Minority Control: An Analysis of British Companies using Voting Power Indices

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March 1999

(Warwick Economic Research Paper Number 529)

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Most of the theoretical work on methods of calculating power indices used in this paper was done during sabbatical leave in 1996/7 and completed during 1997/8 when reliable computer algorithms were devised and implemented. A very preliminary version covering some of the methodological issues were presented to research seminars in the Economics department and the Business School at Warwick in the Autumn Term 1997. I would like to thank the participants for their comments.

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Abstract

An exercise in the empirical use of voting power indices from cooperative game theory applied to ownership data for large companies, this paper contributes in two areas: (1) the analysis of company control based on shareholder voting power, and (2) the empirical use of power indices and understanding of the comparative properties of different indices. New algorithms for calculating power indices, which quantify voting power in weighted voting bodies like company meetings, are applied to detailed data on beneficial ownership of 444 large UK companies without majority control. The results show that the Banzhaf index is, and the Shapley-Shubik index is not, useful for this analysis and a control classification of the firms is obtained.

JEL Codes: G32, G34, C71, C88

Keywords: Ownership and control; power indices; Shapley-Shubik index; Banzhaf index; oceanic games; co-operative game theory.

I. Introduction

The relationship between large shareholdings and the control of the company, or more generally the power of ownership in corporate governance, is an area whose literature is still relatively undeveloped despite its importance and topicality. In general opinions on the role of shareholders in the running of a company tend to vary between extremes represented by some academic economists, who emphasise incentives, and advocates of shareholder activism, who emphasise the legal voting power attached to ownership. The dominant view among economists, deriving from Berle and Means, has tended to be that large companies have very dispersed ownership involving myriad small shareholders none of whom could be said to have any appreciable voting power or control. On this basis economists have tended to emphasise the moral hazard argument that even a large shareholder has little incentive to monitor the performance of the management, to take an interest in the direction of the firm and to vote their shares because their ownership stake – small in percentage terms - gives them only a small entitlement to the returns accruing to their investment in those activities. On the other hand advocates of investor activism and participants in the corporate governance movement base their position on the power of the vote entailed in equity ownership and have painted a different picture, of a world of large institutional investors whose power derives from managing huge pension funds and whose incentives to get the best return for their clients are beyond question. Thus a large shareholder is characterised either: as having the incentive to take part in the strategic direction of a company - and the question is whether he has sufficient voting power to be able to influence or control

it; or as powerful in terms of votes - and the focus of analysis then is whether he has enough of a stake in the company for there to be sufficient of an incentive to bother monitoring and voting..

This paper is a contribution to this literature. While the question of incentives is obviously equally as important as voting power in the analysis, we abstract from it here and concentrate solely on voting power, which is central to the idea of minority control. For present purposes we maintain the assumption that shareholders always do have incentives to take part in monitoring the management and voting their shares. We are here exclusively concerned with the important conceptual and statistical issue of the relation between the size of an ownership stake and the power or control it represents.

Our aim is twofold: to elucidate the separation of ownership and control revealed in comprehensive data on the ownership of a large sample of British companies, and to advance the understanding of the method of voting power indices in empirical application. The paper begins in section II with a discussion of minority control which sets out the general approach. In section III we define the formal approach to the measurement of voting power, present the precise definitions of the classical power indices and discuss the problems of applying them to this data set. In section IV we describe a model of control based on an index of the power of the largest shareholder and in section V discuss the important and open question of how to appraise the adequacy of the different power indices. The results are presented in section VI and section VII draws the conclusions.

Our overall conclusions are that power indices are a useful tool for defining and identifying minority control and that the results give unique insights into the relative properties of the classical power indices in empirical application. We also obtain the quite remarkable result that coalitions of small numbers of large shareholders are frequently capable of control independently of the size of the block of shares they represent.

II. Minority Voting Control

Control of a company can be defined as the right to exercise whatever discretion in strategic decision making exists. More generally the power of a shareholding is the extent to which the votes it commands can influence a decision taken by shareholders under majority voting[1]. Control is the limiting case where the ownership stake is so powerful that its influence becomes complete discretion. Given that it is universally acknowledged that working control can exist on the basis of a block of shares representing a minority of the voting capital, the key question is what kind of ownership structure can be said to entail ownership control, and, specifically, what size of holding can be said to be a controlling one.[2]

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[1] We are not concerned with the propensity of shareholders to attend company meetings or whether they participate or abstain in particular votes. From the perspective of this paper such matters are endogenous manifestations of the distribution of power among the shareholders.

[2] This perspective is essentially static, in that the firm is conceived in terms of a given set of shareholders with a given distribution of shares among them, and must explicitly exclude situations involving transfer of shares such as in corporate restructuring.

The concepts of control and power here are related to the structure of decision making within the firm and not to the preferences or behaviour of any individuals. This is intrinsic to their use as fundamental concepts. The important question is not, for example, whether a shareholder derives control by being able to attract sufficient votes from small shareholders by advocating policies of profit maximisation – in a contested vote both sides would claim to be doing that in any case – but rather whether a large shareholder has enough power to have the right to determine decisions in the abstract. Power is the capacity to influence those decisions to some degree. Control is an absolute property while power exists to a greater or lesser degree.

To use these definitions, couched in *a priori* terms (which are the basis of the technical analysis employed in this study) is only to make explicit what is implicit in common and universally understood language. When we make a statement that a company is controlled by an individual with a 30% ownership stake or that a financial institution is a powerful shareholder because it has 10% of the voting equity, we understand something about voting power in a simple, strategic sense which does not involve preferences or behaviour. It is a normal aspect of analysis of the firm to consider ownership and control and voting power in general, abstract terms.

Voting by shareholders is about the making of a choice, not necessarily only over whether the firm maximises profits with a given technology, but over more strategic matters such as the fundamental nature of the products or the choice of production function. For example a coal mining company, following its loss of markets and the closure of pits, might be faced with a strategic decision whether to restructure itself as a

property company or become a pension fund. A conglomerate might have to decide on a proposal that it divest itself of a subsidiary and concentrate on operating within a single industry against an alternative strategy that it further diversify its activities into new industries. In such cases it is inappropriate to cast the problem facing the shareholders as a simple choice between present values of rival known profit streams. If a shareholder vote is needed to decide the matter this is because there is not unanimity among directors or investors so that the outcome depends on voting power then it is often the case that neither plan can be unambiguously shown to be better. The choice is formally similar to one between candidates or policies in an election or referendum where promises or forecasts about future events have to be evaluated with imperfect information. Methods of measuring control or power deriving from share ownership must therefore be neutral regarding the issue to be decided. Moreover it is precisely in such situations that ownership control and voting power matter in determining the outcome.

In general minority control differs from majority control in that it depends on a range of factors apart from voting power alone. Cases are frequently described where a company is effectively controlled by a group which owns a small equity stake but its influence rests not just on its voting power but also on other circumstances. For example a descendant of the company's founder may own a very small percentage of the equity but have considerable influence, a seat on the board, and so on, amounting to control deriving out of tradition and inheritance. Or a company may be effectively controlled by a shareholder whose ownership stake is enhanced by his vision, his strength of personality, his special knowledge and experience of the business or his

ability to convince the mass of shareholders of the rightness of his point of view. While such special factors are important in many cases they must be studied on an individual, *ad hoc*, basis. We abstract from such factors and seek to investigate the unique role of voting power in a formal and systematic way. That is not to deny the limitations of the analysis or the importance of other factors but formal voting power is an important element in corporate governance and is worth studying. The formal approach to the analysis of voting power is useful as providing a canonical model which gives results which can inform an empirically more complete analysis.[3]

III. The Measurement of Voting Power

Ever since the seminal contribution of Shapley and Shubik (1954) which proposed the use of power indices to measure power within a committee system which used weighted voting, the example of shareholder voting in a corporation has frequently been cited as a possible important application. [4] Despite this there have been only a very few previous empirical studies which have followed this suggestion.[5].

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[3] Economics has many similar examples. Perfect competition, homogeneous goods in imperfectly competitive markets, perfect capital markets, absence of transaction costs, symmetric oligopoly are a few cases of empirical abstraction which lead to useful results.

There is a close parallel to the power indices approach used here (and described in the next section) in the measurement of price changes. Index numbers which use fixed weights are known to be imprecise because the weights do not reflect changes over time in expenditure patterns or quality. Nevertheless they are widely employed and are useful despite this and their limitations are well understood.

[4] Shapley (1961), Milnor and Shapley (1978) studied the question theoretically.

[5] Leech (1988), Pohjola (1988), Rydqvyst (1986).

There seem to be several reasons for this. First, there has been a lack of suitable easily available algorithms for calculating values for large games. Second, at a more fundamental level, there is a lack of consensus on which of the various power indices - which appear to differ only in the technical way in which coalitions are counted - should be preferred. The most widely used alternative to the Shapley-Shubik index, the Banzhaf index (or Banzhaf-Coleman index, Banzhaf (1963), Coleman (1971)) has given similar results in some cases, but very different ones in others. Perhaps had appropriate algorithms been available before, there would have been more empirical studies and a clearer view of the relative value of the indices might have emerged. Both these aspects are central foci of this paper.

A third obstacle to the use of power indices for the analysis of company control is the limited nature of much available ownership data: typically we have a few large shareholdings (usually confined to those which exceed the legal disclosure level and directors' holdings in company reports) and it is necessary to adopt some *ad hoc* means of dealing with the smaller holdings[6]. In this study we use an excellent data set consisting of the sizes of all large shareholdings above 0.25%. Despite this very high level of detail there is still a problem of non-observed data and in order to deal with it we assume two limiting cases - the most concentrated and the most dispersed - in the

[6] There is also the possibility of collecting it directly from share registers but this has in the past proved to be very expensive because of the large numbers involved. Also the problem of identifying the beneficial shareholders from holdings by nominee companies adds another layer of difficulty. This problem will diminish in the future when developments in information technology enable share registers to become available in electronic form.

belief that the results can be regarded as upper and lower bounds. In the terminology of game theory the latter cases are referred to as oceanic games.

III(a). Power Indices

The model underlying the measurement of voting power is that of a game played by n players who co-operate by forming coalitions; that is, they vote for or against a motion in a hypothetical meeting. Coalitions may be winning or losing and each member's power is then defined in terms of quantifying his ability to influence the outcome by changing a coalition from losing to winning by joining it. A power index can be calculated for each player by considering each possible coalition and how frequently he can make the difference between losing and winning.

Four measures are used in this study: three classical power indices and one related measure which is not formally a power index, the degree of control (Cubbin and Leech (1983)); each is applied to the most and least concentrated assumed share distributions. The four indices for which we present analyses are: (1) the Shapley-Shubik index; (2) the normalised and (3) the non-normalised versions of the Banzhaf index. We also apply (4) the degree of control for comparison. New algorithms, which are described in Leech (1999), are used to compute the power indices. Since we think of control in terms of the existence of a powerful dominant block of votes, attention is focused on the value of the index for the largest block, but we also consider the power of potential coalitions of large shareholders.

III(b) Notation

We assume that a company has n shareholders and the individual holdings (voting weights) are denoted $w_1, w_2, ...,w_n, ,$ where $0 \leq w_i \leq 0.5$ for all i. For convenience we assume the weights are ordered in decreasing order of size, so that: $w_i \geq w_{i+1}$. Votes are taken with a decision rule in terms of a quota $q \geq 0.5$[7]. We also conduct an analysis using the data in grouped form to allow for the existence of potential controlling coalitions, where we define $s_j = \sum_{i \leq j} w_i$, as representing a block of votes obtained by grouping the largest j holdings.

III(c) The Problem of Incomplete Data

Information which is collected about company ownership is almost invariably incomplete because of the large number of shareholders there are in a typical major public company. Normally only the observations on the largest shareholdings are easily available and this is often all that is used in discussions of control[8]. It is, however, central to the approach adopted here that control depends not only on the size of the largest holding but also on the dispersion of the smaller holdings and we must explicitly

[7] We must have $q \geq 0.5$ to ensure that there is a unique decision, and that the voting game is "proper". In all the empirical work we take $q = 0.5$, which amounts to assuming that important decisions require a simple majority. There are exceptions to this, however, with some special decisions requiring a supermajority, but it is a broadly satisfactory assumption. In general ordinary decisions taken at company AGMs such as election of directors and passing of resolutions about the direction of the firm and which we might regard as bound up in the ordinary notion of control, are taken by simple majority.

[8] For example much empirical work is based on ownership stakes greater than 5%

deal with these. This incompleteness in the data therefore gives rise to important issues in deciding how to handle the missing observations.

The approach we adopt is to calculate two sets of indices, for two sets of weights - corresponding to the two extremes of “concentrated” and “dispersed” ownership - which are arithmetically feasible given the observed data, as follows. We observe m holdings and do not know anything about the remaining n-m holdings except that they are all smaller than w_m. Nor do we necessarily even know n. We consider two cases: the “dispersed” case where we assume w_i for i > m to be vanishingly small, and the “concentrated” case where we assume n to be as small as possible consistent with the observed data. We shall call the former limiting case D (Dispersed) and the latter limiting case C (Concentrated).

For limiting case C we need to make an assumption about n. If w_m is the smallest holding we observe in the data, then we know that the non-observed holdings are all no greater than w_m. The most concentrated pattern of ownership is assumed when they are equal to w_m. Then we can find the corresponding value of n, call it n', as:

$$n' = \text{integerpart}((1 - s_m)/w_m) + m + 1$$

and we let $w_i = w_m$ for all i = m+1, ..., n'-1 and $w_{n'} = 1 - s_m - (n'-m)w_m$.

Since the data in this study consist of shareholdings no smaller than 0.25% we have taken $w_m = 0.0025$.

III(d) The Shapley-Shubik Index

Voting outcomes are defined in terms of coalitions which are represented by subsets of the set of all players, N = {1, 2, . . , n}. A subset is assumed to cast all its votes in the same way. Let the total combined voting weight of all players in a subset T be w(T); that is $w(T) = \sum_{i \in T} w_i$. If T is a winning coalition $w(T) \geq q$ and for a losing coalition $w(T) < q$.

The power indices are defined in terms of swings: losing coalitions which become winning when a particular player joins. Thus a swing for player i is a losing coalition, T, such that $q - w_i < w(T) < q$. The indices differ in the coalition models assumed.

The Shapley-Shubik index for player i, γ_i , can be defined as

$$\gamma_i = \sum_{T} \frac{t!(n-t-1)!}{n!} \qquad i = 1, \ldots, n \qquad (1)$$

where the summation is over swings, and t is the number of members of T, n the number of members of N. It has a probabilistic interpretation as the probability of a swing for player i when the coalitions are formed by random orderings of the players, the term inside the summation being the probability of T occurring.

The direct evaluation of expression (1) is not feasible when n is large: even for limiting case C the typical values of n are of the order of 300. We calculate the values of γ_i in the two limiting cases using completely different algorithms. For limiting case C we employ the method described in Leech (1999). This provides a very good

approximation for this large finite game. For the limiting case D we are able to compute the expressions for the values given in Shapley and Shapiro (1978) for oceanic games.

The idea of an oceanic game seems to fit the current context very well: it is a game in which there are a finite number, m, of "major" players with fixed voting weights, and a very large number (in the limit an "ocean" of "non-atomic" players) with very small numbers of votes. Then as n goes to infinity the power index for player i converges on the value

$$\gamma_i = \sum_{S \subseteq M_i} \int_a^b u^s (1-u)^{m-s-1} du \qquad i = 1, \ldots, m \qquad (2)$$

where $M = \{1, 2, \ldots, m\}$, the set of major players, $M_i = M - \{i\}$,

$a = \text{median}(0, (q-w(S))/(1-w(M)), 1)$, $b = \text{median}(0, (q-w(S)-w_i)/(1-w(M)), 1)$. This is actually not difficult to evaluate requiring only a minor extension of the algorithm for searching over subsets of M described in Leech (1999) and numerical quadrature.

The Shapley-Shubik index is found in this study for every major shareholder and it has the property that it sums to unity over all the n players. The index can be thought to give a distribution of power among the players. Each index also has a natural interpretation in terms of a model of random coalition formation, as the probability of a swing for the player concerned.

III(e). The Banzhaf Index

The Banzhaf power index, like the Shapley-Shubik index, is based on the idea of counting swings in relation to all the possible voting outcomes, but the model of coalition formation underlying it is different, in that each coalition (corresponding to a

vote) is given the same weight regardless of its size. That is, the way swings are counted is different. The probability of each subset of N, T, assuming random coalition formation, is 2^{1-n} rather than t!(n-t-1)!/n!. The probability of a swing for player i is then

$$\beta_i' = 2^{1-n} \sum_T 1 \qquad i = 1, \ldots, n \qquad (3)$$

This is the Absolute (or Non-normalised) Banzhaf index and it cannot be interpreted as giving a distribution of power among the players since in general it does not sum to unity. Introducing a normalisation and defining

$$\beta_i = \beta_i' / \sum_i \beta_i' \qquad (4)$$

gives an index which does have this property but lacks the probability interpretation. This is the Normalised Banzhaf index.

Expressions (3) and (4) may therefore be taken as alternative and rival versions of an index of power to be considered. Computation of them is considerably easier than that of the Shapley-Shubik index and they have consequently been applied in the political science literature more. They can be approximated by a method based on probabilistic voting similar to that described above for the Shapley-Shubik index. For limiting case C we have used a new algorithm described in Leech (1999.

