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Predicting manhole mixing using a compartmental model

Fred Sonnenwald¹, Ole Mark², Virginia Stovin³, Ian Guymer⁴

¹Research Associate, Department of Civil & Structural Engineering, The University of Sheffield, Mappin Street, Sheffield, S1 3JD, United Kingdom

²Head of Innovation, Krüger A/S, 2860 Søborg, Denmark

³Professor of Green Infrastructure for Stormwater Management, Department of Civil & Structural Engineering, The University of Sheffield, Mappin Street, Sheffield, S1 3JD, United Kingdom

⁴Professor of Civil Engineering, Department of Civil & Structural Engineering, The University of Sheffield, Mappin Street, Sheffield, S1 3JD, United Kingdom

ABSTRACT

Manholes in combined sewers may become surcharged during storm events, resulting in complex mixing conditions. Although manhole hydrodynamics are reasonably well understood, predicting mixing across a surcharged manhole remains a challenge. An analytical compartmental mixing model for manholes, based on jet theory, has been further developed and applied to generate cumulative residence time distributions (CRTDs), which describe mixing. The modelled CRTDs were compared with the experimentally derived CRTDs of over 850 manhole configurations to evaluate how well the new compartmental model represents physical processes. The model underpredicts short-circuiting in manholes with manhole diameter to pipe diameter ratios greater than 4.4 and consequently overestimates mixing. Otherwise, the modelled CRTDs show good agreement with the experimental CRTDs. The new compartmental model represents key manhole hydrodynamics that are not represented in current software modelling packages, which assume manholes are instantaneously well-mixed. The compartmental model provides good predictions of the experimental downstream concentration profiles, although with reduced peak concentrations in those manhole

26 configurations where short-circuiting is not well-predicted. Despite this, the compartmental
27 model still predicts concentrations downstream of a manhole in closer agreement with the
28 recorded data than the complete instantaneously well-mixed assumption. As an analytical
29 model requiring no inputs other than manhole geometry, the new compartmental model applies
30 to a wide range of manhole configurations, is robust, and is useful for predicting manhole
31 mixing in practical applications.

32 Keywords: Water quality, manholes, mixing, short-circuiting, sewers, contaminant transport, travel
33 time, hydraulic structures, residence time, pollutants

34 **INTRODUCTION**

35 Combined sewer networks convey waste water to treatment. Manholes are placed in
36 sewer networks at regular intervals, at junctions, and changes in direction for inspection and
37 maintenance. During storm events, high flows may lead to manholes becoming surcharged,
38 where the water level exceeds the inlet pipe soffit, which in turn could result in combined sewer
39 overflows into receiving waters, or, in extreme cases, flooding (Butler et al., 2018).

40 At present, there is a focus on improving water quality within receiving waters. For
41 example, the Water Framework Directive (European Union Commission, 2000) calls for action
42 to achieve good qualitative and quantitative ecological status of all water bodies. Water quality
43 processes within combined sewer networks are relevant to understanding and reducing the
44 pollutant load on receiving waters. Software modelling packages such as SWMM (Rossman,
45 2015), InfoWorks ICM (Innovyze Inc., 2019), and MIKE URBAN+ (formerly MOUSE) (DHI
46 A/S, 2019) are commonly used to evaluate both water quantity and water quality in sewer
47 networks (Obropta and Kardos, 2007).

48 Significant efforts have been devoted to understanding the hydraulics of sewer networks.
49 Less effort has been dedicated to understanding mixing processes within sewer networks.
50 SWMM, InfoWorks ICM, and MIKE URBAN+ all assume inflow into manholes is

51 instantaneously well-mixed with the water volume. However, Stovin et al. (2013) has shown
52 that, depending on both the surcharge level within the manhole and the manhole geometry,
53 more complex mixing conditions occur, and, in these instances, a well-mixed assumption may
54 be invalid and subsequently lead to erroneous estimations of pollutant concentrations in sewers.
55 This paper presents an improved method of predicting mixing due to manholes of different
56 geometries over a range of surcharge and discharge conditions, suitable for inclusion into
57 software packages.

58 Manholes come in a variety of geometries, including t-shaped, rectangular, and circular
59 (Butler et al., 2018). Manholes may be shallow or deep, with access shafts, and may also have
60 the inlet and outlet at different elevations (stepped). In addition to multiple inlets, differing pipe
61 diameters, or an angle between inlet and outlet, manholes often have a dry weather channel and
62 benching, and in many cases, the pipe may be offset from the manhole centerline. A simple
63 manhole geometry of a circular unbenched straight-through manhole has been most used to
64 study mixing, e.g., Guymer and O'Brien, (2000) and Guymer et al. (2005). These studies
65 applied the Advection-Dispersion Equation (ADE) and Aggregated Dead Zone (ADZ) model
66 (Rutherford, 1994) to investigate mixing within surcharged manholes. Both studies showed the
67 ADE and ADZ model to predict mixing poorly and failed to provide a generalized means of
68 predicting model parameters.

69 Mixing within manholes is a non-Gaussian process that is dependent on surcharge depth
70 and manhole diameter to inlet pipe diameter ratio (D_m/D_p) (Guymer et al., 2005; Guymer and
71 Stovin, 2011; Stovin et al., 2013). For straight $D_m/D_p > 4.4$ manholes, a threshold surcharge
72 depth (S') exists. When surcharge depth S is below-threshold, $S < S'$, mixing is complete, while
73 above-threshold, $S > S'$, short-circuiting occurs. For straight $D_m/D_p \leq 4.4$ manholes, there is no
74 threshold and only short-circuiting flow conditions occur, regardless of surcharge depth. Fig. 1
75 shows laser-induced fluorescence (LIF) images of dye traces in a model manhole. Fig. 1a

76 shows below-threshold flow, which expands throughout the entire manhole volume producing
77 well-mixed conditions. Fig. 1b shows above-threshold flow, where a jet of dye short-circuits
78 from the inlet to the outlet. Dye that does not exit directly recirculates in the manhole volume,
79 which acts as a dead zone. In circular unbent straight-through manholes, Stovin et al.
80 (2010) reported the threshold at $S' = 0.258D_m$ for $D_m/D_p > 4.4$. However, there is evidence that
81 this threshold value is different for more complex manhole geometries, for example in 30-
82 90° angled $D_m/D_p = 4.4$ manholes, Sonnenwald (2014) reported $S' \approx 0.7D_m$.

83 There is a link between mixing and energy losses, as both result from the same
84 hydrodynamic processes; higher energy losses tend to be associated with higher levels of
85 mixing (Guymer et al., 2005). Below-threshold, head-loss coefficients increase with surcharge
86 depth, showing more energy is dissipated by the chaotic well-mixed flow field. Above-
87 threshold head loss coefficients reduce and remain constant, reflecting the more ordered short-
88 circuiting/recirculating flow field (Arao and Kusada, 1999; Stovin et al., 2013). Using head
89 loss data, Beg et al. (2019) reported $S' = 0.33D_m$ in 15-30° angled $D_m/D_p = 3.3$ manholes. In
90 square manholes with $L_m/D_p = 7.8$, where L_m is the length of one side of the manhole, Jimoh et
91 al. (2014) reported $S' = 0.24L_m$. Benched manholes tend to exhibit short-circuiting regardless
92 of geometry and surcharge depth (Sonnenwald, 2014).

93 Residence time distributions (RTDs) describe mixing in manholes independent of the
94 upstream concentration profile (Stovin et al., 2010; Guymer and Stovin, 2011; Stovin et al.,
95 2013). The RTD quantifies the hydrodynamics and hence mixing in a system resulting from an
96 instantaneous input (Danckwerts, 1953; Levenspiel, 1972). Convolution of the RTD with an
97 upstream concentration profile produces a downstream concentration profile. The inverse
98 process, deconvolution, can be used to obtain the RTD from experimental upstream and
99 downstream concentration profiles (Sonnenwald et al., 2015). Deconvolved RTDs can near-

100 perfectly describe the mixing within manholes (Guymer and Stovin, 2011). The cumulative
101 residence time distribution (CRTD), F , is the cumulative sum of the RTD.

102 The RTD and CRTD are commonly analyzed in dimensionless time, tQ/V , where V is
103 the volume of water between the upstream and downstream measurement locations x_1 and x_2 ,
104 and Q is the discharge. The nominal retention time $t_n = V/Q$ occurs at $tQ/V = 1$ and represents
105 the amount of time needed for all particles of water to travel between x_1 and x_2 at a uniform
106 velocity, i.e., plug flow. Dimensionless time has been used to show that mixing in manholes is
107 independent of manhole diameter, flow rate, and surcharge depth when either below- or above-
108 threshold (Stovin et al., 2010; Guymer and Stovin, 2011; Stovin et al., 2013).

109 The hydrodynamics between the upstream and downstream measurement locations can
110 be interpreted by visual inspection of the CRTD. This is illustrated in Fig. 2. Complete
111 instantaneously well-mixed flow (e.g., a continuously stirred tank) results in an exponential
112 CRTD ($F = 1 - \exp(-tQ/V)$). A steep rise in the CRTD, where much of the flow has the same
113 travel time, indicates a preferential flow path. If a steep rise occurs at $tQ/V < 1$, this indicates
114 short-circuiting, with the height of the rise indicative of the amount of short-circuiting. Short-
115 circuiting often occurs when there is a dead zone or recirculation cell. A long gentle CRTD tail
116 indicates dead-zone trapping. When the exchange between the main flow path and dead zone
117 is high, the tail of the CRTD is shorter/steeper.

118 Guymer and Stovin (2011) utilized deconvolution to obtain CRTDs directly from
119 experimental manhole solute trace data. The study synthesized these CRTDs to a single below-
120 and above-threshold CRTD pair that could be used to predict mixing across circular unbenched
121 straight-through $4.4 \leq D_m/D_p \leq 9.1$ manholes, validated for $D_m/D_p = 9.1$. This empirical model
122 has limited transferability as it requires knowledge of the threshold depth, which changes with
123 manhole configuration.

