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A stochastic frontier approach

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SME Investment Determinants and Financing Constraints: A Stochastic Frontier Approach

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Abstract

In this paper, we explore the link between SME investment, firm economic characteristics, and the presence of financing constraints during the post-2008 crisis recovery period in Ireland. We use novel survey data between 2016 and 2018, which disaggregates investment by asset type and allows a granular assessment typically not present in the existing literature. Our approach links investment to the marginal product of capital using a stochastic frontier model to explore, and measure, the presence of constraints. We also test whether liquid assets, indebtedness and investment dissatisfaction impact SMEs investment. We find a clear link between investment and its marginal product with elasticities of between 0.55 and 0.65; a one per cent increase in marginal product leads to a 0.55-0.65 per cent increase in investment. The investment efficiency estimates obtained show the presence of financing constraints. We find evidence of both internal and external finance constraints explaining the investment efficiency of small fixed assets. Higher collateral availability contributes to improve investment efficiency for all fixed assets.

Keywords: Financing constraints; SMEs; investment efficiency; Stochastic Frontier Analysis.
JEL Classification: G31, D25.

1 Introduction

The recovery from the global financial crisis has been characterised by persistently low investment rates in many developed economies including the US, Japan and across Europe. Numerous studies have considered the issue within the context of the underwhelming growth out turn, as well as in terms of the implications for productivity (Dottling et al. 2017, Fay et al. 2017, Kose et al. 2017, Banerjee et al. 2015).

Various explanations relating to the crisis have been put forward, including access to credit, debt overhang and uncertainty. Indeed, credit access and debt overhang have been found to be limiting factors on investment in economies such as Ireland (Gerlach-Kristen et al. 2015a, Lawless et al. 2015). Moreover, a number of structural factors have also been suggested as possible reasons behind the low investment rates. For example, Alexander & Eberly (2018) found sluggish investment could be explained by the transition towards a services-based economy, high-tech industries and off-shoring and automation. Dottling et al. (2017) found that competition may have affected US firms, while the European investment sluggishness was associated with muted demand conditions and uncertainty. Other authors have identified a disconnect between firm profitability and investment, which suggests that the sensitivity of capital investment to its marginal product may have changed (Ogawa et al. 2019).

These findings however may be more relevant for large firms, particularly in sectors where intangible assets are becoming more important; and less relevant for Small and Medium Enterprises (SMEs)[1], who traditionally have been more affected by credit access issues. Indeed, an assessment of the sensitivity of investment to marginal products, as well as an identification and quantification of financing constraints for SMEs, has largely been absent in existing studies. To address this gap in the literature, we use data obtained using a novel survey of SME investment from Ireland to test the

[1]Thought this analysis SMEs are defined as firms that employ 250 people or less and have a turnover of €50 million or less (European Commission 2003).

link between investment and economic fundamentals.[2] We exploit the detailed investment information provided by our dataset, which allows an exploration of the impact of fundamentals on investment rates separately across different asset classes (buildings, machinery and transport equipment, staff and intangibles). It could be expected that, depending on the characteristics of the asset, there may be differences in the sensitivity of investment to profitability. This is an important distinction which has not been explored using SMEs data in past literature.

We include data in our analysis for the period 2016 to 2018, which corresponds to the recovery period following the 2008 Global Financial Crisis. During this period, investment levels suffered a severe hit due to increased risk and uncertainty which resulted in a credit crunch from the lenders perspective, and a reluctance by firms to commit capital to long-term investment(Vermoesen et al. 2013, Demirgüç-Kunt et al. 2020). Irish SMEs constitute an interesting case study. Their investment patterns felt the full effects of the 2008 crisis though the unavailability of credit arising from the banking crisis (Gerlach-Kristen et al. 2015b) and through previous capital structure reliance on high levels of debt (Lawless et al. 2015). Moreover, although the investment activity of Irish SMEs recovered, the use of external finance did not recover, with a continued usage of internal funds observed on the recovery period (Gargan et al. 2018). In an European context, the share of firms investing in Ireland is comparable to the EU average. However, twice as many firms in Ireland reported finance as an investment constraint relative to EU counterparts (EIB n.d.). In this context, an exploration of the size and drivers of SMEs financing constraints in the recovery period constitutes an interesting addition to the literature.

Our estimation approach relies on Stochastic Frontier Analysis (SFA) to estimate SME financing constraints by asset type, and test whether constraints are affected by firm-specific factors such as access to credit, indebtedness or investment dissatisfaction. Our research is the first to apply this methodology to assess financing constraints and its determinants us-

[2]We are grateful to the Irish Department of Finance for granting us access to this data.

ing firm-level data on SMEs (Wang 2003, Bhaumik et al. 2012, Wang & Ho 2010, Islam & Luo 2018).

A number of findings emerge from our analysis. First, we find a strong link between the marginal product of capital and investment. The magnitude of the coefficients indicate that a one per cent increase in the marginal product of capital leads to a 0.55 per cent increase in the investment rate. We find this sensitivity is highest for transport and machinery assets (0.65), and that it is lowest for investment in building assets.

