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MATCH-REFEREE MATCHING MODEL IN FOOTBALL

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Abstract

Referees are assigned to football matches in Turkey by a three-person subcommittee formed by Central Referee Committee, a subsidiary of the Turkish Football Federation, in a non-transparent way and this procedure makes manipulation possible. Therefore, referee appointments are among the topics discussed in the football world. It is a major deficiency that the matching theory, which has been successful in regulating many problematic markets, has not been used to date in football for referee appointments. In this study, a match-referee matching model, using the Gale-Shapley algorithm, for the football matches played in Turkish professional football leagues is proposed. The match-referee matching result offered by this algorithm is the best possible stable matching result for the clubs. This matching result is produced in line with the preferences of the clubs and clubs' satisfaction is at the forefront. Most importantly, it is obtained through a transparent, clear, fair and manipulation-free match-referee matching system.

Keywords: Matching Theory, Gale-Shapley Algorithm, Football referee assignment, Two-sided matching market

JEL Code: C78, L83, D02

1. Introduction

Football has three important elements in a match: players, spectators, and referee. Referee is the only person authorized to enforce the football game rules throughout the match. The referee performs the task of managing the match together with the assistant referees but makes the final decision himself. The decisions made by the referee during the gameplay are final and cannot be changed. Referee decisions during the game have always been a subject of controversy in football. Kilcigil and Partal (2003) found that supporters place biased decisions of the referee

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in the first place in provocation factors causing violence. Similarly, according to Polat and Sonmezoglu (2016), referee decisions during game are in the list of on-field factors that drive supporters to violence in football. In addition to the referee decisions during game, Cantez (2021) stated that Turkish Football Federation's (TFF) inaccurate referee appointments are among the most important factors affecting the supporter violence. Therefore, referee appointments and referee decisions play an active role in fan-originated field incidents during games. The Video Assistant Referee System (VAR) has been implemented in order to reduce the errors in referee decisions and enable a fairer game management. According to Engin and Celik (2019), soccer referees stated that the implementation of the VAR system should be continued in order to ensure transparency without creating any doubt in the referee decisions. However, even the VAR system is now at the center of the dispute on the referee game decisions in the football world.

In all football organizations, national league and cup organizations or international tournaments such as the FIFA World Cup, European Football Championship, UEFA Champions League and UEFA Europa League, referees and their decisions during gameplay have been criticized. For this reason, organizations and people who are responsible from referee assignments are also heavily criticized. The situation is not different in Turkey. Each football season, in professional football leagues and Turkish Cup competitions, there are controversial referee decisions that determine not only the result of the match, but the team that will be the champion or relegated. Some clubs even claim that some referees are biased towards them and systematically make decisions against them, so that these clubs demand that these referees not be assigned to their matches. The lack of information on according to which criteria and how the referee appointments are determined causes further flaring of the discussions. This puts the Central Referee Committee (Merkez Hakem Kurulu, MHK), which works under the Turkish Football Federation (TFF) and is responsible for referee appointments, in a tough position.

There are various researches on referee assignment problem using different methodologies to enhance the transparency and objectivity of the assignment process. Alarcon et al. (2013) proposed an integer linear programming approach that considers balance in the number of matches each referee must officiate, the frequency of each referee being assigned to a given team, the distance each referee must travel over the course of a season, and the appropriate pairings of referee experience or skill category with the importance of the matches. In another work, Yavuz et al. (2007) develops a constructive heuristic and a local search procedure for a fair referee assignment.

Unfortunately, none of the works and studies in the existing literature consider the preferences of the clubs. At this point, we should look into matching theory for a remedy. Matching theory has attracted the attention of many researchers from different scientific fields such as mathematics, computer engineering, finance, and economics. The success of this theory in practice is the

main reason for its active use in many markets. It has been very successful in the placement of medical school graduates to residency programs in the US since 1995 (Roth, 1984; Roth ve Sotomayor, 1990; Roth ve Peranson, 1999), in the placement of middle school graduate students in high schools in New York since 2004 and in Boston since 2006 (Abdulkadiroglu vd., 2005a; Abdulkadiroglu vd., 2005b), in placing university students in dormitories (Chung, 2000; Chen ve Sönmez 2002), and in regulating the kidney transplant market in the northeastern states of the United States and Ohio state since 2005 (Roth, Sonmez ve Unver, 2004).