124 Following Guymer et al. (2005), Stovin et al. (2013) hypothesized that the change in flow
125 regime in manholes with respect to D_m/D_p and surcharge depth, i.e., the occurrence of the
126 threshold, could be explained using jet theory. A submerged circular jet entering a semi-infinite
127 volume takes on the shape of an expanding cone, with water moving less rapidly towards the
128 edges of the cone (Albertson et al., 1950; Rajaratnam, 1976). In the center is a jet core, which
129 is a cone decreasing in diameter but with an approximately uniform velocity equal to the inlet
130 velocity. The expanding cone that makes up the jet, excluding the jet core, is referred to as the
131 jet diffusion zone. It is caused by the momentum that enters with the jet gradually transferring
132 into the surrounding water, expanding at a rate of approximately 1 in m where $m = 5$ (Albertson
133 et al., 1950). The distance the jet core penetrates a volume of water is controlled by the diameter
134 of the jet's inlet, such that $L_j = D_p/(2\alpha_2)$ where L_j is the length of the jet core and α_2 is the rate
135 of jet core dissipation. The values of α_2 reported in the literature vary, with values reported by
136 Gauntner et al. (1970) and Rajaratnam (1976) ranging between $1/15.4 < \alpha_2 < 1/9.4$ with most
137 values around $1/12$. The region where the jet core remains is referred to as the zone of
138 establishment, and the region after the jet core is fully dissipated is referred to as the zone of
139 established flow.

140 Albertson et al. (1950) reported $\alpha_2 = 1/12.4$, giving $L_j = 6.2D_p$. Stovin et al. (2013)
141 therefore concluded, in agreement with their modelling results, that in smaller diameter
142 manholes, $D_m/D_p < 6.2$ or $D_m < L_j$, the jet core would always reach the outlet and short-circuit.
143 At larger diameters, Stovin et al. (2013) proposed that below-threshold the expanding edge of
144 the jet diffusion zone would reach the surface and disturb the manhole volume, leading to the
145 observed complete mixing (Fig. 1a). At higher surcharges above-threshold, they proposed that
146 the overlying surcharge and 3D effects would tend to dampen the flow field and the jet would
147 continue to the outlet uninterrupted and thus short-circuit (Fig. 1b). With the jet diffusion zone

148 expanding at a rate of 1 in 5, the manhole surface would be reached at $0.2D_m$, which Stovin et
149 al. (2013) found to agree reasonably well with $S' = 0.258D_m$.

150 Using jet theory, Mark and Ilesanmi-Jimoh (2017) divided the volume of a manhole into
151 three zones: jet core, jet diffusion, and outer mixing (the remaining manhole volume). They
152 applied the submerged jet theory associated with each zone to create a three-zone
153 compartmental mixing model (Chapra, 1997) where mixing between the zones is driven by
154 mass flux. Velocity at the plane of the outlet of the jet core and jet diffusion zones was
155 calculated based on Albertson et al. (1950) and integrated to calculate flow. By multiplying the
156 integrated flow at the outlet plane with the concentration within the zone, the mass flux out of
157 both zones was calculated. In the Mark and Ilesanmi-Jimoh (2017) model, flow (Q) from the
158 manhole inlet (V_0) enters the manhole in the jet core zone (V_1) and is either transported into the
159 jet diffusion zone (V_3) as $Q_{1,3}$ or continues to the outlet (V_2) as $Q_{1,2}$. Flow from V_1 to V_2 ($Q_{1,2}$)
160 is determined by integrating the jet core velocity at the outlet plane. The remaining outflow
161 comes from the jet diffusion zone ($Q_{3,2}$). The jet diffusion zone flow that does not exit via the
162 outlet is instead exchanged with the outer mixing zone (V_4) as $Q_{3,4}$. Mass-balance in the model
163 is achieved using the return flow from the outer mixing zone to jet diffusion zone $Q_{4,3}$. This is
164 illustrated in Fig. 3a-b. The Mark and Ilesanmi-Jimoh (2017) model is analogous to an ADZ
165 model with multiple zones. While the model is an analytical framework for predicting mixing
166 and was designed for circular unstepped and stepped straight-through manholes, the model
167 only applies to manholes with $D_m < L_j$. Mark and Ilesanmi-Jimoh (2017) compared results
168 against a small dataset of just five solute traces, giving limited confidence in its general
169 applicability. To date, a generalized model for predicting mixing across the wide range of
170 surcharged manhole configurations encountered in practice has not been presented.

171 The Guymer et al. (2020) dataset is a large database of solute traces in manholes,
172 consisting of over 4,000 traces in circular manholes and their corresponding CRTDs for over

173 1,000 experimental conditions, varying both hydraulic conditions (flow rate and surcharge
174 depth) and manhole geometry (diameter, step height, and outlet angle). Although the dataset is
175 incomplete for benched manholes, the unbenched manholes provide a baseline case for model
176 development. Additionally, while design guidance suggests manholes with ratios $D_m/D_p \approx 3$ to
177 be most common in networks (WRC, 2012), the results of Stovin et al. (2013) suggest manholes
178 of this size perform similarly to the $D_m/D_p = 4.4$ manholes covered within the dataset. Larger
179 ratios may also be more common upstream in networks where pipe diameters are smaller and
180 a minimum manhole diameter is fixed. Therefore, using the unbenched manhole data from the
181 Guymer et al. (2020) dataset, this study aims:

- 182 • to further develop the compartmental modelling approach of Mark and Ilesanmi-Jimoh
183 (2017) to describe mixing across manholes for a range of D_m/D_p ratios, stepped
184 manholes, and angled manholes;
- 185 • to confirm compartmental modelled CRTDs are consistent with experimental CRTDs
186 and represent the relevant hydrodynamics across the extensive Guymer et al. (2020)
187 dataset, identifying any limitations; and
- 188 • to demonstrate the compartmental model offers an improved representation of mixing
189 in manholes compared to the instantaneously well-mixed assumption.

190 **EXPERIMENTAL WORK**

191 The Guymer et al. (2020) dataset, as previously mentioned, contains experimental
192 manhole solute traces (upstream and downstream temporal concentration profiles) collected by
193 O'Brien (2000), Dennis (2000), Saiyudthong (2004), Guymer et al. (2005), and Lau (2008)
194 from a variety of circular manhole configurations. The data covers a range of manhole
195 diameters, inlet pipe diameters, flow rates, surcharge depths, outlet step heights, and outlet
196 angles. The range of manhole configurations utilized in this study is outlined in Table 1.
197 Between two and eight repeat traces were conducted with Rhodamine dye and recorded with

198 Series 10 Turner Design fluorometers located on inlet and outlet pipes a short distance
199 upstream and downstream of the manhole. The surcharge level was controlled with a
200 downstream weir and measured with an Armfield Limited H45 mechanical water surface level
201 follower. Both the fluorometers and water level follower were logged by PC using either a
202 Cambridge Electronic Design CED 1401 or Measurement Computing Corporation CIO-
203 DAS802. Flow rate was either measured using a point depth gauge over a calibrated 30° V-
204 notch weir built to BS 3680-4A:1981 (BSI, 1981) or a venturi meter based on BS EN ISO
205 5167-1:2003 (BSI, 2003). Smooth pipes were used with a relative roughness of approximately
206 2×10^{-5} and a friction factor of 0.020.

207 The dataset also contains CRTDs deconvolved from the trace data according to
208 Sonnenwald et al. (2015), which is based on maximum entropy deconvolution (Skilling and
209 Bryan, 1984). To deconvolve, a non-linear optimizer is used to maximize the entropy of a sub-
210 sampled RTD while constrained to goodness-of-fit between the downstream concentration
211 profile predicted using the reconstructed RTD and the measured downstream profile. This may
212 be expressed in a Lagrangian function, which was solved using the MATLAB *fmincon* function
213 (The MathWorks Inc., 2020). The RTD is reconstructed from the sub-sampled RTD using a
214 smoothed interpolation approach. The deconvolution method was modified by Guymer et al.
215 (2020) to permit dynamic sample point number selection and to utilize a slope-based sampling
216 point scheme that allows for RTDs shorter than the recorded data.

217 The trace data have been pre-processed to apply the reported calibrations, filter noise,
218 subtract background concentration, and trim the traces. A low-pass Butterworth filter was
219 applied using the MATLAB *butter* function (The MathWorks Inc., 2020) with a 1 Hz cut-off
220 frequency to remove high-frequency sensor noise. Background was taken as the mean of the
221 first and last 10 seconds of data. A copy of the trace was smoothed with a moving average and

222 used to determine the start time of the trace as 1% of the smoothed peak upstream profile and
223 the end of the trace as 1% of the peak of the smoothed downstream profile.

224 Manhole data with mass-balance less than 0.6 or greater than 1.1 have not been
225 processed, except for 60° outlet angle data where mass-balance was assumed due to missing
226 sensor calibrations (83 traces). Configurations with dissimilar CRTDs between repeat traces
227 have also not been processed (31 configurations). These typically occurred near the threshold
228 and indicate the flow field was changing between repeats. The 800 mm manhole configurations
229 reported by O'Brien (2000) have also not been processed, as the traces were of insufficient
230 duration to record the complete downstream trace (25 configurations). CRTDs were scaled to
231 have a final value of 1 when the final value exceeded 1. Finally, the CRTDs for each repeat
232 trace were averaged to produce a single mean CRTD for each configuration.

233 **MODEL DEVELOPMENT**

234 Several changes have been made and new process descriptions added to the
235 compartmental model of Mark and Ilesanmi-Jimoh (2017). The new developments are intended
236 firstly to improve: estimates of zone volumes; estimates of exchange between zones; and the
237 description of jet deflection in stepped manholes. Secondly, the model has been extended to
238 apply to: very high surcharge levels; large diameter manholes ($D_m \geq L_j$); and angled manholes.
239 Fig. 3 details the compartmental model zones, their notation, relationships between them, and
240 geometry for an unbenched circular manhole.

241 Mark and Ilesanmi-Jimoh (2017) calculated the volume of the jet core (V_1) and jet
242 diffusion zone (V_3) as simple cones, and while the volumes extending beyond the inlet and
243 outlet (V_{1b} and V_{3b}) were accounted for, they did not account for the jet diffusion zone extending
244 below the bottom of the manhole (V_{3c}) or above the water surface (V_{3d}). These volumes have
245 now been accounted for. By considering the bottom of the manhole and water surface, the flow

246 from the jet diffusion zone to the outer mixing zone ($Q_{3,4}$) has also been reduced compared to
247 the Mark and Ilesanmi-Jimoh (2017) model.