For limiting case D we compute the values for the oceanic game. Banzhaf indices for oceanic games were studied by Dubey and Shapley (1974) who showed that they can be obtained as the Banzhaf indices for the modified, finite game consisting only of the major players M with weights $w_1, w_2, \ldots, w_m$ and quota q - (1-w(M))/2. These indices can be obtained by applying the algorithm described in Leech (1999) to the

modified game. The results we obtain for the Banzhaf indices, both absolute and normalised, are more sensitive than the Shapley-Shubik indices to which limiting case we assume, but are still close enough to be informative in most cases.[9]

III(f) The Degree of Control

Cubbin and Leech (1983) defined the degree of control as an index of the voting power of the largest single voting block, shareholding 1, whose size is w_1. We take this shareholder to vote strategically and assume all the other shareholders vote randomly, indifferently and independently with probability 0.5. The number of votes cast by each shareholder is therefore a random variable, x_i , say, with distribution,

$$\Pr(x_i = w_i) = \Pr(x_i = 0) = 0.5, \text{ independent for all } i \geq 2.$$

The total vote in support of shareholder 1 is $y = \sum_{i=2}^{n} x_i$.

The degree of control is defined as the probability of majority support α:

$$\alpha = \Pr(w_1 + y > q). \qquad (5)$$

The distribution of y is approximately normal with

$$E(y) = 0.5(1 - w_1) \text{ , and}$$

$$\text{Var}(y) = 0.25(H - w_1^2) \qquad (6)$$

where $H = \sum_{i=2}^{n} w_i^2$.

The degree of control α is therefore easily obtained as a probability taken from the

[9] More work is needed on which distributional assumption might best characterise the non-observed

normal distribution. The range of values which the degree of control can take is the interval (0.5, 1). The degree of control is based on the same voting model as the Banzhaf index.

IV. Power Indices and Control of the Firm

In previous work (Leech (1987)) we proposed a model of control based on a measure of the formal voting power of the largest block of shares such as a power index or the degree of control. A company is classified as owner-controlled if this index exceeds some level considered high enough to indicate working control. The essential advantage of this approach over the conventional "fixed rules" approach to determining control used by many authors[10] is that the power of a large ownership block depends not only on its percentage of the voting equity but also on the dispersion of the other shareholdings. The fixed rule infers control only from the size of the largest block. Thus, for example, a shareholder with a 20% stake could be regarded as controlling in some cases but not in others on the basis of power indices, while it would always be controlling using a fixed 20% rule.

Figure 1 shows the model of minority voting control described in Leech (1987). The horizontal axis shows the number of members of the potential controlling coalition, starting with the largest and adding successively smaller holdings. A coalition of k members has $s(k)$ shares and its power is measured by its power index, $\varphi(k)$; both

shareholdings. We have used an extreme assumption.

[10] See Short (1994) for a survey. La Porta et. al. (1999) have recently used a fixed rule based on 20%.

functions are shown on the vertical axis[11]. A typical concentrated ownership structure is shown with s(k) represented by AB and the power function φ(k) represented by CD. The coalition has majority control when it has k' members, s(k') = 0.5 and φ(k') = 1. Minority control is assumed to exist when the power of the coalition is high but not quite equal to 1. In the diagram this is represented when the coalition size is k* members and its voting power is φ(k*) = φ*. The threshold φ* is chosen appropriately. This model is the basis of the empirical approach reported in the next section. [12]

V Appraisal of the Power Indices

Very little is known about the relative quality of the different indices as measures of power in general. There is some theoretical literature on their relative properties and a number of empirical applications but there is no body of evidence or settled opinion as to which index might be a better reflection of power relations. Since they measure *a priori* voting power no useful information can be gained by observing voting behaviour. It is well known, for example, that many company AGMs are very badly attended and when ballots are called the votes cast are unrepresentative of the generality of shareholders and often may not reveal the actual control. Many fund managers with substantial holdings have tended to avoid voting as a matter of policy and either back incumbent management or use their influence to bring pressure behind the scenes. The *a*

[11] s(k) is the same as what we have previously defined as s_k .

[12] There is a potential identification problem here since the model can be used to determine control endogenously by choosing the shape of the curve s(k). Therefore we might expect observed ownership structures of actual firms to reflect this.

priori voting power is nevertheless real and a determining factor in the firm's governance.

Since no convincing way of testing the adequacy of the power indices has been devised the best that can be done is to discuss their plausibility either in terms of their theoretical foundations as voting models or in terms of their performance in practical use. As regards the former, we mention Roth (1977), who has shown that differences between the indices can be seen as reflecting different attitudes towards risk, and Straffin (1977), who has characterised them in terms of probabilistic voting with different mechanisms for choosing the voting probability, but neither of these insights leads us at present to conclude that one index is to be preferred.

In this study we hope that the second approach will lead to some progress. By applying the indices to real data, about which we have strong prior expectations and beliefs, we might be able to decide if the results are plausible. For a power index to satisfy this requirement we would expect: first, that it give rise to a classification of companies which might be regarded as under minority control and some which are not, if we believe that such a dichotomy exists, and second, that the results do not conflict with widely accepted criteria of control which are presently in use by real institutions and which have been derived from considerable experience. For example the London Stock Exchange Yellow Book defines a controlling shareholding as one which controls 30 percent or more of the votes at a company meeting.[13] We would therefore require of a suitable index that in most cases where there is such a shareholding it assign a very

[13] London Stock Exchange (1993).

high power to it, and the firm to be classified as minority controlled., although we might also expect there to be exceptions.[14] The widespread use of the 20 percent fixed rule by many researchers has already been remarked on and we would expect a good power index to lead to a classification of control in a substantial proportion of such cases.

Such evidence, in the form of plausible results, if found, would be only suggestive because it would leave open the question of the correct classification of individual firms. Just because the index gives an overall pattern which is consistent with conventional expectations does not mean that it accurately measures power in every case, and argues the need for further research. On the other hand, however, an index which produces results which are at variance with what we know or believe to be the case can be rejected as unfit for purpose and if we can provide evidence of this sort the problem of choice will be simplified. We do provide evidence of this sort for the Shapley-Shubik index.

VI. Results

VI(a) The Data

The indices have been calculated using the algorithms described in Leech (1999) for all the companies in the Leech and Leahy (1991) sample which did not have a majority shareholding. The sample consists of 444 firms, mostly taken from the Times

[14] Such would be where there are two or three very large holdings which would potentially control the company together but their rivalry, detected by the power index, prevents one of them having working control.

1000, and for each firm we have the size of all shareholdings down to 0.25 percent of equity.[15] The data can reasonably be regarded as representing beneficial shareholdings, since details of nominee holdings and names and addresses were used to identify ultimate owners and blocks owned by linked or related individuals or institutions.[16] The year of observation was either 1985 or 1986. The number of shareholdings observed varies across companies between a minimum of 12 and a maximum of 56, with a median of 27.

The data are summarised in Table 1. The table shows the distribution of size of the largest shareholding, w_1, against the second-largest holding, w_2, and also distributions for groupings of large shareholdings in order to indicate the variation in patterns of concentration of ownership in the sample. There is a wide range of patterns of ownership in the sample with 41 companies having relatively concentrated ownership structures with the largest holding w_1 greater than 30% but in the great majority of cases w_1 is less than 30 percent. There is also a wide range of variation in the size of the second largest holding w_2 given w_1. For example in the group of 85 firms where w_1 is between 20 and 30%, w_2 is less than 5% in 12 cases and greater than 20 % in 9 cases; this is expected to give rise to a wide range of control cases as this is reflected in the power indices.

[15] The source and method of construction of the data set are described in Leech and Leahy (1991).

[16] There might remain a very slight underestimation of the true concentration of ownership to the extent this information was incomplete.

The next set of rows in Table 1 show the concentration effect of combining the largest holdings in terms of the size of the block of shares in the combined holding. This shows how dispersed the ownership of many British companies really is. Combining the ten largest holdings gives a majority grouping in only 203 - fewer than half - the companies and even combining the top 20 holdings gives majority control in only 298 cases – 71 percent of the companies in the sample.

The third part of Table 1 gives control classifications (that is the number of firms classified as owner-controlled) using various fixed classification rules. On the basis of the original, uncombined shareholdings using a 10% rule would give owner-control in 259 firms or 59.3% of cases, while the common 20% rule would only give control in 30.2 percent of cases. The Stock Exchange's 30% rule would deem 68 companies or 15.3 percent of the sample to be controlled. The other rows of the table show the results for control on the basis of blocks of combined holdings.

Table 1 The Data

Individual Holdings								
w2	w1:	<5%	5-10%$	10-20%	20-30%	30-40%	40-50%	>50%	Total
<5%		41	46	15	12	2	2		118
5-10%			98	73	26	10	9		216
10-20%				37	38	11	5		91
20-30%					9	4	2		15
30-40%						3	1		4
40-50%							0		0
w1		41	1454	125	85	30	19		444
Combined Holdings									
		<5%	5-10%	10-20%	20-30%	30-40%	40-50%	>50%	Total
s2		4	50	175	83	65	35	32	444
s3		1	14	123	129	58	59	60	444
s4		0	6	83	135	75	56	89	444
s5		0	3	54	114	98	67	108	444
s10		0	0	15	38	93	95	203	444
s20		0	0	4	11	34	71	298	418
Ownership Control Classification: Number Owner-Controlled									
Control Criterion:				>10%	>20%	>30%	>40%	>50%	
Basis: \									
w1	No.			259	134	49	19		
w1	%			58.3	30.2	11.0	4.3		
s2	No.			390	215	130	67	32	
s2	%			87.8	48.4	29.3	15.1	7.2	
s3	No:			429	305	176	119	60	
s3	%			96.6	86.7	39.6	26.8	13.5	
s5	No.			441	387	273	175	108	
s5	%			99.3	87.2	61.5	39.4	24.3	
s10	No.			444	429	391	298	203	
s10	%			100.0	96.6	88.1	67.1	45.7	

VI(b) Results for Illustrative Companies

Tables 2 and 3 present power indices for some illustrative companies. The firms have been selected to span the range of variation in the first two shareholdings. Plessey has the most dispersed ownership with a largest shareholding of under 2% and Associated Newspapers is one of several which are just short of majority control. Two firms have been selected in each range of values for w_1: 10 – 20%, 20 – 30%, 30 – 40%, 40-50%. In each range the two companies are those with relatively large and small values for w_2. The results for these firms might then be taken as illustrative of the effects of ownership concentration in terms both of the size of the largest holding and the relative dispersion of the other holdings as proxied by the second largest. Results are given for representative shareholders numbered 1, 2, 3, 5, 10 and 20.

Table 2 shows the Shapley-Shubik indices. SS(CONC) is the index calculated on the assumption that the non-observed holdings are all as large as theoretically possible given the way the data has been collected. All the non-observed holdings are assumed to be 0.25%; this is an extreme bound which clearly overstates the degree of concentration and understates the number of shareholders. In the case of Plessey for example the data consist of the largest 38 shareholdings ranging from 1.94 down to 0.25 percent with a combined weight of 23.51 percent. The remaining 76.49 percent is distributed among the remaining shareholders and the most concentrated way in which this could theoretically be done would be if there were 305 with 0.25% and one with 0.24. The voting game which is used as the basis of these indices is therefore one with

344 players. These indices were computed using the algorithm described in Leech (1999). The oceanic indices, SS(OCEA), were calculated assuming an oceanic game in which there were 5 players with finite weights, w_1, w_2, w_3, w_4 and w_5, and the remaining votes distributed among an "ocean" of players with infinitessimally small holdings. In the case of Plessey the game assumed would consist of the five weights 1.94, 1.49, 1.29, 1.10, 1.05, totalling 6.87 percent, and the remaining 93.13 percent distributed among an infinity of "non-atomic" players. The algorithm used to compute the indices for this case was that of Shapiro and Shapley (1978)[17].

Despite the methods of calculation used being so completely different for the two cases, the two sets of results reported in Table 2, SS(CONC) and SS(OCEA), are remarkably close. The conclusions which emerge from these results are, first, the striking insensitivity of the indices to the way in which we deal with the incompleteness of the data enables us to place confidence in them for the large shareholders. Secondly, the indices are relatively insensitive to the inequality in the data: comparing Sun Life with Liberty, for example, both of whose largest shareholdings are about 22%, but whose second-largest shareholdings are very different, 3.46% against 22.57%, suggests that differences in w_2 have very little effect on the power of the other shareholders. The power of the largest shareholder falls from

[17] The value 5 for the number of finite players in this game was chosen for reasons of computational speed in calculating indices for all 444 firms but has no effect on the values obtained for the Shapley-Shubik indices. By changing the value of this number and re-calculating for the small sub-sample of companies reported here we have found the results to be practically invariant to the number of finite players.

28% in the case of Sun Life to just under 25% in the case of Liberty. We might expect the fact that the second-largest shareholding in Liberty was almost equal to the largest to have a profound effect.

The third implication to emerge very clearly from Table 2 is the general insensitivity of the indices to differences in ownership concentration in terms of w_1. Although in every case power is more unequally distributed than ownership, the difference is never dramatic and when ownership structure clearly corresponds to what would conventionally be regarded as minority control, as in Securicor or Ropner, the power index of the largest shareholding remains very far below 1. This pattern is typical of the whole sample and prompts the conclusion that the Shapley-Shubik index seriously understates the power of the largest shareholder in such cases.

Table 2 Shapley-Shubik Indices for Illustrative Firms

	Shareholder	1	2	3	5	10	20
Plessey	Weight	0.0194	0.0149	0.0129	0.0105	0.0088	0.0041
	S-S (CONC)	0.0197	0.0151	0.0130	0.0106	0.0088	0.0041
	S-S (OCEA)	0.0198	0.0151	0.0131	0.0106		
Berisford	Weight	0.0579	0.0199	0.0161	0.0094	0.0054	0.0026
	S-S (CONC)	0.0612	0.0202	0.0162	0.0094	0.0054	0.0026
	S-S (OCEA)	0.0614	0.0202	0.0163	0.0094		
Un. Spring & Steel	Weight	0.1228	0.1092	0.0977	0.0368	0.0142	0.0050
	S-S (CONC)	0.1342	0.1171	0.1032	0.0363	0.0137	0.0048
	S-S (OCEA)	0.1348	0.1176	0.1036	0.0364		
Suter	Weight	0.1275	0.0646	0.0534	0.0308	0.0174	0.0088
	S-S (CONC)	0.1432	0.0666	0.0544	0.0306	0.0171	0.0086
	S-S (OCEA)	0.1442	0.0670	0.0547	0.0308		
Sun Life	Weight	0.2216	0.0346	0.0187	0.0132	0.0087	0.0045
	S-S (CONC)	0.2826	0.0326	0.0174	0.0122	0.0080	0.0041
	S-S (OCEA)	0.2837	0.0327	0.0174	0.0122		
Liberty	Weight	0.2263	0.2257	0.0894	0.0498	0.0181	
	S-S (CONC)	0.2475	0.2465	0.0894	0.0460	0.0162	
	S-S (OCEA)	0.2486	0.2475	0.0922	0.0460		
Securicor	Weight	0.3163	0.0730	0.0531	0.0289	0.0164	0.0077
	S-S (CONC)	0.4479	0.0592	0.0433	0.0234	0.0133	0.0062
	S-S (OCEA)	0.4513	0.0587	0.0430	0.0234		
Bulgin	Weight	0.3097	0.2216	0.0450	0.0275	0.0090	0.0026
	S-S (CONC)	0.3563	0.1738	0.0485	0.0284	0.0089	0.0025
	S-S (OCEA)	0.3553	0.1715	0.0493	0.0288		
Ropner	Weight	0.4103	0.0598	0.0497	0.0198	0.0116	0.0028
	S-S (CONC)	0.6757	0.0287	0.0252	0.0109	0.0065	0.0016
	S-S (OCEA)	0.6799	0.0277	0.0245	0.0107		
Steel Brothers	Weight	0.4250	0.2132	0.0384	0.0303	0.0070	0.0026
	S-S (CONC)	0.6159	0.0546	0.0348	0.0277	0.0064	0.0024
	S-S (OCEA)	0.6183	0.0515	0.0349	0.0278		
Assoc Newspapers	Weight	0.4995	0.0263	0.0213	0.0207	0.0128	0.0056
	S-S (CONC)	0.9839	0.0003	0.0003	0.0003	0.0003	0.0002
	S-S (OCEA)	0.9976	0.0000	0.0000	0.0000		

Table 3 contains the Banzhaf indices for the same 11 illustrative companies. For each firm it gives both the normalised and the non-normalised power indices for the same shareholders as in Table 2, under the same assumptions about the non-observed shareholdings, that they are as concentrated as possible. These are labelled BZ and BZ(NN) respectively and have been calculated using the algorithm described in Leech

(1999). It also contains the normalised indices for the oceanic game representing the limiting-dispersion case, calculated according to Dubey and Shapley (1979). The non-normalised indices are not presented for this case because the oceanic Banzhaf indices are dominated by the particular assumptions underlying the assumed game. For example, in the case of Plessey, the oceanic Banzhaf indices are equal to 20% for each of the five assumed finite players and zero for all the others. Arbitrarily assuming larger number of finite players changes the results completely. In many other cases, the oceanic Banzhaf index often assigns all power to the largest shareholder, for example Beresford where the largest shareholding is less than 6 percent. We therefore question the use of the Banzhaf indices for the oceanic case.

The values of the indices in Table 3, both normalised and non-normalised, are very sensitive to differences in ownership structure and vary widely. The normalised index, BZ, is interpreted, in the same way as the Shapley-Shubik index, as a share in decision-making power, and the values obtained for shareholders in these firms appear to conform to widely held a priori notions of the power of shareholding blocks of a given size in relation to others. Where ownership is widely dispersed as in the case of Plessey, power is almost equally dispersed. Where it is highly concentrated, as in Ropner, Steel Brothers or Associated Newspapers, with a shareholding over 40%, giving working control, the index reflects this. In other cases the Banzhaf index gives a richer variety of power distributions in response to differences in ownership structure.