The aim of this study is to propose a method for match-referee matching that will enable the appointment of referees to football matches in a systematic and transparent manner using the matching theory in which the preferences of the clubs are at the center of interest. While the great knowledge gained from the studies on matching theory in the literature is successfully applied in many problematic markets, it is a very important deficiency that it is not used in solving the problems in the football market, which creates an economic magnitude of billions of US dollars. In particular, with the method proposed in this study, which is an application of the algorithm examined by Celik and Knoblauch (2005, 2007) and Celik (2009), appointments of referees will be determined transparently in line with the preferences of the clubs. Hence, complaints and discussions of club managers and fans on referee assignments will be largely avoided. The proposed methodology here is very important to be examined and evaluated, since it is applicable to all sports played in league style.

2. Matching Theory

In order to understand the match-referee matching methodology and its results proposed in this study for the appointment of referees to manage football matches, it is necessary to address some issues and important points of matching theory. Matching theory made great progress with the algorithm (Gale-Shapley algorithm) developed for solving the marriage problem in the work of two mathematicians David Gale and Lloyd Shapley in 1962 and became the focus of attention of researchers from many different scientific fields. The marriage problem is a two-sided matching problem in which elements from two disjoint sets are matched. In the marriage matching problem, how to match n number of men, $M = \{m_1, m_2, ..., m_n\}$, and n number of women, $W = \{w_1, w_2, ..., w_n\}$, with each other considering their preference lists for marriage is examined. Each man creates a preference list by ranking the women he wants to marry from the most preferred to the least preferred. Likewise, women list the men they want to marry in their preference lists, starting with the one they prefer the most. A man/woman cannot place two different candidates at the same rank in the preference lists. Preference lists should be complete, strict, consistent and transitive. The Gale-Shapley algorithm uses these preference lists to match men and women for marriage. Accordingly, the matching solution, μ, can be written as a one-to-one function.

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$$\mu : M \cup W \rightarrow M \cup W$$

$$\forall w, \mu(w) \in M \cup \{w\} \text{ and } \forall m, \mu(m) \in W \cup \{m\}$$

$$\mu(m) = w \Leftrightarrow \mu(w) = m, m \in M, w \in W,$$

$\mu(m)$ represents the women than men are matched with to marry under the matching result and $\mu(w)$ represents the men that women are matched with. In case of $\mu(m) = m$, it means that the man did not match with any woman as a result of the matching and remained single. Similarly, $\mu(w) = w$ is the situation where the woman is not matched with any man and remains single. There are two versions of the Gale-Shapley algorithm: i) men make the marriage proposal and ii) women make the proposal. If we consider the men-propose version of the algorithm, it performs the following steps in order.

i. Each man proposes to the first woman in his preference list.
ii. Each woman who received a proposal keeps the offer from the man she prefers the most and rejects the others. Each woman is matched tentatively with the man who she has not rejected yet. These matched couples are called "engaged" at this stage of the algorithm. If the woman who receives a proposal is already engaged, she will also consider her fiancée while evaluating the marriage proposals. If she receives an offer from a man he prefers to her fiancée, she breaks up with her fiancée and holds the proposal of the most preferred man, that is, she gets engaged with this person.
iii. Each rejected man proposes to the next woman in his preferences list. At this stage, the algorithm repeats the steps ii. and iii. until there is no rejected man.
iv. When there is no rejection, the algorithm stops and engaged couples are matched for marriage.

