2021

A continuous model for coordinated pricing of mixed access modes to transit

Nicholas Fournier

Eleni Christofa

Eric J. Gonzales

Transportation Research Part C 128 (2021) 103208

Contents lists available at ScienceDirect

Transportation Research Part C

journal homepage: www.elsevier.com/locate/trc

A continuous model for coordinated pricing of mixed access modes to transit

Nicholas Fournier [a,*], Eleni Christofa [b], Eric J. Gonzales [b]

[a] University of California Berkeley, McLaughlin Hall, Berkeley, CA 94720, USA
[b] University of Massachusetts Amherst, 130 Natural Resources Road, Amherst, MA 01003, USA

ARTICLE INFO

Keywords:
Discrete choice
Continuum approximation
Access mode
Pricing
Equity

ABSTRACT

The land-use pattern for many cities is a central business district surrounded by sprawling suburbs. This pattern can lead to an inefficient and congestion-prone transportation system due to a reliance on automobiles. This is because high-capacity transit is inefficient in low-density areas where insufficient travelers can access transit. This also poses an equity concern as the monetary cost of faster and more expensive travel disproportionately burdens low income travelers, especially when fixed congestion pricing is imposed. This paper presents a deterministic approximation of a discrete choice model for mixed access and mainline transportation modes, meaning that travelers may use different modes to access a mainline system, such as transit. The purpose is to provide a tractable computationally efficient model to address the first/last mile problem using a system-wide pricing policy that can account for heterogeneous values of time; a problem that is difficult to solve efficiently using a stochastic model. The model is structured for a catchment area around a central access point for a mainline mode, approximating choice by comparing modal utility costs. The underlying utility model accommodates both fixed prices (e.g., parking, fixed tolls, and fares) and distance-based unit prices (e.g. taxi fare, bike-share, and distance tolls) that may be set in a coordinated way with respect to value of time. Using numerical analysis to assess accuracy, the deterministic model achieved results within 3% of a stochastic logit-based model, and within 7% of measured values. The optimization of prices using the final model achieved a 22% reduction in generalized travel time and a 30% improvement in the Gini inequity measure from 0.2 to 0.14.

1. Introduction

For many developed nations, the era of fast paced construction of transportation infrastructure to keep up with demand has largely ended. Yet many cities remain plagued with congestion caused by inefficient utilization of infrastructure, compounded by sprawling suburban regions surrounding central business districts (CBD). This creates a persistent challenge in how to serve a concentrated demand from a spatially distributed source using high-capacity transit. In low population density areas, the number of travelers within walking distance of transit often does not support fixed route transit. Colloquially called the first/last mile problem, it is the challenge in getting travelers to and from a transit access point. Although travelers may travel by car to transit, either by driving to a park-n-ride or using an e-hailing service operated by a transportation network company (TNC), there is little incentive to transfer to transit when

* Corresponding author.
E-mail address: nick.fournier@berkeley.edu (N. Fournier).

https://doi.org/10.1016/j.trc.2021.103208
Received 22 March 2021; Accepted 3 May 2021
Available online 25 May 2021

completing the whole trip by driving is perceived as more convenient. This often results in overly congested roads and underutilized transit capacity (Schaller, 2017; Gan and Ye, 2018). Alternatively, bicycling is often cited as an excellent alternative access mode serving populations between walking and driving. However, bicycling has its own set of social and infrastructural challenges limiting it, such as physical impedance, discomfort, safety, and social stigmas (Boarnet et al., 2017).

The complications in modeling such multi-modal systems come from the need to realistically represent both the supply and demand, and to do so at the scales of the local neighborhood and broader urban region. Travel demand depends on the behavioral characteristics of the population that are difficult to model or predict due to lack of data and the uncertainty associated with behavior and choice. The demand patterns, in turn, affect available supply. Thus, mode choice should be considered in the context of equilibrium models that represent the inter-related nature of transportation supply and demand.

Setting policies to influence demand with respect to supply is not only a political challenge, but also a modeling challenge. Realistic stochastic demand models introduce mathematical hindrances in finding an optimal solution for the system. Conversely, generalized deterministic system models can more easily find a system optimum, but lack user heterogeneity (e.g. assume uniform travelers with a single value of time (VOT)). Although the lack of heterogeneity may be justified by using a mean value, the resulting pricing policies may exacerbate, rather than mitigate, existing inequities in the population.

This paper presents a hybrid mode choice system model using a deterministic model for mixed access (e.g., walk, bike, and drive) and mainline modes (e.g. highway, in other words drive on the highway, and rail). The proposed model is based on comparing modal utility over a continuous catchment area around an access point and can accommodate heterogeneity among users in terms of their location (i.e., distance from the neighborhood center) and value of time. By presenting a deterministic approximation that can be formulated as a convex minimization with linear constraints, the global system-wide optimal pricing can be identified efficiently. This facilitates decision making about pricing and analysis of equity impacts. This pricing policy incentivizes mode-shift to achieve a certain objective, such as minimum average travel time. Furthermore, the pricing inputs are provided as both fixed and distance based prices independently for each mode, providing an opportunity to integrate TNCs and bike-share systems. The continuous approximation model is validated by comparing its results against a stochastic logit-based counterpart. A numerical example for which we are aiming at minimizing average total travel time of all commuters across a varying VOT distribution is presented.

The contributions of this paper are not only in developing a computationally efficient multi-model pricing model, but by also exploring the equity impact (i.e., the distribution of time spent travelling) of pricing with a heterogeneous population across income. The results reveal that with income-specific prices, equity can be improved while still achieving congestion and travel time reduction. The paper is outlined as follows. First a discussion of the background and supporting literature to introduce the need for this model is presented. Then, the model concept and formulation are developed in the methodology section. The results section contains a numerical validation of the deterministic model against a stochastic model using aggregated data for a case study of commuting between Worcester and Boston, Massachusetts, USA. The numerical application is then further extended to a varying VOT distribution. The paper concludes with a discussion of the results, implications, and potential future research.

2. Background

The desire to improve multi-modal transportation system efficiency through pricing is a decades old ambition. Since Vickrey's model (Vickrey, 1969), many have extended this work to include competing modes (Tabuchi, 1993; Huang, 2000; Kraus, 2003; Gonzales and Daganzo, 2012; Gonzales and Daganzo, 2013), and more recently to include population heterogeneity through discrete choice models (Huang, 2002). Much of this work, especially in transportation economics, has focused on a single corridor or a simplified representation of a city as a linear system. For example, recognizing that transportation infrastructure requires space itself, Solow and Vickrey (1971) analyze traffic patterns in an idealized linear city to find an economic equilibrium allocation of space to balance land value and congestion costs. These models have been developed to identify the costs and externalities of transportation (Solow, 1972; Solow, 1973; Wheaton, 1998) as well as equilibrium and optimum urban land use patterns (Anas et al., 1998; Anas and Xu, 1999; Rossi-Hansberg, 2004).

A significant body of literature has focused on equilibrium traffic assignment problems that consider detailed mode and route choices on specific links in a network. Equilibrium traffic models are necessary for estimating demand to be used as input in pricing optimization. These include stochastic equilibrium (Daganzo and Sheffi, 1977), dynamic user equilibrium (Ben-Akiva et al., 1986; de Palma et al., 1983), experience-based models (Iida et al., 1992; Fujii and Kitamura, 2000; Polak, 1998), boundedly rational user equilibrium (Lou et al., 2010), late arrival penalized user equilibrium (Watling, 2006), and multi-objective equilibrium models (Dial, 1996; Chen et al., 2010; Zhang et al., 2013; Levinson and Zhu, 2013; Wang et al., 2014).

There is extensive literature advocating marginal social cost prices (also known as "first-best tolls"), which include costs of externalities like congestion and pollution, to achieve optimal utilization of the transportation system (Pigou, 1920; Beckmann et al., 1956; Arnott and Small, 1994; Yang and Huang, 2005). In a system with fixed demand, Hearn and Ramana (1998) that there exist sets of prices other than the marginal social cost prices that also achieve system optimum. Thus, there are an opportunities to identify other "first-best tolls" that are both system optimal and achieve other objectives, such as maintaining revenue neutrality or minimizing the maximum toll, even with variable demand (Yildirim and Hearn, 2005). Recent work includes examples of pricing analysis for heterogeneous values of time (Long and Szeto, 2019) and spatially distributed demand (Ren et al., 2020). This paper brings together multiple dimensions of heterogeneity to consider prices across modes that are more equitable for diverse populations.

A separate body of literature has been developed around modeling transit networks in regions. Analytical continuous approximation models date back to (Holroyd, 1967) and have been recently used to compare the performance of radial, grid, and hybrid network structures, as well as feeder-bus transit systems (Daganzo, 2010; Badia et al., 2014; Badia et al., 2017; Chen et al., 2015;

Sivakumaran et al., 2012). The advantages of these models is that the costs for users and operating agencies can be associated with the performance of a transit system based on a small number of design variables (e.g. route spacing, stop spacing, and service headway) and the number of people using the system. This aggregated approach of modeling transit, as well as e-hailing services, fits well with the network-level perspective of the Macroscopic Fundamental Diagram (MFD), because the characteristics of the traveling public and region can be described in terms of high-level, aggregate quantities.

With the continued advancement and availability of high performance computing, computationally intensive stochastic behavioral models have been at the forefront of transportation research. Stochastic models (e.g., discrete choice) utilize abundant data for empirically fit models based on random utility theory, providing generalizable and realistic results (Ben-Akiva and Lerman, 1985; McFadden, 1978; Train, 1978). Stochastic behavioral models are well suited for user-side demand estimation models, but often lack a closed-form solution, obscuring parameter relationships and require heuristic solutions that do not guarantee global optimality (Sivakumaran et al., 2012). Alternatively, deterministic continuum approximation models can provide robust closed-form solutions, revealing the relationships between input variables and their optimal values (Daganzo and Sheffi, 1977; Newell, 1973; Yang and Bell, 1997). However, deterministic models tend to be more general and their simplified nature typically only allows for a select few competing modes. Moreover, accounting for a mixture of access modes has yet to be accounted for in a pricing optimization model.

Recent research has demonstrated the potential for personalized and optimized incentive programs to improve system performance (Azevedo et al., 2018). In order to support effective decision making for policies related to infrastructure investment, operations planning, and demand management (e.g., through incentives), there is a need for simplified models that consider the variations across different travelers, e.g., varying values of time to account for modal preferences and socio-demographic factors. This includes considering the effect of distance from transit stations on mode choice decisions as well effects of other types of heterogeneity of travel preferences across the population.

3. Methodology

The proposed methodology is described in seven subsections: The basic concept of the model as it relates to access mode and discrete choice utility is presented in Section 3.1. Then in Section 3.2, the model is mathematically formulated to include a chosen objective function (minimize travel time in this case) and necessary constraints. Next, the mainline mode choice is introduced as a spatial interpretation of conditional choice depending upon access mode in Section 3.3. Building upon the mainline mode, congestion is introduced into the model in Section 3.4, which is then compiled into a final continuous objective function in Section 3.5. This objective function provides the optimal allocation of demand for minimum total generalized travel time, which is then realized through a pricing policy determined in Section 3.6. This pricing is presented separately for access mode and mainline mode in Sections 3.6.1 and 3.6.2, with a brief discussion on pricing implementation in Section 3.6.3. Access mode pricing yields the deficit cost in monetary prices needed to shift the observed demand to become optimal demand. The mainline mode pricing is a decomposition of a nested logit model, which extracts the pricing necessary to achieve the desired optimal mode split given the access mode split determined by the optimization in Section 3.5. Section 3.7 then extends the demand and pricing models to account for a varying VOT distribution as well as a means of measuring the model's impacts on equity using a Gini coefficient.