A comparison of Sun Life and Liberty, for example, reveals that the index is sensitive to the size of the second largest shareholding in a way which the Shapley-

Shubik index is not. The 22% largest shareholding in Sun Life has a Banzhaf power index of 98% suggesting that it can be regarded as a controlling holding in our sense and reflecting the relatively high dispersion of ownership of the other 78% of shares. In the case of Liberty, however, both the largest two holdings are above 22% which must mean that the largest shareholder is not much more powerful than the second-largest. The Banzhaf index gives this result. A similar finding emerges for companies which have a shareholding of between 30 and 40 percent. A shareholder with a 31% ownership stake has almost 93% of the voting power in Securicor where there are no other large owners. On the other hand a similar-sized stake in Bulgin would have only 37% of the voting power because of the presence of a large second shareholder with 22%. These results are entirely plausible in conforming with conventional ideas about minority control.

The normalised Banzhaf index by definition provides a power distribution among all the voters in the game, in this case the shareholders, both real and assumed. The indices are normalised to sum to unity over all n members because they are each member's share of decisive swings out of all theoretically possible decisive swings of votes. The normalised Banzhaf indices do not therefore take into account the decisiveness of the voting body, that is its capacity to act, which is dependent on the distribution of voting weights, as well as the majority rule, under the coalition model assumed. The non-normalised Banzhaf indices are not subject to this limitation since they are proportions of all voting outcomes in which members are decisive. The alternative name for the non-normalised Banzhaf index is the swing probability. This latter index cannot give a power distribution but may be useful as a basis for a

definition of control by considering its value for the controlling shareholder only, taking into account not only its share of power within voting body but also the latter's power to act. In Table 3 the values of the non-normalised Banzhaf index for the largest shareholding behave in much the same way as already described for the normalised index, and therefore the index might be a suitable basis for a definition of minority control. It does have the advantage of being easier to compute since it does not require the normalising constant.

Table 3 Banzhaf Power Indices for Illustrative Firms

	Shareholder	1	2	3	5	10	20
Plessey	Weight	0.0194	0.0149	0.0129	0.0105	0.0088	0.0041
	Bz	0.0200	0.0152	0.0131	0.0106	0.0088	0.0041
	Bz(NN)	0.2535	0.1917	0.1652	0.1338	0.1118	0.0518
	BZ (OCEA)	0.2000	0.2000	0.2000	0.2000		
Berisford	Weight	0.0579	0.0199	0.0161	0.0094	0.0054	0.0026
	BZ (CONC)	0.0796	0.0194	0.0157	0.0092	0.0053	0.0025
	Bz(NN)	0.6841	0.1663	0.135	0.0789	0.0453	0.0218
	BZ (OCEA)	1.0000	0.0000	0.0000	0.0000		
Un. Spring & Steel	Weight	0.1228	0.1092	0.0977	0.0368	0.0142	0.0050
	BZ (CONC)	0.1431	0.1235	0.1116	0.0334	0.0132	0.0047
	Bz(NN)	0.5015	0.4329	0.3913	0.1172	0.0464	0.0164
	BZ (OCEA)	0.3333	0.3333	0.3333	0.0000		
Suter	Weight	0.1275	0.0646	0.0534	0.0308	0.0174	0.0088
	BZ (CONC)	0.1691	0.0601	0.0512	0.0294	0.0166	0.0084
	Bz(NN)	0.692	0.246	0.2093	0.1204	0.0679	0.0343
	BZ (OCEA)	0.6364	0.0909	0.0909	0.0909		
Sun Life	Weight	0.2216	0.0346	0.0187	0.0132	0.0087	0.0045
	BZ (CONC)	0.9809	0.0004	0.0003	0.0003	0.0002	0.0001
	Bz(NN)	0.9996	0.0004	0.0003	0.0003	0.0002	0.0001
	BZ (OCEA)	1.0000	0.0000	0.0000	0.0000		
Liberty	Weight	0.2263	0.2257	0.0894	0.0498	0.0181	
	BZ (CONC)	0.2025	0.2013	0.1121	0.0534	0.0189	
	Bz(NN)	0.5013	0.4982	0.2775	0.1321	0.0468	
	BZ (OCEA)	0.2857	0.2857	0.1429	0.1429		
Securicor	Weight	0.3163	0.0730	0.0531	0.0289	0.0164	0.0077
	BZ (CONC)	0.9295	0.0032	0.0032	0.0027	0.0018	0.0009
	Bz(NN)	0.9966	0.0034	0.0034	0.0029	0.002	0.001
	BZ (OCEA)	1.0000	0.0000	0.0000	0.0000		
Bulgin	Weight	0.3097	0.2216	0.0450	0.0275	0.0090	0.0026
	BZ (CONC)	0.3716	0.0593	0.0527	0.0340	0.0109	0.0032
	Bz(NN)	0.8624	0.1376	0.1223	0.0789	0.0254	0.0073
	BZ (OCEA)	0.6364	0.0909	0.0909	0.0909		
Ropner	Weight	0.4103	0.0598	0.0497	0.0198	0.0116	0.0028
	BZ (CONC)	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Bz(NN)	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BZ (OCEA)	1.0000	0.0000	0.0000	0.0000		
Steel Brothers	Weight	0.4250	0.2132	0.0384	0.0303	0.0070	0.0026
	BZ (CONC)	0.9914	0.0004	0.0004	0.0004	0.0002	0.0001
	Bz(NN)	0.9996	0.0004	0.0004	0.0004	0.0002	0.0001
	BZ (OCEA)	1.0000	0.0000	0.0000	0.0000		
Assoc Newspapers	Weight	0.4995	0.0263	0.0213	0.0207	0.0128	0.0056
	BZ (CONC)	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Bz(NN)	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	BZ (OCEA)	1.0000	0.0000	0.0000	0.0000		

The graphs in Figure 2 show comparative results for shareholders 1 to 10 for the normalised indices except the oceanic Banzhaf index. There is clear agreement among the indices in the extreme cases of Plessey and Associated Newspapers. The disagreement between them becomes clear in comparing Sun Life with Liberty and Securicor with Bulgin, where the Shapley-Shubik index seems to be much less plausible than the Banzhaf index. Likewise the plots for Ropner and Steel Brothers lead to the same conclusion.

VI(c) The Complete Sample

Results for the full sample are shown in Figures 3 and 4. Figure 3 shows the respective power indices for the largest shareholding, of size w_1, separately for each index. Figure 4 shows the equivalent plots after combining the largest 4 shareholdings into a single block. These plots are useful for giving an insight into the respective behaviour of the power indices and their potential as a basis for identifying minority control.

Figures 3(a) and 3(b) show the Shapley-Shubik indices for the most concentrated and oceanic assumptions. They are very close with only very slight differences apparent for very large values of w_1. The incompleteness of the data is virtually not a problem for this index.[18] The plots are bounded above by the function $w_1/(1-w_1)$ which is the value of the index for player 1 in an oceanic game with only one major player with weight w_1.[19] Where the index is less than this it reflects the existence of more than

[18] This is perhaps not surprising given the quality of the data in terms of detail.

[19] Shapley and Shapiro (1978).

one large shareholding. However the fact that power is almost always relatively low and that in only 2 or 4 companies does it exceed 90% means that this index is of limited utility for revealing minority voting control.

Figures 3(c) and 3(d) show the corresponding plots for the Banzhaf indices for the concentrated assumption. Here there is much more variation suggesting that the index may be capturing the effect of different ownership structures. Figure 3(c) shows the normalised Banzhaf index β_1 for the largest shareholding against its size w_1. There is very little effect up to about 15% but after that power varies widely. These results suggest that shareholdings between 20 and 30 percent can be said to have voting control in many cases but not in many others. Most (but not all) holdings greater than 35 percent have a power index equal to or almost equal to 1. Figure 3(d) presents the same indices without the normalisation: the index is the probability that shareholder 1 can swing a vote, ignoring the capacity of the others to do the same. These indices are generally much higher than the normalised ones which is a reflection of the fact that a simple majority rule, assumed to apply generally to all firms in the sample, makes decisions easy to take. The variation between firms which is observed suggests that this index may be useful as a guide to control on the basis of individual shareholding data. Figure 3(e) presents the degree of control for both concentrated DC(CONC) and dispersed DC(DISP) extremes; the general picture which emerges is similar. From all these results the Banzhaf indices and the degree of control suggest that voting control is possible with a holding smaller than 20% but that such cases are not very frequent.

VI(d) Potential Controlling Blocks: The Complete Sample

Figures 4 and 5 report the results of analysing scenarios where we have calculated the power indices assuming the existence of blocks or groupings of large shareholdings. They provide evidence on whether control might hypothetically be exercised by such a coalition. We can construct many arbitrary coalitions but we have confined attention to those in which voting power is maximised for a given number of members which means grouping the largest shareholdings. This approach is consistent with the common way of discussing ownership in which such hypothetical groupings are assumed. In some cases this grouping process leads very quickly to a voting majority but in others the effect is a disproportionate enhancement of power of a minority coalition..

In Figures 4 and 5 the Shapley-Shubik indices tell essentially the same story as before and we conclude that combining shareholdings has really only a slight effect. The effect on the Banzhaf indices is different because this index is affected by both the increase in size of the largest holding and also the reduction in concentration of the other holdings. Therefore under this scenario there are more firms classified as minority controlled because their largest block of shares is larger, but also there are more so classified on the basis of a shareholding of given size, for example when the block's combined ownership is between 20 and 30 percent.

VI(e) Potential Controlling Blocks: Illustrative Companies

Figure 6 extends the analysis of the last section by considering the relationship between the build-up of the coalition in terms of its size and its resulting power.

Results are shown for illustrative companies in which the power indices have been calculated for each assumed coalition from 1 shareholder to 20[20]. This analysis is intended to cast light on the model of control in which coalitions are formed endogenously as a voting block in order to achieve a given level of power.

Each plot shows the size of the coalition, j, on the horizontal axis and its ownership stake, s_j, and associated power indices on the vertical axis. No plots are shown for the cases where s_1 is greater than 40 percent since grouping very quickly gives majority control. These plots give insight into the question: if a small group of large shareholders combined, could it have working control with a minority block? The answer is very strongly that it depends on which index is used. The Shapley-Shubik index (for which the graphs for the two extreme bounds are not distinguishable because they are so close) is very unresponsive to the formation of the blocks. These results suggest that this particular power index is not useful for this purpose.

The Banzhaf index, by contrast, is very sensitive to the formation of blocks and often indicates control on the basis of a minority block of shares much smaller than 50 percent. The general pattern which is obtained using this index is plausible.

VI(f) Control Classifications

Table 4 shows some classifications of companies using the different indices for the largest shareholder as the criterion for identifying minority control. A classification has been carried out for three of the normalised indices, the two extremes of the

[20] 16 for Liberty.

Banzhaf index and the Shapley-Shubik index, and for the non-normalised Banzhaf indices and the degree of control. A separate classification is presented for the original data and the hypothetical controlling blocks of the top two, three and five shareholders. The figures in the body of the table are the numbers of firms which might be classified as minority owner controlled on the basis that the relevant index exceeds the threshold value. For this exercise we assume minority control in the case of the normalised indices if the shareholder has 90 percent or more of the power and in the case of the non-normalised indices, which are probabilities of either winning or swinging a vote, if the relevant probability is at least 90 percent. The results are cross-tabulated against the size of the shareholding concerned. These results are presented in order to compare the performance of the different indices in terms of our appraisal criteria.

The inadequacy of the Shapley-Shubik index for the purpose is evident from Table 4 where it is clearly doing nothing more than indicating the concentration of the leading shareholding.

Both versions of the normalised Banzhaf index show the capacity to discriminate between firms as the basis of a classification, but the differences between the concentrated and oceanic indices are substantial: the oceanic Banzhaf indices are asymptotic extremes and tend to find controlling shareholders with fairly small holdings at times. However as we have indicated above, these oceanic indices should be regarded as really reflecting the assumed model and to a very large extent arbitrary. The Banzhaf indices assuming concentrated holdings seem to give much more plausible results. We conclude that while the Banzhaf index appears to be capable of giving

plausible results, the bounds assumed lead to too great an area of ignorance and further work is needed.[21]

The second part of Table 4 shows the results obtained using the non-normalised Banzhaf index as well as the degree of control. There is fairly good agreement between the concentrated Banzhaf index and the degree of control (for the concentrated case) and both produce plausible results for the over 30% group. Both assign control to shareholdings of less than 20 percent in a number of cases which could be plausible. The results for the oceanic Banzhaf index do not appear very useful.

[21] It would be better to replace the clearly unrealistic assumption that non-observed holdings are all equal to 0.25% by an assumption about their distribution. An alternative assumption which was tried experimentally was that the non-observed holdings decline linearly. This still considerably overestimated the concentration of the smaller holdings but did not greatly change the results, giving only slightly more concentrated power distribution. This suggests that the Banzhaf index is perhaps less sensitive to minor differences in data than the wide gap between these and the computed oceanic indices suggests and that the problem is more to do with the oceanic indices.

Table 4 A Comparison of Control Classifications by Different Indices

Normalised Indices > 0.9								
S1	<10%	10-20%	20-30%	30-40%	40-50%	Minority	>50%	Total
Banzhaf (conc.)	0	0	7	14	18	39	0	39
Banzhaf (ocea.)	1	13	35	23	18	90	0	90
Shapley-Shubik	0	0	0	0	3	3	0	3
Total Firms	185	125	85	30	19	444	0	444
S2	<10%	10-20%	20-30%	30-40%	40-50%	Minority	>50%	Total
Banzhaf (conc.)	0	1	15	41	35	92	32	124
Banzhaf (ocea.)	6	56	65	62	35	224	32	256
Shapley-Shubik	0	0	0	0	4	4	32	38
Total Firms	54	175	83	65	35	412	32	444
S3	<10%	10-20%	20-30%	30-40%	40-50%	Minority	>50%	Total
Banzhaf (conc.)	0	0	30	49	59	138	60	198
Banzhaf (ocea.)	14	114	128	58	59	373	60	433
Shapley-Shubik	0	0	0	0	7	7	60	67
Total Firms	15	123	129	58	59	384	60	444
S5	<10%	10-20%	20-30%	30-40%	40-50%	Minority	>50%	Total
Banzhaf (conc.)	0	2	78	98	67	245	108	353
Banzhaf (ocea).	3	54	114	98	67	336	108	444
Shapley-Shubik	0	0	0	0	12	12	108	120
Total Firms	3	54	114	98	67	336	108	444
Non-normalised Indices > 0.9								
S1	<10%	10-20%	20-30%	30-40%	40-50%	Minority	>50%	Total
Banzhaf (conc.)	0	11	32	24	19	86	0	86
Banzhaf (ocea.)	39	30	11	6	4	90	0	90
Degree ofControl	0	13	49	27	18	107	0	107
Total Firms	185	125	85	30	19	444	0	444
S2	<10%	10-20%	20-30%	30-40%	40-50%	Minority	>50%	Total
Banzhaf (conc.)	0	25	64	63	35	187	32	219
Banzhaf (ocea.)	32	94	50	43	19	238	32	270
Degree ofControl	2	63	79	65	35	244	32	276
Total Firms	54	175	83	65	35	412	32	444
S3	<10%	10-20%	20-30%	30-40%	40-50%	Minority	>50%	Total
Banzhaf (conc.)	1	59	125	58	59	302	60	362
Banzhaf (ocea.)	5	121	122	58	59	365	60	425
Degree ofControl	4	111	129	58	59	361	60	421
Total Firms	15	123	129	58	59	384	60	444
S5	<10%	10-20%	20-30%	30-40%	40-50%	Minority	>50%	Total
Banzhaf (conc.)	1	54	114	98	67	334	108	442
Banzhaf (ocea.)	3	54	114	98	67	336	108	444
Degree ofControl	2	54	114	98	67	335	108	443
Total Firms	3	54	114	98	67	336	108	444

VII Conclusions

In this paper we have reported on an exercise in the use of voting power indices to measure formal shareholder voting power and identify control in a large sample of British companies. The fundamental methodological assumption was that voting power matters and is an important component of the system of governance of the firm.

New accurate algorithms for computing power indices in large finite voting bodies and oceanic games have been devised and applied. Two main methodological questions were addressed: whether this approach is feasible as a basis for identifying control, and how the different power indices compare in empirical application. The paper has succeeded in demonstrating the feasibility of the approach. The power indices have been used to classify companies according to whether there is a powerful minority shareholder who has working control and we have been able to exploit the particular nature of the data set to carry out a comparative appraisal of the different classical power indices. This latter is a unique contribution to the literature on empirical power indices.

Our principal conclusions are:

(1) The approach is feasible as the basis for the definition of minority control and we have obtained control classifications of British companies on the basis of whether there exists a sufficiently powerful shareholder. There remains a question of choice of criterion by which to measure the voting power of this shareholder.

(2) In terms of research into power indices, the results indicate that the Shapley-Shubik index is clearly unsuitable for the measurement of power in shareholder voting games.

(3) The results obtained using the Banzhaf index satisfied our appraisal criteria (although those obtained for the oceanic game were clearly unsatisfactory) and this index performed very well. There is still an open question of choice between the normalised and absolute Banzhaf indices.

(4) An important and striking result which emerged very clearly was that in a substantial proportion of companies a quite small group of leading shareholdings can combine to produce a very powerful, controlling block even with a minority of the shares.

