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A Nonlinear Model Order Reduction Framework for Dynamic Vapor Compression Cycles via Proper Orthogonal Decomposition

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A Nonlinear Model Order Reduction Framework for Dynamic Vapor Compression Cycles via Proper Orthogonal Decomposition

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ABSTRACT

A computationally efficient and accurate modeling approach is critically important for designing and evaluating controls and fault detection and diagnosis (FDD) algorithms. This paper proposes a reduced order modeling approach for vapor compression cycles (VCC) that involves application of nonlinear model order reduction (MOR) methods to dynamic heat exchanger (HX) models to generate reduced order HX models. A reformulated finite volume HX model was first developed that matches the baseline MOR model structure. Then, a nonlinear MOR framework based on Proper Orthogonal Decomposition (POD) and a Discrete Empirical Interpolation Method (DEIM) was developed for generating nonlinear reduced order HX models. The proposed approach was implemented within a comprehensive VCC model. Reduced order HX models were constructed for a centrifugal chiller system and coupled to quasi-static models of a compressor and expansion valve to complete the reduced order VCC model. The reduced cycle model was implemented within the Modelica-based platform and used to predict load-change transients over a wide range of operating conditions for comparison with measurements. The proposed reduced order modeling approach is computationally efficient and accurately captures cycle dynamics.

1. INTRODUCTION

Since transient models for vapor compression cycles (VCC) are particularly important for designing and evaluating control and fault detection diagnostic (FDD) algorithms, transient modeling of VCC has become an active area of research over past decades. The finite volume (FV) and moving boundary (MB) methods are the two dominant approaches to capture the complex thermo-fluid dynamic behavior of VCC (Rasmussen, 2012). The fundamental difference between them is in the discretization schemes for solving the governing conservation equations. The FV method divides a heat exchanger into a number of fixed control volumes, while the MB method segments a heat exchanger based on thermodynamic phases of the working fluid, and moves control volumes as the length of each phase section changes.

In the literature, the MB method has received significant attention for control applications (He *et al.*, 1998; Rasmussen and Alleyne, 2004) because of its lower dimensionality and faster execution speed. However, the model complexity and inherent discontinuities associated with switching model representations when a phase region disappears or reappears can result in simulation failure and limit capability of the MB for advanced control and FDD purposes (Kim *et al.*, 2020).

In this paper, we present an alternative modeling strategy which applies a well-developed nonlinear model order reduction method from other fields (e.g., turbulent flow, applied mathematics and control). The proposed process starts from a high fidelity FV heat exchanger model. A nonlinear model order reduction approach is applied in order to generate reduced order models (ROMs), and then the ROMs are coupled with quasi-static models of other components to complete a reduced order VCC model. This approach is attractive since a reduced order VCC model could be extracted from the high fidelity FV models in a systematic manner, and the inherent discontinuities of the MB method can be naturally avoided. Very few studies on this topic can be found in the literature. Henrik and Olsson

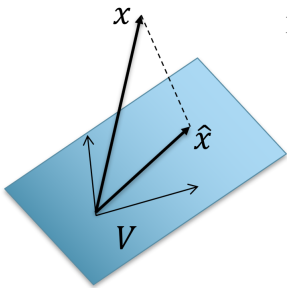

Figure 1: Projection of states vector onto an orthonormal basis.

(2005) discussed feasible applications of linear and nonlinear MOR methods for heat exchanger modeling. Linear reduced order models were derived using the balanced truncation and Krylov subspace methods from a linearized heat exchanger model. In addition, a Proper Orthogonal Decomposition (POD) method was applied to produce nonlinear reduced order models. Dynamic responses of those models were compared under a perturbation of the evaporating pressure. It was reported that different models showed similar accuracy and the POD model had the lowest dimension. No comparison of simulation speed was provided. Recently, Xu *et al.* (2018) developed reduced order heat exchanger models from high-fidelity FV models for an Organic Rankine Cycle using POD. A reduced order evaporator model was implemented for a waste heat recovery system, and its performance was compared with the full order FV model. However, the methodology presented in the study made too strict assumptions for simplifying the problem, i.e. fixed refrigerant flow rate and pressure across a heat exchanger, and therefore the resulting model is not generally applicable for most vapor compression and expansion systems. This paper proposes a nonlinear model order reduction framework for generating reduced order VCC models. The propsoed framework is applied to a VCC and validation results are provided.