We find financing constraints are present across all asset classes, indicating that Irish SMEs are in generally under-investing relative to the maximum possible investment frontier. This was suggested by the average investment efficiency scores, which were in all cases below 1. For all asset classes, average investment efficiency was 0.73. For buildings, the average efficiency was 0.73, for machinery and transport equipment it was 0.75, and finally for staff and intangibles it was 0.80. In terms of the determinants of investment efficiency, we find evidence that firms with higher debt-to-turnover and liquidity-to-turnover ratios have a higher investment efficiency. Given that 40 per cent of SMEs in the sample have 0 debt, it is likely to be capturing the fact that firms with higher credit access invest more. The estimates also highlight the importance of collateral availability in improving investment efficiency. Finally, we find a statistically significant and negative effect of investment dissatisfaction on investment efficiency.

The rest of this paper is structured as follows: Section 2 presents the methodological approach. Section 3 outlines the data and presents some summary statistics. Section 4 presents the main empirical findings and Section 5 concludes.

2 Methodology

The empirical estimation of financing constraints has been typically reliant on linear models based on Q investment or Euler equations, estimated using standard econometric techniques (for example, Whited (1992), Gilchrist & Himmelberg (1995)). However, it is also possible to assess the effect of financing constraints by using frontier estimation approaches. These techniques have been used extensively in production economics to evaluate firm performance through the estimation of measures such as technical efficiency and productivity. Two main methodologies have traditionally been used in order to estimate firm-specific technical efficiency scores, data envelopment analysis (DEA) and SFA. The former is a non-parametric method that involves programming; while the latter is a parametric technique that relies on econometric estimation. For our empirical application, SFA is preferred due to its capacity to accommodate factors such as the effects of random shocks or data measurement errors.

SFA is a parametric technique that makes use of econometric estimation techniques, therefore requiring the assumption of a given functional form to reflect the relationship between dependent (Y) and independent variables (X). SFA was originally and independently developed by Aigner et al. (1977) and Meeusen & van Den Broeck (1977). Its main feature is the inclusion of a composed error term that allows the separation of technical inefficiency from other stochastic variation:

$$Y = f(\beta X)e^{v-u} \tag{1}$$

where v is a symmetric random error that accounts for statistical noise and u is a non-negative component associated with technical inefficiency. In this framework, the measure of firm-specific technical efficiency is obtained as the ratio of observed output to the stochastic frontier output (Kumbhakar & Lovell 2000):

$$TE = \frac{Y}{f(\beta X)e^{v}} = e^{-u} \tag{2}$$

Wang (2003) set up the framework for the analysis of financing constraints using the SFA methodology. Wang (2003) showed that the constrained investment rate $ln(I/K_{t-1})$ is a combination of the optimal (or efficient) investment rate $ln(I/K_{t-1})^*$ represented by the frontier, and the one-sided constraint effect -*u* that characterises frontier estimation:

$$ln(I/K_{t-1}) = ln(I/K_{t-1})^* - u \tag{3}$$

This specification relies on the assumption of imperfect capital markets, due to factors such as information asymmetries, which in turn result in a shortage of financing (i.e. in the presence of financing constraints, the observed investment rate will be less than the efficient rate). The optimal investment rate can be defined using a Q investment equation:

$$ln(I/K_{t-1})^* = \beta_0 + \beta_1 ln(Sales/K_{t-1}) + \beta D + \beta_t + v \tag{4}$$

For unlisted companies (such as the sample of Irish SMEs we are using in our analysis) the Tobin Q is not available, therefore we use the sales to capital ratio as an alternative measure of the return to capital (Galindo et al. 2007). A vector of dummy variables (*D*) capturing firm characteristics and a set of year dummies are also included as controls in equation (4). These variables are described in more detail in Section 3. By substituting and reorganising equations (3) and (4) we get:

$$ln(I/K_{t-1}) = \beta_0 + \beta_1 ln(Sales/K_{t-1}) + \beta D + \beta_t + v - u \tag{5}$$

This specification has the same structure as a standard SFA model outlined in equation (1), albeit expressed in natural logarithms, including the two error term structure already described above. In this framework, the -*u* term in equation (5) measures the investment shortfall. In other words, the difference between the optimal investment rate represented by the frontier and the observed investment rate (i.e. $-u$) is attributed to financing constraint. This firm-specific investment efficiency score is recovered post-estimation following Battese & Coelli (1988):

$$IE = E[exp(-u)|v - u] \tag{6}$$

The obtained estimate of efficiency takes a value between 0 and 1. Therefore, the difference between each firm specific investment efficiency score and 1 can be interpreted as the percentage shortfall of investment from its optimal (frontier) level due to the presence of financing constraints.

The SFA approach also allows us to directly estimate the impact of several financing factors on the degree of financing constraints, rather than estimating the impact of these factors on investment of the average firm and infer from it whether they contribute to financing constraints. This can be done by extending the basic model proposed above to accommodate additional explanatory variables through the distribution assumed of the inefficiency term $-u$[3]. Numerous ways to incorporate these variables have been developed in the literature. Here we apply a model proposed in Caudill et al. (1995), which has the advantage that it allows accounting for heteroskedasticity in u in the estimation. In the Caudill et al. (1995) specification the distribution of the inefficiency term u is assumed to have zero mean and variance σ_u^2. Variance σ_u^2 is an exponential function of a vector of financing factors assumed to affect the degree of financing constraints (Z), with δ being a set of parameters to be estimated (these variables are described in Section 3):

[3]The effect of these variables is estimated in one step using Maximum Likelihood, and not in two separated steps.

$$\begin{aligned} & u \sim N^{+}(0, \sigma_u^2) \\ & \sigma_u^2 = exp(\delta_0 + \delta_k Z_k) \\ & v \sim N(0, \sigma_v^2) \end{aligned} \tag{7}$$

This specification also satisfies the scaling property, as noted in Alvarez et al. (2006). This property implies that the basic distribution $N^{+}(0,1)$ can be interpreted as the firm base investment efficiency, where the ultimate level of investment efficiency also depends on a function of the observed factors captured by the variables in Z (i.e. $\sigma(\delta Z)$). Finally, v is assumed to be distributed normally, with 0 mean and constant variance σ_v^2.

