subjective time is a refutable matter and so it the existence of a time-dependent RTP. Empirical work in the area of the existence of subjective time, time-preference, and consumer behavior in subjective time should be a very fruitful area of future research.

Conclusions

The existence of subjective time or the sense of the speed at which people view time as passing could lead to interesting implications for consumer behavior. The model of systematic impatience shows that if a speeding of subjective time implies an increasing rate of discounting future utility, than optimal consumption profiles could be 'hump-shaped', or growing (or shrinking) at a variety of rates dependent on the functional form of the discounting parameter which logically would have the same form as the subjective function of time.

If consumers did maximize their lifetime consumption according to subjective time then the life-cycle model of consumption shows that the optimal consumption path in real time is highly dependent upon the functional form the subjective time function.

Because consumer behavior has the possibility of a high rate of dependence on the existence of subjective time, future areas of study that appear to be fruitful are: The existence of subjective time, to what degree do consumers base consumption and economic decisions on subjective time, and what is the relationship between subjective time and the rate of time preference.

Appendix 1

This appendix presents the derivation of consumption growth in the life cycle model found in Butler (2001). It is included so that the reader may have access to the derivation of the fundamental growth equations of the life cycle model and also because the derivation itself is instructive.

Let $\rho(t)$ be a continuous function of time. Let y(t) be the consumers income stream, c(t) their consumption stream, and a(t) be the consumers wealth beginning with a positive wealth a_0 so that $a(0) = a_0$. We assume that the consumer will retire at time t=T with a positive wealth a(T) such that $a(T) \geq 0$. We let the interest rate, r(t) vary across time and also be a continuous function of time. Hence the change in wealth for any period of time can be written: $\dot{a}(t) = a(t) \bullet r(t) + y(t) - c(t)$. The consumer will choose to maximize

$$\Omega(0) = \int_{t=0}^{t=T} \rho(t)U(c(t))dt \tag{a1}$$

subject to

$$\dot{a}(t) = a(t) \bullet r(t) + y(t) - c(t) \tag{a2}$$

$$a(0) = a_0 \tag{a3}$$

$$a(T) \geq 0 \tag{a4}$$

This forms a standard problem in optimal control theory, the Hamiltonian to this optimization problem is given by

$$\Pi(t) = \rho(t)U(c(t)) + \lambda(t)\{a(t)r(t) + y(t) - c(t)\} \tag{a5}$$

with first order conditions

$$\frac{\partial \Pi(t)}{\partial c(t)} = \rho(t)U'(c(t)) - \lambda(t) = 0 \tag{a6}$$

$$-\frac{\partial \Pi(t)}{\partial a(t)} = \dot{\lambda}(t) = -\lambda(t)r(t) \tag{a7}$$

$$\dot{a}(t) = \frac{\partial \Pi(t)}{\partial \lambda(t)} = a(t)r(t) + y(t) - c(t) \tag{a8}$$

Rearranging (a6) we to obtain an equation for the marginal utility of consumption

$$U'(c(t)) = \frac{\lambda(t)}{\rho(t)} \Rightarrow \ln(U'(c(t)) = \ln(\lambda(t)) - \ln(\rho(t)) \qquad \text{(a9)}$$

Take the time derivative of this (using the chain rule) equation yields:

$$\dot{c}(t) \bullet \frac{U''(c(t))}{U'(c(t))} = \frac{\dot{\lambda}(t)}{\lambda(t)} - \frac{\dot{\rho}(t)}{\rho(t)} \qquad \text{(a10)}$$

Using (a7) and solving for $\dot{c}(t)$

$$\dot{c}(t) = \frac{U''(c(t))}{U'(c(t))}\left\{-r(t) - \frac{\dot{\rho}(t)}{\rho(t)}\right\} \qquad \text{(a11)}$$

Using the definition of the EIS, ε, given earlier

$$\frac{\dot{c}(t)}{c(t)} = -\frac{U''(c(t))}{c(t)U'(c(t))}\left\{r(t) + \frac{\dot{\rho}(t)}{\rho(t)}\right\} = \varepsilon(t)\left\{r(t) + \frac{\dot{\rho}(t)}{\rho(t)}\right\} \qquad \text{(a12)}$$

In the case where $\rho(t) = e(-\rho t)$, then $\frac{\dot{\rho}(t)}{\rho(t)} = \rho$ a constant rate of time preference and if the EIS and interest rate are also constant then the model reduces to the standard form of the life cycle model with constant growth of consumption.

$$\frac{\dot{c}(t)}{c(t)} = \varepsilon\{r + \rho\} \qquad \text{(a13)}$$

References

Attanasio, Orazio, James Banks, Costas Meghir, and Guglielmo Weber (1999), Humps and Bumps in Lifetime Consumption. Journal of Business and Economic Statistics 17(1), 22-35.

Attanasio, Orazio and Martin Browing (1995), Consumption over the Life Cycle and over the Business Cycle. American Economic Review 85(5), 1118-1137.

Becker, Gary (1997), The Endogenous Determination of Time Preference. Quarterly Journal of Economics 112(3), 729.

Browning, Martin and Thomas F. Crossley (2001), The Life-cycle Model of Consumption and Saving. Journal of Economic Perspectives 15(3), 3-22.

Browning, Martin, Angus Deaton and Margaret Irish (1985), A Profitable Approach to Labor Supply and Commodity Demands Over the Life Cycle. Econometrica 53, 503-544.

Browning, Martin and Mette Ejrnaes (2000), Consumption and Children. Working paper, University of Copehagen.

Bullard, James and James Feigenbaum (2005), A Leisurely Reading of the Life-cycle Consumption Data. Working paper.

