Polyphase Radar Signals with ZACZ Based on p-Pairs D-Code Sequences and Their Compression Algorithm

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***Abstract* — In modern synthetic-aperture radars, signals with the linear frequency modulation (LFM) have found the practical application as probing signals. Utilization of LFM-signals was formed historically since they were the first wideband signals, which found application in radar technology, and their properties have studied a long time ago and in detail. However, the LFM-signals have the "splay" ambiguity function, which results the ambiguity in range. The question of the probing signal choice is also relevant in connection with the problem of weak echoes detection, which are closed by the side lobes of ACF of the strong echoes. In this paper, the polyphase (*p*-phase, where *p* is the prime integer number) radar signal, which has an area of zero side lobes in a vicinity of the central peak of autocorrelation function, has been synthesized. It is shown that this signal represents a train from *p* coherent phase-code-shift keyed pulses, which are coded by complementary sequences of the *p*-ary D-code. The method of ensemble set formation of the *p*-ary D-code for signal synthesis is suggested. Correlation characteristics of the synthesized signal are discussed. The compression algorithm of this signal is considered including in its structure the combined algorithm of Vilenkin-Chrestenson and Fourier fast transform.**

***Index Terms* — Autocorrelation function, complementary sequences, polyphase signal, pulse train, Vilenkin-Chrestenson functions, zero autocorrelation zone.**

I. Introduction

For accurate determination of the distance (range) and speed of a variety of small-size space objects on the near-Earth orbit, for resolution of separate elements of complex space objects and also for resolution of small-size objects on the Earth surface, it is necessary to use the wideband probing signals, which have high resolution on the slant range $\Delta r = c/(2F_s)$, where F_s is the signal spectrum width, and c is the radial speed. To obtain the high angular resolution $\Delta\theta$ of Earth surface elements and targets located on this surface, radars are used, which are installed on the quickly-moved aircraft-space carriers with the direct aperture synthesis. High resolutions on the slant and transverse $\Delta r_\perp = r_0\Delta\theta$ ranges, where r_0 is the slant range to observing resolution element, permit to obtained of two-dimensional target patterns in distance. Ensuring of the high angular resolution of small-size space objects or elements of complex space objects is based on the effect of the inverse synthesis of the antenna aperture [1]. For resolution on the Doppler frequency equaled to $\Delta F_D = 1/T_s$, where T_s is the probing signal duration (time of coherent accumulation), the angular resolution $\Delta\theta = \lambda/(2V\sin\theta_0)\Delta F_D$ is provided, where V is the ground speed of object motion, θ_0 is the angle between the ground speed vector and the pointing direction. The transverse resolution is provided by turning of the target velocity vector with regard to the pointing direction and is realized by processing of the sequence of complex samples, which arrive from each target element resolved on the slant range.

It follows from the above-mentioned that for providing of high resolutions on the slant Δr and transverse $\Delta r_\perp$ ranges, it is necessary to use the probing signals with the wide spectrum and the long duration.

As research shows, for these purposes can use the train of linear-frequency-modulated (LFM) pulses with the high repetition frequency [2, 6]. Nevertheless, as we know, the LFM signals have the "splay" ambiguity function, which results the ambiguity in range. The ambiguity peaks are appeared on autocorrelation function (ACF) of the train of LFM pulses.

The question of the probing signal choice is also relevant in connection with the problem of weak echoes detection, which are closed by the side lobes of ACF of the strong echoes. To suppress the side lobes of ACF echoes, one can apply the intra-pulse and inter-pulse weighting [3, 4]. However, at that, the spreading of the main ACF lobe occurs together with the loss in SNR.

To solve the stated tasks, we can use the phase-code-shift keyed (PCSK) signals, which are free from shortcomings of FM and FSK signals. In [4 - 7], the radar PCSK signals are considered, which have the zero correlation zone in the region of the central peak of aperiodic ACF (Zero Autocorrelation Zone - ZACZ). These signals represent the periodic sequence from $M \gg 1$ coherent pulses coding (or phase-shift keyed) by the ensembles of complementary or orthogonal sequences. PCSK signals with ZACZ solve the problem of weak echo detection on the background of strong echoes. However, the relative ZACZ width of these signals is

$$\varepsilon = Z/L = (q-1)/(q(M-1)+1) \ll 1. \tag{1}$$

where Z is the ZACZ width; L is the signal duration [8].

In addition, at formation and processing of the PCSK signal with the large number of pulses in the train, it is

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difficult enough to keep their coherence. The polyphase PCSK signals discussed in [4 - 7] (Frank or P4), also have the large alphabet of phases equaled to the number of contiguous time slices (each of duration T_0) in the pulse.