2. MODEL ORDER REDUCTION METHODOLOGIES

2.1 Proper Orthogonal Decomposition

The Proper Orthogonal Decomposition (POD) is a popular model reduction method, which produces optimal low order basis functions from ensembles of data, or namely *snapshots* in the state space. A POD reduced order model can inherit the original system dynamics from samples of state trajectories of the baseline model which can be obtained from either a numerical simulation of the full-order model or experimental observations. The nature of the POD is similar to the Fourier modes: functions of interest are projected onto a set of basis functions or modes thus providing a finite set of scalar coefficients that represent the underlying functions. The POD method produces a particular set of modes that form an optimal, orthonormal basis for describing the finite set of samples. More precisely, consider a standard nonlinear system,

$$\dot{x} = f(x, u) \tag{1}$$

where $x \in \mathbb{R}^n$ denotes the dynamic states, $u \in \mathbb{R}^m$ denotes the control inputs. Suppose the system is perturbed with some input profiles, and take p snapshots of state responses. Let X be the ensemble of the snapshots as follows,

$$X = \begin{bmatrix} x^{(1)} & x^{(2)} & \dots & x^{(p)} \end{bmatrix} \ (\in \mathbb{R}^{n \times p}), \tag{2}$$

where each column of the snapshots matrix represents states at a time instance.

The POD seeks to find a special basis $V_k (= \begin{bmatrix} v_1 & v_2 & \dots v_k \end{bmatrix} \in \mathbb{R}^{n \times k})$ which solves the following optimization problem:

$$\begin{aligned} &\min \|X - V_k V_k{}^T X\|_F^2 \\ &\text{s.t.} \quad V_k{}^T V_k = I_k \end{aligned} \tag{3}$$

where $\|\cdot\|_F$ denotes the matrix Frobenius norm. It can be shown that the objective function is equal to

$$\sum_{i=1}^{p} \|x^{(i)} - V_k V_k{}^T x^{(i)}\|^2$$

which represents the sum of the orthogonal projection errors. Note that $V_k V_k^T x$ $(=: \hat{x})$ is the orthogonal projection of x onto the span of column vectors of V_k as depicted in Figure 1. The constraint indicates that V_k represents an orthonormal basis. Kunisch and Volkwein (1999) demonstrated the close connection between POD and the Singular Value Decomposition (SVD) in constructing a reduced basis, and showed that the solution to the minimization problem in (3) is a truncation of the left singular vectors of the snapshots matrix X with the approximation error indicted by the singular values,

$$\sum_{i=1}^{p} \|x^{(i)} - V_k V_k{}^T x^{(i)}\|^2 = \sum_{j=k+1}^{N} \sigma_j^2 \tag{4}$$

where $k < n$ and σ_j is the j^{th} singular value. This facilitates solving the POD problem, since there are efficient and reliable tools to compute SVD of any rectangular matrix, for instance, the MATLAB svd() function. The SVD of a snapshot matrix X is

$$X = V \Sigma U^T \tag{5}$$

where all left singular vectors are stored in $V \in \mathbb{R}^{n \times n}$.

As mentioned before, the POD basis is a truncation of the left singular vectors V by selecting the first k columns according to the first k dominant singular values. The choice of the reduced basis dimension k is certainly of central importance in applying POD, as a trade-off between approximation accuracy and computational savings. Observing the magnitude of singular values can be a natural criterion, since POD approximation error is connected with singular values which are formed in a descent order, as shown in (4). Practically, the ratio of amount of energy captured by the POD basis to the total energy is usually used to determine the dimension (Kunisch and Volkwein, 1999; Kerschen *et al.*, 2005),

$$\mathcal{E}(k) = \frac{\sum_{i=1}^{k} \sigma_i^2}{\sum_{i=1}^{N} \sigma_i^2}. \tag{6}$$

A threshold for determining dimension k can be set for the value of $\mathcal{E}(k)$, where 99.99% is adopted in this work.

Once the reduced order basis V_k is generated, corresponding POD reduced order models are constructed by applying Galerkin projection. The state space is first approximated by a linear combination of the reduced basis,

$$x(t) \approx \sum_{i=1}^{k} \tilde{x}_i(t) v_i = V_k \tilde{x}(t) \tag{7}$$

where the coefficients vector $\tilde{x} \in \mathbb{R}^k$ will be the reduced states. Then projecting the governing equations of (1) onto the reduced basis results in a reduced system,

$$\dot{\tilde{x}} = V_k{}^T f(V_k \tilde{x}, u) \tag{8}$$

2.2 Stabilization of Reduced Order Models

Preserving the stability of the original high-fidelity model is crucially important for model order reduction methods. Unfortunately, projection-based model reduction methods (e.g., POD) often result in an unstable reduced order model although the baseline system is (locally) stable. Therefore, stabilization methods should follow as a *posterior* process after POD reduced order models are obtained. We adopt the methodology proposed by Amsallem and Farhat (2012). The method is motivated by semidefinite programming, and requires primarily the solution of a convex optimization problem.