Questions for further research include how to calculate the power index given the problem of non-observed smaller shareholdings; it is not clear from this study how sensitive the Banzhaf index is to this issue. There is also the question of whether other power indices might be useful in this context: we have confined attention to the classical power indices. The assumption that a firm's ownership structure is exogenously given, which has been maintained in this study, is clearly untenable in practice and further analysis in which it is an endogenous part of the firm's financing is needed. Finally there is the important question of the relation of all this with efficiency. Further research will use the results obtained to study the relationships between minority control and company performance.

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Figure 1. Minority Control by a Shareholding Block

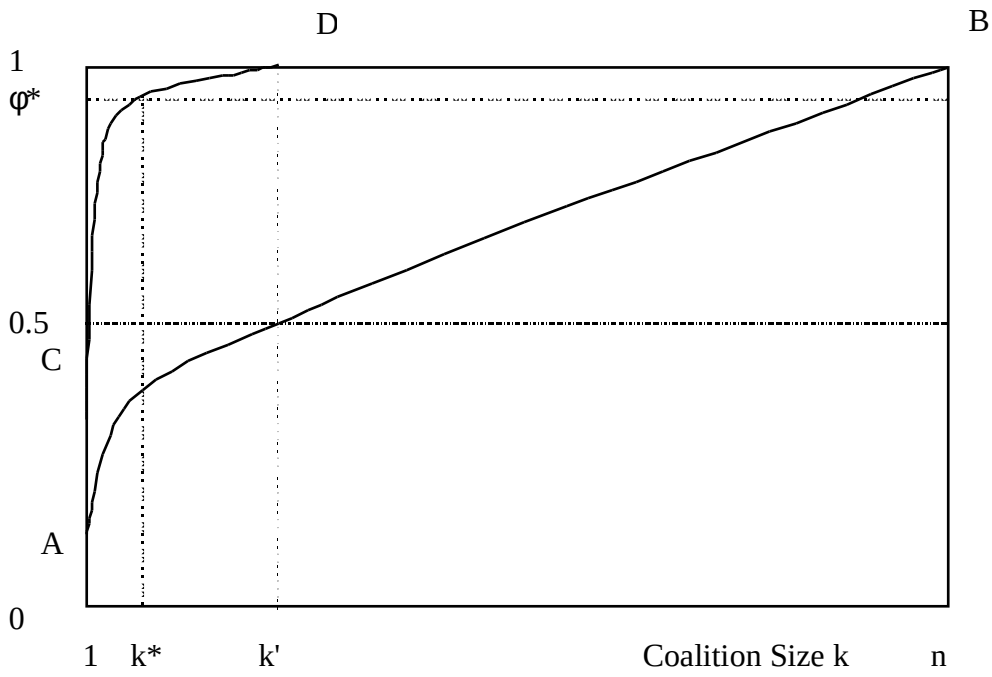
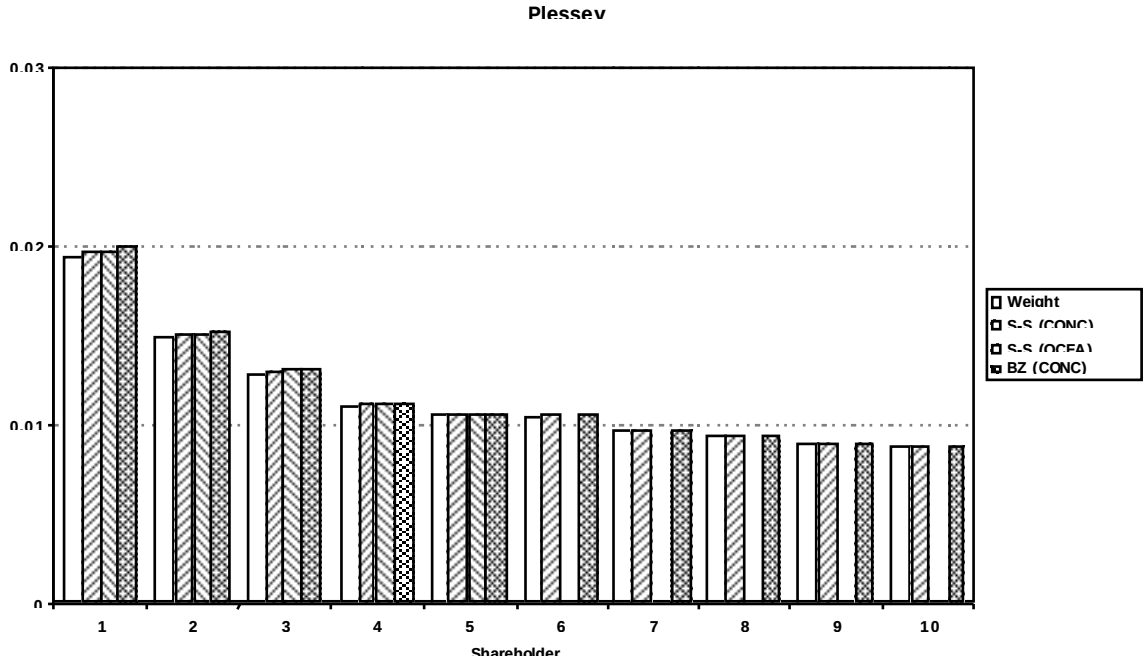


Figure 2 Power Indices for Illustrative Firms

Fig 2(a)



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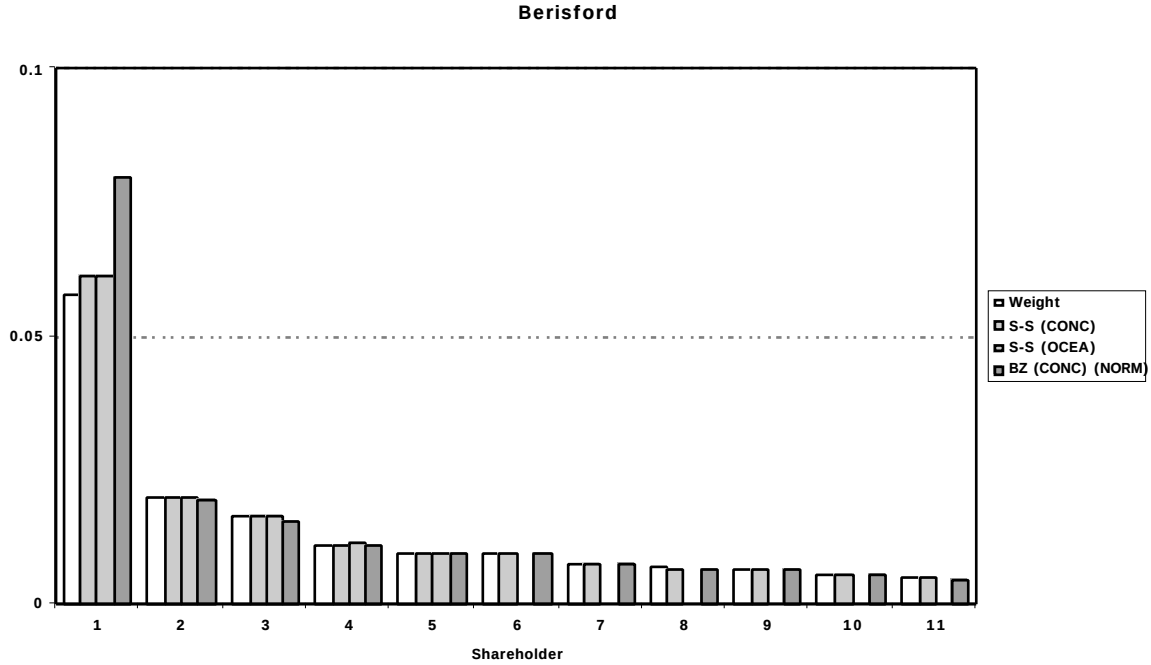

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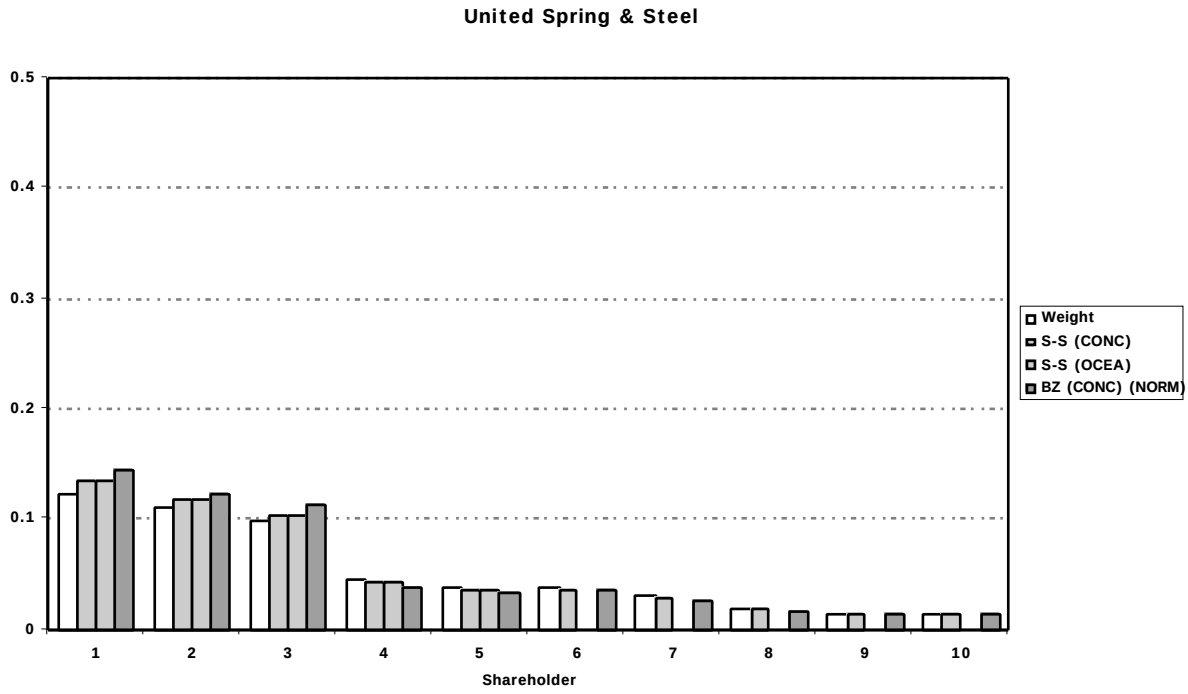

United Spring & Steel
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
7
8
9
10
Shareholder
Weight
S-S (CONC)
S-S (OCEA)
BZ (CONC) (NORM)

Fig 2(d)

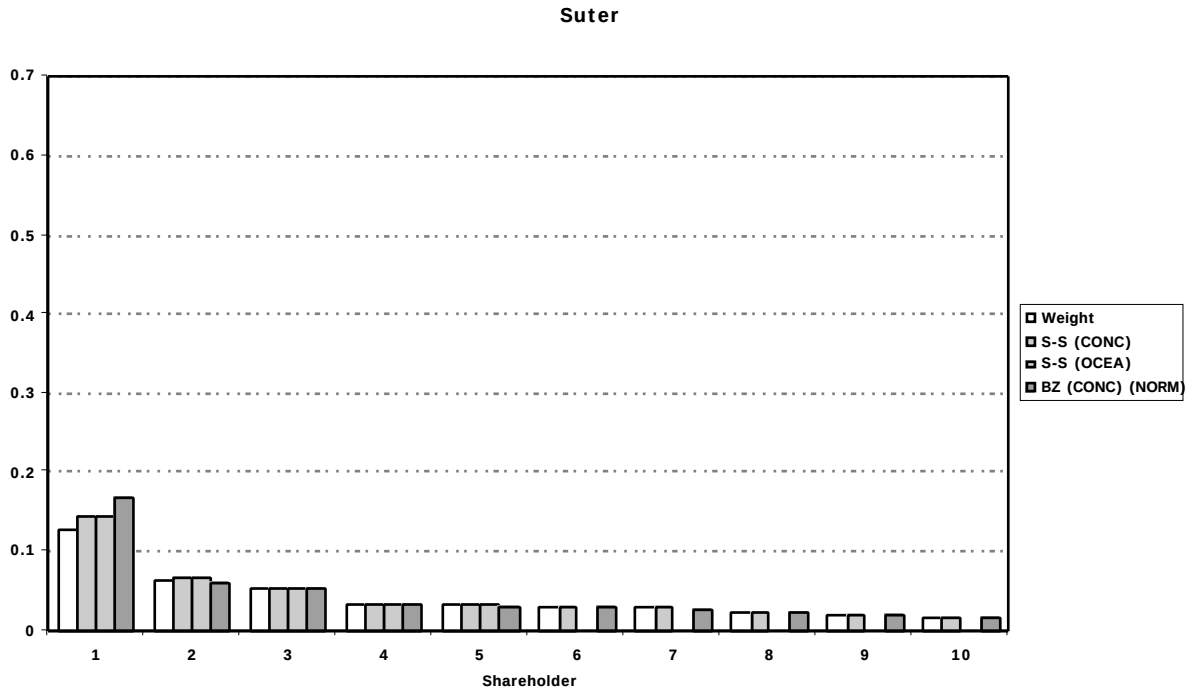

Suter
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
7
8
9
10
Shareholder
Weight
S-S (CONC)
S-S (OCEA)
BZ (CONC) (NORM)

Fig 2(e)

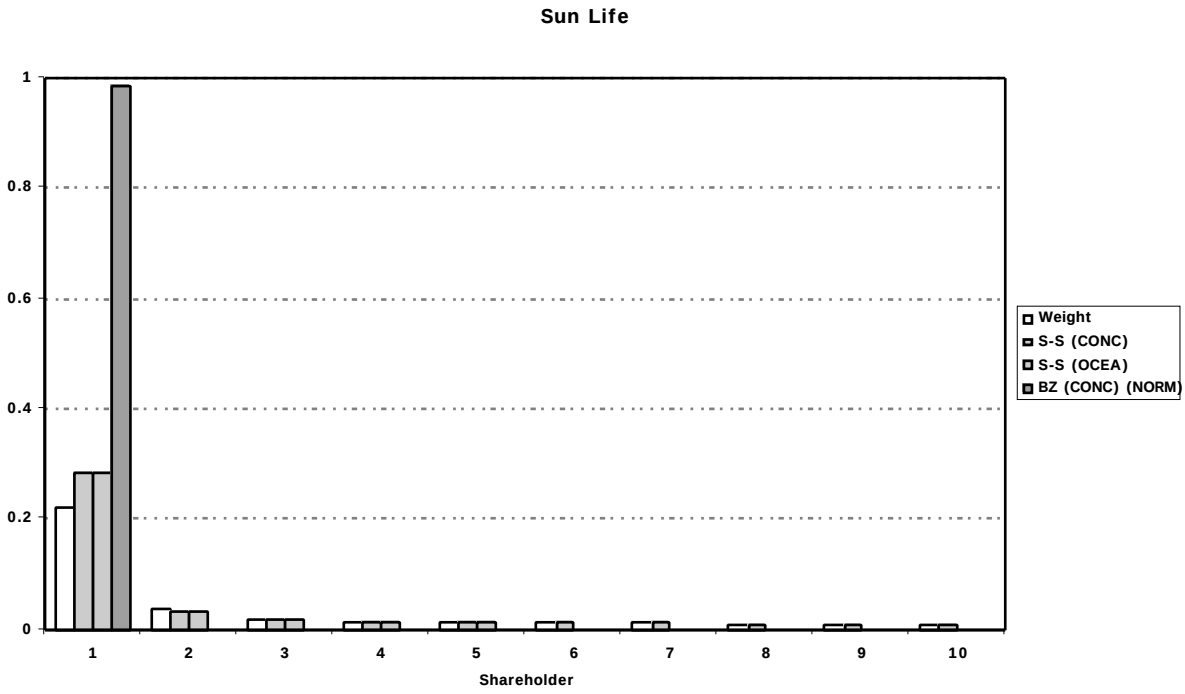

Sun Life
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
Shareholder
Weight
S-S (CONC)
S-S (OCEA)
BZ (CONC) (NORM)

Fig 2(f)

