Performance Comparison of Particle Filter in Small Satellite Attitude Estimation

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Small satellite missions with higher complexities tend to demand more sophisticated requirements, which push the limits of classical attitude estimation methods.

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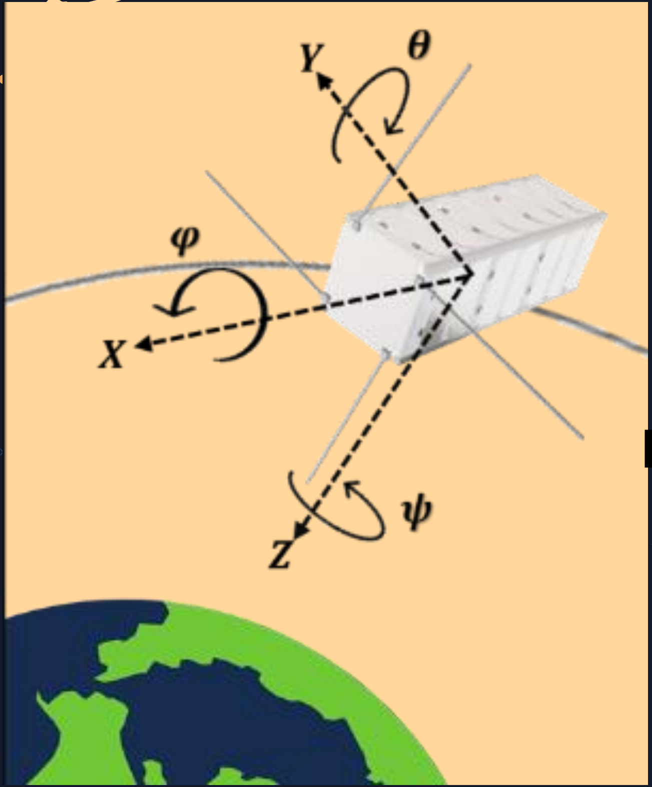
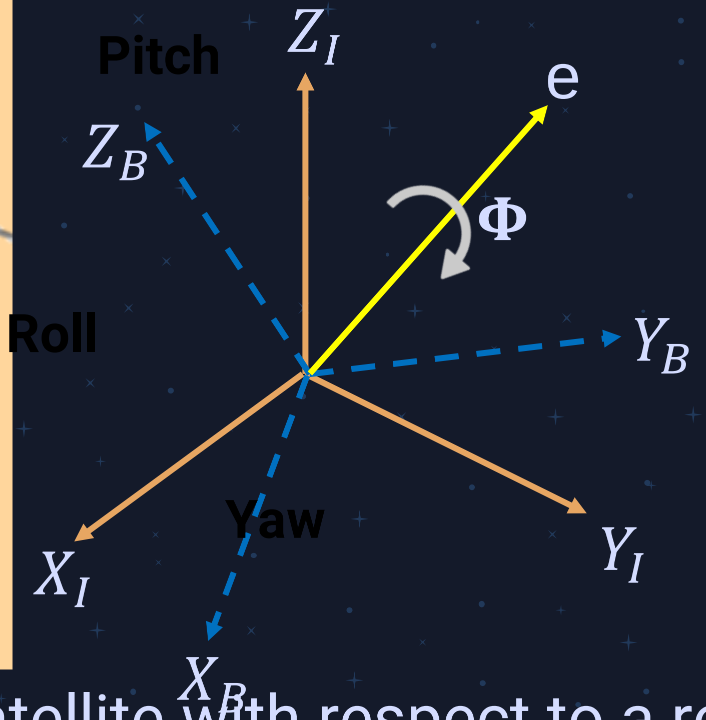
Introduction

