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IMPROVED MODEL OF POWDER BLEND COMPACTING IN A ROLL COMPACTOR

Abstract. A new mathematical model of mineral fertilizer compacting using a roll compactor is developed. This model is based on the transition to the values of stress tensor components averaged over the cross-sectional area of the powder mixture flow. To define these stresses, equations of equilibrium of the elementary layer determined in the mixture by two planes perpendicular to the flow direction are composed. To obtain relatively simple analytical relations in the calculations, the hypothesis of a power-law dependence of hydrostatic pressure on mixture density, accepted in the framework of the Johansen model, was used. In order to take into account changes in the mechanical characteristics of the mixture (angle of internal friction, coefficient of external friction, transverse strain coefficient) while compacting, we approximated the known experimental dependencies of the corresponding characteristics on the density. The inter-particle cohesion parameter was taken to be proportional to the hydrostatic pressure. The model allows calculating the gap between the rolls surfaces for a given initial bulk density and the required flake density. With the known gap value, the distribution of the axial average stresses in the powder mixture, the normal and shear stresses on the rolls' surfaces are determined. The results of the calculations of the rolls surface gap and the normal roll pressure diagram are compared with the experimental data given in the literature for the urea compacting process.

Keywords: compacting, powder mixture, roll compactor, equilibrium equations, stress tensor components, internal friction angle, inter-particle cohesion

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УСОВЕРШЕНСТВОВАННАЯ МОДЕЛЬ ПРЕССОВАНИЯ ПОРОШКОВОЙ СМЕСИ В ВАЛКОВОМ ПРЕССЕ

Аннотация. Разработана математическая модель прессования минерального удобрения на валковом прессе. Данная модель основана на переходе к усредненным по площади поперечного сечения потока порошковой смеси значениям компонент тензора напряжений. Для определения этих напряжений составляются уравнения равновесия элементарного слоя, выделяемого в смеси двумя плоскостями, перпендикулярными к направлению потока. Для обеспечения возможности получения относительно простых аналитических соотношений при расчетах использована принятая в рамках модели Йохансена гипотеза о степенной зависимости гидростатического давления от

плотности смеси. Для учета изменения механических характеристик смеси (угла внутреннего трения, коэффициента внешнего трения, коэффициента поперечной деформации) в процессе прессования производилась аппроксимация известных экспериментальных зависимостей соответствующих характеристик от плотности. Параметр межчастичного сцепления принимался пропорциональным гидростатическому давлению. Модель позволяет вычислить значение зазора между поверхностями валов при заданных значениях исходной насыпной плотности смеси и требуемой плотности плитки. При известном значении зазора устанавливаются распределения осевых усредненных напряжений в порошковой смеси, нормального и сдвигового напряжений на поверхности валов. Результаты расчетов зазора между поверхностями валов и эпюры нормального давления на вал сопоставлены с приведенными в литературных источниках экспериментальными данными для процесса прессования мочевины.

Ключевые слова: прессование, порошковая смесь, валковый пресс, уравнения равновесия, компоненты тензора напряжений, угол внутреннего трения, межчастичное сцепление

Introduction. The modern stage of technological equipment development is characterized, in particular, by increased requirements for the quality of manufactured products. In addition, in a competitive environment, the role of premier choice of design and process values, providing the required level of quality while meeting the reliability criterion and reducing the costs, is high. This indicates the relevance of further improvement of calculation methods that allow predicting the power and energy equipment load during the technological processes.

At present, the technology of dry granulation of powder bulk materials by compacting without use of binding agents is widely used in production of mineral fertilizers. The most important and expensive equipment in the process lines, however, is a roll compactor (Figure 1), that works as follows: the initial bulk material is fed through the feed system in the area between two rotating against each other rolls, where the material is compacted and formed into sheet (flakes), which are further crushed, classified and additionally treated. Further optimization of process parameters for compacting the bulk materials and improvement of roll compactors' design is an important scientific and technical task, which cannot be successfully solved without relevant mathematical modeling of compacting processes of powder bulk materials.

When describing the process of mineral fertilizers compacting on roll compactors [1], the simplified Johansen model [2, 3] is currently mainly used. The main disadvantages of this model are the following.

1. A plane stress state of a powder mixture is considered. The axial component of the stress tensor corresponding to the direction of the compacting rolls is neglected.