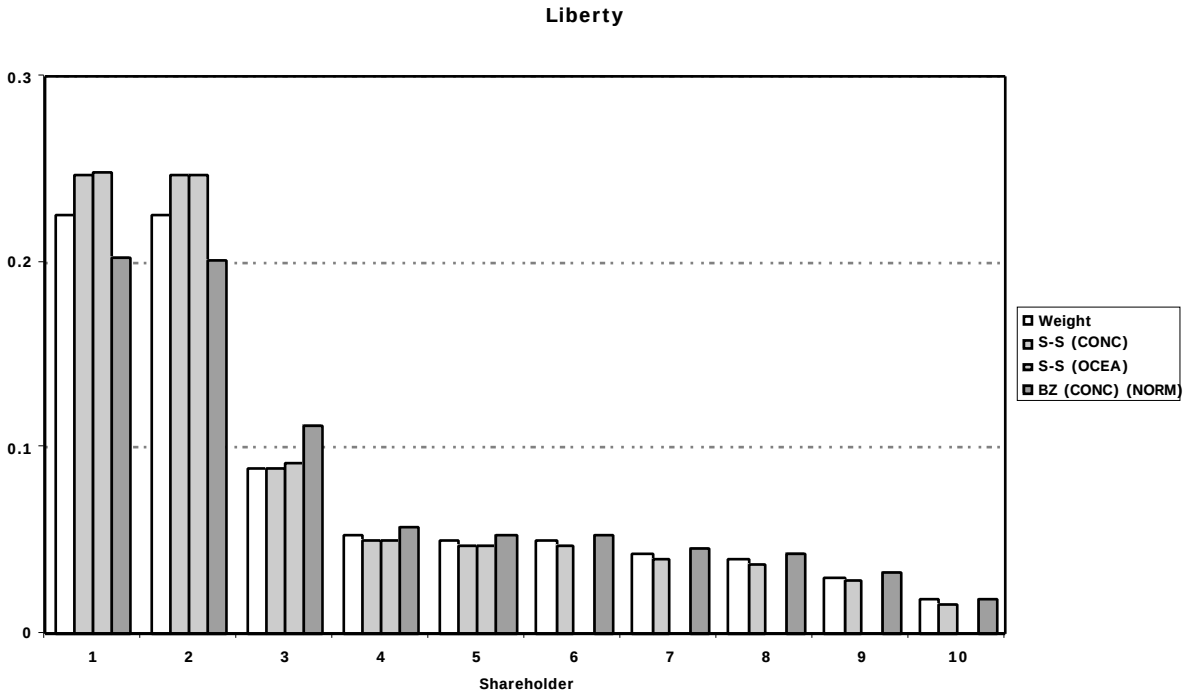

Liberty
0.3
0.2
0.1
0
1
2
3
4
5
6
7
8
9
10
Shareholder
Weight
S-S (CONC)
S-S (OCEA)
BZ (CONC) (NORM)

Fig 2(g)

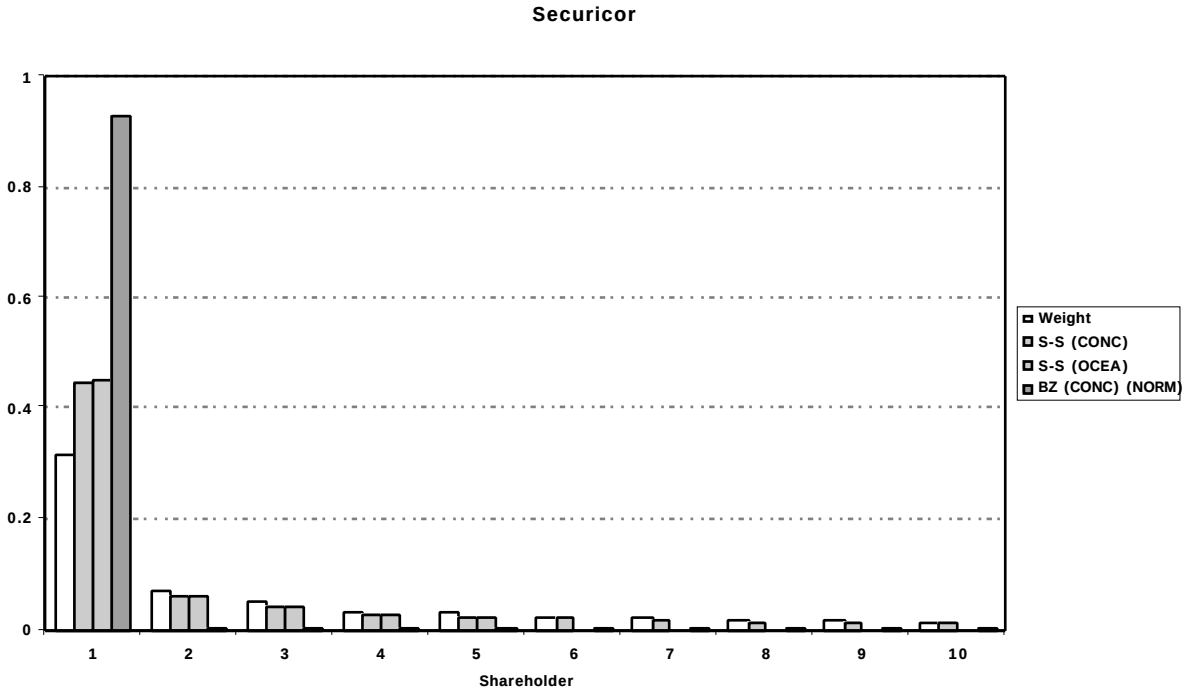

Securicor
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
Shareholder
Weight
S-S (CONC)
S-S (OCEA)
BZ (CONC) (NORM)

Fig 2(h)

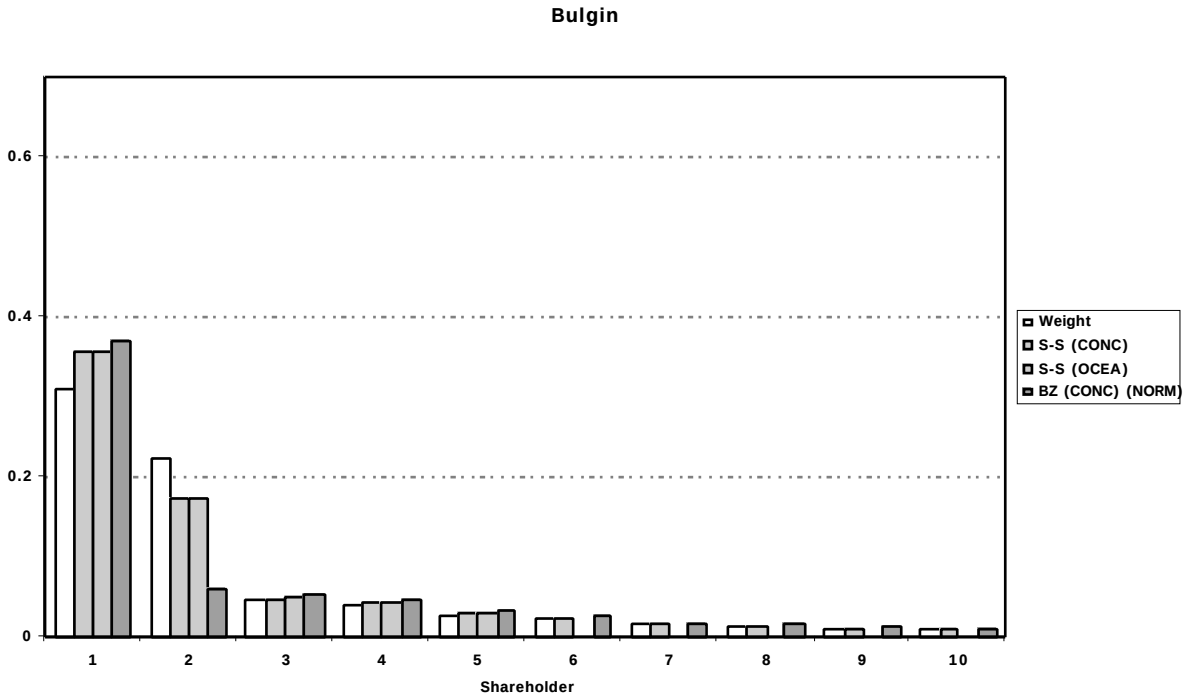

Bulgin
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
Shareholder
Weight
S-S (CONC)
S-S (OCEA)
BZ (CONC) (NORM)

Fig 2(i)

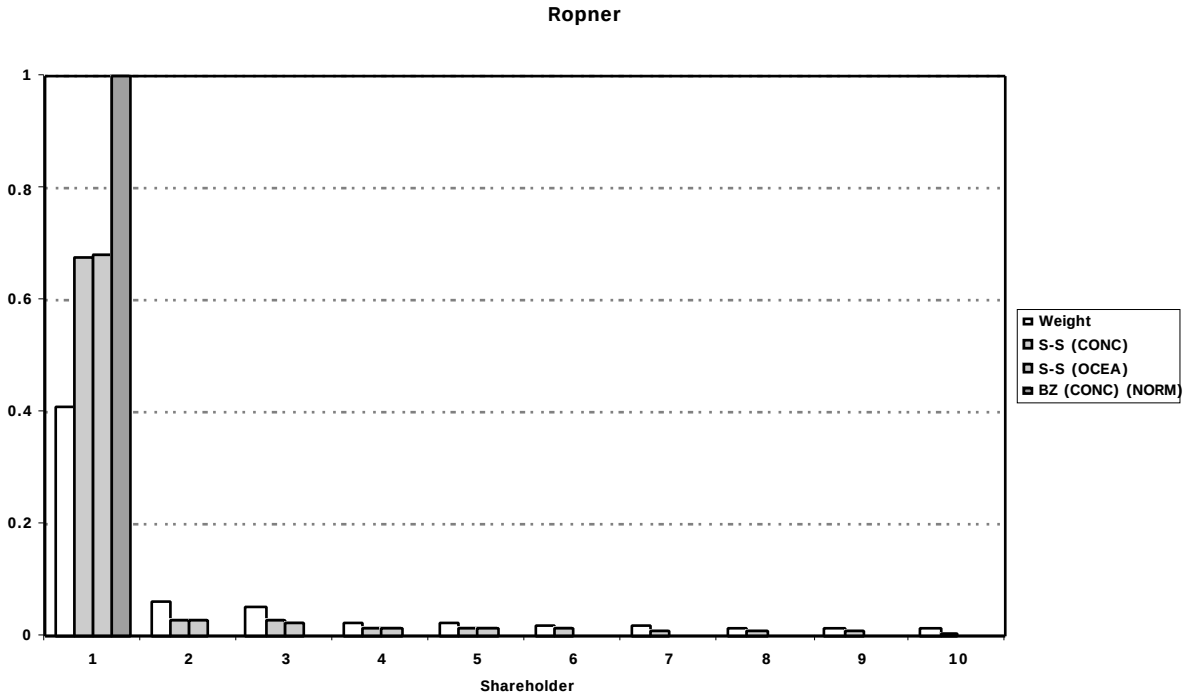

Ropner
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
Shareholder
Weight
S-S (CONC)
S-S (OCEA)
BZ (CONC) (NORM)

Fig 2(j)

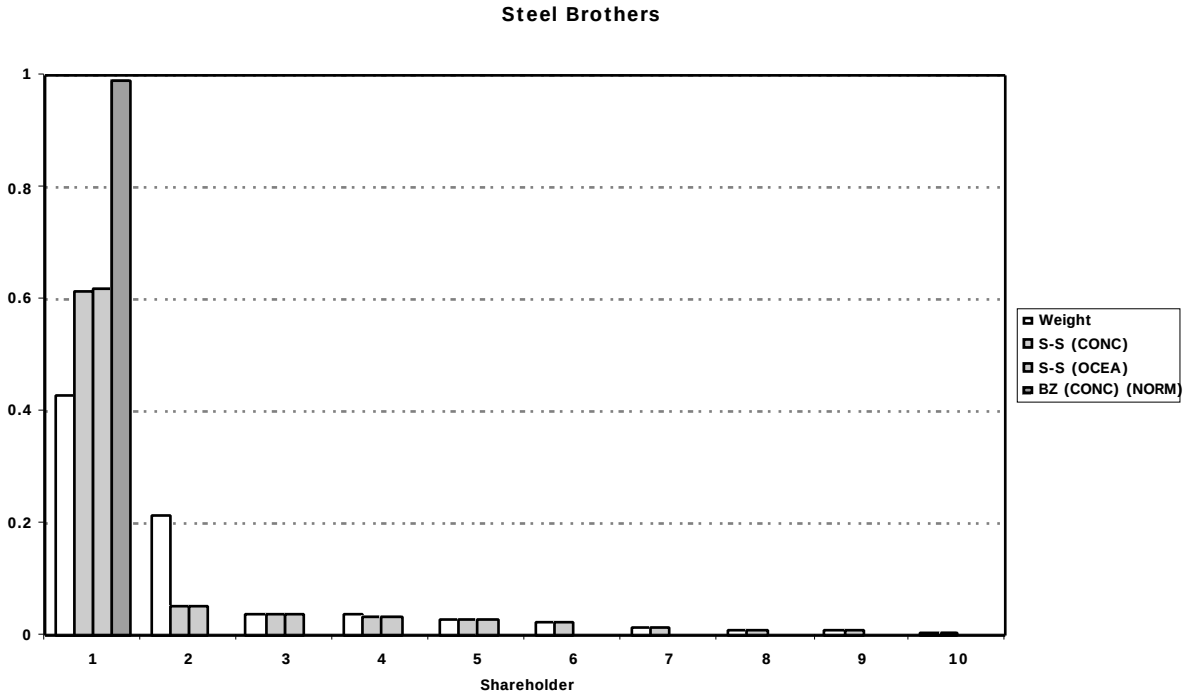

Steel Brothers
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
Shareholder
Weight
S-S (CONC)
S-S (OCEA)
BZ (CONC) (NORM)

Figure 3. Power Indices for the Complete Sample

Fig 3(a)

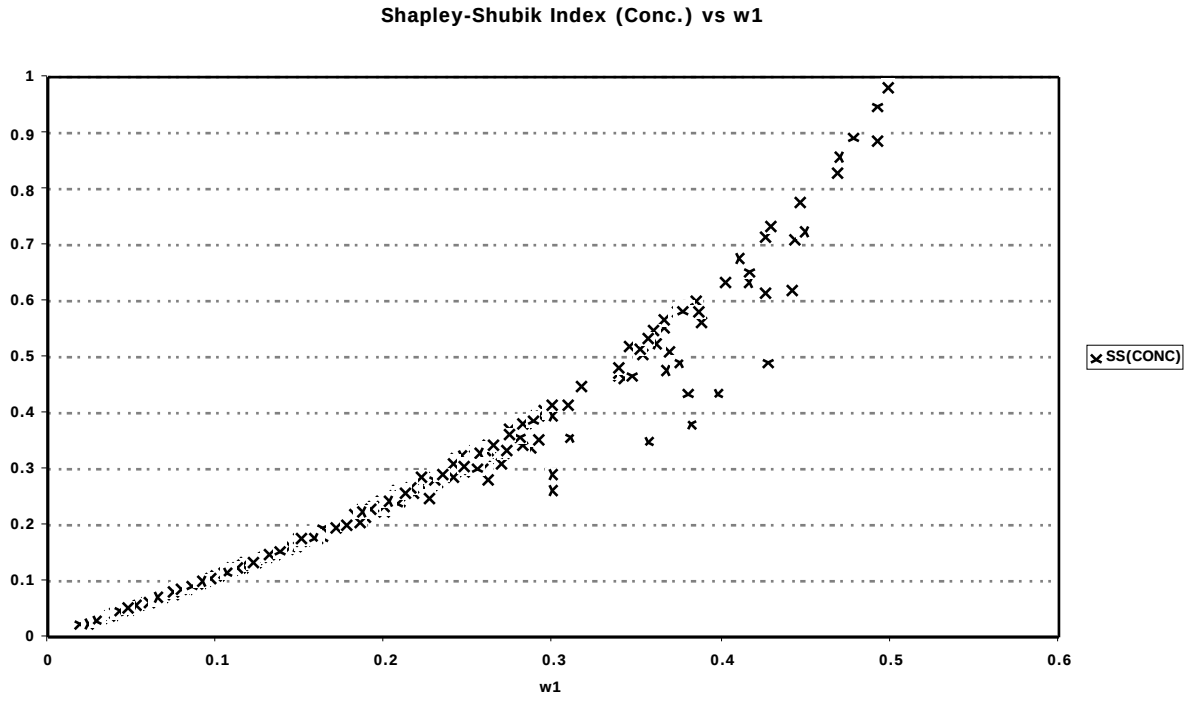


Fig 3(b)

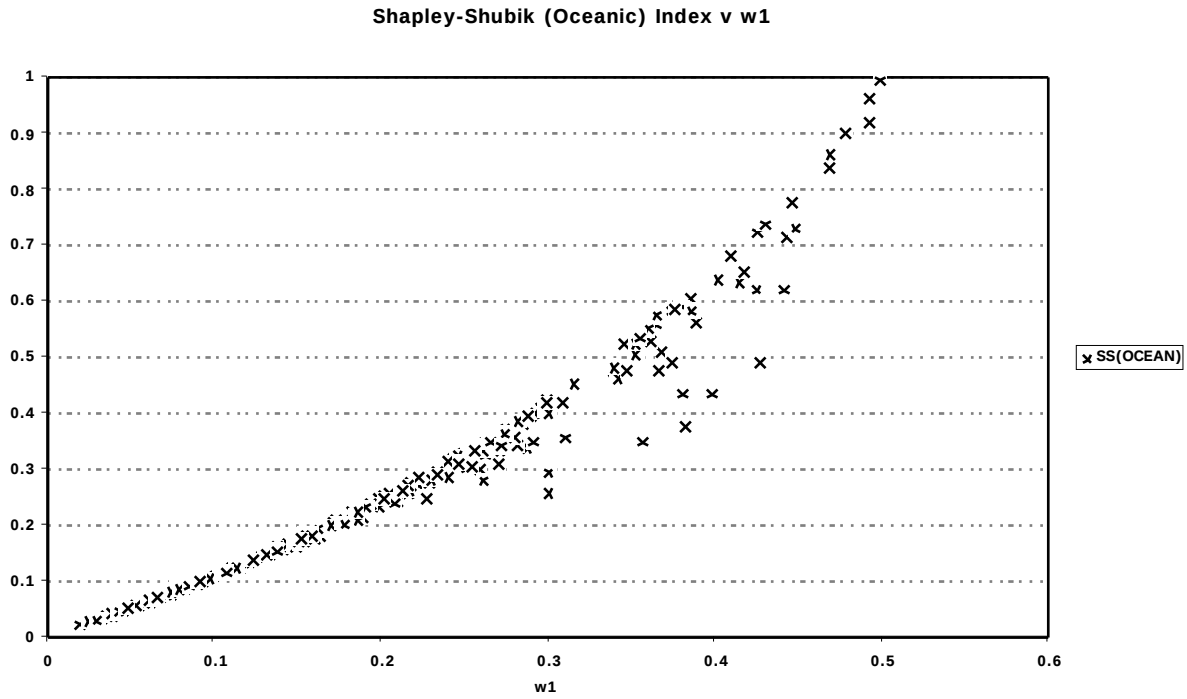


Fig 3(c)