Applying this estimation approach to the analysis of financing constraints has several advantages. First, it allows to model the relationship between the investment shortfall (i.e. financing constraints) and firm-level characteristics in a straightforward and intuitive way, since these constraints are made dependant on the vector of observable variables Z in equation (7). This overcomes the difficulties of both linear regression and structural Euler equation models face in terms of accommodating the relation between real and financial variables (Wang 2003). Second, most analyses that explored financing constraints in the past (Gilchrist & Himmelberg (1995), Campello et al. (2010), Guariglia & Mateut (2010), for example), have relied on the separation of constrained and unconstrained firms in a given sample using arbitrary criteria, which in turn may generate potential endogenous selection issues (Wang 2003). The SFA methodology explores financing constraints without the need of separating the sample, reducing the aforementioned endogeneity problems.

Our analysis is closely related to a handful of other empirical applications. Wang (2003) and Wang & Ho (2010) used a panel of Taiwanese traded firms in the manufacturing sector, to explore financing constraints using SFA for the first time. Bhaumik et al. (2012) focused on large private Indian manufacturing firms, also based on a Q investment model. Islam & Luo (2018) used panel of listed Canadian forest firms. We contribute to this literature in several ways. First, previous analyses include large or traded firms. However, our dataset includes only SMEs, which are likely to have differing patterns of investment shortfall, as information asymmetry issues are more severe for small firms. Second, since we use survey data rather than information provided by traded firms or balance sheet data, we can disaggregate total investment by different asset categories. Therefore, we are able explore potentially divergent financing constraints across different types of assets. In order to do this we estimate the model described in equation (5) defining the investment rate for different types of assets. And third, the dataset used also includes information related to firms' attitudes and perceptions towards investment, which to our best knowledge have not been incorporated before to an analysis of financing constraints of this kind.

3 Data and Summary Statistics

The data used in this analysis comes from an investment module that is appended to the Irish Ministry of Finance's bi-annual Credit Demand Survey (CDS). The CDS is a telephone survey of approximately 1,500 firms that is weighted by size, sector and region. It has been running since 2009 and aims to collate information on firms' applications for credit, financing needs and views on the financial environment. Since 2017, a module has been added to the survey once a year which incorporates a range of questions on investment activity, the sufficiency of investment, capital stock, employment and indebtedness. More details on the survey and the questions can be found in Gargan et al. (2018) which presents the module in detail.

For the purpose of this analysis, we use a small panel dataset which is extracted from the main dataset. We also focus only on those firms who have positive investment activity. The total number of investing firms in our panel is 448, which results in 721 observations. The CDS provides detailed information on investment by type of asset (i.e. buildings, transport, machinery, intangibles and staff), and also on the value of firms' total assets. This information is used to build investment rates for different types of assets, which are used as dependent variables in equation (5). We group together investment in intangibles and staff, due to the low number of SMEs which invested in the former type of asset. We also group investment in machinery and transport assets. This way we compare the investment activity of SMEs in large fixed assets (by including buildings as a single asset category) and smaller fixed assets. As a result, separate regressions are run for each of the three asset categories described (buildings, transport/machinery, and intangibles/staff), as well as for the aggregate total investment rate.

In terms of the controls included in equation (5), an extensive body of literature linking investment to fundamentals for large firms have used the Tobin's Q model (Erickson & Whited 2000, Hennessy et al. 2007), approximated by average Q, to describe the relation between investment and the marginal product value of capital. The average Q is defined as the ratio of the book to the market value of the firm, and it is generally interpreted as a proxy for the unobservable shadow returns to capital. However, using the average Q is not feasible when analysing SMEs, since they do not have a market value that can be used in the calculation. By using a direct proxy for the marginal value product of capital, our research overcomes this limitation and provides a direct link between the profitability of capital and investment for small firms. We approximate marginal products by using the sales to capital ratio as in Galindo et al. (2007)[4]. The vector of dummy variables D in equation (5) includes dummies that control for firm size (micro firms), age (less than 10 years of operation), and sector (industry sector). We also include year dummies to capture shifts in the investment frontier over time.

[4]Galindo et al. (2007) also use the profits to capital ratio as a proxy for the marginal value product of capital. However, due to insufficient data we are not able to produce a consistent profits series so must instead use the sales to capital ratio.