248 In stepped manholes, vertical jet deflection towards the outlet was observed in some LIF
249 images presented by Dennis (2000). Mark and Ilesanmi-Jimoh (2017) proposed to calculate
250 the jet angle as the resultant of mean downwards velocity and mean horizontal jet velocity,
251 giving jet deflection on the order of 10° . However, empirical relationships for jet crossflow
252 (where a jet enters a perpendicular flow) given by Rajaratnam (1976) suggest jet deflection of
253 less than 1° . Agelin-Chaab and Tachie (2011) described the behavior of round offset jets, which
254 are analogous to the inlet jet in a stepped manhole, in a semi-infinite volume. They found the
255 edge of the jet diffusion zone to hit the bed at a relatively short distance from the inlet,
256 spreading to the bed more rapidly than into an infinite volume. Although the jet core similarly
257 deflects towards the bed, at small jet offsets ($B/D_p \leq 1$) the jet core does not deflect significantly
258 until 10 jet diameters ($10D_p$) from the inlet. As the jet inlet moves further from the bed
259 (increasing jet offset or manhole step height) the distance before jet deflection increases. As
260 with jets into a semi-infinite volume, this behavior is independent of flow rate, scaling with
261 inlet diameter and jet offset. Both crossflow jet and offset jet behavior suggests that jet
262 deflection due to a step would be minimal within the diameter of a manhole. To be consistent
263 in the use of jet theory and to simplify model geometry, we, therefore, have not included jet
264 deflection in either stepped or angled manholes.

265 In a stepped manhole, the outlet invert is at a lower elevation B than the inlet invert.
266 Without jet deflection then, the jet core and outlet are not aligned and as B increases eventually
267 the jet diffusion zone will not entirely overlap the outlet (Fig. 3f). In this case, as outlined by
268 Mark and Ilesanmi-Jimoh (2017), the position of the outlet relative to the jet core and jet
269 diffusion zone is used to determine the proportion of flow each zone contributes to the outlet.
270 Any flow not from the jet core or jet diffusion zone comes from the outer mixing zone ($Q_{4,2}$).

271 Fig. 4a shows a recorded downstream concentration profile compared to a downstream
272 concentration prediction made using the Mark and Ilesanmi-Jimoh (2017) model and a
273 prediction made using the new estimates of zone volume and exchange. Fig. 4b shows the
274 effect of assuming no vertical jet deflection. Goodness-of-fit between the experimental and
275 modelled downstream concentration profiles has been evaluated using the R_t^2 correlation
276 coefficient (Young et al., 1980). In both examples, the changes made for the new
277 compartmental model increase the quality of the downstream prediction.

278 The new compartmental model incorporates a new ‘high-surge storage’ zone (V_5)
279 when surge depth exceeds manhole diameter. LIF images of dye in high-surge
280 manholes, presented by Dennis (2000), suggest that in such conditions little dye reaches the
281 upper level of the surge. Habib et al. (2005) investigated flow in dead-end pipe legs and
282 showed that counter-rotating recirculation cells could form in similar hydraulic conditions.
283 Assuming counter-rotation, recirculation in the outer mixing zone would be mirrored in the
284 high-surge storage volume. Hence, the upwards and downwards exchange between the
285 outer mixing zone and the high-surge zone ($Q_{4,5}$ and $Q_{5,4}$) have been assumed to be equal
286 to the downwards exchange between the outer mixing zone to the jet diffusion zone ($Q_{4,3}$).

287 In a large diameter manhole ($D_m > L_j$), the jet core no longer reaches the outlet (shown
288 in Fig. 3e) and extending the reasoning applied to a stepped manhole, the jet core zone
289 contributes entirely to the jet diffusion zone. In turn, the latter contributes entirely to the outlet.
290 To calculate contributions to the outlet and flow between zones in angled manholes, with an
291 outlet turned through θ degrees (shown in Fig. 3g), again a similar approach to a stepped
292 manhole was used. If the outlet is 90° to the inlet, the outlet is only in contact with the outer
293 mixing zone, and flow must pass through the jet core, jet diffusion zone, and outer mixing zone
294 to reach the outlet. In general, the location of the outlet should be compared with the locations
295 of the jet core and jet diffusion zones to determine proportional flow contributions to the outlet.

296 For comparison of model results to the experimental data, it is necessary to account for
297 the fluorimeters being located on the inlet and outlet pipes just before and after the manhole,
298 rather than directly at the inlet and outlet. To account for the varying length of pipe between
299 experimental configurations, we assume pipe dispersion is negligible compared to mixing
300 caused by the manhole, and thus it only necessary to account for the advection within the pipe.
301 This may be achieved with a time delay term, which Rutherford (1994) defined as the
302 difference in the first arrival time of a tracer between an upstream and downstream
303 measurement location. Time delay has been estimated as the length of the pipe sections divided
304 by peak pipe velocity U_{max} , i.e., $(x_2 - x_1 - D_m)/U_{max}$, with $U_{max} \approx 1.14U$ assuming a logarithmic
305 turbulent pipe velocity profile. This can be converted to dimensionless time, similar to the
306 CRTD, using tQ/V , where V is the total volume of water between x_1 and x_2 . The mean
307 experimental dimensionless time delay for the experimental data was 0.15, taken as t_1 from the
308 CRTD, the time at which 1% of material entering the manhole has exited. The mean estimated
309 dimensionless time delay was 0.19. This is an overestimate of 23% compared to the
310 experimental value but given experimental manhole CRTDs typically have a dimensionless
311 time duration of 3 or longer, the difference in time delay is around only 1% of CRTD duration
312 and thus an acceptable approximation.

313 The new compartmental model has been implemented in MATLAB (The MathWorks
314 Inc., 2020). V_3 has been approximated as a cone that partially extends beyond the edge of the
315 manhole, shown outside the boundaries of the manhole in Fig. 3c. The flow rates between
316 zones relating to V_3 are affected by this simplification. The solution used to estimate the
317 reduction in flow due to the bottom of the manhole is also an approximation. These
318 approximations, when compared to using high-resolution numerical integration affect their
319 respective quantities by 2-3%, depending on the exact manhole geometry. Fig. 5 shows the
320 effect of changing V_3 and $Q_{3,4}$ by $\pm 5\%$, represented by the shaded areas, on a compartmental

321 modelled CRTD. The CRTD was calculated directly from the compartmental model as the
322 cumulative response to a single time step upstream pulse input. Given the variation in α_2 values
323 reported in the literature, Fig. 5 also shows the effect of changing α_2 by $\pm 5\%$. Of the three, the
324 new compartmental model is most sensitive to changes in $Q_{3,4}$. The model is insensitive to
325 small changes in V_3 and α_2 . Sonnenwald et al. (2021) provide the code used for the new
326 compartmental model, as well as further details on calculating the zone volumes, the exchange
327 between zones, and model solution.

328 RESULTS

329 Comparison of experimental and modelled CRTDs

330 Fig. 6 compares modelled CRTDs with the deconvolved experimental CRTDs for the
331 straight-through $D_m/D_p = 4.4$, straight-through $D_m/D_p = 9.1$, stepped $B/D_p = 1.5$, and 90°
332 angled outlet manholes (the latter two are both $D_m/D_p = 4.4$). The remaining CRTDs are shown
333 in Figs. S1–S3 in the supplementary material. In all cases, the trend of decreasing tQ/V at the
334 start of the CRTD with increasing surcharge is reproduced. The modelled CRTDs also become
335 more angled with increasing surcharge, similar to the experimental CRTDs.

336 The agreement between the experimental and modelled CRTDs is particularly good for
337 the straight-through $D_m/D_p = 4.4$ manholes, Figs. 6a and 6b. As manhole diameter increases to
338 $D_m/D_p = 9.1$ (Figs. 6c and 6d), the experimental CRTDs fall into two clear groups above and
339 below $S/D_m \approx 0.27$, corresponding to below-threshold and above-threshold flow conditions. As
340 surcharge increases, the modelled CRTDs do indicate increased levels of short-circuiting.
341 However, this is far less obvious than in the experimental data. In both the stepped $B/D_p = 1.5$
342 (Figs. 6e and 6f) and 90° angled (Figs. 6g and 6h) manholes, there is no clear separation of
343 below- and above-threshold experimental or modelled CRTDs. In both sets of experimental
344 data, though, there is a slight deviation around $F \approx 0.4$ that is not reproduced by the model.

345 Fig. 7 shows t_{10} , t_{50} , and t_{70} residence times (when 10%, 50%, and 70% of the material
346 entering the manhole has exited) for the experimental deconvolved and modelled CRTDs with
347 respect to surcharge ratio S/D_m . For the straight-through $D_m/D_p > 4.4$ manholes, the difference
348 between the experimental and modelled values for t_{10} (Fig. 7a) is generally small and consistent
349 with the differences in estimated time delay. However, the experimental values of t_{50} (Fig. 7d)
350 show a discontinuity at $S/D_m \approx 0.27$, consistent with the results of Stovin et al. (2013). The
351 model does not predict the sharp drop in t_{50} at the threshold surcharge depth, which is consistent
352 with the lack of differentiation between below- and above-threshold CRTDs in Fig. 6d. At t_{70}
353 (Fig. 7g) the scatter in the experimental residence times for straight-through $D_m/D_p = 4.4$
354 manholes corresponds to the plateau shown in the experimental CRTDs in Fig. 6a at $F \approx 0.7$.

355 In the stepped manholes, t_{10} (Fig. 7b) is estimated well, except for $B/D_p = 2.0$. t_{50} and t_{70}
356 (Figs. 7e and 7f) are also overestimated for the largest step height configurations. Modelled t_{50}
357 and t_{70} both decrease with $S/D_m > 1$, reflecting the influence of the high-surcharge storage zone.

358 Residence times are predicted reasonably well for angled manholes, except for t_{70}
359 (Fig. 7i) in the 30° angled manhole. t_{10} (Fig. 7c) is overestimated in the angled manholes at
360 $S/D_m \lesssim 0.4$, which could be a result of the flow path cutting the corner between the inlet and
361 the outlet. The experimental t_{50} (Fig. 7f) values for the 30° angled and t_{10} for the 60° and 90°
362 angled manholes show a systematic decrease above $S/D_m \approx 0.7$ that is consistent with the
363 threshold between well-mixed and short-circuiting flow conditions reported by Sonnenwald
364 (2014).