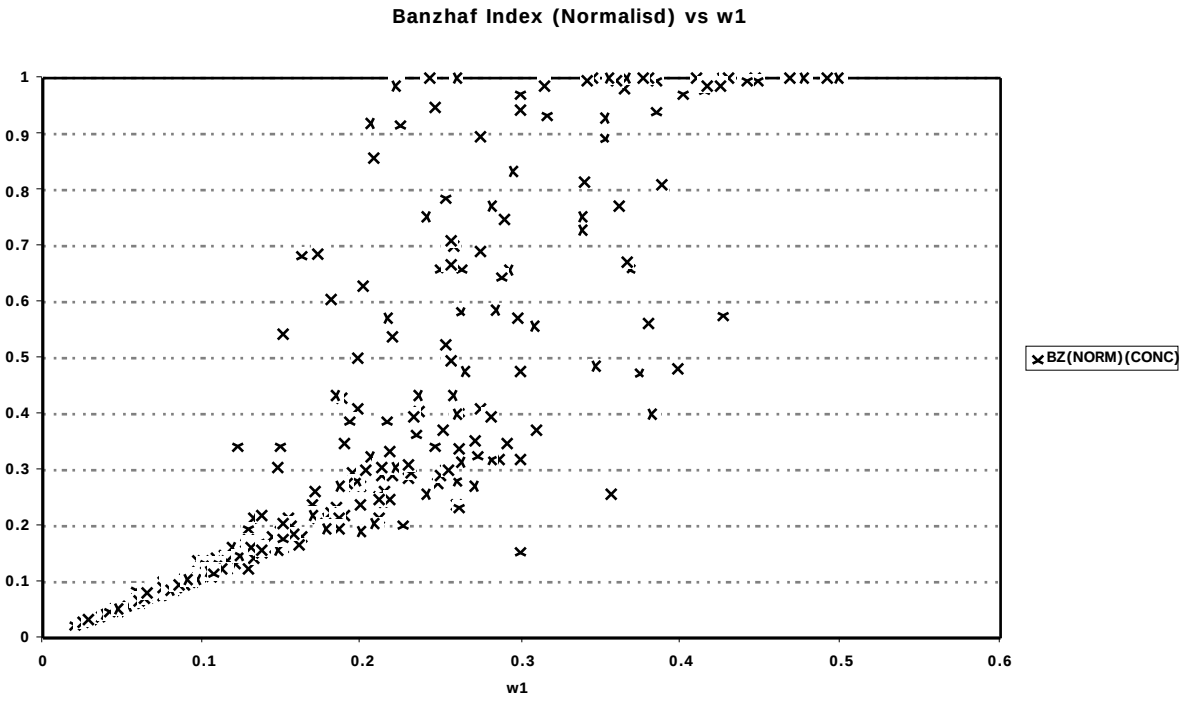

Banzhaf Index (Normalisd) vs w1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
w1
BZ(NORM)(CONC)

Fig 3(d)

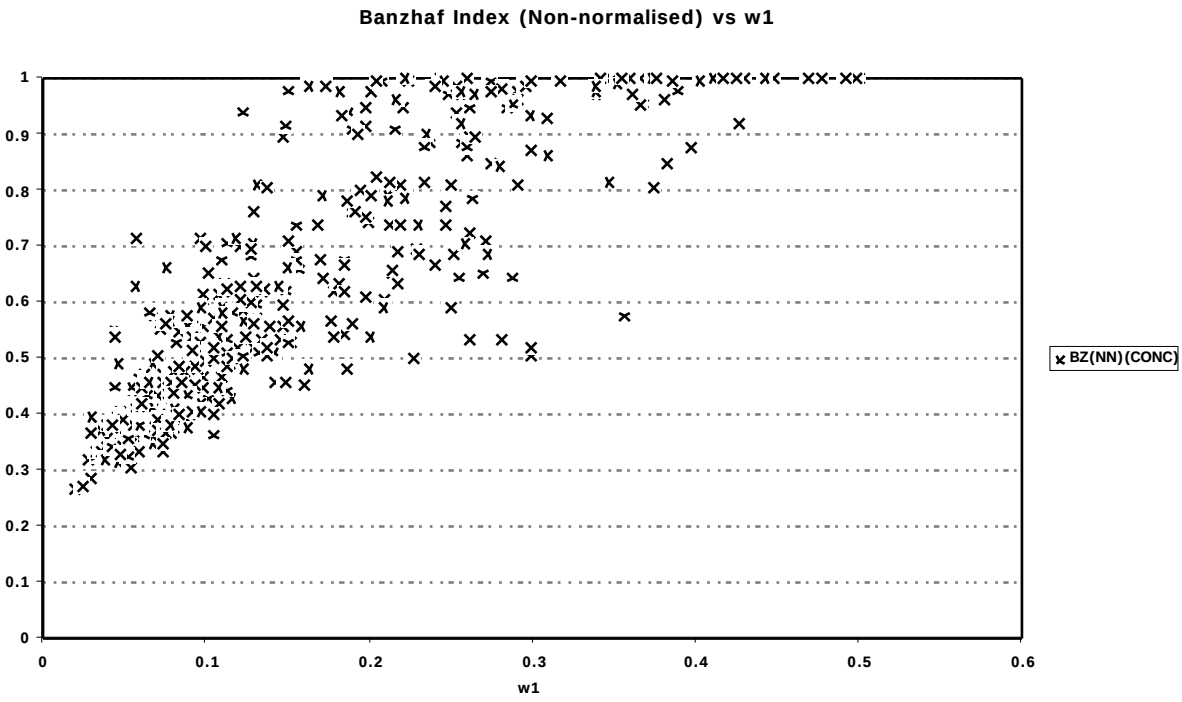

Banzhaf Index (Non-normalised) vs w1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
w1
BZ(NN)(CONC)

Fig 3(e)

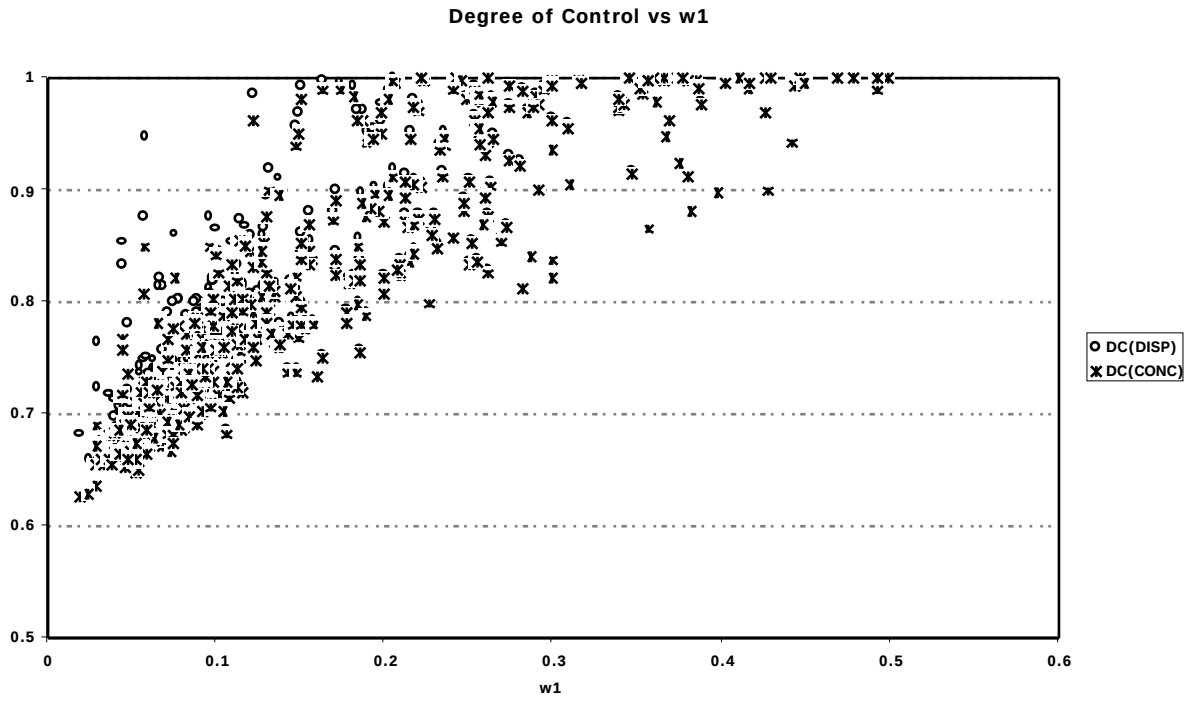

Degree of Control vs w1
1
0.9
0.8
0.7
0.6
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
w1
DC(DISP)
DC(CONC)

Figure 4 Power Indices for Complete Sample, Two Shareholder Blocks

Fig 4(a)

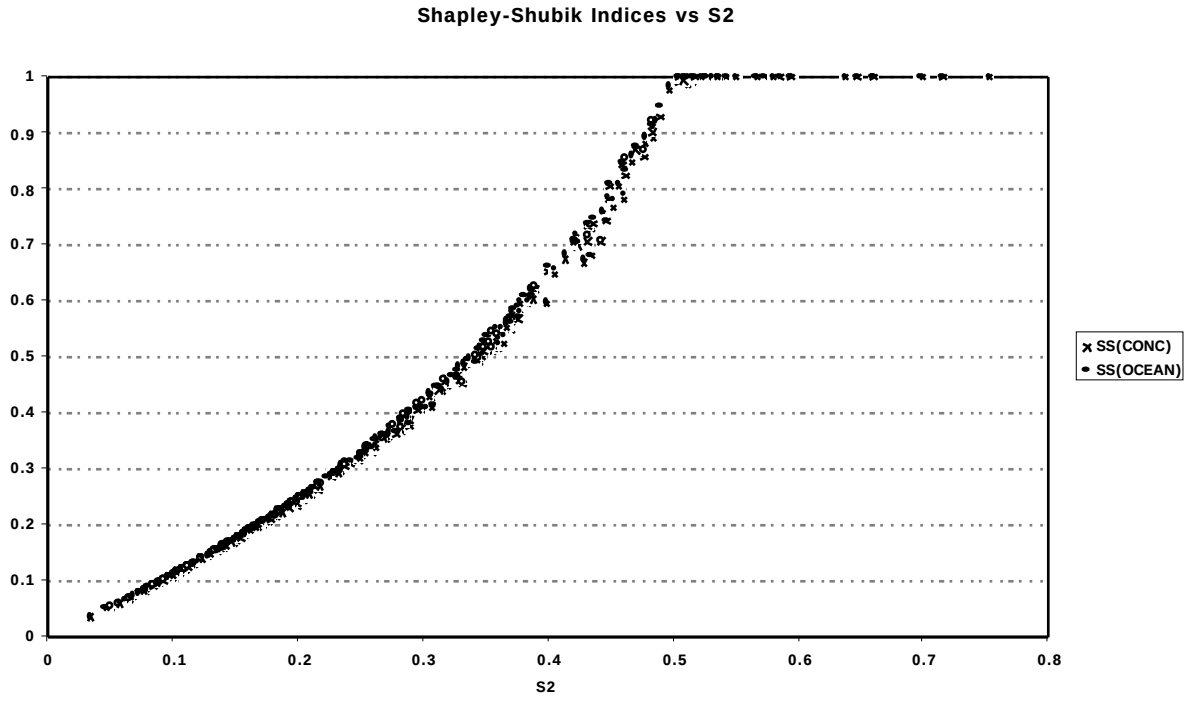


Fig 4(b)

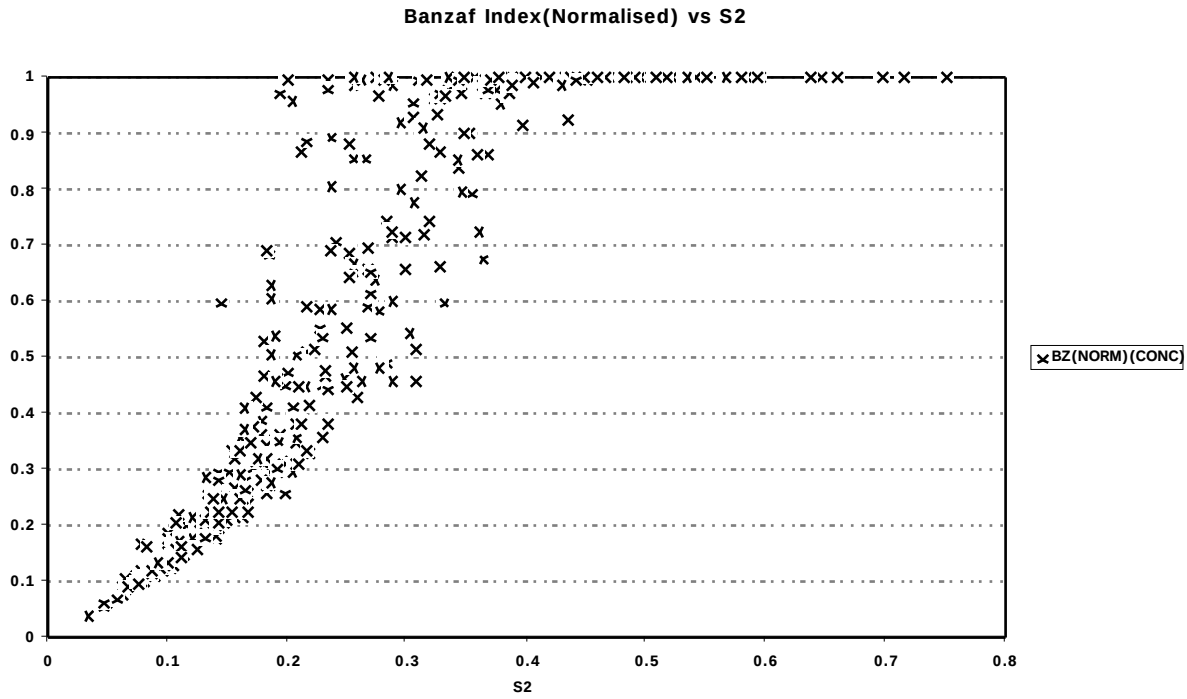


Fig 4(c)

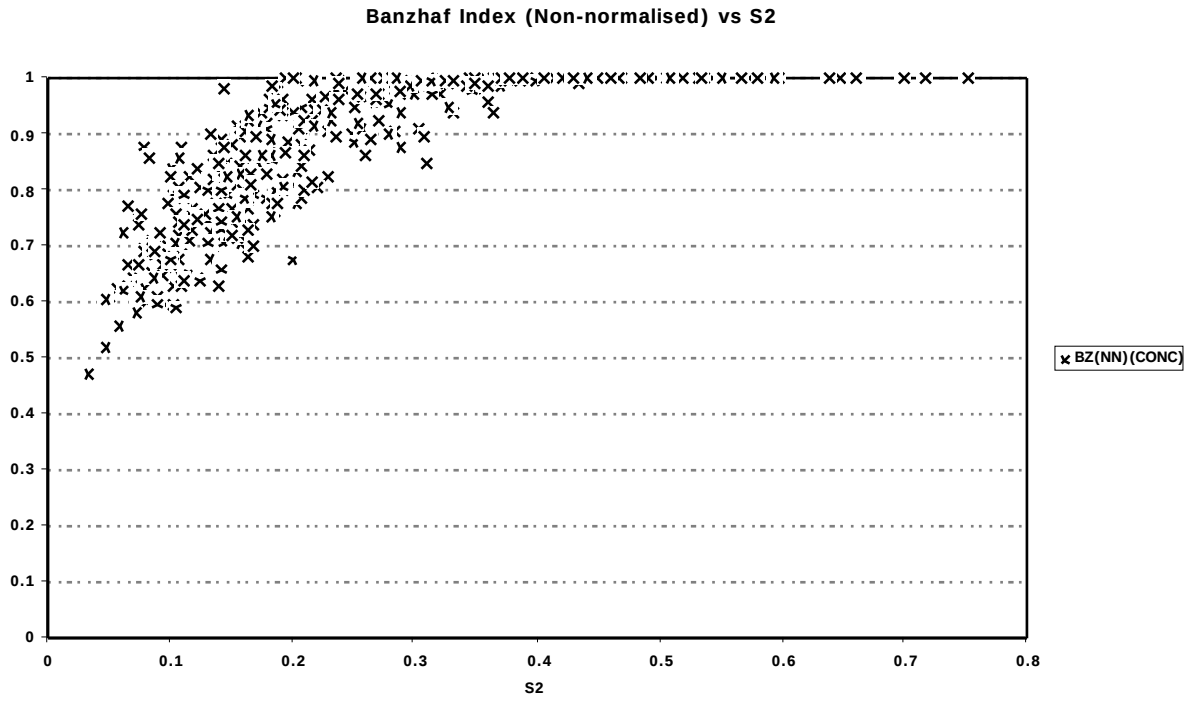

Banzhaf Index (Non-normalised) vs S2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
S2
BZ(NN)(CONC)

Fig 4(d)