Recently there were considerably great attention attracted to reduction of the detection possibility of radar stations (RS) by the means of the radio-electronic reconnaissance and by the self-guided anti-radar missiles [9 - 11]. In RS with low probability of emission detection, the special measures are anticipated to increase of the RS operation secrecy. Among them: the low spectral density of emission, variation of probing signal parameters according the random law, operation in the wide frequency band, control of the emission power. Applied wideband probing signals – LFM signals or bi-phase PCSK signals – do not provide the RS operation secrecy. So, LFM signals can be easily recognized by means of reconnaissance on the phase variation speed; while bi-phase PCSK signals – with the help of quadratic detection circuits. The emission secrecy can be increased by a great extent through the utilization of the polyphase PCSK signals [11]. Polyphase pulse signals can be formed by the wide set of p-ary codes and differ by the low spectral density and the low level of ACF side lobes.

In this paper, to solve problems of high resolution of the variety of small-size space objects on the near-Earth orbit, the separate elements of the complicated space object, as well as the small-size objects on the Earth surface, the polyphase (p-phase, where p is the prime integer number) radar signal is synthesized, which has ZACZ. This signal represented the train from p coherent PCSK pulses coding by complementary sequences of the p-ary D-code [13, 14]. It has low pulse number p in the train, the small alphabet of phases equaled to p, and an approach to code formation allows usage of the fast transform algorithm for its compression in the matched filter.

II. THE SYNTHESIS OF THE P-ARY D-CODE AND THE POLYPHASE COHERENT ADDITIONAL SIGNAL

Sequences $\{d_n^1\},\{d_n^2\},...,\{d_n^i\},...,\{d_n^p\}$, $(n=1,2,...,N)$ of the length $N=p^k$, where $k\geq 2$ is the integer number, are called complementary [6, 12, 13] if

$$r_m^1+r_m^2+...+r_m^i+...+r_m^p=\begin{cases}pN;\ m=0,\\0;\ m=\pm 1,...,\pm(N-1),\end{cases} \tag{2}$$

where $r_m^i=r_m^{i,i}=\sum_{n=m+1}^{N} d_n^i d_{n-m}^{*i}$ is the aperiodic ACF of the sequence $\{d_n^i\}$, * is the complex conjugation operation.

N/p sets of complementary (additional) sequences with the N length satisfying to (2) form a matrix of p-phase additional sequences (MAS) with dimensions $N\times N$. In publications this matrix is called the ensemble of Golay complimentary sequences at $p=2$ [6, 12]. We introduce the generalized concept of the Golay sequences for $p>2$ [14].

Let $\tilde{\mathbf{D}}_N=\left\|\tilde{d}_{i,n}\right\|_1^N$ be a matrix of p-ary D-codes [13, 14], $\tilde{d}_{i,n}=0,1,...,p-1$; $N=p^k$, p is the prime number. Then MAS of the k-th order (dimensions $N\times N$) will have a form:

$$\mathbf{D}_N=\left\|d_{i,n}\right\|_1^N,\ d_{i,n}=\exp\left\{j\frac{2\pi}{p}\tilde{d}_{i,n}\right\}. \tag{3}$$

Let us call sequences $\mathbf{D}_{1,N}^i=\left\|d_{i,n}\right\|_{n=1}^N$ and $\mathbf{D}_{1,N}^j=\left\|d_{j,n}\right\|_{n=1}^N$ p-paired if

$$(i-1)_p\oplus(j-1)_p=(\Delta)_p,\ i,j=1,2,...,N, \tag{4}$$

where i, j are numbers of sequences in the D-code or numbers of MAS rows; $(a)_p$ is a number a in the p-ary form; $\oplus$ is the operation of adding modulo p; $\Delta=p^{k-1}$.

p-paired sequences are complimentary, i.e., for them (2) holds true.

Let $\mathbf{D}_N$ be MAS (3), and $\mathbf{H}_N=\left\|h_{i,n}\right\|_1^N$ be a matrix of the system of Vilenkin-Chrestenson-Kronecker (VC-Kronecker) functions [15]. It is known that the system of VC-Kronecker is multiplicative Abelian group [16]. Since the variety consisting of the MAS rows is the adjacent class in the sub-group, elements of which are rows of the VC-Kronecker matrix, and the first MAS row is the leader of the adjacent class, then we may write:

$$\mathbf{D}_N=\mathbf{H}_N\mathbf{d}_N, \tag{5}$$

where $\mathbf{d}_N=\mathrm{diag}\{d_{1,1},d_{1,2},...,d_{1,n},...,d_{1,N}\}$ is the diagonal matrix with elements from the first row of $\mathbf{D}_N$.

At $p=2$, the VC-Kronecker matrix is transformed into the Hadamard matrix [17].

It follows from (5) that to construct of MAS $\mathbf{D}_N$, it is necessary to form its first row $\mathbf{D}_{1,N}^1$.