There are some features of the matching result that this algorithm produces for the marriage problem. First of all, every matching obtained as a result of this algorithm is stable. In stable matches, there is no man and no woman who prefers each other over his/her partner. Otherwise, man and woman who prefer each other to their current spouses would separate from matched mates and get together for marriage. Therefore, the matching result produced by the algorithm would have been violated. Stability of the matching result is an important requirement for a successful organization of the market to be regulated. If equality between candidates is allowed in the preference lists, a stable matching result may not exist. Preference lists should be complete and strict, ensuring that at least one stable matching exists, that is, the set of stable matchings is not empty (Gale ve Shapley, 1962). There can be more than one stable matching in a marriage problem, and the number of different stable matching solutions is $e^{-1}.n.\ln(n)$ on average (Pittel, 1989).

Matching result produced by the men-propose version of the Gale-Shapley algorithm is the most preferred one for all men among all stable matching results

and this matching result is called men-optimal matching. In the men-optimal matching, men are matched with their best possible option and since the matching is stable, a better matching is not possible for men. In addition, although women accept or reject the proposals, the men-optimal matching is the least preferred one for all women among all matchings in the set of stable matchings. Similarly, the stable matching produced by the women-propose version of the algorithm is women-optimal and it is the most preferred matching for all women. The women-optimal matching is also the least preferred stable matching for all men (Knuth, 1976).

Some men in the marriage problem can be more popular among women, so they may be more preferred than other men. For this reason, these more preferred candidates will appear at the top of all women's lists and other less preferred men will always be ranked lower. This situation causes similarities, i.e. correlation, in the preference lists of women. Similarly, if there are women who are more popular for all men, there will be similarities, i.e. correlation, in men's preference lists. As the level of correlation in the preference lists increases, the number of stable matching results decreases. If all men have exactly the same preference list (perfect correlation among men) and all women have exactly the same preference list (perfect correlation among women), there is only one stable matching result. In this case, men-optimal and women-optimal matchings are the same (Celik and Knoblauch, 2005) and both versions of the Gale-Shapley algorithm will produce the same matching result. Therefore, it doesn't matter which version of the algorithm is used.

In their work on correlated preference lists, Calderelli and Capucci (2001) found that more popular candidates are matched with higher ranked candidates in their preference lists compared to their matched mates, where all the preference lists are randomly created, i.e., the correlation among the preference lists is zero. In more detailed studies on this subject, Celik and Knoblauch (2007) and Celik (2009) revealed that the correlation in the preference lists is an important factor affecting the satisfaction of men and women from the result of the matching. According to these works, as the correlation in the men's preference lists increases (meaning there are popular women), men's satisfaction from the matching result decreases and women's satisfaction increases. This is because men compete to be matched with a few popular women, so that men place those popular women high on their preference lists. Similarly, as the correlation in women's lists increases (meaning there are popular men), women's satisfaction from the matching result decreases, while men's satisfaction increases. The unequal number of men and women in marriage matching markets does not cause any change in the information we have mentioned so far from the existing literature. In markets where the number of men and the number of women are different, some candidates will not be matched, that is, they will remain single.

Finally, Celik (2009) introduces correlation in the preference lists in the roommate's problem of students who are matched according to their preference lists to share the same room in a dormitory. Roommates' problem is a one-sided matching problem where elements are selected from the same set for a matching. According to Celik (2009), as the correlation in preference lists increases, students' satisfaction from the matching result decreases. In the light of the information we have collected from the studies in the matching theory literature so far, we will be able to understand the features of the match-referee matching system that we propose for referee assignments to football matches much better.

