Opinion Formation among Mobile Agents

A dissertation submitted for the partial fulfilment

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To my lovely family
for their patience and support.

Abstract

The evolution of public opinion has been widely studied to understand how atomic interactions between individuals cause opinions to evolve. However, while many studies have paid attention to the influence and interaction mechanisms, the vast majority of the literature assumes a static representation of immobile agents, ignoring the effect that physical proximity and mobility has on interactions, as observed in real-life.

Mobility provides humans with the opportunity to meet and locally interact with a diverse range of people, which can heavily influence opinion spread in human societies. Considering both opinion and location dynamics on widely used opinion models, such as the Bounded Confidence model, can therefore result in more realistic understanding of the drivers that cause agreement and diversity.

This thesis investigates both directed and random mobility, inspired by two fundamental concepts from psychology: homophily and cognitive dissonance. These theories can drive the response behaviours to agreement and disagreement in humans. We translate these as attraction and repulsion forces in our mobility model. Through incorporating these phenomena, we quantify the different outcomes that arise and propose new evaluation metrics for analysis in this context that capture the formation of opinions and communities, reflecting the self-organisation among the populations.

Extensive simulation results demonstrate the impact of the random and directed mobility. The main findings show that opinion formation is highly insensitive to random mobility, showing similarity in behaviour to static modelling. This is a very important result because the literature usually applies this approach. Furthermore, we find that alternative psychological theories, as incorporated into mobility, impact differently on both the opinion and spatial organisation of the agents. As these parameters are varied, we find a distinct transition in behaviour. Finally, by combining and analysing all the results, we propose a novel classification approach for different outcomes of self-organisation in opinion models.

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Contents

List of Figures

List of Tables

List of Algorithms

List of Acronyms

ABM Agent-Based Modelling

DW Guillaume Deffuant, David Neau, Frederic Amblard, and Gérard Weisbuch

DBSCAN Density-based Spatial Clustering of Applications with Noise

HM Hybrid Mobility

RM Repulsive Mobility

AM Attractive Mobility

RRM Random Repel Mobility

PRM Pure Random Mobility

Chapter 1

Introduction

In this chapter we discuss the motivation of this research and identify the novel contributions to the literature. We also provide an outline to clarify the points covered in each chapter. Finally, we present the list of publications which have resulted from the research in this thesis.

1.1 Background and motivation

Human behaviour, and particularly the interactions we have with our peers, has a profound effect on the nature of consensus that emerges within social groups. Due to the rise of on-line social networks, it is increasingly easy to widely share opinion on a given subject, and thereby further influence peers. This general phenomena is represented by the field of opinion dynamics, which is rapidly increasing in relevance to society [16, 13, 68, 14]. Opinion dynamics is traditionally studied via agent based modelling [16, 103], which involves understanding how the individual interactions between agents leads to the formation of shared opinions across groups and sub-populations. This is important because it can influence behaviour (e.g., around political/voting decisions [104]) and is closely related to social representations and pursuits, such as the problems of competing cultures [9], use of different languages [17, 4] or even searching for a buzz word around a set of vocabularies [11].

In this thesis we explore opinion dynamics to investigate different drivers that impact opinion evolution, specifically concerning social attraction and repulsion, originating from human psychology, and how these manifest themselves through agent mobility. Many of the previous investigations in this area are physics based, using techniques and methods inspired from physical systems. Therefore these models are often expressed as analogies of stochastic processes which were originally applied the properties of matter rather than human beings. For example, some of the techniques used percolation theory [105], synergistic [107], differential or partial derivative equations [78, 102], and Boltzmann-like equations [52].

However an alternative approach, as considered by this thesis, is to take inspiration from behaviours that are observable in humans. For example, in sociology, empirical evidence has shown that the frequency or probability of an interaction occurring between two people is dependent on their proximity [73]. Another theory that supports this is propinquity theory, which states that physical proximity increases the frequency of encounters on a regular basis. Consequently this raises the chances of being friends [36]. The power of propinquity explains how influence is created through relationships, and it is an important factor leading to interpersonal attraction. Also, [87] noted the fact that proximity is dynamic and the distance fluctuates with time due to peoples movement over time. These explicit relationships reflect restricted interactions when the world is more open for different possibilities and encounters.

When it comes to modelling assumptions, opinion models have often been criticised for neglecting a number of realistic features of social interaction, perhaps most significantly at a local level, the possibility for agents to actually move in physical space, in relation to others [103, 102, 16, 116, 53]. For example, [23] used proximity and homophily in order to recommend a new contact. This work also showed that social relationships can explain 10-30% of human movement while periodic or pattern based movement explains more then 50%. Given substantial social psychological research on the relationship between impact and distance (i.e., the proximity-influence relationship), there appears to be limited research on opinion evolution in settings where mobility is captured between the participants as a natural feature - i.e., personal mobility is influenced by psychology. Indeed, most research in this area is conducted on static settings where agents location are not dynamic [43, 9, 29].

Considering features such as proximity and mobility is especially important in scenarios where large numbers of individuals congregate but a social network is not present, for example, in a conference or the first day at university, etc. In these types of scenarios there isn't a pre-existing network, but instead people walk around and interact with each other and eventually form groups.

The model presented in this thesis abstracts fundamental principles of psychology (i.e., attraction and repulsion towards others) and applies this through response behaviours as a consequence of opinions. One of the interesting things about this approach is the co-evolution of opinion clusters and segregated communities that fluctuate in a spatial context. This can be complex to accurately interpret and therefore we have developed a robust multi-dimensional framework that allows the relationship between different key parameters to be assessed. In developing this work through agent-based modelling, we have observed that there is a substantial lack of rigorous methodologies that allow agent-based models to be understood in full detail. As a consequence our approach to analysis aims to resolve this, and delivers a categorisation of models. This provides an additional contribution to the field.

1.2 Contributions

Below we summarise the main contributions made in this thesis.

1. *Free-space opinion formation.* We propose a novel opinion model in a 2D free space environment via agent-based modelling. This environment enables analysis of the **impact of distance** (instead of explicit or fixed links) on interactions and community formation, which **captures features not previously considered**

in previous studies. The model allows an in-depth assessment of the groups that form through pairwise local interactions.

2. *Criteria for evaluation.* We propose new evaluation metrics that assesses the behaviour of the model and return quantitative results that can be compared against other models with rigour. This includes the development of functions that capture stability in the models for both opinion and movement, in particular, **measuring stability in movement** is a novel addition to the field that allows new insight into the structures that emerge between agents. Also, other functions are developed to measure opinion clusters in a quantifiable manner, communities formation in geographical space and tolerance ratio to different others. These metrics are evaluated by the characteristics of opinion in geographical space.

3. *Systematic review on mobility.* We identify why mobility within opinion models has received limited attention and identify the gaps in the field, through a systematic review focusing on mobility models. We analyse the models in the literature and break down the related algorithms in detail to come up with a **categorised table of mobility models for opinion formation**. Mobility has been applied in the opinion modelling literature in a limited manner. However, the associated literature is scattered and mobility isn't generally the main consideration under investigation. This means that the literature is fragmented with weak paths of citation between related papers.

4. *Alternative forms of mobility.* We demonstrate how alternative models of mobility allow more realistic behaviour to emerge, in contrast to the complete consensus often found in the literature, by proposing **new mobility models that are inspired by psychological theories**, where the mechanisms are directional in response to an interaction (instead of random mobility).

5. *Classification of opinion dynamics*. We identify and categorise the forms of self-organisation that occur through the co-evolution of opinion and mobility. This

involves identifying the commonality between a range of parameters. We specify this behaviour by synthesising results from different models. We generate a **classification of six types of behaviours that describe the emergence of self-organisation**.