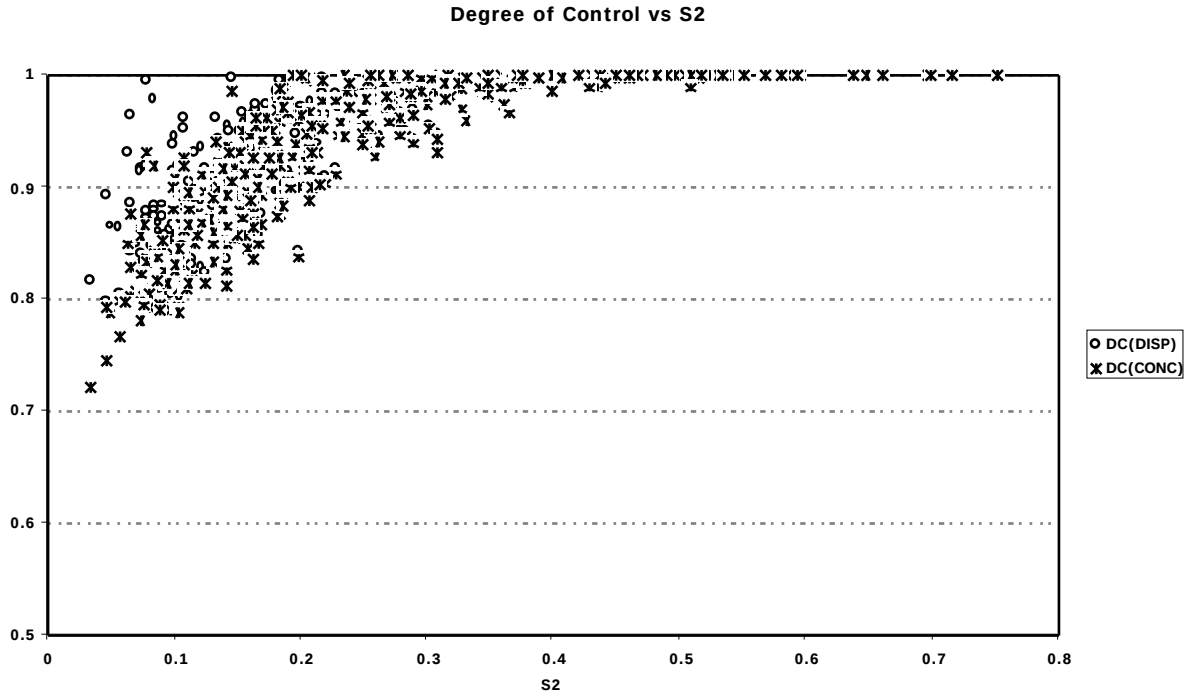
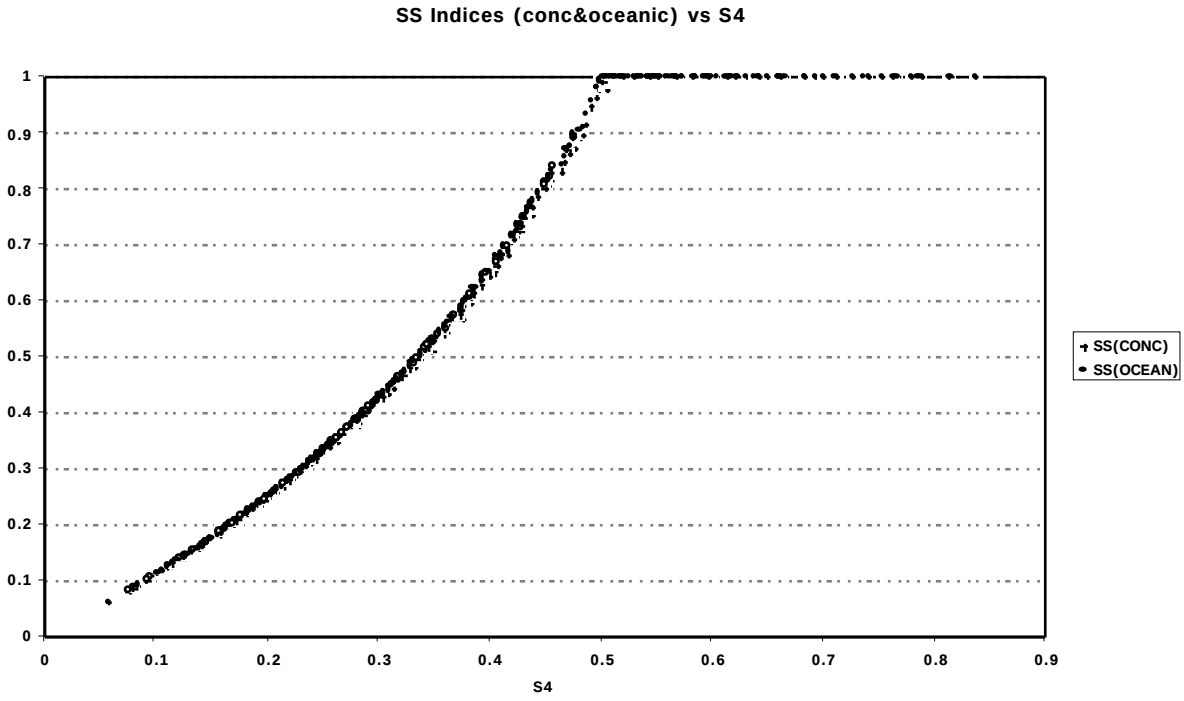

Degree of Control vs S2
1
0.9
0.8
0.7
0.6
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
S2
DC(DISP)
DC(CONC)

Figure 5 Power Indices for Complete Sample, 4-Shareholder Blocks

Fig 5(a)



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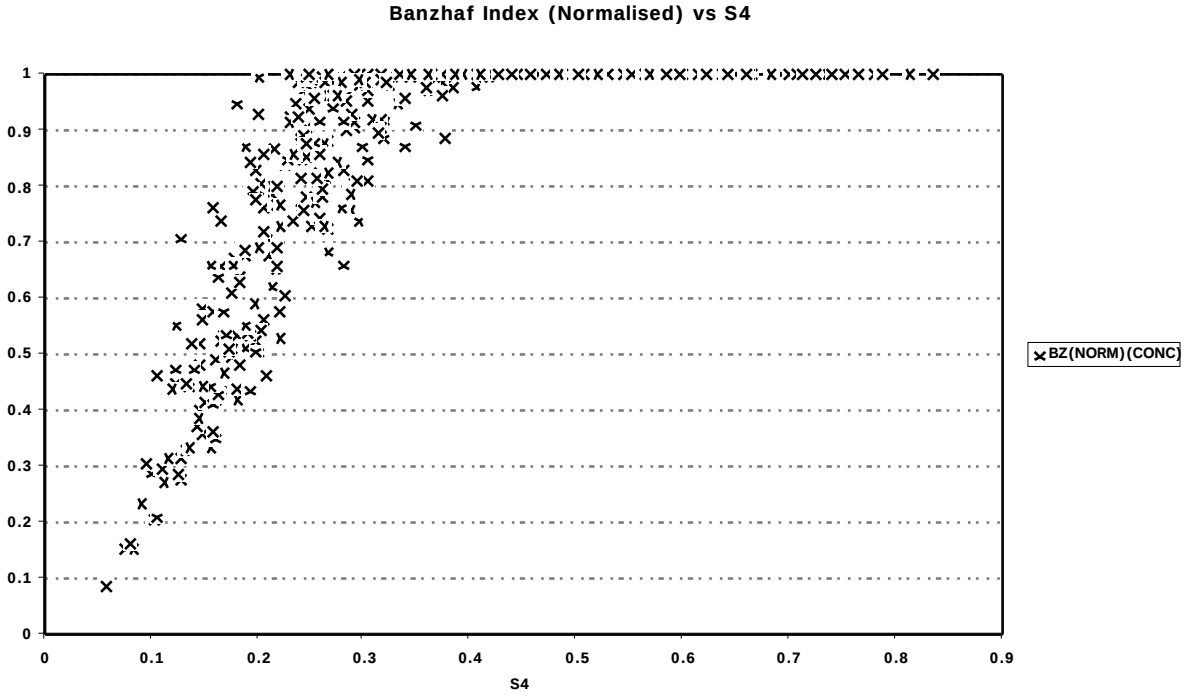


Fig 5(c)

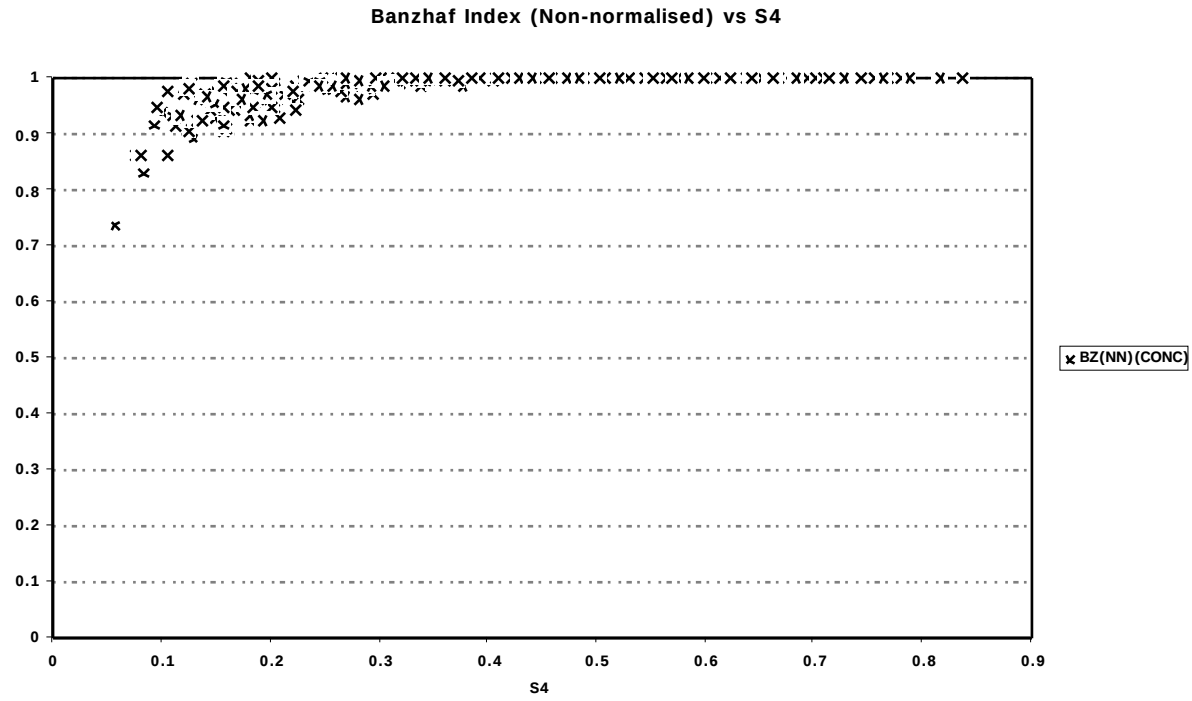

Banzhaf Index (Non-normalised) vs S4
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
S4
BZ(NN)(CONC)

Fig 5(d)

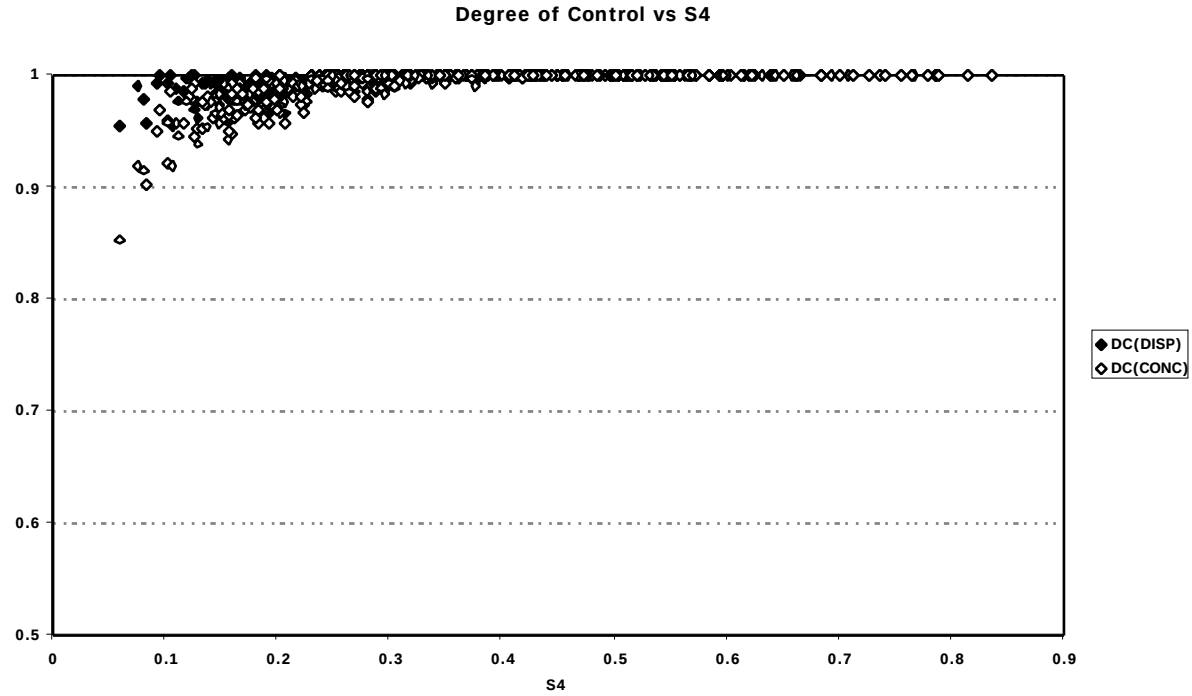

Degree of Control vs S4
1
0.9
0.8
0.7
0.6
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
S4
DC(DISP)
DC(CONC)

Figure 6 Power of Potential Shareholder Blocks, Illustrative Firms

Fig 6(a)

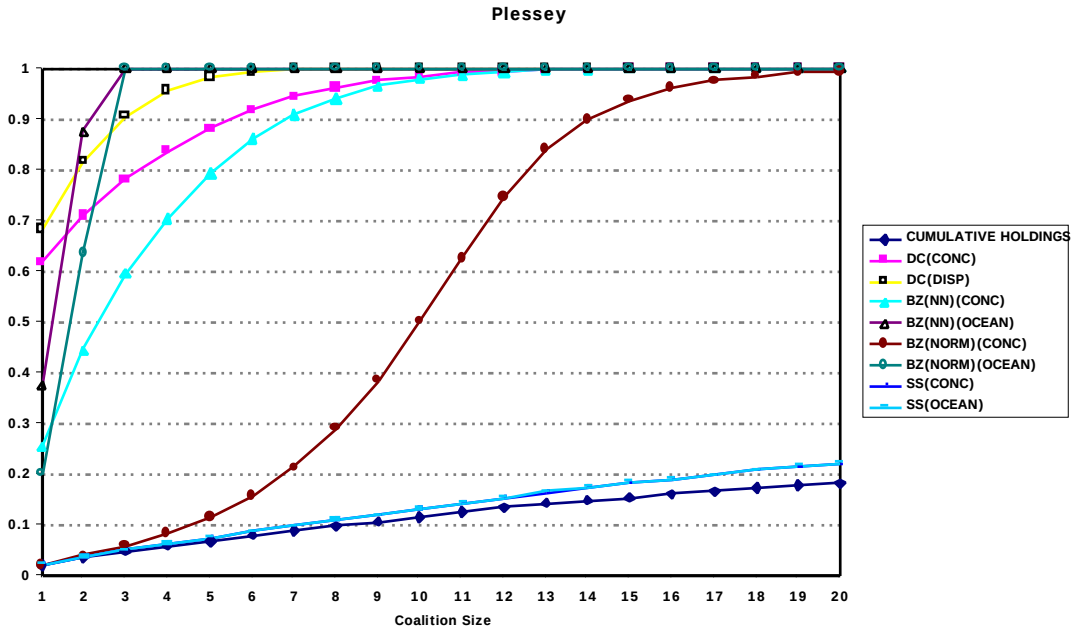


Fig 6(b)

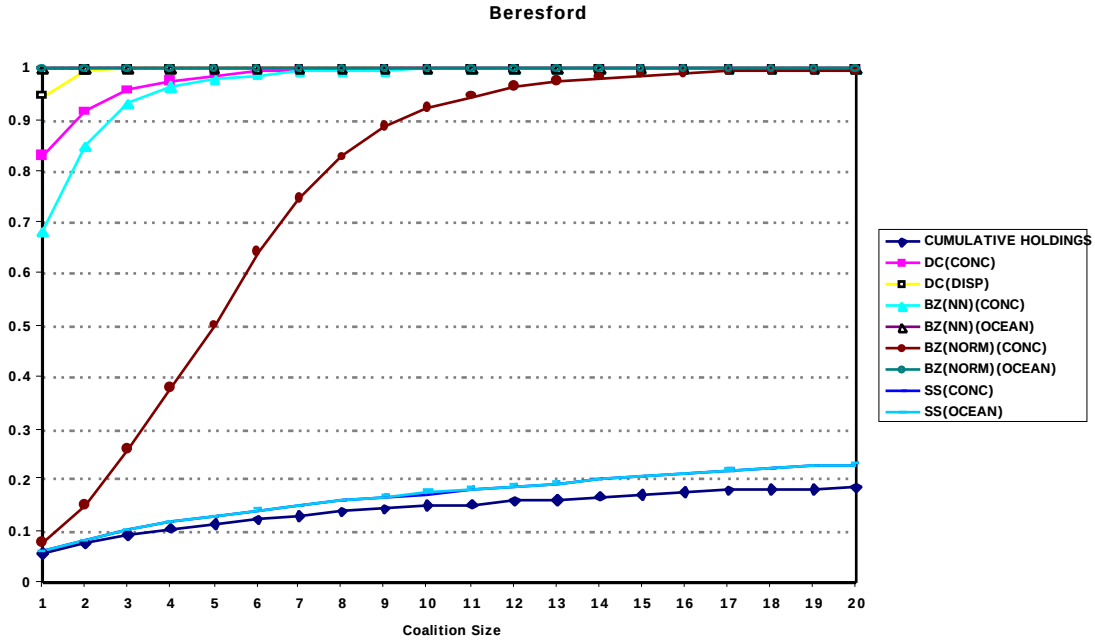


Fig 6(c)

