Applying Game Theory to Mortgage Framework

Abdul Shukur Abu Hasan[1], **Nurulhuda Firdaus Mohd Azmi**[1], **Nilam Nur Amir Sjarif**[1], **Haslina Md Sarkan**[1], **Yazriwati Yahya**[1] **and Suraya Ya'acob**[1]

[1]Advanced Informatics Department, Razak Faculty of Technology and Informatics, UTM, Kuala Lumpur, Malaysia.

E-mail: huda@utm.my

Abstract. To some, the retail banking market is considered an oligopoly. This is not healthy for a country's economy and its people because it is important for competition to thrive in the banking industry. Any form of market failure or anti-competitive behaviour affects productive efficiency, consumer welfare and economic growth. With the introduction of the Base Rate (BR) to replace the Base Lending Rate (BLR), Bank Negara Malaysia (BNM) seeks to inject transparency into the mortgage framework. Under this system, banks will have to disclose their margins and are not allowed to loan below the BR just to attract customers and boost loan growth. This new framework will allow customers to make better financial decisions whilst having little impact on borrowers. Even though this only applies to personal home loans, it still poses a challenge to competing banks. Those with a seemingly strong niche in consumer financing will be able to offer more attractive and competitive BR leading to better Effective Lending Rates (ELR). This paper proposes using Game Theory (GT) as a method to predict probable outcomes in strategic interactions for BR revision. Monte Carlo Simulation (MCS) will then provide a means of weighing the probabilities of the contributing factors. With the ability to anticipate the competition's next moves in a quantifiable manner, strategists will be better prepared in understanding and navigating the complex banking system.

1. Introduction

In 2015, Bank Negara Malaysia (BNM) replaced the Base Lending Rate (BLR) with the Base Rate (BR) for new residential property loans in a bid to promote transparency and healthy competition. Unlike the BLR, banks set their own BR according to BNM's guideline and are free to revise them at their discretion. The challenge is to set a BR at a competitive rate and still make a healthy profit. This paper proposes Game Theory (GT) as a method of determining whether to increase, maintain or reduce the BR in relation to the actions of other banks. Laffont [4] states that non-cooperative GT has revolutionized the industrial revolution with empirical work making use of GT to study oligopolistic market. Implementing major strategy is always subjected to uncertain reactions from competitors. Counter-moves from competitors may be an important determining factor for a successful strategy. According to Sagi [1], companies within social and economic domains share a common trait due to the existence of conflicting interest. By pre-empting the reactionary moves of competitors, GT helps the decision-making process by shaping the competitive dynamics. In their paper, Li et al. [9] applied MCS before GT to optimize evacuation clearance time while this paper seeks to apply MCS after all possible actions that could be taken by the competitor have been identified. Just as in most mathematical process, changing priorities in calculations applies to different problems and will usually result in totally different outcomes. In the case of BR, there are only three possible decisions and they are: Increase, Maintain and Reduce the BR with results ranging from Gain, Loss and Maintain of market shares and revenues.

This paper proposes dividing the process of managing these conflicts into 2 parts. Game Theory is first used to evaluate all possible moves and outcomes while Monte Carlo Simulation (MCS) is then used to quantify all the probabilities of the competitor's possible moves. The final section concludes the paper and discussed the future implementation.

2. Base Rate (BR)

To fully appreciate the importance of setting the right BR, it is necessary to break it down to its component parts. BR is a floating rate which could go up or down depending on the benchmark cost and liquidity of the bank plus the Statutory Reserve Requirement (SRR) set by BNM. Depending on their lending efficiencies, banks can review their BR anytime even if there are no changes to the Overnight Policy Rate (OPR) by BNM. Sub section below explained the component parts described in BR.

2.1. Statutory Reserve Requirement

This is the minimum level of cash reserves each bank is required to retain before lending out. This cash reserve is used to pay out depositors' withdrawals requests. If the central bank increases the SRR, the cost of lending increases and inherently affects the BR as well.