Elements of the MAS first row are defined as follows [18]:

$$d_{1,y+1}=\exp\left\{j\frac{2\pi}{p}\sum_{i=1}^{k-1}y_{i+1}y_{l_i}\right\}, \tag{6}$$

where $y+1=n$ is a number of the MAS column; $(y)_p=(y_k\ \ y_{k-1}\ \ ...\ \ y_i\ \ ...\ \ y_1)$ is a number of the MAS column in the p-ary form; $y_i=0,1,...,p-1$; $y=0,1,...,p^k-1$; $l_i=1,2,...,i$; $i=1,2,...,k-1$; $l_{k-1}\neq l_{k-2}\neq\ldots\neq l_2$.

Adding in (6) is performed modulo p. This approach allows formation of 2^{k-2} ensembles of the D-code of k-th order. Let further γ - number of ensembles of the D-code of k-th order, i.e. $\gamma=2^{k-2}$.

A. Example formation of binary D-code

Consider an example of the formation of a binary D-code of size 16×16 [17]. The D-code of order $k=4$ has

$\gamma = 2^{k-2} = 4$ ensembles.

At $k = 4$, we obtain:

$$i = 1, 2, 3 \Rightarrow l_1 = 1\,; l_2 = 1, 2\,; l_3 = 1, 2, 3$$
$$y_1, y_2, y_3, y_4 = 0, 1\,;\; y = 0, 1, ..., 15\,;$$
$$(y)_2 = (y_4 \quad y_3 \quad y_2 \quad y_1)$$

In this case, the condition $l_3 \neq l_2$ must be satisfied.

Let us write the sum from (6) for four ensembles:

$$\tilde{d}_{1,y+1} = \sum_{i=1}^{3} y_{i+1} y_{l_i} = y_2 y_1 + y_3 y_1 + y_4 y_2\,;$$
$$\tilde{d}_{1,y+1} = \sum_{i=1}^{3} y_{i+1} y_{l_i} = y_2 y_1 + y_3 y_1 + y_4 y_3\,;$$
$$\tilde{d}_{1,y+1} = \sum_{i=1}^{3} y_{i+1} y_{l_i} = y_2 y_1 + y_3 y_2 + y_4 y_1\,;$$
$$\tilde{d}_{1,y+1} = \sum_{i=1}^{3} y_{i+1} y_{l_i} = y_2 y_1 + y_3 y_2 + y_4 y_3\,.$$

Having performed additions for all columns of the CSM, from (6) we obtain the first sequences of four D-code ensembles of order $k = 4$.

(0 0 0 1 0 1 0 0 0 0 1 0 0 1 1 1);
(0 0 0 1 0 1 0 0 0 0 0 1 1 0 1 1);
(0 0 0 1 0 0 1 0 0 1 0 0 0 1 1 1);
(0 0 0 1 0 0 1 0 0 0 0 1 1 1 0 1).

The remaining rows of the D-code matrix are obtained from the first row by element-wise modulo 2 addition with the corresponding rows of the Hadamard matrix.

B. Example formation of ternary D-code

Consider now an example of the formation of the ternary D-code of order $k = 3$ with the length of code words $N = 3^3 = 27$, which allows you to form a $\gamma = 2^{3-2} = 2$ ensembles of D-code using method (5) - (6) [18].

We form the first rows of two different matrices (ensembles) of the D-code of order $k = 3$. The remaining rows of the D-code matrix are obtained from the first row by element-wise addition modulo p with the corresponding rows of the VC-Kronecker matrix.

In this case, from (6) we obtain:

$$\tilde{d}_{1,y+1} = \sum_{i=1}^{2} y_{i+1} y_{l_i} = \left(y_2 y_{l_1} + y_3 y_{l_2}\right);$$
$$(y)_3 = (y_3 \quad y_2 \quad y_1);\; y_1, y_2, y_3 = 0,1,2;\; y = 0,1,...,26;$$
$$i = 1,2;\, l_1 = 1;\, l_2 = 1,2.$$

From where for two ensembles we obtain:

$$\tilde{d}_{1,y+1} = y_2 y_1 + y_3 y_1 \text{ and } \tilde{d}_{1,y+1} = y_2 y_1 + y_3 y_2\,.$$

At $y = 0,1,...,26$, for the first ensemble we obtain the following first row of the matrix of the D-code:

(0 0 0 0 1 2 0 2 1 0 1 2 0 2 1 0 0 0 0 2 1 0 0 0 0 1 2).

For the second ensemble, the first row of the D-code matrix is as follows:

(0 0 0 0 1 2 0 2 1 0 0 0 1 2 0 2 1 0 0 0 0 2 0 1 1 0 2).

C. Determination of coherent additional signal

Let us refer as the polyphase coherent additional signal (CAS) of the train of p PCSK pulses encoded by p-paired sequences of the D-code [14]. We call this signal "coherent" because the coherence of PCSK pulses must be maintained in the train. In addition, we call it "additional" because the pulses are encoded by complementary (additional) sequences.