3.1. Concept

The proposed model is structured for a catchment area around a central access point, which serves a mainline mode, as a simple catchment zone around an access point; see Fig. 1. This mainline mode may be part of a larger more complex network system, but in a simplistic case it can be represented as a direct link to a central point, such as the central business district (CBD).

Mode choice can then be broken down into two basic sub-models for access mode and mainline mode. The respective utility cost, or dis-utility since it is a cost not a benefit, may be formulated as

$$U = Y + Z = \beta\left[\xi\left(t_a + \frac{r}{v_a}\right) + rC_a^d + C_a^f + \xi\left(t_m + \frac{L}{v_m}\right) + C_m\right] \tag{1a}$$

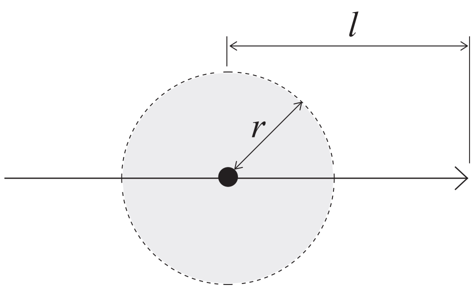


Fig. 1. Illustration of model concept.

$$Y = \beta\left[\xi_a\left(t_a + \frac{r}{v_a}\right) + rC_a^d + C_a^f\right] \tag{1b}$$

$$Z = \beta\left[\xi_m\left(t_m + \frac{L}{v_m}\right) + C_m\right] \tag{1c}$$

$$\begin{aligned} &a \in \{w, b, d\} \text{ for walk, bike, drive} \\ &m \in \{p, h\} \text{ for park-n-ride and highway} \\ &\beta < 0 \end{aligned}$$

where U is the dis-utility for the combined access and mainline modes, Y is the access mode dis-utility, and Z is the mainline mode dis-utility. β is the scaling parameter for the model, providing the relative magnitude of dis-utility as well as ensuring a negative utility value (i.e., dis-utility) in each case, where a user will choose the alternative of least cost. The parameters used are as described in Table 1. Although value of time can vary by mode (i.e., ξ_a and ξ_m), a single value of time, ξ, is used in this paper for simplicity. A more general formulation allowing different values of time for each instance of ξ in Eq. (1) would require more data for implementation.

This basic dis-utility function may be applied to independent access modes, such as walk, bike, and drive; as well as mainline modes of highway and train. A conventional approach to mode choice is to enter these dis-utility functions into the general logit function in Eq. (2) to yield the probability P_j of the discrete mode choice $j \in J$, where J is the set of available modes, and β is an additional scaling parameter.

$$P_j = \frac{e^{-\beta U_j}}{\sum_J e^{-\beta U_j}} \tag{2}$$

By varying the radial distance, the linear dis-utility and choice probability may be represented visually in Figs. 2a and b, respectively. The total demand for each mode $\lambda_a^{(s)}$ from a stochastic solution, is then the product of population density δ, and the area under the probability curve P_a (see Fig. 2b). This can be achieved radially by integrating probability as a function of r from 0 to the maximum radius R using the shell integration method, expressed as

$$\lambda_a^{(s)} = 2\pi\delta\int_0^R P_a\left(r\right)dr \tag{3}$$

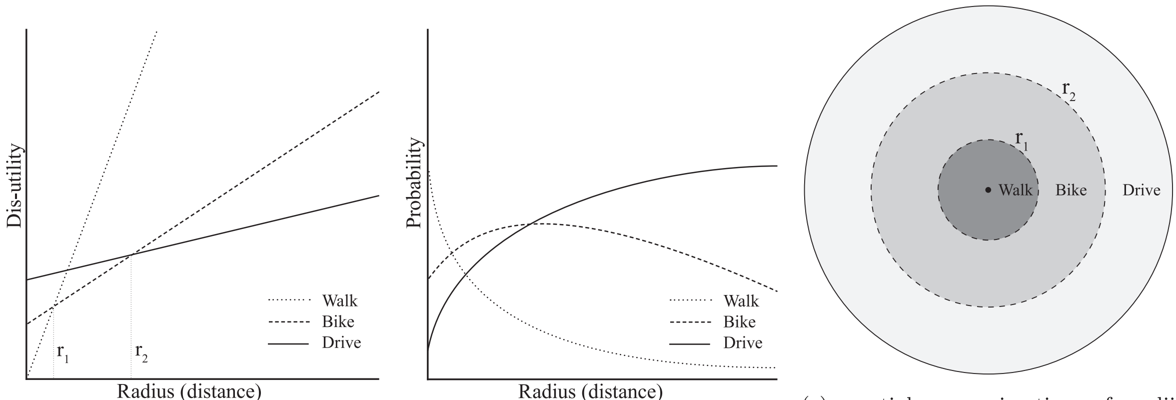
The problem, however, is that the logit function is not a closed form expression, making the optimization of pricing to incentivize demand difficult. Alternatively, the basic geometry of the model in Fig. 2c may be exploited to approximate the logit model, while remaining in an analytical form that can be optimized efficiently. Assuming that travelers choose the mode with the least dis-utility, all travelers between radius 0 and r_1 will walk, all travelers between r_1 and r_2 will bike, and all travelers between r_2 and R will drive in the deterministic case as shown in Fig. 2a. Demand $\lambda_a^{(d)}$ for each access mode in the deterministic case (d) is simply the product of the population density δ, and the ring areas between the tipping points at radii r_1 and r_2. In other words, the area under each probability curve in Fig. 2b is approximately equal to the area formed between the respective radii that determine the boundaries of choosing that mode. For example, the sum of probabilities under the walk curve is equal to the area formed between 0 and r_1.

3.2. Mathematical model

Various objectives may be formulated to include other costs, such as agency cost; however, a simple case of minimizing total travel time TT, will be used for demonstration. Generically this may be formulated in Eq. (4) as the sum of the products of demand in number

Table 1
Utility model parameters.

Parameter/Variable	Unit	Description
β	*unitless*	Scaling parameter
ξ_a	\$/time	access mode value of time (VOT)
ξ_m	\$/time	mainline mode value of time (VOT)
r	distance	access distance
L	distance	mainline distance
v_a	distance/time	access speed
v_m	distance/time	mainline speed
t_a	time	access mode startup time
t_m	time	mainline mode delay time
C_a^d	\$/distance	unit cost per distance for access mode
C_a^f	\$	fixed cost for access mode
C_m	\$	fixed cost for mainline mode



(a) access mode dis-utility (b) access mode logit probability (c) spatial approximation of radii boundaries for access mode choice

Fig. 2. Access mode choice model comparison.

of travelers $\lambda_a^{(d)}$ and average travel time T_a, respectively, for each access mode $a \in A$, where A is the set of all access modes.

$$TT = \sum_A T_a \lambda_a^{(d)} \tag{4}$$

The deterministic travel demand $\lambda_a^{(d)}$, between two radii r_i and r_{i+1}, is derived from the area of a circle multiplied by the population density as follows

$$\lambda_a = \pi\delta\left(r_{i+1}^2 - r_i^2\right) \tag{5}$$

The average travel time T_a is the average radial distance $\bar{r}_a$ divided by average speed $\bar{v}_a$, which is assumed to be constant across the network. The average speed $\bar{v}_a$ is calculated as the average distance divided by the sum of startup time t_a (e.g., time to unlock bicycle or start and un-park automobile) and travel time $\bar{r}_a/v_a$.

$$\bar{v}_a = \frac{\bar{r}_a}{t_a + \frac{\bar{r}_a}{v_a}} \tag{6}$$

The average radial distance $\bar{r}_a$ may be calculated using the population's probability distribution function $f(r)$. Assuming a uniformly distributed population, the probability density function increases linearly with $r, f(r) = kr$. The value of k is determined using the law of total probability to solve $\int_{r_{min}}^{r_{max}} krdr = 1$, implying that the probability density function is

$$f\left(r\right) = \frac{2r}{r_{max}^2 - r_{min}^2} \quad \text{for } r \in [r_{min}, r_{max}] \tag{7}$$

The expected radius is $E(r) = \int rf(r)dr = \int kr^2dr$, which yields a formula for average radius, $\bar{r}_a$, for points within a ring assuming uniform density. For access mode a serving the population between r_i and r_{i+1}, the average distance is

$$\bar{r}_a = \frac{2\left(r_{i+1}^3 - r_i^3\right)}{3(r_{i+1}^2 - r_i^2)} \tag{8}$$

Recalling that average travel time is $T_a = \bar{r}_a/\bar{v}_a$, and substituting for $\bar{r}_a$ and $\bar{v}_a$, a formula for travel time as a function of r_i and r_{i+1} is expressed as

$$T_a = \frac{\bar{r}_a}{\bar{v}_a} = \left[t_a + \frac{2\left(r_{i+1}^3 - r_i^3\right)}{3v_a(r_{i+1}^2 - r_i^2)}\right] \tag{9}$$

which can then be used to formulate an objective function for minimizing total travel time of all users in the system

$$TT_a\left(r_i, r_{i+1}\right) = \lambda_a T_a = \pi\delta\left[t_f\left(r_{i+1}^2 - r_i^2\right) + \frac{2}{3v_a}\left(r_{i+1}^3 - r_i^3\right)\right] \tag{10}$$

where t_f is the fixed additional travel time specific to the combination of access and mainline mode (i.e., startup time t_a plus mainline travel time T_m), and r_i and r_{i+1} are the decision variables. The radii subscripts i and $i+1$ reflect whether the relevant distances are 0 to r_1 for walk, r_1 to r_2 for bike, or r_2 to R for drive. Global monetary costs (e.g., tolls, fares, parking, etc.) experienced by the user may also be included into the model to express the generalized cost. This can be done by converting monetary costs to generalized time (GT) cost using the VOT as in the expression

$$GT_a\left(r_i, r_{i+1}\right) = \lambda_a\left[T_a + \frac{\bar{r}C_a^d + C_a^f}{\xi_a}\right] = \pi\delta\left[\left(r_{i+1}^2 - r_i^2\right)\left(t_f + \frac{C_a^f}{\xi_a}\right) + \frac{2}{3}\left(r_{i+1}^3 - r_i^3\right)\left(\frac{1}{v_a} + \frac{C_a^d}{\xi_a}\right)\right] \tag{11}$$

where C_a^d and C_a^f are the existing fixed or distance-based monetary costs. A final caveat of this model is that a basic hierarchical constraint must be imposed such that

$$\text{Vehicle speed}: \frac{1}{v_W} + \frac{C_W^d}{\xi_a} \geqslant \frac{1}{v_B} + \frac{C_B^d}{\xi_a} \geqslant \frac{1}{v_D} + \frac{C_D^d}{\xi_a} \tag{12a}$$

$$\text{Vehicle startup time}: t_W + \frac{C_W^f}{\xi_a} \leqslant t_B + \frac{C_B^f}{\xi_a} \leqslant t_D + \frac{C_D^f}{\xi_a} \tag{12b}$$

meaning that the distance *cost* of walking is greater than that of biking, which is greater than that of driving, and the fixed *cost* of driving is greater than that of biking, which is greater than that of walking.

3.3. Conditional choice of mainline mode

Since highway travel time ultimately depends upon the number of drivers that use the highway λ_H rather than park-and-ride to the train λ_D, the mode-share $\theta = \frac{\lambda_H}{\lambda_D}$ of highway drivers to all drivers must also be determined. This may be conceptualized spatially in Fig. 3a where the portion of walk, bike, and drive are determined by the radii r_1 and r_2, but highway and park-and-ride are subsequently split within the drive portion. This is analogous to the nested logit illustrated in Fig. 3b.