2.2. Overnight Policy Rate

An Overnight Policy Rate (OPR) is the interest rate set by BNM that will be charged for inter-bank borrowing. Depending on their customers' withdrawals and deposits, banks may face shortage or surplus of the cash-at hand known as SRR. When a bank experiences a shortage of cash, it will borrow from other banks that has a surplus or even BNM to maintain the SRR. These borrowings will be subjected to the interest set by the central bank in the form of OPR. For obvious reasons, the OPR is always lower than banks' offer rates to depositors.

2.3. Effective Landing Rate (ELR)

Adding the spread to the BR gives us the Effective Lending Rate (ELR). This is essentially the interest rate that a borrower agrees to when taking a loan.

From the explanations above, we can see that the BR and spread plays a big role in determining a bank's profitability. Since the BR is fixed and set by the bank, they will have to strategically decide the BR which is non-negotiable. Shi et al. [2] proposed an economic model to analyse prices between two competing providers and derive equations to calculate each expected pay-offs. Many may argue that this falls in the realm of risk analysis and Decision Theory is just as capable as dealing with the strategic interactions as Game Theory. However, Bier et al. [11] believe that this type of interactions which is partly controlled by regulators is better modelled in Game Theory than Decision Theory.

3. Game Theory (GT)

According to Slimani et al. [3], game theory is a mathematical tool and a modelling language that deals with strategic behaviour of rational competing players. The players are deemed to be rational because they only seek to defend their own interests by maximizing their profits and not make

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decisions solely to place the competitor at a disadvantage. This competitive interaction can be deemed a good approximation for larger commercial markets. In a classical economic theory of markets, individuals' trade is based on competing prices. They accepted given prices and do not consider that their actions or those of other individuals can result in different terms of trade. This paper seeks to show that GT can provide the necessary means to predict possible outcomes of such interactions.

GT can be divided into cooperative and non-cooperative games. An excellent example of a co-operative game is a paper by Chen et al. [7] where the players consist of local and central governments. Each government arm has separate interest but technically strive for a common public goal. The non-cooperative game is the more common form of GT. It usually applies to the business world where players act and behave to their own advantage. There is another facet to GT which is played with complete and incomplete information. This refers to the players having necessary information to base their assumptions on. This changes the dynamic aspect of the games. Theoretically, a game G is a social interaction between two or more (N) individuals who are called players. The players are assumed to be rational and make intelligent decisions solely in pursuit of their own objectives. The players choose their strategies, Ci, which yields payoffs to each player and these payoffs are assumed to be random. In GT, the payoff is measured in utility scale, u_i and each player will strive only to maximize the expected value of his payoff. 'Strategy' means the actions of the players and each player's objective is to maximize the expected value of his own payoff, which is measured in utility scale.

3.1. Prisoner's Dilemma

A simple and well-known example that illustrates Game Theory is called Prisoners' Dilemma. In this instance, 2 persons are suspected of committing a robbery. The investigating authorities have no definite proof and need to rely on their confessions, if they can get any. The robbers are interrogated individually in separate rooms. Each of them has two options and they are: confess or deny. However, the outcome of their fate does not rely only on their own actions but also that of their partner-in-crime. The payoff matrix can be seen in Figure 1 where, C represent confess and D represent deny.

		Prisoner 2	
		C	**D**
Prisoner 1	**C**	(6,6)	(3,6)
	D	(3,6)	(0.1,0.1)

Figure 1. The payoff matrix for prison dilemma

In the matrix as shown in Figure 1, the prisoners can expect to be remanded for a month should they both deny committing the robbery while each will be imprisoned for 6 years should they both confess. However, should one confess and the other denies, the person who confesses will get 3 years while the one who denied will get 6 years.