1.3 Thesis outline

The following chapters are organised as follows:

Chapter 2. In this introductory chapter we summarise the field of opinion modelling, and provide a clear framework for easy discussion of the literature. We identify the key papers in opinion modelling as well as the field's challenges. This identifies the gaps and limitations of the field that frames our work. Specifically, we provide a detailed literature review for the previous works that have considered mobility (or some form of change in their neighbourhood structure), noting details on the rules for *when* and *how* mobility is applied.

Chapter 3. We introduce our model and explain how opinions are incorporated with mobility. We include a free space environment to reflect the physical proximity as a factor for communication. We believe restriction in distance is more realistic for describing potential interaction opportunities instead of an explicit pre-defined network structure that is static. Mobility and free space are rarely considered in the opinion formation literature. Therefore, we include alternative mobility mechanisms, enabling a detailed investigation of the impact of mobility on the co-evolution of both opinion and the location of agents, providing a new spatial perspective on opinion formation. We take into consideration the importance of making the psychological theories as a basis for development of the model (instead of the analogy of particles in physical matter). Also, we propose new evaluation metrics that include geographic aspects for a quantitative analysis, which are often not clearly defined in the existing literature. For detailed analysis we provide both a visual and quantitative description of the results, introducing clustering to assess the structural configurations that are present in agents positioning. This chapter provides us with a basis to conduct experiments and explore the agent's co-evolution of opinion and location simultaneously.

Chapter 4. In this chapter we undertake an initial evaluation of the model and the proposed evaluation parameters across a range of experiments. This gives us benchmark information that will be useful in other chapters. In particular, since the simulations may be sensitive to the input parameters, we therefore use this investigation to validate our choice for default evaluation parameters. We continue exploring the chosen opinion model to present the mechanics of the original Deffuant-Weisbuch opinion model before incorporating our extension.

Chapter 5. In this chapter we explore the outcomes of mobility that represents the psychological behaviours identified above. These mobility models are inspired by homophily theory and cognitive dissonance theory. We define two types of mobility: one is a directed mobility model that moves based on decision-making related to the peer's opinion, and the other is triggered by disagreement and subsequent repulsion to a random location. We compare and synthesise the differences between these mobility models, noting that mobility itself hasn't caught light in the wider opinion formation literature. This work has been published in [5].

Chapter 6. In this chapter we explore the robustness of the mobility model to noise. This represents the uncertainty and randomness that occurs when individuals express their opinion, important aspects of human decision making in a real world context. We provide a detailed literature review as a basis for these experiments and their context. This work has been published in [6].

Chapter 7. In this chapter we compare between two models, one where the movement is triggered by psychological theories (i.e., a social driver) and the other analogous to particles where agents constantly move at random. We study these models and compare them against static models (i.e., as commonplace in the literature), investigating how the agent interaction range affects the formation of structure. We also study how significant random mobility is, in comparison to static models. Finally, we study the convergence and stability of the directed mobility model to establish the model's characteristics, and to determine whether it requires further consideration.

Chapter 8. In this chapter we conduct a study across all the proposed mobility models as presented in Chapter 3. We widen the parameter space to an extent that hasn't been studied before in the literature. We identify a number of key scenarios that describe how agents self-organise themselves by synthesising between common parameters that behave similarly. This leads us to propose a new classification approach that enables comparison between different mobility models. The literature has not previously considered mobility as a significant ingredient in opinion modelling and neither their geographic location, therefore, this structured classification approach sheds new light on the different ways self-organisation can appear via a spatial approach. This work is in preparation for The Journal of Artificial Societies and Social Simulation.

Chapter 9. In this chapter we summarise the thesis and limitations of this work, as well as highlighting proposals for future work.

1.4 List of publications

1. Alraddadi, E.E., Allen, S.M. and Whitaker, R.M., 2019, September. Homophily, mobility and opinion formation. In International conference on computational collective intelligence (pp. 130-141). Springer, Cham.

2. Alraddadi, E.E., Allen, S.M., Colombo, G.B. and Whitaker, R.M., 2020. The role of homophily in opinion formation among mobile agents. Journal of Information and Telecommunication, 4(4), pp.504-523.

Chapter 2

Background and Literature Review

The methodology of the study in this thesis follows the same three principles as used in Axelrod's model of social influence and culture [9]:

Agent based modelling We will draw conclusions from computational simulation of a population of agents, whose interactive behaviour is specified by simple rules.

No central authority There is no central coordinating agent or external influence in the model. Agents only have knowledge of their own opinion and the opinion of an individual agent that they interact with. After being assigned an initial opinion value, agent's can only change their opinion through pairwise interactions with their peers.

Adaptive rather than rational agents Agents only follow the simple rules defined in the model, and do not have wider goals, costs or benefits. In particular, they do not "game" their behaviour in order to promote their own opinion more widely.

In this chapter we provide a summary of the literature relevant to this approach, beginning with the general approach of Agent-Based Modelling which is commonly used to study opinion dynamics. After providing an overview of the individual components used in opinion modelling, we provide a detailed literature review specifically on opinion models that incorporate mobility. Finally we discuss the challenges that arise which motivate our study in this thesis.

2.1 Agent-Based Modelling

Agent-Based Modelling (ABM) is a powerful tool for research, commonly used to model the outcomes that arise from the actions and interactions of different agents. In this context, an agent is an entity defined by a number of attributes/properties which is autonomous, modular and social [81]. ABM consists of a population of rule-based interacting agents with autonomous behaviour in the context of some environment. Commonly, agents emulate simple versions of human individuals as well as other virtual or living entities, and the environment is the 'space' in which they interact. ABM can be used to observe the emergence of higher level systems behaviour from local micro-level rules carried out by an agent. A thorough explanation of ABM simulations can be found in [49, 50].

Classic examples of the resulting emergent behaviours include Boids [99] and the Game of Life [46]. In both cases simple rules carried out by individuals result in complex collective phenomena. These models have been implemented for different purposes, for example, the Boids model was originally used to explain the behaviour of flocking birds. The algorithm has three different functions that control the overall behaviour of the agents movement: cohesion, separation and alignment. Further research has used the model for crowd simulation and other applications [67]. [86] extends the boids algorithm to demonstrate time-varying data sets. Other purposes for using ABM include simulation of emergency evacuation, pedestrian behaviour and activity in auction-type markets [21].

Opinion dynamics is also an important application for ABM such as in [9, 61, 29]. The field is quite complex with many human characteristics and uncertainty involved, combined with the *lack of available data* for modelling or validation. However, for the last couple of decades, computation power has increased as well as processing speed and memory capacity. This has enabled ABM to be used more to study social simulation, where actions and interactions can be represented [13], so that foresight can be made. With opinion dynamics, the agents represent individuals holding opinions, and rules are defined to simulate the sharing, updating and spreading of opinion. In this sense, ABM is an approach that works as electronic laboratory, especially for cases where high quality data of opinion spread is not available [14, 10, 73].

This makes ABM a *common approach* used to simulate opinion dynamics under different conditions and assumptions. Alongside simulation, there are also some theoretical studies with analytical results [76, 78, 12, 94], that are more concerned about the macroscopic features such as density. Such models deal with differential equations, however they are analytically focused and not as flexible in the scope of experiments that they can consider.