To describe the approach, let's denote the linearized system description of (8) at an equilibrium point as

$$V_k \dot{\tilde{x}} = A V_k \tilde{x}. \tag{9}$$

For stability analysis and notational simplicity, the term associated with inputs is omitted. Let's consider another k-dimensional subspace represented by W_k $(\in \mathbb{R}^{n \times k})$ where each column vector of W_k indicates a basis vector of the new subspace. The projection of the dynamics of (9) onto it forms new dynamics described as follows.

$$W_k^T V_k \dot{\tilde{x}} = W_k^T A V_k \tilde{x} \tag{10}$$

According to the Lyapunov stability theorem for a linear time-invariant (LTI) descriptor system, the asymptotic stability criteria can be written as

$$V_k^T W_k P W_k^T A V_k + V_k^T A^T W_k P W_k^T V_k \prec 0 \tag{11}$$

where P is symmetric positive definite. The main idea of the stabilization method is to find W_k which minimizes the deviation from the POD subspace, i.e. V_k, while aiming at preserving the asymptotic stability of the newly projected system (10).

Given a matrix $Y_{k+p} \in \mathbb{R}^{n\times(k+p)}$, W_k can be parameterized as

$$W_k = Y_{k+p} Z, \tag{12}$$

where $Z \in \mathbb{R}^{(k+p)\times k}$ is the coordinates of W_k with respect to Y_{k+p}. Theoretically Y_{k+p} could be any matrix in $\mathbb{R}^{n\times(k+p)}$ but it is natural to choose the first $k+p$ $(\leq n)$ column vectors of V in (5), since it is optimal to represent the empirical data.

Then, the stability constraint (11) can be written as

$$V_k^T Y_{k+p} Z P Z^T Y_{k+p}^T A V_k + V_k^T A^T Y_{k+p} Z P Z^T Y_{k+p}^T V_k \prec 0. \tag{13}$$

Note that constraint (13) is a Quadratic Matrix Inequality in the variable Z which can not be solved by convex solvers (Boyd *et al.*, 1994). In order to apply techniques from convex optimization, the change of variable

$$\tilde{P} = ZPZ^T \tag{14}$$

is introduced, which leads to a Linear Matrix Inequality. With this parameterization, the objective function can be expressed as

$$f(Z) = \| Y_{k+p} Z - V_k \| \tag{15}$$

where $\|\cdot\|$ denotes any matrix norm. Detailed formulation of the optimization algorithm can be found in Amsallem and Farhat (2012).

It is important to mention that it is possible that no feasible solution exists for all $p \in \{0, \cdots, N-k\}$ to this problem. That is, although the method in general increases the chance of obtaining a stable reduced order model substantially, it might fail for doing that depending on the dynamics of the original system A and the POD basis V_k. In that case, the only option is to attempt modifying V_k through trials and errors.

2.3 Discrete Empirical Interpolation Method

The standard POD model reduction technique reduces the dimension of internal states, and is computationally efficient for LTI systems (Kalashnikova *et al.*, 2014). However, when dealing with a system of general nonlinear ordinary differential equations (ODEs), the computational complexity of evaluating the full order nonlinear equations remains even though the number of states is reduced. We adopt the solution proposed by (Chaturantabut and Sorensen, 2010), namely the discrete empirical interpolation method (DEIM), which has been successfully applied to improve computational efficiency of projection-based nonlinear reduced order models. It constructs specially selected interpolation indices to minimize a certain upper bound of the approximation error. It is applicable to ODEs arising from finite difference or finite volume discretization of time dependent partial differential equations (PDEs). The DEIM approach can be viewed as a combination of projection and interpolation. The nonlinear functions, i.e. f in (1), are approximated by projecting them onto a reduced basis of dimension $m \ll n$. Recall that the POD basis is generated from snapshots of state trajectories. The same procedure is adopted to extract a subspace from snapshots of nonlinear dynamics:

$$F = \begin{bmatrix} f^{(1)} & f^{(2)} & \cdots & f^{(q)} \end{bmatrix} \in \mathbb{R}^{n\times q}. \tag{16}$$