Meanwhile, Nash Equilibrium is attained when neither side has any incentive to change their strategy nor in this instance, the equilibrium is attained when they both deny. The dynamic of the game changes greatly when the payoff matrix is revised as Figure 2 below. Similarly, C represent confess and D represent deny. In Figure 2, confessing gets you a better payoff while there is much more to lose if you deny. The dynamics of the game has changed but the difficulty in decision-making remains. Nash Equilibrium is again attained when they both deny the crime. There may be instances in a game where each player has more than one move. Applying the same treatment to banks deciding on their BR decision may result in the following payoff matrix as illustrate in Figure 3. Although an increase in market share may seem a good thing, it hardly makes any sense if it affects the profitability of the bank. The matrix above is just an indication of all possible outcomes. It would be more useful if the degree of gains and losses can be predicted as well. This will allow banks to balance out the payoff between market share and profitability. Clearly Game Theory can be a good decision-making and prediction tool. To keep things simple, it is advised that the game is kept to 2 players. The process is repeated when comparison between other banks are needed.

		Prisoner 2	
		C	**D**
Prisoner 1	**C**	(3,3)	(0,10)
	D	(10,0)	(0.1,0.1)

Figure 2. The revised payoff matrix for prison dilemma

		Bank B		
		I	**M**	**D**
Bank A	**I**	(N, N)	(L, G)	(L, G)
	M	(G, L)	(N, N)	(L, G)
	D	(L, G)	(G, L)	(N, N)

Figure 3. Applying the payoff matrix of prison dilemma to banks

where;
I: Increase, M: Maintain, D: Decrease, N: No Change, G: Gains, L: Losses

4. Monte Carlo Simulation for Non-Cooperative Game Theory

Monte Carlo Simulation is a computerized mathematical technique in quantitative analysis. It allows decision-makers to account for risk with extreme possibilities ranging from the best-case to the worst-case scenarios and everything in between. Cordero et al. [5] used MCS as a data generation process to produce results that are adequately representative while Follain et al. [6] used MCS to generate predicted housing. MCS is not a new method in the banking industry. A typical Monte Carlo Simulation is an iterative process that repeatedly calculates a model up to hundreds and thousands of times with each time using different randomly-selected values. The simulation results are used to describe the likelihood or probability of reaching various points in the model. According to Min Chen et al. [8], it is especially suited to be used when it is unfeasible to compute an exact result with a deterministic algorithm. As an example, consider throwing a dart at Figure 4. From Figure 4, the probability of having the dart landing in the circle can be calculated as: Area of circle / square from Equation (1).

$$\frac{\pi r^2}{(2r)^2} = \frac{\pi}{4} = 0.785 \quad (1)$$

By running a simulation in R, we get the results as shown below. From the result, we can see that the more throws we simulate, the closer the probability gets to the mathematically calculated figure. This is good display of law of large numbers at work. Probability is a way to bracket the volatility of short-term forecasts using seemingly random data. A perfect example would be the forecast of a stock price. Suppose we use the following Equation (2) to model the price of a stock:

$$S_{t+1} = S_t\,(1 + \boldsymbol{\mu}\Delta \boldsymbol{t} + \boldsymbol{\sigma\varepsilon}\sqrt{\Delta \boldsymbol{t}}) \tag{2}$$

where, S_t : stock price in time t , $\boldsymbol{\mu}$: drift , $\Delta \boldsymbol{t}$: one period (1 day), $\boldsymbol{\sigma}$: volatility and $\boldsymbol{\varepsilon}$: random number between 0 and 1.

```
no.of.throws probability
          10         0.6
no.of.throws probability
         100        0.79
no.of.throws probability
        1000       0.778
no.of.throws probability
       10000      0.7843
```

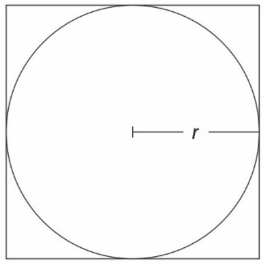

Figure 4. Throwing dart situation

In R code, S_t is set at RM100, $\boldsymbol{\mu}$ = 0.1, $\Delta \boldsymbol{t}$ = 20 and $\boldsymbol{\sigma}$ = 0.2. Using sets of random numbers as input, MCS produces the following outcome:

```
      1%       5%
67.91530 77.86578
```

