Co-opetition in Service Clusters with Waiting-Area Entertainment

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Problem Definition: Unoccupied waiting feels longer than it actually is. Service providers operationalize this psychological principle by offering entertainment options in waiting areas. A service cluster with a common space provides firms with an opportunity to cooperate in the investment for providing entertainment options while competing on other service dimensions.

Academic / Practical Relevance: Our paper contributes to the literature by being the first to examine co-opetition in a service setting, in addition to developing a novel model of waiting-area entertainment. It also sheds new light on the emerging practice of service clusters and small-footprint retailing.

Methodology: Using a queueing theoretic approach, we develop a parsimonious model of co-opetition in a service cluster with a common space.

Results: By comparing the case of co-opetition with two benchmarks (monopoly, and duopoly competition), we demonstrate that a service provider that would otherwise be a local monopolist can achieve higher profitability by joining a service cluster and engaging in co-opetition. Achieving such benefits, however, requires a cost-allocation scheme that properly addresses an efficiency-fairness tradeoff—the pursuit of fairness may backfire and lead to even lower profitability than under pure competition.

Managerial Implications: We show that as much as co-opetition facilitates resource sharing in a service cluster, it heightens price competition. Furthermore, as the intensity of price competition increases, surprisingly, service providers may opt to charge *higher* service fees, albeit while providing a higher entertainment level.

Key words: service co-opetition; waiting-area entertainment; marketing/operations interfaces; service clusters; common spaces

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1. Introduction

Seemingly endless waiting "destroys the soul" (Kolbert 2014, p. 19). An extensive literature examines managing customer waiting, with a focus on managing queueing discipline to ease congestion. An equally important—though less explored—aspect in service management entails reducing customers' *perceived* waiting time by operationalizing the principles of "the psychology of waiting lines" (Maister 1985). One of these principles is that occupied waiting feels shorter than unoccupied waiting. As a case in point, Walt Disney Parks and Resorts famously pioneered the practice of providing entertainment options (or diversions) for customers waiting for rides, which has been

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widely mimicked across the service industry (Larson 2011). Sewell Mini, a car dealer in Plano, Texas, "created a waiting area that was four times bigger than the original and includes a quiet office area with computers, a kids' play space, and a lounge-type arcade area" (Dizik 2011). As another example, OTG Management, an operator of airport restaurants, installed 6,000 iPads in dining and waiting areas at the United Airlines' terminal in Newark Liberty International Airport (White 2015). Complementing this practice, the consumer psychology literature (e.g., Kellaris and Kent 1992; Borges et al. 2015) has explored the role of waiting-area entertainment in reducing customers' perceived waiting time and increasing their service satisfaction. Waiting-area entertainment can represent a significant portion of firms' operational costs, as exemplified by HaiDiLao, a restaurant chain with a market capitalization of US$18.7 billion as of April 5, 2019, which operates restaurants with waiting spaces that account for as much as a third of their total spaces (Harvard Business School 2015).

Clustering, on the other hand, is one of the most intriguing socio-economic phenomena that has become increasingly prevalent. As a cultural byproduct of the rise of e-commerce sites such as Amazon, a growing number of brick-and-mortar retailers operate smaller-footprint stores that depend on the coexistence and usage of common spaces (Florida 2017; Smith 2016).[1] Clusters are "geographic concentrations of interconnected companies and institutions in a particular field" (Porter 1987). In a service cluster, where multiple firms offer services of a similar nature and share a common space, an opportunity exists for them to cooperate in the often costly investment needed to offer and maintain the space. This simultaneously competitive and cooperative relationship among service providers is known as "co-opetition" (Van Wassenhove 2016). One notable example of service clustering is "boardwalks"— pedestrian walkways built in tourist destinations that facilitate enjoyment for customers waiting for services (e.g., dining and drinking) provided by multiple, often competing, service providers (e.g., restaurants). As another example, various airports feature airport car-rental facilities shared by multiple car-rental companies that promise to improve customers' waiting experiences. In 2016, El Paso International Airport (ELP) opened a $46 million rental car facility, the cost of which is split among car-rental companies through a customer facility charge of $3.50 per car rental per day and a concessionaire fee of 10% of their car-rental revenues (Wysocky 2016). When the Miami International Airport levied a $3.25 charge per car rental per day to fund its "much-needed new car rental facility," and the Orlando International Airport was contemplating a similar charge, the car-rental companies actively lobbied the airports to drop or postpone such charges that "could have a devastating effect on the car rental industry" (Huxley and Coulter 2004). In both of these instances, the car-rental companies operating at the airport,

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[1] We thank the Associate Editor for suggesting this explanation for the increasing prevalence of service clusters and for suggesting the connection of our modeling framework to broadly defined common spaces.

while welcoming the improved facilities, view these charges as a significant burden on their business and make them less competitive than those operating off the airport.

Firms in service clusters often experience significant costs of constructing and maintaining common spaces. A situation where the problem arises is in commercial properties such as shopping malls and business improvement districts (BIDs). Common-area maintenance (CAM) fees are usually stipulated in lease contracts, in which the cost-allocation methods are among the key provisions (Noor and Pitt, 2009). In the US, a typical shopping mall collects yearly CAM fees that account for 40%-50% of its total operating expenses, more than its property taxes and insurance fees (Linneman 2016). CAM fees are attributed to high operating costs of shopping malls, which affect the occupancy-cost ratio, an important performance measure considered by Moody's in evaluating the credit quality of regional malls (Daniels and McDonnell 2003). The costs incurred from maintaining the common spaces affect tenants' renting experiences (Halvitigala 2018) and have become a leading source of tension between landlords and tenants (Mclinden 2017), as exemplified by a lawsuit filed by Gap Inc. against the high-end-mall operator Westfield over CAM fees (Cherney 2018). In India, high (and uncertain) CAM fees have contributed to conflicts between mall operators and their tenants (Bailay 2017; Kuruvilla and Ganguli 2008). In addition, as aforementioned, brick-and-mortar retailers increasingly operate with small footprints and rely on common spaces (Florida 2017; Smith 2016). For these small-footprint stores, the cost of maintaining such common spaces can account for a significant proportion of their revenues.

Although the overarching rationale of co-opetition in a service cluster is fairly straightforward, there is a paucity of analytical models and theory linking the intra-firm service operations and inter-firm strategic interactions. A particularly interesting setting is one in which the cost of maintaining common spaces is a strategic decision. This case, which our paper focuses on, requires more "equitable," "reasonable" or "good faith" cost-allocation schemes that more closely tie each tenant's share to the costs associated with its revenue-generating activities (Boyle and Novack 2015).[2] At a more fundamental level, despite the widespread usage of waiting-area entertainment in the service industry, a more systematic understanding of the practice is called for. To gain a deep understanding of service clustering with congestion and entertainment options, we model various market structures; analyzing and comparing these structures provide interesting insights into managing co-opetition, a dyadic, war-and-peace relationship.

We start with a focused view of the design of entertainment options, through analyzing the case with a service provider that is a local monopolist. We characterize the service provider's

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[2] By contrast, when the total cost of maintaining common areas is exogenous, it has been a common practice to adopt a simple fixed-share cost-allocation scheme under which each tenant pays a fixed amount or share of the total cost (Lynn 2010; Nash 2015).

optimal decisions, and find that as the entertainment options become more effective in alleviating consumers' disutility from waiting, the service provider will be able to charge a higher service fee, but the optimal entertainment level may either increase or decrease, meaning the service provider chooses a high entertainment level only when the entertainment options are moderately effective. Furthermore, one may expect that entertainment options and service capacity are substitutes for each other; that is, the service provider would choose a high entertainment level when building capacity is costly. By contrast, we show the opposite is true—as expanding capacity becomes increasingly costly, the arrival rate in equilibrium has to be lower to maintain the waiting-time standard; the service provider would thus choose a lower entertainment level.

Building on the local monopolist's problem of determining its entertainment options, we next analyze the scenario in which the firm joins a service cluster that also consists of a competitor in the same service category. We compare two situations involving competition and co-opetition, respectively. The impact of co-opetition on the firm's service-operations decisions and performance crucially depends on the way the service providers share the cost for providing and maintaining entertainment options. We consider a volume-based cost-allocation scheme and show that, if properly executed, co-opetition can help service providers achieve a profit higher than under monopoly, demonstrating that a service provider, which would otherwise be a local monopolist, can achieve higher profitability by joining a service cluster and engaging in co-opetition. Our numerical results also suggest co-opetition is most lucrative when (i) the market size is small, (ii) the cost of expanding capacity is high, and (iii) consumers are highly sensitive to waiting.

The benefits of co-opetition, however, are not guaranteed. We find the pursuit of fairness in cost sharing can backfire and completely eliminate the cost-sharing advantage, alluding to a fairness-efficiency tradeoff that is behind several counterintuitive results. For instance, we find that contrary to the case of duopoly competition, when price competition becomes more intense, charging *higher* service fees might be optimal for co-opeting service providers.

Our paper constitutes an initial attempt to understand how waiting-area entertainment interacts with pricing and capacity decisions in a service setting. Through analyzing the scenarios of monopoly, competition, and co-opetition, we demonstrate the benefit of co-opetition in service operations, and provide managerial insights into operational execution and strategic inter-firm interactions under co-opetition.

1.1. Literature

Our study builds on and advances two streams of literature, namely, competition among service providers, and co-opetition in manufacturing and supply-chain settings.

The first stream of literature focuses on the effect of competition on service providers' operational decisions. To incorporate waiting time as a basis for competition, the literature relies on queueing

models to account for customers' "waiting costs;" see Hassin and Haviv (2003) for a comprehensive survey of the relevant queueing literature, and Allon and Federgruen (2007) for an account of the prevalence of waiting-time standards in various service industries. Hall and Porteus (2000) consider a situation in which demand depends solely on waiting time. Ho and Zheng (2004) model the competition between service providers based on waiting-time announcements, in which demand is also influenced by service quality. Several studies empirically examine the impact of waiting time on demand for services. Png and Reitman (1994), for example, study the impact of waiting time on the demand at gas stations, identifying service time as a key factor in driving consumer demand. Savva et al. (2018) study a yardstick-competition scheme in which each local monopolist is compensated by its service performance relative to its peer service providers.

Several papers study competition in terms of price, capacity, and service quality. Li and Lee (1994) consider price and delivery-time competition between two service providers. Lederer and Li (1997) investigate competition between two service providers surrounding their pricing and capacity decisions. In most of these service-competition models, a customer's choice is based on the *full price* of the service, defined as the sum of the service fee and the expected waiting cost. Cachon and Harker (2002) consider competition between two service providers, where each provider's demand depends on its own as well as its competitor's full prices.

Departing from the aforementioned full-price models, So (2000) develops an attraction model of the competition based on both prices and waiting-time standards. In his model, each firm has an attraction value that is a function of its price and waiting-time standard, and its market share is proportional to that attraction value. Allon and Federgruen (2007, 2008) consider price and service competition based on a general demand model that is a separable function of price and service level. Our consumer-demand modeling approach is aligned with these models.

Our paper departs from and contributes to the first stream of literature in that we emphasize the role of entertainment options in shaping consumer demand and, in turn, the firm's other service decisions. In contrast to the service operations literature with quality considerations, where service quality is directly driven by the service rate or provision of services (see, e.g., Anand et al. 2011; Dai et al. 2017, 2018; Dai and Singh 2019; Guo et al. 2017; Debo and Veeraraghavan 2014; Veeraraghavan and Debo 2009; Zhan and Ward 2014), in our model, entertainment options essentially function as an auxiliary service that helps to reduce customers' psychological anxiety from waiting. To the best of our knowledge, our paper is the first to analytically study entertainment options in the service industry.

The second stream of relevant literature involves co-opetition in manufacturing and supply-chain settings. Venkatesh et al. (2006) consider a manufacturer of proprietary component brands in the end-product market, and show the manufacturer may benefit from being a "co-optor,"

that is, a component supplier for another brand as well as a producer of its own branded end-product. Gurnani et al. (2007) model co-opetition between a supplier and a buyer under demand uncertainty, where the supplier decides the product quality and the wholesale price and the buyer decides the retail price and the demand-boosting selling effort before the demand uncertainty is resolved. Nagarajan and Sošić (2007) model coalition formation among competitors who set prices, and characterize the equilibrium behavior of the resultant strategic alliances. Casadesus-Masanell and Yoffie (2007) model the simultaneously competitive and cooperative relationship between two manufacturers of complementary products, such as Intel and Microsoft, on their R&D investment, in addition to the pricing and timing of new product releases. Bakshi and Kleindorfer (2009) model co-opetition between a supplier and a retailer in investment decisions to mitigate the losses from supply chain disruptions, where the level of vulnerability to disruptions of the supplier is private information. Chen and Roma (2011) consider co-opetition between two retailers procuring from a common manufacturer. The two retailers compete for the market size through setting their retail prices. At the same time, they may cooperate in ordering decisions to take advantage of the manufacturer's quantity-discount scheme. Huang et al. (2015) study the formation of alliances among upstream suppliers serving the same downstream manufacturer. They show coalitions help soften competition, but the competition-reduction effect itself does not facilitate the formation of large coalitions. Mantovani and Ruiz-Aliseda (2015) consider a scenario in which competing manufacturers are engaged in co-building an ecosystem of innovation. Guo and Wu (2018) study the co-opetition between two manufacturing firms that share their production capacity through a randomized-rationing rule.

Our paper advances the second stream of literature in that it is the first to study co-opetition in a *service* (as opposed to manufacturing and supply-chain) setting. Cooperation is "vertical" in a supply-chain setting yet "horizontal" in the service context we study in this paper. The service setting presents a vastly different set of operational challenges, including, for example, the need to enforce a waiting-time standard that is instrumental in influencing demand. These differences allow us to generate novel insights that have not been reported in the literature. For example, we show that as price competition becomes more intense, under co-opetition, the service providers may charge higher, instead of lower, prices. Additionally, our work highlights how co-opetition impacts price competition, and how their compound effect drives the results. Thus, our research significantly expands the breadth and depth of the literature on co-opetition.

Lastly, the marketing literature has examined how co-opetition, for example, in sharing the same advertising agency (Villas-Boas 1994), shapes a firm's competitive landscape. A recent study by Lu and Shin (2016) examines the problem of marketing a new product category for which the market does not yet exist and educating consumers and gathering consumer demand can be costly. Lu

and Shin (2016) show that a firm may benefit from cooperating with its competitors by disclosing its key innovations and inducing others to exert demand-sided effort. Our paper shares the co-opetition aspect but focuses on the design of service operations with consumers who are sensitive to waiting times, leading to a distinctive set of managerial implications.