365 **Comparison with the instantaneously well-mixed model**

366 Fig. 8 compares CRTDs generated using the new compartmental model with both the
367 experimental CRTDs and the instantaneously well-mixed model CRTDs for selected manhole
368 configurations (see Table S1 in the supplementary material). Mean pipe travel time has been
369 added to the well-mixed model CRTDs to account for the experimental fluorometer placement

370 on pipe lengths upstream and downstream of the manhole. Figs. 8a and 8b show results for
371 straight-through manholes, Figs. 8c and 8d stepped manholes, and Figs. 8e and 8f angled
372 manholes. The left-hand plots show below-threshold CRTDs, while the right-hand plots show
373 above-threshold CRTDs. Note, for straight and stepped manholes the threshold depth of
374 $S' = 0.258D_m$ has been used, and for angled manholes $S' = 0.7D_m$.

375 Overall, it is clear that the new compartmental model provides significantly better
376 estimates of the experimental CRTDs than the well-mixed exponential model, which does not
377 estimate any of the observed short-circuiting effects.

378 As the manhole geometry deviates from a small diameter straight-through manhole, the
379 experimental CRTDs tend to show more complete mixing at low surcharges. This is
380 demonstrated in Fig. 8a, where the fit of the well-mixed model improves as the D_m/D_p ratio
381 increases for below-threshold cases. However, as highlighted in the discussion of Fig. 6, the
382 compartmental model is not fully able to capture the short-circuiting effects that were
383 experimentally observed in larger diameter above-threshold conditions (Fig. 8b).

384 In below-threshold geometries when the manhole is similar to a straight-through narrow
385 manhole, e.g., the stepped $B/D_p = 0.5$ manhole, short-circuiting occurs, and the new
386 compartmental model reflects this (Fig. 8c). The transition to complete mixing in stepped
387 manholes is likely driven by the jet deflecting slightly for small changes in outlet alignment
388 (Dennis, 2000; Beg et al., 2019) until it no longer deflects and instead impacts the far wall of
389 the manhole, disrupting the flow field (Sonnenwald, 2014).

390 Whilst there appear to be some inconsistencies in the compartmental model's ability to
391 predict the tail of the CRTD, i.e., the amount of dead-zone exchange, compared to the
392 experimental CRTDs, it is believed that this reflects experimental limitations, either in the
393 premature termination of experimental traces or due to insufficient sensitivity in the
394 measurement systems.

395 **Concentration profiles**

396 The new compartmental model and the well-mixed model were applied to make
397 downstream predictions for 3,941 recorded solute traces. Overall, the compartmental model
398 underpredicted peak concentrations by 10% on average, compared with 60% by the well-mixed
399 model. Goodness-of-fit between the measured and modelled downstream concentration
400 profiles was evaluated using R_t^2 . The compartmental model created good engineering
401 predictions for the available data with a mean R_t^2 of 0.881, a standard deviation of 0.119, and
402 a median of 0.917, compared with 0.667, 0.180 and 0.697 respectively for the complete mixing
403 assumption. Across all traces, the compartmental model R_t^2 tended to decrease with increasing
404 manhole diameter, step, and angle, i.e., as manhole geometry deviated further from an ideal
405 straight-through small diameter manhole. This is consistent with the trends observed in the
406 modelled CRTDs.

407 Fig. 9 shows a selection of recorded downstream traces compared with both the new
408 compartmental and well-mixed model predictions (the upstream traces are not shown for
409 clarity). These traces match the manhole configurations of the CRTDs shown in Fig. 8.
410 Corresponding R_t^2 values are given in Table 2. The tails of some experimental traces, e.g.,
411 Fig. 9d and 9e, show oscillations that indicate recirculation within the manhole (Stovin et al.,
412 2010). As expected, the compartmental model does not reproduce this behavior, but the
413 otherwise good agreement between the experimental and modelled tails offers some evidence
414 that the compartmental model predicts dead-zone exchange well.

415 Below-threshold, the new compartmental modelled concentration profiles are generally
416 good fits, whilst peak concentrations are underpredicted in some above-threshold cases (e.g.,
417 Fig. 9g). In contrast, the well-mixed model provides significantly poorer predictions,
418 underpredicting peak concentrations in all cases. The example traces shown in Fig. 9 and

419 corresponding R_t^2 values in Table 2 confirm that the new compartmental model offers a definite
420 improvement over the well-mixed model.

421 **DISCUSSION**

422 **Model limitations**

423 The compartmental model has been shown to predict mixing for a wide range of circular
424 unbenched $4.4 \leq D_m/D_p \leq 9.1$, $0 \leq B/D_p \leq 2.0$, and $0^\circ \leq \theta \leq 90^\circ$ manhole configurations. The
425 main limitation of the compartmental model is the underestimation of short-circuiting in high-
426 surcharge wide, stepped, and angled manholes, which leads to underestimated peak
427 concentrations. The compartmental model does not account for the more complex
428 hydrodynamics occurring in these instances, and the underestimation of short-circuiting is to
429 be expected given the model's relatively large zones used and the assumption of instantaneous
430 mixing in each zone.

431 In the case of above-threshold large diameter manholes, the limitation noted above could
432 potentially be addressed by increasing the length of the jet core assumed in the model, although
433 further work would be required to justify such a modification. Within stepped and angled
434 manholes, further investigation of jet behavior may also offer additional insight.

435 Benched manholes and manholes with multiple inlets and outlets or differing inlet and
436 outlet diameter have not been considered here, however, the concepts of the new
437 compartmental mixing model could be applied to these conditions by considering the manhole
438 geometry in relation to the inlet jet.

439 **Practical application**

440 Although short-circuiting is underestimated by the new compartmental model, the well-
441 mixed model currently used in commercial sewer network modelling packages predicts no
442 short-circuiting at all and thus will tend to underestimate peak concentrations associated with

443 the transport of transient pulses of pollutants. The new compartmental model, therefore,
444 represents a significant improvement, predicting higher, more realistic concentrations.
445 Notably, the new compartmental model will predict short-circuiting in $D_m/D_p \leq 4.4$ manholes,
446 which are common in practice.

447 A further application of the modelling approach described here lies in its potential use
448 for estimating energy losses due to manholes. Pedersen and Mark (1990) used a related jet-
449 theory derived approach to estimate head losses in manholes, and their approach could be
450 combined with the compartmental model to investigate the link between head losses and
451 mixing previously highlighted by Stovin et al. (2013).

452 **CONCLUSIONS**

453 Previous investigations into mixing in surcharged manholes have failed to develop a
454 robust model for predicting the effects of different geometry, surcharge, and discharge on
455 mixing conditions. Current commercial modelling approaches assume instantaneous complete
456 mixing, which cannot predict short-circuiting and underestimates peak pollutant
457 concentrations. In this paper, a compartmental model for predicting mixing within manholes,
458 based on submerged jet theory, has been further developed to account for a range of inlet to
459 manhole diameter ratios (D_m/D_p), outlet steps, and outlet angles, with varying surcharge and
460 discharge. The model performance was evaluated with over 850 manhole configurations.

461 The new compartmental model was shown to predict cumulative residence time
462 distributions (CRTDs) consistent with experimental CRTDs. The model reproduced the
463 expected short-circuiting flow conditions below- and above-threshold in straight-through small
464 D_m/D_p manholes. As D_m/D_p , step, or outlet angle increased, below-threshold the model
465 predicted the observed complete mixing. Above-threshold, the model showed less short-
466 circuiting than observed. In general, as manhole geometry deviated from a straight-through

467 small D_m/D_p manhole, or from below-threshold conditions, the agreement between
468 experimental and modelled CRTDs decreased.

469 The new compartmental model successfully predicted downstream concentration profiles
470 both below- and above-threshold. Peak concentrations were underpredicted when the
471 compartmental model underestimated short-circuiting. The mean goodness-of-fit, using R_t^2 ,
472 was 0.881 for the new compartmental model, showing significant improvement compared to
473 0.661 for the well-mixed model. Compared to the well-mixed model which estimates the most
474 mixing possible, the new compartmental model will estimate more realistic, higher
475 concentrations, for overflow impact assessments.

476 The new compartmental model is completely analytical and has been validated for a
477 range of geometries. It requires no assumptions regarding mixing conditions or threshold depth,
478 is insensitive to small changes in geometry, and as such is suitable for practical applications.
479 For manholes outside the range investigated in this study, the jet theory used to develop the
480 model remains relevant, as well as for other structures where an inlet jet dominates the flow
481 field. The new compartmental model provides a robust framework to predict mixing in
482 manholes.

483 **DATA AVAILABILITY STATEMENT**

484 Data and code used during this study are available in a repository online in accordance
485 with funder data retention policies from Guymer, I., Stovin, V., O'Brien, R., Dennis, P.,
486 Saiyudthong, C., Lau, S.-T. D., and Sonnenwald, F. (2020)
487 <https://doi.org/10.15131/shef.data.13373039> and Sonnenwald, F., Mark, O., Stovin, V., and
488 Guymer, I. (2021) <https://doi.org/10.15131/shef.data.14160884>.

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491 **NOTATION LIST**

492 *The following symbols are used in this paper:*

493 B = manhole outlet step size;

494 C = concentration;

495 C_{\max} = peak upstream concentration;

496 D_m = manhole diameter;

497 D_p = inlet/outlet pipe diameter;

498 F = the cumulative residence time distribution;

499 L_j = the length of the jet core;

500 m = rate of jet diffusion zone expansion;

501 Q = manhole inflow or outflow rate, discharge;

502 $Q_{1,2}$ = flow rate from jet core to outlet;

503 $Q_{1,3}$ = flow rate from jet core to jet diffusion zone;

504 $Q_{3,2}$ = flow rate from jet diffusion zone to outlet;

505 $Q_{3,4}$ = flow rate from jet diffusion zone to outer mixing zone;

506 $Q_{4,2}$ = flow rate from outer mixing zone to outlet;

507 $Q_{4,3}$ = flow rate from outer mixing zone to jet diffusion zone;

508 $Q_{4,5} = Q_{5,4}$ = flow rate from outer mixing zone to high-surge storage zone and back;

509 R_t^2 = Young's correlation coefficient;

510 S = surcharge depth;

511 S' = threshold depth;

512 t = time;

513 $t_1, t_{10}, t_{50}, t_{70}$ = time at which $F = 0.01, F = 0.1, F = 0.5, F = 0.7$;

514 t_{50} = median residence time;

515 tQ/V = dimensionless time;

516 U = mean pipe velocity;
517 V_0 = manhole inlet;
518 V_1 = jet core zone;
519 V_{1b} = volume of jet core cone extending beyond manhole outlet;
520 V_2 = manhole outlet;
521 V_3 = jet diffusion zone;
522 V_{3b} = volume of jet diffusion zone cone extending beyond manhole inlet;
523 V_{3c} = volume of jet diffusion zone extending beyond manhole floor;
524 V_{3d} = volume of jet diffusion zone extending beyond water surface;
525 V_4 = outer mixing zone;
526 V_5 = high-surge storage zone;
527 x_1 = upstream measurement location;
528 x_2 = downstream measurement location;
529 α_2 = the rate of jet core dissipation; and
530 θ = manhole outlet angle.