Satellite Attitude

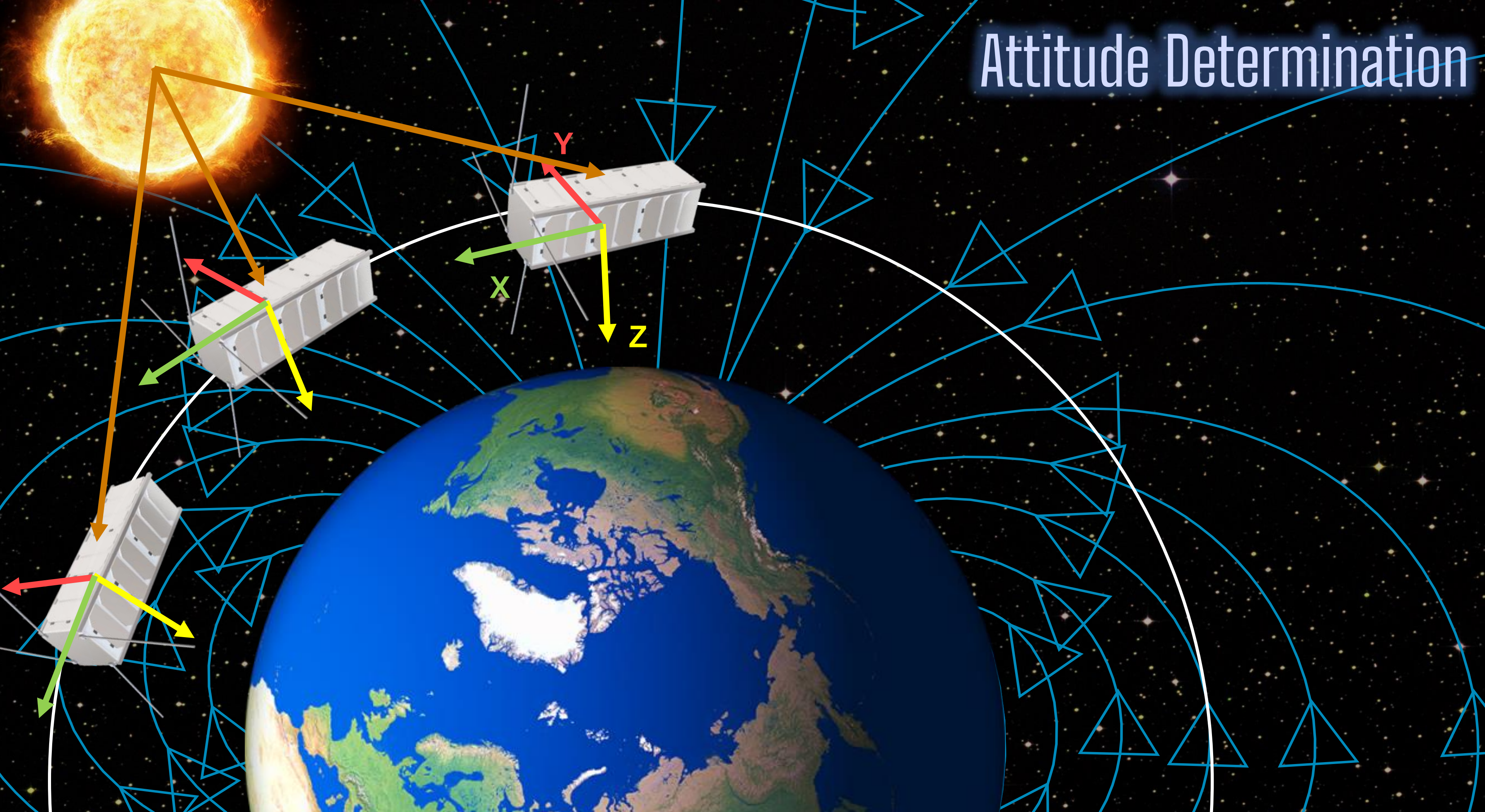




$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} \cos\frac{\Phi}{2} \\ e_x \sin\frac{\Phi}{2} \\ e_y \sin\frac{\Phi}{2} \\ e_z \sin\frac{\Phi}{2} \end{bmatrix}$$

The orientation of a satellite with respect to a reference frame.

Attitude Determination



Source: Facchinetti G. Small Satellites: Economic trends. 2016;130(December):1–102. Available from: http://www.defencesa.com/upload/Facchinetti G. Small Satellites

Attitude Estimation Algorithms

- TRIAD, Q-Method, Least Squares
- Kalman Filter – Extended Kalman Filter (EKF)
- Particle Filter
- H-Infinity Filter

- The Particle Filter has been proposed for satellite attitude estimation and has shown improved performance compared with traditional approaches.

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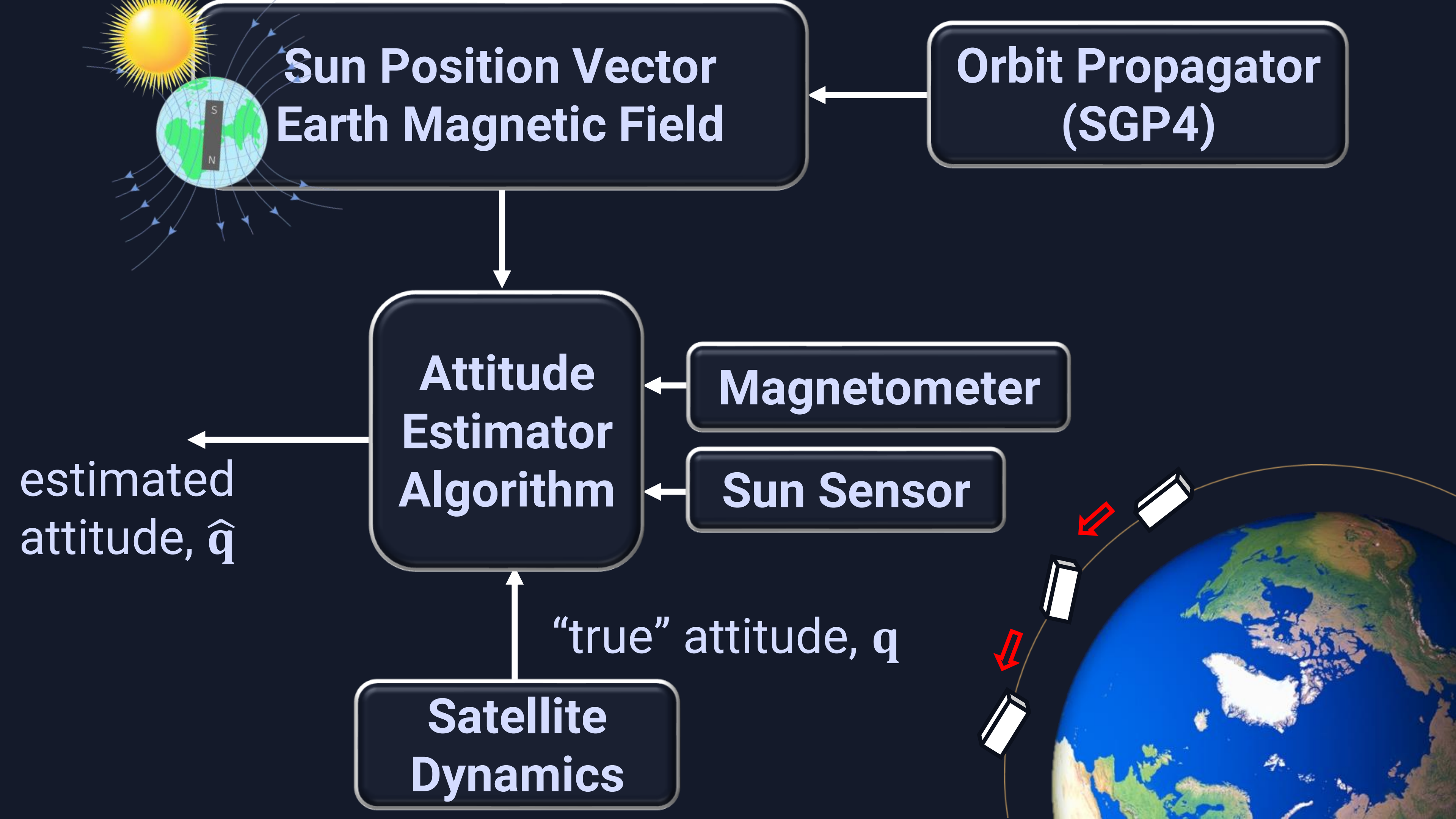
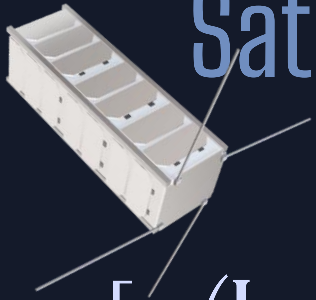
Modeling & Simulation