2. The cohesion between the particles in the powder mixture is not taken into account. In the classical Johansen model, the limit state of a material is defined by one characteristic, the angle of internal friction. In later modifications of this model [4] the angle between the coordinate axis and the slip line is also introduced. In this case, along with the minimum (residual) angle of internal friction, the effective angle of internal friction is included in the limit state equation. However, even in the paper [4] the calculations are made without taking into account the cohesion of the powder mixture particles.

3. The contact area between the powder mixture and the rollers surfaces is divided into a feed and sealing area. This does not consider the final

Figure 1. Roll compactor PVP 1000×650MG

extrusion area in which the already formed flakes are displaced. By neglecting this area, the calculated value of the longitudinal (in the direction of the mixture movement) stress, when the flake comes out of rolls contact, reaches its maximum value. However, in the absence of extrusion, the cross-sectional average of this stress should be zero.

4. The change in mixture characteristics with a change in density is not taken into account. It was experimentally found out [5] that during compacting such characteristics as internal friction angle and external friction coefficient (with the roll surface) change by a factor of two or more.

The noted weaknesses of the Johansen model are eliminated in the frame of the Katashinsky–Stern model, described in detail, in particular, in the paper [6]. However, this refined model also has a number of weaknesses.

1. The real region of space occupied by the powder mixture during compacting is replaced by a prismatic area whose width is equal to the gap between the roll surfaces. This assumption allows the model to be used only in the range of small angles of powder capture.

2. In the sealing area, using Katashinsky–Stern model, Coulomb's law for shear stress on rolls surface is assumed to be fulfilled. At the same time, the sealing area is characterized by the cohesion of powder particles to the roll surface and the shear stress within this area changes its direction.

In relation to the above, *the aim of this study* is to develop a combined mathematical model of a roll compactor, which would take advantage and eliminate the disadvantages of the existing calculation methods.

Description of the calculation methods. The operation of a roll compactor is schematically shown in Figure 2. The following designations are adopted in the figure: R – roll radius; p_0 – feed pressure; h_s – gap between roll surfaces; ω – angular velocity of rolls; θ – current angle, varying from 0 to α; α – angle defining the beginning of feed area; γ – angle defining the boundary between feed and sealing areas; β – angle defining the boundary between sealing and extrusion areas. Roller-compaction is described in Cartesian coordinates. The x-axis is vertical, equispaced from the surfaces of neighbouring rolls and directed opposite to the movement of the powder mixture. The y-axis is horizontal and runs through the centres of the neighbouring rolls. The z-axis is perpendicular to the pattern plane. The thickness of the rolls (dimension along the z-axis) is H.
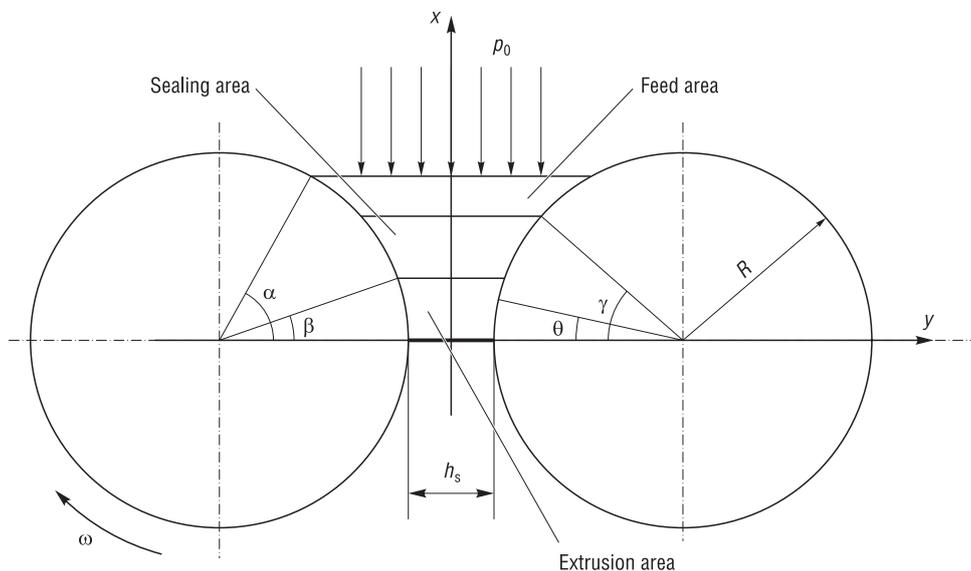

Figure 2. Operation mode of a roll compactor

The limit state of the mixture for all areas will be described by the equation [5]

$$\frac{1}{4}\left(\sigma_y - \sigma_x\right)^2 + \tau_{xy}^2 = \frac{1}{4}\sin^2\delta\left(\sigma_y + \sigma_x + 2k\mathrm{ctg}\delta\right)^2. \tag{1}$$

Here σ_x, σ_y are the axial components of the stress tensor; τ_{xy} is the shear component of the stress tensor; δ is effective angle of internal friction; k is the inter-particle cohesion parameter.