531 **SUPPLEMENTARY MATERIALS**

532 Table S1 and Figs. S1–S3 are available online in the ASCE Library (ascelibrary.org).

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681

682 **Table 1.** Circular manhole configurations in Guymmer et al. (2020) dataset

Manhole diameter D_m (mm)	Pipe Diameter D_p (mm)	Centerline distance between fluorometers $x_2 - x_1$ (mm)	D_m/D_p	Flow Rate (l/s)	Surcharge Depth S (mm)	S/D_m	Outlet Angle	Step Height B (mm)	B/D_p
218	24	736	9.1	0.25–0.50	7–100	0.04–0.46	0°	0	0
388	88	2700	4.4	1.00–8.00	1–1241	0–3.20	0°, 30°, 60°, 90°	0, 44, 88, 132, 176	0, 0.5, 1.0, 1.5, 2.0
500	88	2700	5.7	0.97–7.45	7–268	0.02–0.54	0°	0	0
600	88	2700	6.8	0.87–8.72	27–308	0.04–0.51	0°	0	0
800	88	2700	9.1	0.88–8.77	27–332	0.03–0.41	0°	0	0

683

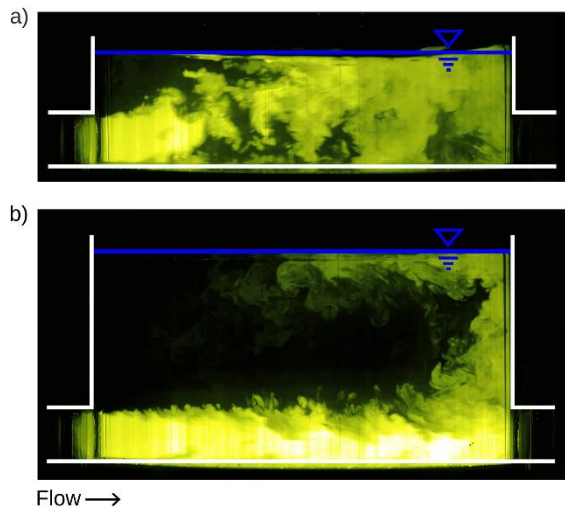
684

685 **Table 2.** Illustrative R_t^2 values comparing goodness-of-fit between recorded downstream
 686 profiles to new compartmental model and well-mixed exponential CRTD predictions,
 687 matching Fig. 9

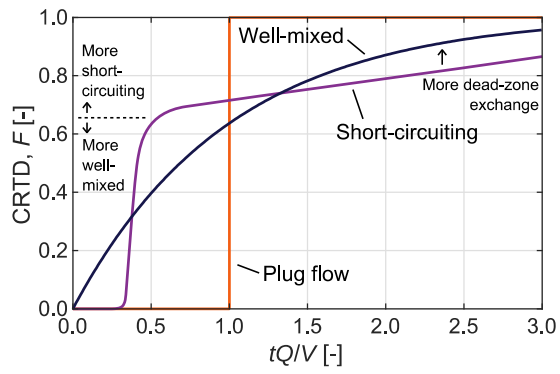
Configuration		New compartmental model		Well-mixed exponential CRTD	
		Below-threshold	Above-threshold	Below-threshold	Above-threshold
Straight D_m/D_p	4.4	0.968	0.982	0.492	0.481
	6.8	0.932	0.863	0.847	0.324
	9.1	0.928	0.627	0.782	0.175
Stepped B/D_p	0.5	0.968	0.964	0.596	0.223
	1.0	0.956	0.896	0.729	0.460
	1.5	0.988	0.915	0.878	0.548
Angled	30°	0.909	0.707	0.556	0.254
	60°	0.732	0.510	0.687	0.530
	90°	0.944	0.792	0.863	0.833
Mean		0.927	0.925	0.806	0.714

688

689



691 **Figure 1.** Vertical central plane laser-induced fluorescence imaging snapshot of dye traces
692 through a $D_m/D_p = 9.1$ manhole a) below-threshold and b) above-threshold
693

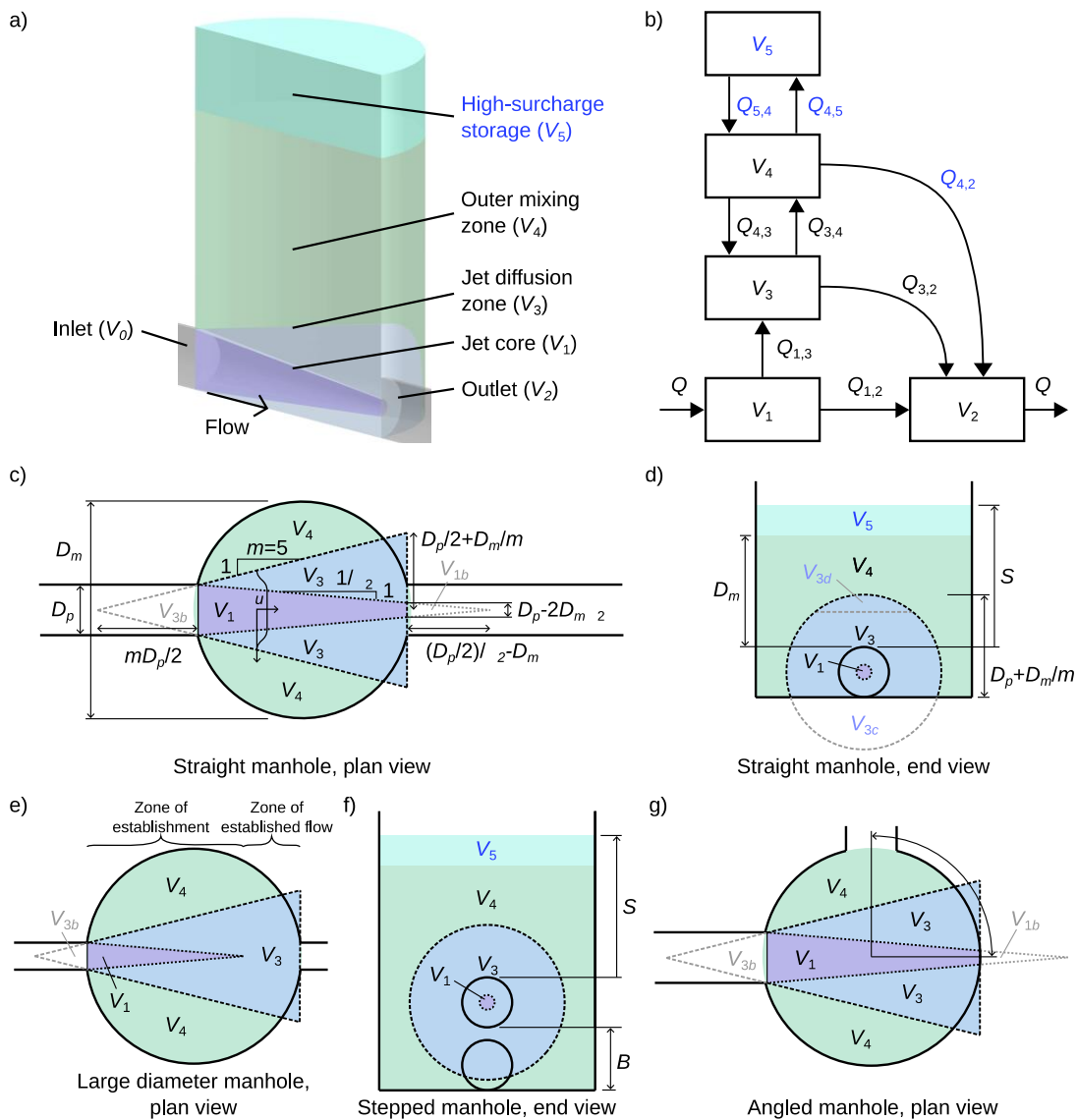


694

695 **Figure 2.** Example CRTDs representing different mixing processes, the short-circuiting curve

696 indicates around 65% of the flow short-circuits ($F \approx 0.65$)

697



698

699 **Figure 3.** a) Cross-section indicating main model zones in a simple circular manhole, b) flow

700 relationships between model zones, c) plan view (also illustrating the jet velocity profile) and

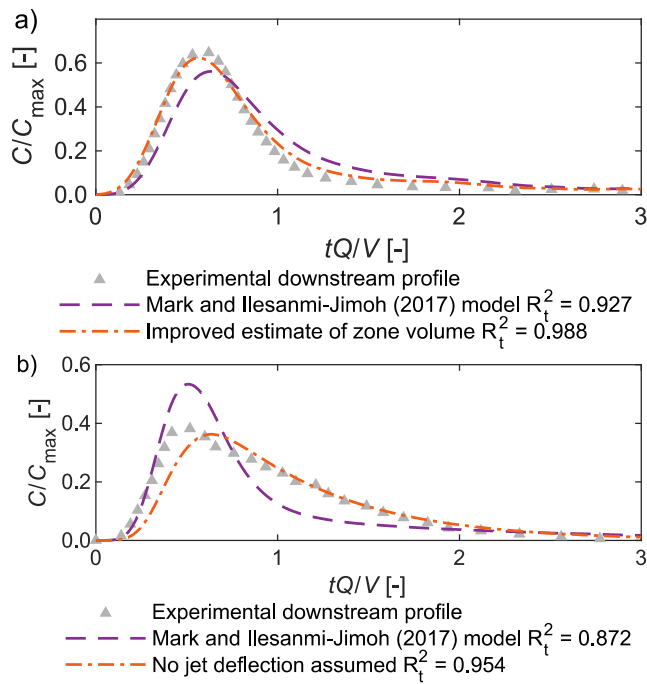
701 d) end view of a manhole with $D_m < L_j$, e) plan view of a large manhole with $D_m > L_j$, f) end

702 view of a stepped manhole, and g) plan view of an angled manhole, the gray areas indicate

703 subtracted volumes, the blue text indicates new compartmental model developments, diagrams