It should be noted that the snapshots of nonlinear functions can be obtained from the POD procedure, and hence no additional computational cost is added to the original POD procedure although more computer memory is required to store values of nonlinear functions during simulation. Nonlinear functions are approximated by projection onto a reduced basis

$$f(t) \approx Tc(t) \tag{17}$$

where $T \in \mathbb{R}^{n\times m}$ is obtained by applying POD to the nonlinear function snapshots matrix F in (16), and $c \in \mathbb{R}^m$ represents the vector of corresponding time coefficients. To calculate the coefficients from the overdetermined system in (17), m distinguished rows are selected to form a well-posed system. Consider an interpolation scheme:

$$P^T f(t) \approx P^T T c(t) \tag{18}$$

where P is a matrix whose i^{th} column e_{ζ_i} is identified by an interpolation index ζ_i, which means

$$e_{\zeta_i} = \begin{bmatrix} 0 & \dots & 0 & 1 & 0 \dots & 0 \end{bmatrix}^T \tag{19}$$

contains the ζ_i^{th} columns of the identity matrix $I \in \mathbb{R}^{n\times n}$. Suppose $P^T T$ is nonsingular, then the coefficients can be uniquely determined,

$$c(t) = (P^T T)^{-1} P^T f(t). \tag{20}$$

The final interpolation approximation of the nonlinear functions is expressed as

$$\tilde{f}(t) = T(P^T T)^{-1} P^T f(t). \tag{21}$$

Note that P and T are constant matrices, so the matrix multiplication can be pre-computed before the online simulation. $P^T f(t)$ can be viewed as the evaluation of a partial set of functions from the original high order system at the specified interpolation indices, which will significantly reduce the computational load. To complete the DEIM, Chaturantabut and Sorensen (2010) proposed an algorithm for determining interpolation indices (matrix P) inductively from the basis T and derived an approximated error bound for the DEIM algorithm:

$$\left\| f - \tilde{f} \right\|_2 \approx \left\| (P^T T)^{-1} \right\|_2 \sigma_{m+1} \tag{22}$$

where σ_{m+1} is the $(m+1)^{th}$ leading singular value of the nonlinear snapshots matrix in (16). In practice, it can be used in determining the number of interpolation points. As more functions are evaluated, the approximation error is smaller since singular values are arranged in a descending order by SVD. When there is a significant gap between the magnitude of two adjacent singular values, the number of indices m could be selected at that point.

3. REDUCED ORDER MODELING FOR VCC

3.1 Reformulation of Finite Volume Model

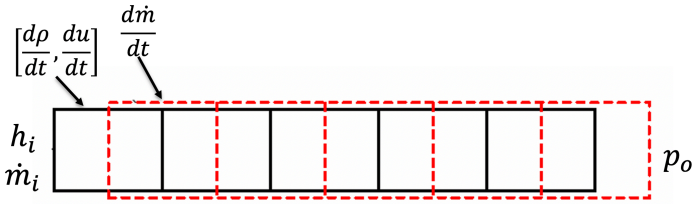

Figure 2: Staggered grid for heat exchanger discretization.

To enable the use of nonlinear model order reduction techniques, a typical FV model (Bendapudi *et al.*, 2008; Rasmussen, 2012), which applies refrigerant pressure and enthalpies as dynamic states, needs to be converted to the standard ODE form as shown in (1). This is because the FV formulation consisting of refrigerant pressure and enthalpies as states as well as an elimination of interface mass flow rates, has a descriptor form $E(x)\dot{x} = f(x,u)$ due to the partial derivatives of refrigerant density with respect to pressure and enthalpy in the mass balance and energy balance. This model structure (denoted as the standard FV model in this paper) can not be used as for the baseline nonlinear model reduction. Therefore a reformulated FV model selecting refrigerant density, internal energy and interface mass flow rate as dynamic states was developed. A momentum balance is integrated into the governing equations to evaluate dynamics of refrigerant mass flow rates. A counter-flow arrangement is assumed for heat exchanger model development. Incompressible liquid water is employed as the secondary fluid in this work, leading to a system of governing equations consisting of the refrigerant mass, momentum and energy balances as well as the tube wall and water energy

balances. The heat exchanger is divided into N equal control volumes. Integrating the governing equations over the length of each control volume yields a system of ODEs.