The condition of plain strain is also fulfilled for all areas. Namely, the axial strain corresponding to the z-axis $(\varepsilon_z = 0)$ and the shear strains in the xz and yz planes are equal to zero.

The axial component of the stress tensor z is related to the values of σ_x and σ_y

$$\sigma_z = \mu\left(\sigma_x + \sigma_y\right). \tag{2}$$

Here is the transverse strain coefficient (Poisson's ratio) of the material. In the Katashinski–Stern model, the average axial stress is expressed as follows

$$\sigma = \frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right) = \frac{1}{3}(1+\mu)(\sigma_x + \sigma_y). \tag{3}$$

This value corresponds to the actual hydrostatic pressure.

To keep the elegance of analytical relationships derived from the Johansen model, we use the approach proposed in paper [7]. Namely, we determine a thin layer of powder mixture by two planes parallel to the yz plane and placed at a small distance $dx = R \cdot d\theta$ from each other. We replace the real values of the axial stresses σ_x, σ_y with their y-coordinate averaged values σ_{xav}, σ_{yav}, which will not depend on y. Let us use the averaged stress components in generating the equilibrium equations of the elementary layer. Without repeating the transformations described in detail in the paper [7], we give only two relations obtained in this paper. The equilibrium equation in projections on the x-axis appears as the equality

$$\frac{d\sigma_{xav}}{d\theta}(1+s-\cos\theta) + \sigma_{xav}\sin\theta = p\sin\theta - \tau_f\cos\theta. \tag{4}$$

Where p, τ_f are normal and shear stress acting on the layer from the surface side of the roll; $s = h_s/(2R)$ – design value introduced for brevity of further entries.

The equilibrium equation for the part of elementary layer in projections onto the y-axis can be transformed to the form

$$\sigma_{yav} = p + \tau_f \operatorname{tg}\theta. \tag{5}$$

In the paper [7] the transition to the averaged components of the stress tensor is not explicitly specified. However, the equations (4), (5) are composed exactly for the averaged components, which do not depend on the y-coordinate.

The structure being considered and its loading mode are symmetrical against the xz plane. Consequently, the shear stress value τ_{xy} averaged over the y-coordinate is zero $\tau_{xyav} = 0$. Using jointly relations (1) and (3) for the averaged axial components of the stress tensor we obtain

$$\sigma_{xav} = \frac{3(1-\sin\delta)}{2(1+\mu)}\sigma_{av} - k\cos\delta, \quad \sigma_{yav} = \frac{3(1+\sin\delta)}{2(1+\mu)}\sigma_{av} + k\cos\delta. \tag{6}$$

Equations (4)–(6) will be used for all three areas in the field where the powder is in contact with the rolls. Let us consider each area separately.

Feed area. In the feed area the density of the powder mixture varies slightly and can be assumed equal to the initial bulk density ρ_b. Consequently, the mixture characteristics δ, μ and k, which in general depend on density, can be assumed constant. In the feed area, Coulomb's law is true for the shear stress τ_f on the roll surface

$$\tau_f = fp. \tag{7}$$

Here f is the coefficient of friction between the powder mixture particles and the roll surface. The value of f generally depends on the density of the mixture. For the feed area, the friction coefficient takes the value f_b corresponding to the density ρ_b.

For characteristics δ, μ and f of most known mineral fertilizers the experimental dependence on density is known and given, in particular, in the paper [5]. The dependence on average axial stress is more often used [5] for the inter-particle cohesion parameter

$$k = c\sigma + c_0. \tag{8}$$

Here c, c_0 are constants determined by approximation of the experimental dependencies. If baking and caking effects can be neglected for a given material, then $c_0 = 0$. Later on we will only use the constant c.