We explore three paths through which firms' financing choices can potentially affect the investment efficiency identified by the SFA model (i.e. captured by the variables in Z in equation (7)). We control for the effect of access to external and internal finance sources by including in Z a direct measure of the internal funds available to each firm (using a liquid assets-to-turnover ratio), and also an indicator of indebtedness (using a debt-to-turnover ratio as in Lawless et al. (2015)). It has been established in the literature that SMEs have different financing patters compared to large and traded firms. Their capital structures are typically more weighted towards internal resources due to external debt or equity being more difficult to access (Beck et al. 2008), or being more costly due to information asymmetries. In this regard, the pecking order theory is considered to be the most widely applicable framework to small firm financing (Myers 1984). This theory builds on the premise that information asymmetries drive a wedge between the internal and external costs of capital. The theory suggests that small firms make financing choices in a hierarchical fashion, first using internal finance sources (when/until available) as opposed to external sources due to the relative cost differences (Berger & Udell 1998). Since bank lending to SMEs is predominately based on the availability of collateral due to information opacity, we also include a measure of collateral availability, built as the ratio of fixed assets on total assets. Besides detailed information on investment by asset type, the use of survey data also has the advantage of providing firms' qualitative information, which would not be possible to obtain otherwise (i.e. when using datasets based on balance sheet data). This survey provides a measure of firm-specific investment dissatisfaction, which is self-reported by each firm. As a fourth control, we include a dummy indicator in Z which equals one if firms reported that they have invested less than they wanted, and zero otherwise. A table containing a detailed definition of all variables mentioned in this section is presented in Appendix A.

Some summary statistics for each of the main variables described above are presented in Table 1. The average rate of investment is 13 per cent of total assets. Looking across asset types, the highest rate is in buildings, while the lowest is in intangibles and staff at 0.02 (i.e. 2 per cent of the value of total assets is invested in staff or intangibles in any one year). In terms of the composition of the sample, 42 per cent are micro firms (defined as firms with less than 10 employees), 25 per cent operate in industrial sectors (such as manufacturing and construction), and 14 per cent were operating for less than 10 years. The value of sales is on average quite high relative to firms' value of capital in the previous year[5]. The average level of the debt to turnover ratio is 10.7 per cent, which reflects the fact that many firms have no debt. The mean liquid assets to turnover ratio stands at 0.29, suggesting liquidity reserves are equivalent to almost one third of firms' turnover.

Table 1: Descriptive Statistics

	Mean	Median	Obs.	Min.	Max.
Total inv. rate	0.131	0.070	721	0.003	1.222
Inv. rate, Buildings	0.105	0.059	181	0.003	0.714
Inv. rate, Transport+Machinery	0.099	0.053	674	0.002	1.222
Inv. rate, Intang.+Staff	0.024	0.009	712	0.001	0.460
Sales/K_{t-1}	2.707	2.057	986	0.127	10
Age less than 10 years (D)	0.141	0	1,281	0	1
Micro firm (D)	0.420	0	1,281	0	1
Industry sectors (D)	0.252	0	1,281	0	1
Liquid assets/Turnover	0.293	0.216	1,013	0.013	1.500
Debt/Turnover	0.107	0	1,184	0	1
Collateral availability	0.526	0.500	1,083	0	1
Inv. dissatisfaction (D)	0.147	0	1,199	0	1

In order to provide more detail on investment patterns of Irish SMEs during the period analysed, Table 2 displays the mean and median investment rate by type of asset, time and selected firm categories. In all cases the median is lower than the mean, indicating that the distribution of total investment, as well as investment by asset is skewed to the left. The aver-

[5]This variable can take values over 1 as the scalar is total assets in the previous period.

age rate of total investment remained relatively constant between 2016 and 2018 at between 12 and 14 per cent of total assets. With the exception of buildings, average investment rates declined in 2017 across assets, but increased again in 2018. Younger firms operating for less than 10 years have a higher average investment rate, except for investment in intangibles and staff. Firms operating in industry sectors also had higher average investment rates, except for investment in transport also. Finally, micro firms display higher average investment rates in all assets.

Table 2: Investment rates overview

		Total	Buildings	Machinery+Transport	Staff+Intang.
Years					
2016	Mean	0.144	0.082	0.113	0.029
	Median	0.081	0.038	0.064	0.008
2017	Mean	0.125	0.125	0.089	0.020
	Median	0.071	0.067	0.046	0.008
2018	Mean	0.132	0.087	0.106	0.026
	Median	0.064	0.058	0.053	0.009
Sector categories					
Industry	Mean	0.137	0.122	0.101	0.026
	Median	0.072	0.035	0.053	0.009
Services	Mean	0.133	0.100	0.100	0.024
	Median	0.077	0.062	0.053	0.008
Other	Mean	0.111	0.089	0.093	0.021
	Median	0.061	0.035	0.042	0.008
Age categories					
Less than 10 years	Mean	0.163	0.122	0.133	0.028
	Median	0.106	0.060	0.099	0.012
10 to 25 years	Mean	0.143	0.116	0.106	0.030
	Median	0.075	0.056	0.056	0.009
More than 25 years	Mean	0.111	0.093	0.084	0.017
	Median	0.059	0.059	0.036	0.007
Size categories					
Micro	Mean	0.156	0.106	0.136	0.029
	Median	0.107	0.073	0.087	0.013
Small	Mean	0.121	0.105	0.090	0.025
	Median	0.065	0.060	0.053	0.009
Medium	Mean	0.112	0.104	0.067	0.014
	Median	0.053	0.048	0.027	0.006

4 Empirical Results

4.1 SFA Estimates

Several models are available for the estimation of stochastic frontiers in a panel data setting. Previous empirical applications have relied on a fixed effect approach to account for the panel structure of the data when estimating the investment equation (Wang 2003, Bhaumik et al. 2012, Islam & Luo 2018). We attempted to estimate our investment model using the True Fixed Effects approach proposed in Greene (2005a), with unsatisfactory results. This was likely due to the structure of the panel dataset we use, which includes a large N dimension but only 3 years of data. When using ML estimation, this feature may cause the incidental parameters problem (Greene 2002, 2005b). This is an inferential issue that arises when the number of units is relatively large compared with the length of the panel, as it is in this case. As a result, the firm specific effects estimated are inconsistent, which in turn leads to unreliable variance parameters on which the inefficiency estimates are based (Belotti & Ilardi 2018). For this reason, we estimated a pooled model instead. This approach simply implies that the time-invariant unobserved heterogeneity that would have been disentangled though using fixed effects estimation is incorporated in the inefficiency estimates. Despite this limitation, we expect that some of the unobserved heterogeneity is controlled for though the incorporation of firm characteristics dummies in the frontier specification.