However the ABM approach is better suited to exploring different forms of complexity. In particular, the ABM approach presents *'abstract models that allow for clarification or development of new theories or mechanisms'* [14]. *'The goal is not to reproduce existing patterns but to develop a new way of thinking about a problem and provide a great deal of theoretical stimulation for existing empirical research'* [14]. This makes the approach well suited to understanding human issues through social simulation. In this thesis we apply an ABM approach to opinion formation. Although critics may consider this as 'too simple' to capture the complexity of human behaviour, the advantage is that particular issues can be considered in isolation, so that understanding can be built. Therefore it is important to note that we are not aiming to predict human behaviour - instead we are analysing simulation outcomes to shed light on the underlying mechanisms that may control the behaviour of a system, with the aim of classifying scenarios where certain behaviours or outcomes may occur.

2.2 Opinion modelling

The field of opinion dynamics is a key sub-field of complex social systems, which aims to explain how the opinions held by individuals are changed through interaction with their peers, leading to global patterns and consensus. Opinion modelling is utilised to understand the mechanisms and conditions where opinions are influenced and certain behaviours and structures emerge in response.

The propagation of rumours or news is an instance from the vast class of social spreading phenomena, which includes the diffusion of fads, the adoption of technological innovations, and the success of consumer products mediated by word of mouth [16]. This makes opinion modelling of interest and applicable in a number of different real-world contexts. In the literature it has been used to describe political choices [29, 107, 45], the preference of different groups for residential [100], opinion formation [61], culture dissemination [9], the competition of different products in an open market [108] and the occurrence of information cascades in social and economic systems [113].

To provide structure for the literature review in this thesis, we will adopt a framework that represents a generalised process of opinion modelling, as represented in Figure 2.1. For easier discussion, this framework provides a basis that divides different aspects of the model into sections. This helps to overcome one of the challenges in the opinion modelling field, as previously stated in [16], *'the development of opinion dynamics so far has been uncoordinated and based on individual attempts, where social mechanisms considered reasonable turned into mathematical rules, without a general shared framework and often with no reference to real sociological studies'*. This general framework allows us to draw parallels and distinctions between the many different published approaches. Also, we must note that there has been a substantial increase in papers on opinion models, often with minimal changes to the model and little benchmarking against the literature. Therefore, in this thesis we have been selective, in only considering those that are most relevant.

Building a model of opinion dynamics that is consistent with existing social theories is challenging. However, the framework we use divides the complex modelling problem into five main parts to assess opinion models in the literature, and each describes one aspect of the opinion model (see Figure 2.1). Note that for static opinion models, of which there are many, the framework proposed in Figure 2.1 does not include the mobility influence component, as no movement is applied, which reflects on the majority of the literature. However, for completeness we incorporate *Mobility Influence*, which reflects on opinion models that consider the impact on the agents changing their locations.

For the purpose of this thesis to highlight the breadth of literature we'll be following the components of this framework. The first part of an opinion model is to define the *Opinion Representation*, whether it's by discrete numbers, real numbers, or some other form. Then *Social Interaction* is the process of selecting an agent to communicate or interact with. The main factor in opinion dynamics models is then *Social Influence*, the process by which individuals adapt their opinion, revise their beliefs, or change their behaviour as a result of their social interactions with others [89] and therefore we divide it into two categories. Firstly, *Opinion Influence* which describes the factors that impact the change in opinion. After that, the *Mobility Influence* reflects the agents response to that opinion by choosing their preferred location. Lastly, *Update* refers to updating the agent's properties for both their opinion's and location's accordingly. In the following Sections we explain each of the components in Figure 2.1.

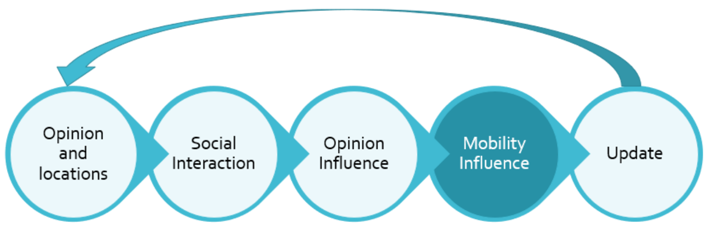


Figure 2.1: General framework for opinion models. Note that many models do not include the influence of mobility (highlighted).

2.2.1 Opinion Representation

Opinion has been represented in different ways. The mainstream models that are widely used, are discrete models [43, 107], continuous [29, 61] or a vector of discrete opinions [9, 40]. However, models have studied adding more features in the opinion representation. For example, instead of having a linear continuous opinion $[0, 1]$, some authors [98, 22] suggest a circular opinion where no extremes exist, so that 0 and 1 are the same, which can make modelling easier, but moves further from practical applications such as political preference. The paper [22] explains that if there are opinions A and B, A is not always chosen because it's preferred, but because they don't like B. In other words, the opinions may have dependencies. Different opinion representations depend in the type of motivating problem. For example, the [29] model was originally proposed to model political situations, specifically to represent the political spectrum of an individual which is not necessarily restricted to an extreme right or left wing but also positions in between [29, 77, 16], also each side of the spectrum can represent an extreme [82]. [78] explains that continuous opinions are more related to negotiation problems or fuzzy attitudes which can't be specified as yes or no decisions, whereas discrete models are more explicit to one opinion or the other. Despite the wide variation in approaches, the majority of papers can be traced back as variations to a handful of the most commonly applicable and well studied papers, which are summarised in Section 2.3 and Table 2.1.

2.2.2 Social Interaction

One of the important features in opinion formation models is the interaction scheme, which defines which group of agents are selected to take part in an interaction. Approaches in the literature include random selection on a global basis [29, 61] (some refer to it as homogeneous mixing or mean-field approach) in which a pair of agents are chosen uniformly randomly from the entire population. Others use various measures of local selection [93, 121, 102, 52, 61, 107, 29, 105, 9, 45, 71, 53, 54] and some use a combination of both local and global selections [48, 57]. Global interactions such as in [29], mean that all agents have the same probability to interact with anybody within the entire population, lattice or network, counter to sociology theories that indicate more structured interactions. For example, empirical evidence highlights geographical proximity as an indicator of increased interactions between peers [73, 72] or increased probability of friendship [36].

Many opinion models do take locality into consideration, however, with limitations on a fixed size neighbourhood [107, 43, 60] or only explicit group links [48, 52, 71, 54]. Most of the models use the Von Neumann approach to describe a neighbourhood, which explicitly presents a fixed neighbourhood. Usually group sizes differ and encounters can also be made with others that are not explicitly a group member. [87] noted the fact that proximity is dynamic and the distance fluctuates with time due to peoples movement over time, motivating our inclusion of a mobility component. These explicit relationships reflect restricted interactions when the world is more open for different possibilities and encounters.

2.2.3 Opinion Influence

This component defines how the information received by an agent during an interaction is processed and affects them in regards to their own attributes. In this section we will highlight different ways opinion influence is implemented.

Different models assume different rules of opinion adaptation:

- Imitation: where an opinion is copied exactly the same, e.g. [9, 107]
- Compromise: Where opinions of a peer gets closer to each other in opinion space, e.g. [29, 61]
- Physics-related equations: Usually described in deferential equations etc. eg. [102]

Some adaption rules depend on different size of influencers, such as following a group [43, 61], following a similar pair [107] or a single peer [29].

Other models incorporate more detail to the agents properties. For example [93] introduces social ties where their strength increases via frequency of interactions and therefore provides higher power to influence. [122] studies the opinion leaders strength in opinion formation. [97] demonstrates opinion leaders and followers under dynamic confidence levels. [47] demonstrates external media pressure on homophilic network structure. [119] introduces agents with personalities. [91] presents a representation of personality knowledge based on personality theories processable in fuzzy logic.

In real-life scenarios people's encounters and exposure to others is dynamic and therefore provides influence as well. This relates to mobility and interactions, providing motivation for our research in this thesis.