Sun Position Vector
Earth Magnetic Field
Orbit Propagator
(SGP4)
Attitude
Estimator
Algorithm
Magnetometer
Sun Sensor
estimated
attitude, $\hat{\mathbf{q}}$
"true" attitude, $\mathbf{q}$
Satellite
Dynamics

Satellite Dynamic & Kinematic Equations

Parameter	Value	
Small Satellite Properties	CubeSat Type	3U
	Mass	3.5 kg
	Dimensions	$0.34\,m \times 0.10\,m \times 0.10\,m$
	Inertia matrix	$\begin{bmatrix} 0.0058 & 0 & 0 \\ 0 & 0.0366 & 0 \\ 0 & 0 & 0.0366 \end{bmatrix} kgm^2$

Satellite Dynamic & Kinematic Equations



$$\dot{\omega} = \begin{bmatrix} -\left(\frac{I_z - I_y}{I_x}\right)\omega_y\omega_z + \frac{\tau_x}{I_x} \\ -\left(\frac{I_x - I_z}{I_y}\right)\omega_x\omega_z + \frac{\tau_y}{I_y} \\ -\left(\frac{I_y - I_x}{I_z}\right)\omega_x\omega_y + \frac{\tau_z}{I_z} \end{bmatrix}$$

Attitude Dynamics

$$\dot{q} = \frac{1}{2}\begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

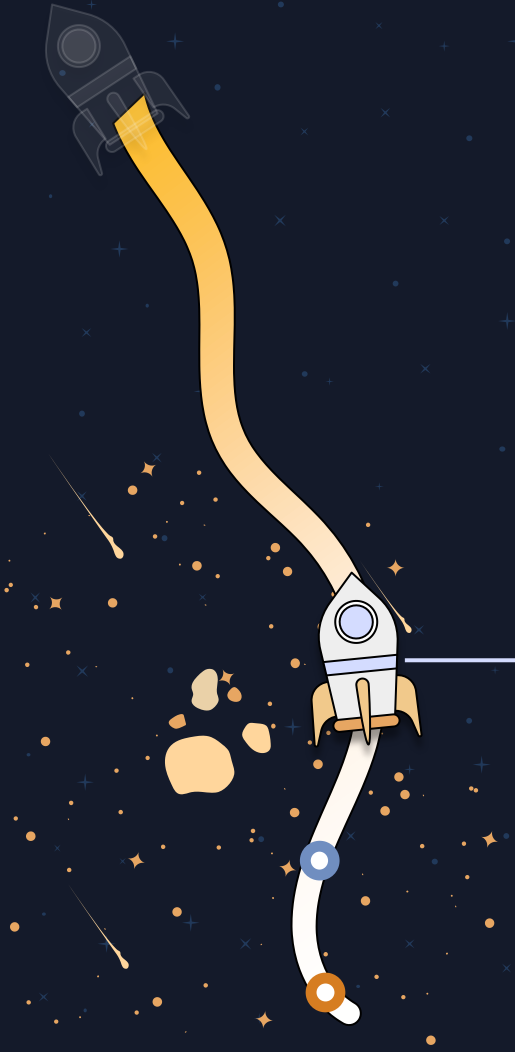
Attitude Kinematics

Sensors – Mathematical Models

Magnetometer $\mathbf{B}_{\mathrm{meas}} = \mathrm{A}(\mathrm{q})\mathbf{B}_{\mathrm{ECI}} + \mathbf{v}_{\mathrm{mag}}$

Sun Sensor $\mathbf{S}_{\mathrm{meas}} = \mathrm{A}(\mathrm{q})\mathbf{S}_{\mathrm{ECI}} + \mathbf{v}_{\mathrm{sun}}$

$$\mathrm{A}(\mathrm{q}) = \left(\mathrm{q}_4^2 - \mathrm{q}_{13}^{\mathrm{T}}\mathrm{q}_{13}\right)\mathrm{I}_{3\times3} - 2\mathrm{q}_4[\mathrm{q}_{13}\,\times] + 2\mathrm{q}_{13}\mathrm{q}_{13}^{\mathrm{T}}$$



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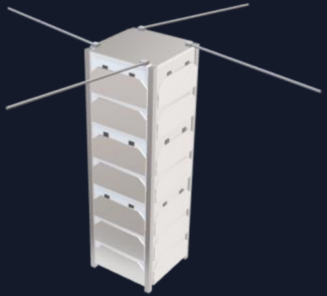
Particle Filtering

The PF is a nonlinear estimation algorithm that approximates nonlinear functions using a set of random particles.