3. Referee Assignments by Turkish Football Federation (TFF) According to the Regulations of Central Referee Committee (MHK)

Turkish Football Federation (TFF) is the governing body of football in Turkey. The authority to assign, directly or through the relevant boards, the referee, assistant referees, fourth and additional assistant referees, observers and mentors required for all football, futsal, beach football and HiF (Football for All) competitions organized or permitted by FIFA, UEFA or TFF is given to Central Referee Committee (MHK), a sub-committee of the Turkish Football Federation, by Turkish Football Federation Central Referee Committee Instructions (May 2019). MHK consists of a president and eight members appointed upon the proposal of the TFF President and the approval of the TFF board of directors. MHK meets with absolute majority as often as necessary, and decisions are taken by absolute majority. In case of equality of votes, the vote of the MHK president is decisive. However, assignments of referee, observer and mentor to the matches of professional football leagues, Turkish Cup and Super Cup are made by Professional Competition Executive Board (Profesyonel Musabaka Icra Kurulu, PMIK), a subsidiary of MHK, consisting of MHK president and two members. MHK president is the chairman of this board and two members of PMIK are appointed upon the proposal of the president of the MHK and the approval of the TFF board of directors.

Referees are categorized by MHK according to some criteria such as age, experience, observer score, number of matches, success in exams and physical proficiency tests. These categories are regional referee category, classification referee category, top class referee category, and FIFA referee category. Only referees from top class referee category can be assigned to the matches of the highest tier two professional football leagues (Turkish Super League and TFF 1. League) and the final rounds of Turkish Cup. Among the referees in the top class referee category, those who meet certain criteria are elected to the FIFA referee category and given a FIFA badge with the recommendation of MHK and the approval of the TFF board. In addition to football matches in Turkey, FIFA referees

can be assigned to the matches organized by FIFA, governing body of football in the world, and UEFA, the governing body of football in Europe. Therefore, the highest quality referees in professional football in Turkey are the referees with FIFA badge in FIFA referee category. MHK has the authority to decide to send a referee to any sub-category or to promote to any higher category. There is no transparency in what criteria MHK president and two members in PMIK consider, what evaluations they make and how they give their decision when assigning referees to matches in the professional football leagues. Therefore, the club managers and presidents who have close relationship with these three people in PMIK may cause an injustice in the league competition by having the referees they prefer assigned to their matches and/or preventing the assignment of referees they do not want. Nobody can guarantee that such situations do not occur in referee assignments made by a non-transparent method. For this reason, the establishment of an objective and transparent referee appointment system that clubs, football players and fans know how to apply will remove the MHK and referees from being the focus of many discussions in the football world.

4. Match-Referee Matching Model for Referee Assignments

If we consider the assignment of referees to the matches in Turkish Super League as a matching market, one of the two disjoint sets from which the members will be matched is the set formed by Turkish Super League matches played every week, and the other set is formed by referees who are competent to manage these matches. In order for the Gale-Shapley algorithm to work, we need to know the preference lists of the elements in these two sets. It is important that the preference lists are complete and strict so that we can find a stable matching result. In 2021, there are seven FIFA referees in Turkey ("MHK, FIFA kokartı takmaya aday hakemleri FIFA'ya bildirdi" https://www.tff.org/default.aspx?pageID=248&ftxtID=34186.). In addition to seven FIFA referees, there are 47 referees in the top class category who can be assigned to matches in the top two professional leagues (Turkish Super League and TFF 1. League) in the 2020-2021 season (Referee and Observer List for the 2020-2021 Season). For this reason, football clubs in the Turkish Super League can include a maximum of 54 referees in their preference lists. Since 10 matches are played every week in the Turkish Super League, each referee's preference list will consist of 10 matches.

Every week, each club manager will submit his/her club's preference list on referees to the MHK by listing the referees from the most preferred to the least preferred one that (s)he wants to be assigned to the match they will play in that week of the league. Since there are two teams playing a match, there will be two preference lists submitted for each match. MHK will combine these two lists with a method and creates a single preference list for each match. The methodology to

be used for combining these preference lists should be determined in the light of the information that will be obtained from academic studies and simulations on the subject of referee assignments. This topic is excluded from this study as another detailed research topic. The methodology to be determined should be fair, taking into account the preferences of both clubs, and it should be shared with the public in order to prevent manipulation and provide transparency.