The Maximum Likelihood estimates obtained from the pooled estimation of equations (5) and (7) are displayed in Table 3. The coefficients of the sales to capital ratio variable all have a positive sign, and are also statistically significant at the 1 per cent significance level. The magnitude of the coefficients range from 0.653 for transport and machinery to 0.548 for buildings. This indicates that a 1 per cent increase in the sales to capital ratio can cause between a 0.55 per cent and 0.65 per cent increase in the investment rate, depending on the type of asset in which firms invest. It is clear that the estimate obtained when using total investment rate (0.558) therefore masks the variation that exists across different assets.

The coefficients of the dummy variables capture investment differences across firms in each of the different categories included. Size appears to be the most important firm characteristics determining differences in investment, as the coefficient is statistically significant in all cases except for investment in buildings. The positive sign indicates that micro firms achieved higher investment rates, with the same level of sales over capital. Younger firms also display higher total investment rates in transport and machinery. This is also the case for firms operating in industry sectors. Finally, the year dummies are statistically insignificant, indicating that the investment frontier did not shift upwards or downwards between 2016 and 2018.

Table 3: SFA Estimates

	(1) Total	(2) Buildings	(3) Transport+Machinery	(4) Intang.+Staff
ln(Sales/K_{t-1})	0.558***	0.548***	0.653***	0.575***
	(0.070)	(0.140)	(0.072)	(0.078)
Age less than 10 years (D)	0.328**	0.134	0.282**	0.145
	(0.134)	(0.240)	(0.133)	(0.136)
Micro (D)	0.414***	0.072	0.686***	0.520***
	(0.101)	(0.185)	(0.102)	(0.104)
Industry sectors (D)	0.116	0.081	0.264**	0.129
	(0.109)	(0.212)	(0.109)	(0.114)
Year 2017 (D)	-0.007	0.309	-0.166	0.005
	(0.132)	(0.252)	(0.131)	(0.137)
Year 2018 (D)	0.130	0.363	0.072	0.211
	(0.138)	(0.272)	(0.138)	(0.142)
Constant	-3.070***	-3.147***	-3.503***	-5.242***
	(0.149)	(0.293)	(0.148)	(0.243)
Inv. constraint drivers				
ln(Liq. assets/Turnover)	-0.833***	-0.808	-0.896**	-0.975
	(0.319)	(0.773)	(0.349)	(0.812)
Debt/Turnover	-9.425**	-7.565	-8.311*	1.733
	(4.315)	(5.801)	(4.849)	(2.028)
Inv. dissatisfaction (D)	1.027*	-0.358	1.396**	-0.069
	(0.525)	(1.268)	(0.576)	(0.923)
ln(Collateral avail.)	-0.952***	-1.311**	-1.103***	-0.395
	(0.244)	(0.571)	(0.269)	(0.446)
Constant	-3.427***	-3.122	-4.000***	-4.655
	(1.037)	(2.021)	(1.146)	(3.616)
Observations	606	161	567	588
Log likelihood	-946.143	-232.019	-867.106	-923.998

Standard errors in parentheses. *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$. (D) indicates a dummy variable.

Some variables included in vector Z in equation (7) are statistically significant, as displayed in the bottom panel in Table 3. The sign of the coefficients indicate the effect of each variable on investment inefficiency (i.e.

financing constraints); therefore, the reversed signs indicate their relationship with investment efficiency. As for the case of the coefficient of the marginal product of capital proxy, the coefficients obtained for the drivers of total investment efficiency and by type of asset differ. Investment efficiency is positively correlated with the debt to turnover ratio for investment in machinery and transport. This result implies that a higher debt to turnover ratio decreased financing constraints. This is also the case for the liquidity ratio, which is also positively correlated with investment efficiency in machinery and transport. The effect of these two ratios on investment efficiency on buildings and intangibles and staff is statistically insignificant. These estimates might suggest that both internal and external financing sources have played an important role in reducing financing constraints in smaller fixed asset, as opposed to large fixed assets and intangibles. Lawless et al. (2015) using similar data found that debt financing had an important role to play in firms by facilitating investment which appears to be in line with our estimates. More recently, La Rocca et al. (2019) found that when SMEs have investment opportunities, having higher liquidity positively contributes to the decision to invest, though the relaxation of financing constraints. Collateral availability had a positive and statistically significant influence on investment efficiency for all types of fixed assets. This suggests that having more collateralisable assets reduces financing constraints, highlighting SMEs high reliance on collateral found in previous literature (Mac an Bhaird & Lucey 2010).

Finally, firms that reported being dissatisfied with their investment have higher financing constraints. This effect could be linked to increased economic uncertainty. Despite the years included in the sample corresponding to the recovery period after the 2008 financial crisis, they are also coincidental with the Brexit referendum and the EU-UK trade negotiations which constituted an important source of uncertainty for businesses.