The analytical expression of the complex envelope (CE) of CAS has a form:

$$\dot{S}(t) = \sum_{i=1}^{p} \sum_{n=1}^{N} S_0\Big(t - \big(n + (i-1)Nq - 1\big)T_0\Big) d_{i,n}, \tag{7}$$

where $S_0\left(t - (n-1)T_0\right) = \begin{cases} 1, \; (n-1)T_0 \le t \le nT_0 \\ 0, \text{ at other } t \end{cases}$ is the envelope of the n–th slice of CAS; T_0 is the slice duration; $q \ge 2$ is the off-duty factor; $\left\| d_{i,n} \right\|_1^N = \mathbf{D}_{1,N}^i$ are elements of i-th p-paired sequence.

CE of CAS in the vector form will have the following form:

$$\mathbf{S}_{1,N((p-1)q+1)} = \Big(\mathbf{D}_{1,N}^1 \quad \mathbf{0}_{1,N(q-1)} \quad \mathbf{D}_{1,N}^2 \quad \mathbf{0}_{1,N(q-1)} \quad \dots$$
$$\dots \quad \mathbf{D}_{1,N}^i \quad \mathbf{0}_{1,N(q-1)} \quad \dots \quad \mathbf{D}_{1,N}^p\Big), \tag{8}$$

where $\mathbf{0}_{1,N(q-1)} = \left(0_1 \quad 0_2 \quad \dots \quad 0_n \quad \dots \quad 0_{N(q-1)}\right)$ is the zero vector- row with length $N(q-1)$.

III. CORRELATION CHARACTERISTICS OF POLYPHASE COHERENT ADDITIONAL SIGNALS

An analysis of CAS correlation characteristics is performed in [14, 17].

The aperiodic cross-correlation function (CCF) of sequences $\left\{d_n^i\right\}$ and $\left\{d_n^j\right\}$ is defined as:

$$r_m^{i,j} = \sum_{n=m+1}^{N} d_n^i d_{n-m}^{*j} \text{ at } i \neq j;\; m = 0, \pm 1, ..., \pm(N-1), \tag{9}$$

where $r_m^{i,j} = 0$ at $m = 0$, because complimentary sequences built according to (5) – (6), are orthogonal.

In the vector form, ACF of the polyphase CAS will have a form [14]:

$$\mathbf{R}_{1,2N((p-1)q+1)-1}^{s} = \Bigg(\sum_{i=1}^{p-(p-1)} \mathbf{R}_{1,2N-1}^{i+p-1,i} \quad \dots.$$
$$\dots \quad \sum_{i=1}^{p} \mathbf{R}_{1,2N-1}^{i} \quad \mathbf{0}_{1,N(q-2)} \quad 0 \quad \sum_{i=1}^{p-1} \mathbf{R}_{1,2N-1}^{i,i+1} \quad \dots \tag{10}$$
$$\dots \quad \sum_{i=1}^{p-j} \mathbf{R}_{1,2N-1}^{i,i+j} \quad \mathbf{0}_{1,N(q-2)} \quad 0 \quad \dots \quad \sum_{i=1}^{p-(p-1)} \mathbf{R}_{1,2N-1}^{i,i+p-1}\Bigg),$$

where $\mathbf{0}_{1,N(q-2)} = \left(0_1 \quad 0_2 \quad \dots \quad 0_n \quad \dots \quad 0_{N(q-2)}\right)$ is the zero vector-row with the length $N(q-2)$;

$$\mathbf{R}^{i,j}_{1,2N-1} == \left(r^{i,j}_{-N+1} \quad r^{i,j}_{-N+2} \quad \ldots \quad r^{i,j}_{-1} \quad r^{i,j}_{0} \quad r^{i,j}_{1} \quad \ldots \right.$$
$$\left. \ldots \quad r^{i,j}_{m} \quad \ldots \quad r^{i,j}_{N-2} \quad r^{i,j}_{N-1} \right).$$

For $i = j$, $\mathbf{R}^{i,j}_{1,2N-1} = \mathbf{R}^{i}_{1,2N-1}$ and according to (2):

$$\sum_{i=1}^{p} \mathbf{R}^{i}_{1,2N-1} = \left(0_{-N+1} \quad 0_{-N+2} \quad \ldots \quad pN \quad \ldots \quad 0_{N-2} \quad 0_{N-1} \right), \qquad (11)$$

and at $i \neq j$ $r^{i,j}_0 = 0$.

It follows from (10) and (11) that the ZACZ width (from both sides of the central peak of ACF) of the polyphase CAS is equal to $2Z = 1 + N(q-2) + 2N - 1 - 1 + N(q-2) + 1 =$ $= 2N(q-1)$, and taking into account the slice duration T_0 :

$$Z = NT_0(q-1). \qquad (12)$$

The relative width of ZACZ is defined as:

$$\varepsilon = Z/LT_0 = (q-1)/(q(p-1)+1), \qquad (13)$$

where $L = N((p-1)q+1)$ is the slices number in CAS.