With T_H as highway travel time and T_T as the train travel time, the total travel time for driving is estimated as in Eq. (13). It is essentially the product of the total drive demand λ_D and the sum of the proportional mainline travel times $\theta T_H, (1-\theta)T_T$, and drive access time T_D.

$$TT_D = \lambda_D[\theta T_H + (1-\theta)T_T + T_D] \tag{13}$$

3.4. Congestion

Thus far in the paper, congestion has been absent from the model. To be more realistic, once the mainline mode is incorporated, a model for congested mainline travel time must be considered. The following section describes this formulation and justification for utilizing a simpler continuous function for congested travel time.

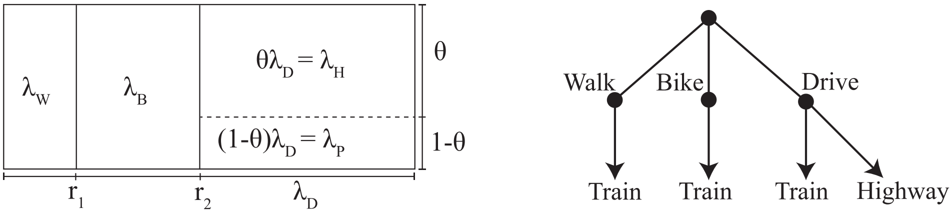
Mainline dis-utility is a fixed constant time cost that may be added to t_a in Eq. (10). The effect of the mainline mode is a vertical shift in dis-utility while the slope remains unchanged. This provides a generic input for the mainline travel time, allowing for flexibility in mainline travel time estimation. Assuming that train transit is reliable and does not experience delay due to congestion, then the train travel time T_T in Eq. (14), depends on the distance, L; speed, v_T; and any fixed delay, t_T, such as the waiting time associated with transit headway.

$$T_T = t_T + \frac{L}{v_T} \tag{14}$$

The travel time for driving, which is a congestible mode, requires accounting for the network's capacity to move car traffic (i.e., flow) and account for the effect of traffic volumes (i.e., demand) on speed. The Macroscopic Fundamental Diagram (MFD) provides an aggregate representation of network traffic conditions that can be used to characterize traffic speeds. A generic mathematical model for the flow-density relationship is a function $q(k)$ for traffic flow q in vehicles per time, and density k in vehicles per distance. The average travel time T_H, including the effects of congestion, can then be derived by solving for k and substituting into the inverse of speed $\frac{1}{v} = \frac{k}{q}$ to yield units of time per distance per lane (e.g., hours/km/lane). However, when modeling travel time for all traffic states over the entire U-shaped MFD, the resulting travel time function yields multiple values for a given flow, complicating optimization. Steady-state conditions can only be maintained for highway demand that does not exceed the network capacity, $\lambda_H \leqslant q_c$. Each flow point is associated with an "uncongested" state with density less than the critical density associated with capacity, k_c, and a "congested" state with density exceeding k_c.[1]

Since the overall objective is to identify system optimum pricing strategies, travel times can be modeled using the lower "uncongested" part of the travel time function because uncongested conditions use fewer resources to serve traffic demand and are therefore always preferable to congestion. Pricing strategies should aim at maintaining uncongested conditions. Thus, travel time for highway users can be modeled using a continuous non-decreasing function of demand $T_H(\lambda_H)$ for cases when pricing or demand management will keep demand from exceeding the highway's capacity. Although this is a major simplification of traffic flow characteristics, the model purpose is demand-side management for optimal supply utilization, not a full traffic flow model (May, 2019).

[1] These regions are referred to as "congested" and "hypercongested," respectively, in the economics literature (Gonzales, 2015).



(a) spatial interpretation of nests (b) logit nests

Fig. 3. Nested mode choice.

3.5. Optimal mode split

Mainline travel times for train and highway in Eq. (14), and $T_H(\lambda_H)$ can be incorporated into the respective access travel time functions in Eqs. (11) and (13). Once expanded to include mainline travel time, the walk, bike, and drive travel times are calculated as in Eqs. (15a–15c), respectively.

$$GT_W\left(0, r_1\right) = \pi\delta\left[\left(r_1^2 - 0^2\right)\left(t_W + t_T + \frac{L}{v_T} + \frac{C_W^f + C_T^f}{\xi_a}\right) + \frac{2}{3}\left(r_1^3 - 0^3\right)\left(\frac{1}{v_W} + \frac{C_W^d}{\xi_a}\right)\right] \tag{15a}$$

$$GT_B\left(r_1, r_2\right) = \pi\delta\left[\left(r_2^2 - r_1^2\right)\left(t_B + t_T + \frac{L}{v_T} + \frac{C_B^f + C_T^f}{\xi_a}\right) + \frac{2}{3}\left(r_2^3 - r_1^3\right)\left(\frac{1}{v_B} + \frac{C_B^d}{\xi_a}\right)\right] \tag{15b}$$

$$\begin{aligned} GT_D\left(r_2, \theta\right) = \pi\delta\Bigg[&\left(R^2 - r_2^2\right)\left(t_D + \theta T_H(\lambda_H) + \left(1 - \theta\right)\left(t_T + \frac{L}{v_T}\right) + \frac{C_D^f + \theta C_H^f + (1-\theta)C_T^f}{\xi_a}\right) \\ &+ \frac{2}{3}\left(R^3 - r_2^3\right)\left(\frac{1}{v_D} + \frac{C_D^d}{\xi_a}\right)\Bigg] \end{aligned} \tag{15c}$$

where the highway travel time T_H is a function of highway demand λ_H. Highway demand is calculated as the portion of total drive access users taking the highway given by the θ ratio, calculated as

$$\lambda_H = \theta\lambda_D = \theta\pi\delta\left(R^2 - r_2^2\right) \tag{16}$$

The final objective function then becomes the summation of the three generalized travel time functions with three decision variables of r_1, r_2, and θ in Eq. (17).

$$\min GT(r_1, r_2, \theta) = GT_W(0, r_1) + GT_B(r_1, r_2) + GT_D(r_2, \theta) \tag{17}$$

Theorem 1. *The total generalized cost, $GT(r_1, r_2, \theta)$, with conditions (12a) and (12b) is a unimodal function over the feasible space $r_1, r_2 \geqslant 0, \theta \in (0, 1)$. The values of $\widehat{r}_1$ and $\widehat{r}_2$ that satisfy the first order necessary conditions correspond to the global minimum.*

The proof is provided in the Appendix. Since $GT(r_1, r_2, \theta)$ is a continuous unimodal function, this can be solved efficiently for the optimal values. The optimal parameters, $\widehat{r}_1, \widehat{r}_2$, and $\widehat{\theta}$ yield the allocation of demand to access and mainline modes that minimizes the generalized total travel time while accounting for congestion.

3.6. Pricing

To achieve the optimal mode share determined by Eq. (17), society might need to impose additional policies to incentivize users through the intervention price parameters $\widehat{C}_a^f, \widehat{C}_a^d$, and $\widehat{C}_m$. This would adjust the dis-utility functions shown in Fig. 2a such that the intersections for r_1 and r_2 are shifted from their observed position to match the optimal values (i.e., $\widehat{r}_1$ and $\widehat{r}_2$). First the observed "effective" radii r' and highway ratio θ' must be determined from an observed mode share P'. This can be done using the proportions in Fig. 3a from the proportional demand P_{am} and the known total radius R to calculate the effective proportional radii r_1' and r_2', expressed more fully as

$$r_1' = P_{WT}' \cdot R \tag{18a}$$

$$r_2' = \left(P_{WT}' + P_{BT}'\right) \cdot R \tag{18b}$$

$$\theta' = \frac{P'_{DH}}{P'_{DT} + P'_{DH}} \tag{18c}$$

3.6.1. Access mode pricing

The point of intersection can be solved when the costs are known by setting Eq. (1b) for the access modes' dis-utilities equal to each other (i.e., $Y_a = Y_{a+1}$) and solving for r_i. This yields Eq. (19). Without any intervention, the given parameters yields a null tipping point radii r_i° calculated as

$$r_i^{\circ} = \frac{\xi_{a+1}t_{a+1} - \xi_a t_a + C^f_{a+1} - C^f_a}{\frac{\xi_a}{v_a} - \frac{\xi_{a+1}}{v_{a+1}} + C^d_a - C^d_{a+1}} \tag{19}$$

where i is the tipping point radius between incremental access modes a and $a+1$ (i.e., r_1 is tipping point between walk and bike and r_2 is the tipping point between bike and drive). Incorporating intervention prices $\widehat{C}^f_a$ and $\widehat{C}^d_a$ yields the cost *differential* needed to shift from the null radii r_i°, to some target radii r_i^*. However, we seek to find the differential from the optimal to the existing, not from the null. This total differential is simply calculated as

$$r_i^* = r_i' + r_i^{\circ} - \widehat{r}_i \tag{20}$$

where $\widehat{r}_i - r_i^{\circ}$ accounts for the difference between the null and optimal condition, which can then be used to find the differential intervention prices with

$$r_i^* = \xi(t_{a+1} - t_a) + C^f_{a+1} - C^f_a + \frac{\widehat{C}^f_{a+1} - \widehat{C}^f_a}{\frac{\xi_a}{v_a} - \frac{\xi_{a+1}}{v_{a+1}} + C^d_a - C^d_{a+1} + \widehat{C}^d_a - \widehat{C}^d_{a+1}} \tag{21}$$

However, when the price variables are not known, an "optimal" set of prices must be found to achieve the desired radii. This can be done by reformulating Eq. (21) for r_1 and r_2 as constraints in an optimization problem as shown in Eqs. (22b) and (22c). An infinite number of solutions may exist, thus a reasonable objective function in Eq. (22a) might be to minimize the square magnitude of costs to shift the radii as in Eq. (22a). From a social perspective, this would minimize the quantity of money changing hands. Eqs. (22d) and (22e) impose the hierarchical constraints required for the deterministic model from Eq. (12). An additional constraint could also be set to maintain revenue neutrality (Eq. 22f).

$$\min \sum_J C_j^2 \tag{22a}$$

$$\text{s.t.} r_1^* \widehat{C}^d_W - r_1^* \widehat{C}^d_B + \widehat{C}^f_W - \widehat{C}^f_B = \xi_B t_B - \xi_W t_W + r_1^* \left(\frac{\xi_B}{v_B} - \frac{\xi_W}{v_W} \right) + r_1^* \left(C^d_W - C^d_B \right) + C^f_B - C^f_W \tag{22b}$$

$$r_2^* \widehat{C}^d_B - r_2^* \widehat{C}^d_D + \widehat{C}^f_B - \widehat{C}^f_D = \xi_D t_D - \xi_B t_B + r_2^* \left(\frac{\xi_D}{v_D} - \frac{\xi_B}{v_B} \right) + r_2^* \left(C^d_B - C^d_D \right) + C^f_D - C^f_B \tag{22c}$$

$$C^d_W + \widehat{C}^d_W + \frac{\xi}{v_W} \geqslant C^d_B + \widehat{C}^d_B + \frac{\xi_B}{v_B} \geqslant C^d_D + \widehat{C}^d_D + \frac{\xi_D}{v_D} \tag{22d}$$

$$C^f_W + \widehat{C}^f_W + \xi_W t_W \leqslant C^f_B + \widehat{C}^f_B + \xi_B t_B \leqslant C^f_D + \widehat{C}^f_D + \xi_D t_D \tag{22e}$$

$$\sum C_j = 0 \tag{22f}$$

3.6.2. Mainline mode pricing

The optimal prices from solving Eq. (22) yield only the access mode costs. The equation does not yet consider the two mainline modes. Walk and bike access users are limited to only the train for mainline mode, so determination of the relevant radii is sufficient to determine the number traveling by each mode. However, drive access users have an additional choice between train or highway, so in addition to determining the number of drivers by radius, they must be further distinguished by mainline mode (see Fig. 3). Recalling the conditional mode choice model described in Section 3.3 where $\theta = \lambda_H/\lambda_D$, the analogous nested choice may be decomposed to extract the cost differential necessary to achieve the desired mainline mode split θ. Assuming revenue neutrality, the individual mainline mode prices for highway C_H, and train C_T, can be solved analytically as a fixed cost. Although in practice mainline mode pricing may be implemented as a distance based cost (e.g., distance-based tolling), from an access mode perspective it is a fixed cost parameter because the total cost associated with the mainline part of the trip will always be the same for the constant mainline distance L.