704 not to scale

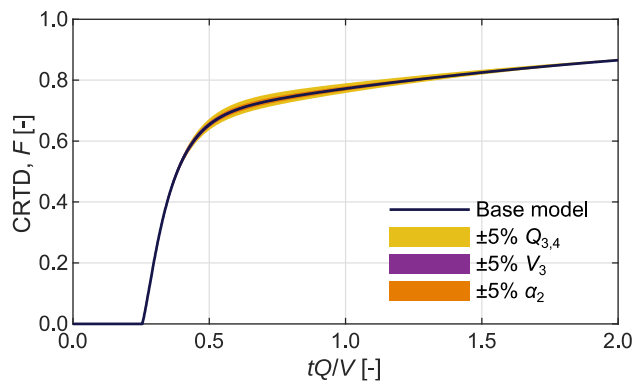
705



706

707 **Figure 4.** Illustration of a) new estimates of zone volume and exchange (in a
 708 $D_m/D_p = 4.4$ manhole) and b) assumption of no jet deflection (in a $B/D_p = 1.5$ stepped manhole)
 709 made by comparing an experimental downstream manhole concentration profile to predictions,
 710 scaled by peak upstream concentration, with R_t^2 values

711



712

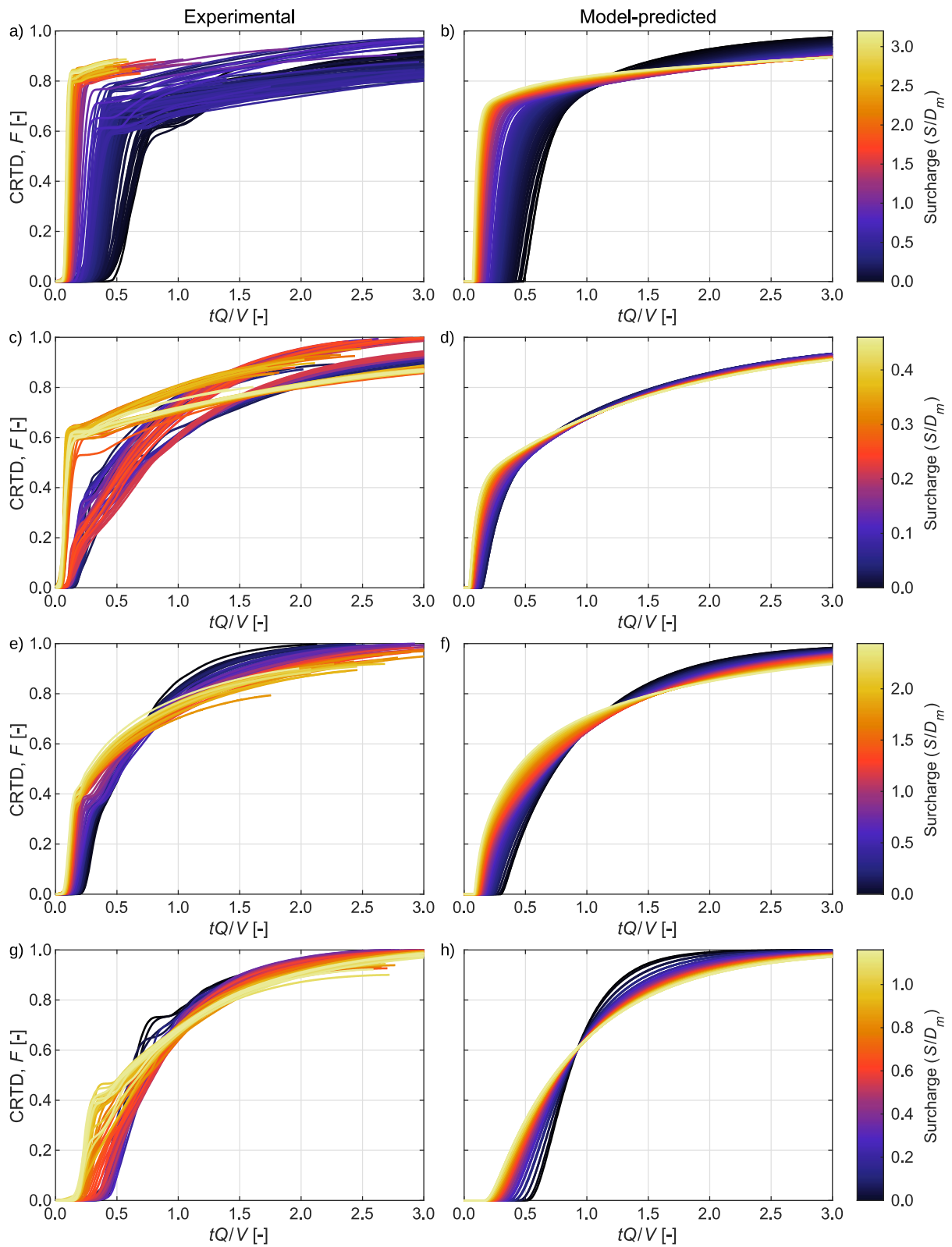
713 **Figure 5.** The sensitivity of new compartmental modelled CRTDs to key model parameters

714 $Q_{3,4}$ controlling exchange between jet diffusion zone and outer mixing zone, V_3 volume of jet

715 diffusion zone (and hence also affecting the volume of outer mixing zone), and α_2 affecting jet

716 core length and hence the amount of flow directly reaching outlet

717



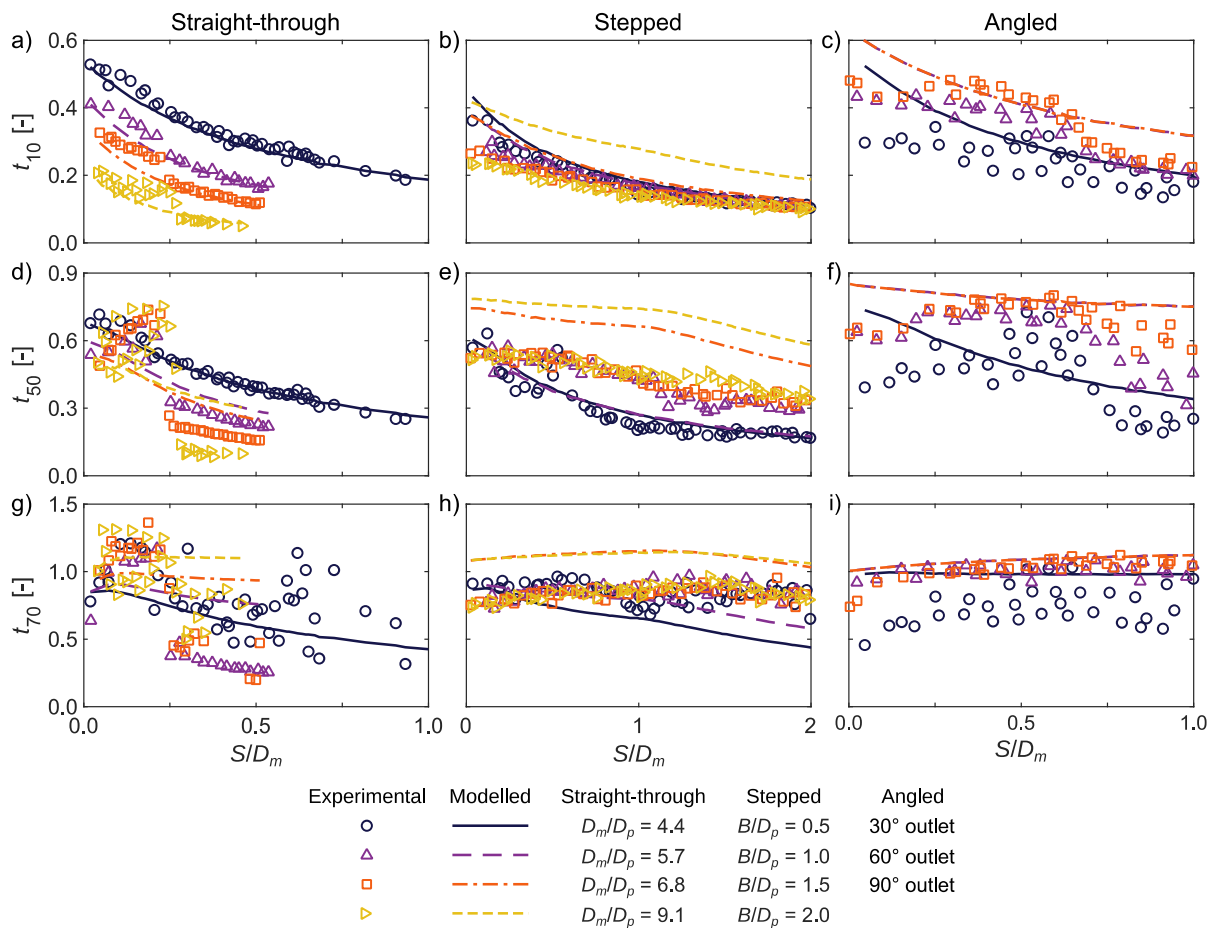
718

719 **Figure 6.** Comparison of experimental deconvolved CRTDs (left) with new compartmental
 720 modelled CRTDs (right) for a) and b) straight-through $D_m/D_p = 4.4$, c) and d) straight-through

721 $D_m/D_p = 9.1$, e) and f) stepped $B/D_p = 1.5$, and g) and h) 90° angled outlet manholes, CRTDs

722 groups separated by $S/D_m \approx 0.27$ in c)