The rest of the paper is organized as follows. In §2, we analyze a scenario with a monopoly service provider. In §3, we analyze a scenario with two service providers competing with each other. Building on these benchmarks, in §4, we study the full scenario in which two competing service providers cooperate on entertainment options. In §5, we compare the three scenarios and generate managerial insights. In §6, we examine a benchmark without queueing considerations, and use it to shed light on the effect of queueing considerations. In §7, we consider several extensions of our main model to explore the boundary of our main model. We conclude the paper and discuss future research opportunities in §8. All technical proofs are relegated to the appendix.

2. Local Monopolist: Service Design with Waiting-Area Entertainment

We start with the case of a service provider being a local monopolist and not part of a service cluster. This case provides a focused view of the decision on the entertainment options and how it interacts with other service decisions.

With a waiting-time standard w, service fee p, and the level of entertainment options α, the service provider faces a customer arrival rate of

$$\lambda(p, \alpha; w) = B - hwe^{-\delta\alpha} - \beta_0 p, \tag{1}$$

where B is the maximum demand rate, β_0 measures a customer's price sensitivity, and h is the waiting cost per unit of waiting time, which captures customers' aversion to waiting.[3] Note this type of demand function is along the line of Allon and Federgruen's (2007, 2008) and has been commonly used in the literature. As reflected in the customer arrival rate, the entertainment options reduce customers' disutility from waiting such that each customer has an effective waiting-cost rate of $he^{-\delta\alpha}$, where $\delta > 0$ measures the effectiveness of the entertainment options. In §7.1, we extend the above demand function (1) by replacing the term $hwe^{-\delta\alpha}$ with a general function $E(w, \delta, \alpha)$. In providing an entertainment level of α, the service provider incurs a cost of $C(\alpha)$, which is assumed to be convex increasing in α, as is consistent with the notion of diseconomies of scale arising in cases where technology of production is non-scalable (Anand and Mendelson 1997). Without loss of generality, we assume $C(\alpha) = \frac{1}{2}c\alpha^2$, where c is a positive constant.

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[3] Broadly speaking, the above demand system also applies to a more generic scenario in which the service provider invests and maintains a "common space" to improve customers' service experiences. In that case, one may interpret w is the total time spent in the system, and α is the intensity of investment in the common space.

The service provider builds its capacity (i.e., service rate) μ at a marginal cost of γ. For simplicity of representation, we assume customer arrivals follow a Poisson process, and each customer's service time is exponentially distributed. The service process is therefore an $M/M/1$ queue. Our key findings extend to alternative queueing disciplines (e.g., $M/G/1$ and $G/G/1$). To maintain an expected waiting time of w, the service provider sets the service rate at

$$\mu = \lambda(p, \alpha; w) + 1/w.$$

This service rate is referred to as the system's *volume-based capacity*, a term coined by Allon and Federgruen (2007). It ensures a steady-state expected waiting time of $w = 1/(\mu - \lambda(p, \alpha; w))$ in equilibrium.

Figure 1   **Entrance of Pappas' Famous Crab Cake in Baltimore, Maryland, displaying its waiting-time standard.**

For simplicity of analysis, we assume w is exogenous, consistent with the commonly observed phenomenon of service providers announcing their waiting-time standards; Figure 1 shows an example of such an announcement. We relax this assumption in §7.5 by allowing such a waiting-time standard to be endogenous. The service provider's problem consists of choosing the service fee p and the level of entertainment options α to maximize its expected profit represented by

$$\Pi = p \cdot \lambda(p, \alpha; w) - C(\alpha) - \gamma\mu,$$

or, equivalently,

$$\Pi = (p - \gamma)\left(B - hwe^{-\delta\alpha} - \beta_0 p\right) - \frac{1}{2}c\alpha^2 - \frac{\gamma}{w}.$$

We assume $2\beta_0 c - \delta^2(B - \beta_0\gamma)^2 \geq 0$ to ensure the profit function is jointly concave in $p \geq \gamma$ and $\alpha \geq 0$; this assumption means the effectiveness of entertainment options (δ) cannot be overly large. We use the superscript M (short for "monopoly") to denote the decisions and performance under this setting. We present the service provider's optimal decision in the following proposition.

PROPOSITION 1. *Given the announced waiting-time standard w, the optimal entertainment level α^M uniquely satisfies*

$$\delta hw(B - hwe^{-\delta\alpha} - \beta_0\gamma) - 2\beta_0 c\alpha e^{\delta\alpha} = 0 \ \ at \ \alpha = \alpha^M, \tag{2}$$

and the optimal service fee is $p^M = (B - hwe^{-\delta\alpha^M} + \beta_0\gamma)/(2\beta_0)$.

Building on Proposition 1, we examine the impact of the effectiveness of entertainment options.

COROLLARY 1. *For a monopoly service provider, as the entertainment options become more effective (i.e., as δ increases), the optimal service fee p^M always increases, but the optimal entertainment level α^M first increases and then decreases.*

As δ increases, the entertainment options become more effective in reducing customers' disutility from waiting; the service provider in turn can market the service at a higher price. The impact of δ on α^M, however, is non-monotone. Specifically, if δ is small, offering a high entertainment level in the hope of attracting a high demand is not cost-effective for the service provider; thus, the service provider will choose a low entertainment level. As δ increases, the service provider will increase the entertainment level. Once δ becomes sufficiently large, the entertainment options are so effective that even a moderate entertainment level would lead to a boost in the arrival rate. The service provider will respond to an increasing δ by curbing its entertainment offerings. Figure 2 illustrates the impact of δ on α^M and p^M.

Corollary 2 below shows the impact of the unit capacity cost γ. To guarantee $p^M \geq \gamma$ for any $\alpha \geq 0$, we assume that the cost of capacity (γ) is not overly high; that is, $\gamma \leq (B - hw)/\beta_0$.

COROLLARY 2. (i) α^M *monotonically decreases in γ.*

(ii) *If $\delta^2 h^2 w^2 < \beta_0 c$, p^M monotonically increases in γ; otherwise, a unique γ^c exists such that p^M increases in γ when $\gamma < \gamma^c$, and decreases in γ if $\gamma \geq \gamma^c$.*

(iii) Π^M *monotonically decreases in γ.*

One may expect that entertainment options and service capacity are substitutes for each other such that as expanding capacity becomes more costly, the service provider would choose a higher entertainment level. On the contrary, Corollary 2(i) states the optimal level of entertainment options decreases in γ. To understand the basic intuition behind this result, note that as capacity-expanding becomes more costly, the service provider would respond by choosing a low capacity.
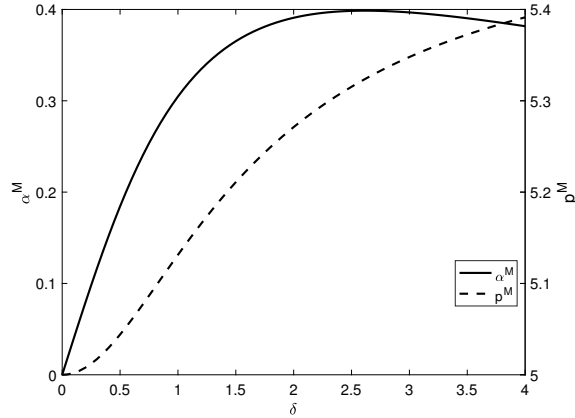
Figure 2   **The impact of δ on α^M. Parameters:** $B = 10$, $\gamma = 1$, $\beta_0 = 1$, $c = 10$, $h = 5$, $w = 0.2$.

To maintain its waiting-time standard, the service provider has to serve a lower demand rate at equilibrium. Thus, the service provider serves a lower demand rate, which sustains even under relatively limited entertainment options.

Another result one might expect is "cost externalizing;" that is, as expanding capacity becomes more costly, the service provider would pass the cost on to consumers by charging a higher service fee. Corollary 2(ii) suggests it is not necessarily the case. In response to a more costly capacity, the service provider may opt for a lower service fee. This counter-intuitive result comes from the interaction among the service provider's decisions on its entertainment level, service fee, and capacity. According to Proposition 1, the service provider's optimal service fee is $p^M = (B - hwe^{-\delta\alpha^M})/(2\beta_0) + \gamma/2$, which is driven by (1) the *margin-compensation effect*—a larger γ reduces the profit margin from offering the service and thus calls for a higher fee to compensate for it, and (2) the *demand-requirement effect*—as γ increases, a decreased demand due to a smaller α^M requires a lower price through the term $(B - hwe^{-\delta\alpha^M})/(2\beta_0)$. When γ is small such that $\gamma \leq \gamma^c$, the margin-compensation effect dominates, which leads to a higher price as γ increases. When γ increases beyond γ^c, the entertainment level α^M becomes sufficiently low such that the demand-requirement effect will dominate, leading to a decreased service charge as γ increases. Figure 3 illustrates the impact of γ on α^M and p^M.

Lastly, we characterize the effect of a waiting-time standard on the optimal entertainment level.

COROLLARY 3. *Under the monopoly setting, assuming $w \in [\underline{w}, \bar{w}]$ and $B - \beta_0\gamma \geq h\bar{w}$, as the waiting-time standard increases, the optimal entertainment level may either monotonically increase, or first increase and then decrease.*

Corollary 3 indicates the effect of the waiting-time standard on the optimal entertainment level is non-monotone. The non-monotonicity alludes to a low entertainment level when the waiting-time
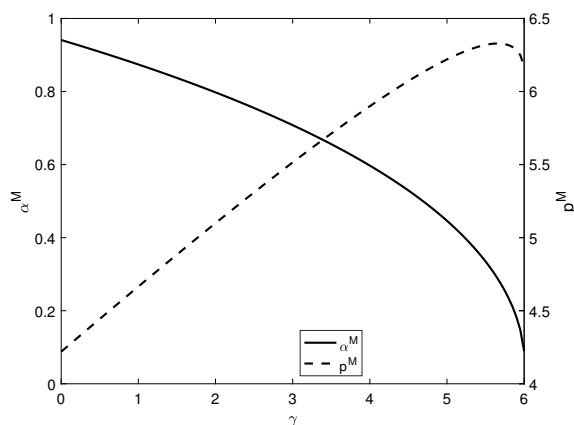
Figure 3 The impact of γ on p^M, where γ^c is calculated as 5.5691. **Parameters:** $B = 10$, $\beta_0 = 1$, $c = 7$, $h = 4$, $w = 1$, $\delta = 1$ and $\gamma \in [0, \bar{\gamma}]$, where $\bar{\gamma} = (B - hw)/\beta_0 = 6$.

standard is either very large or very small, and a high entertainment level when the waiting-time standard is intermediate. We provide some intuition for this result. The impact of entertainment on the arrival rate and the profit depends on the waiting-time standard, w, through the term $hwe^{-\delta\alpha}$ in the arrival-rate function. When the waiting-time standard is small, the marginal benefit of providing entertainment options is low, which has a limited demand-inducing effect. Thus, the service provider has little incentive to provide a high entertainment level. As w increases, the service provider will counteract long waiting times by providing richer entertainment options. When the waiting-time standard becomes sufficiently large, however, further enriching the entertainment offering is no longer cost effective. The service provider therefore chooses to compensate customers' disutility by reducing the service fee. Figure 4 illustrates the non-monotone case in Corollary 3.
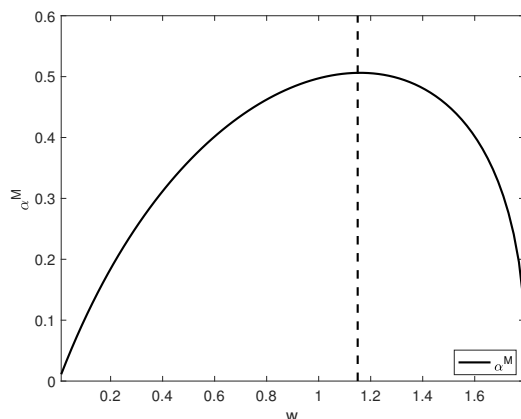
Figure 4 The impact of w on α^M. **Parameters used:** $B = 10$, $\gamma = 1$, $\beta_0 = 1$, $c = 10$, $h = 5$, $\delta = 0.5$, $\underline{w} = 0.1$, $\bar{w} = 1.8$.

The insights from Corollaries 1–3, albeit derived from the case of monopoly, qualitatively carry over after we have incorporated competition (in §3) and co-opetition (in §4). For conciseness of exposition, in the rest of the paper, we refrain from restating results similar to these corollaries, and instead focus on examining the strategic interactions (i.e., competition and cooperation) between firms in a service cluster.

3.   Service Clustering with Competition

In the previous section, we study a local monopolist's service-design problem. We now examine the case in which the firm joins a service cluster with competition only. Specifically, we model the duopoly competition between firms offering the same type of service. Each firm determines its own entertainment options dedicated solely to its own customers. Note that competition within a service cluster is not uncommon.

Without loss of generality, we assume the two service providers are symmetric: Each of the two service providers, indexed as $i = 1, 2$, has the same potential market size, denoted by B, and follows an industry-wide waiting-time standard w. The demand rate of service provider i, given the price charged by the other service provider, p_j, is given by

$$\lambda_i = B - hwe^{-\delta\alpha_i} - \beta_0 p_i + \theta(p_j - p_i), \tag{3}$$

where $\theta \geq 0$ captures the price-competition intensity such that a larger θ indicates more intense price competition. In §7.3, we generalize the above demand system by allowing the competition between service providers to depend on both their service fees and entertainment levels.

For ease of exposition, we write $P_i = p_i - \gamma$ and $A = B - \beta_0\gamma$. In addition, we write $D(\alpha) = A - hwe^{-\delta\alpha}$ and $\beta = \beta_0 + \theta$. Given the pre-announced waiting-time standard, w, we represent the demand and profit rates of service provider i, $i = 1, 2$ as

$$\lambda_i = D(\alpha_i) - \beta P_i + \theta P_j \text{ and } \Pi_i = P_i\lambda_i - C(\alpha_i) - \gamma/w.$$

We focus on the case in which each service provider determines its own service fee and entertainment level simultaneously. The key results (especially those vis-à-vis the co-opetition case in §4) hold qualitatively under alternative decision sequences. We assume $2\beta_0 c - \delta^2(B + \theta p^M - \beta_0\gamma)^2 \geq 0$ such that both service providers' objective functions are jointly concave in $\alpha \geq 0$ and $\gamma \leq p \leq p^M$ for any $\theta \geq 0$, where p^M is the optimal price in the monopoly case. The assumption resembles that for Proposition 1 and means the effectiveness of entertainment options is not overly large. (Note we can prove that under duopoly competition, a dominated strategy is for a service provider to set its service fee above p^M.) In Proposition 2, we use the superscript C (short for "competition") to denote the decisions and performance under this setting, and characterize the equilibrium of the duopoly-competition scenario. We define each service provider's utilization rate as the ratio of its arrival rate to its service rate.