$$\frac{d\rho_j}{dt} = \frac{1}{V_j}(\dot{m}_{r,k} - \dot{m}_{r,k+1}) \tag{23}$$

$$\frac{d\dot{m}_{r,k}}{dt} = \frac{1}{L}\left(\dot{m}_{r,j-1}v_{j-1} - \dot{m}_{r,j}v_j + A_c(p_{j-1} - p_j) - F_f\right) \tag{24}$$

$$\frac{du_j}{dt} = \frac{1}{V_j\rho_j}\left(\dot{m}_{r,k}h_{j-1} - \dot{m}_{r,k+1}h_j - \dot{Q}_{r,j} + u_j(\dot{m}_{r,k+1} - \dot{m}_{r,k})\right) \tag{25}$$

$$\frac{dT_{t,j}}{dt} = \frac{\dot{Q}_{r,j} - \dot{Q}_{w,j}}{M_{t,j}c_{pt}} \tag{26}$$

$$\frac{dT_{w,j}}{dt} = \frac{\dot{m}_w c_{pw}(T_{w,j+1} - T_{w,j}) + \dot{Q}_{w,j}}{M_{w,j}c_{pw}} \tag{27}$$

It should be noted that a staggered grid scheme was utilized to derive the above equations which has been commonly used in dynamic modeling of thermo-fluid systems to decouple the momentum balance from the mass and energy balances (Elmqvist *et al.*, 2003; Laughman and Qiao, 2019). As shown in Figure 2, equations of refrigerant mass and energy balances (indices j) are solved in the volume cells (solid black line), and the momentum balances (indices k) are solved in the flow cells (dashed red line), which are staggered by half of one volume cell.

For the reformulated FV heat exchanger model, the dynamic states are

$$x = \left[\rho_1, \ldots, \rho_N, \dot{m}_1, \ldots, \dot{m}_N, u_1, \ldots, u_N, T_{t,1} \ldots, T_{t,N}, T_{w,1}, \ldots, T_{w,N}\right]^T (\in \mathbb{R}^{5\times N}), \tag{28}$$

and inputs or boundary conditions to the heat exchanger model consist of refrigerant inlet mass flow rate and enthalpy $(\dot{m}_i, h_i)$, outlet pressure (p_o), and the inlet mass flow rate and temperature of the secondary fluid.

Refrigerant pressure and enthalpy are evaluated as functions of density and internal energy of each control volume $p_j(\rho_j, u_j), h_j(\rho_j, u_j)$. For those evaluations, tabular or regression approaches may be utilized based on a pressure and enthalpy database obtained from CoolProp (Bell *et al.*, 2014). In this paper, an approach utilizing Neural Networks was applied.

3.2 POD-DEIM Heat Exchanger Model

Internal state of the reformulated heat exchanger model consists of refrigerant density, mass flow rate, internal energy, tube temperature and secondary fluid temperature for each control volume as shown in (28). To generate reduced order heat exchanger models, snapshots of the state trajectories as well as nonlinear functions are taken from numerical simulations of the full-order model under a perturbation of inputs. Then reduced order heat exchanger models are constructed using the POD and stabilization methods introduced in Section 2. Instead of directly applying the POD method to the entire state trajectory matrix, it was split into four matrices according to the thermodynamic and fluid properties of $\rho, \dot{m}, u$ and T, and then the POD was applied to each sub-matrix. This results in four sets of reduced order basis corresponding to the different physical properties as follows.

$$\rho \approx V_\rho\tilde{\rho} \quad \dot{m} \approx V_m\tilde{m} \quad u \approx V_u\tilde{u} \quad T \approx V_T\tilde{T} \tag{29}$$

where $T = \left[T_t^T \quad T_w^T\right]^T$, and the resulting reduced states are

$$\tilde{x} = \left[\tilde{\rho}^T \quad \tilde{m}^T \quad \tilde{u}^T \quad \tilde{T}^T\right]^T. \tag{30}$$

For a linearized model at an equilibrium point, internal states are projected onto these bases. Then a stabilizaing basis W_T is searched in the range of POD basis V_T to stabilize the linearized reduced model.