Butler, Monika (2001), Neoclassical Life-cycle Consumption: A Textbook Example. Economic Theory 17, 209-221.

Caliendo, Frank and Kevin X.D. Huang (2007), Rational Procrastination and Macroeconomic Consumption Puzzles. Working paper.

Caliendo, Frank and Kevin X.D. Huang (2006), Overconfidence and Consumption. Working paper.

Carroll, Christopher D. (1994), How Does Future Income Affect Current Consumption? Quarterly Journal of Economics 109(1), 111-147.

Carroll, Christopher D. (1997), Buffer-Stock Saving and the Life-Cycle/Permanent Income Hypothesis. Quarterly Journal of Economics 112(1), 1-55.

Chung, S. H. and Richard Herrnstein (1967), Choice and Delay of Reinforcement. Journal of the Experimental Analysis of Behavior 10,67-64.

Epstein, Larry G. (1983), Stationary Cardinal Utility and Optimal Growth Under Uncertainty. Journal of Economic Theory 31, 133-152.

Feigenbaum, James (2005), Can Mortality Risk Explain the Consumption Hump? Working paper.

Fernandez-Villaverde, Jesus and Dirk Krueger (2001), Consumption and Saving over the Life Cycle: How Important are Consumer Durables? Working paper, University of Pennsylvania.

Gourinchas, Pierre-Olivier and Jonathan A. Parker (2002), Consumption over the Life Cycle. Econometrica 71, 47-89.

Hansen, Gary D. and Selahattin Imrohoroglu (2005), Consumption over the Life Cycle: The Role of Annuities. UCLA working paper.

Heckman, James (1974), Life Cycle Consumption and Labor Supply: An Explanation of the Relationship Between Income and Consumption Over the Life Cycle. American Economic Review 64(1), 188-194.

Hubbard, R.G., J. Skinner and S.P. Zeldes (1994), The Importance of Precautionary Motives in Explaining Individual and Aggregate Saving. Carnegie-Rochester Conference Series on Public Policy 40, 59-125.

Mas-Colell, Andreu, M. D. Whinston, and J. R. Green (2002), Microeconomic Theory, pp 732-787. New York, NY: Oxford University Press.

Modigliani, Franco and R. Brumberg (1954), Utility analysis and the consumption function: an interpretation of cross-section data. In: Kurihara, K.K. (ed.) Post-keynesian economics, pp. 388-436. New Brunswick, NJ: Rutgers Unversity Press.

Nagatani, Keizo (1972), Life Cycle Saving: Theory and Fact. American Economic Review 62(3), 344-353.

Ogaki, Masao and A. Atkeson (1997), Rate of Time Preference, Intertemporal Elasticity of Substitution, and Level of Wealth. The Review of Economics and Statistics 79(4), 564-572.

Romer, David (2006), Advanced Macroeconomics, pp.346-386. Boston, MA: Irwin McGraw-Hill.

Samuelson, Paul (1977), Speeding Up of Time with Age in Recognition of Life as Fleeting. In H. Nagatani and K. Crowley (eds.), The Collected Papers of Paul A. Samuelson, vol. 4, 318-333. The MIT Press, Cambridge, Massachusetts.

Silberberg, Eugene and W. Suen (2001), The Structure of Economics, pp. 368-392. Boston, MA: Irwin McGraw-Hill.

Tobin, James (1967), Life-cycle Savings and Balanced Growth. In W. Fellner et al. (eds.), Ten Economic Studies in the Tradition of Irving Fisher, New York.

Thurow, Lester (1969), The Optimum Lifetime Distribution of Consumption Expenditures. American Economic Review 59, 324-30.

Uzawa, Hirofumi (1968), Time Preference, the Consumption Function, and Optimum Asset Holdings. In James N. Wolfe (ed.), Value, Capital and Growth: Papers in honor of Sir John Hicks. Edinburgh: Edinburgh University Press.

Yaari, Menahem E. (1964), On the Consumer's Lifetime Allocation Process. International Economic Review 5, 304-317.

Yaari, Menahem E. (1965), Uncertain Lifetime, Life Insurance, and the Theory of the Consumer. Review of Economic Studies 32(2), 137-150.

Author's Biography

Michael Bailey was born and raised in Logan, UT and graduated from Logan High School in 2001. He has known that he wanted to study economics since high school and enrolled at USU as an economics major eventually specializing in economic theory. He will graduate spring 2007 summa cum laude (4.0 GPA) with a dual BA in economics and mathematics with University Honors and Honors in Economics and will have completed 181 undergraduate credits and 18 credits of PhD-level economics classes. Michael has been conducting research in economics his entire undergraduate career, researching in areas such as optimal control theory, international trade and GDP convergence, and life-cycle consumption theory and presented an honors paper on tuition and undergraduate enrollment to the provost. The only break he has taken from studying economics was for a two-year mission to France between his freshman and sophomore years. Michael worked for three years as a research and teaching assistant for Dr. L. Dwight Israelsen and as a data analyst for two years at the Institute for Anti-viral Research. He has taught numerous lectures in the economics department including for the following classes: introduction to microeconomics, introduction to macroeconomics, international economics, labor economics, and the MBA economics core. Honors and scholarships include: 2006-2007 College of Business Scholar of the Year, Stanford University Graduate Fellowship, 1st place in Economic analysis and decision making at the national PBL conference in 2006, 1st place in Economics at the PBL state competition numerous times, Seely-Hinckley Scholarship, Rae N. and Orson A. Scholarship, Dean. William Wanlass Scholarship, Science Scholarship, and two 'A' pins. Michael will be entering the PhD program in economics at Stanford university fall 2007 with a 5-year 'full-ride' fellowship. He plans to study Industrial Organization and the economics of strategy and eventually obtain a tenure track position at a research university.