PROPOSITION 2. *(i) In the case of duopoly competition, a unique equilibrium exists in which each service provider chooses an entertainment level α^C that uniquely satisfies*

$$\delta h w D(\alpha) - (2\beta_0 + \theta)c\alpha e^{\delta\alpha} = 0 \ at \ \alpha = \alpha^C,$$

and charges a service fee $p^C = D(\alpha^C)/(2\beta_0 + \theta) + \gamma$.

(ii) In equilibrium, each service provider's arrival rate, utilization level, and expected profit are $\lambda^C = (\beta_0 + \theta)D(\alpha^C)/(2\beta_0 + \theta)$, $\rho^C = 1 - 1/(w\lambda^C + 1)$, and $\Pi^C = P^C\lambda^C - C(\alpha^C) - \gamma/w$, respectively.

We now investigate the impact of price-competition intensity (θ) on the equilibrium entertainment and pricing decisions, and state the result in Corollary 4 below.

COROLLARY 4. *Under the duopoly-competition setting, as the intensity of price competition θ increases, each service provider chooses a lower entertainment level and a lower service fee, leading to a lower profit.*

As price competition becomes more intense, entertainment options become less effective in boosting demand. As one would expect, both service providers respond by charging higher service fees and curbing the entertainment offerings, leading to a lower profit. Later, in the case of co-opetition, we present a contrasting result (Proposition 5).

In the corollary below, we compare and contrast the cases of monopoly and duopoly competition.

COROLLARY 5. *Compared to the case of monopoly, under duopoly competition,*

(i) each service provider chooses a lower service fee and a lower level of entertainment options; that is, $p^C \leq p^M$ and $\alpha^C \leq \alpha^M$;

(ii) each service provider has a lower expected profit: $\Pi^C \leq \Pi^M$;

(iii) each service provider has a higher utilization if $(\alpha^C + (\beta_0 + \theta)(1 + \alpha^C\delta) \cdot \partial\alpha^C/\partial\theta) \geq 0$, and a lower utilization otherwise.

Parts (i) and (ii) of Corollary 5 follow from Corollary 4 and suggest that due to competition, the service providers must charge lower prices and provide a lower entertainment level, leading to lower profits. Furthermore, Corollary 5(iii) states that each service provider may experience a higher utilization level than in the monopoly case, which occurs when the optimal entertainment level is sufficiently high. From consumers' perspective, the net effect of the service providers' decisions is that they are now facing lower full prices. Thus, both the demand rate and the system utilization increase. Taken together, Corollary 5 shows that competition among service providers discourages the use of entertainment options, leading to lower profits and higher utilization levels.

4. Service Clustering with Co-opetition

In this section, we continue to examine a service-cluster setting, albeit in this case under co-opetition; that is, firms cooperate on providing entertainment options. We characterize the equilibrium and analyze the firm-level performance, which sheds light on the design of cost-allocation schemes for co-opetition.

Under co-opetition, firms jointly decide the entertainment level (α), with a total cost of $C(\alpha) = \frac{1}{2}c\alpha^2$ that is shared between the service providers. We focus on the case in which the cost of providing the same level of waiting-area entertainment options does not significantly increase when different service providers share the entertainment options. For example, the entertainment may be provided by a piano player, a live music group, or a line-dancing demonstration. In such cases, the cost is largely independent of audience size. Our key insights on service design still apply to the case in which the cost increases in the audience size. As in §3, we continue to focus on the symmetric scenario. Each service provider i's demand rate λ_i is $D(\alpha) - \beta P_i + \theta(P_j - P_i)$, for $i, j \in \{1, 2\}$ and $i \neq j$, where the first term captures the demand-inducing effect of the entertainment options, and the second and third terms capture the effect of price competition.

Because the entertainment options are shared among all the customers, one commonly used method to finance the entertainment options is to divide the cost between the service providers based on their respective market sizes. As such, we focus on a linear cost-sharing scheme, in which the total cost of providing entertainment options is split between the service providers according to their demand rates such that service provider i is responsible for a cost share of

$$\phi(\lambda_i, \lambda_j) = \frac{1}{2} + 2t\left(\lambda_i - \frac{\lambda_1 + \lambda_2}{2}\right)$$
$$= \frac{1}{2} + t \cdot \frac{\lambda_i - \lambda_j}{2},$$

where $i, j \in \{1, 2\}$, $i \neq j$, and $t \geq 0$. We refer to t as the *cost-sharing factor*, which measures the sensitivity of each service provider's share of the total cost to its realized demand. At $t = 0$, the cost for providing entertainment options is evenly split, and each service provider's share is independent of its actual market size. A larger t implies each service provider's share increasingly depends on its actual market size relative to the other service provider. This type of linear cost-allocation scheme, resembling the idea of "yardstick competition" (Savva et al. 2018) and satisfying the axioms of demand monotonicity (DM) and upper bound for homogeneous goods (UBH) proposed by Friedman and Moulin (1999), is quite simple to implement in practice. Later, in §7.4, we show our main findings qualitatively hold under an alternative volume-based cost-allocation scheme in which each service provider's cost share is the same as its customer share.

Because both service providers jointly determine the entertainment level, anti-trust considerations preclude each service provider from deciding its entertainment level and service fee simultaneously. We assume each service provider first sets its own service fee, and then both service providers jointly decide on entertainment level; under an alternative sequence of events in which the entertainment decisions are made before prices are set, we can numerically show our key results hold qualitatively. The profit function for service provider i can be written as follows:

$$\Pi_i = \pi(\alpha, P_i, P_j) = P_i \lambda_i - \phi(\lambda_i, \lambda_j) C(\alpha) - \gamma/w \text{ for } i = 1, 2.$$

In deriving the optimal solution, we first derive the entertainment level that maximizes the joint profit of the two service providers, $\Pi_1 + \Pi_2$, for a given pair (P_1, P_2). With the optimal solution $\alpha(P_1, P_2)$, we then identify the equilibrium service fees. Proposition 3 below provides the optimal service prices and entertainment levels for both service providers. We use the superscript O (indicating "co-opetition") to denote the decisions and performance under this setting.

PROPOSITION 3. *(i) In the case of co-opetition, the optimal entertainment level α^O satisfies*

$$2\left(D(\alpha) + t(\beta_0 + 2\theta)C(\alpha)\right)hw\delta - (2\beta_0 + \theta)c\alpha e^{\delta\alpha} = 0 \text{ at } \alpha = \alpha^O,$$

and the equilibrium price is $p^O = P^O + \gamma$, where $P^O = \frac{D(\alpha^O) + t(\beta_0 + 2\theta)C(\alpha^O)}{2\beta_0 + \theta}$.
(ii) The equilibrium arrival rate for each service provider is $\lambda^O = \frac{(\beta_0 + \theta)D(\alpha^O) - t\beta_0(\beta_0 + 2\theta)C(\alpha^O)}{2\beta_0 + \theta}$.
The profit and the utilization of each service provider are $\Pi^O = P^O\lambda^O - C(\alpha^O)/2 - \gamma/w$, and $\rho^O = 1 - 1/(w\lambda^O + 1)$, respectively. In equilibrium, each service provider's optimal cost share is $\frac{1}{2}$.

Proposition 3 states that each of the two symmetric service providers has a cost share of $\frac{1}{2}$, regardless of the value of t. Nevertheless, t influences each service provider's cost through influencing its service and pricing decision. We first examine the effect of the cost-sharing factor t on the optimal decisions, α^O and P^O. Corollary 6 illustrates our findings.

COROLLARY 6. *In the case of co-opetition, both the optimal entertainment level α^O and each firm's optimal service fee P^O increase in the cost-sharing factor t.*

Corollary 6 suggests that under co-opetition, a larger cost-sharing factor induces better waiting-area entertainment and more expensive service charges. Under the volume-based cost-allocation scheme, two effects drive the service providers' decisions about the entertainment level and service fees. On the one hand, the *price-competition effect* induces a service provider to charge a lower price. On the other hand, driven by the tension between value creation and value division often associated with a co-opetitive relationship (Brandenburger and Nalebuff 1997), a service provider is incentivized to charge a higher fee to lower its own demand rate and to increase its competitor's demand rate, in order to have a smaller market share and thus a smaller proportion of the

entertainment cost. We refer to this effect as a *cost-sharing effect*. When the cost-sharing factor t is large, the cost-sharing effect becomes more significant and the optimal price is higher, calling for richer entertainment options.

Proposition 4 below demonstrates the role of the cost-sharing parameter t in driving the equilibrium profit Π^O.

PROPOSITION 4. *In the case of co-opetition, the optimal cost-sharing factor t^* and the optimal entertainment level α^O satisfy*

$$\theta D(\alpha) = 2t\beta_0(\beta_0 + 2\theta)C(\alpha) \text{ and } 2\delta h w \left(D(\alpha) + \theta D(\alpha)/(2\beta_0) \right) = (2\beta_0 + \theta)c\alpha e^{\delta\alpha}$$

at $(t, \alpha) = (t^*, \alpha^O)$. Furthermore, Π^O increases in t if $0 \leq t \leq t^*$, and decreases in t if $t > t^*$.

Proposition 4 suggests a service provider's profit increases in the cost-sharing factor (t) when t is small, and decreases in it when t is sufficiently large. The underlying intuition is that under a large t (i.e., $t > t^*$), each service provider's incentive to charge a higher fee to avoid a large share of the cost of providing entertainment options can reduce demand to such an extent that it hurts its bottom line. This result alludes to a caveat to pursuing fairness in designing the cost-allocation scheme.

Recall from Corollary 4 that in the case of duopoly competition, both the equilibrium price and the entertainment level decrease in the price-competition intensity, θ. However, under co-opetition, this result no longer holds, as demonstrated in Proposition 5 below.

PROPOSITION 5. *Under co-opetition, a threshold $\hat{t} = e^{\delta\alpha^O}/(2hw\delta\alpha^O)$ exists such that when $t \geq \hat{t}$, both the equilibrium price p^O and entertainment level α^O increase in θ; the opposite holds when $t < \hat{t}$. Furthermore, $\hat{t} \geq t^*$.*

Proposition 5 suggests both the service fee and entertainment level may increase in θ if $t \geq \hat{t} \geq t^*$. This result is rather surprising, because one might expect the firms to charge a lower price as the price-competition level (θ) increases, as is the case under the competition setting. To understand the intuition behind this result, note that when t is sufficiently large, due to the cost-sharing effect, a firm responding to a higher θ by reducing the service fee will incur a larger share of the total cost for entertainment options. This increase in cost-sharing can more than offset the benefit of a higher demand induced by the lower service fee. As such, increasing the service fee becomes a more lucrative option for each service provider.

Proposition 5 also states that $\hat{t} \geq t^*$, meaning the service fee increases in θ only when t is larger than the optimal cost-sharing factor t^*. This result suggests a high cost-sharing factor may induce a type of nonintuitive competitive behavior.

5. Comparison across Scenarios

In this section, we compare across the three scenarios—monopoly (§2), competition (§3), and co-opetition (§4). We begin by comparing the service providers' optimal decisions and profits under the competition and co-opetition settings, and present the results in Proposition 6.

PROPOSITION 6. *All else being equal,*

(i) the equilibrium price and entertainment level under co-opetition are always higher than under competition; that is, $p^O \geq p^C$ and $\alpha^O \geq \alpha^C$;

(ii) a threshold $t^c > 0$ exists such that when $0 \leq t < t^c$, the profit under co-opetition is greater than under competition; that is, $\Pi^O > \Pi^C$; otherwise, the opposite holds; that is, $\Pi^O \leq \Pi^C$.

By sharing the cost of providing waiting-area entertainment, the two service providers can co-invest in a higher entertainment level, which allows them to charge a higher price without compromising on the demand rate. Cost sharing, however, is a double-edged sword. On the one hand, it helps the service providers offer a high entertainment level that they would otherwise not be able to afford individually. On the other hand, the cost-sharing scheme can induce the service providers to charge a high service fee when t is large, in order to avoid a significantly higher entertainment cost. This strategic response can in turn hurt their profitability, and makes co-opetition even less desirable than under competition. Specifically, a threshold cost-sharing factor exists above which the benefit from co-opetition disappears.

The implication drawn from our above analysis is that in designing a cost-allocation scheme for co-opetition, the service providers need to carefully weigh a fairness-efficiency tradeoff. The pursuit of fairness, through increasing the cost-sharing factor t, may backfire and completely eliminate the benefit from resource sharing such that neither service provider benefits from co-opetition. This perhaps explains why shared entertainment options are not as prevalent as one would expect.

Note that Proposition 6 is based on a setting in which both service providers share the same waiting-time standards. If the service providers possess different waiting-time standards, we can show the proposition holds. However, the asymmetric setting yields several different results. For example, we can show that in the case of service clustering with competition, the profit of the firm with a higher waiting-time standard may increase in the price-competition intensity when the price-competition intensity is low, and then decrease if price competition becomes more intense. The intuition is that the firm with a lower waiting-time standard can charge a higher price than that of the competitor with a higher wait-time standard. When the price-competition intensity is low, as it increases, the firm with a higher waiting-time standard, due to its low service fee, can benefit from an increased demand despite its lower service fee. As the price-competition intensity

becomes sufficiently large, however, both firms will suffer from a decreased margin due to reduced service fees.

Proposition 7 below compares the entertainment levels and profits under the co-opetition and monopoly settings when the entertainment cost is split evenly.

PROPOSITION 7. *Suppose $t = 0$ (i.e., when the entertainment cost is split evenly between the two service providers).*

(i) Both α^O and p^O as well as Π^O decrease in θ.

(ii) A threshold θ^c exists such that if $\theta \leq \theta^c$, then $\Pi^O \geq \Pi^M$; that is, the profit under co-opetition is higher than that under monopoly, and the opposite holds if $\theta > \theta^c$.