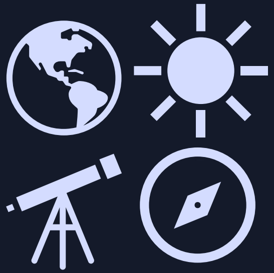
Models in PF Attitude Estimation



Attitude Dynamics Model

System Model

$$x_{k+1} = f_k(x_k, w_k) \leftrightarrow p(x_{k+1}|x_k)$$



Simulated Sensors

Measurement Model

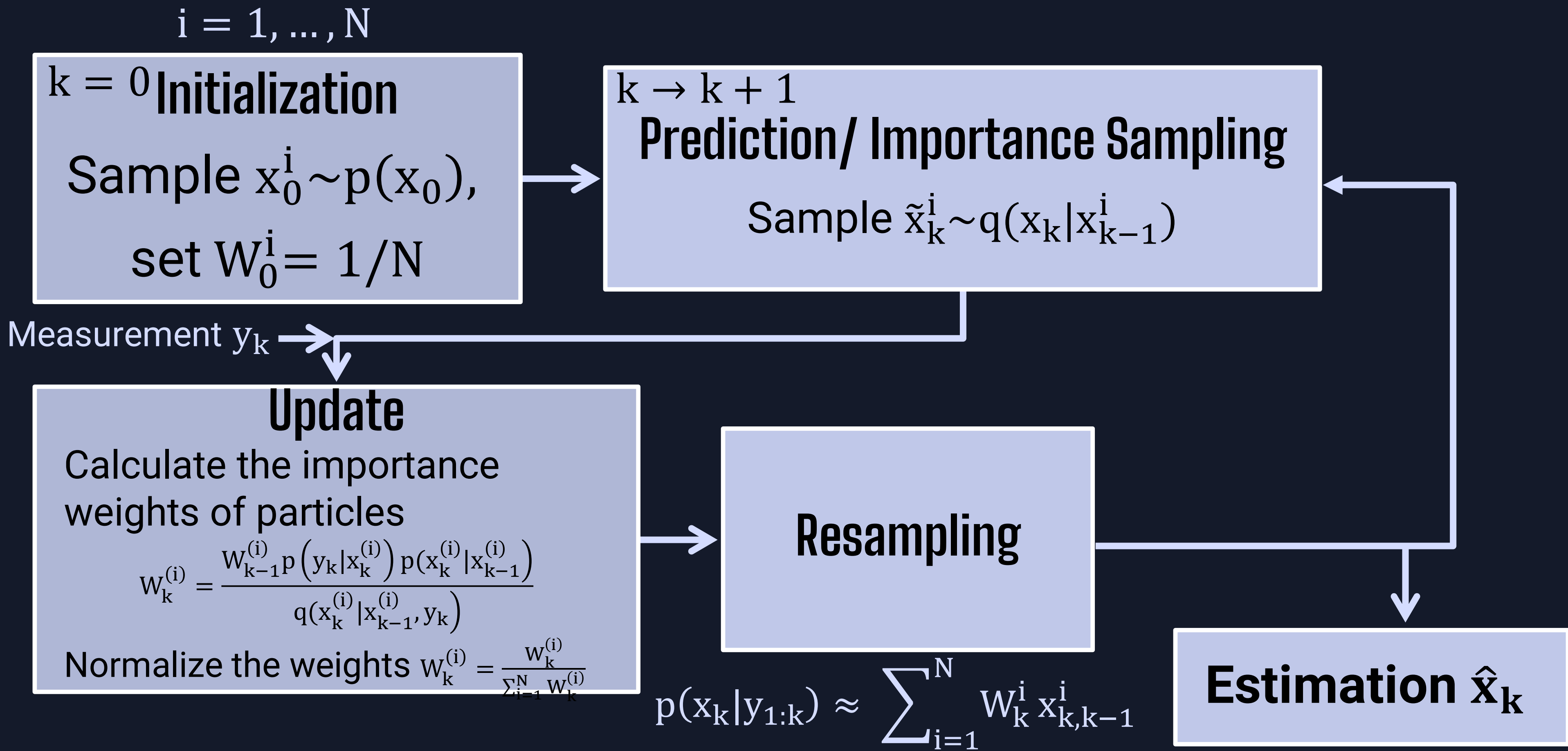
$$y_k = h_k(x_k, v_k) \leftrightarrow p(y_k|x_{k+1})$$

Models in PF Attitude Estimation

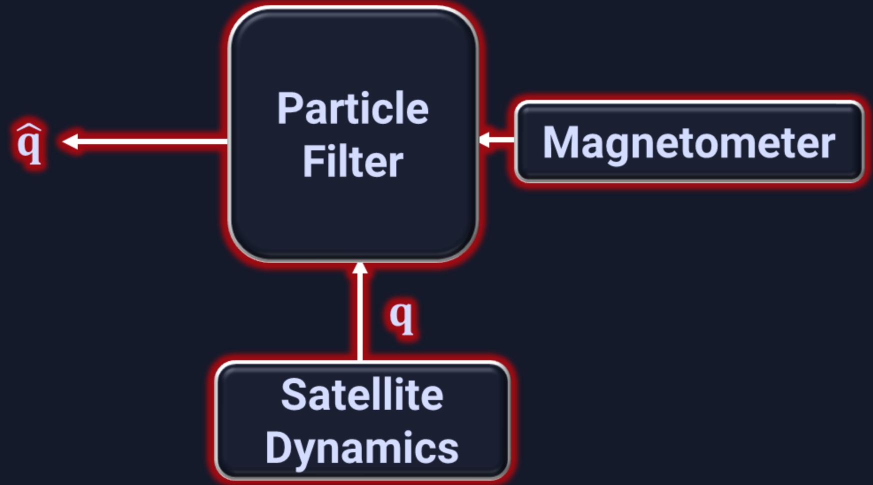
$$\mathrm{x} = [\mathrm{q} \quad \omega]^{\mathrm{T}} = [\mathrm{q}_1 \quad \mathrm{q}_2 \quad \mathrm{q}_3 \quad \mathrm{q}_4 \quad \omega_{\mathrm{x}} \quad \omega_{\mathrm{y}} \quad \omega_{\mathrm{z}}]^{\mathrm{T}}$$