2.2.4 Mobility Influence

Particularly for mobility and influence, the literature is challenging to read due to the fact that many researchers appear not aware of the others work, evidenced by a lack citation links. In fact some have published in the same journal, for example, [105, 102] and [83, 53] without cross referencing each other. Therefore, for ease of discussion we categorise the different aspects of mobility into two parts, namely the *environment* that describes how agents are structured in space to perform mobility and the *mechanism* for their movement.

2.2.4.1 Environment

Several approaches to mobility have been taken in the literature on opinion dynamics. The most structured of these consider agents that are located on a lattice (e.g. [121, 93, 53, 105]), in which agents move to an empty space on the lattice (when available). Although this allows computationally efficient simulations, the limited discrete spaces available greatly constrain movement, which impacts on the formation of groups with any significant similarity. Also, lattice-based simulations typically have fixed-distance radii of communication, whereas implementing simulation in a continuous space still generally has a radius of communication, but is non-trivial [67]. Continuous space might be thought non-trivial due to the lack of experiments under these settings, however, it adds flexibility to the interaction.

While agents in a network allow more realistic social structures, they share similar issues to lattices, such as the absence of freedom in group formation. This is especially an issue given that most network studies are spatially constrained, so geographic distance isn't considered as one of the influential factors [15]. [112] has shown that mobility measures has more power in predicting links than network-based measures. Furthermore, [74] has studied proximity impact with empirical data and conclude that the average number of interactions people find noteworthy or *memorable*, being is proportional to the inverse of the distance at which individuals live. This shows that geographical distance is a factor for the people we choose to interact with frequently.

There are only limited approaches in the literature that consider unconstrained or free-space movement such as [102, 60], which may overcome some of the spatial limitations in previous studies. Although there is a breadth of research on the relationship between impact and distance (proximity-influence relationship) [74, 73, 36, 95], to date there is shortage of works specifically on opinion evolution in more unconstrained spatial settings. Research around opinion models seem to follow a norm under the same settings such as a lattice or a social network, the focus has been on adding variation to the models such as features resembling human factors, different approaches of interaction etc. However, to the best of our knowledge, geographic distribution, distance and proximity have not been considered in detail.

In summary, limitations of previous models concerning the spatial environment focus on:

1. Fixed neighbourhood size; members in a community are fixed.

2. Available empty sites; if an empty site is not found that will lead an agent to

move to another space that was not intentional due to the limited space that's occupied.

3. Discrete space restricts degrees of freedom interaction; free space gives wider flexible range of interaction outside explicit links.

4. Static neighbourhood members; the interacting peers are the same throughout the simulation.

2.2.4.2 Mobility mechanisms

Across the different mobility mechanisms that are proposed, the simplest approach to applying mobility is *random mobility* in which agents move constantly at random, such as in [121, 105, 45]. Others (e.g., [102]) present a model of discrete opinions based on Brownian motion, while [93, 53, 100] consider a lattice model in which movement is triggered by disagreement. Disagreement is also used to *trigger changes* in network structure (e.g., [48, 71, 64, 96]), however this is based on social group membership without including location.

While different forces of influence are often studied, the mechanism of when/how to move is rarely studied. In fact in everyday life our movement often follows an incentive. For example, in the university, we might change our lunch table because we disagree or don't fit with those who are around us, with the change resulting in naturally being exposed to new people. To the best of our knowledge, these dynamics have not been explored in the literature to date. We believe mobility influence can be modelled and mapped on to agents and locations, just as opinions are.

2.3 Key papers

In Table 2.1 we present an overall view of the most noteworthy and heavily cited opinion models that have been influential in the field. These are all static models that con-

Model	Opinions	Social interaction		Social influence		
		Selection	Influence	Opinion	Environment	Mobility
Voter model [27, 63]	Discrete	Immediate neighbour	Peer	Imitate	Lattice	N/A
Axelrod model [9]	Discrete (vector)	Immediate neighbour	Peer	Imitate	Lattice	N/A
Ising model [107]	Discrete	Immediate neighbour	pair	Imitate	Lattice	N/A
Deffuant et al. [29]	Continuous	Global	Peer	Compromise	Lattice	N/A
Hegselmann et al. [61]	Continuous	Global	Group	Compromise	Lattice	N/A
Majority rule [43]	Discrete	Immediate neighbour	Group	Imitate	Lattice	N/A

Table 2.1: Key opinion models from the literature

sider individual agents which do not move, however some do have structure for agents location, either in a lattice or as nodes in a graph. In the next section we provide a detailed review considering mobility. This Table identifies the key papers and describes the opinion models following the framework presented previously (Section 2.2). [61] has noted the jump from linear models to nonlinear models, stating that linear models such as [30, 41], were carried out with mathematical tools such as matrix theory, Markov chains and graph theory. They continue to clarify that nonlinear models are described where the structure of the model changes with the states of the model given by the opinions of the agents. In this section we focus only on models that take an agent based approach.

One of the main differentiation points between these models is in their representation of opinions. The first type of representation is *discrete* opinions, usually shown as two alternative opinions (such as positive or negative) [107, 43]. Much of the existing modelling work about opinion dynamics has been addressed from a physics-based point of view, where the basic mechanisms of social influence are derived from analogies with physical systems [16, 106], in particular with spin systems. Some features presented in these models are too simplified to resemble society. For example, these models often lead to a complete consensus which is rare in many social scenarios. In reality we do not all become alike, but the forces in the models prevents continuous disagreement [3]. Differences is a fundamental situation to incorporate to represent a society where complete consensus is not always the norm. These models also assume that every agent has an equal chance to interact with others, however in reality a person might choose to not interact with another, for example, due to their difference in beliefs or from a lack of opportunity.

Well studied models that actually take into consideration opinion differences as a driver for interactions include the Bounded Confidence (BC) models [29, 61], which are characterised by opinions over a *continuous* interval. The two models are similar but they differ in their updating scheme, where [61] shows simultaneous update rather than a series of sequential pairwise interactions [29]. The need to calculate opinion averages of large groups of agents makes computer simulations of the [61] model rather lengthy as compared to [29] model ([16]). The models have been used to simulate both formal group meetings [61] and face-to-face meetings [29]. These models demonstrate a probability of influence, that is restricted between only those of similar interest. With continuous opinions, divergence is readily modelled (contrary to the discrete), allowing different forms of opinion clusters to emerge. Alternative configurations of opinion formation can emerge, usually described as single opinion of *complete consensus*, *polarisation* into two dominant opinion clusters or *fragmentation* of multiple opinion clusters.

Another highly cited model that used a similar threshold to have selective interactions is the Axelrod model [9]. This introduced a vector of multiple discrete opinions for each agent. This model studied the mechanics of the dynamics of cultural assimilation and diversity. Originally the model described cultures defined in terms of multiple features, but this model is equally used in opinion modelling.

We have identified the key papers of opinion models and explained the modelling process. We have shown that models differ in the smallest details of their interaction, for example whom has influence on whom. Each model suits a different problem depending on the context studied. In the next section we will discuss the challenges in opinion modelling.

2.4 Mobility in opinion models

In this section we shed light on the features of mobility approaches and mechanisms that have been applied to opinion models. We will focus on the different components of mobility models and provide a thorough review on the mechanisms they lead to.

Table 2.2 summarises work that considers mobility in opinion dynamics (to the best of our knowledge). Many fields became interested in researching opinion models, from sociologists and computer scientists to physicists, where an unexpectedly large body of research has been motivated by considering the mobility and interaction of agents as gas particles. Many of the papers that incorporate mobility into opinion models, as reviewed in Table 2.2, were published in physics journals[1] with [100, 57] as exceptions from mathematical journals and [19] from the social sciences.