Finally, the DEIM approximation is applied to the reformulated heat exchanger model to reduce the computational cost that depends on the full order nonlinear functions in the reduced order model. Similar to splitting the state trajectories snapshots, the nonlinear function snapshots are grouped based on reduced states shown in (30). Then interpolation indices are constructed for each group of equations. Note that refrigerant mass balances are linear in refrigerant mass

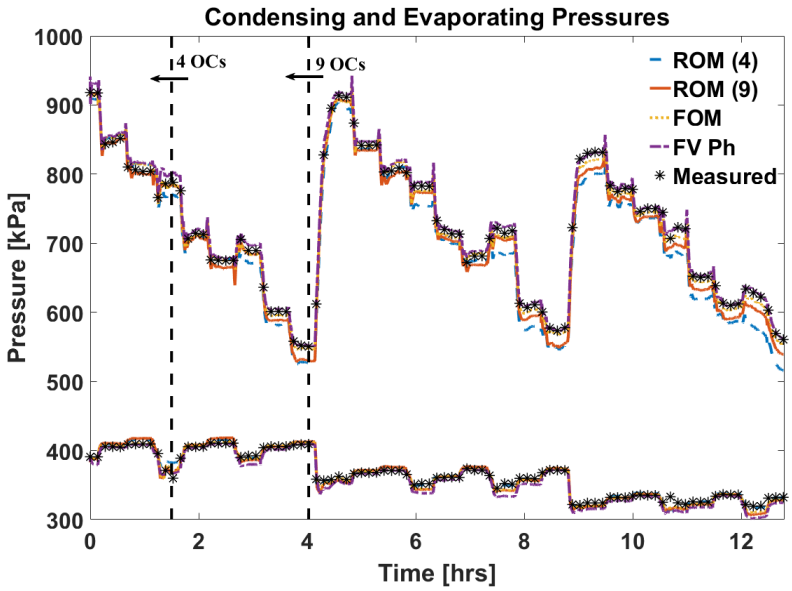

Figure 3: Validations of refrigerant pressures

flow rates, thus there is no need to apply the DEIM approximation, and the number of reduced mass balances to be evaluated completely depends on the dimension of refrigerant density basis V_ρ. As shown in (22), the DEIM approximation error bound is indicted by singular values of the snapshots matrix. Therefore, the ratio of energy captured by the POD basis can also be adopted here to determine the number of interpolation points.

4. VALIDATIONS

The proposed methodology was applied to a R134a centrifugal chiller system. The system description, a system model based on the standard FV heat exchanger formulation and validation with experimental data are presented in Bendapudi *et al.* (2008). Reduced order condenser and evaporator models were generated and then coupled with other component models to form a reduced order cycle model. The resulting reduced order model was compared with 1) the standard FV model, 2) full-order reformulated model, and 3) experimental data over a wide range of operating conditions.

The reformulated full-order HX models (15 control volumes) were simulated for collecting snapshots data of state and function trajectories for the POD and DEIM processes. The simulations were carried out in the Dymola environment with the Radau IIa solver and default relative error tolerance of 10^{-4}. Comparisons between two reduced order models obtained from different snapshots were also investigated. Two data sets were extracted from the snapshots data. One, namely **ROM(4)**, is associated with the first four operating conditions out of the 26 operating conditions, while the other, namely **ROM(9)**, corresponds to the first nine operating conditions. The final POD-DEIM reduced order condenser and evaporator models are summarized in Table 1. It can be seen that for the ROM(4) more than half of the states are reduced and roughly one third of the nonlinear ODEs are eliminated by the DEIM scheme. On the other hand, the ROM(9) leads to less reduction in states and ODEs. This is because the ROM(9) covers a wider range of operating conditions which would require additional dimensions to explain them.

Table 1: Dimension of the reduced order condenser and evaporator models

HX	Cond States	Cond ODEs	Evap States	Evap ODEs
Full order	75	75	75	75
ROM(4)	34	53	32	53
ROM(9)	43	61	38	66

The reduced order cycle models for ROM(4) and ROM(9) respectively, were simulated to predict cycle transient re-