4.2 Investment efficiency

Table 4: Investment efficiency estimates

	Mean	Median	Obs.	Min.	Max.
Total assets	0.734	0.743	606	0.062	0.998
Buildings	0.728	0.743	161	0.118	0.992
Transport/Machinery	0.753	0.773	567	0.059	0.997
Intangibles/Staff	0.800	0.816	588	0.468	0.932

Table 4 displays the descriptive statistics for the investment efficiency estimates. The mean efficiency for total assets is 0.734, indicating a loss of roughly 26.6 per cent of investment due to financing constraints. However, this aggregate figure of financing constraints does mask some variation depending on the asset type. Buildings emerged as the asset for which the loss in investment efficiency due to constraints is the highest (average of 27.2 per cent). Financing constraints seem to be less severe for machinery and transport, with median investment efficiencies of 0.728. This result points to some differences in the financing constraints faced by large versus smaller fixed assets. Finally, investment in intangibles and staff emerges as the investment category with the lowest financing constraints, with a 20 per cent investment efficiency loss.

The histograms displayed in Figure 1 indicate that the distributions of the investment efficiency scores estimated for each type of asset are skewed to the right, particularly for transport and machinery, and intangibles and staff.

Figure 1: Investment efficiency, All

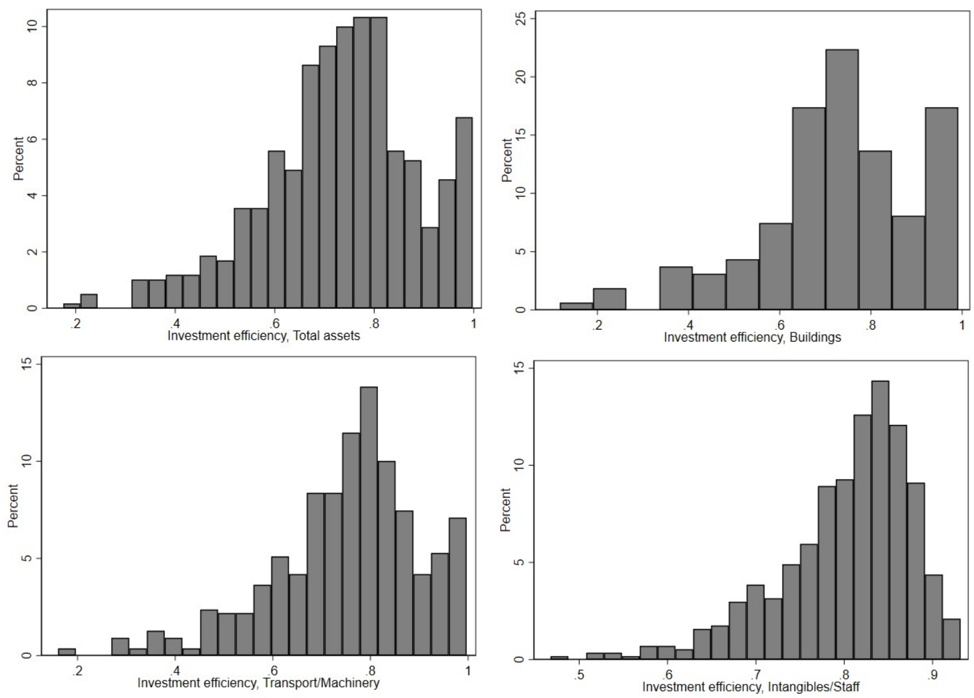

Table 5 displays the mean and median investment efficiency by age and size categories for total assets and small assets (i.e. machinery and transport assets). A Kruskal-Wallis test did not reject that the mean investment efficiency for other assets (buildings, intangibles and staff), as well as other categories (year, sectors), came from different populations, therefore these further comparisons were not possible. Average investment efficiency is higher for older SMEs, operating for more than 25 years, and for those in the small size category.

Table 5: Investment efficiency estimates by categories

	Total assets		Transport/Machinery	
	Mean	Median	Mean	Median
Age category				
Less than 10 years	0.705	0.699	0.721	0.730
10-25 years	0.723	0.729	0.745	0.753
More than 25 years	0.749	0.774	0.768	0.804
Size category				
Micro	0.709	0.733	0.725	0.746
Small	0.751	0.762	0.775	0.792
Medium	0.737	0.732	0.753	0.753

5 Conclusions

The empirical analysis of firm-level investment, credit constraints, and their relationship with firm fundamentals and other firm characteristics has typically relied on the estimation of linear equation models, based on Euler or Q investment equations. In this paper we implemented an alternative, and seldom used, approach based on SFA estimation. Moreover, we use novel and detailed investment data obtained through a survey that includes only SMEs, which have been typically neglected from this type of analysis. Therefore, although our sample includes SMEs located in Ireland, the estimates presented in this paper lead to a number of interesting conclusions. The sales to capital ratio emerged as the most significant explanatory factor of investment rates, which is indicative of the well known link between investment and its marginal product. The elasticities of sales to capital range between 0.55 and 0.65, depending on the type of asset. We also uncovered the presence of investment shortfalls for Irish SMEs. However, the severity of the shortfall varies depending on the type of asset in which firms invest. Average investment efficiency was the lowest for buildings, indicating financing constraints were more pronounced for large fixed assets. These differences across assets might suggest that legacy effects from the 2008 global financial crisis are still present when firms invest on larger fixed assets, as this type of investment, normally larger in size, is likely to be more reliant on the availability of external funding sources (i.e. bank credit).