Firstly we will reflect on the different definitions of the term *mobility* across the literature. The simplest approaches locate agents at discrete locations on a lattice, with mobility either changing the locations of individuals [100, 102, 45, 105, 93, 121, 53, 60, 98, 96] or swapping places occupied by pairs of agents [82]. Others refer to mobility in terms of allowing an agent to interact with a far away agent, even though neither agent will actually change their location [57]. A distinctive approach is presented in [34], which applies mobility on a toroidal grid, however in this work opinion and location don't co-evolve, as the agents first move to organise themselves, after which the opinion dynamics start.

Most of the work that has taken mobility into consideration in opinion modelling has applied purely uniform random mobility without considering the direction of movement ([105, 121, 98, 83, 45]). Furthermore, the trigger or reason for movement has typically been ignored and executed as an analogy to moving particles. Similarities and differences have often been used as triggers that cause a change in location (e.g. [18, 88]). At a more general level, studies have shown how the preference of people

[1]Based on the classifications of SJR Scimago Journal & Country Rank

Reference	Opinion model	Environment	Interaction	Mobility		
				Trigger	Dynamic	Inspiration
Alraddadi et al. [6]	BC	Free space	Neighbour	Agreement and disagreement	Move closer or away from a peer	Homophily [84] and Cognitive Dissonance [35]
Centola et al. [19]	Axelrod model [9]	Lattice	Neighbour	Disagreement	Augment neighbourhood	N/A
Galam et al. [45]	Voting [42]	Lattice	Neighbour	Random	Move (to unoccupied)	Reaction-diffusion automata [26]
Gargiulo and Huet [48]	BC	Network	Local and external	Disagreement	Re-linking	Cognitive Dissonance [35]
Gracia-Lázaro et al. [53]	Axelrod model [9]	Lattice	Neighbour	Disagreement	Moving (to unoccupied)	Intolerance [100]
Guo et al. [57]	Majority rule	Network (small world) [90]	Local and global	N/A	N/A	Levy flights [51]
Hamann [60]	[44] and [59]	Free space	Neighbour	Random	Move	Swarms
Holme and Newman [64]	Voter [27, 63]	Network (random)	Neighbour	Disagreement	Re-linking	N/A
Kozma and Barrat [71]	BC	Network (random)	Neighbour	Disagreement	Re-linking	N/A
Martins [83]	Voter [27, 63]	Lattice and network (small world)	Neighbour	Random	Swap	N/A
Pfau et al. [93]	Axelrod model [9]	Lattice	Distance/link strength	Disagreement	Move (to unoccupied)	[18]
Qiang et al. [96]	BC	Lattice and network (scale free)	Neighbour	Disagreement	Move (to unoccupied)	N/A
Ree [98]	BC	Lattice	Neighbour	Random	Move	N/A
Schelling [100]	N/A	Lattice	Neighbour	Disagreement	Move (to unoccupied)	Discrimination
Schweitzer and Hołyst [102]	Social Impact Theory [1, 75]	2D spatial structure	Social distance	Agreement and disagreement	Move	Brownian particles, Langevin equations
Sousa et al. [105]	Sznajd model [107]	Lattice (various)	Neighbour	Random	Move (to unoccupied)/swap	Lattice gas [8]
Zhang et al. [121]	BC	Lattice	Neighbour	Random	Move (to unoccupied)	N/A

Table 2.2: Opinion models with mobility

holding different ideologies to co-locate can lead to segregation, as they move out of a certain community or neighbourhood to a more similar one. [100].

Furthermore, there is a lack of consistency and comparison between the published opinion models that incorporate mobility, and the works are largely independent. To address these shortcomings, Table 2.2 summarises the opinion models with some sort of explicit mobility. However, because the literature with mobility is small we have added the *dynamic network* (rewiring links) models to give some further perspective on the literature.

Therefore, we add a few models that have a change in their *network structure*. In these dynamic networks, agents change their linking relationships depending on the policy or reaction toward the model. More work has been done on dynamic networks but we will only highlight a few in this table because it is not the main focus of this thesis. In addition, to extend the depth of mobility models we have included [100], although the authors do not model changes in opinion, but only apply mobility based on the opinions that individuals hold.

In the table, we categorise the details of the models rules, including the *mobility trigger* which describes the reason of an agent to take action. Some models perform movement after disagreement is encountered and others move entirely randomly at each time step. As for, *mobility dynamics* describes the execution of movement (changing location, re-linking etc.). Following this, we describe the inspiration behind the dynamics, which is typically based on physical phenomena rather than psychological concepts.

This section helps highlight the different features of mobility mechanisms and provides a strong basis to conduct an in-depth study of our model. A majority of works apply randomness in their decision making, either in their *mobility trigger* or their *mobility dynamics*. There are also some models that are triggered by disagreement. However, the agent's choice of location does not reflect the disagreement but it is simply randomly chosen, with exception of [48, 93, 64]. Therefore, we find it important to explore the area and investigate the dynamics of the mobility forces that would reflect on the emergence of opinions and communities.

2.5 Challenges in opinion modelling

From examining the literature, there are a number of challenges and gaps in the field of opinion dynamics that we will explain in more detail in this section. Many opinion models have been proposed that add different drivers and rules to previous models in an attempt to add more realism. For example, some have added personality traits to agents, such as openness [55, 66], while others have sought to represent the influence of knowledge through experts and lay people [85, 89, 111].

Extensive work has been undertaken to develop this field of opinion modelling, as found across several surveys [16, 78, 117, 116, 3, 32]. Unfortunately, although the number of surveys is increasing, they are largely covering the same material and not adding new insight over the years. For example, [32] is the latest survey, however, only the same mainstream opinion models are reviewed. Furthermore, in a way they only provide a list of variations, however, the difference between them or the conclusions of these models are not synthesised or reported, which makes these surveys of less use to the community. While it is understood that the number of published work in the field is very large and diverse, so it is hard to draw consistent and clear conclusions, however a consolidation of modelling and evaluation approaches that would make the field develop between the researchers more effectively. For this thesis we will focus and refer to the selection of works that are most relevant to our study.

2.5.1 Evaluation and real world data

One of the main issues in opinion modelling is that there is no consensus in the way opinion models are evaluated. For example, for convergence in opinion the interest is largely in the measurement of stability which has been very widely studied, however, it is not quantified in order to enable easy comparisons between other models or the function is not defined as in [121]. Another example, is the number of opinions, where works often plot individual opinion values against time and report the final opinions such as in [70, 115, 29, 22], however, there is frequently no function defined to quantify the actual numbers of opinion. Furthermore, many opinion models have been proposed but either details of the simulations are omitted, without enough detail of metrics [96] or with weak assumptions [98, 105, 57]. This shows that rigour is needed in the field and comprehensive evaluation methods are required to state the findings.

One of the main criticisms on opinion modelling as a field is that models are not validated by real world data [16, 103, 89]. Some attempts have considered using data but with limited success. Use of data is quite rare in the field, because it is generally very difficult to obtain at a suitable scale, over a long enough time period to show opinions changing, or at a sufficient level of detail. To some extant the ABM approach mitigates the lack of data, through computational studies with simplified models over a much wider range of scenarios, and allowing isolated facets to be investigated in detail.

We discuss the few models that have taken some type of approach to validate their opinion models. In many cases, the underlying data is not publicly available and the way the data has been collected and processed is not entirely clear or reproducible [52, 107]. One of the rare pieces of work found on collecting data is [89]. Their model is based on controlled experiments with around 50 participants, showing how participants answering factual questions revise their initial judgements after being exposed to the opinion and confidence level of others. Then, a simple process model is derived from their observations to demonstrate how opinions in a group of interacting people can converge or split over repeated interactions. However, their model has not been validated, and focuses on objective facts rather than more subjective opinions (e.g. *What is the length of the river Oder in kilometres?*). Another attempt by [52] collected a database received from an on-line game server, and used it to build a social network. However the data collected does not reflect on the opinions nor their dynamics, but only the network structure that is used for an agent based approach.