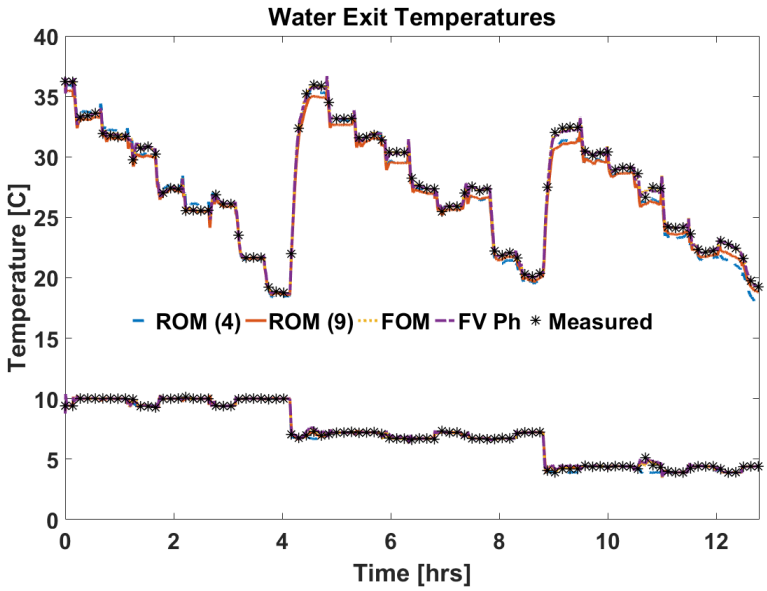

Figure 4: Validations of water exit temperatures

sponses over the 26 operating conditions. The two baseline models for comparisons are 1) the standard FV model utilizing refrigerant pressure and enthalpy as states, namely FV Ph, and 2) the full-order reformulated FV model, namely FOM. Remember that ROM(4) and ROM(9) were generated from a part of the 26 operating conditions. Therefore inputs and trajectories for the rest of the conditions were unexplored in generating the reduced order models.

Fig. 3 to 5 show result comparisons of refrigerant condensing and evaporating pressures, water exit temperatures, and compressor motor power. The results show that the FV-Ph and FOM are nearly identical and that they agree well with the measurements. Discrepancies between the ROMs and measurements are typically observed for the operating conditions that were not included in the generation of the ROMs, that is after 4 hours. This makes sense because the limited input perturbations are likely not sufficient to simulate dynamics that cover the entire *controllable subspace*. Nonetheless, the ROMs are still capable of capturing the essential features of the chiller system. It can be clearly seen that the ROM(9) yields a higher accuracy than ROM(4) in predicting the condensing pressure and motor power. It was observed that predictions of the motor power using the reduced order models had significant discrepancies compared to measurements. This is because the compressor model, specifically the refrigerant flow rate map, is sensitive to the refrigerant pressures. Errors in the compressor controller model amplified prediction errors of the refrigerant mass flow rate.

Simulation speed is measured by the real time factor (RTF) Pangborn *et al.* (2015):

$$\text{RTF} = \frac{\textit{length of CPU time taken to run simulation}}{\textit{length of time that is simulated}}. \tag{31}$$

For real-time simulation applications, a model having RTF less than 1 is generally required. Simulation speeds of the ROM(4), ROM(9), FOM, and FV-Ph are 0.005, 0.0064, 0.0137, 0.03, respectively. All these models run faster than the real time. Compared with the FOM, the ROM(4) and ROM(9) require less than half the computation times. ROM(9) requires about 28% more simulation time than ROM(4). Recall that the total number of dynamic states and ODEs to be solved for ROM(9) is greater than ROM(4), which play an important role in determining the model execution speed. This confirms the trade-off between the computational efficiency and accuracy for generating reduced order models.

5. CONCLUSIONS

This paper presented a nonlinear model order reduction framework for vapor compression systems. A series of methodologies were described to generate reduced order heat exchanger models. The feasibility of applying the nonlinear

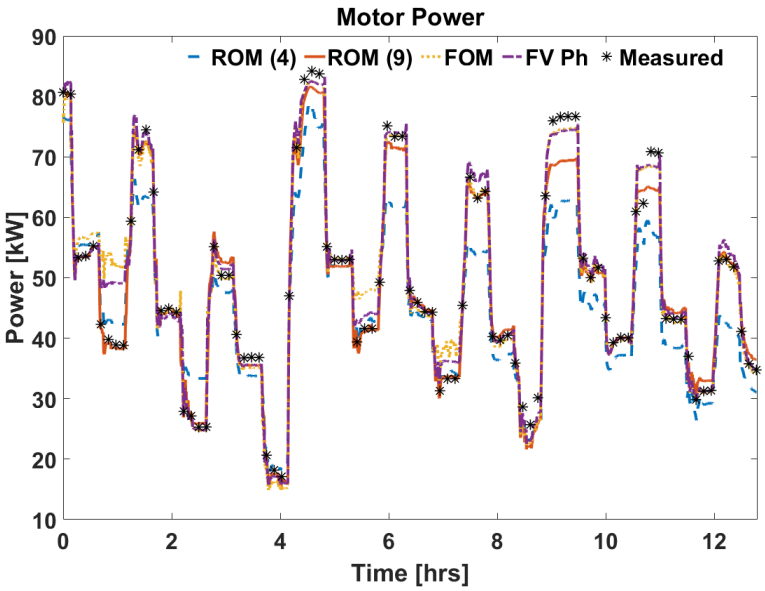

Figure 5: Validations of motor power

model order reduction method to refrigeration cycle systems for a centrifugal chiller was presented. Using the POD-DEIM model reduction scheme, dynamic states as well as differential equations to be solved were reduced, leading to an 80% computation time reduction compared to a standard finite volume modeling approach.