In the feed area, the k value is taken as constant and corresponding to the maximum value of the average axial stress σ_γ for this area, which is achieved at $\theta = \gamma$

$$k_b = c\sigma_\gamma. \tag{9}$$

Using relations (5)–(7) and making mathematical transformations, we obtain the differential equation for the averaged hydrostatic pressure

$$\frac{d\sigma_{av}}{d\theta} = \sigma_{av} Q_{1b}(\theta) + w_b \sigma_\gamma Q_{2b}(\theta). \tag{10}$$

In equation (10), for brevity, the functions of angle θ

$$
\begin{aligned}
Q_{1b}(\theta) &= \left(\frac{(1+\sin\delta_b)(\sin\theta - f_b\cos\theta)}{(1-\sin\delta_b)(1+f_b\operatorname{tg}\theta)} - \sin\theta \right) \frac{1}{1+s-\cos\theta}, \\
Q_{2b}(\theta) &= \left(\frac{\sin\theta - f_b\cos\theta}{1+f_b\operatorname{tg}\theta} + \sin\theta \right) \frac{1}{1+s-\cos\theta},
\end{aligned}
\tag{11}
$$

and combination of constants are given

$$w_b = \frac{2(1+\mu_b)c\cos\delta_b}{3(1-\sin\delta_b)}.$$

Here δ_b, μ_b are the values of the powder mixture characteristics corresponding to the bulk density ρ_b.

According to the first equality (6), the value of the average axial stress $\sigma\alpha$ at angle $\theta = \alpha$ is given by the relation

$$\sigma_\alpha = \frac{2(1+\mu_b)(p_0 + c\sigma_\gamma\cos\delta_b)}{3(1-\sin\delta_b)}. \tag{12}$$

The value of the angle α limiting the contact area between the powder mixture and the roll surface is determined from the continuity of the pressure gradient, which is zero outside the contact area

$$p_0 Q_{1b}(\alpha) + c\sigma_\gamma\cos\delta_b(Q_{1b}(\alpha) + Q_{2b}(\alpha)) = 0. \tag{13}$$

To determine the angle α from this equation with a non-zero value of the coefficient c and an unknown value σ_γ is impossible. The value α will be determined during considering the sealing area.

The solution of differential equation (10) is

$$\sigma_{av}(\theta) = \left[\sigma_\alpha - w_b\sigma_\gamma \int_\theta^\alpha Q_{2b}(\zeta) \exp\left(\int_\zeta^\alpha Q_{1b}(\eta)d\eta \right) d\zeta \right] \exp\left(-\int_\theta^\alpha Q_{1b}(\eta)d\eta \right). \tag{14}$$

Knowing the function $\sigma_{av}(\theta)$, we determine the angle θ dependence of the averaged axial stresses on the formulas (6) and (2). The material characteristics δ, μ, k take the values δ_b, μ_b and k_b. Using relations (5) and (7) together for the values p and τ_f we obtain

$$p(\theta) = \frac{\sigma_{yav}(\theta)}{1+f_b\operatorname{tg}\theta}, \quad \tau_f(\theta) = f_b p(\theta). \tag{15}$$

Sealing area. In the sealing area, the mixture density increases from ρ_b to the final flake density ρ_d. In order to keep the possibility of obtaining relatively simple analytical relations for this area, we use one of the basic assumptions of the Johansen model relating to the power law dependence of the average axial stress on the density

$$\sigma \sim \rho^K. \tag{16}$$

Here K is the compaction ratio in the Johansen model, which is a characteristic of the powder mixture. Using assumption (16), a differential equation [4, 7] can be derived for the average axial stress in the sealing area

$$\frac{d\sigma_{av}}{d\theta} = \sigma_{av} Q(\theta). \tag{17}$$

An angle function θ is introduced here

$$Q(\theta) = K\operatorname{tg}\theta\,\frac{1+s-2\cos\theta}{1+s-\cos\theta}. \tag{18}$$

The boundary condition for the average axial stress in the sealing area follows from the requirements of stress continuity σ and gradient $d\sigma/d\theta$ in the transition from the feed area to the sealing area ($\theta = \gamma$). Initially the gradient continuity condition is drawn up

$$Q_{1b}(\gamma) + w_b Q_{2b}(\gamma) = Q(\gamma). \tag{19}$$

Solving this non-linear equation, we determine the angle γ. According to the function (14) taking into account relations (12) and (13) at a known value γ we set up expression for voltage σ_γ

$$w_b\left(1 - \int_\gamma^\alpha Q_{2b}(\zeta)\exp\left(\int_\zeta^\alpha Q_{1b}(\eta)d\eta\right)d\zeta\right) - \exp\left(\int_\gamma^\alpha Q_{1b}(\eta)d\eta\right) = w_b\left(1 + \frac{Q_{2b}(\alpha)}{Q_{1b}(\alpha)}\right). \tag{20}$$