The following two sub-sections describe the decomposition of conditional mode choice to yield highway and train prices using both a deterministic and a stochastic logit based solution.

Stochastic mainline mode pricing. Since the mainline mode travels a constant distance L, this means that the dis-utility cost does not vary. Thus a deterministic approximation of the discrete choice would unrealistically assign all trips to the lowest cost mainline mode unless the costs were exactly equal, yielding an all or nothing result. For the model to predict non-zero numbers of highway and transit users in uncongested conditions, a stochastic logit model is needed. Fortunately in this case, the constant distance does not introduce the same computational complexity as for access mode and can be conveniently solved analytically. A logit model does introduce the problem of independence of irrelevant alternatives (IIA), but is often solved by nesting the choices using a nested logit model as in Eqs. (23).

$$P_n = P_i \cdot P_{n|i} \tag{23a}$$

$$P_i = \frac{e^{-Y_i}}{\sum_J e^{-Y_j}} \tag{23b}$$

$$P_{n|i} = \frac{e^{-Z_n+\mu IV_n}}{\sum_K e^{-Z_k+\mu IV_k}} \tag{23c}$$

$$IV_n = ln\left(\sum e^{-Y_i}\right) \tag{23d}$$

where P_n is the probability of the mainline nest (i.e. highway or train), $P_{i|n}$ is the probability of access mode i given a mainline nest n, IV_n is the logsum for nest n, and μ is the logsum coefficient. When $\mu = 1$, there is no correlation, and when $\mu = 0$, there is perfect correlation among alternatives. It can be assumed that the choices are perfectly correlated because highway is exclusive to driving, reducing the conditional probability of choosing highway given driving as the access mode to $P_{H|D} = \frac{e^{-Z_H}}{e^{-Z_H}+e^{-Z_T}}$ and the conditional probability of choosing train given walk or bike access modes to $P_{W|T} = P_{B|T} = 1$, because users that user walk or bike as their access mode must take the train. Thus, $P_H = P_D \cdot P_{H|D} = \frac{\lambda_H}{\lambda} = \frac{\lambda_D}{\lambda} \cdot \frac{e^{-Z_H}}{e^{-Z_H}+e^{-Z_T}}$ the logit can be solved to extract the cost differential $\Delta C_m = C_H - C_T$ as in Eq. (24a), where $\widehat{C}_T$ and $\widehat{C}_T$ are the intervention prices/subsidy imposed in addition to the existing monetary costs C_H and C_T. Subsequently if revenue neutrality is maintained across the mainline modes as well, then $\lambda_H(C_H+\widehat{C}_H)+\lambda_T(C_T+\widehat{C}_T) = 0$ may be used to solve for the specific cost values.

$$\left(C_H+\widehat{C}_H\right) - \left(C_T+\widehat{C}_T\right) = ln\left(\frac{\lambda_D-\lambda_H}{\lambda_D}\right) + \xi T_T - \xi T_H \tag{24a}$$

$$0 = \lambda_H\left(C_H+\widehat{C}_H\right) + \lambda_T\left(C_T+\widehat{C}_T\right) \tag{24b}$$

$$\widehat{C}_H = \frac{\lambda_T}{\lambda_T+\lambda_H}\left[ln\left(\frac{\lambda_D-\lambda_H}{\lambda_H}\right) + \xi T_T - \xi T_H\right] - C_H \tag{24c}$$

Deterministic mainline mode pricing. Moving away from stochastic discrete choice, the problem can also be formulated more generally as a classical equilibrium model of generalized cost between highway and train travel, as shown in Fig. 4. With the total driving demand being $\lambda_D = \lambda_{H|D} + \lambda_{T|D} = \frac{\lambda_H}{\theta}$, the dis-utility of highway usage increases monotonically with highway demand while transit remains constant. The equilibrium point is where the dis-utility functions (see Eq. (1)) for highway Z_H and train Z_T are equal.

To achieve the desired equilibrium given the optimal θ, additional intervention prices $\widehat{C}_T$ and $\widehat{C}_T$ can be imposed. Setting the functions equal and again assuming revenue neutrality, the intervention price $\widehat{C}_H$ can be solved for analytically in Eq. (25).

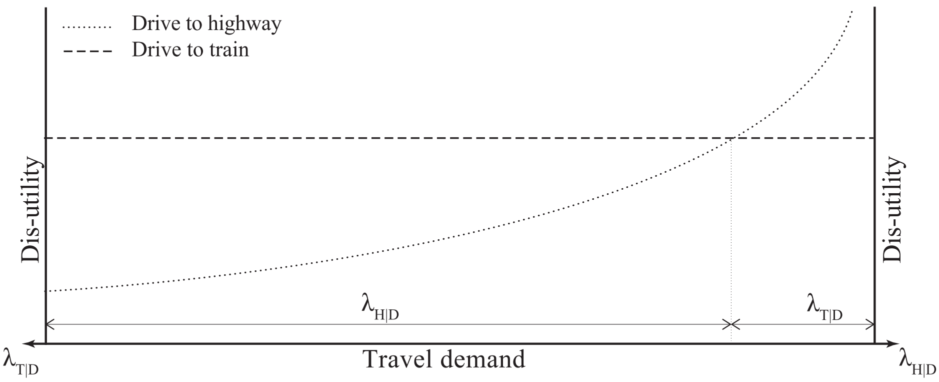


Fig. 4. Equilibrium model of mainline dis-utility.

$$\left(C_H+\widehat{C}_H\right)-\left(C_T+\widehat{C}_T\right)=\xi T_T-\xi T_H \tag{25a}$$

$$0=\lambda_H\left(C_H+\widehat{C}_H\right)+\lambda_T\left(C_T+\widehat{C}_T\right) \tag{25b}$$

$$\widehat{C}_H=\frac{\lambda_T}{\lambda_T+\lambda_H}(\xi T_T-\xi T_H)-C_H \tag{25c}$$

3.6.3. Pricing implementation

The flexibility of the model's pricing inputs as both fixed and distance-based prices provides potential for multi-modal integration. Fixed prices may be attributed to more classical features, such as parking, transit fare, bridge tolls, etc., whereas distance based costs can help integrate future technologies, such as gantry mounted all electronic tolling (AET), Vehicle Miles Traveled (VMT) fees, or even bike-share and e-hailing taxi fares. Moreover, with increasingly inter-operable payment systems, it is possible to promote an efficient multi-modal pricing system through price collaboration, such as providing a discount when transferring to transit.

3.7. Income, equity, and the value of time

Thus far, the mode choice model uses a homogeneous population with a single VOT. This is not only unrealistic, but poses equity concerns considering that VOT is often correlated with income (Small et al., 2005; Hensher, 1976; Li and Hensher, 2012; Hensher, 2001). To ensure that the transportation system is both efficient and equitable, the VOT must also be a parameter to account for varying distributions of income or preferences. Since Eq. (17) is convex, further summation remains convex, enabling the population to be segmented into any number of discrete population groups optimized for the total generalized travel time in Eq. (26).

$$\min\sum_K GT_k\left(r_{1k},r_{2k},\theta_k\right) \tag{26}$$

The population may then be segmented by VOT ξ_k for K discrete segments with a population density of δ_k, modeled by some population probability density function. Such a function might follow a log-normal distribution commonly used to model income distributions. Similarly, the pricing policy can also be optimized by extending Eq. (22) with a set of access pricing variables C^d_{ak},C^f_{ak}, and the mainline prices of C_{Hk} and C_{Tk}, associated with each population segment k with Eq. (27). The same constraints hold, but with a corresponding set for each k.

$$\min\sum_K\sum_J C^2_{jk} \tag{27a}$$

$$\text{s.t.}\, r^*_{1k}\widehat{C}^d_{Wk}-r^*_{1k}\widehat{C}^d_{Bk}+\widehat{C}^f_{Wk}-\widehat{C}^f_{Bk}=\xi_k\left[t_B-t_W+r^*_{1k}\left(\frac{1}{v_B}-\frac{1}{v_W}\right)\right]+r^*_{1k}\left(C^d_{Wk}-C^d_{Bk}\right)+C^f_{Bk}-C^f_{Wk} \tag{27b}$$

$$r^*_{2k}\widehat{C}^d_{Bk}-r^*_{2k}\widehat{C}^d_{Dk}+\widehat{C}^f_{Bk}-\widehat{C}^f_{Dk}=\xi_k\left[t_D-t_B+r^*_{2k}\left(\frac{1}{v_D}-\frac{1}{v_B}\right)\right]+r^*_{2k}\left(C^d_{Bk}-C^d_{Dk}\right)+C^f_{Dk}-C^f_{Bk} \tag{27c}$$

$$C^d_{Wk}+\widehat{C}^d_{Wk}+\frac{\xi_k}{v_W}\geqslant C^d_{Bk}+\widehat{C}^d_{Bk}+\frac{\xi_k}{v_B}\geqslant C^d_{Dk}+\widehat{C}^d_{Dk}+\frac{\xi_k}{v_D} \tag{27d}$$

$$C^f_{Wk}+\widehat{C}^f_{Wk}+\xi_k t_W\leqslant C^f_{Bk}+\widehat{C}^f_{Bk}+\xi_k t_B\leqslant C^f_{Dk}+\widehat{C}^f_{Dk}+\xi_k t_D \tag{27e}$$

$$\sum C_{jk}=0 \tag{27f}$$

Empirical estimation of the VOT in transport is typically achieved by comparing the coefficients β, for time and cost fitted in a discrete choice model (Hensher, 1976; Li and Hensher, 2012; Hensher, 2001; Small et al., 2005). Estimating VOT varying across income is accomplished by estimating a multinomial logit choice model with income as fixed effects,

$$V_{ijk}=\alpha_j+\beta^{time}_k t_{ik}+\beta^{\$}_k C_{ik} \tag{28}$$

where V_{ijk} is the mode choice utility, α_j is the intercept for choice alternative j, and C_{ik} and t_{ik} are the costs imposed on the individual i in money and time, respectively; with income group k as the fixed effect in the model. From this, discrete VOT can be calculated for each income group k as

$$\xi_k=\frac{\beta^{time}_k}{\beta^{\$}_k} \tag{29}$$

In order to provide any number of discrete segment breaks in a VOT distribution, a smooth continuous distribution function can be fitted to a sample. However, such a sample distribution does not exist and must be estimated. This is done by further modeling the correlation between income and VOT itself. A simple linear model would follow the form $\xi = \alpha + \beta Income$. Applying the linear VOT-income model to a sufficiently large sample distribution of income, the distribution can be fitted using the appropriate parametric model, such as a log-normal distribution. Once fitted, any number of discrete segments may be created from the distribution for analysis.

Accounting for varying income groups presents the prospect of evaluating equity in the system, in addition to efficiency. The Gini coefficient is a common comparative measure of equity in the system (Gini, 1912; Litchfield, 1999). Moreover, it can be used with discrete probability distributions with Eqs. (30).

$$G = \frac{1}{2\mu}\sum_{k=1}^{n}\sum_{h=1}^{n} f\left(y_k\right) f\left(y_h\right)|y_k - y_h| \tag{30a}$$

$$\mu = \sum_{i=1}^{n} y_k f\left(y_k\right) \tag{30b}$$

where G is the Gini coefficient which ranges from 0 to 1, 0 being perfectly equitable and 1 being perfectly inequitable. $f(y_k)$ is the discrete probability function for the percent of the population with income y_k. The Gini coefficient is calculated as one half the total absolute relative difference in income normalized by sum of all costs in the population, μ. In this paper, instead of income we use average generalized cost $\overline{GT}_k$, for each VOT group k.