NOMENCLATURE

$\dot{m}$	Mass flow rate	[kg/s]
$\dot{Q}$	Heat transfer rate	[kW]
A	Area	[m^2]
c_p	Specific heat	[kJ/(kg · C)]
h	Specific enthalpy	[kJ/kg]
M	Mass	[kg]
p	Pressure	[kPa]
T	Temperature	[C]
t	Time	[s]
u	Specific internal energy	[kJ/kg]
V	Volume	[m^3]
ρ	Density	[kg/m^3]

Subscript

i	inlet
j	index of volume cell
k	index of flow cell
o	outlet
r	refrigerant
t	tube
w	water

REFERENCES

Amsallem, D., and Farhat, C. (2012). Stabilization of projection-based reduced-order models. *International Journal*

for Numerical Methods in Engineering, *91*(4), 358–377.

Bell, I. H., Wronski, J., Quoilin, S., and Lemort, V. (2014). Pure and pseudo-pure fluid thermophysical property evaluation and the open-source thermophysical property library coolprop. *Industrial & engineering chemistry research*, *53*(6), 2498–2508.

Bendapudi, S., Braun, J. E., and Groll, E. A. (2008). A comparison of moving-boundary and finite-volume formulations for transients in centrifugal chillers. *International journal of refrigeration*, *31*(8), 1437–1452.

Boyd, S., El Ghaoui, L., Feron, E., and Balakrishnan, V. (1994). *Linear matrix inequalities in system and control theory* (Vol. 15). Siam.

Chaturantabut, S., and Sorensen, D. C. (2010). Nonlinear model reduction via discrete empirical interpolation. *SIAM Journal on Scientific Computing*, *32*(5), 2737–2764.

Elmqvist, H., Tummescheit, H., and Otter, M. (2003). Object-oriented modeling of thermo-fluid systems. In *3rd international modelica conference* (pp. 269–286).

He, X.-D., Liu, S., Asada, H. H., and Itoh, H. (1998). Multivariable control of vapor compression systems. *HVAC&R Research*, *4*(3), 205–230.

Henrik, K., and Olsson, A. (2005). Model order reduction of a heat exchanger model.

Kalashnikova, I., Arunajatesan, S., Barone, M. F., van Bloemen Waanders, B. G., and Fike, J. A. (2014). Reduced order modeling for prediction and control of large-scale systems. *Sandia National Laboratories Report, SAND*(2014-4693).

Kerschen, G., Golinval, J.-c., Vakakis, A. F., and Bergman, L. A. (2005). The method of proper orthogonal decomposition for dynamical characterization and order reduction of mechanical systems: an overview. *Nonlinear dynamics*, *41*(1-3), 147–169.

Kim, D., Ma, J., Braun, J. E., and Groll, E. A. (2020). Fuzzy modeling approach for transient vapor compression and expansion cycle simulation. *International Journal of Refrigeration*.

Kunisch, K., and Volkwein, S. (1999). Control of the burgers equation by a reduced-order approach using proper orthogonal decomposition. *Journal of optimization theory and applications*, *102*(2), 345–371.

Laughman, C. R., and Qiao, H. (2019). On closure relations for dynamic vapor compression cycle models. In *Proceedings of the american modelica conference 2018, october 9-10, somberg conference center, cambridge ma, usa* (pp. 67–76).

Pangborn, H., Alleyne, A. G., and Wu, N. (2015). A comparison between finite volume and switched moving boundary approaches for dynamic vapor compression system modeling. *International Journal of Refrigeration*, *53*, 101–114.

Rasmussen, B. P. (2012). Dynamic modeling for vapor compression systems—part i: Literature review. *HVAC&R Research*, *18*(5), 934–955.

Rasmussen, B. P., and Alleyne, A. G. (2004). Control-oriented modeling of transcritical vapor compression systems. *J. Dyn. Sys., Meas., Control*, *126*(1), 54–64.

Xu, B., Yebi, A., Hoffman, M., and Onori, S. (2018). A rigorous model order reduction framework for waste heat recovery systems based on proper orthogonal decomposition and galerkin projection. *IEEE Transactions on Control Systems Technology*.

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