723



724

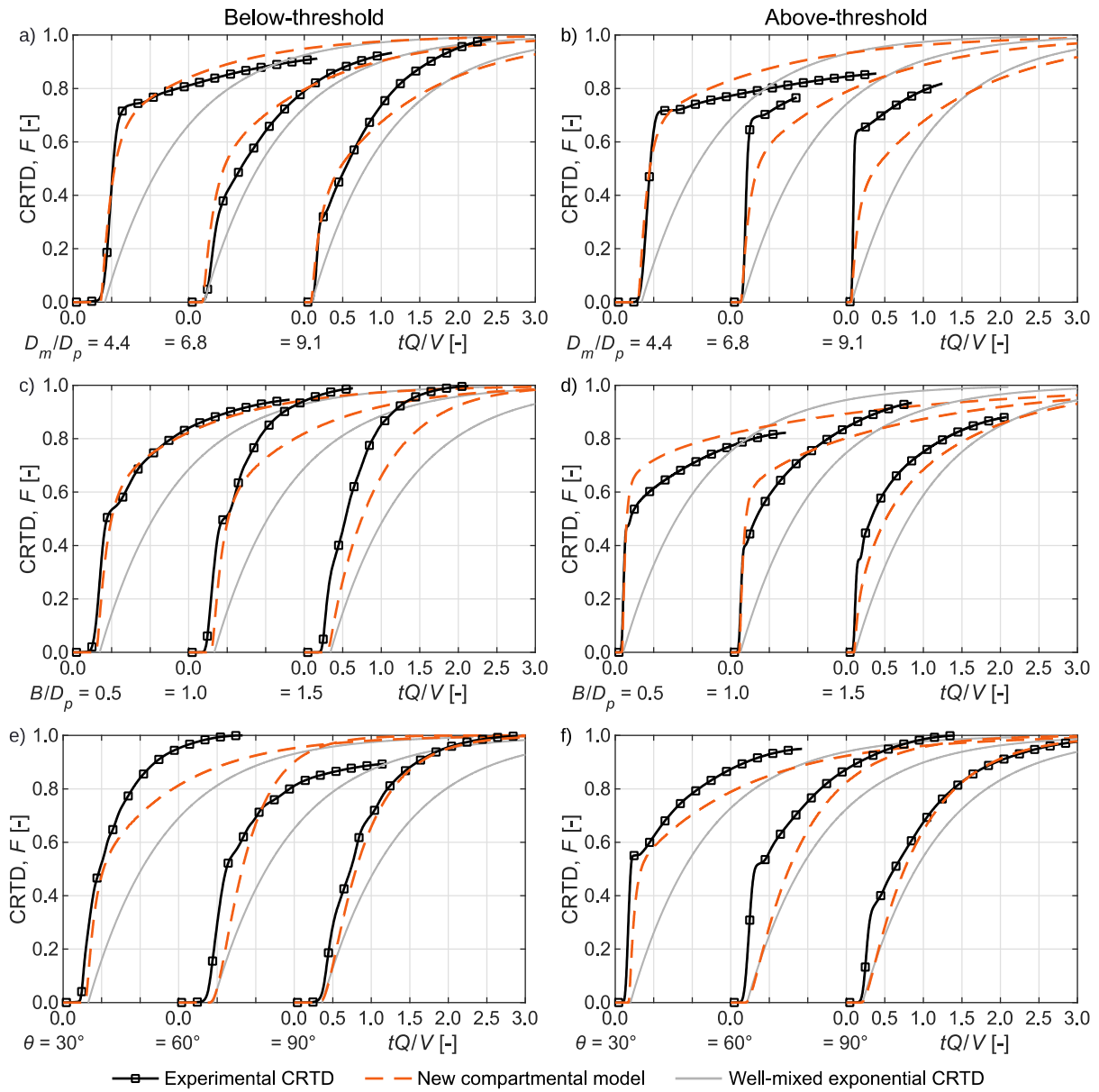
725 **Figure 7.** Comparison of experimental and new compartmental modelled dimensionless a-c)

726 t_{10} , d-f) t_{50} , and g-i) t_{70} for a), d), g) straight-through, b), e), h) stepped, and c), f), i) angled

727 outlet manholes (error bars on experimental data not shown, but on the order of 0.005, 0.020,

728 and 0.050 for t_{10} , t_{50} , and t_{70} respectively)

729



730

731 **Figure 8.** Characteristic experimental, new compartmental model, and complete

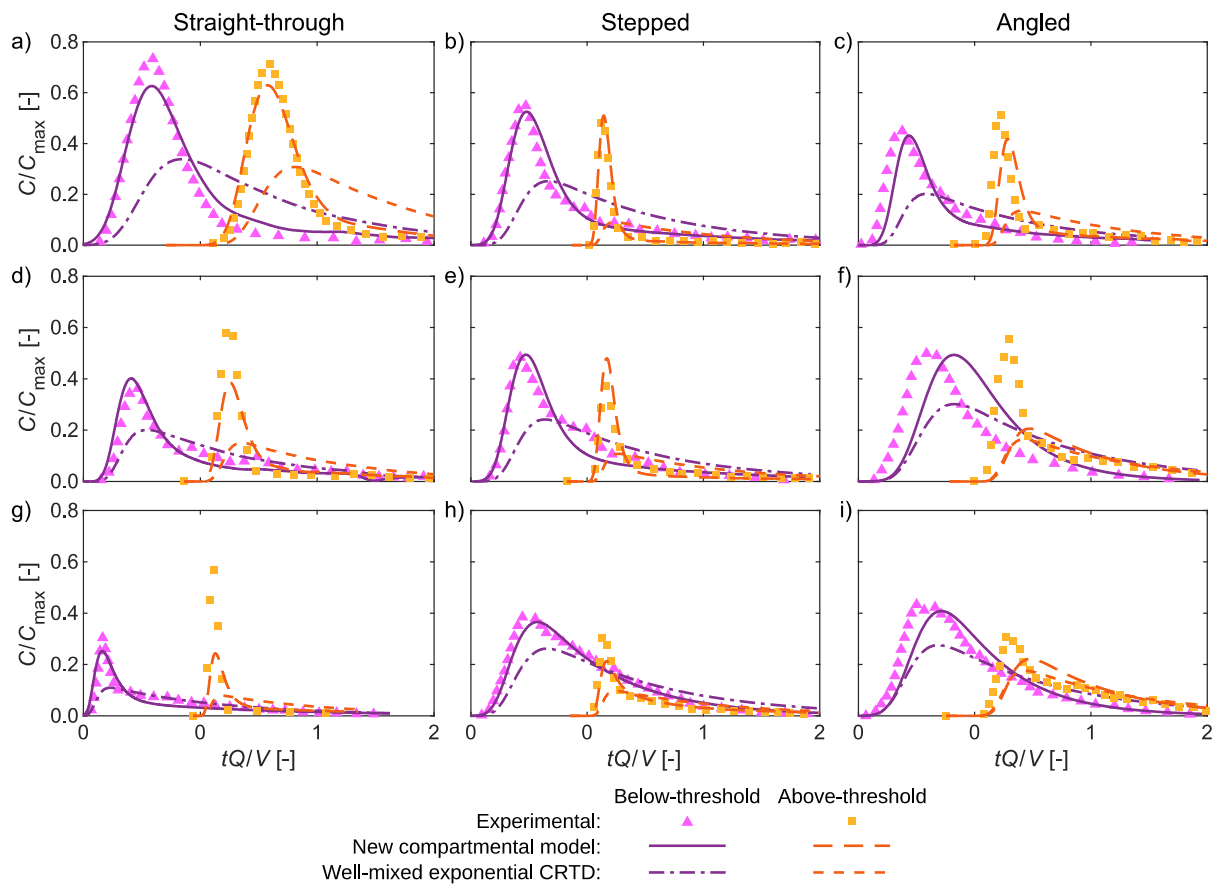
732 instantaneously well-mixed exponential model a) below- and b) above-threshold straight-

733 through $D_m/D_p = 4.4$, $D_m/D_p = 6.8$, and $D_m/D_p = 9.1$ manhole CRTDs, c) low and d) high-

734 surcharge stepped $B/D_p = 0.5$, $B/D_p = 1.0$, and $B/D_p = 1.5$ manhole CRTDs, e) below- and f)

735 above-threshold 30° , 60° , and 90° angled manhole CRTDs

736



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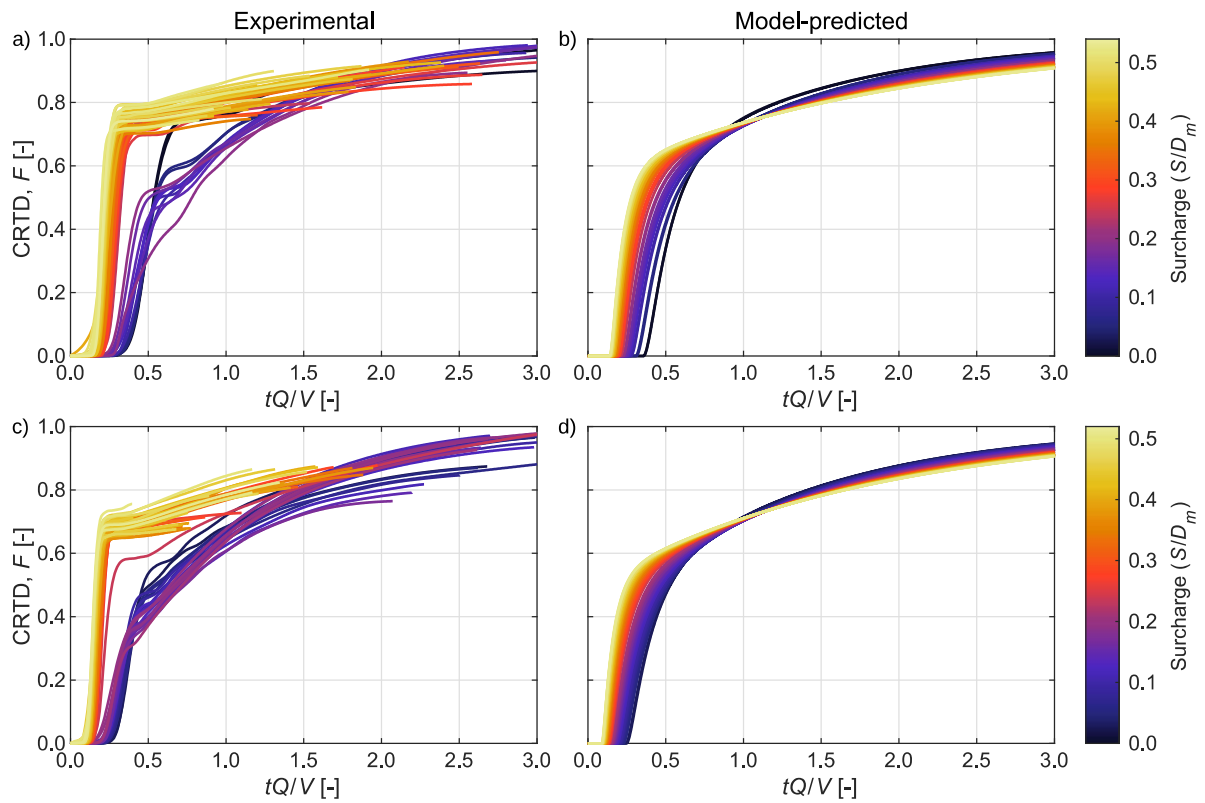
738 **Figure 9.** Example straight-through a) $D_m/D_p = 4.4$, d) $D_m/D_p = 6.8$, and g) $D_m/D_p = 9.1$;
 739 stepped b) $B/D_p = 0.5$, e) $B/D_p = 1.0$, and h) $B/D_p = 1.5$; and angled c) 30° , f) 60° , and i) 90°
 740 downstream below- and above-threshold/low- and high-surge (the latter offset by
 741 $tQ/V = 1$) downstream manhole concentration profiles with new compartmental model
 742 predictions, traces correspond to the CRTDs shown in Fig. 8, scaled by peak upstream
 743 concentration, R_t^2 values given in Table 2