From this non-linear equation we find out the angle α. Then from the equation (13) we find out σ_γ

$$\sigma_\gamma = -\frac{p_0 Q_{1b}(\alpha)}{c\cos\delta_b\left(Q_{1b}(\alpha)+Q_{2b}(\alpha)\right)}. \tag{21}$$

Knowing γ and σ_γ, the solution of the differential equation (17) is as follows

$$\sigma_{av}(\theta) = \sigma_\gamma \exp\left(-\int_\theta^\gamma Q(\eta)d\eta\right). \tag{22}$$

With a known function $\sigma_{av}(\theta)$, the ratio (16) allows determining the dependence of the mixture density on the angle θ

$$\rho(\theta) = \rho_b\left(\frac{\sigma_{av}(\theta)}{\sigma_\gamma}\right)^{1/K}. \tag{23}$$

When using relations (2) and (6) to determine the averaged axial stresses in the sealing area, the density functions δ_ρ, μ_ρ, f_ρ must be substituted for the specific values of characteristics δ, μ, f. In accordance with the equality (23) these dependencies can be presented by functions of angle θ: $\delta_\rho = \delta(\rho) = \delta(\rho(\theta)) = \delta(\theta) = \delta_\theta$.

The function corresponding to the inter-particle cohesion parameter k_θ is defined according to (8) at $c_0 = 0$: $k_\theta = k(\theta) = c\sigma_{av}(\theta)$. Thus, in the sealing area the axial stresses are given by the relations

$$\sigma_{xav}(\theta) = \sigma_{av}(\theta)\left(\frac{3(1-\sin\delta_\theta)}{2(1+\mu_\theta)} - c\cos\delta\right),\quad \sigma_{yav}(\theta) = \sigma_{av}(\theta)\left(\frac{3(1+\sin\delta_\theta)}{2(1+\mu_\theta)} + c\cos\delta\right). \tag{24}$$

As noted above, Coulomb's law for the shear surface stress τ_f is not met in the sealing area. Therefore, we use relations (4) and (5) to determine the surface distributed forces p and τ_f with known functions $\sigma_{xav}(\theta)$ and $\sigma_{yav}(\theta)$. After performing mathematical transformations we obtain

$$\tau_f(\theta) = (\sigma_{yav}(\theta) - \sigma_{xav}(\theta))\sin\theta\cos\theta - \frac{d\sigma_{xav}(\theta)}{d\theta}(1+s-\cos\theta)\cos\theta,$$
$$p(\theta) = \sigma_{yav}(\theta) - \tau_f(\theta)\operatorname{tg}\theta. \tag{25}$$

The roll compactor is designed in such a way that ensures the required flake density value ρ_d. Therefore, the value of the compaction factor $z = \rho_d/\rho_b$ z is a given one.

Knowing z and using the relation (16), the value of the average axial stress at the transition to the extrusion area ($\theta = \beta$) can be determined

$$\sigma_\beta = \sigma_\gamma z^K. \tag{26}$$

To ensure the continuity of the mixture flow, an inverse relationship must be maintained between the density ρ_θ in a given cross-section and the elementary volume V_θ, corresponding to that cross-section

$$\rho_\theta \sim 1/V_\theta. \tag{27}$$

Here the elementary volume is defined by the ratio

$$V_\theta = (h_s + 2R(1 - \cos\theta))Hdx = (1 + s - \cos\theta)2R^2 H \cos\theta d\theta. \tag{28}$$

We use the ratio (27) taking into account (28) for the cross-sections $\theta = \gamma$ (ρ_b) and $\theta = \beta$ (ρ_d)

$$\frac{\rho_b}{\rho_d} = \frac{(1 + s - \cos\beta)\cos\beta}{(1 + s - \cos\gamma)\cos\gamma}. \tag{29}$$

Let us solve the equation (29) with respect to the angle β

$$\beta = \arccos\left[\frac{1+s}{2}\left(1 + \sqrt{1 - \frac{4\cos\gamma}{z(1+s)}\left(1 - \frac{\cos\gamma}{1+s}\right)}\right)\right]. \tag{30}$$

Formulas (26) and (30) can be considered as boundary conditions for the extrusion area.