This simply means that at 95% confidence the stock at time $t+1$ (S_{t+1}) is higher than RM77.87 and with 99% confidence is higher than RM67.92. From the example above, we can see that MCS can produce a range of probability values. S Siddiqui [10] performed a similar experiment using 5 years of historical data placed into a MCS block. This is necessary to provide a certain degree of precision. To understand the examples given better, we explore the theoretical concept of Monte Carlo Simulation as found below. Consider the following Equation (3):

$$\boldsymbol{\alpha} = \int_0^1 \boldsymbol{f(x)dx} \tag{3}$$

The expectation E|f(U)|, with U uniformly distributed between 0 and 1. Supposed we have a mechanism for drawing points $U_1, U_2, \ldots$ independently and uniformly from [0,1]. Evaluating the function f at n of these random points and averaging the results produces the Monte Carlo estimate;

$$\widehat{\boldsymbol{\alpha}}_n = \frac{\boldsymbol{1}}{\boldsymbol{n}} \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \boldsymbol{f}(\boldsymbol{U}_i) \tag{4}$$

If f is indeed integral over [0,1] then, by strong law of large numbers, $\widehat{\boldsymbol{\alpha}}_n \rightarrow \boldsymbol{\alpha}$ with probability 1 as n → ∞. If f is in fact integral and we set as,

$$\boldsymbol{\sigma_f^2} = \int_0^1 (\boldsymbol{f(x)} - \boldsymbol{\alpha})^2\, \boldsymbol{dx} \tag{5}$$

then, the error $\widehat{\boldsymbol{\alpha}}_n \rightarrow \boldsymbol{\alpha}$ in the Monte Carlo estimate is approximately normally distributed with mean 0 and standard deviation $\boldsymbol{\sigma_f} \,/\, \sqrt{\boldsymbol{n}}$, the quality of this approximation improving with increasing n. The parameter $\boldsymbol{\sigma_f}$ would typically be unknown in a setting in which $\boldsymbol{\sigma}$ is unknown, but it can be estimated using the sample standard deviation. The law of large numbers ensures that this estimate converges to the correct value as the number of draws increases. The central limit theorem provides information about the likely magnitude of the error in the estimate after a finite number of draws.

5. Conclusions

In this paper, a non-cooperative game is proposed to solve the problem of setting the Base Rate for a bank. In the game where Bank A and Bank B are known players, each bank will assess the probable result of both players' actions. The result can be further visualised in a tree as illustrate in Figure 6. From Figure 5, with all probabilities listed down, Monte Carlo Simulation is then used to quantify the values of e1 to e9. With this, it is expected that better and more calculated decisions can be made according to the probabilities of each path. The usefulness of GT has been criticized on a basis of three big impediments to its validity: first, it is argued that game theory focuses on games with asymmetric information when the individuals may discuss their steps before the game begins, although cannot come to an agreement beforehand, second, game theory proposes solutions that in real life do not have the same outcomes the theory predicts, third, that theory regards situations where players act rationally, although people cannot always have complete information and be rational.

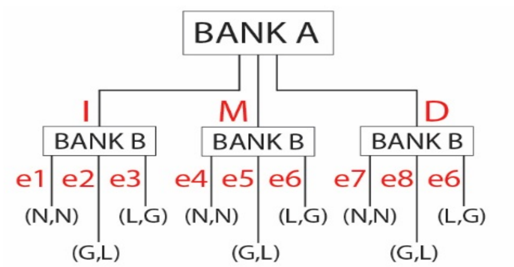

Figure 5. Non-Cooperative GT for base rate applied in Bank

While we have only looked at the possible results of increasing, maintain or lowering the BR, Goncharov et al. [12] took a step further in using MCS to compute mortgage rates. This can be examined in the future.

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