It follows from (13) that

$$1/(2p-1) \leq \varepsilon < 1/(p-1), \qquad (14)$$

and at $p = 2$, $1/3 \leq \varepsilon < 1$ [17].

In (1), $M \geq N = p^k$, $k \geq 2$ is the integer number, therefore, for signals considered in [4 - 7], $\varepsilon \ll 1$.

The polyphase CAS can be considered as the signal formed by the sequence from the ZACZ-ensemble [8, 17] with parameters:

$$ZACZ(J, L, Z),$$

where $J = \gamma \dfrac{N}{p}$ is a number of sequences in the ensemble.

The set of sequences forming p CAS and formed from the adjacent sets of p-paired sequences of the D-code can be considered as the ZCZ-ensemble [17] with parameters:

$$ZCZ(p, L, Z).$$

Fig. 1 and 2 show, relatively, a part of the two-dimensional ambiguity function $|R(\tau, F)|$ of the three-phase CAS with the number of slices of $N = 243$ in the pulse and with the off-duty factor $q = 3$ and its section by the plane $F = 0$, i.e., ACF of CE of CAS at complete filter matching with echoes in frequency.

The width of CAS ZACZ with given parameters in relative units is $Z/T_0 = 486$. From Fig. 1 it is seen that in the region of the central peak, the ambiguity function has the clearly expressed rectangular region of zero correlation along the whole frequency axis F at $N - 1 < |\tau/T_0| \leq N(q-1)$, which is caused by a presence of the vector $\mathbf{0}_{1,N(q-2)}$ in (10). The dimensions of this region does not depend on the law of the phase-shift keying and on mismatching in frequency, but depends only on the off-duty factor q. The region of zero correlation at $0 < |\tau/T_0| \leq N - 1$ near the central peak of ACF, which is caused by the property of complementary sequences (2), takes a place only at complete filter matching with the echoes in frequency.

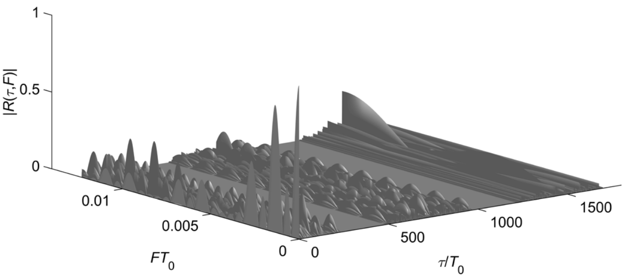

Fig. 1. Ambiguity function of the three-phase CAS

The ambiguity function section of CAS by the plane $\tau = 0$ has the envelope of the form $|\sin x/x|$ with the main lobe width on the zero level $2/(NT_0)$ and the internal comb structure. The spectrum combs are spaced from each other in F by the value $1/(qNT_0)$. The comb width on the zero level is $2/(pqNT_0)$, and the total number of combs within the main lobe of the amplitude-frequency spectrum envelope of the CAS CE square is equal to $2q - 1$. The side lobes with the width $1/(pqNT_0)$ on the zero level occur between combs and the total number of side lobes is equal to $p - 2$.

ZACZ exists only at complete filter matching with echoes in the Doppler frequency [14]. At mismatch ΔF in the frequency in ZACZ near the main ACF peak, the side lobed appear, the greatest of them is compared in the level with the maximal side lobe outside ZACZ at $\Delta F = 0.3/pqNT_0$.

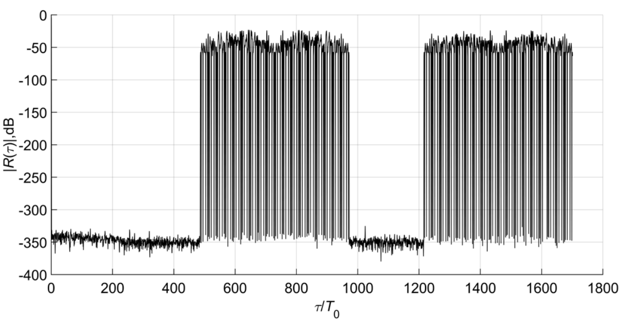

Fig. 2. Autocorrelation function of the three-phase CAS

CAS is assumed to use at radar target tracking in resolution modes for accurate measurements (specification) of the Doppler frequency, when the target rough estimation is already known from the preliminary target detection. At that, the compression device for CAS should be multi-channel in the Doppler frequency with the necessary channel width.

IV. THE COMPRESSION DEVICE OF POLYPHASE COHERENT ADDITIONAL SIGNALS

For compression of the coherent pulse sequence, the correlation-filtering processing is usually used, at which the reflected signal modulation is first removed and then, with the help of the fast Fourier transform (FFT), the Doppler frequency is defined [4, 6].