When $t = 0$, the price-competition effect dominates the cost-sharing effect, so we expect the equilibrium service fees to decrease in the intensity of competition θ. This decrease alleviates the need to offer a high entertainment level. Furthermore, when θ is low, the benefit of a high entertainment level can more than offset the profit loss from competition, leading to a performance superior to the case of monopoly.

PROPOSITION 8. *Given $t \geq 0$, if $\theta \in [0, 2\beta_0]$, the equilibrium entertainment level under co-opetition is larger than the monopoly decision; that is, $\alpha^O \geq \alpha^M$. The opposite holds when $\theta \in (2\beta_0, \infty)$. Furthermore, given $\theta \geq 0$, a threshold $t^m(\theta)$ may exist such that if $t \in [0, t^m(\theta)]$, then $\Pi^O \geq \Pi^M$, whereas if $t > t^m(\theta)$, $\Pi^O < \Pi^M$.*

When the intensity of price competition, θ, is low, high equilibrium service fees call for a high entertainment level to reduce customers' disutility from waiting, and cost sharing is likely to drive the optimal entertainment level higher than under monopoly. Proposition 8 also shows that if the cost-sharing factor is properly specified, co-opetition can benefit the competing service providers and help restore their monopoly profit levels. This result, together with Propositions 6 and 7, formally establish the benefit of co-opetition.

Numerical Study. To gain an overall sense of the improvement achieved through co-opetition, we conduct a numerical study with the following combinations of parameters: $w \in \{0.3, 0.5, 0.7\}$, $B \in \{8, 10, 12\}$, $\gamma \in \{0.5, 1, 1.5\}$, $\beta_0 \in \{0.8, 1, 1.2\}$, $\delta \in \{0.3, 0.5, 0.7\}$, $c \in \{8, 10, 12\}$, and $\theta \in \{0.3, 0.5, 0.7\}$. For each of these parameters, we refer to the three possible values as "low value," "medium value," and "high value," respectively. In this study, $h = 5$ and $u = 0.5$. As such, this setup provides $3^7 = 2,187$ instances. After dropping 180 instances with parameters violating our assumptions, we have 2,007 instances.

For each instance, we identify the optimal cost-sharing factor t^* and calculate the service providers' profits. Table 1 summarizes the statistics on the profit gain of co-opetition over monopoly.

We observe that compared to the monopoly case, a service provider under co-opetition can gain a profit that is 7.65% higher on average, with a maximum of 77.40%. Furthermore, in 92.92% of the instances, co-opetition outperforms monopoly, leading to a service provider gaining a profit that is 8.23% higher on average. Although not presented in the table, we also find that each service provider's profit under co-opetition is always higher than under duopoly competition, with an average profit gain of 14.95% and a maximum of 98.14%.

Table 1   **Summary of the instances.**

Sample	Number	mean, %	median, %	std. dev, %	min, %	max, %
All instances	2007	7.65	5.42	9.75	-19.59	77.40
Instances with positive gains	1865	8.45	5.86	9.62	0.02	77.40
Instances with negative gains	142	-2.86	-2.05	3.01	-19.59	-0.01

As Figure 5 illustrates, our numerical study further shows co-opetition is the most lucrative when (i) the market size is small, (ii) the cost of building capacity is high, (iii) consumers' price-sensitivity is medium, (iv) the cost of providing entertainment options is low, (v) entertainment options are highly effective in alleviating consumers' pain from waiting, (vi) price competition is low, and (vii) consumers are highly sensitive to waiting.
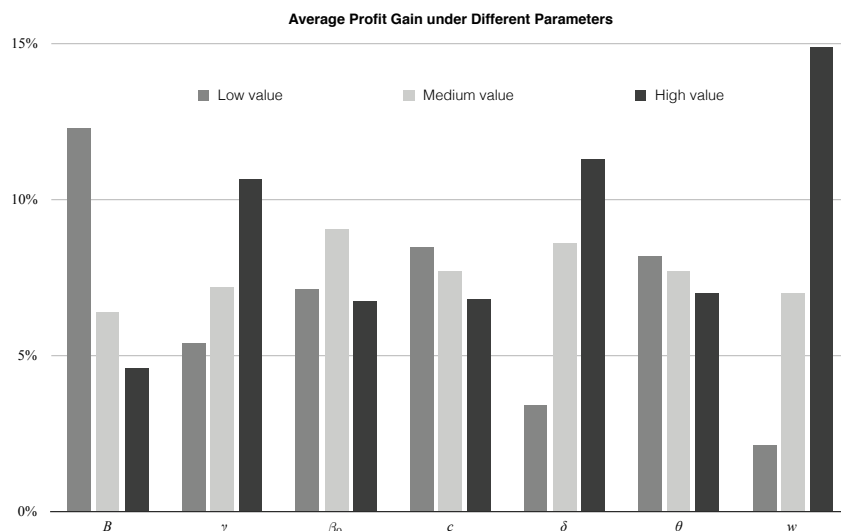
Figure 5   **Profit gain of co-opetition relative to monopoly with respect to parameters.**

6.   Effect of Queueing Considerations

So far, in this paper, we have focused on a service setting in which queueing considerations are instrumental. In this section, we develop a benchmark without queueing considerations. For ease of comparison, we choose a setup and notation system in a way that is as close to our main model

as possible. In this benchmark, entertainment options help directly boost demand *without* through reducing customer disutility from waiting. Then, comparing the results from such a benchmark with those from our main model helps us understand the effect of queueing considerations. Corresponding to our main model, we analyze three scenarios of this benchmark, concerning (1) a local monopolist, (2) service clustering with competition, and (3) service clustering with co-opetition, in §§6.1–6.3, respectively.

6.1. Local Monopolist: Service Design with Waiting-Area Entertainment

We first consider a local monopolist who faces a demand system given by

$$\lambda = B + \delta\alpha - \beta_0 p,$$

where α denotes the entertainment level provided by the firm, and $\delta \geq 0$ captures the effectiveness of entertainment options in boosting demand. As in our main model, the cost of providing entertainment is $C(\alpha) = \frac{1}{2}c\alpha^2$. In this setup, the service provider's expected profit is $\pi = p(B + \delta\alpha - \beta_0 p) - \frac{1}{2}c\alpha^2$. We assume $\beta_0 c \geq \delta^2$ so that the profit function is jointly concave in p and α. The following proposition provides the service provider's optimal service fee and entertainment level.

PROPOSITION 9. *Without queueing considerations, in the case of a local monopolist, the optimal price, denoted as p^M, and entertainment level, denoted as α^M, are given by*

$$\alpha^M = \frac{\delta B}{2\beta_0 c - \delta^2} \ \ and \ \ p^M = \frac{cB}{2\beta_0 c - \delta^2}.$$

Proposition 9 implies, among other findings, that as entertainment options become more effective (i.e., as δ increases), both the optimal service fee and entertainment level increase. Comparing this result with that in our main model, particularly Corollary 1, gives the following observation: With the queueing effect, p^M always increases in δ, but α^M first increases and then decreases in δ; without the queueing effect, however, both α^M and p^M monotonically increase in δ. Thus, incorporating queueing considerations leads to a non-monotone impact of the effectiveness of entertainment options on the optimal entertainment level.

6.2. Service Clustering with Competition

Next, mirroring §3, we consider two competing service providers in the same service cluster. For each firm i, given its own price p_i and entertainment level α_i, as well as its competitor's price p_j, the demand function is given by $\lambda_i = B + \delta\alpha_i - \beta_0 p_i + \theta(p_j - p_i)$. Each service provider's expected profit function can be expressed as $\pi_i = p_i(B + \delta\alpha_i - \beta_0 p_i + \theta(p_j - p_i)) - \frac{1}{2}c\alpha_i^2$. The following proposition gives each service provider's service fee and entertainment level in equilibrium.

PROPOSITION 10. *Without queueing considerations, in the case of service clustering with competition, each service provider chooses the following equilibrium price (p^C) and an entertainment level (α^C):*

$$\alpha^C = \frac{\delta B[2(\beta_0 + \theta)c - \delta^2 + \theta]}{[2(\beta_0 + \theta)c - \delta^2]^2 - c\theta^2} \quad and \quad p^C = \frac{cB[2(\beta_0 + \theta)c - \delta^2 + \theta]}{[2(\beta_0 + \theta)c - \delta^2]^2 - c\theta^2}.$$

We arrange the expression of p^C in Proposition 10 as

$$p^C = \frac{cB}{[2\beta_0 c - \delta^2 + c\theta] + \frac{(c-1)}{\frac{1}{\theta} + \frac{1}{(2(\beta_0 + \theta)c - \delta^2)}}}.$$

We observe from the above expression that both the optimal service fee p^C and the optimal entertainment level α^C decrease in θ. In addition, similar to the case of a local monopolist in §6.1, both p^C and α^C decrease in δ.

Compared to the result from our main model (especially Corollary 4), we see that with and without the queueing effect, both p^C and α^C always decrease in θ. A key difference is that without queueing considerations, both p^C and α^C decrease in δ. By contrast, with queueing considerations, p^C decreases in δ, but α^C first increases and then decreases in δ (a result not reported in §3 for conciseness). Thus, incorporating queueing considerations leads to non-monotonicity of the optimal entertainment level in terms of the effectiveness of entertainment options.

6.3.   Service Clustering with Co-opetition

We now consider the case of service clustering with co-opetition. For each firm i, given its own price p_i, the jointly determined entertainment level α, and the competitor's price p_j, each service provider's demand is $\lambda_i = B + \delta\alpha - \beta_0 p_i + \theta(p_j - p_i)$. We use the same piecewise linear cost-sharing function as in §4. The cost-sharing function can be reformulated in terms of the prices, denoted as $\phi(p_i, p_j)$ (see the proof of Proposition 3). The profit function of each service provider i is given by $\pi_i = p_i(B + \delta\alpha - \beta_0 p_i + \theta(p_j - p_i)) - \phi(p_i, p_j)C(\alpha)$.

Given p_i and p_j, the two firms jointly determine the entertainment level, denoted as $\alpha(p_i, p_j)$, to maximize the joint profit. By the first-order condition, given p_i and p_j, the optimal entertainment level satisfies $(p_i + p_j)\delta - c\alpha = 0$, which gives $\alpha = \frac{(p_i + p_j)\delta}{c}$. Thus, we have

$$\alpha_i' = \frac{\partial \alpha}{\partial p_i} = \frac{\delta}{c} = \frac{\partial \alpha}{\partial p_j} = \alpha_j'.$$

We assume $\beta_0 c \geq \delta^2$ to guarantee the concavity of the profit function π_i in p_j: On the one hand, if $\phi(p_i, p_j) = 1$, it is straightforward to show $\beta_0 c \geq \delta^2$ guarantees the concavity of the profit function. On the other hand, if $\phi(p_i, p_j) = 0$, again, we can show $\beta_0 c \geq \delta^2$ guarantees the concavity of the profit function. Because $0 \leq \phi(p_i, p_j) \leq 1$, the condition $\beta_0 c \geq \delta^2$ suffices to guarantee the concavity of the profit function.

PROPOSITION 11. *Without queueing considerations, in the case of service clustering with coope-tition, in equilibrium, each service provider chooses an entertainment level (α^O) that satisfies*

$$B + \left(\delta - \frac{(2\beta_0 + \theta)c}{2\delta}\right)\alpha + \frac{1}{2}c\alpha^2 t(\beta_0 + 2\theta) = 0 \ at \ \alpha = \alpha^O,$$

and a service fee of $p^O = \frac{c\alpha^O}{2\delta}$.

The optimal entertainment level α^O can be solved as below:

$$\alpha^O = \frac{\left(\frac{(2\beta_0 + \theta)c}{2\delta} - \delta\right) - \sqrt{\left(\delta - \frac{(2\beta_0 + \theta)c}{2\delta}\right)^2 - 2Bct(\beta_0 + 2\theta)}}{2ct(\beta_0 + 2\theta)}, \tag{4}$$

where $(2\beta_0 + \theta)c > 2\delta^2$ holds because $\beta_0 c \geq \delta^2$. In addition, the upper bound of the cost-sharing factor t, denoted as $\bar{t}$, is calculated as

$$\bar{t} = \frac{\left(\delta - \frac{(2\beta_0 + \theta)c}{2\delta}\right)^2}{2Bc(\beta_0 + 2\theta)}. \tag{5}$$

Denote the condition in Proposition 11 as

$$F(\alpha, t, \theta) = B + \left(\delta - \frac{(2\beta_0 + \theta)c}{2\delta}\right)\alpha + \frac{1}{2}c\alpha^2 t(\beta_0 + 2\theta),$$

with $F(\alpha^O, t, \theta) = 0$, and by the concavity of the profit function

$$F_\alpha = \left.\frac{\partial F(\alpha, t, \theta)}{\partial \alpha}\right|_{\alpha = \alpha^O} < 0.$$

Thus, α^O increases in t followed by

$$F_t = \frac{\partial F(\alpha, t, \theta)}{\partial t} > 0, \ \frac{\partial \alpha^O}{\partial t} = -\left.\frac{F_t}{F_\alpha}\right|_{\alpha = \alpha^O} > 0.$$

The equilibrium price p^O also increases in t.

COROLLARY 7. *In the case of co-opetition, both the optimal entertainment level α^O and the optimal price p^O in equilibrium increase with the cost-sharing factor t.*

The optimal cost-sharing factor t is given by the following result:

COROLLARY 8. *In the case of co-opetition, the optimal cost-sharing factor t^* is*

$$t^* = \frac{(2 + 2\delta^2 - (2\beta_0 + \theta)c)(\beta_0 c - \delta^2)}{c\delta^2 B(\beta_0 + 2\theta)}.$$

We next consider the impact of θ on α^O and p^O. If $t = 0$, then $\alpha^O = \frac{2\delta B}{(2\beta_0 + \theta)c - 2\delta^2}$, which serves as the lower bound, and we can see α^O decreases in θ. For $t > 0$, we have

$$F_\theta = \frac{\partial F(\alpha, t, \theta)}{\partial \theta} = \frac{c\alpha}{2\delta}(2\delta\alpha t - 1).$$

Thus, if $2\delta\alpha^O t \leq 1$, then α^O decreases in θ as

$$\frac{\partial \alpha^O}{\partial \theta} = -\frac{F_\theta}{F_\alpha}\bigg|_{\alpha=\alpha^O} \leq 0,$$

and vice versa. By Corollary 7, α^O increases in t, where we conclude $\hat{t}$ exists such that if $t \in [0, \hat{t}]$, then $2\delta\alpha^O t \leq 1$, implying both α^O and p^O decrease in θ. On the other hand, if $t > \hat{t}$, both α^O and p^O increase in θ.