4. Data

As an application of the proposed model, an ideal case study is the Worcester to Boston commute in Massachusetts, USA; shown in Fig. 5. Worcester is a satellite city to Boston with competing mainline transportation modes of tolled highway and a commuter rail line. Both the highway and commuter rail line originate at relatively central locations in Worcester and Boston.

The value of time (VOT) was modeled by first estimating the VOT for eight discrete household income groups using data from the Massachusetts Household Travel Survey (MTS) containing 61,777 trips reported by 15,828 travelers (Massachusetts Department of Transportation, 2012). The monetary costs for the respective modes and their alternatives were determined by either imputing known prices based on location for commuter rail and transit fares, or by using travel distance with an average cost of \$0.31 per km (≈ \$0.50 per mile) for driving. The travel times were obtained using the estimated travel time matrices from the Boston Region Metropolitan Planning Organization (MPO)/Central Transportation Planning Staff (CTPS) (Massachusetts Central Transportation Planning Staff, 2018). Once a VOT was determined for each of the income groups, a linear model was constructed using the median income value of each group, shown in Fig. 6a.

This VOT model was then applied to a much larger data set using the 2016 Massachusetts Public Use Microdata Sample (PUMS) containing 343,615 individuals, provided by the U.S. Census (U.S. Census Bureau, 2016). This larger distribution was then used to fit a continuous log-normal distribution used to model population density δ_k for k income groups, shown in Fig. 6b. The mean VOT was determined to be \$17.21 per hour, with a standard deviation of \$1.93 per hour. This mean value is used as the VOT for the single VOT validation example.

For the purposes of demonstrating the application of the model, the travel time function for the highway between Worcester and Boston is characterized by Eq. (31),

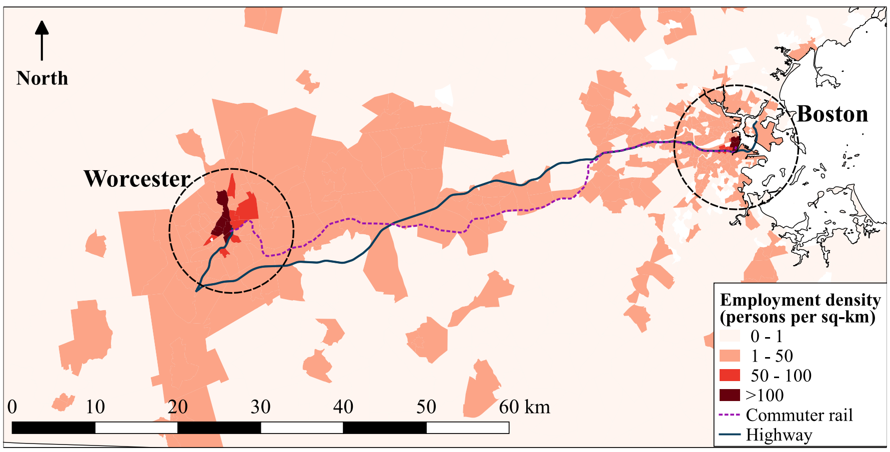
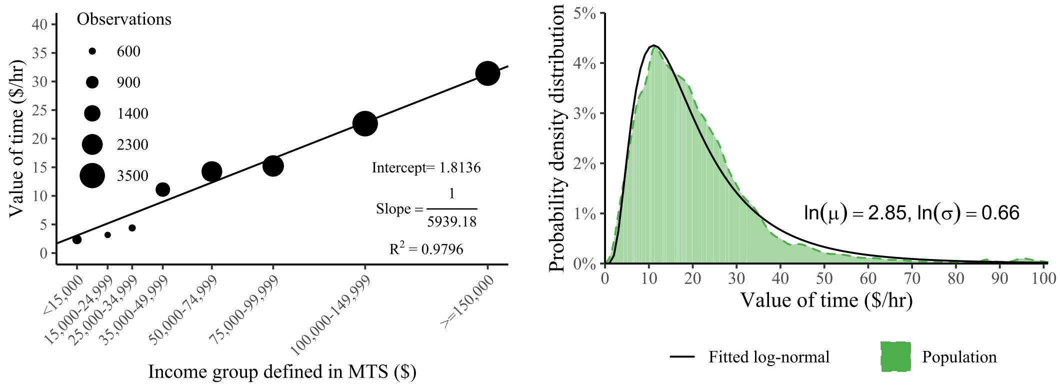


Fig. 5. Worcester residents employment density.



(a) Value of time and income in Massachusetts MTS (b) Value of time distribution in PUMS

Fig. 6. Value of time distribution.

$$T_H\left(\lambda_H\right) = t_H + \frac{L}{v_H}\left[1 + \alpha\left(\frac{\lambda_H}{\gamma}\right)^{\phi}\right] \tag{31}$$

where λ_H is the highway demand, and α,ϕ, and γ are model parameters. Although a hard constraint for capacity is not set, it is assumed that the model results will never exceed capacity. This expression represents only the uncongested lower part of the travel time vs. flow relation. A summary of the basic parameters used for this numerical example are detailed in Table 2, which includes both the general system parameters, as well as vehicle specific parameters.

5. Results

The results section is split into two main parts using data for the Worcester-Boston commute example case. First, the deterministic model itself is validated against a logit-based counterpart for a simplified case with a single average VOT. Second, the deterministic model is extended to include multiple values of time across the distribution.

5.1. Validation

Before the deterministic model is used to find optimal prices, it is first validated against the stochastic logit-based model to

Table 2
Worcester–Boston commuter parameters.

System Parameters

Parameter	Value	Units	Description
δ	49.14	pax/km^2	Average commuting population density
ξ	17.21	$\$/hour$	Value of time (VOT)
β	−1	*unitless*	Stochastic model scaling parameter
R	7.5	*km*	Approximate access radius
L	70	*km*	Approximate distance to Boston
α	0	–	Congestion model parameter
ϕ	4	–	Congestion model parameter
γ	4,400	veh/hr	Highway throughput parameter
Commuters	3%	–	Share of population that travels from Worcester to Boston
Train boardings	1,000	pax	Average daily passenger boardings at Worcester
Train auto parking	700	pax	Number of auto parking spaces available
Train bike parking	30	pax	Number of bike parking spaces
Train walk access	270	pax	Remaining number of passengers boarding minus parking

Vehicle Specific Parameters

Parameter	Units	Description	Walk	Bike	Drive	Train	Highway
v	$km/hour$	Speed	5	10	40	50	100
t_0	hour	Startup time	0	5/60	10/60	0	0
C^d	$\$/km$	existing distance prices	0	0.1	0.31	0	0.31
C^f	$	existing fixed prices	0	0	5	12.25	2.5

investigate how varying the value of time ξ, the stochastic model's scaling parameter β, and the pricing scheme (e.g., fixed and/or distance based prices) can affect the results. The validation is done by directly comparing estimated demand for each mode using the deterministic and logit models, $\lambda^{(d)}$ and $\lambda^{(s)}$, respectively.

Since the logit model does not provide a closed form solution for the access mode choice, a comparison must be evaluated numerically. This is done by generating 10,000 random price combinations that range between -\$10 to \$10 for the fixed prices C_a^f, and distance-based prices C_a^d. The resulting demand estimated from the deterministic and stochastic models using the 10,000 combinations are then compared using percent root-mean square error (RMSE) as an overall measure of fit, expressed in Eq. (32) as

$$\%RMSE = \frac{100}{\lambda_{total}} \cdot \sqrt{\frac{\sum_{i=1}^{N} (\lambda^{(d)} - \lambda^{(s)})^2}{n}} \tag{32}$$

where $\lambda^{(s)}$ and $\lambda^{(d)}$ are the stochastic and deterministic estimation for demand, respectively. Alternatively, the comparison could be using measured demand versus modeled demand.

This process is performed three times for a given value of time ξ, and scaling parameter β. Once with only fixed prices varying, another with only distance prices varying, and a third with both prices varying. These three pricing comparisons are then repeated across a range of VOTs, and scaling parameters. The VOT ranges from 0 to 50 with 5 $\$/hour$ increments and the scaling parameter ranges from -10^{-1} to -10^{1} (-0.1 to -10) using logarithmic increments of $-10^{0.25}$. The results of the VOT and scaling parameter error are shown in Fig. 7.

The overall analysis results in Fig. 7a show that the greatest loss in accuracy occurs at lower VOTs, but the magnitude of the loss differs when using the different price sets. With fixed costs only, the model accuracy increases from 17% to 2% RMSE over a range of 5 to 50$\$/hour$. The same is true for distance prices only, but to a lesser degree, ranging from 6% to 2% RMSE over the same VOT range. However, when using a combination of pricing schemes, different values of time have little effect on the model's accuracy, ranging from 4% to 2% RMSE, with an average of 2.56% RMSE.

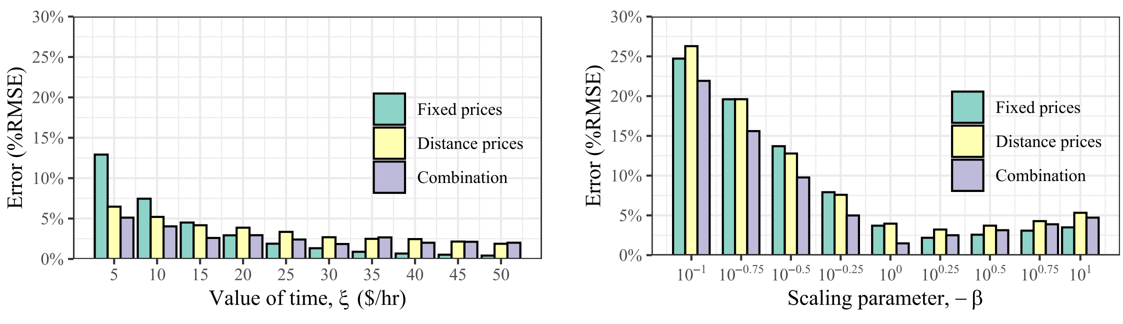
The effect that the scaling parameter has on accuracy is much more severe than that of varying VOT. As the magnitude of β approaches 0 from -1, the error rapidly increases to approximately 25% from less than 5% error (see Fig. 7b. This error is likely to continue increasing as β continues towards 0 from 0.1. The reason for this is that as the magnitude of the scaling parameter shrinks, the difference in dis-utility between the discrete choices becomes relatively small, thus affecting how probability is calculated in the logit. So as β approaches zero, the logit approaches equal probability for all choices. Meanwhile, the deterministic model is unaffected as it does not contain the β scale parameter. However, it is interesting that the trend does not continue as β exceeds -1 and approaches -10. Instead the error gradually increases as well, but to a lesser degree, with a low point at approximately -1. Suggesting other sources of error are exaggerated, such as the nested mainline mode choice.

5.2. Single value of time – optimal mode split

Using the observed mode-share in Table 2, the effective "observed" radii and θ are calculated using Equation set (18). The optimal radii and θ are then determined through optimization of Eq. (17). The calculated observed radii and optimal radii are presented in Table 3.

The shape of the objective function is shown in Fig. 8. In Fig. 8a the gradient contours for the travel time are plotted by varying the access mode radii while holding the third term θ as constant. A unique optimal point is visible at the bottom of the distorted bowl shape. Conversely, in Fig. 8b the travel time function is plotted by varying mainline θ and holding the radii constant. Two surfaces are present in Fig. 8b because the optimal radii are used for the optimal case and the observed effective radii are used for the observed case. Regardless of radii, there appears to be a unique optimum at the bottom of the curve.