Extrusion area. Within this area, the density of the object to be compacted remains unchanged and is ρ_d. Therefore, the characteristics δ, μ, f and k take the values δ_d, μ_d, f_d and k_d corresponding to ρ_d. In this case $k_d = c\sigma_\beta$. Coulomb's law is fulfilled for the contact stresses p and τ_f in the extrusion area

$$\tau_f = -fp. \tag{31}$$

The minus sign in the last equation reflects the change in direction of the shear contact stress compared to the feed area.

Performing the same transformations as for the feed area, we obtain a differential equation similar to (10)

$$\frac{d\sigma_{av}}{d\theta} = \sigma_{av}Q_{1d}(\theta) + w_d\sigma_\beta Q_{2d}(\theta). \tag{32}$$

The functions $Q_{1d}(\theta)$ and $Q_{2d}(\theta)$ are defined by the relations

$$Q_{1d}(\theta) = \left(\frac{(1 + \sin\delta_d)(\sin\theta + f_d\cos\theta)}{(1 - \sin\delta_d)(1 - f_d tg\theta)} - \sin\theta\right)\frac{1}{1 + s - \cos\theta},$$

$$Q_{2d}(\theta) = \left(\frac{\sin\theta + f_d\cos\theta}{1 + f_d tg\theta} + \sin\theta\right)\frac{1}{1 + s - \cos\theta}. \tag{33}$$

Similarly to the feed area, a combination of characteristic values is also entered

$$w_d = \frac{2(1 + \mu_d)c\cos\delta_d}{3(1 - \sin\delta_d)}. $$

The solution of equation (32) taking into account the boundary condition $\sigma_{av}(\beta) = \sigma_\beta$ is

$$\sigma_{av}(\theta) = \sigma_\beta\left[1 - w_d\int_\theta^\beta Q_{2d}(\zeta)\exp\left(\int_\zeta^\beta Q_{1d}(\eta)d\eta\right)d\zeta\right]\exp\left(-\int_\theta^\beta Q_{1d}(\eta)d\eta\right). \tag{34}$$

As noted above, in the absence of extrusion, the averaged axial stress σ_{xav}, when the flake leaves the contact with the roll $(\theta = 0)$, is zero. Consequently

$$\sigma_0 = \sigma_{av}(0) = w_d\sigma_\beta.$$

By writing down the function (34) for $\theta = 0$, we obtain the following equation (34)

$$\int_0^\beta Q_{2d}(\zeta)\exp\left(\int_\zeta^\beta Q_{1d}(\eta)d\eta\right)d\zeta + \exp\left(\int_0^\beta Q_{1d}(\eta)d\eta\right) = \frac{1}{w_d}. \tag{35}$$

If the density of the flake is given, then the equation (35) is used to determine the design parameter s that provides the required compaction factor z.

An example of the calculation method application. As an example of the application of the developed mathematical model we consider the process of compacting the urea with rolls of radius $R = 0.04$ m and thickness $H = 0.03$ m. In the absence of additional feed pressure the value of p_0 is taken as equal to 0.1 MPa. For this process, the experimental data are given in paper [5].

The experimental dependencies [5] of the internal friction angle tangent tgδ, transverse strain factor μ and external (with roll surface) friction coefficient f on powder mixture density in the range from $\rho_b = 1.0$ kg/m^3 to $\rho_{max} = 2.1$ kg/m^3 will be approximated by the function

$$\mu_\rho = \mu_b + A\left(\frac{\rho - \rho_b}{\rho_{max} - \rho_b}\right)^{0.5} + B\left(\frac{\rho - \rho_b}{\rho_{max} - \rho_b}\right). \tag{36}$$

Here A, B are least-squares approximation coefficients. Ratios similar to (36) are also written down for tgδ and f.

The results of the approximation are shown in the Table 1. The table 1 also shows the values of correlation coefficients χ of the experimental values and the results of using the functions (36) to assess the accuracy.

The coefficient determining the growth rate of the inter-particle cohesion parameter is $c = 0.29$. The compaction index K for the Johansen model is determined by approximation of the experimental dependence of density on average axial stress $\rho(\sigma)$, given in [5], in the range of densities from ρ_b to ρ_{max}. The value was $K = 8.32$ with correlation coefficient $\chi = 0.82$.