Others have proposed models that attempt to mimic collected data. A piece of work performed by [107] who collected data over a couple years based on asking “do you think the future will be good?". They used the data to validate their models prediction, however, not validating their micro-rules of social influence, such as their assumptions on the interaction or influence process. Furthermore, details about the nature of the data was not presented. For example, it is unclear if the people answering the question are the same ones after a period of time has passed, or is this a measure of collective opinion formation over the years. In other words are the dynamics per individual or collective. [118] has taken a similar approach, where they have collected data from twitter and recorded some observations. They then developed a model that produced similar properties to the actual data. Again, the data collected does not appear to track the changing viewpoints of individual users, but rather the population as a whole.

This shows tracking the actual change in opinion is quite challenging (see surveys such as [16, 103]), specifically concerning the measuring and tracking influence and to pinpoint the main factors. For this thesis our focus in on data consisting of both opinion and locations that is generated through simulation, based on specific assumptions. Through simulations that are extensively studied, we form rigorous conclusions against the assumptions.

2.5.2 Psychological features

Another potential criticism of previous opinion formation work is the absence of a link to psychological behaviour [16, 117]. As highlighted in [16], it is important that the mathematical rules used to model social mechanisms and simulate opinion dynamics are referenced to psychological and/or sociological studies, because there is usually strong empirical evidence that these can influence an individuals behaviour. [103] also emphasised on the poor linking between models and reality.

Many theories of social interaction are based on interpersonal communication and characterised by mutual attraction and proximity among local individuals sharing similar characteristics, such as age, gender or social class. [110] provides an extended explanation of social influence theories. One of the most well-known psychological theories is *homophily* [84], which describes the tendency of an individual to interact with other peers that share similar interests. This is often expressed by the proverb "birds of a feather flock together". A further profound concept of relevance is social impact theory [73], which explains that the amount of influence a person experiences in group settings depends on (a) the strength (power or social status) of the group, (b) immediacy (physical or psychological distance) of the group, and (c) the number of people in the group exerting the social influence (i.e., number of sources). Another theory that is very important and is more related to geographical distance is *propinquity* [36], that's referred to the similarity between individuals with respect to geographical space acting as a factor to interpersonal attraction. Self-categorisation theory [109] governs a person's sense of who they are based on their group membership (eg. family, sports etc.), where communication is based on "them" and "us".

The final psychological theory we note is that of cognitive dissonance [35], which describes our subconscious desire for internal consistency. More specifically, it is the cognitive discomfort experienced by a person who has two contradicting beliefs. Due to this psychological discomfort, a person tends to act to reduce their cognitive dissonance, either adding new parts to the cognition, by actively avoiding social situations (for example, by moving away) or contradicting received information.

It is evident that some opinion formation work from the literature has been inspired by sociological theories such as homophily [47, 64], cognitive dissonance [48, 92, 56], conformity [114, 56] and social impact theory [52, 102, 65]. We can highlight a number of interesting results from incorporating psychological theories. One study [47] considered the effect of propaganda or a central media on the spread of opinion across a network made up of links added through both popularity and similarity. They show that homophily favours the formation of consensus, and also mitigates the influence of dominant media. Another study [104] investigates the emotional state that is linked with certain opinions. They suggest if persuasion is needed toward a certain opinion (e.g. by a politician), discretized as between positive, negative or neutral, then it's easier to convince the *calm* agents then to convince the *agitated* ones to change their original opinion.

In this thesis, we will use psychological theories as drivers to make the model relevant to humans. We do not aim to formalise any theories of psychology, but instead use some very broadly stated principles that are frequently used in the field. Homophily is one of the most used theories to represent a mechanism of interaction (eg.[9, 29, 47, 64]), and cognitive dissonance or intolerance has been used to translate change in agent's locations or links [48, 53, 100] (shown in Table 2.2). In contrast to studies in many areas that consider our desire for diversity, our approach of consider homophily also covers the human trait of seeking similarity. We believe just as agreement/disagreement impact on how individuals naturally process information and influence opinion change, so does the incentive to move and explore.

2.5.3 Mobility

Models of opinion dynamics have been criticised for ignoring the fundamental principle of human mobility [103, 102, 16, 116, 53]. The inclusion of mobility has not been the primary focus in the investigation of opinion models (as discussed in Section 2.4), however, their use within models of other social contexts have shown successful applications that highlight their potential importance.

One obvious scenario where this have been applied is the study of pedestrian crowd behaviour [62], which started off using models based on cellular automata and successfully shifted to models where agents can move in continuous space [16]. Others propose a two-dimensional factorisation of perceived personality in crowd simulations with mobile agents [58]. Some researchers have also studied the links between relationships and geographical locations. [25] found that human mobility is influenced via the social network structure. They state that the short-range travel is less impacted by the social network structure, however, long distance travel is more influenced by an pre-existing friend (social ties). [28] found that users made increased visits to certain places frequented by their social friends. [120] examines the strength of influence and friendship creation and how much are they stimulated by the types of venues (e.g., coffee shops, airport etc.).

Some real world application have tried to increase the chances of meeting other attendees in a conference to expand their social network [24]. They used proximity and homophily in order to recommend a new contact. This work also shown that social relationships can explain 10-30% of the human movement while periodic or pattern movement explains more then 50%.

This highlights the importance of mobility in opinion modelling. Influence depends on your encounters and encounters depend on your location. In the next Chapter 3 we propose different mobility mechanisms to explore the impact on opinion evolution.

2.5.4 Noise and uncertainty

When modelling human behaviour and social agents noise should be a realistic ingredient of the model, representing the natural variability demonstrated in an individual's behaviour. This randomness can be introduced in the form of a social "temperature" or pure noise. Noise can resemble different features of real world interactions, such as misreading the opinions that an individual expresses, or misattribution of an opinion.

Different opinion models have applied noise, as we will discuss in more detail in Chapter 6. In this section we will briefly go through the main techniques of including noise. The term noise has been used in a number of different ways across the literature. Some referred to it as an expression of a random behaviour that forces the model not to follow the rules strictly, others refer to it as the opinions that are outliers and have not converged. In this thesis we use the term *noise* to refer to the random behaviour resembling human uncertainty. We highlight three notable approaches to modelling noise from the literature.

In addition to proposing their own approach, [54] provides a useful summary of the different approaches to incorporating noise in an opinion model. One of the ideas that was proposed by [71], is to add a probability that two agents might actually interact and influence each other even if they are in disagreement. A second form of noise was introduced by [94], which describes the consistent presence of dynamic behaviour through time. They model this using a probability to assign a random opinion to a random agent over time. As a consequence, there always remains a chance that individual opinions will reappear, and form new clusters.

The final type of noise is proposed in [54] as an improvement to the model in [71], which suggests adding a thermal factor, analogous to Glauber dynamics, where the thermal noise is a function of the difference in opinion instead of energy. The scale of the probability to change opinion is inversely proportional to this difference, with volatility controlled by a temperature parameter.

Given the different types of noise shows different approaches to relax the rules to demonstrate uncertainty and study the robustness of the model. We will tackle this in Chapter 6.