The structural circuit of the compression device for polyphase CAS is shown in Fig. 3 and represents the equivalent structural diagram of the matched filter of polyphase CAS at the known Doppler frequency or the equivalent structural diagram of the matched filter in the single frequency channel.

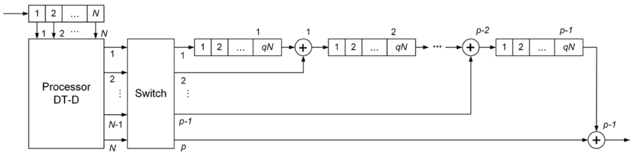

Fig. 3. Structural diagram of the single-channel compression device of CAS

The compression device consists of the input register on N memory cells, the processor of the discrete D-transform with N inputs and N outputs, the switching block, $p-1$ similar shift registers on qN memory cells and $p-1$ similar summation units of complex number, where q is the off-duty factor, $N = p^k$ is the D-code length.

The switching block performs connection of p from its N inputs with p outputs according to expression (4), i.e., in accordance with the row numbers, in which the p-paired D-codes are situated.

At $p = 2$, we obtain the compression device of bi-phase (binary) CAS.

The basing element of this device is the processor of discrete D-transform (the processor DT-D), which operation algorithm is described by the following mathematical expression:

$$\mathbf{G}_{N,1} = \mathbf{D}_N \mathbf{S}_{1,N}^T, \tag{15}$$

where $\mathbf{S}_{1,N}$ is the vector of input signal samples of the discrete D-transform; T is operation of the vector transposition.

Substituting (5) in (15), we obtain:

$$\mathbf{G}_{N,1} = \mathbf{H}_N \mathbf{d}_N \mathbf{S}_{1,N}^T. \tag{16}$$

It is known that the VC-Kronecker matrix can be factorized by the Good method [19], i.e., the discrete D-transform (16) can be reduced to FFT in the basis of VC-Kronecker function system (FTVC), which has the form:

$$\mathbf{G}_{N,1} = \mathbf{C}_{k_N} \mathbf{C}_{k-1_N} \ldots \mathbf{C}_{j_N} \ldots \mathbf{C}_{1_N} \mathbf{d}_N \mathbf{S}_{1,N}^T, \tag{17}$$

$$\mathbf{C}_{k_N} = \mathbf{E}_p \otimes \mathbf{1}_p \otimes \ldots \otimes \mathbf{1}_p;$$
$$\mathbf{C}_{k-1_N} = \mathbf{1}_p \otimes \mathbf{E}_p \otimes \ldots \otimes \mathbf{1}_p;$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\mathbf{C}_{j_N} = \mathbf{1}_p \otimes \ldots \otimes \mathbf{E}_p \otimes \ldots \otimes \mathbf{1}_p;$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\mathbf{C}_{1_N} = \underbrace{\mathbf{1}_p \otimes \ldots \otimes \mathbf{1}_p \otimes \mathbf{E}_p}_{k},$$

where $\otimes$ is the operation of the Kronecker product; $\mathbf{1}_p$ is the unitary matrix with dimensions $p \times p$; $\mathbf{E}_p$ is the matrix of discrete exponential functions (DEF) with dimensions $p \times p$.

From expression (17), it follows that the DT-D processor in the diagram in Fig. 3 can be replaced by the FTVC processor with addition of weight coefficients (the matrix $\mathbf{d}_N$ in expression) in the processor input, which are elements of the first row of MAS $\mathbf{D}_N$. Then the structural diagram of the compression device of the polyphase CAS will have the form presented in Fig. 4.

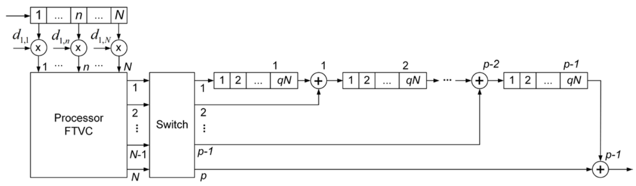

Fig. 4. Structural diagram of the single-channel compression device of CAS with FTVC

For $p = 2$, the compression algorithm of polyphase CAS presented in the form of the structural diagram in Fig. 4 is transformed into the compression algorithm of binary CAS, and the FTVC processor is transformed into the processor of the fast Walsh transform.