We next show the condition $2\delta\alpha^O t > 1$ can never hold: By (4), we can calculate

$$
\begin{aligned}
2\delta\alpha^O t &= \delta\frac{\left(\frac{(2\beta_0+\theta)c}{2\delta} - \delta\right) - \sqrt{\left(\delta - \frac{(2\beta_0+\theta)c}{2\delta}\right)^2 - 2Bct(\beta_0 + 2\theta)}}{c(\beta_0 + 2\theta)} \\
&\leq \delta\frac{\left(\frac{(2\beta_0+\theta)c}{2\delta} - \delta\right)}{c(\beta_0 + 2\theta)} = \frac{(2\beta_0 + \theta)c - 2\delta^2}{2c(\beta_0 + 2\theta)} < 1.
\end{aligned}
\tag{6}
$$

Based on the assumption $\beta_0 c \geq \delta^2$, the following result summarizes the above discussion,

COROLLARY 9. *Without queueing considerations, in the case of service clustering with co-opetition, in equilibrium, both the service fee and the entertainment level decrease in t.*

Compared to the result from our main model, one key differentiating result is that without queueing considerations, both the entertainment level and the service fee (weakly) decrease in the intensity of price competition θ; with queueing considerations, both the entertainment level and the service fee can increase in the intensity of price competition θ. This comparison demonstrates that our finding from Proposition 5 that the service fee may increase in the intensity of price comparison is a differentiating result due to queueing considerations. In other words, incorporating queueing considerations leads to the counterintuitive finding that the service fee may increase in the intensity of price comparison.

7. Extensions

In this section, we discuss several extensions to our main model to explore its boundary. In §7.1, we analyze the case with a general demand function. In §7.2, we consider an alternative formulation reflecting the effect of customer waiting on the firm's objective. In §7.3, we generalize our model of service clustering with competition by allowing customers to use both the service fee and entertainment options in choosing a service provider. In §7.4, we consider an alternative cost-sharing scheme in which each service provider's cost share is identical to its market share. In §7.5, we numerically examine the case in which the waiting-time standard is endogenous.

7.1.   General Demand Function

In the case of a monopolist service provider, for simplicity of analysis, we assume a specific form of demand function, that is, $\lambda(p, \alpha; w) = B - hwe^{-\delta\alpha} - \beta_0 p$. We now consider a more general setting in which

$$\lambda = B - E(w, \delta, \alpha) - \beta_0 p,$$

where $E(w, \delta, \alpha)$ captures the impact of the announced waiting-time standard w, the entertainment level α, and the entertainment discount factor δ on the arrival rate. We assume $E(w, \delta, \alpha)$ increases in w, whereas it decreases and is convex in δ and α; that is, $E_w > 0$, $E_\delta < 0$, $E_\alpha < 0$, $E_{\delta\delta} > 0$, and $E_{\alpha\alpha} > 0$. In other words, waiting-area entertainment helps reduce the disutility from waiting but has a declining marginal effect. In addition, for a given w, we assume $E_{\alpha\delta}(w, \delta, \alpha) > 0$; that is, the effectiveness of entertainment options (δ) and the entertainment level (α) are substitutes.

Under the general demand function, we can characterize the optimal service fee and optimal entertainment level in the proposition that follows. We assume $B - E(w, \delta, 0) - \beta_0\gamma > 0$ to maintain a positive demand in the case in which no entertainment option is offered (i.e., $\alpha = 0$) and the price is set as low as the marginal capacity cost (i.e., $p = \gamma$). Given the waiting-time standard w, the optimal entertainment level uniquely satisfies

$$(B - \beta_0\gamma - E(w, \delta, \alpha))E_\alpha(w, \delta, \alpha) + 2\beta_0 c\alpha = 0, \text{at } \alpha = \alpha^M.$$

The optimal price is given by $p^M = (B - E(w, \delta, \alpha^M) + \beta_0\gamma)/(2\beta_0)$.

We can verify that all our main findings from §2 (i.e., Corollaries 1–3) continue to hold. Additionally, our key findings from the settings in §§3–5 hold qualitatively.

7.2.   An Alternative Approach to Modeling the Effect of Waiting

In our main model, we consider the case in which the waiting time is reflected in the customer arrival rate. An alternative approach to modeling the effect of the waiting is to incorporate waiting time as a cost term in the service provider's objective function. In the case of the service provider being a local monopolist, it solves the following problem:

$$\max_{0 \leq \lambda < \mu, \alpha \geq 0} \quad \Pi = p\lambda - C(\alpha) - \gamma\mu - hwe^{-\delta\alpha}\lambda$$
$$s.t. \quad \lambda = B - \beta_0 p, \tag{7}$$
$$w = \frac{1}{\mu - \lambda}.$$

In the above formulation, the firm charges a net price of $\hat{p} \triangleq p - hwe^{-\delta\alpha}$. After substituting its two constraints into the objective function, we can rewrite (7) as

$$\max_{p \geq 0, \alpha \geq 0} \Pi = (\hat{p} - \gamma)[B - \beta_0(\hat{p} + hwe^{-\delta\alpha})] - C(\alpha) - \frac{\gamma}{w}. \tag{8}$$

Assuming $2c - \beta_0 h^2 \delta^2 w^2 \geq 0$, the profit function is jointly concave in p and α. By the first-order condition, the optimal net price $\hat{p}^M$ is

$$\hat{p}^M = (B - \beta_0 hwe^{-\delta\alpha^M} + \beta_0\gamma)/(2\beta_0),$$

and the optimal entertainment level α^M satisfies

$$hw\delta \left(B - \beta_0 hwe^{-\delta\alpha} - \beta_0\gamma\right) - 2c\alpha e^{\delta\alpha} = 0, \text{ at } \alpha = \alpha^M. \tag{9}$$

Note the above two equations resemble those in Proposition 1. Indeed, we can proceed to show the above modeling approach and our approach in the main model are roughly equivalent in that they lead to qualitatively equivalent results.

7.3. Competition Based on Both Price and Entertainment Level

In our model of the scenario of service clustering with competition in §3, we consider a demand system, represented by (3), that only reflects price competition. As an additional practical consideration, the relative magnitude of entertainment levels across service providers may also play a role in influencing consumer demand. Accordingly, we now extend our main model by considering a demand system reflecting the competition based on both the service fees and entertainment levels chosen by the two service providers.

Specifically, we consider the following demand system:

$$D(\alpha_i, \alpha_j) = A - hwe^{-\delta\alpha_i} + d\left(e^{\kappa(\alpha_i - \alpha_j)} - 1\right), \tag{10}$$

where $\kappa > 0$ captures the effect of the entertainment competition, which functions similarly as the price-competition intensity θ. The parameter $d \geq 0$ captures how the cross-provider difference in waiting-area entertainment drives the competition. For example, if $\alpha_i > \alpha_j$, firm i will attract more demand while firm j will lose demand, compared to the case without the entertainment-competition effect (i.e., $\kappa = 0$ or $d = 0$).

Given α_j, we assume $D(\alpha_i, \alpha_j)$ is concave increasing in $\alpha_i \geq 0$. The concavity assumption in $\alpha_i \geq 0$ ensures a unique pair of optimal entertainment level and service fee exists given the other firm's entertainment level and price decisions. Under this duopoly-competition setting, we can show a unique equilibrium entertainment level α^C exists that satisfies

$$\left(\delta hwe^{-\delta\alpha} + d\kappa\right) D(\alpha, \alpha) - (2\beta_0 + \theta)c\alpha = 0, \text{ at } \alpha = \alpha^C,$$

and the unique price in equilibrium is $p^C = D(\alpha^C, \alpha^C)/(2\beta_0 + \theta) + \gamma$, where $D(\alpha, \alpha) = A - hwe^{-\delta\alpha}$.

The above result gives the following corollary:

COROLLARY 10. *Under the duopoly competition, the impacts of the price and entertainment competition on the equilibrium entertainment level and the price as well as the profit are given as follows:*

(i) Both the entertainment level and the price in equilibrium increase in $\kappa \geq 0$; the equilibrium profit also increases in $\kappa \geq 0$.

(ii) Both the entertainment level and the price in equilibrium decrease in $\theta \geq 0$; the equilibrium profit also decreases in $\theta \geq 0$.

The results in Corollary 10 are consistent with those in §3. Note that because the new demand system (10) is irrelevant to the case of a local monopolist (§2) or service clustering with co-opetition (§4), it suffices to check that our key insights hold in the case of duopoly competition.

7.4. An Alternative Volume-Based Cost-Sharing Scheme

In our main analysis of the case of co-opetition, we examine a case in which the two service providers use a linear transfer payment scheme to determine their shares of the cost of providing waiting-area entertainment. We now extend the model to an alternative volume-based cost-sharing scheme, under which service provider i's share of the cost is given by

$$\phi(\lambda_i, \lambda_j) = \frac{\lambda_i}{\lambda_i + \lambda_j}$$

for $i, j \in \{1, 2\}, i \neq j$.

We assume the service providers first set the service fees individually, and then jointly determine the entertainment level. Given P_i and P_j, the entertainment is determined at the level that maximizes the joint profit of the two service providers. Specifically, the entertainment level is solved by the following:

$$\max_{\alpha \geq 0} \pi(\alpha, P_i, P_j) = P_i(D(\alpha) - \beta P_i + \theta P_j) + P_j(D(\alpha) - \beta P_j + \theta P_i) - C(\alpha) - \frac{2\gamma}{w},$$

where the optimal entertainment level, denoted as $\alpha^O = \alpha(P_i, P_j)$, satisfies the first-order condition

$$(P_i + P_j)hw\delta e^{-\delta \alpha^O} - c\alpha^O = 0. \tag{11}$$

We observe from (11) that α^O increases in both P_i and P_j.

Next, each service provider determines the optimal service fee. Given P_j, the optimal price P_i can be solved by the following pogram:

$$\max_{P_i} \pi(P_i, P_j) = P_i(D(\alpha^O) - \beta P_i + \theta P_j) - \frac{\lambda_i}{\lambda_i + \lambda_j}C(\alpha^O) - \frac{\gamma}{w} \tag{12}$$

$$s.t. \quad (P_i + P_j)hw\delta e^{-\delta \alpha^O} - c\alpha^O = 0$$

$$\lambda_i = D(\alpha^O) - \beta P_i + \theta P_j$$

$$\lambda_j = D(\alpha^O) - \beta P_j + \theta P_i.$$

In the case of co-opetition, we show the optimal entertainment level α^O satisfies

$$2hw\delta D(\alpha^O) - (2\beta_0 + \theta)c\alpha^O e^{\delta\alpha^O} + (2hw\delta)^2 \frac{(\beta_0 + 2\theta)C(\alpha^O)}{4(2hw\delta D(\alpha^O) - \beta_0 c\alpha^O e^{\delta\alpha^O})} = 0, \text{ at } \alpha = \alpha^O,$$

and the equilibrium price is $p^O = P^O + \gamma$, where P^O satisfies

$$D(\alpha^O) - (2\beta_0 + \theta)P^O + \frac{(\beta_0 + 2\theta)}{4(D(\alpha^O) - \beta_0 P^O)}C(\alpha^O) = 0.$$

We can proceed to show our main findings in the case of service clustering with co-opetition, as presented in §§4–5, hold. For example, we can show scenarios exist in which the equilibrium entertainment level and the price in the co-opetition case may increase in the price-competition intensity. In addition, we can show our main findings carry over to an alternative decision sequence whereby the two service providers first decide the entertainment level α, and then each service provider individually makes its pricing decision.

7.5. Endogenous Waiting-Time Standard

So far, we have focused on the setting with exogenous waiting-time standards, which is a realistic assumption in many scenarios where service providers often share the same industry-wide standard for customer waiting times. As a robustness check, we relax this assumption, considering the case in which the waiting-time standard is endogenous, for all three cases (monopoly, duopoly competition, and co-opetition). We can numerically show all our findings extend to the case with an endogenous waiting-time standard.

8. Concluding Remarks

In the service industry, firms commonly use entertainment to reduce customers' disutility from waiting. In a service cluster with a common space, an opportunity exists for service providers to cooperate with each other in providing waiting-area entertainment. Whereas the service operations literature has extensively examined service decisions under competition, the scenario in which service providers cooperate in providing entertainment options while competing with each other has not been previously studied. Likewise, co-opetition has been studied in the manufacturing and supply-chain settings, but not in a service setting. Several research questions naturally arise, with no immediately clear answers at hand: (1) Can service providers benefit from co-opetition? (2) How should service providers share the cost of providing entertainment options? (3) How does the intensity of price competition affect service providers' pricing behavior in equilibrium?

To answer these questions, we first analyze a benchmark with a local monopolist deciding on its entertainment level, service fee, and capacity. This benchmark helps us understand how various factors drive a service provider's entertainment-level decision. In another benchmark, we consider

two firms in the same service cluster competing for customers and independently making price, capacity, and entertainment-level decisions. Jointly, these two benchmarks show that intense competition among service providers necessitates heavy investment in entertainment options and erodes firm profits. We then build a full model in which two service providers compete for customers but cooperate in providing entertainment options. We show co-opetition may help service providers obtain higher profits than under a monopoly.

In investigating the co-opetition case, we analyze a linear-type cost-allocation scheme, and demonstrate the crucial role of a parameter, namely, the cost-sharing factor. The cost-sharing factor indicates how sensitive each service provider's proportion of cost sharing is to its ex-post market size. A larger cost-sharing factor would seem to be fairer, because it requires service providers to contribute to the total cost based on the benefit they receive from the entertainment options. As a result, one may expect that a higher cost-sharing factor is beneficial to the co-opetiting service providers. Our analysis reveals, surprisingly, the opposite may be true. Specifically, a threshold cost-sharing factor exists above which both service providers may suffer from co-opetition. One key insight from our analysis is that in designing cost-sharing contracts for co-opetition, a fairness-efficiency tradeoff occurs that must be carefully incorporated—the pursuit of fairness may backfire and completely eliminate the benefit from resource sharing.

Due to the fairness-efficiency tradeoff characterized in our paper, we also find that as price competition becomes more intense, under a high cost-sharing factor, service providers may choose higher—not lower, as one would expect—service fees. This result is counterintuitive and does not arise in the absence of co-opetition.