Another perspective of the results is the highway travel time as users are loaded onto the highway, as shown in Fig. 9. One might

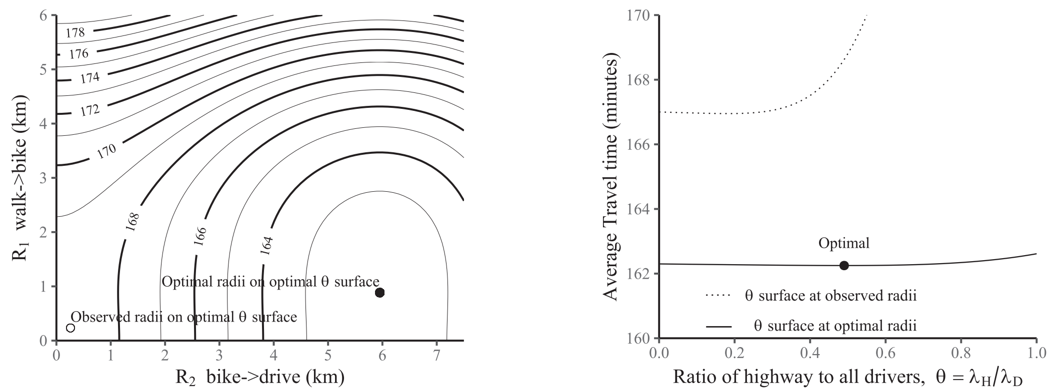


(a) Effects of value of time (ξ) on model accuracy (b) Effects of β scaling parameter on model accuracy

Fig. 7. Effects of scaling parameter (β) and value of time (ξ) variation on model accuracy.

Table 3
Optimal and observed radii.

	r_1 (km)	r_2 (km)	θ	Average travel time (minutes)
Observed	0.23	0.26	0.92	207.63
Optimal	0.88	5.95	0.49	162.25



(a) Travel time across radii with optimal θ (b) Travel time across θ with optimal radii

Fig. 8. Optimal demand allocation.

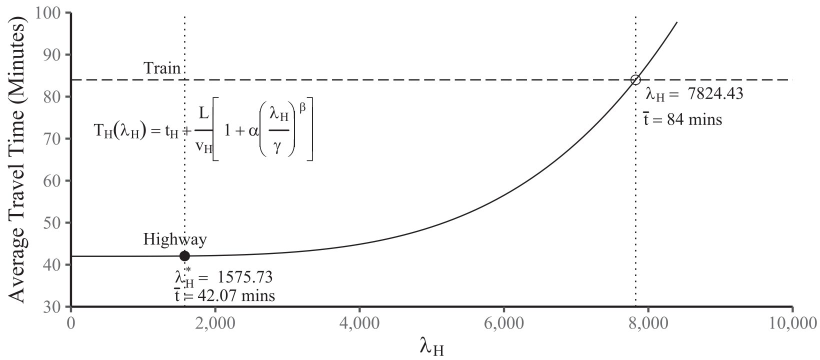


Fig. 9. Mainline travel time with congestion.

assume that the optimal highway utilization would be when the highway travel time is equal to the train travel time. However, a more global optimal point actually leaves the highway with excess capacity. This is because the access mode is being considered in the proposed model. System performance is improved by maintaining a free-flowing highway since more drivers take the train, as well as by leveraging the shorter start up times and access distances for walk and bike.

5.3. *Single value of time – optimal prices*

Using the proposed optimization approach in Eq. (22), optimal access mode prices are determined for the three price sets of fixed

Table 4
Optimal pricing.

Price set	Access prices						Mainline prices		Model
	$\widehat{C}_W^d$	$\widehat{C}_B^d$	$\widehat{C}_D^d$	$\widehat{C}_W^f$	$\widehat{C}_B^f$	$\widehat{C}_D^f$	$\widehat{C}_H$	$\widehat{C}_T$	
Fixed only	–	–	–	-$1.06	-$0.02	$1.08	$3.64	-$5.52-$5.81	Stochastic Deterministic
Distance only	-$4.37	$0.06	$4.31	–	–	–	$3.83		
Combination	-$0.23	-$0.03	$0.26	-$1.00	-$0.01	$0.26			

prices only, distance prices only, and the combination of the two. Each of the three price sets are optimized using a revenue neutral scheme. The mainline prices are calculated using the nesting approach proposed in Eq. (24), also using a revenue neutral scheme. The optimal prices are presented in Table 4 for the three sets of prices. The table also presents the optimal mainline prices.

The effect on results for each pricing set are compared by modeling the demand for the observed condition using stochastic and deterministic methods as well as for the optimal condition without additional pricing. The results of this are presented in Table 5. A percent RMSE is calculated for each model's fit against the observed condition, as well as between model fit (i.e., difference between deterministic and stochastic). The reason that the observed condition is modeled with pricing is because the optimal point is when costs are zero. The additional pricing costs reflect the unobserved utility "deficit" required to move the observed condition to the optimal one. Modeling the observed condition also provides a useful measure of overall accuracy, not just relative precision between models.

As a demonstration of how the dis-utility functions are modified by the prices, Fig. 10 displays both the original and optimized functions for dis-utility and subsequent logit curves using a combination price set. In both Figs. 10a and b the original functions without any additional costs are shown in gray, and the optimized functions are shown in black. The dis-utility plot in Fig. 10a shows how the functions are adjusted to intersect at the optimal radii, shown as the vertical dotted lines. The original dis-utility functions all nearly intersect at the same point, reflecting the small number of bicyclists. In the logit plot in Fig. 10b, the curves for walk and bike grow to represent a much larger share while the driving curve shrinks.

An artifact of the model is that the dis-utility functions intersect exactly where the logit functions intersect. However, unlike the all-or-nothing deterministic case, the intersection merely represents where the probabilities intersect. This corresponds to the results in Table 5 where the deterministic model always yields the same result, but the stochastic model can vary. This is because the deterministic model simply finds the radii where dis-utility intersects to approximate demand. Thus, as long as dis-utility intersects at the desired radii the result is the same, regardless of slope or intercept. However, the stochastic model determines demand by integrating the area under the non-linear probability curve across the radii. The steepness of the curves, or the magnitude of the difference in dis-utility, will then affect the results.

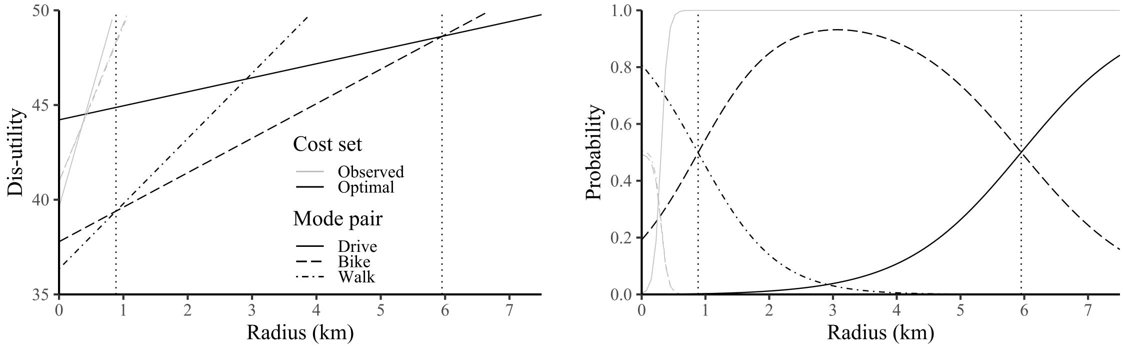
5.4. Distributed value of time – optimal mode split

Using the continuous log-normal distribution function for VOT, a range of discrete probability distribution segments is created. The product of this probability distribution and the population density δ provides a set of densities δ_k, for each VOT segment in the population. From this, a set of optimal parameters for r_{1k}, r_{1k}, θ_k, are determined for each segment k in the initial parameter optimization step. In this paper, a discrete distribution size of thirty equal-length segments were chosen for optimization. Although this is a fairly large number of segments, it was chosen in order to provide a reasonably smooth result for analysis and graphical representation. The results of the parameter optimization step are shown in Fig. 11.

The horizontal axis in Fig. 11 is the range of VOT and the vertical axis shows the tipping point radii for the corresponding VOT (i.e., the point at which users change mode choice). The gradual descent of bike users makes intuitive sense, because as the users' VOT increases, the time savings of the much faster drive mode begins to outweigh the cheaper but slower bike mode. This is also true for the walk mode, but at a much steeper slope, indicating that all but the poorest users will choose another mode if they travel further than approximately 2/3 of a kilometer. A particularly interesting result is the severe change of drive users from train to highway. Unlike the single VOT case where θ is between zero and one, here we see that the shift from train to highway is fairly severe depending upon VOT. This is because from the perspective of an individual VOT group, the choice between train and highway doesn't vary by distance and only depends on whether the VOT is high enough to justify the travel time savings. A θ between zero and one would occur when a VOT group spans the range of VOT where the choice between drive to highway and drive to train occurs, as is the case when the mean VOT is used for the entire system.

Table 5
Model demand estimation comparison.

Price scheme	Model	λ_W (veh/hr)	λ_B (veh/hr)	λ_D (veh/hr)	$\lambda_{D\|T}$ (veh/hr)	$\lambda_{D\|H}$ (veh/hr)	TT (mins)	RMSE (%) Deterministic vs. Stochastic	RMSE (%) Observed vs. model
Matching observed condition									
	Measured	270.0	30	8,383	700.0	7,683.0	103.99	–	–
Combination	Deterministic	8.0	2	8,673	1.0	8,672.0	122.86	1.29%	6.56%
	Stochastic	29.0	132	8,522	1.0	8,521.0	118.28		5.82%
Distance	Deterministic	8.0	2	8,673	0.0	8,673.0	122.86	0.17%	6.56%
	Stochastic	9.0	20	8,654	0.0	8,654.0	122.26		6.46%
Fixed	Deterministic	8.0	2	8,673	1.0	8,672.0	122.86	1.84%	6.56%
	Stochastic	36.0	188	8,459	1.0	8,458.0	116.51		5.58%
Optimal condition									
–	Deterministic	107.0	83	8,492	4,246.0	4,246.0	100.83	2.57%	–
	Stochastic	130.0	384	8,169	4,084.0	4,084.0	68.15		



(a) Optimized access mode dis-utility (b) Optimized access mode logit

Fig. 10. Optimal logit and dis-utility comparison.

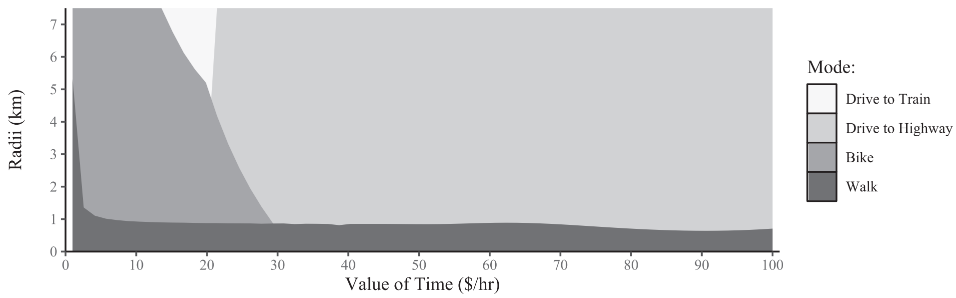


Fig. 11. Optimal radii by VOT.