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{q}} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\boldsymbol{\Omega}(\mathbf{q}) \\ \mathbf{I}^{-1}(\boldsymbol{\tau} - \boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega}) \end{bmatrix} \mathbf{x}$$

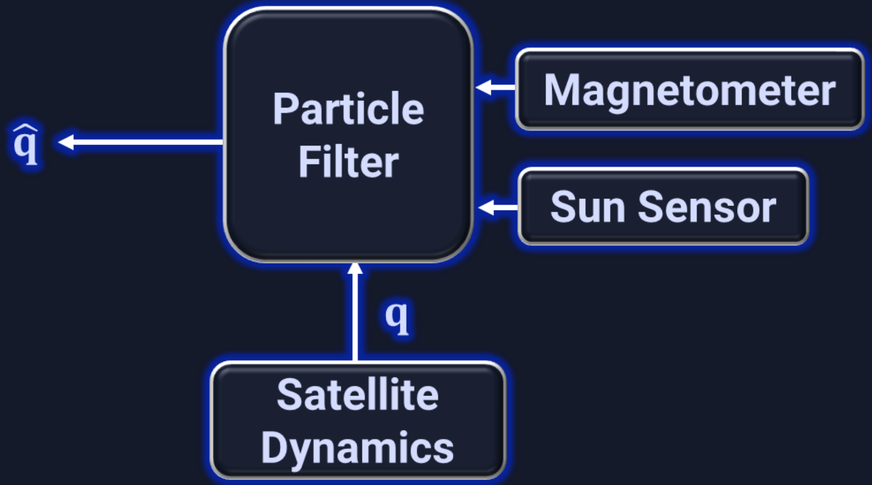
Particle Filter – Schematic

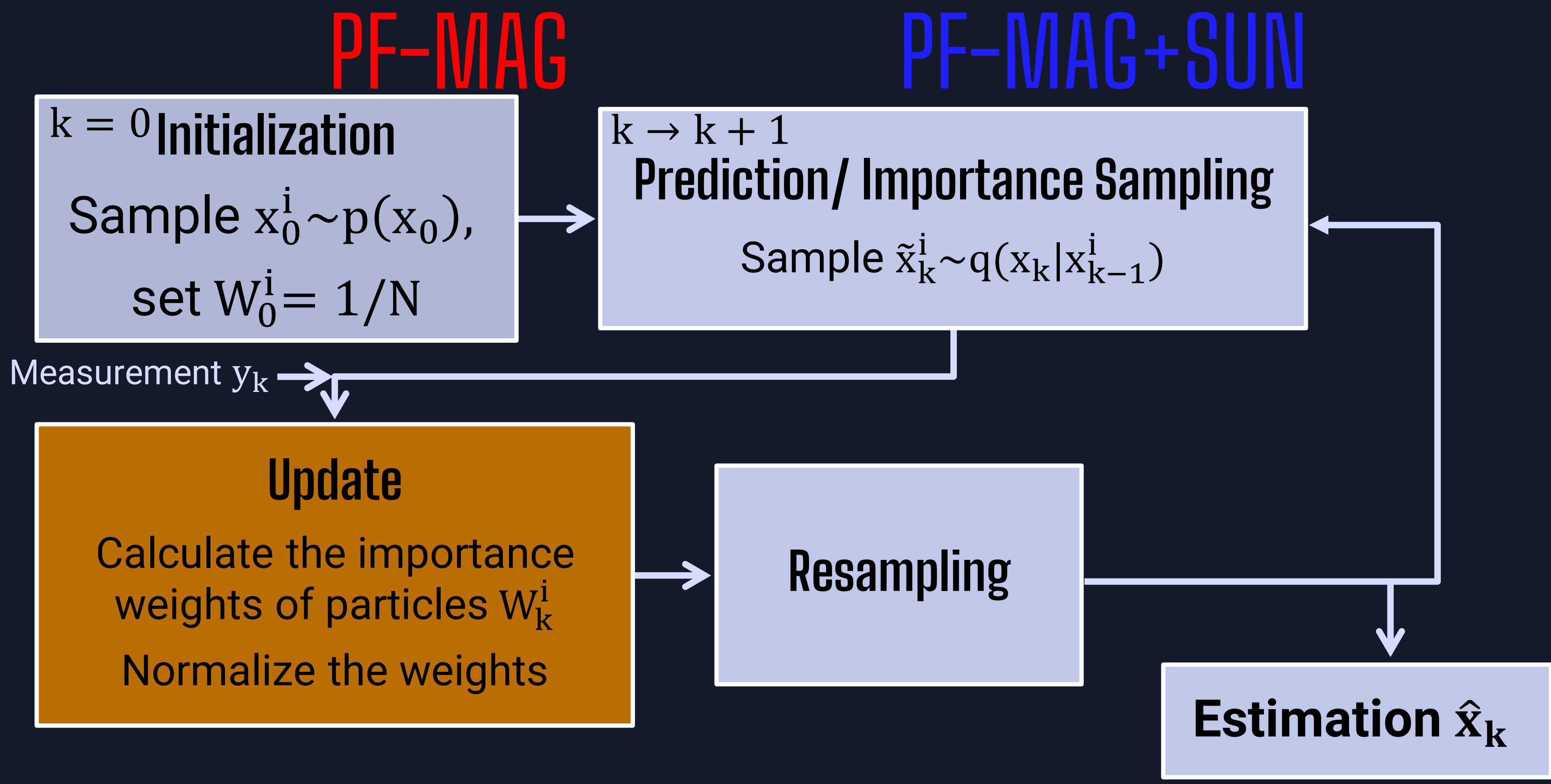


PF–MAG



PF–MAG+SUN




PF-MAG
PF-MAG+SUN
$k = 0$ Initialization
Sample $x_0^i \sim p(x_0)$,
set $W_0^i = 1/N$
$k \to k+1$
Prediction/ Importance Sampling
Sample $\tilde{x}_k^i \sim q(x_k | x_{k-1}^i)$
Measurement y_k
Update
Calculate the importance weights of particles W_k^i
Normalize the weights
Resampling
Estimation $\hat{x}_k$