2.6 Conclusions

In this chapter we discuss the widely used ABM approach and how it is commonly utilised in opinion modelling. We provided a framework for clear discussion of relevant elements and comparison of the literature. Also, we identified the key papers in opinion modelling. Following that we have provided a more detailed literature review which categorises the relatively few opinion models that incorporate some form of mobility. To identify gaps in the treatment of movement, we broke down the algorithms in terms of *when* and *how* mobility is performed, and their inspiration for such mechanisms

(Section 2.4, Table 2.2). Finally, we summarised the field's challenges, highlighting the key gaps in opinion models.

Mobility has a significant impact on the evolution of opinion. It can impact the formation of groups, the exposure of individuals to new opinions and change the people whom your surrounding. One can see in every day life that if one moved from one university to another, some of their beliefs change through time due to the exposure on a new environment. Also, the action to move must have a reasoning as human beings. Our choices are not formed at random but decided as a function of our personality and relationships. Incorporating psychological behaviours to the simulations makes the simulation more relatable.

These challenges motivate the proposed models and investigations in the remainder of this thesis. The next two chapters focus on developing the basis for this thesis, presenting the model and approach to evaluation. In Chapter 3 we propose the opinion model that incorporates mobility. Mobility under free space has not been thoroughly considered in the literature, therefore, we propose evaluation metrics that will assess the model in geographical terms as well as in opinion space. We specify detailed evaluation such as identifying communities as well as those that don't belong to a community. In Chapter 4 we present an initial investigation of the model parameters and simulation in order to give confidence of the validity of our results.

The following chapters focus on detail simulations, results and discussion that contribute in different ways to the literature by addressing the challenges listed above. In Chapter 5 we investigate the significance of mobility on opinion modelling, in particular, with the hypothesis that taking into account psychological theories that trigger movement will provide more interesting scenarios. We define two types of mobility that are triggered by psychological theories, with one performing a *random mobility* and the other a *directed mobility* that moves toward a preferred location, based on the concepts of cognitive dissonance and homophily. In Chapter 6 we incorporate noise to test the robustness of our directed mobility model. We continue to Chapter 7 to explore how different mobility mechanism under different triggers behave. To do this we compare between the different mobility mechanism and expand the parameter space for a thorough exploration. In Chapter 8 we classify the behaviour of each mobility model based on the different form they structure themselves geographically. This synthesise all of the metrics under a large parameter space and extensive experiments. Finally, we present a classification that describes each models behaviour in self-organising themselves.

Chapter 3

Model

This chapter introduces the model that is used as a basis for investigation throughout the thesis. The model considers the general framework of opinion modelling based on the popular the Deffuant-Weisbuch (DW) opinion model [29]. Importantly, we extend this to include mobility, which is a fundamental aspect of human behaviour that is also linked to psychological and sociological properties such as attraction, homophily and crowds. We present the associated algorithms that define this extended model, including input parameters that will be used to assess the model in experimentation conducted in the following chapters. Together with the evaluation metric that will assess the opinion distribution in both opinion space and geographical space. These metrics will highlight the structure of opinion within a community and those that don't "belong" to a community as well.

3.1 Introduction

In Chapter 2 we identified an important gap in the literature concerning opinion modelling. In particular most opinion formation models ignore agent mobility. Instead, modelling assumes that the opportunities for agents to communicate and share opinions are represented by links in a network or graph (with nodes representing the agents). In some cases, the links are formed based on the underlying positions of the agents, often using a regular grid or lattice (e.g. [29, 61, 9, 107, 43]), while in other work, these networks represent more abstract social networks (e.g. [39, 7, 31, 121, 47]). We are interested in directly modelling agent mobility, to assess in detail how this affects the dynamics and structures that evolve. To demonstrate this we apply a free space topology - this is a two-dimensional representation of a region with a boundary containing agents that are free to move.

This model can be a generalisation of the filter bubble problem, where information is usually shared within the group. We understand how hard it is to change opinion and how misinformation spread. Therefore using this model of mobility and opinion influence, while starting from a clear canvas can show why we see this in real-life.

As raised by [37], often the reasons why certain modelling configurations are chosen are not explained, and therefore we highlight the following reasons for our model. Firstly, in contrast to a lattice or network, free space topology overrides the limitation of not being able to find a discrete empty site in a lattice. Secondly it eliminates forced re-wiring of a fixed size neighbourhood in a network, and replaces this with a representation based on interaction due to proximity. In our model initiating an interaction and choosing to move location is potentially an action without consequences, depending on the locality. This gives the chance for both communities or isolated agents to arise without rules that force the structure of an agent's neighbourhood. For example, some models will force another agent to join a group if one has just left [71, 48].

The neighbourhood element of modelling has the main influence on opinion formation, however, it is currently not well-understood what will happen to opinion diffusion if agents can move around freely. This leads to a dynamic neighbourhood with potentially more exposure to new agents due to mobility. Also, in a free space environment unlike having discrete locations, an agent isn't restricted to only interact with its immediate neighbours. For example, they are able to interact with someone outside their immediate 'bubble' if they are still sufficiently nearby. Therefore, interactions are more flexible and new communications are introduced without being forced to only interact with those characterised by a set of explicit links. Moving from assuming a small discrete set of locations and neighbours for an agent gives a wider range of simulation configurations to explore and more variation for different behaviours to arise.

Following the overarching framework for our study presented in Figure 2.1, we propose a new model for the co-evolution of opinion and location, inspired by psychological theories concerning homophily. Agents in our model do not form or break explicit links, but interact with their nearby peers at any point in time, and are free to move in Euclidean space. A fixed interaction radius specifies the range within peers of an agent can be selected for communication, and these neighbourhoods of agents change as agents move freely within the space. The use of Euclidean space contrasts with forced interactions with the same neighbours for the entire simulation as in the static opinion formation models such as those proposed in [29, 61, 107] or the network/lattice models of social structures proposed in [39, 7, 31].

3.2 Model formulation

In this work we use the widely studied DW opinion model proposed by [29]. This represents opinion in a continuous interval, which better represents the varied opinions that emerge in real social situations when compared to alternative discrete models with only two opinions, such as the Ising model [107], Majority-rule model [43], etc. The DW model is applied on a lattice and allows peer-based interaction that can't change their locations. The model implements a global interaction in a mean-field-like scenario where each agent is allowed to interact with all the others regardless of it's geographical limitations. We will use this model as a benchmark to assess our extended mobility model (denoted in the remaining of the thesis, as *static*).

Definition 1. *The **static model** is the DW original opinion model where agents do not change their locations ($p = 0$). Agents' peer selection for an interaction is across an unrestricted, global interaction range* [1]

[1]For this thesis, it's evaluated with the maximum length of interaction $r_s = L$ for convenience

The DW model has been very widely studied, with a considerable variety of extended studies undertaken to understand the underlying features. For example, this includes studying heterogeneous distributions of the opinion threshold over agents [115, 70, 121, 79], exploring the rising of complete consensus in different environments [39], the propagation of extremism [7], and the impact of cautious levels that would prevent complete consensus [80]. In this section we will present an extension of the DW model which incorporates mobility.

3.2.1 The Deffuant-Weisbuch (DW) model

We consider a population of n agents, $A = \{a_1, \ldots, a_n\}$, where each agent a_i is defined by a location $xy_i = (x_i, y_i)$ and opinion $op_i \in [0, 1]$.

Following the DW model [29], a pair of *similar agents* a_i, a_j will interact if and only if their respective opinions (op_i, op_j) are within an *opinion threshold* ϵ, in which case they both adjust their opinion to be more similar, governed by a global parameter μ (termed *convergence rate* in the original model [29]).