Our paper is the first to study co-opetition in a service setting with entertainment options that help relieve customers of their pain from unoccupied waiting. Our paper highlights the strategic interactions among service providers engaged in a simultaneously competitive and cooperative relationship, leading to a novel characterization of the fairness-efficiency tradeoff that is essential in guiding the design of a cost-allocation scheme for co-opetition.

Rather than providing an exhaustive analysis of co-opetition in any service setting, this work is the first step toward understanding co-opetition in a uniquely compelling scenario—service clustering with waiting-area entertainment. Our research can be extended in a number of directions. For instance, in practice, co-opetition may occur among more than two adjacent service providers. An examination of whether the results continue to hold as we move to oligopolies could help shed light on whether co-opetition works better as the number of competitors increases. Another direction for future research would be to examine the case with asymmetric service providers. We expect our key insights to hold directionally, but the asymmetry in service parameters itself may lead to

interesting implications. Lastly, our key findings from the paper, especially those relevant to the co-opetition setting, may be tested in a laboratory or in the field.

Acknowledgments

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Online Supplement for "Co-opetition in Service Clusters with Waiting-Area Entertainment"

Proof of Proposition 1. For ease of presentation, we write $F(\alpha) = \delta hw(B - hwe^{-\delta\alpha} - \beta_0\gamma) - 2\beta_0 c\alpha e^{\delta\alpha}$. Given w, the first-order conditions with respect to p and α are $B - hwe^{-\delta\alpha} - 2\beta_0 p + \beta_0\gamma = 0$ and $\delta hw(p - \gamma)e^{-\delta\alpha} - c\alpha = 0$, respectively, which may be rewritten as $F(\alpha) = \delta hw(B - hwe^{-\delta\alpha} - \beta_0\gamma) - 2\beta_0 c\alpha e^{\delta\alpha} = 0$. Denote the optimal decisions as p^M and α^M. If the profit function $\pi(p, \alpha; w)$ is jointly concave at (p^M, α^M), the first-order conditions are sufficient and necessary for the optimality of p and α. The profit function is concave in p and α, respectively, with the second-order derivatives in terms of p and α as

$$\pi_{pp} = -2\beta_0, \ \pi_{\alpha\alpha} = -\delta^2 hw(p - \gamma)e^{-\delta\alpha} - c,$$

and the cross derivative, $\pi_{p\alpha} = \delta hwe^{-\delta\alpha} > 0$. We have $\phi(\alpha) = \pi_{pp}\pi_{\alpha\alpha} - \pi_{p\alpha}^2 = 2\beta_0(\delta^2 hw(p - \gamma)e^{-\delta\alpha} + c) - \delta^2 h^2 w^2 e^{-2\delta\alpha}$, which is reduced as $\phi(\alpha^M) = 2\beta_0(\delta c\alpha^M + c) - \delta^2 h^2 w^2 e^{-2\delta\alpha^M}$ at (p^M, α^M). Clearly, $\phi(\alpha^M)$ is increasing in α^M. Therefore, if $\alpha^M \geq \alpha^c$, such that $\phi(\alpha^c) = 2\beta_0(\delta c\alpha^c + c) - \delta^2 h^2 w^2 e^{-2\delta\alpha^c} = 0$, then $\phi(\alpha^M) \geq 0$, which indicates $\pi(p, \alpha; w)$ is jointly concave at (p^M, α^M). Clearly, $F(\alpha)$ is concave in α, with $F_\alpha = \delta^2 h^2 w^2 e^{-\delta\alpha} - 2\beta_0 c e^{\delta\alpha} - 2\beta_0 c\alpha\delta e^{\delta\alpha}$ and $F_{\alpha\alpha} = -\delta^3 h^2 w^2 e^{-\delta\alpha} - 4\beta_0 c\delta e^{\delta\alpha} - 2\beta_0 c\alpha\delta^2 e^{\delta\alpha} < 0$, and $F(0) = \delta hw(B - hw - \beta_0\gamma) > 0$, $F_{\alpha|\alpha=\alpha^c} = 0$. Because $F(\alpha^M) = 0$, we have $\alpha^c < \alpha^M$. Thus, $\phi(\alpha^M) > 0$, which implies $\pi(p, \alpha; w)$ is jointly concave at (p^M, α^M) and $F_{\alpha|\alpha=\alpha^M} < 0$. Therefore, the first-order conditions, thus $F(\alpha) = 0$, are sufficient and necessary for the optimality of p^M and α^M. Furthermore, α^M is the unique solution to $F(\alpha) = 0$. *Q.E.D.*

Proof of Corollary 1. Given α, the first-order derivative of $F(\alpha)$ with respect to δ as $F_\delta = hw(B - hwe^{-\delta\alpha} - \beta_0\gamma) + \alpha\delta h^2 w^2 e^{-\delta\alpha} - 2\beta_0 c\alpha^2 e^{\delta\alpha}$ with $\lim_{\delta\to 0} F_\delta = hw(B - hw - \beta_0\gamma) \geq 0$ and $\lim_{\delta\to\infty} F_\delta = hw(B - \beta_0\gamma) - 2\beta_0 c\alpha^2 e^{\delta\alpha} < 0$. Therefore, by the derivative of α^M with respect to δ, denoted as α'_δ, we have $\alpha'_\delta = -\frac{F_\delta}{F_\alpha} \geq 0$ if $\delta \to 0$ and $\alpha'_\delta = -\frac{F_\delta}{F_\alpha} < 0$ if $\delta \to \infty$, implying a critical δ-value denoted as δ^c exists such that if $\delta \leq \delta^c$, α^M is increasing in δ, whereas if $\delta > \delta^c$, α^M is decreasing in δ. Mathematically, δ^c satisfies $F(\alpha) = 0$ and $F_\delta(\alpha) = 0$. The derivative of p^M with respect to δ, is denoted as $p'_\delta = \frac{hwe^{-\delta\alpha}(\alpha + \delta\alpha'_\delta)}{2\beta_0}|_{\alpha=\alpha^M}$, where we have $\alpha + \delta\alpha'_\delta = \frac{-2\beta_0 c\alpha e^{\delta\alpha} - \delta hw(B - hwe^{-\delta\alpha} - \beta_0\gamma)}{F_\alpha} > 0$, implying $p'_\delta > 0$. Therefore, p^M is always increasing in δ. By the envelop theorem, the profit π^M is always increasing in δ because the derivative of π^M with respect to δ is $\pi'_\delta = (p^M - \gamma)hw\alpha^M e^{-\delta\alpha^M} > 0$. *Q.E.D.*

Proof of Corollary 2. First, note α^M is decreasing in γ: $\alpha'_\gamma = \frac{\beta_0\delta hw}{F_\alpha}|_{\alpha=\alpha^M} < 0$ because $F(\alpha)$ is decreasing in γ. The derivative of p^M with respect to γ, is denoted as $p'_\gamma = \frac{\delta hwe^{-\delta\alpha}\alpha'_\gamma + \beta_0}{2\beta_0}|_{\alpha=\alpha^M}$, where $\delta hwe^{-\delta\alpha}\alpha'_\gamma + \beta_0 = \beta_0(\frac{\delta^2 h^2 w^2 e^{-\delta\alpha}}{F_\alpha} + 1)$. Because $\delta^2 h^2 w^2 e^{-\delta\alpha} + F_\alpha = 2e^{-\delta\alpha}(\delta^2 h^2 w^2 - \beta_0 c(e^{2\delta\alpha} + \alpha\delta e^{2\delta\alpha}))$, if $\delta^2 h^2 w^2 \geq \beta_0 c$, a unique α^o exists such that $\delta^2 h^2 w^2 = \beta_0 c(e^{2\delta\alpha^o} + \alpha^o\delta e^{2\delta\alpha^o})$. Because α^M is decreasing in γ, a unique γ^c exists such that at γ^c, $\alpha^M = \alpha^o$. If $\gamma \leq \gamma^c$, then $p'_\gamma \geq 0$, and thus p^M is increasing in γ, whereas if $\gamma > \gamma^c$, then $p'_\gamma < 0$, and thus p^M is decreasing in γ. However, holding the other parameters fixed, because $\gamma \in [0, \frac{B - h\bar{w}}{\beta_0}]$, if $\gamma^c > \frac{B - h\bar{w}}{\beta_0}$, p^M is always increasing in $\gamma \in [0, \frac{B - h\bar{w}}{\beta_0}]$. If $\delta^2 h^2 w^2 < \beta_0 c$, then $\delta^2 h^2 w^2 e^{-\delta\alpha} + F_\alpha < 0$ for any $\alpha \geq 0$, which implies $p'_\gamma > 0$ and p^M is always increasing in γ. By the envelop theorem, we have that the profit π^M is always decreasing in γ based on the derivative $\pi'_\gamma = -(B - hwe^{-\delta\alpha^M} - \beta_0 p^M) - \frac{1}{w} < 0$. *Q.E.D.*

Proof of Corollary 3. By the implicit function theorem, the derivative of α^M with respect to w, denoted as α'_w, is given as

$$\alpha'_w = \frac{\partial \alpha^M}{\partial w} = -\frac{F_w}{F_\alpha} = \frac{\delta h^2 w e^{-\delta \alpha} - \delta h (B - hw e^{-\delta \alpha} - \beta_0 \gamma)}{\delta^2 h^2 w^2 e^{-\delta \alpha} - 2\beta_0 c e^{\delta \alpha} - 2\beta_0 c \alpha \delta e^{\delta \alpha}}\bigg|_{\alpha = \alpha^M}.$$

Because F_w is decreasing in w while increasing in α, if $F_w|_{w=\bar{w}}(0) \geq 0$, that is, $2h\bar{w} \leq B - \beta_0 \gamma$, then $F_w(\alpha) \geq 0$ for any $w \in [\underline{w}, \bar{w}]$ and $\alpha \geq 0$. Therefore, if $2h\bar{w} \leq B - \beta_0 \gamma$, we have $\alpha'_w \geq 0$, which implies α^M is increasing in w for any $w \in [\underline{w}, \bar{w}]$. However, if $F_w|_{w=\bar{w}}(0) < 0$, that is, $h\bar{w} \leq B - \beta_0 \gamma < 2h\bar{w}$, then a $w^0 \in [\frac{\bar{w}}{2}, \bar{w}]$ exists that satisfies $B - \beta_0 \gamma = 2hw^0$, such that for $w \in [\underline{w}, w^0]$, $F_w(w, \alpha) \geq 0$ for any $\alpha \geq 0$, and thus $\alpha'_w \geq 0$, which implies α^M is increasing in $w \in [\underline{w}, w^0]$. If $w \in (w^0, \bar{w}]$, a critical $\alpha(w) > 0$ exists that is increasing in w and satisfying $F_w(\alpha(w)) = 0$, such that if $\alpha \leq \alpha(w)$, then $F_w(\alpha) \leq 0$, whereas if $\alpha > \alpha(w)$, $F_w(\alpha) > 0$. Because for any $w \in [\underline{w}, \bar{w}]$, a unique $\alpha^M > 0$ exists such that $F(\alpha^M) = 0$ from Proposition 1, if $\alpha^M \leq \alpha(w)$, then $F_w(\alpha^M) \leq 0$, thus $\alpha'_w \leq 0$, which implies α^M is decreasing in w. If $\alpha^M > \alpha(w)$, $F_w(\alpha^M) > 0$, and thus $\alpha'_w > 0$, which implies α^M is increasing in w. Therefore, if a $w^c \in (w^0, \bar{w}]$ exists where $F(\alpha^M)|_{w=w^c} = 0$ and $F_w(\alpha^M)|_{w=w^c} = 0$, we conclude that if $w > w^c$, $F_w(\alpha^M) < 0$, and thus $\alpha'_w < 0$; that is, α^M is decreasing in $w \in (w^c, \bar{w}]$. If $w \leq w^c$, $F_w(\alpha^M) \geq 0$, and thus $\alpha'_w \geq 0$; that is, α^M is increasing in $w \in [\underline{w}, w^c]$. If such $w^c \in (w^0, \bar{w}]$ exists, that is, $w^c > \bar{w}$, $F_w(\alpha^M) \geq 0$, then α^M is always increasing in $w \in [\underline{w}, \bar{w}]$.   *Q.E.D.*

Proof of Proposition 2. Given service provider j's price P_j and entertainment level α_j, service provider i maximizes its profit through P_i and α_i. Similar to the monopoly case, the optimal entertainment level α_i satisfies the first-order condition,

$$\frac{\partial \Pi_i}{\partial \alpha_i} = P_i hw\delta e^{-\delta \alpha_i} - c\alpha_i = 0,$$

and the best response of price P_i satisfies the following equation:

$$D(\alpha_i) - 2\beta P_i + \theta P_j = 0.$$

The optimal entertainment level and the best response of price P_j for service provider j given service provider $i's$ price P_i and entertainment level α_i can be similarly derived. Thus, the equilibrium price and entertainment level can be solved through the following system of equations:

$$D(\alpha_i) - 2\beta P_i + \theta P_j = 0$$
$$D(\alpha_j) - 2\beta P_j + \theta P_i = 0$$
$$P_i hw\delta e^{-\delta \alpha_i} - c\alpha_i = 0$$
$$P_j hw\delta e^{-\delta \alpha_j} - c\alpha_j = 0,$$

where we can first solve the equilibrium price as a function of α_i and α_j as

$$P_i = \frac{2\beta D(\alpha_i) + \theta D(\alpha_j))}{4\beta^2 - \theta^2}$$

and then get the best-response function of α_i in terms of α_j as

$$G(\alpha_i, \alpha_j) = (2\beta D(\alpha_i) + \theta D(\alpha_j)) hw\delta - (4\beta^2 - \theta^2) c\alpha_i e^{\delta \alpha_i} = 0.$$

Clearly, given $\alpha_j \geq 0$, $G(\alpha_i, \alpha_j)$ is concave in α_i, and $G(0, \alpha_j) > 0$. Therefore, a unique $\alpha_i^e(\alpha_j)$ exists as the best response, such that $G(\alpha_i^e(\alpha_j), \alpha_j) = 0$, and

$$\frac{\partial G(\alpha_i^e(\alpha_j), \alpha_j)}{\partial \alpha_i} = 2\beta(hw)^2\delta^2 e^{-\delta\alpha_i} - (4\beta^2 - \theta^2)(ce^{\delta\alpha_i} + \delta c\alpha_i e^{\delta\alpha_i})|_{\alpha_i = \alpha_i^e(\alpha_j)} < 0.$$

Because the two service providers are symmetric, in equilibrium, the two service providers must choose the same price and entertainment level. Denote the equilibrium price and entertainment level as P^C and α^C, respectively. Thus, we have the equilibrium condition $G(\alpha^C, \alpha^C) = 0$, which is simplified as

$$\delta hw(A - hwe^{-\delta\alpha^C}) - (2\beta_0 + \theta)c\alpha^C e^{\delta\alpha^C} = 0, \ \ P^C = \frac{D(\alpha^C)}{2\beta_0 + \theta} = \frac{A - hwe^{-\delta\alpha^C}}{2\beta_0 + \theta}.$$

Correspondingly, the equilibrium arrival rate is given as $\lambda^C = D(\alpha^C) - \beta P^C + \theta P^C = \beta P^C = \frac{(\beta_0 + \theta)D(\alpha^C)}{2\beta_0 + \theta}$, and the utilization is calculated as $\rho^C = 1 - \frac{1}{w\lambda^C + 1}$.   Q.E.D.