The preference lists of the referees can be created randomly to ensure equality among the referees, or a preference list creation algorithm can be developed to allow FIFA referees to rank the matches with higher difficulty higher in their preference lists. So that, the likelihood of a more difficult match to be managed and an experienced FIFA referee to be matched will be higher. A methodology similar to the one in Celik (2009), giving more weight to some of the possible choices can be used to create such a preference list for each FIFA referee.

After a preference list for each element in both sets, matches and referees, is created, MHK can make the necessary changes to finalize these lists. However, MHK should announce these changes publicly with their reasons. Referees who have health problems, report a personal excuse, have been sanctioned by MHK for any reason, or have to manage another international match very soon may be removed from the combined preference lists of the clubs for that week. Similarly, in order to prevent the same referee from managing a team's match for two consecutive weeks, the matches of the teams managed by a referee in the previous week can be removed from the preference list of that referee. This is why some referees' preference lists may consist of 8 matches instead of 10.

After the preference lists are finalized, the Gale-Shapley algorithm is employed to produce a matching result. We know that the match-referee matching result obtained from this algorithm will be stable. The question to be asked at this stage is, which version of the algorithm is going to be used. In the match-referee matching system that is tried to be established here, clubs-offer version should be used, since the priority should be the satisfaction of the clubs, not the satisfaction of the referees. This version produces the club-optimal matching result, which is the best possible stable matching for all clubs.

5. Discussion and Conclusion

In this study, a match-referee matching model has been proposed that allows the appointment of referees to professional football matches made by three people in the PMIK, the sub-committee of the MHK, in a transparent, fair and clearly understandable way in the light of valuable information provided by the matching theory literature. In the referee assignment model proposed here, satisfaction of the clubs will be at the highest possible level as the match-referee matching will be made by the Gale-Shapley algorithm in line with the preferences of the clubs.

Since the preference lists of the clubs are not created randomly, there will be similarities, i.e. correlation, in the preference lists of the clubs since the beliefs and preferences about referees are shaped by the clubs' own perceptions, experiences and thoughts. Especially, FIFA referees and referees who have had good performance and shown good management in the recent weeks will appear at the top of the preference lists of all clubs while new and inexperienced referees or referees who have recently shown bad management will be ranked lower. In such a case, according to the results of Celik and Knoblauch (2007) and Celik (2009), as the correlation in the preference lists of clubs increases, clubs will be matched with the lower ranked referees in the clubs' preference lists compared to the matching result when club preferences are created randomly. In other words, the satisfaction of the clubs from the matching result will be less. Celik (2009) explains this as everyone competes for a few most preferred candidates. Again, based on the findings obtained from these studies, as the correlation in the preference lists of the referees increases, clubs will matched with the referees who are ranked higher in their preference lists compared to the matching result produced by the randomly created preference lists for referees. Hence, the satisfaction of the clubs from the matching result will increase.

Ability of each club to affect the match-referee matching is limited to the changes they deem necessary in their own preference list before each referee assignment procedure but they cannot predict what the outcome will be. It should be noted that even a single change in one of the preference lists used in the algorithm can change all matches in the matching result. A club can reduce the likelihood of an unwanted referee to be assigned to its match, but cannot avoid it, by ranking that referee in the last place in its preference list. Although the probability is low, this referee may be assigned to this club's match depending on the preference lists of other clubs and referees. In such a case, no one can blame the three members of PMIK for acting maliciously and assigning the unwanted referee to the match. Similarly, a club can rank a very much desired referee in the first place in its preference list to increase the likelihood of the referee to be assigned to its match but can not guarantee the assignment. The reason for this is that the match-referee matching result that the Gale-Shapley algorithm generates for referee assignments depends on the preference lists of other clubs and referees. In the system proposed here, it is necessary to control many variables in order to manipulate a referee assignment, which is almost impossible. Therefore, manipulations that can be done through three PMIK members in the referee assignment system used today will not be possible in the model proposed here.

How to create the preference lists for referees and how to apply a method to combine the preference lists of the two clubs that will meet in a match require more detailed academic research and simulations. These methods should be shared with the public in the name of transparency after they are determined as a result of academic studies and calibrations.

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