V. THE MULTI-CHANNEL COMPRESSION DEVICE OF POLYPHASE COHERENT ADDITIONAL SIGNALS

In [20] the multi-channel compression device for CAS is described, which allows simultaneous removal of the modulation of polyphase pulse signals encoded by complimentary sequences and determine the Doppler frequency in restricted Doppler frequency range according to preliminary target detection. This device consists of the processor of fast D-transform-Fourier (FT-D-F_K, K is a number of used frequency channels), using the combination of the FFT algorithms in basis-matrices of additional sequences and DEF by means of bit-by-bit multiplication of each MAS row with dimensions of $N \times N$. The MAS matrix here is the matrix of pulse characteristics of CAS pulses. The block matrix with dimension $NK \times N$ obtained at that represents the set of matrices of pulse characteristics on K different frequencies, i.e., rows of DEF matrix play the role of frequency channels. In the FT-D-F_K algorithm, the MAS matrix itself is factorized.

The multi-channel compression device of CAS described in

[20], can be built on the base of FTVC using (5). The structural diagram of such a device is presented in Fig. 5. The controllable local oscillator of the radar receiving device, according to rough estimation of the Doppler frequency $\hat{F}_{\mathrm{D}}$ obtained in the mode of target detection, retunes its frequency so as the value $\hat{F}_{\mathrm{D}}$ falls in the frequency range, which is covered by frequency channels of the compression device of CAS. To CAS compression, the processor of fast Vilenkin-Chrestenson-Fourier (FTVC-F$_K$, K is a number of used frequency channels) using of combination of FFT algorithms in the basis of the VC-Kronecker and DEF function system by means of the bit-by-bit multiplication of each row of the VC-Kronecker matrix with dimension $N \times L$, $L = N((p-1)q+1)$ (the matrix rows are prolonged by repeating of each element or zero padding), by each from K rows of DEF matrix with dimension $L \times L$.

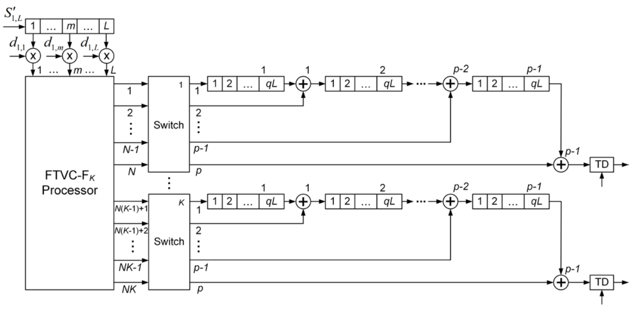

Fig. 5. Structural diagram of the multi-channel compression device of CAS with FTVC-F$_K$

CE of the signal reflected from the target can be written as:

$$\dot{S}(t,\hat{F}_{\mathrm{D}}) = \dot{S}(t)\exp(-j2\pi\hat{F}_{\mathrm{D}}t),$$

where $\dot{S}(t)$ is CAS CE (7). The signal is fed from the ADC outputs in the quadratic channels to the input shift register of the compression device. Having transferred from analog quantities to discrete ones, i.e., at $t \to t_n = (n-1)T_0$, $\hat{F}_{\mathrm{D}} \to \hat{F}_{\mathrm{D}_k} = (k-1)\Delta F$, $\Delta F = 1/LT_0$ is the mismatch between frequency channels, $n = 1, 2, ..., L$, $k = 1, 2, ..., K$, we obtain the echo CE in the discrete form:

$$\dot{S}(n,k) = \dot{S}(n)\exp\left(-j\frac{2\pi}{L}(n-1)(k-1)\right).$$

Hence, the DEF matrix should have the dimension $L \times L$.

Because MAS has N columns, and a number of MAS and DEF columns should be equal, the rows of MAS matrix and CAS pulses need to be prolonged, for example, owing to repeat of each samples by $L/N = (p-1)q+1$ times or to use zero padding.

Then the discrete D-transform-Fourier (DT-D-F$_K$) has a form:

$$\mathbf{G}_{KN,1} = \mathbf{D}'_{N,L}\begin{pmatrix}\mathbf{E}_{1_L} & \mathbf{E}_{2_L} & \ldots & \mathbf{E}_{k_L} & \ldots & \mathbf{E}_{K_L}\end{pmatrix}^T \mathbf{S}'^T_{1,L} = \\ = \mathbf{D}'_{N,L}\mathbf{E}_{KL,L}\mathbf{S}'^T_{1,L}, \quad (18)$$

where $\mathbf{S}'_{1,L}$ is the sample vector of the prolonged input signal; $\mathbf{E}_{k_L} = \mathrm{diag}\left\{W^{0(k-1)} \quad W^{1(k-1)} \quad \ldots \quad W^{(n-1)(k-1)} \quad \ldots \quad W^{(L-1)(k-1)}\right\}$, $W = \exp\left(-j\,2\pi/L\right)$, is a diagonal matrix with elements from the k-th row of the DEF matrix, which is included in the structure of the block matrix $\mathbf{E}_{KL,L}$; $\mathbf{D}'_{N,L}$ is MAS with prolonged rows.