Proof of Corollary 4. Because the equilibrium entertainment level α^C satisfies the optimality condition $G(\alpha^C, \alpha^C) = \delta hw D(\alpha^C) - (2\beta_0 + \theta)c\alpha^C e^{\delta\alpha^C} = 0$, where $G(\alpha, \alpha) = \delta hw D(\alpha) - (2\beta_0 + \theta)c\alpha e^{\delta\alpha}$ is concave in α and $G(0,0) > 0$, we have $G'_\alpha = \frac{\partial G(\alpha,\alpha)}{\partial\alpha}|_{\alpha=\alpha^C} < 0$, and $G'_\theta = \frac{\partial G(\alpha,\alpha)}{\partial\theta}|_{\alpha=\alpha^C} = -c\alpha^C e^{\delta\alpha^C} < 0$. Holding β_0 fixed, we have $\frac{\partial\alpha^C}{\partial\theta} = -\frac{G'_\theta}{G'_\alpha} = \frac{c\alpha^C e^{\delta\alpha^C}}{\delta^2(hw)^2 e^{-\delta\alpha^C} - (2\beta_0 + \theta)ce^{\delta\alpha^C}(1 + \delta\alpha^C)} < 0$, which indicates α^C is decreasing in θ. Thus, the equilibrium price P^C is also decreasing in θ. The profit of each service provider in the equilibrium is calculated as $\Pi^C = P^C(D(\alpha^C) - \beta_0 P^C) - C(\alpha^C) - \frac{\gamma}{w}$, with the first-order derivative with respect to θ as

$$\frac{\partial\Pi^C}{\partial\theta} = \frac{\partial P^C}{\partial\theta}(D(\alpha^C) - \beta_0 P^C) + P^C(hw\delta e^{-\delta\alpha^C}\frac{\partial\alpha^C}{\partial\theta} - \beta_0\frac{\partial P^C}{\partial\theta}) - c\alpha^C\frac{\partial\alpha^C}{\partial\theta}$$
$$= \frac{\partial P^C}{\partial\theta}(D(\alpha^C) - 2\beta_0 P^C) \leq 0$$

because $\frac{\partial P^C}{\partial\theta} < 0$ and $D(\alpha^C) - 2\beta_0 P^C = \frac{\theta P^C}{2\beta_0 + \theta} \geq 0$.   Q.E.D.

Proof of Corollary 5. By Corollary 4, in the case of duopoly competition, the equilibrium price, entertainment level, and profit in equilibrium all decrease in the price-competition intensity θ. Thus, $p^C \leq p^M$, $\alpha^C \leq \alpha^M$, and $\Pi^C \leq \Pi^M$. The equilibrium arrival rate is $\lambda^C = \frac{\beta_0 + \theta}{2\beta_0 + \theta}D(\alpha^C)$, and by the optimality condition of α^C, we have $\lambda^C = \frac{(\beta_0 + \theta)c\alpha^C e^{\delta\alpha^C}}{hw\delta}$, and the first-order derivative with respect to θ is

$$\frac{\partial\lambda^C}{\partial\theta} = ce^{\delta\alpha^C}\left(\alpha^C + (\beta_0 + \theta)(1 + \alpha^C\delta)\frac{\partial\alpha^C}{\partial\theta}\right).$$

Thus, if $\left(\alpha^C + (\beta_0 + \theta)(1 + \alpha^C\delta)\frac{\partial\alpha^C}{\partial\theta}\right) \geq 0$, λ^C increases in θ, and ρ^C increases in θ; that is, $\rho^C \geq \rho^M$ for $\theta \geq 0$.   Q.E.D.

Proof of Proposition 3. Given the prices of the two service providers, P_i and P_j, under co-opetition, the optimal entertainment level is determined to maximize the joint profit of the two service providers as

$$\max_{\alpha} \pi(\alpha, P_i, P_j) = P_i(D(\alpha) - \beta P_i + \theta P_j) + P_j(D(\alpha) - \beta P_j + \theta P_i) - C(\alpha) - \frac{2\gamma}{w},$$

where the optimal entertainment level, denoted as $\alpha^O = \alpha(P_i, P_j)$, satisfies the first-order condition

$$(P_i + P_j)hw\delta e^{-\delta\alpha^O} - c\alpha^O = 0,$$

where α^O is increasing in P_i and P_j. We denote the partial derivative of α^O in terms of P_i and P_j as $\frac{\partial \alpha^O}{\partial P_1}$ and $\frac{\partial \alpha^O}{\partial P_2}$, respectively. Clearly,

$$\frac{\partial \alpha^O}{\partial P_1} = \frac{\partial \alpha^O}{\partial P_2} = \frac{hw\delta e^{-\delta \alpha^O}}{(P_i + P_j)hw\delta^2 e^{-\delta \alpha^O} + c} > 0.$$

Given service provider j's decision, P_j, the optimal price of service provider i is determined by solving the following profit-maximizing problem

$$\max_{P_i} \pi(P_i, P_j) = P_i(D(\alpha^O) - \beta P_i + \theta P_j) - \phi(\lambda_i, \lambda_j)C(\alpha^O) - \frac{\gamma}{w}$$

$$s.t. \quad (P_i + P_j)hw\delta e^{-\delta \alpha^O} - c\alpha^O = 0$$

$$\lambda_i = D(\alpha^O) - \beta P_i + \theta P_j$$

$$\lambda_j = D(\alpha^O) - \beta P_j + \theta P_i.$$

Substituting λ_i and λ_j, the entertainment cost-sharing function $\phi(\lambda_i, \lambda_j)$ is simplified as $\phi(\lambda_i, \lambda_j) = \frac{1}{2} + t(\beta + \theta)(P_j - P_i)$. Based on the above reformulation, the optimal price, denoted as P_i^*, as the best response of P_j satisfies the following first-order condition:

$$D(\alpha(P_i^*, P_j)) - 2\beta P_i^* + \theta P_j + P_i^* hw\delta e^{-\delta\alpha(P_i^*, P_j)}\frac{\partial\alpha(P_i^*, P_j)}{\partial P_i}$$

$$+ t(\beta + \theta)C(\alpha(P_i^*, P_j)) - \phi(\lambda_i, \lambda_j)c\alpha(P_i^*, P_j)\frac{\partial\alpha(P_i^*, P_j)}{\partial P_i} = 0.$$

Similarly, the best response of P_j with respect to P_i satisfies the following first-order condition:

$$D(\alpha(P_i, P_j^*)) - 2\beta P_j^* + \theta P_i + P_j^* hw\delta e^{-\delta\alpha(P_i, P_j^*)}\frac{\partial\alpha(P_i, P_j^*)}{\partial P_j}$$

$$+ t(\beta + \theta)C(\alpha(P_i, P_j^*)) - \phi(\lambda_j, \lambda_i)c\alpha(P_i, P_j^*)\frac{\partial\alpha(P_i, P_j^*)}{\partial P_j} = 0.$$

Because the two service providers are identical, in equilibrium, the two service providers will choose the same price denoted as P^O. Thus, adding up the above two first-order conditions, after simplification, we have

$$D(\alpha(P^O, P^O)) - (2\beta_0 + \theta)P^O + t(\beta_0 + 2\theta)C(\alpha(P^O, P^O)) = 0,$$

with $2P^O hw\delta e^{-\delta\alpha^O} = c\alpha^O$, implying the following result in terms of the equilibrium price P^O and the entertainment level α^O in Model D-SE:

$$2\delta hw\left(D(\alpha^O) + t(\beta_0 + 2\theta)C(\alpha^O)\right) - (2\beta_0 + \theta)c\alpha^O e^{\delta\alpha^O} = 0, \quad P^O = \frac{D(\alpha^O) + t(\beta_0 + 2\theta)C(\alpha^O)}{2\beta_0 + \theta},$$

where $\alpha^O = \alpha(P^O, P^O)$.

Correspondingly, the arrival rate, the profit, and the utilization of each service provider under co-opetition are $\lambda^O = \frac{\beta D(\alpha^O) - t(\beta^2 - \theta^2)C(\alpha^O)}{2\beta - \theta}$, $\pi^O = P^O\lambda^O - \frac{1}{2}C(\alpha^O) - \frac{\gamma}{w}$, and $\rho^O = 1 - \frac{1}{w\lambda^O + 1}$, respectively.   *Q.E.D.*

Proof of Corollary 6. For ease of exposition, we define

$$V(\alpha) = 2\left(D(\alpha) + t(\beta_0 + 2\theta)C(\alpha)\right)hw\delta - (2\beta_0 + \theta)c\alpha e^{\delta\alpha}. \tag{13}$$

We have from (13) that $\frac{\partial V(\alpha)}{\partial t} > 0$. The derivative of $V(\alpha)$ with respect to α is

$$\frac{\partial V(\alpha)}{\partial \alpha} = 2hw\delta[hw\delta e^{-\delta\alpha} + t(\beta_0 + 2\theta)c\alpha] - (2\beta_0 + \theta)(ce^{\delta\alpha} + c\alpha\delta e^{\delta\alpha}),$$

and with $V(0) > 0$, we conclude $\frac{\partial V(\alpha)}{\partial \alpha}|_{\alpha=\alpha^O} \leq 0$. Thus, we have

$$\frac{\partial \alpha^O}{\partial t} = -\frac{\partial V(\alpha)/\partial t}{\partial V(\alpha)/\partial \alpha}\bigg|_{\alpha=\alpha^O} \geq 0.$$

It is straightforward from Proposition 3 that P^O also increases in t.   *Q.E.D.*

Proof of Proposition 4. Denote $V(\alpha, t) = 2\delta hw\left(D(\alpha) + t(\beta_0 + 2\theta)C(\alpha)\right) - (2\beta_0 + \theta)c\alpha e^{\delta\alpha}$, and the profit under co-opetition as $\Pi^O = \pi(t, \alpha^O)$, which can be rewritten as

$$\pi(t, \alpha^O) = \frac{(\beta_0 + \theta)[D(\alpha^O)]^2}{(2\beta_0 + \theta)^2} + \frac{\kappa(t, \theta, \alpha^O)}{(2\beta_0 + \theta)^2} - \frac{1}{2}C(\alpha^O) - \frac{\gamma}{w},$$

where

$$\kappa(t, \theta, \alpha^O) = t(\beta_0 + 2\theta)C(\alpha^O)\left[\theta D(\alpha^O) - t\beta_0(\beta_0 + 2\theta)C(\alpha^O)\right],$$

and $D(\alpha^O) = A - hwe^{-\delta\alpha^O}$. Clearly, α^O depends on t. Then, the derivative of $\pi(t, \alpha^O(t))$ with respect to t is given as

$$\frac{d\Pi^O}{dt} = \frac{\partial \pi(t, \alpha^O)}{\partial t} + \frac{\partial \pi(t, \alpha^O)}{\partial \alpha}\frac{\partial \alpha^O}{\partial t}.$$

We first argue that α^O increases in t. We have $V(\alpha^O, t) = 0$ and $V(0, t) > 0$. Then, $\frac{\partial V(\alpha, t)}{\partial \alpha}|_{\alpha=\alpha^O} < 0$. By the implicit function theorem, we have

$$\frac{\partial \alpha^O}{\partial t} = -\frac{\partial V(\alpha, t)/\partial t}{\partial V(\alpha, t)/\partial \alpha}|_{\alpha=\alpha^O} > 0,$$

because $\frac{\partial V(\alpha, t)}{\partial t} > 0$, which implies

$$\frac{\partial \alpha^O}{\partial t} > 0.$$

We have

$$\frac{\partial \pi(t, \alpha^O)}{\partial t} = \frac{(\beta_0 + 2\theta)C(\alpha^O)\left[\theta D(\alpha^O) - 2t\beta_0(\beta_0 + 2\theta)C(\alpha^O)\right]}{(2\beta_0 + \theta)^2}$$

and

$$\frac{\partial \pi(t, \alpha^O)}{\partial \alpha} = \frac{1}{(2\beta_0 + \theta)^2}\left\{hw\delta e^{-\delta\alpha^O}\left[2(\beta_0 + \theta)D(\alpha^O) + \theta t(\beta_0 + 2\theta)C(\alpha^O)\right]\right.$$
$$\left. + t(\beta_0 + 2\theta)c\alpha^O\left[\theta D(\alpha^O) - 2t\beta_0(\beta_0 + 2\theta)C(\alpha^O)\right]\right\} - \frac{1}{2}c\alpha^O.$$

From the optimality condition of α^O, we have

$$2t(\beta_0 + 2\theta)C(\alpha^O) = \frac{(2\beta_0 + \theta)c\alpha^O e^{\delta\alpha^O} - 2\delta hwD(\alpha^O)}{\delta hw},$$

which gives

$$\frac{\partial \pi(t, \alpha^O)}{\partial t} = \frac{(\beta_0 + 2\theta)C(\alpha^O)\left[\delta hwD(\alpha^O) - \beta_0 c\alpha^O e^{\delta\alpha^O}\right]}{\delta hw(2\beta_0 + \theta)},$$

and

$$\frac{\partial \pi(t, \alpha^O)}{\partial \alpha} = \frac{\delta hwD(\alpha^O) - \beta_0 c\alpha^O e^{\delta\alpha^O}}{(2\beta_0 + \theta)}\left\{e^{-\delta\alpha^O} + \frac{(2\beta_0 + \theta)c\alpha^O e^{\delta\alpha^O} - 2\delta hwD(\alpha^O)}{\alpha^O \delta^2(hw)^2}\right\}.$$

Thus, if $\delta hwD(\alpha^O) - \beta_0 c\alpha^O e^{\delta\alpha^O} > 0$, then $\frac{\partial \pi(t, \alpha^O)}{\partial \alpha} > 0$ and $\frac{\partial \pi(t, \alpha^O)}{\partial t} > 0$, and thus $\frac{d\pi(t, \alpha^O)}{dt} > 0$. That is, the equilibrium profit under co-opetition increases in t; otherwise, the equilibrium profit under co-opetition decreases in t.