Let us use the developed model to calculate the design parameter s for given flake density ρ_d in the range of 1.5 to 2.1 kg/m^3. At the same time five functions Q_{1b}, Q_{2b}, Q, Q_{1d}, Q_{2d} of the angle θ and the required parameter s are initially made using the formulas (11), (18) and (33). Then non-linear equation (19) is solved and the angle γ is found out as a function of the parameter s. Then the function $\beta(s)$ is defined by formula (30). Once done, non-linear equation (35) is solved and parameter s is found out.

Figure 3 shows the results of the calculations and their comparison with the experimental relationship borrowed from paper [5]. An acceptable accuracy of the calculated estimates can be noted.

By determining the parameter s at a given value of sealing coefficient, the values of angles γ and β can be calculated in accordance with the developed model. After that by solving the non-linear equation (20) we determine the angle α. Under the formula (21) we calculate the tension σ_γ, and using the formula (26) we calculate σ_β. Table 2 shows the calculation results of the above-mentioned parameters of compaction process for the analyzed process at $z = 2.07$. Parameter $s = 0.00434$.

The values given in the table and the parameter s are sufficient to establish an explicit form of dependence of the average axial stress on the angle for the feed, sealing and extrusion areas, respectively, using the basic data from the formulas (14), (22) and (34). Then, in each area, the dependencies on the averaged axial stresses θ are determined. In addition, the functions $p(\theta)$ and $\tau_f(\theta)$

T a b l e 1. **Approximation results of the experimental dependencies of the powder mixture (urea) characteristics on density**

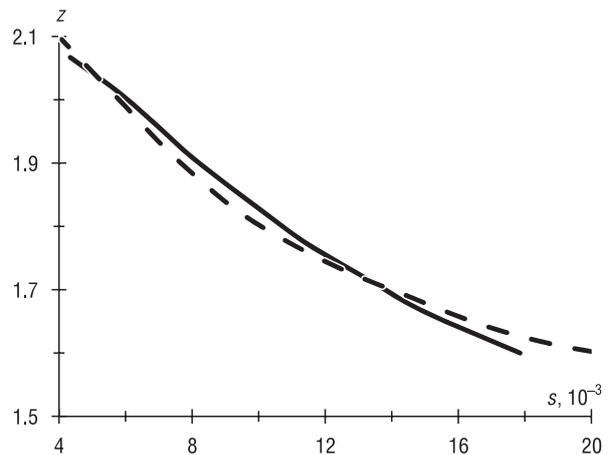| Characteristic	$\rho = \rho_b$	A	B	χ
tgδ	0.52	−0.20	−0.50	0.92
μ	0.21	0.12	0.20	0.89
f	0.16	−0.08	−0.03	0.96

Figure 3. Dependence of the sealing factor z on the relative (referred to the roll's diameter) thickness of the gap between the roll surfaces. The block curve – experimental dependence from [5]; the dashed curve – calculated dependence obtained using the developed model

T a b l e 2. **Results of calculating urea rolling parameters at $z = 2.07$**

Boundary	α	γ	β
Angle, degree	18.52	10.96	4.09
σ, MPa	0.12	0.25	112.91

are determined also. For this example (urea at $z = 2.07$) the normal contact pressure stress diagram is given in paper [5]. Figure 4 compares this stress diagram with the results of the developed model. As for the graphs in Figure 3, we can speak about acceptable accuracy of the contact pressure prediction. The slightly lower estimates for the sealing area are due to the use of assumption (16) borrowed from Johansen's model.
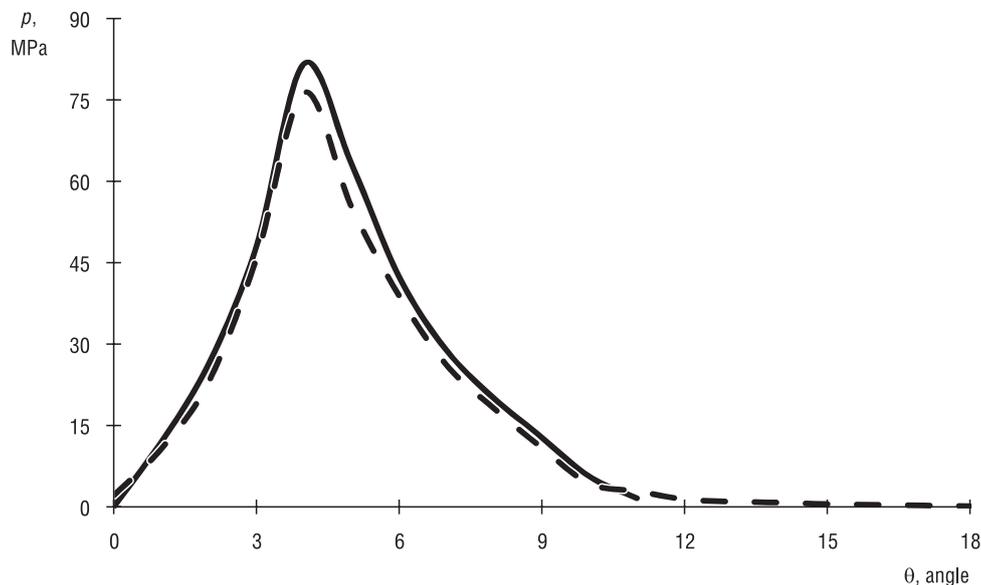