Extended Kalman Filter (EKF)

EKF

$$\hat{\mathbf{x}}_0 ,\ \mathbf{P}_0$$

Kalman Gain

$$\mathbf{K} = \mathbf{P}_k^{-}\mathbf{H}_k^{T}(\mathbf{H}_k\mathbf{P}_k\mathbf{H}_k^{T} + \mathbf{R}_k)^{-1}$$

Prediction

$$\hat{\mathbf{x}}_k^{-} = \mathbf{f}_{k-1}(\hat{\mathbf{x}}_{k-1}^{+}) + \mathbf{n}_k$$

$$\mathbf{P}_k^{-} = \mathbf{F}_{k-1}\mathbf{P}_{k-1}^{+}\mathbf{F}_{k-1}^{T} + \mathbf{Q}_{k-1}$$

Update

$$\hat{\mathbf{x}}_k^{+} = \hat{\mathbf{x}}_k^{-} + \mathbf{K}(\mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}_k^{-})$$

$$\mathbf{P}_k^{+} = (\mathbf{I} - \mathbf{K}\mathbf{H}_k)\,\mathbf{P}_k^{-}$$

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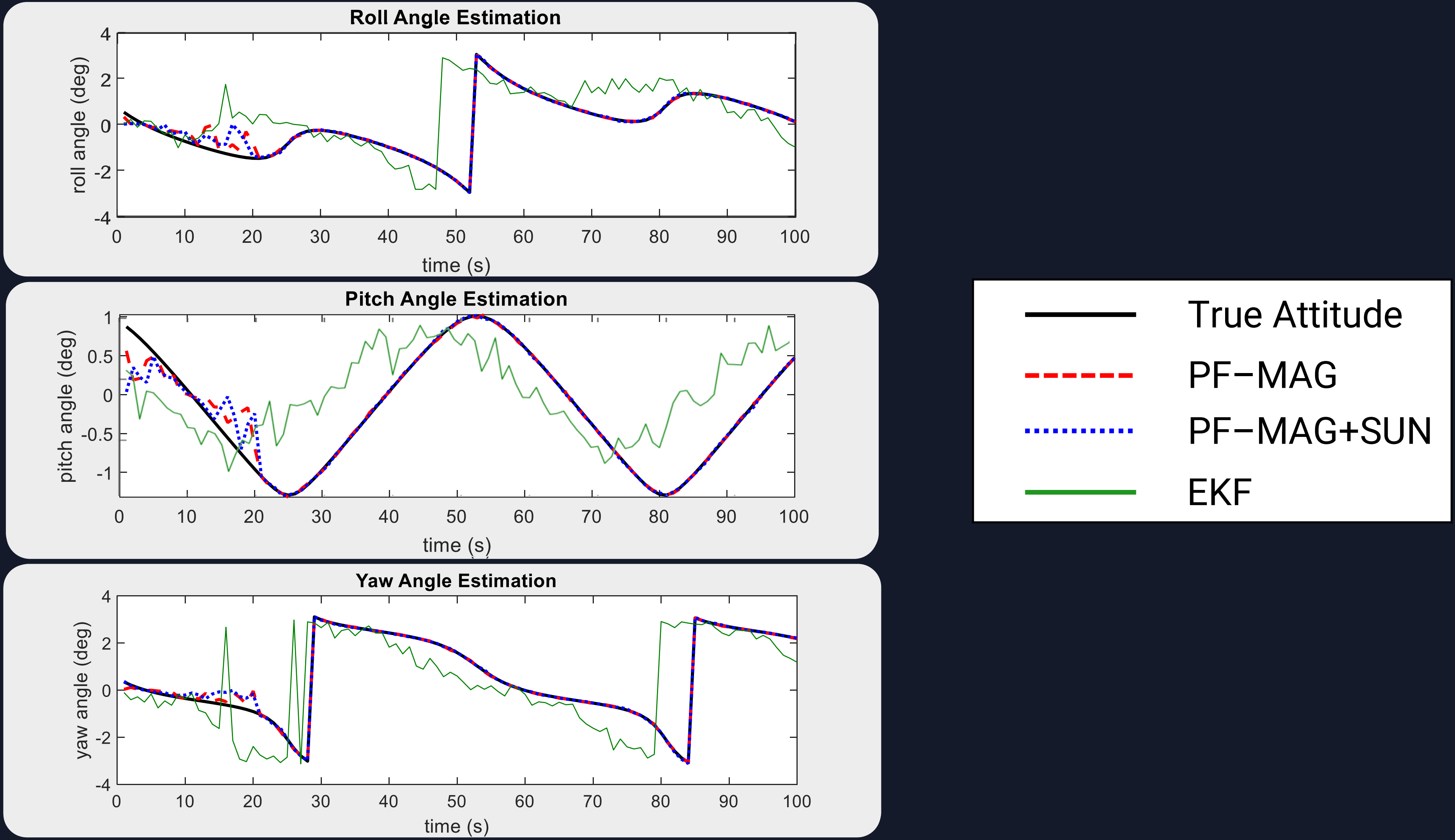
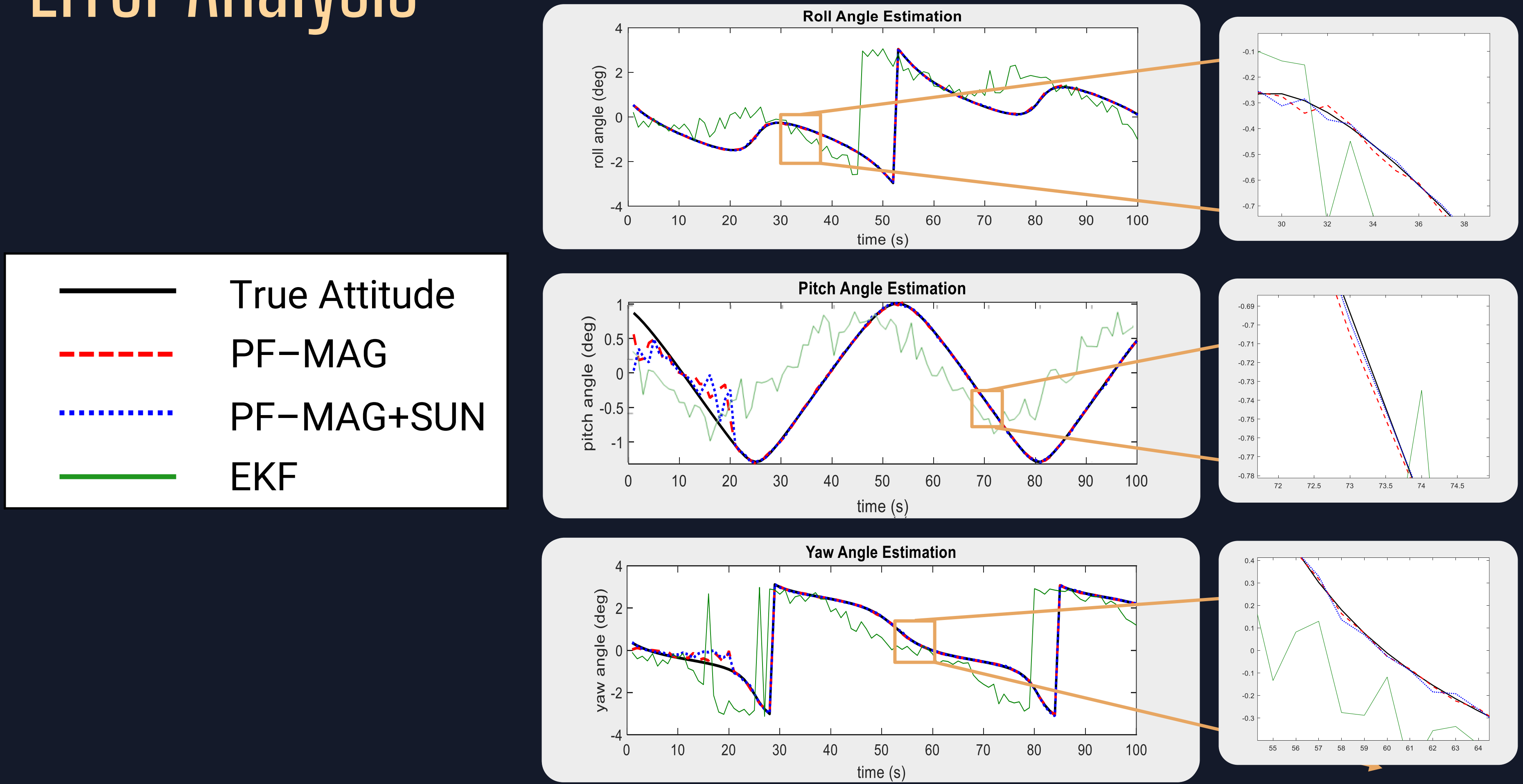
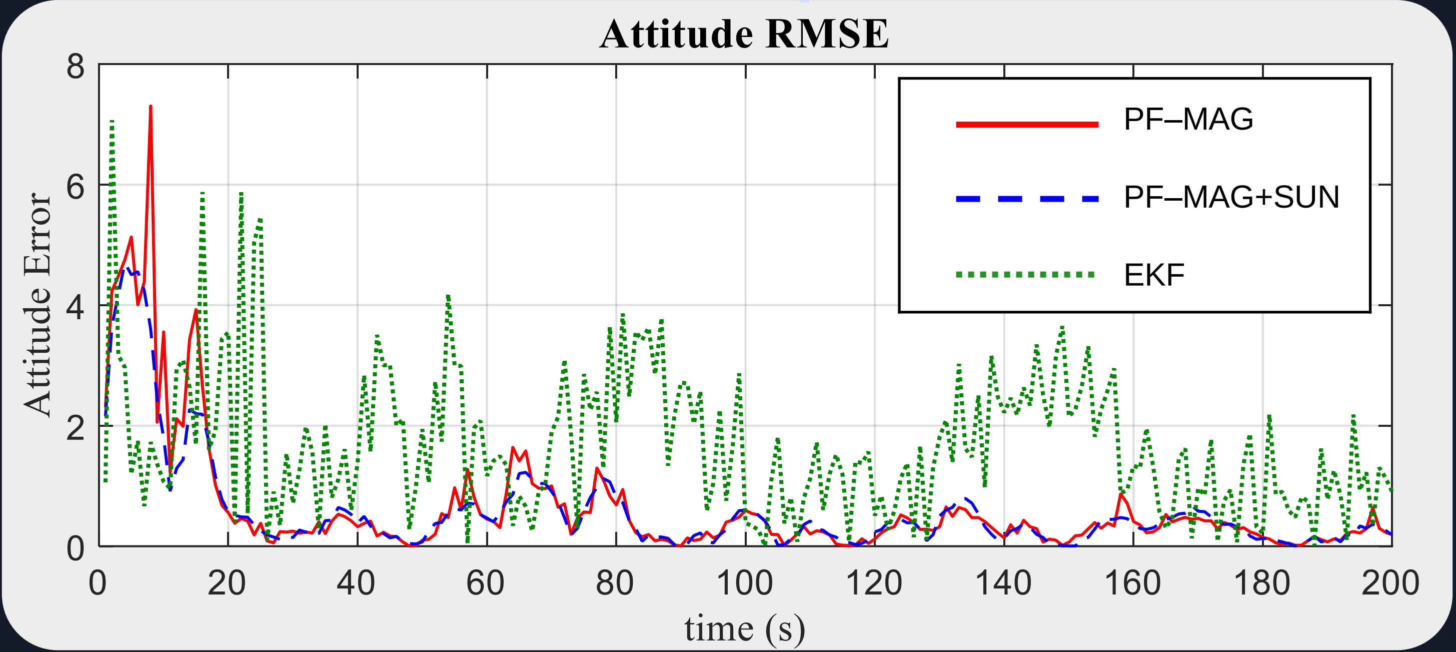
Simulation Results