5.5. *Distributed value of time – optimal pricing*

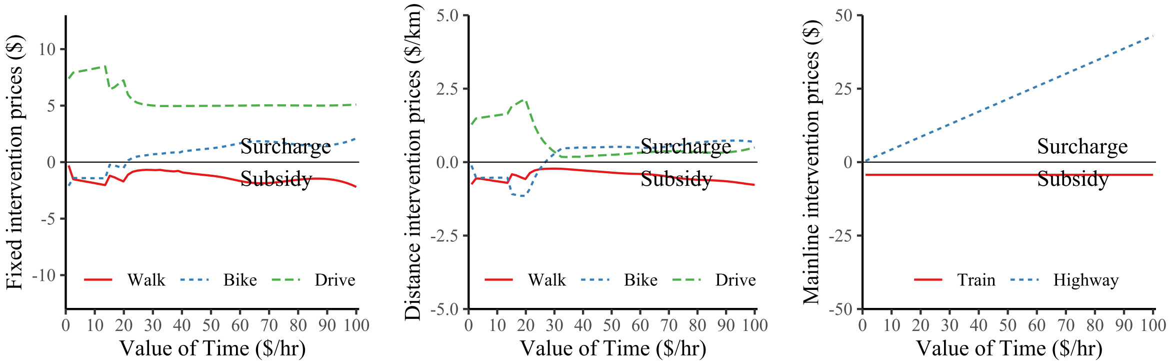
Although the results of a single mean VOT system optimization may simply be the aggregated result of a distributed VOT system, the pricing policy and generalized cost experienced do not affect users equally. A set of prices for all VOT groups was determined simultaneously using Eq. (27), minimizing the total amount of money changing hands while achieving the desired radii and θ parameters for all k. The results of this are presented in Fig. 12, displaying the costs of fixed only, distance only, and combination across VOT.

When the VOT is near or below the mean VOT, the fixed- and distance-based prices for walk, bike, and train are subsidized, while drive and highway have a surcharge. This largely reflects the results obtained in the single VOT case. However, as VOT increases, the subsidies gradually decrease (prices increase) while driving becomes cheaper. While having driving be cheaper for higher VOTs makes little intuitive sense from an equity perspective, this result is because the model is merely attempting to keep the system revenue neutral. It is plausible that in a revenue positive case, the price for driving could also increase, or at least stay constant, while still achieving the desired modal distribution. The pricing results are similar for the mainline mode price where train is subsidized, while highway prices are a surcharge that increases with VOT.

5.6. *Distributed value of time – equity*

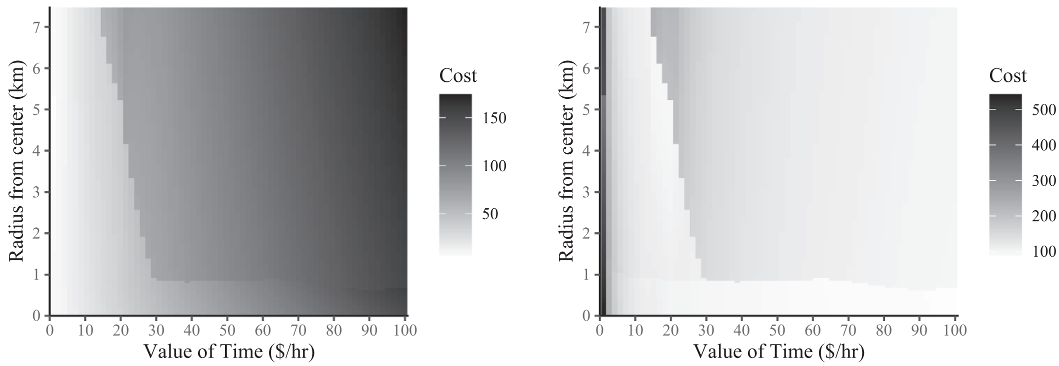
A fundamental disadvantage of a system optimum is the potential for users to experience inequitable costs. These costs can be experienced across two user variables, VOT or radii, and is measured either as the generalized time cost or as the generalized monetary cost. These costs are presented graphically by VOT and radii with generalized monetary cost in Fig. 13a and generalized time cost in Fig. 13b.

When measuring generalized cost as a monetary cost, the wealthier VOT users appear to experience the greatest cost burden, but when viewed from a temporal lens, the poorest travelers experience the greatest burden. This is not just because wealthier users experience greater pricing costs, but because the poorer users' VOT is so low that their burden appears low from a monetary perspective. When measuring equity, time can be considered as a more robust and objective measure because time is a non-transferable and equally finite resource. Meaning that time cannot be redistributed the way money can, and once it is expended it cannot be redeemed. In addition, all users possess the same initial "wealth" of time by sharing a 24-h day. For these reasons, time will be used as the measure of equity, rather than the monetary cost. Furthermore, it makes moral sense to use the measure that exposes the cost burden on lower income users, not the inverse. The equity measure used is the Gini coefficient, which is calculated using Eq. (30)



(a) Fixed prices by VOT (b) Distance based prices by VOT (c) Mainline prices by VOT

Fig. 12. Optimal pricing varying by VOT.



(a) Generalized monetary cost ($) (b) Generalized time cost (minutes)

Fig. 13. Comparison between generalized monetary and temporal user costs.

across VOT. It is calculated across VOT for two reasons. First, because users can feasibly choose their location in the radii but they cannot so easily choose their VOT. Second, radii is a physical cost based on location, it would make little sense to subsidize users to live further away.

The overall results of the distributed VOT optimization achieved a 22% reduction in average generalized travel time from 225 min to 176 min per traveler. The resulting average generalized time costs are higher than the single VOT validation example in Table 3. This is because now the average cost accounts for the much higher costs experienced by low VOT users whereas before it only considered the average VOT and the average travel time. Despite this, the Gini equity measure improved by 30% from 0.2 to 0.14. Fig. 14 is a

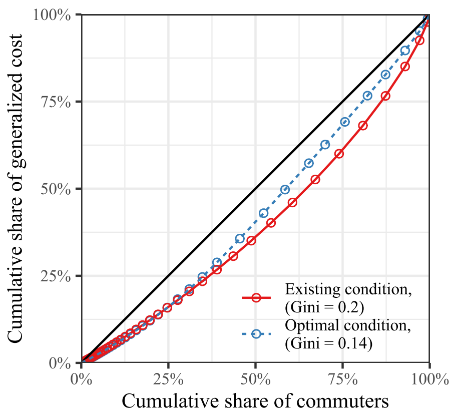


Fig. 14. Lorenz plot at observed and optimal conditions.

graphical representation of the Gini coefficient as the ratio of the area under the Lorenz curve to the area under the diagonal. A perfectly equitable society (i.e., Gini $= 1$) would be a diagonal line and a perfectly inequitable society (i.e., Gini $= 0$) would be along the bottom edge.

This is an interesting result considering that neither objective function directly contains any equity factor. This is because when minimizing the total generalized *time* cost, the burden on low VOT users is taken into account effectively since money is used as a transferable resource to achieve this minimized overall cost. This is apparent in Fig. 12c where low VOT users are actually subsidized as an incentive to take the faster highway mode.

6. Conclusions

This paper presents a deterministic approximation of a discrete choice model for mixed-modal access and mainline mode choice that can account for VOT equity. The purpose is to develop a choice model that can be efficiently optimized to obtain pricing schemes; a current challenge using stochastic logit-based models. This is achieved by first optimizing demand using a spatial model, then subsequently determining the pricing necessary to achieve the optimal demand. The added complexity of nested mode choice is modeled using an auxiliary spatial model to determine the mainline cost differential. In addition to the model's efficient optimization, it also possesses the ability to account for varying values of time and the flexibility to set both fixed and distance based pricing policies for specific modes. An important insight from the performed analysis is that efficiency (minimizing total system cost) is not necessarily at odds with equity among traveler experiences, and pricing can be used to make an efficient outcome more equitable.

Overall, the deterministic model provides a reasonable approximation of its logit-based counterpart within 3% error. Although error was observed, the majority of this error is largely systematic with respect to each cost type when dissected by fixed and distance based costs. It is possible that future efforts could correct this systematic error while remaining deterministic. Despite the error, a major benefit of the simplistic model is that it requires very little computational resources, allowing for future applications in larger networks. One such application could include a corridor of stations as in Fig. 15a that feed into a central business district in a commuter rail type model. At a more macroscopic level, the model could be applied to a network of cities, as in Fig. 15b, that could be optimized for inter-city travel. In any case, it may also be further used to incorporate agency cost into the model to account for the cost of operation, not merely user cost.

Appendix A

Theorem 1. *The total generalized cost, $GT(r_1, r_2, \theta)$, with conditions (12a) and (12b) is a unimodal function over the feasible space $r_1, r_2 \geqslant 0, \theta \in (0,1)$. The values of $\widehat{r}_1$ and $\widehat{r}_2$ that satisfy the first order necessary conditions correspond to the global minimum.*

Proof. We first identify the radii that satisfy the first order necessary conditions for optimizing GT, and then we show that GT is unimodal with this point representing a global minimum.

$GT(r_1, r_2, \theta)$ is a continuously differentiable function over the feasible space, so the first order necessary condition for optimality of $GT(r_1, r_2, \theta)$ is that $\nabla GT(\widehat{r}_1, \widehat{r}_2, \widehat{\theta}) = 0$.

$$\frac{\partial GT}{\partial r_1} = 2r_1\left[t_W + \frac{C_W^f}{\xi_a} - t_B - \frac{C_B^f}{\xi_a}\right] + 2r_1^2\left[\frac{1}{v_W} + \frac{C_W^d}{\xi_a} - \frac{1}{v_B} - \frac{C_B^d}{\xi_a}\right] = 0 \tag{33a}$$

$$\begin{aligned}\frac{\partial GT}{\partial r_2} &= 2r_2\left[t_B + \frac{C_B^f}{\xi_a} - t_D - \frac{C_D^f}{\xi_a} + \theta\left(t_T + \frac{L}{v_T} + \frac{C_T^f}{\xi_a} - T_H\left(\lambda_H\right) - \frac{C_H^f}{\xi_a}\right)\right] \\ &+ 2r_2^2\left[\frac{1}{v_B} + \frac{C_B^d}{\xi_a} - \frac{1}{v_D} - \frac{C_D^d}{\xi_a}\right] = 0\end{aligned} \tag{33b}$$

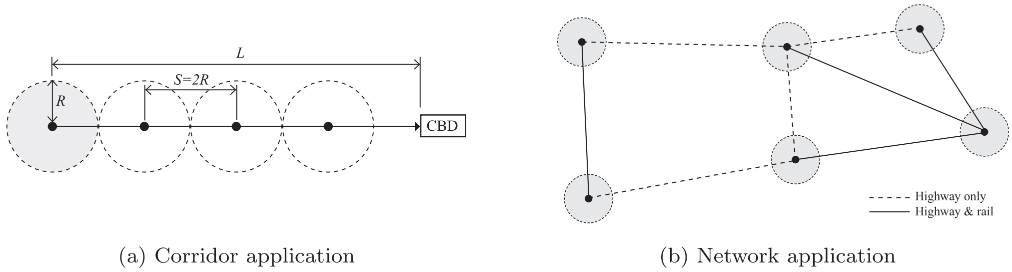


Fig. 15. Possible model applications.

$$\frac{\partial GT}{\partial \theta} = 2r_2\left(t_T + \frac{L}{v_T} + \frac{C_T^f}{\xi_a} - T_H\left(\lambda_H\right) - \frac{C_H^f}{\xi_a}\right) = 0 \tag{33c}$$

For any non-zero r_2, (33c) requires that expression in parentheses is 0, which implies that the cost of mainline travel by highway and transit are equivalent. This expression is also the coefficient of θ in (33b), so $\partial GT/\partial r_2$ can be expressed as

$$\frac{\partial GT}{\partial r_2} = 2r_2\left[t_B + \frac{C_B^f}{\xi_a} - t_D - \frac{C_D^f}{\xi_a}\right] + 2r_2^2\left[\frac{1}{v_B} + \frac{C_B^d}{\xi_a} - \frac{1}{v_D} - \frac{C_D^d}{\xi_a}\right] = 0 \tag{34}$$

The first order conditions for r_1 and r_2 are univariate functions, so each can be solved independently to identify the optimal values $\widehat{r}_1$ and $\widehat{r}_2$.

$$\widehat{r}_1 = \frac{t_B + \frac{C_B^f}{\xi_a} - t_W - \frac{C_W^f}{\xi_a}}{\frac{1}{v_W} + \frac{C_W^d}{\xi_a} - \frac{1}{v_B} - \frac{C_B^d}{\xi_a}} \tag{35a}$$

$$\widehat{r}_2 = \frac{t_D + \frac{C_D^f}{\xi_a} - t_B - \frac{C_B^f}{\xi_a}}{\frac{1}{v_B} + \frac{C_B^d}{\xi_a} - \frac{1}{v_D} - \frac{C_D^d}{\xi_a}} \tag{35b}$$

The numerators and denominators in (35a) and (35b) are all positive based on the conditions (12a) and (12b), so $\widehat{r}_1, \widehat{r}_2 \geqslant 0$.