Figure 4. Dependence of normal pressure p on the roll on the angle θ while urea compacting at sealing factor $z = 2.07$: block curve – experimental dependence from [5]; dashed curve – calculated dependence obtained using the developed model

Conclusion. A mathematical model of mineral fertilizer compacting using a roll compactor is developed. Unlike the previously used Johansen model, it allows to take into account comprehensively the presence of three non-zero axial stresses in a powder mixture, the influence of inter-particle cohesion and its dependence on hydrostatic pressure, changes of mixture characteristics when density changes, the presence of three characteristic areas (feeding, sealing and extrusion), where the mixture contacts the roll. The model developed enables to derive relatively simple analytical relationships for the parameters that determine the power load of the material to be compacted and of the rolls. The calculation is the solution of three non-linear equations, followed by the use of the derived functional relationships. Within the framework of the developed model, it is possible to establish the dependence of the sealing factor on the relative gap between the roll surfaces. Previously while designing, the corresponding experimental dependencies were used. Comparison of the usage results of the model with the empirical $z(s)$ relationship for urea showed acceptable accuracy of the evaluated estimates calculated. Slightly overestimated gap values for small (up to 1.7) sealing coefficients are due to the neglect of backing and caking effects (the inter-particle cohesion parameter is assumed to be zero in the absence of hydrostatic compression), and using the hypothesis of a power-dependence of hydrostatic pressure on density in the sealing area (Johansen model assumption). The same assumptions lead to slightly underestimated pressure on the roll from the material being compacted. In this case, the calculated diagram of this pressure describes the known experimental data for the urea compaction with an acceptable accuracy.

References

1. Osokin A. V., Sevostyanov M. V. Analysis of existing methods and technological means for compacting the technogenic materials. *Aktual'nye problemy gumanitarnykh i estestvennykh nauk* [Current Issues in the Humanities and Sciences], 2013, no. 5, pp. 89–93 (in Russian).
2. Johanson J. R. A rolling theory for granular solids. *Journal of Applied Mechanics*, 1965, vol. 32, no. 4, pp. 842–848. https://doi.org/10.1115/1.3627325
3. Johanson J. R. Redusing air entrainment problems in your roll press. *Powder and Bulk Engineering*, 1989, pp. 43.

4. Peciar P., Macho O., Eckert M., Fekete R., Kotora P., Juriga M., Kabát J., Gabrišová Ľ., Peciar M. Design of particulate material compactor rolls diameter. *Acta Polytechnica*, 2017, vol. 57, iss. 4, pp. 263–271. https://doi.org/10.14311/ap.2017.57.0263

5. Generalov M. B., Klassen P. V., Stepanova A. R., Shomin I. P. *Calculation of Equipment for Granulating the Mineral Fertilizers*. Moscow, Mashinostroenie Publ., 1984. 192 p. (in Russian).

6. Katashinsky V. P. Stress-strain state of powder during rolling in the sealing area. I. Mathematical model of rolling in the sealing area. *Poroshkovaya metallurgiya= Powder Metallurgy*, 1983, no. 11 (251), pp. 17–21 (in Russian).

7. Kondratchik N. Yu. Equipment load assessment for compacting the granular fertilizers. *Sovremennye tekhnologii. Sistemnyi analiz. Modelirovanie* [Modern Technologies. System Analysis. Modelling], 2017, no. 1 (53), pp. 20–25 (in Russian).

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