Roll Angle Estimation
roll angle (deg)
4
2
0
-2
-4
0
10
20
30
40
50
60
70
80
90
100
time (s)
Pitch Angle Estimation
pitch angle (deg)
1
0.5
0
-0.5
-1
0
10
20
30
40
50
60
70
80
90
100
time (s)
Yaw Angle Estimation
yaw angle (deg)
4
2
0
-2
-4
0
10
20
30
40
50
60
70
80
90
100
time (s)
True Attitude
PF−MAG
PF−MAG+SUN
EKF

Error Analysis

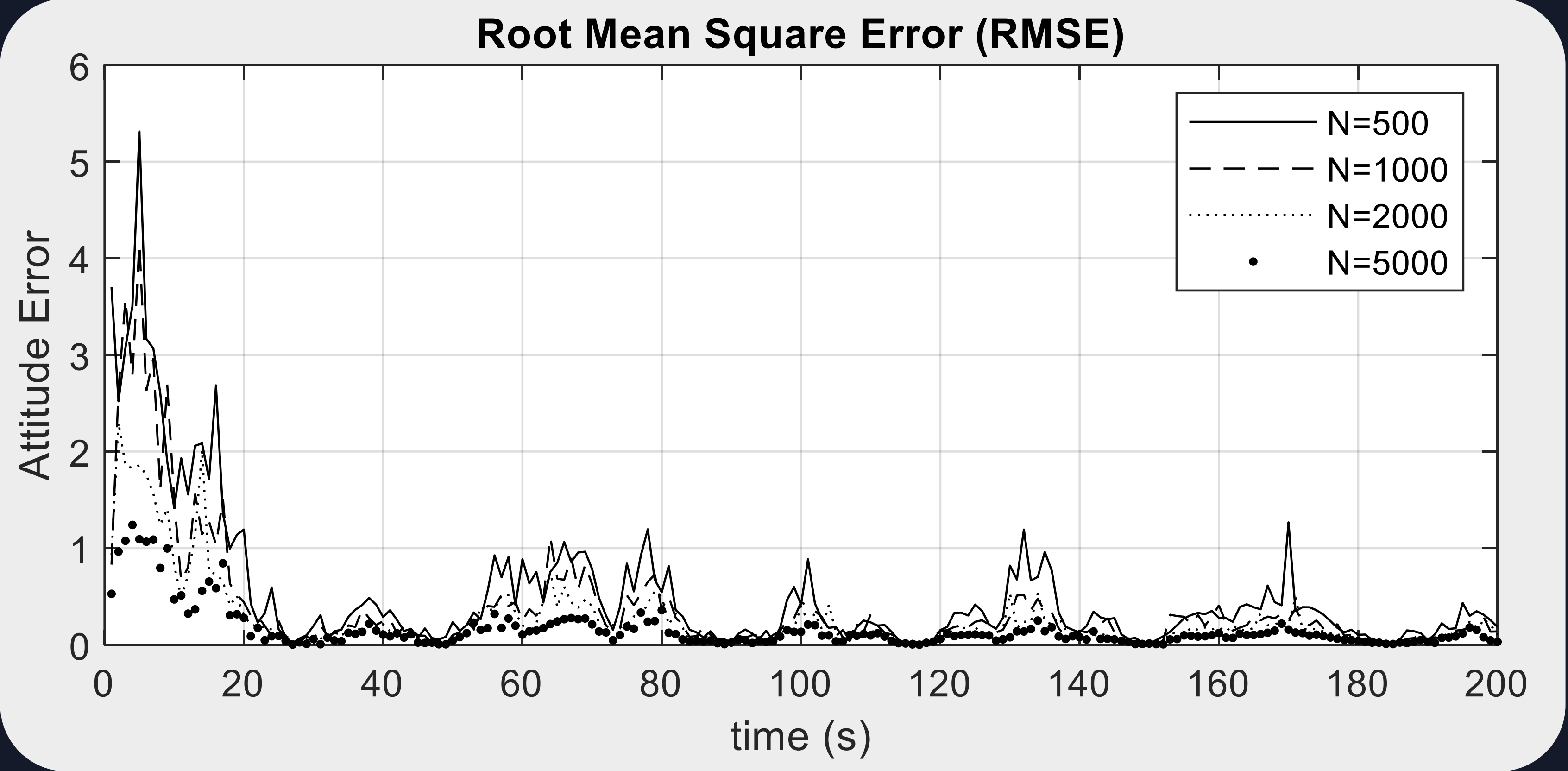


Root Mean Square Error



- High accuracy attitude estimation achieved with PF (±0.01°), compared to EKF (±1°)
- Improvements in PF estimation with 2 sensors

Computation Burden


Root Mean Square Error (RMSE)
Attitude Error
time (s)
0
1
2
3
4
5
6
0
20
40
60
80
100
120
140
160
180
200
N=500
N=1000
N=2000
N=5000

5

Conclusions

- High accuracy attitude estimation achieved with PF (±0.01°), compared to EKF (±1°).
- Two different attitude sensor configurations for PF were simulated.
- The inclusion of a sun sensor improved the PF attitude estimation, but the computational burden was higher.

A comparison for attitude estimation via PF in small satellite estimation where two different sets of attitude measurements are available.

Thank You!

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