The methodology implemented also has the advantage that allowed the effect of firm-level financial characteristics and attitudes directly on financing constraints to be explored. For the case of constraints in machinery and transport investment, liquidity and indebtedness ratios also contributed towards the reduction of financing constraints. Previous empirical applications of SFA to analyses firm-level financial constraints had exclusively focused on traded and large companies, therefore the comparability of our estimates is limited. Wang (2003), Islam & Luo (2018) or Bhaumik et al. (2012) also found evidence pointing to the importance of liquid assets easing financing constraints of large firms, however they did not explore the contribution of indebtedness (with the only exception of Islam & Luo (2018), who also uncovered a negative relation with constraints). Collateral availability significantly contributed to reduce financing constraints for fixed assets, however not for intangibles.

The analysis presented in this paper faces two main limitations. First, the estimates presented in Section 4 are only based on firms which invested in the time frame analysed have been included in the estimations. Selection might be a concern when estimating investment equations, since there is a portion of the total sample of firms which did not invest in any or some of the years included in the sample. The treatment of selection in SFA estimation is still in its early stages, and it is a complex topic given that the typical Heckman type of solution for selection using the Inverse Mill's Ratio as an additional dependent variable does not work in SFA estimation due to the non-linear nature of these models (Kumbhakar et al. 2009, Greene 2010). Second, an assumption behind the SFA approach lies on the asymmetric effects of financing constraints, which implies that observed investment rates are always below the optimal investment rate represented by the frontier, and that therefore firms not over-investing. Given the time period covered in our analysis we believe this is a reasonable assumption, however the presence of over investment could be explored in future work by resorting to two-tier SFA as suggested in Xie & Li (2018).

A Variables

Variables description

Dependent variables	
ln(Total investment rate)	Log of the ratio of the value of investment in buildings, transport, machinery and intangibles over the value of total assets in the previous year.
ln(Investment rate, Buildings)	Log of the ratio of the value of investment in buildings over the value of total assets in the previous year.
ln(Investment rate, Machinery/Transport)	Log of the ratio of the value of investment in machinery and transport assets over the value of total assets in the previous year.
ln(Investment rate, Intangibles/Staff)	Log of the ratio of the value of investment in staff and intangibles over the value of total assets in the previous year.
Independent variables	
ln(Sales/K_{t-1})	Log of the ratio of total firm turnover over the value of total assets in the previous year.
Age less than 10 years	Dummy variable which equals 1 if the firm has been operating for less than 10 years; and 0 otherwise.
Micro	Dummy variable which equals 1 if the firm has less than 9 employees; and 0 otherwise.
Industry sectors	Dummy variable which equals 1 if the firm operates in the manufacturing or construction sectors; and 0 otherwise.
ln(Liq. assets/Turnover)	Log of the value of firm liquid assets over the value of total turnover.
Debt/Turnover	Ratio of the value of firm total debt over total firm turnover value.
Collateral availability	Ratio of the value of firm fixed assets over total assets.
Investment dissatisfaction	Dummy variable which equals 1 if the firm reported to be dissatisfied with the level of investment undertaken in the previous year; and 0 otherwise.

References

Aigner, D., Lovell, C. & Schmidt, P. (1977), 'Formulation and estimation of stochastic', Review of Economics and Statistics **80**(3), 454–465.

Alexander, L. & Eberly, J. (2018), 'Investment Hollowing Out', IMF Economic Review **66**(1), 5–30.

Alvarez, A., Amsler, C., Orea, L. & Schmidt, P. (2006), 'Interpreting and testing the scaling property in models where inefficiency depends on firm characteristics', Journal of Productivity Analysis **25**(3), 201–212.

Banerjee, R., Kearns, J. & Lombardi, M. (2015), '(why) is investment weak?', BIS Quarterly Review .

Battese, G. E. & Coelli, T. J. (1988), 'Prediction of firm-level technical efficiencies with a generalized frontier production function and panel data', Journal of Econometrics **38**(3), 387–399.

Beck, T., Demirgüç-Kunt, A. & Maksimovic, V. (2008), 'Financing patterns around the world: Are small firms different?', Journal of financial economics **89**(3), 467–487.

Belotti, F. & Ilardi, G. (2018), 'Consistent inference in fixed-effects stochastic frontier models', Journal of Econometrics **202**(2), 161–177.

Berger, A. N. & Udell, G. F. (1998), 'The economics of small business finance: The roles of private equity and debt markets in the financial growth cycle', Journal of banking & finance **22**(6-8), 613–673.

Bhaumik, S. K., Das, P. K. & Kumbhakar, S. C. (2012), 'A stochastic frontier approach to modelling financial constraints in firms: An application to india', Journal of Banking & Finance **36**(5), 1311–1319.

Campello, M., Graham, J. R. & Harvey, C. R. (2010), 'The real effects of financial constraints: Evidence from a financial crisis', Journal of financial Economics **97**(3), 470–487.

Caudill, S. B., Ford, J. M. & Gropper, D. M. (1995), 'Frontier estimation and firm-specific inefficiency measures in the presence of heteroscedasticity', Journal of Business & Economic Statistics **13**(1), 105–111.

Demirgüç-Kunt, A., Peria, M. S. M. & Tressel, T. (2020), 'The global financial crisis and the capital structure of firms: Was the impact more severe among smes and non-listed firms?', Journal of Corporate Finance **60**, 101514.