Taking (5) into consideration, we obtain from (18) the discrete Vilenkin-Chrestenson-Fourier transform (DTVC-F$_K$):

$$\mathbf{G}_{KN,1} = \mathbf{H}'_{N,L}\mathbf{d}'_L\mathbf{E}_{KL,L}\mathbf{S}'^T_{1,L} = \mathbf{H}'_{N,L}\mathbf{E}_{KL,L}\mathbf{d}'_L\mathbf{S}'^T_{1,L}, \quad (19)$$

where $\mathbf{H}'_{N,L}$ is the VC-Kronecker matrix with prolonged rows; $\mathbf{d}'_L = \mathrm{diag}\{\mathbf{D}'^1_{1,L}\}$ is the diagonal matrix with elements from the first rows of the $\mathbf{D}'_{N,L}$ matrix.

From [19] we know that column repeating of the VC-Kronecker matrix with dimension $N \times N$, $N = p^k$, p^l times is equivalent to row decimation of the VC-Kronecker matrix with dimension $L \times L$, where $L = p^{k+l}$ to the rectangular matrix with dimension $N \times L$. In other words, in (19), the $\mathbf{H}'_{N,L}$ matrix can be replaced by the VC-Kronecker matrix with dimension $L \times L$, we can factorize it, and necessary values of the signal spectrum can be obtained from the known decimated row numbers.

Thus, from (19) we have the expression for FTVC-F$_K$:

$$\mathbf{G}_{KL,1} = \mathbf{H}_L\mathbf{E}_{KL,L}\mathbf{d}'_L\mathbf{S}'^T_{1,L} = \mathbf{C}_{k+l_L}\mathbf{C}_{k+l-1_L}\ldots\mathbf{C}_{j_L}\ldots\mathbf{C}_{1_L}\mathbf{E}_{KL,L}\mathbf{d}'_L\mathbf{S}'^T_{1,L}, \quad (20)$$

where $\mathbf{C}_{j_L}$ is the weakly-filled matrix from the Good factorization algorithm (17), $j = 1, 2, ..., k+l$.

At $p = 2$, FTVC-F$_K$ transforms into the fast Walsh-Fourier (FTW-F$_K$) and $L = N(q+1)$ [8]. To achieve the maximal FFT effectiveness, the DEF matrix dimension should be equal to the power of 2. For this, we introduce the quantity $l = \lceil \log_2(q+1) \rceil$, where $\lceil x \rceil$ is the operation of the number x rounding to the larger value. Then $L = 2^l N = 2^{k+l}$. The rectangular matrix in (19)

$$\mathbf{H}'_{N,L} = \left\| h_{i,\lfloor (m-1)/2^l \rfloor+1} \right\|_{i=1,m=1}^{N,L},$$

where $\left\| h_{i,n} \right\|_1^N = \mathbf{H}_N$ is the Hadamard matrix with dimension $N \times N$, $\lfloor x \rfloor$ is the operation of integer part extraction of the number x, which is obtained from the Hadamard matrix $\mathbf{H}_L$ with dimension $L \times L$ in (20) be means of its rows decimation. The diagonal matrix in (19) and (20) is

$$\mathbf{d}'_L = \mathrm{diag}\left\{d_{1,1} \quad d_{1,1} \quad \ldots \quad d_{1,\lfloor (m-1)/2^l \rfloor+1} \quad \ldots \quad d_{1,N} \quad d_{1,N}\right\},$$
$$m = 1, 2, ..., L,$$

where $\left\| d_{1,n} \right\|_1^N = \mathbf{D}^1_{1,N}$. The processor FTW-F$_K$ has NK outputs (decimated rows). The first N outputs represent the result of multiplication of the pulse characteristics matrix (MAS) by the processor input signal samples in the first frequency channel, the second N outputs – in the second frequency channel and so on, the last N outputs – in the K-th frequency channel. The switching block in each frequency channel performs the connection of two from its N inputs with two outputs in accordance with the rows number, in which the paired or adjacent sequences of the D-code are located. In adder of each channel, the summation of ACF samples of CAS pulses is performed owing to the samples delay of the ACF in the shift register by the repetition period of pulses qL. According to the number of the threshold device (TD in Fig. 5) ($k = 1, 2, ..., K$), in which the threshold is exceeded, the Doppler frequency shift ($\hat{F}_{D_k} = (k-1)\Delta F$) is determined.

VI. Conclusions

The method of polyphase radar signal with ZACZ is offered in this paper. At that, this signal represents the train of p PCSK pulses encoded by p-ary complementary sequences and is called the coherent additional signal. ZACZ takes place only at complete matching of the filter with echoes in the Doppler frequency. At mismatch in frequency, the level of the ACF main peak decreases and side lobes appear in ZACZ. The multi-channel compression device of this signal is studied. It is shown that the method of D-code formation allows utilization of algorithms of the fast transform for signal compression in the matched filter.

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