1 **SUPPLEMENTARY MATERIALS**

2 **Table S1.** Manhole configurations details and Guymmer et al. (2020) dataset Configuration ID
 3 (CID) for mean CRTDs used in Fig. 8, along with dataset File ID (FID) for the matching
 4 solute traces used in Fig. 9 (all have 88 mm diameter pipe and 2 l/s flow rate)

Fig. 8	D_m/D_p	B/D_p	Outlet Angle	S/D_m	Dataset CID	Fig. 9	Dataset FID
a	4.4	0	0°	0.21	18	a	851
	6.8	0	0°	0.15	180	d	893
	9.1	0	0°	0.15	1064	g	4484
b	4.4	0	0°	0.39	25	a	19
	6.8	0	0°	0.28	186	d	923
	9.1	0	0°	0.27	1069	g	4507
c	4.4	0.5	0°	0.24	321	b	1597
	4.4	1.0	0°	0.16	412	e	2052
	4.4	1.5	0°	0.03	496	h	2472
d	4.4	0.5	0°	2.32	329	b	1637
	4.4	1.0	0°	1.70	418	e	2082
	4.4	1.5	0°	1.84	503	h	2507
e	4.4	0	30°	0.41	880	c	3903
	4.4	0	60°	0.15	937	f	4074
	4.4	0	90°	0.31	1003	i	4272
f	4.4	0	30°	1.02	887	c	3924
	4.4	0	60°	0.92	947	f	4104
	4.4	0	90°	0.92	1011	i	4296

5



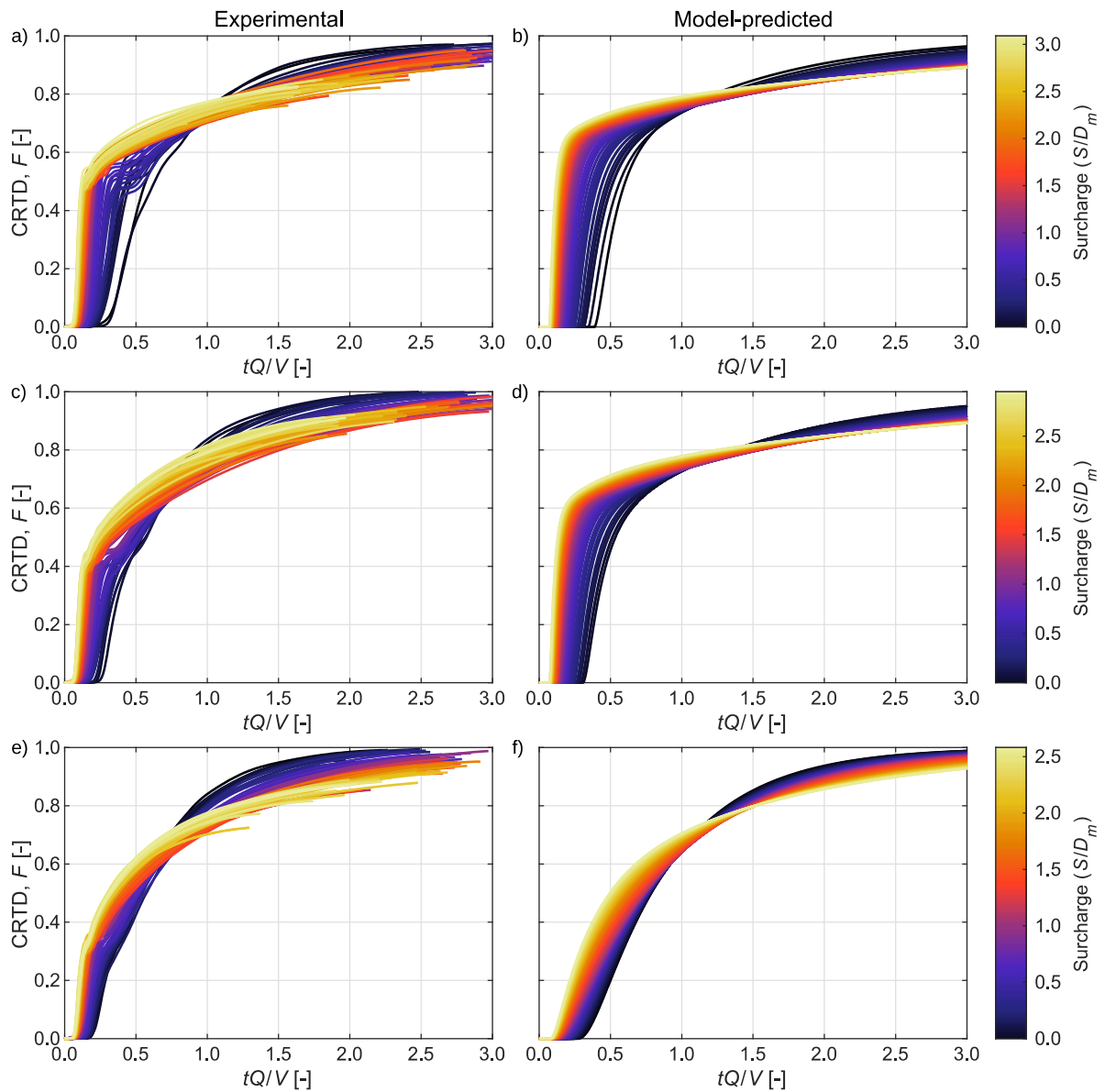
6

7 **Fig. S1.** Comparison of experimental deconvolved CRTDs (left) with new compartmental

8 model predicted CRTDs (right) for straight-through a) and b) $D_m/D_p = 5.7$ and c) and d)

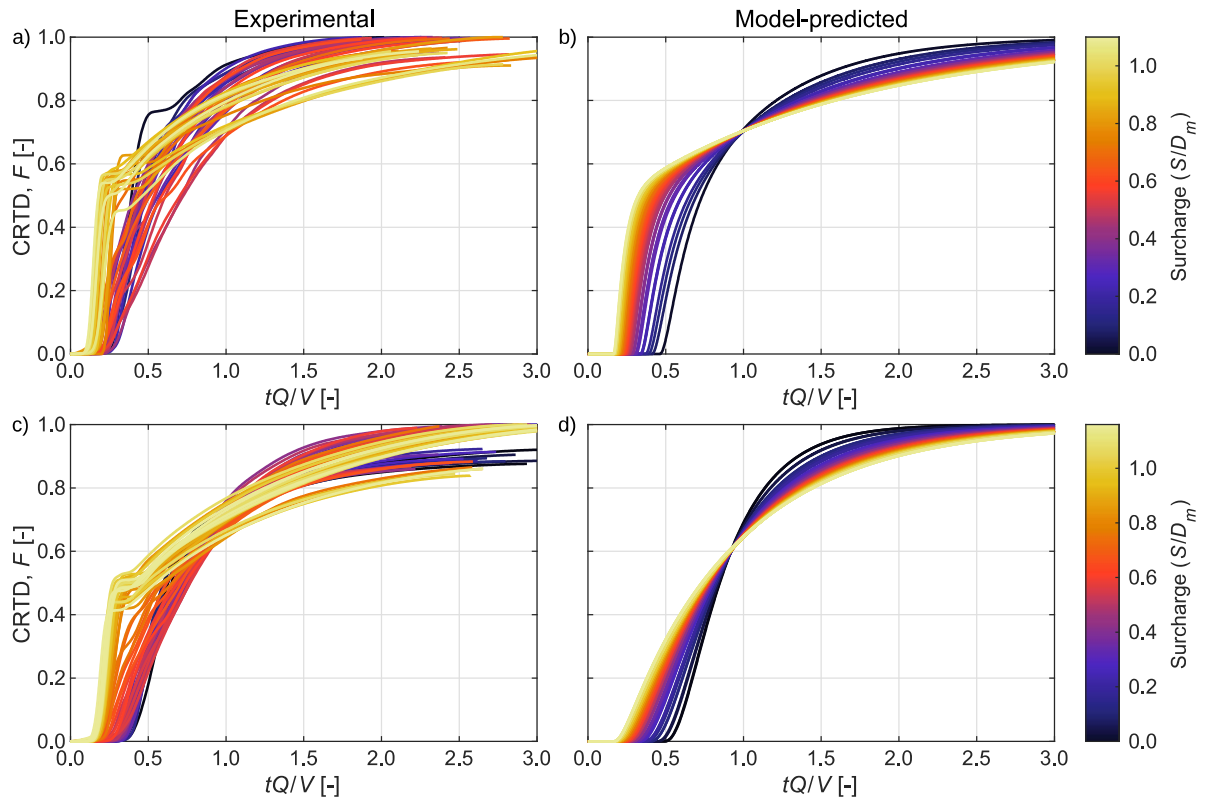
9 $D_m/D_p = 6.8$ manholes, CRTDs groups separated by $S/D_m \approx 0.24$ and $S/D_m \approx 0.26$ in a) and c)

10 respectively



11

12 **Fig. S2.** Comparison of experimental deconvolved CRTDs (left) with new compartmental
 13 model predicted CRTDs (right) for stepped a) and b) $B/D_p = 0.5$, c) and d) $B/D_p = 1.0$, and e)
 14 and f) $B/D_p = 2.0$ manholes



15

16 **Fig. S3.** Comparison of experimental deconvolved CRTDs (left) with new compartmental
 17 model predicted CRTDs (right) for angled a) and b) 30° and c) and d) 60° outlet manholes,
 18 CRTDs groups separated by $S/D_m \approx 0.71$ and $S/D_m \approx 0.78$ in a) and b) respectively