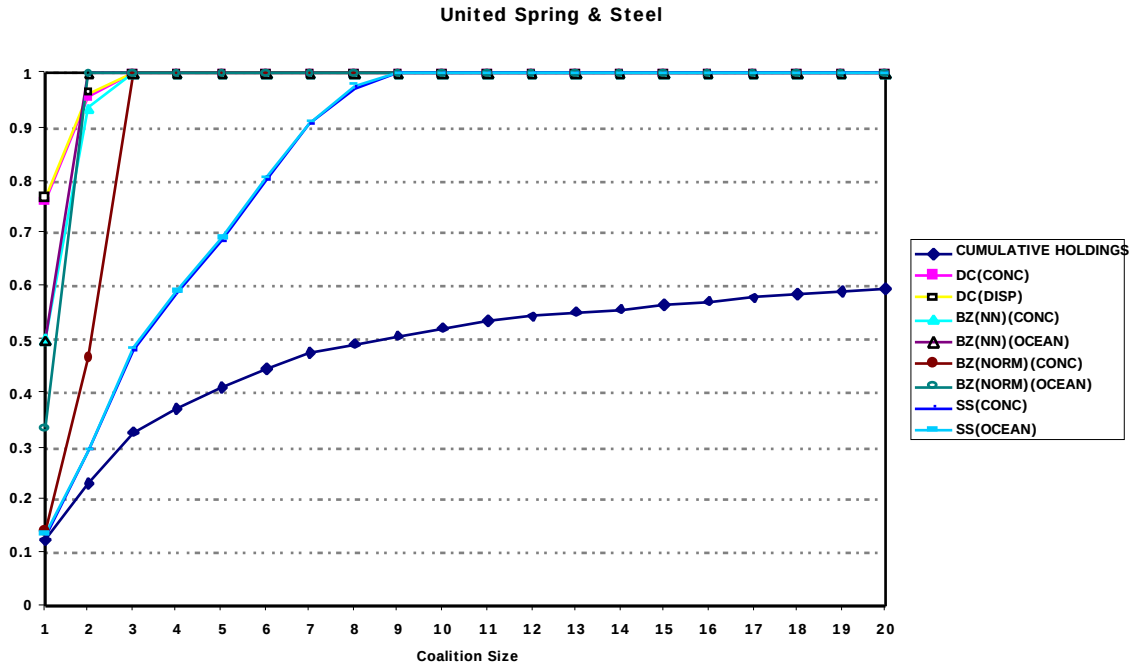

United Spring & Steel
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Coalition Size
CUMULATIVE HOLDINGS
DC(CONC)
DC(DISP)
BZ(NN)(CONC)
BZ(NN)(OCEAN)
BZ(NORM)(CONC)
BZ(NORM)(OCEAN)
SS(CONC)
SS(OCEAN)

Fig 6(d)

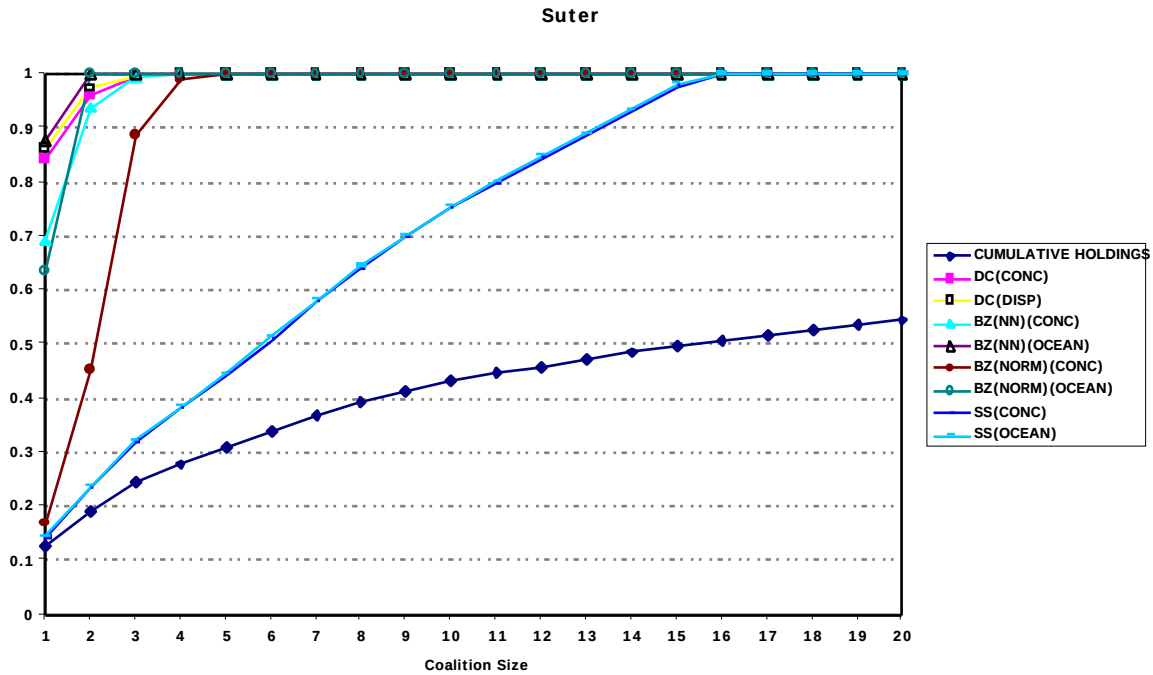

Suter
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Coalition Size
CUMULATIVE HOLDINGS
DC(CONC)
DC(DISP)
BZ(NN)(CONC)
BZ(NN)(OCEAN)
BZ(NORM)(CONC)
BZ(NORM)(OCEAN)
SS(CONC)
SS(OCEAN)

Fig 6(e)

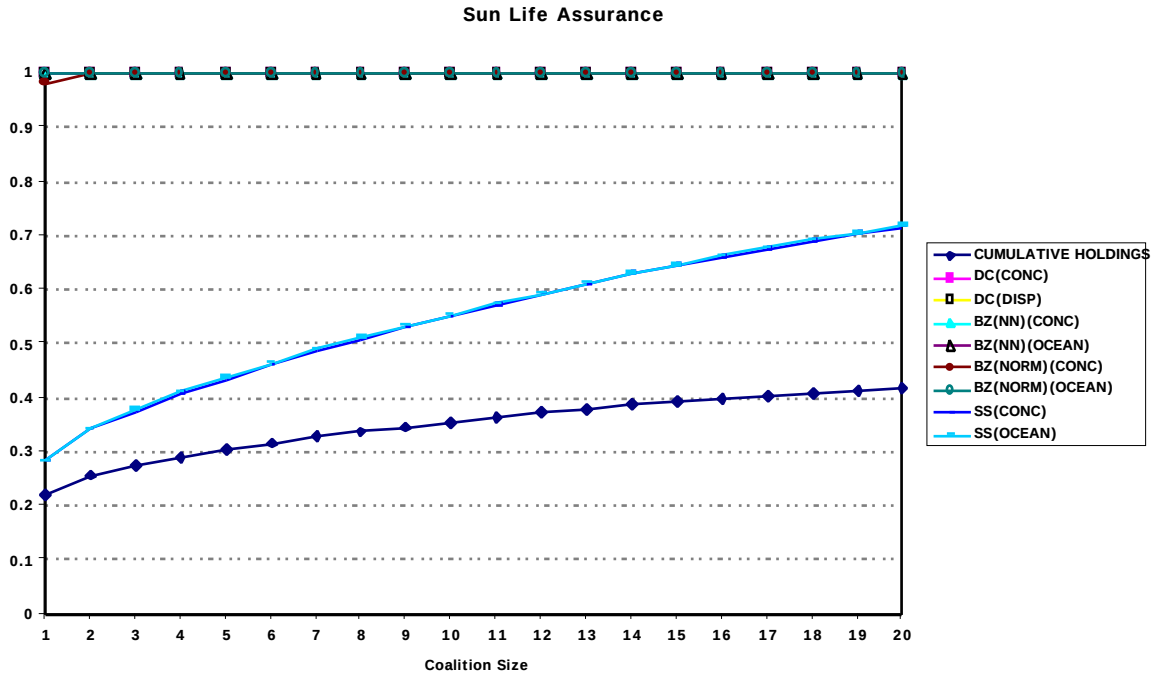

Sun Life Assurance
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Coalition Size
CUMULATIVE HOLDINGS
DC(CONC)
DC(DISP)
BZ(NN)(CONC)
BZ(NN)(OCEAN)
BZ(NORM)(CONC)
BZ(NORM)(OCEAN)
SS(CONC)
SS(OCEAN)

Fig 6(f)

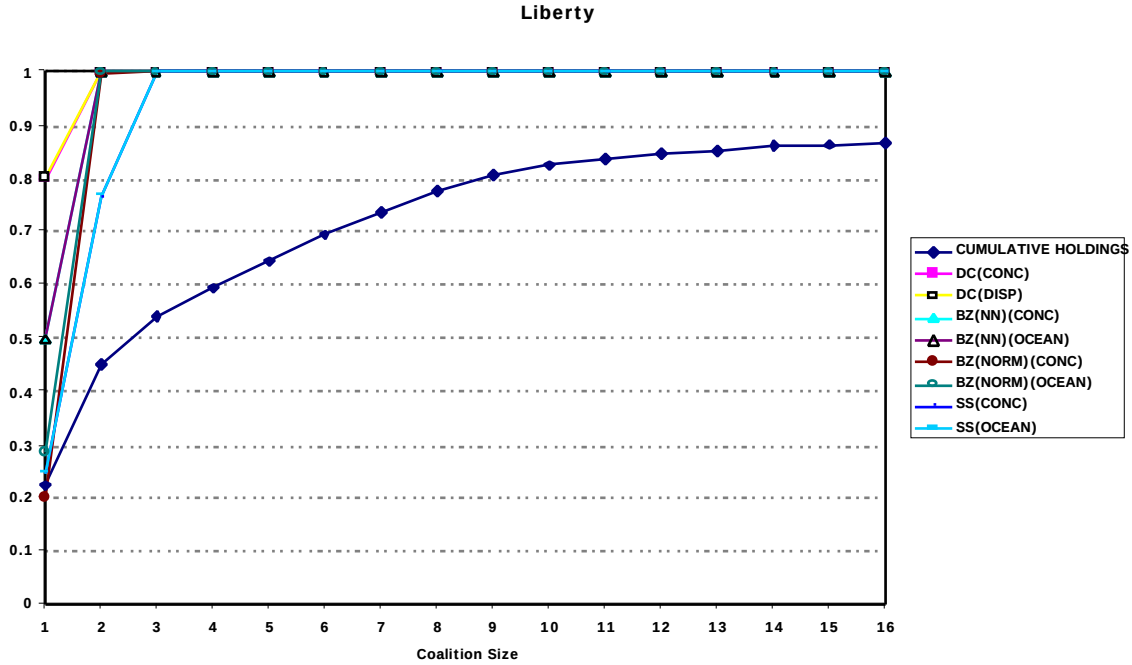

Liberty
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
Coalition Size
CUMULATIVE HOLDINGS
DC(CONC)
DC(DISP)
BZ(NN)(CONC)
BZ(NN)(OCEAN)
BZ(NORM)(CONC)
BZ(NORM)(OCEAN)
SS(CONC)
SS(OCEAN)

Fig 6(g)

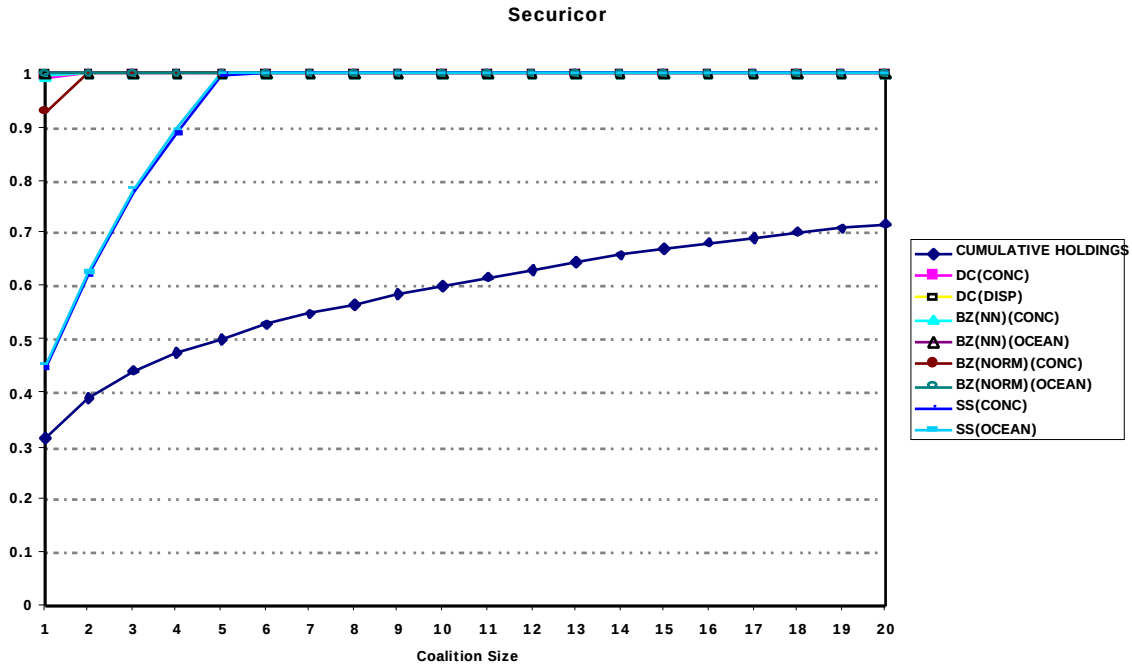
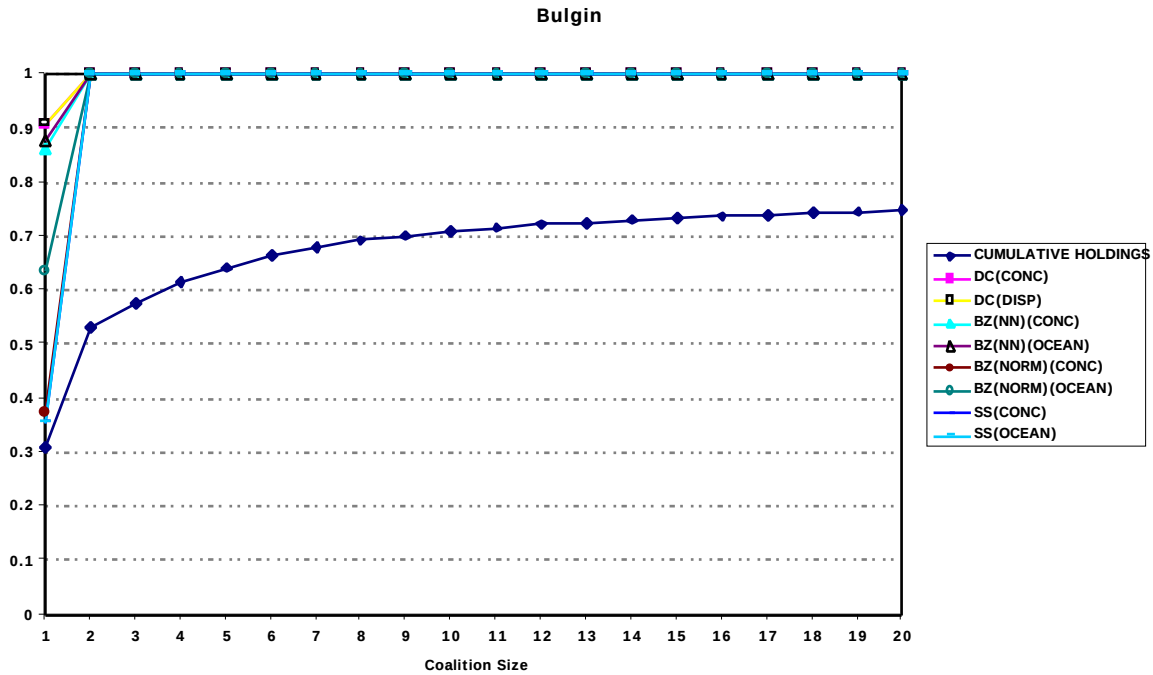

Securicor
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Coalition Size
CUMULATIVE HOLDINGS
DC(CONC)
DC(DISP)
BZ(NN)(CONC)
BZ(NN)(OCEAN)
BZ(NORM)(CONC)
BZ(NORM)(OCEAN)
SS(CONC)
SS(OCEAN)

Fig 6(h)


Bulgin
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Coalition Size
CUMULATIVE HOLDINGS
DC(CONC)
DC(DISP)
BZ(NN)(CONC)
BZ(NN)(OCEAN)
BZ(NORM)(CONC)
BZ(NORM)(OCEAN)
SS(CONC)
SS(OCEAN)