Dottling, R., Gutierrez Gallardo, G. & Philippon, T. (2017), Is there an investment gap in advanced economies? if so, why?, Technical report, ECB Forum.

EIB (n.d.), Ireland overview eib investment survey, Technical report, EIB.

Erickson, T. & Whited, T. (2000), 'Measurement error and the relationship between investment and q', Journal of Political Economy **108**(5), 1027–1057.

European Commission (2003), 'Commission recommendation of 6 may 2003 concerning the definition of micro, small and medium-sized enterprises', Official Journal of the European Union **46**(L 124), 36–41.

Fay, R., Guénette, J.-D., Leduc, M. & Morel, L. (2017), 'Why Is Global Business Investment So Weak? Some Insights from Advanced Economies', Bank of Canada Review **2017**(Spring), 56–67.

Galindo, A., Schiantarelli, F. & Weiss, A. (2007), 'Does financial liberalization improve the allocation of investment?: Micro-evidence from developing countries', Journal of Development Economics **83**(2), 562–587.

Gargan, E., Lawless, M., Martinez-Cillero, M. & O'Toole, C. (2018), 'Exploring sme investment patterns in ireland: New survey evidence', Quarterly Economic Commentary: Special Articles .

Gerlach-Kristen, P., O'Connell, B. & O'Toole, C. (2015a), 'Do credit constraints affect sme investment and employment?', The Economic and Social Review **46**(1), 51–86.

Gerlach-Kristen, P., O'Connell, B. & O'Toole, C. (2015b), 'Do credit constraints affect sme investment and employment?', The Economic and Social Review **46**(1, Spring), 51–86.

Gilchrist, S. & Himmelberg, C. P. (1995), 'Evidence on the role of cash flow for investment', Journal of monetary Economics **36**(3), 541–572.

Greene, W. (2002), 'Alternative panel data estimators for stochastic frontier models', Unpublished manuscript (Septemebr 1, 2002), Department of Economics, New York University .

Greene, W. (2005a), 'Fixed and random effects in stochastic frontier models', Journal of Productivity Analysis **23**(1), 7–32.

Greene, W. (2005b), 'Reconsidering heterogeneity in panel data estimators of the stochastic frontier model', Journal of Econometrics **126**(2), 269–303.

Greene, W. (2010), 'A stochastic frontier model with correction for sample selection', Journal of Productivity Analysis **34**(1), 15–24.

Guariglia, A. & Mateut, S. (2010), 'Inventory investment, global engagement, and financial constraints in the uk: evidence from micro data', Journal of Macroeconomics **32**(1), 239–250.

Hennessy, C. A., Levy, A. & Whited, T. (2007), 'Testing q theory with financing frictions', Journal of Financial Economics **83**(3), 691–717.

Islam, A. R. M. & Luo, R. H. (2018), 'Financing constraints and investment efficiency: evidence from a panel of canadian forest firms', Applied Economics **50**(48), 5142–5154.

Kose, A., Ohnsorge, F., Ye, L. S. & Islamaj, E. (2017), Weakness in investment growth: causes, implications and policy responses, Policy Research Working Paper Series 7990, The World Bank.

Kumbhakar, S. C. & Lovell, C. K. (2000), Stochastic frontier analysis, Cambridge University Press.

Kumbhakar, S. C., Tsionas, E. G. & Sipiläinen, T. (2009), 'Joint estimation of technology choice and technical efficiency: an application to organic and conventional dairy farming', Journal of Productivity Analysis **31**(3), 151–161.

La Rocca, M., Staglianò, R., La Rocca, T., Cariola, A. & Skatova, E. (2019), 'Cash holdings and sme performance in europe: the role of firm-specific and macroeconomic moderators', Small Business Economics **53**(4), 1051–1078.

Lawless, M., O'Connell, B. & O'Toole, C. (2015), 'Sme recovery following a financial crisis: Does debt overhang matter?', Journal of Financial Stability **19**(C), 45–59.

Mac an Bhaird, C. & Lucey, B. (2010), 'Determinants of capital structure in irish smes', Small business economics **35**(3), 357–375.

Meeusen, W. & van Den Broeck, J. (1977), 'Efficiency estimation from cobb-douglas production functions with composed error', International Economic Review pp. 435–444.

Myers, S. C. (1984), Capital structure puzzle, Technical report, National Bureau of Economic Research.

Ogawa, K., Sterken, E. & Tokutsu, I. (2019), Why is investment so weak despite high profitability? a panel study of japanese manufacturing firms, Discussion papers, Research Institute of Economy, Trade and Industry (RIETI).

Vermoesen, V., Deloof, M. & Laveren, E. (2013), 'Long-term debt maturity and financing constraints of smes during the global financial crisis', Small Business Economics **41**(2), 433–448.

Wang, H.-J. (2003), 'A stochastic frontier analysis of financing constraints on investment: the case of financial liberalization in taiwan', Journal of Business & Economic Statistics **21**(3), 406–419.

Wang, H.-J. & Ho, C.-W. (2010), 'Estimating fixed-effect panel stochastic frontier models by model transformation', Journal of Econometrics **157**(2), 286–296.

Whited, T. M. (1992), 'Debt, liquidity constraints, and corporate investment: Evidence from panel data', The Journal of Finance **47**(4), 1425–1460.

Xie, C. & Li, L. (2018), 'The empirical test on investment efficiency and influence of equity incentive in supply-side structural reform: Based on the two-tier stochastic frontier approach', Applied Stochastic Models in Business and Industry **34**(5), 730–745.