We now show that GT is unimodal with respect to r_1. Using expression in $\partial GT/\partial r_1$ from (33a) and the expression for $\widehat{r}_1$ from (35a), the values r_1 satisfying $\partial GT/\partial r_1 < 0$ can be expressed as $r_1 < \widehat{r}_1$, which is the range over which GT is decreasing. Similarly, the values of r_1 satisfying $\partial GT/\partial r_1 > 0$ can be expressed as $r_1 > \widehat{r}_1$, which is the range over which GT is increasing. By definition, this makes GT a unimodal function of r_1 with a global minimum at $\widehat{r}_1$. The same logic can be used with (34) and (35b) to show that GT is a unimodal function of r_2 with a global minimum at $\widehat{r}_2$. □

References

Anas, A., Arnott, R., Small, K., 1998. Urban spatial structure. J. Econ. Literature 36, 1426–1464.
Anas, A., Xu, R., 1999. Congestion, land use, and job dispersion: A general equilibrium model. J. Urban Econ. 45, 451–473.
Arnott, R., Small, K., 1994. The economics of traffic congestion. Am. Sci. 82, 446–455.
Azevedo, C.L., Seshadri, R., Gao, S., Atasoy, B., Akkinepally, A.P., Christofa, E., Zhao, F., Trancik, J., Ben-Akiva, M., 2018. Tripod: Sustainable Travel Incentives with Prediction, Optimization, and Personalization. In: 97th Annual Meeting of the Transportation Research Board 1, 19.
Badia, H., Argote-Cabanero, J., Daganzo, C.F., 2017. How network structure can boost and shape the demand for bus transit. Transport. Res. Part A: Policy Practice 103, 83–94.
Badia, H., Estrada, M., Robusté, F., 2014. Competitive transit network design in cities with radial street patterns. Transport. Res. Part B: Methodol. 59, 161–181.
Beckmann, M., McGuire, C., Winston, C., 1956. Studies in the Economics of Transportation. Yale University Press, New Haven, CT.
Ben-Akiva, M., De Palma, A., Kanaroglou, P., 1986. Dynamic model of peak period traffic congestion with elastic arrival rates. Transport. Sci. 20, 164–181.
Ben-Akiva, M.E., Lerman, S.R., 1985. Discrete Choice Analysis: Theory and Application to Travel Demand, vol. 9. MIT Press.
Boarnet, M.G., Giuliano, G., Hou, Y., Shin, E.J., 2017. First/last mile transit access as an equity planning issue. Transport. Res. Part A: Policy Practice 103, 296–310.
Chen, A., Oh, J.S., Park, D., Recker, W., 2010. Solving the bicriteria traffic equilibrium problem with variable demand and nonlinear path costs. Appl. Math. Comput. 217, 3020–3031.
Chen, H., Gu, W., Cassidy, M.J., Daganzo, C.F., 2015. Optimal transit service atop ring-radial and grid street networks: a continuum approximation design method and comparisons. Transport. Res. Part B: Methodol. 81, 755–774.
Daganzo, C.F., 2010. Structure of competitive transit networks. Transport. Res. Part B: Methodol. 44, 434–446.
Daganzo, C.F., Sheffi, Y., 1977. On stochastic models of traffic assignment. Transport. Sci. 11, 253–274.
Dial, R.B., 1996. Bicriterion traffic assignment: basic theory and elementary algorithms. Transport. Sci. 30, 93–111.
Fujii, S., Kitamura, R., 2000. Anticipated travel time, information acquisition, and actual experience: Hanshin expressway route closure, osaka-sakai, japan. Transport. Res. Rec.: J. Transport. Res. Board 79–85.
Gan, H., Ye, X., 2018. Will commute drivers switch to park-and-ride under the influence of multimodal traveler information? a stated preference investigation. Transport. Res. Part F: Traffic Psychol. Behav. 56, 354–361.
Gini, C., 1912. Variabilità e mutabilità. Reprinted in Memorie di metodologica statistica (Ed. Pizetti E, Salvemini, T). Rome: Libreria Eredi Virgilio Veschi.
Gonzales, E.J., 2015. Coordinated pricing for cars and transit in cities with hypercongestion. Econ. Transport. 4, 64–81.
Gonzales, E.J., Daganzo, C.F., 2012. Morning commute with competing modes and distributed demand: user equilibrium, system optimum, and pricing. Transport. Res. Part B: Methodol. 46, 1519–1534.
Gonzales, E.J., Daganzo, C.F., 2013. The evening commute with cars and transit: duality results and user equilibrium for the combined morning and evening peaks. Transport. Res. Part B: Methodol. 57, 286–299.
Hearn, D.W., Ramana, M.V., 1998. Equilibrium and Advanced Transportation Modeling. Kluwer Academic Publishers. chapter Solving congestion toll pricing models. pp. 109–124.
Hensher, D.A., 1976. The value of commuter travel time savings: Empirical estimation using an alternative valuation model. J. Transport Econ. Policy 167–176.
Hensher, D.A., 2001. The valuation of commuter travel time savings for car drivers: Evaluating alternative model specifications. Transportation 28, 101–118.
Holroyd, E., 1967. The optimum bus service: a theoretical model for a large uniform urban area, in. In: Proceedings of the Third International Symposium on the Theory of Traffic FlowOperations Research Society of America.
Huang, H.J., 2000. Fares and tolls in a competitive system with transit and highway: the case with two groups of commuters. Transport. Res. Part E: Logist. Transport. Rev. 36, 267–284.
Huang, H.J., 2002. Pricing and logit-based mode choice models of a transit and highway system with elastic demand. Eur. J. Oper. Res. 140, 562–570.
Iida, Y., Akiyama, T., Uchida, T., 1992. Experimental analysis of dynamic route choice behavior. Transport. Res. Part B: Methodol. 26, 17–32.
Kraus, M., 2003. A new look at the two-mode problem. J. Urban Econ. 54, 511–530.

Levinson, D., Zhu, S., 2013. A portfolio theory of route choice. Transport. Res. Part C: Emerg. Technol. 35, 232–243.
Li, Z., Hensher, D.A., 2012. Estimating values of travel time savings for toll roads: Avoiding a common error. Transp. Policy 24, 60–66.
Litchfield, J.A., 1999. Inequality: Methods and Tools. Text for World Bank's website on Inequality, Poverty and Socio-economic performance. URL http://siteresources.worldbank.org/INTPGI/Resources/Inequality/litchfie.pdf.
Long, J., Szeto, W., 2019. Congestion and environmental toll schemes for the morning commute with heterogeneous users on parallel routes. Transp. Res. Part B 129, 305–333.
Lou, Y., Yin, Y., Lawphongpanich, S., 2010. Robust congestion pricing under boundedly rational user equilibrium. Transport. Res. Part B: Methodol. 44, 15–28.
Massachusetts Central Transportation Planning Staff, 2018. Massachusetts Travel Demand Model [data]. URL http://www.ctps.org/.
Massachusetts Department of Transportation, 2012. Massachusetts Travel Survey [data file]. Technical Report. Massachusetts Department of Transportation. URL https://www.mass.gov/files/documents/2018/05/24/MTSAppendices.pdf.
May, A.D., 2019. Measure Selection for Urban Transport Policy. Springer International Publishing, Cham. pp. 307–324. URL https://doi.org/10.1007/978-3-319-74983-9_16, doi:10.1007/978-3-319-74983-9_16.
McFadden, D., 1978. Modeling the choice of residential location. Transp. Res. Rec.
Newell, G.F., 1973. Scheduling, location, transportation, and continuum mechanics: some simple approximations to optimization problems. SIAM J. Appl. Mathe. 25, 346–360.
de Palma, A., Ben-Akiva, M., Lefevre, C., Litinas, N., 1983. Stochastic equilibrium model of peak period traffic congestion. Transport. Sci. 17, 430–453.
Pigou, A.C., 1920. The Economics of Welfare. MacMillan, New York.
Polak, J., 1998. The influence of alternative traveller learning mechanisms on the dynamics of transport systems. In: Transportation Planning Methods. Proceedings of seminar D held at AET European Transport Conference, Loughborough University, U.K., pp. 14–18.
Ren, T., Huang, H., Liu, T., Nie, Y., 2020. Some analytical results on spatial price differentiation in first-best congestion pricing schemes. Transp. Res. Part C 114, 425–445.
Rossi-Hansberg, E., 2004. Optimal urban land use and zoning. Rev. Econ. Dynam. 7, 69–106.
Schaller, B., 2017. Empty seats, full streets. fixing manhattan's traffic problem. Schaller Consulting, December 1.
Sivakumaran, K., Li, Y., Cassidy, M.J., Madanat, S., 2012. Cost-saving properties of schedule coordination in a simple trunk-and-feeder transit system. Transport. Res. Part A: Policy Practice 46, 131–139.
Small, K.A., Winston, C., Yan, J., 2005. Uncovering the Distribution of Motorists' Preferences for Travel Time and Reliability. Econometrica 73, 1367–1382.
Solow, R., 1972. Congestion, density and the use of land in transportation. Swedish J. Econ. 74, 161–173.
Solow, R., 1973. Congestion cost and the use of land for streets. Bell J. Econ. Manage. Sci. 4, 602–618.
Solow, R., Vickrey, W., 1971. Land use in a long narrow city. J. Econ. Theory 3, 430–447.
Tabuchi, T., 1993. Bottleneck Congestion and Modal Split. J. Urban Econ. 34, 414–431.
Train, K., 1978. A validation test of a disaggregate mode choice model. Transp. Res. 12, 167–174.
U.S. Census Bureau, 2016. 2012–2016 5-year Public Use Microdata Sample. https://www.census.gov/programs-surveys/acs/data/pums.html.
Vickrey, W.S., 1969. Congestion Theory and Transport Investment. Am. Econ. Rev. 59, 251–260.
Wang, G., Jia, N., Ma, S., Qi, H., 2014. A rank-dependent bi-criterion equilibrium model for stochastic transportation environment. Eur. J. Oper. Res. 235, 511–529.
Watling, D., 2006. User equilibrium traffic network assignment with stochastic travel times and late arrival penalty. Eur. J. Oper. Res. 175, 1539–1556.
Wheaton, W., 1998. Land use and density in cities with congestion. J. Urban Econ. 43, 258–272.
Yang, H., Bell, M.G., 1997. Traffic restraint, road pricing and network equilibrium. Transport. Res. Part B: Methodol. 31, 303–314.
Yang, H., Huang, H.J., 2005. Mathematical and Economic Theory of Road Pricing. Elsevier.
Yildirim, M.B., Hearn, D.W., 2005. A first best toll pricing framework for variable demand traffic assignment problems. Transp. Res. Part B 39, 659–678.
Zhang, K., Mahmassani, H.S., Lu, C.C., 2013. Dynamic pricing, heterogeneous users and perception error: Probit-based bi-criterion dynamic stochastic user equilibrium assignment. Transport. Res. Part C: Emerg. Technol. 27, 189–204.