To show a unique cost-sharing factor t^* exists, we proceed as follows. Denote $\varphi(\alpha) = \delta h w D(\alpha) - \beta_0 c \alpha e^{\delta \alpha}$, with

$$\frac{\partial \varphi(\alpha)}{\partial \alpha} = \delta^2 (hw)^2 e^{-\delta \alpha} - \beta_0 c e^{\delta \alpha} - \beta_0 \delta c \alpha e^{\delta \alpha}, \quad \frac{\partial^2 \varphi(\alpha)}{\partial \alpha^2} = -\delta^3 (hw)^2 e^{-\delta \alpha} - 2\beta_0 \delta c e^{\delta \alpha} - \beta_0 \delta^2 c \alpha e^{\delta \alpha} < 0;$$

that is, $\varphi(\alpha)$ is concave in α. Because $\varphi(0) > 0$, a unique α^* exists such that $\varphi(\alpha^*) = 0$. By $\frac{\partial \alpha^O}{\partial t} > 0$, we conclude a unique t^* exists where

$$\theta D(\alpha^*) = 2t^* \beta_0 (\beta_0 + 2\theta) C(\alpha^*),$$

such that if $t \in [0, t^*)$, $\alpha^O < \alpha^C$, implying $\varphi(\alpha^O) > 0$, and if $t > t^*$, $\alpha^O > \alpha^*$, implying $\varphi(\alpha^O) < 0$. Thus, if $t \in [0, t^*)$, $\pi^O(t, \alpha^O)$ increases in t, whereas if $t > t^*$, $\pi^O(t, \alpha^O)$ decreases in t, and $\pi^O(t, \alpha^O)$ is maximized at t^* with $\alpha^O = \alpha^*$.   *Q.E.D.*

Proof of Proposition 5. The derivative of $V(\alpha)$ with respect to θ is given as

$$\frac{\partial V(\alpha)}{\partial \theta} = 4hw\delta t C(\alpha) - c\alpha e^{\delta \alpha},$$

and with $\frac{\partial V(\alpha)}{\partial \alpha}$, we have

$$\frac{\partial \alpha^O}{\partial \theta} = -\frac{\partial V(\alpha)/\partial \theta}{\partial V(\alpha)/\partial \alpha}\bigg|_{\alpha = \alpha^O} = -\frac{4hw\delta t C(\alpha) - c\alpha e^{\delta \alpha}}{2hw\delta[hw\delta e^{-\delta \alpha} + t(\beta_0 + 2\theta)c\alpha] - (2\beta_0 + \theta)(ce^{\delta \alpha} + c\alpha\delta e^{\delta \alpha})}\bigg|_{\alpha = \alpha^O}.$$

Because $\frac{\partial V(\alpha)}{\partial \alpha}\big|_{\alpha = \alpha^O} \leq 0$, if $4hw\delta t C(\alpha^O) - c\alpha^O e^{\delta \alpha^O} \geq 0$, then $\frac{\partial \alpha^O}{\partial \theta} \geq 0$; that is, α^O increases in θ. Because $P^O = \frac{c\alpha^O e^{\delta \alpha^O}}{2hw\delta}$, we have

$$\frac{\partial P^O}{\partial \theta} = \frac{ce^{\delta \alpha^O} + c\alpha^O \delta e^{\delta \alpha^O}}{2hw\delta} \frac{\partial \alpha^O}{\partial \theta} \geq 0,$$

if $\frac{\partial \alpha^O}{\partial \theta} \geq 0$.

From the condition of t^*, we have

$$\frac{2\delta hw\theta D(\alpha^O)}{\beta_0(\beta_0 + 2\theta)} = 4\delta hwt^* C(\alpha^O).$$

By the condition of α^O, we have

$$c\alpha^O e^{\delta \alpha^O} = \frac{2\delta hw(D(\alpha^O) + t^*(\beta_0 + 2\theta)C(\alpha^O))}{2\beta_0 + \theta},$$

which implies if $\theta \leq \beta_0$, then

$$\frac{2\delta hw\theta D(\alpha^O)}{\beta_0(\beta_0 + 2\theta)} \leq \frac{2\delta hw(D(\alpha^O) + t(\beta_0 + 2\theta)C(\alpha^O))}{2\beta_0 + \theta};$$

that is, $4\delta hwt^* C(\alpha^O) \leq c\alpha^O e^{\delta \alpha^O}$, or $2\delta hwt^* \alpha^O - e^{\delta \alpha^O} \leq 0$.   *Q.E.D.*

Proof of Proposition 6. We first prove that given the same θ, $P^O \geq P^C$ and $\alpha^O \geq \alpha^C$. Recall the equilibrium entertainment level in competition α^C satisfies

$$G(\alpha^C) = \delta H(A - hwe^{-\delta \alpha^C}) - (2\beta_0 + \theta)c\alpha^C e^{\delta \alpha^C} = 0,$$

and the equilibrium entertainment level in co-opetition α^O satisfies

$$V(\alpha^O) = 2\delta hw\left(A - hwe^{-\delta \alpha^O} + t(\beta_0 + 2\theta)C(\alpha^O)\right) - (2\beta_0 + \theta)c\alpha^O e^{\delta \alpha^O} = 0.$$

For any $t > 0$, we have $G(0) < V(0)$, and for any $\alpha \geq 0$, $V(\alpha) > G(\alpha)$ and $V'(\alpha) > G'(\alpha)$, which indicates $\alpha^O > \alpha^C$. Because $P^O = \frac{D(\alpha^O) + t(\beta_0 + 2\theta)C(\alpha^O)}{2\beta_0 + \theta}$ and $P^C = \frac{D(\alpha^C)}{2\beta_0 + \theta}$, with $\alpha^O \geq \alpha^C$, we have $P^O > P^C$.

Based on Proposition 4, we now argue a t^c exists in the proposition. If $t = 0$, that is, each service provider splits the entertainment cost equally, then the co-opetition case is equivalent to the competition case in which each service provider's entertainment cost is only $\frac{1}{2}C(\alpha)$, given α. Thus, each service provider's profit under co-opetition with $t = 0$ is always larger than in competition. Because the equilibrium profit under co-opetition Π^O decreases in $t > t^*$, a threshold t^c exists such that if $0 \leq t < t^c$, $\Pi^O > \Pi^C$, whereas if $t \geq t^c$, $\Pi^O \leq \Pi^C$. $\hfill Q.E.D.$

Proof of Proposition 7. If $t = 0$, the equilibrium entertainment level α^O under co-opetition satisfies

$$2\delta hwD(\alpha^O) - (2\beta_0 + \theta)c\alpha^O e^{\delta\alpha^O} = 0,$$

and the price is $P^O = \frac{D(\alpha^O)}{2\beta_0 + \theta}$. The profit in equilibrium is

$$\Pi^O = \frac{(\beta_0 + \theta)[D(\alpha^O)]^2}{(2\beta_0 + \theta)^2} - \frac{1}{2}C(\alpha^O) - \frac{\gamma}{w}.$$

Similarly to the proof in Proposition 2, α^O, P^O and Π^O decrease in θ.

Because when $t = 0$, Π^O decreases in θ, and when $t = 0$ and $\theta = 0$, $\Pi^O > \pi^M$, a threshold θ^c exists such that if $\theta \in [0, \theta^c]$, $\Pi^O \geq \Pi^M$, whereas if $\theta > \theta^c$, $\Pi^O < \Pi^M$. $\hfill Q.E.D.$

We now present a lemma that will be used for the proof for Proposition 8.

LEMMA 1. *If $\theta = 0$, a threshold t^m exists such that if $t > t^m$, the profit under co-opetition is smaller than in the monopoly case.*

Proof of Lemma 1. If $\theta = 0$, under co-opetition in equilibrium, the entertainment level is solved by

$$V(\alpha^O) = 2\left(D(\alpha) + t\beta_0 C(\alpha^O)\right)hw\delta - 2\beta_0 c\alpha^O e^{\delta\alpha^O} = 0,$$

and the price and the profit are given as

$$P^O = \frac{D(\alpha^O) + t(\beta_0 + 2\theta)C(\alpha^O)}{2\beta_0 + \theta}, \quad \Pi^O = \frac{[D(\alpha^O)]^2 - t^2\beta_0^2[C(\alpha^O)]^2}{4\beta_0} - \frac{1}{2}C(\alpha^O) - \frac{\gamma}{w}.$$

We denote $\Pi^O = \pi(t, \alpha^O)$, and its derivative with respect to t is calculated as

$$\frac{d\Pi^O}{dt} = \frac{\partial\pi(t, \alpha^O)}{\partial t} + \frac{\partial\pi(t, \alpha^O)}{\partial\alpha}\frac{\partial\alpha^O}{\partial t}.$$

In addition,

$$\frac{\partial\pi(t, \alpha^O)}{\partial\alpha} = \frac{e^{-\delta\alpha^O}}{4\beta_0}\left[2hw\delta D(\alpha^O) - 2t^2\beta_0^2 C(\alpha^O)c\alpha^O e^{\delta\alpha^O} - 2\beta_0 c\alpha^O e^{\delta\alpha^O}\right].$$

Because α^O satisfies $V(\alpha^O) = 0$, we conclude $\frac{\partial\pi(t,\alpha^O)}{\partial\alpha} < 0$, which, together with $\frac{\partial\pi(t,\alpha^O)}{\partial t} < 0$, implies $\frac{d\Pi^O}{dt} < 0$; that is, Π^O decreases in t. If $t = 0$ and $\theta = 0$, Π^O is the optimal profit where two service providers split the entertainment cost, implying $\Pi^O > \pi^M$. Therefore, a threshold t^m exists such that if $t \in [t, t^m]$, $\Pi^O \geq \pi^M$, whereas if $t > t^m$, $\Pi^O < \pi^M$. $\hfill Q.E.D.$

Proof of Proposition 8. Using Propositions 1 and 3, we have α^M satisfies $F(\alpha^M) = \delta hwD(\alpha^M) - 2\beta_0 c\alpha^M e^{\delta\alpha^M} = 0$, or $F(\alpha^M) = \delta hwD(\alpha^M)(1 + \frac{\theta}{2\beta_0}) - (2\beta_0 + \theta)c\alpha^M e^{\delta\alpha^M} = 0$; and α^O satisfies $V(\alpha^O) = 2\left(D(\alpha^O) + t(\beta_0 + 2\theta)C(\alpha^O)\right)hw\delta - (2\beta_0 + \theta)c\alpha^O e^{\delta\alpha^O} = 0$. We have

$$F(\alpha^M) - V(\alpha^M) = D(\alpha^M)(\frac{\theta}{2\beta_0} - 1) - 2t(\beta_0 + 2\theta)C(\alpha^M).$$

Therefore, if $\theta \le 2\beta_0$, $F(\alpha^M) - V(\alpha^M) \le 0$, $V(\alpha^M) \ge 0$. Because $V(\alpha^O) = 0$ and $V(0) > 0$, we conclude $\alpha^O \ge \alpha^M$. Likewise, we can show that if $\theta > 2\beta_0$, $\alpha^O < \alpha^M$.

By Proposition 4, given θ, if

$$\theta D(\alpha^O) = 2t^* \beta_0(\beta_0 + 2\theta)C(\alpha^O),$$

Π^O increases in $t \in [0, t^*]$, and decreases in $t > t^*$. By Lemma 1, if $\theta = 0$, Π^O decreases in t, and by Proposition 7, if $t = 0$, Π^O decreases in θ. Obviously, the profit function Π^O is continuous in t or θ. Therefore, we conclude that given θ, a threshold $t^m(\theta)$ exists such that if $t \in [0, t^m(\theta)]$, the profit in the case of co-opetition is larger than in the case of monopoly, and vice versa.   *Q.E.D.*

Proof of Proposition 9. Because the profit function is jointly concave in p and α, using the first order condition

$$\frac{\partial \pi}{\partial p} = B + \delta\alpha - 2\beta_0 p = 0, \ \frac{\partial \pi}{\partial \alpha} = p\delta - c\alpha = 0,$$

the optimal price and entertainment level can be derived as in Proposition 9.   *Q.E.D.*

Proof of Proposition 10. We can obtain the equilibrium service fee and entertainment level by jointly solving

$$p_i\delta - c\alpha_i = 0,$$
$$p_j\delta - c\alpha_j = 0,$$
$$B + \delta\alpha_i + \theta p_j - 2(\beta_0 + \theta)p_i = 0, \ \text{and}$$
$$B + \delta\alpha_j + \theta p_i - 2(\beta_0 + \theta)p_j = 0.$$

Hence proposition 10.   *Q.E.D.*

Proof of Proposition 11. The equilibrium price and entertainment level can be solved by the following equation system:

$$(p_i + p_j)\delta - c\alpha = 0,$$
$$B + \delta\alpha + \theta p_j - 2(\beta_0 + \theta)p_i + p_i\delta\alpha_i' + t(\beta_0 + 2\theta)C(\alpha) - \phi(p_i, p_j)c\alpha\alpha_i' = 0,$$
$$B + \delta\alpha + \theta p_i - 2(\beta_0 + \theta)p_j + p_j\delta\alpha_j' + t(\beta_0 + 2\theta)C(\alpha) - \phi(p_j, p_i)c\alpha\alpha_j' = 0,$$

where the second and the third equations are derived by the first order condition.

Adding up the second and the third equations, with $\phi(p_i, p_j) + \phi(p_j, p_i) = 1$, in the symmetric equilibrium, the optimal entertainment level, denoted as α^O, satisfies $B + \delta\alpha - \frac{c\alpha}{2\delta}(2\beta_0 + \theta) + \frac{1}{2}c\alpha^2 t(\beta_0 + 2\theta) = 0$, at $\alpha = \alpha^O$. *Q.E.D.*

Proof of Corollary 8. In equilibrium, each firm will share half of the entertainment cost, and the profit for each firm is

$$\pi^O = p^O(B + \delta\alpha^O - \beta_0 p^O) - \frac{1}{4}c(\alpha^O)^2.$$

By the first-order condition, $\alpha^O = \frac{\delta B}{\beta_0 c - \delta^2}$, and substituting α^O into the condition in Proposition 11, the optimal t^* can be solved.   *Q.E.D.*