Definition 2. *An* ***interaction*** *occurs when a pair of selected agents share their respective opinions (and potentially influence each other). In contrast to the original DW model, we consider the trigger for an interaction to be asymmetric, each having an* ***inviting agent*** *who initiates the interaction and an* ***invited agent*** *who responds.*

Definition 3. *A pair of agents a_i and a_j are said to be* ***similar*** *if their respective opinions are close enough to influence each other, i.e. their difference is within the* ***opinion threshold****, ϵ:*

$$|op_i - op_j| \leq \epsilon$$

Definition 4. *An interaction between two similar agents a_i and a_j results in both adjusting their opinions to be closer, based on a global parameter μ, termed the* ***conver-***

gence rate. *The updated opinions for i and j are denoted op'_i and op'_j, defined as:*

$$
\begin{aligned}
op'_i &= op_i + \mu(op_j - op_i) \\
op'_j &= op_j + \mu(op_i - op_j)
\end{aligned}
\tag{3.1}
$$

Figure 3.1 represents a single interaction in the DW model. The steps follow the general framework we previously presented in Figure 2.1, Chapter 2. First, Figure 3.1 presents the *opinion spectrum* which is a continuous interval of $[0, 1]$ represented by different colours.

Definition 5. *An* ***opinion spectrum*** *is the continuous interval $[0, 1]$ which represents the range of views an agent has on a given topic or statement. For example, 1 may represent complete agreement with a statement, while 0 represents complete disagreement.*

Second, the *social interaction* happens, where a random pair of agents are selected to interact with each other (orange and green opinion agents). Then, *social influence* happens, where both agents are influenced to change their opinion to compromise between each other, if the opinions are close to each other. Following this, if interaction is successful then both their *opinions are updated* (to become yellow).

Pseudo-code for our implementation of the DW model is provided in Algorithm 1. Note that the model doesn't have any underlying structure or network topology governing the choice of agents to interact. Instead, every agent is free to interact with any other. Also note that the time step count is also advanced if the chosen agents do not successfully interact.

3.2.2 Introducing mobility

We modify the DW model by only allowing interactions between agents that are close not only opinion, but also in their location, and similarly updating both opinions and

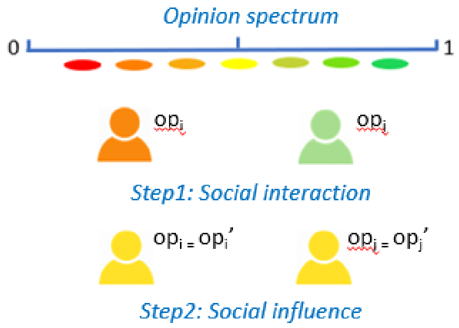


Figure 3.1: The framework of DW opinion model - continuous opinions are assigned to agents and interactions are between randomly chosen peers and therefore try to influence each other to compromise toward an opinion in between the both of them.

Algorithm 1 DW model

Require: Input parameters $(n, limit, r_s, \epsilon, \mu)$
Require: Initial population A of n agents
  for $limit$ time steps **do**
    $a_i \leftarrow U(A)$ ▷ Select random *inviting agent*
    $a_j \leftarrow U(A - \{a_i\})$ ▷ Select random *invited agent*
    if $|op_i - op_j| \leq \epsilon$ **then** ▷ DW model [29]
      $op'_i \leftarrow op_i + \mu(op_j - op_i)$ ▷ Successful interaction: Opinions influenced
      $op'_j \leftarrow op_j + \mu(op_i - op_j)$
    else
      $op'_i \leftarrow op_i$ ▷ Unsuccessful interaction: opinion unchanged
      $op'_j \leftarrow op_j$
    end if
    $op_i \leftarrow op'_i; op_j \leftarrow op'_j$ ▷ Update opinions
  end for

location following an interaction. *When* and *Where* to move, are important questions that need to be addressed in a mobility mechanism. However, in the small number of scientific articles that applied mobility (discussed in Chapter 2, Section 2.4) this has not been addressed. Therefore we provide a wide experimentation of mobility mechanisms to study each mobility's impact on the co-evolution of both opinion and location. We investigate different mobility models when their function is called, as highlighted in Algorithm 2 in red.

Definition 6. *The **neighbourhood** of an agent a_i, denoted $N(xy_i, r_s)$, is defined as the set of all other agents within a Euclidean distance r_s. Let $d(xy_i, xy_j)$ denote the Euclidean distance between agents a_i and a_j, then:*

$$N(xy_i, r_s) = \{a_j \in A - \{a_i\} : d(xy_i, xy_j) \leq r_s\}$$

The model has been implemented in Python 3.7.6. The environment in which agents move is a confined Euclidean space of $L \times L$ (i.e. a simple box). The agents are able to move freely within this region, however if their movement would take them outside of the space, reflection is applied to their movement vector at the border so that they bounce back.

Algorithm 2 DW model with mobility framework

```
Require: Input parameters (n, limit, r_s, ε, μ, p, λ)
Require: Initial population A of n agents
  for limit time steps do
    a_i ← U(A)                          ▷ Select random inviting agent from population
    if N(xy_i, r_s) ≠ ∅ then
      a_j ← U(N(xy_i, r_s))             ▷ Select random invited agent from neighbourhood
      if |op_i − op_j| ≤ ε then                                   ▷ DW model ([29])
        op'_i ← op_i + μ(op_j − op_i)   ▷ Successful interaction: Opinion influenced
        op'_j ← op_j + μ(op_i − op_j)
      else
        op'_i ← op_i                    ▷ Unsuccessful interaction: opinion unchanged
        op'_j ← op_j
      end if
      if U([0, 1]) < p then                                 ▷ Apply mobility rate
        Update xy'_i ← mobility(a_i, a_j)                  ▷ Apply mobility model
      else
        xy'_i ← xy_i                                              ▷ No movement
      end if
      op_i ← op'_i; op_j ← op'_j                                 ▷ Update opinions
      xy_i ← xy'_i                                               ▷ Update location
    end if
  end for
```

As in Algorithm 2, we increment the time step counter for each selection of an inviting agent, irrespective of whether a successful interaction took place. This includes cases where it is not possible to find a suitable agent to invite, i.e. where $N(xy_i, r_s) = \emptyset$. The mobility model is only applied to the inviting agent, and the invited agent always remains in the same location.

Under mobility, movement is applied with probability p. If $p = 0$ agents are *static* and consequently interact with the same set of neighbours throughout the simulation. The parameter $\lambda \in [0, 1]$ is used to control the scale of movement, with $\lambda = 0$ leading to no distance to be moved with, and $\lambda = 1$ denoting that a_i moves to it's full range of distance.

Definition 7. *Simulations are termed* ***local static*** *when the input parameters prevent any movement by agents, i.e. where $p = 0$ (or $\lambda = 0$), but the influencing peers are still restricted by the interaction range r_s instead of having a global interaction across the population.*

Two main categories of mobility models are described below: random mobility and directed mobility.

3.2.2.1 Random mobility behaviour

In the literature, many of the proposed mobility models are analogous to statistical physics. These models usually share certain features such as, agents are seen as particle, holding one of two states, in a lattice environment, under homogeneous mixing where any agent can interact to any other ignoring the presence of a reason or trigger to behave in a certain way. And these models are observed until a phase transition is found and an equilibrium state is reached.

To investigate the random mobility we implement two different mechanisms (Figure 3.2). One mobility applies constant random mobility and the other inspired by a social driver. First, we apply a Pure Random Mobility (PRM) which is applied after every interaction, irrespective of the agents' respective opinions. Even though it's not actual mobility as we will discuss in Chapter 7, but the concept of the dynamics was considered previously in [105, 121, 98, 83, 45]. Second, we propose the Random Repel Mobility (RRM) model, inspired by the *cognitive dissonance theory* [35], where agents avoid those that are different. This only triggers a random movement whenever two interacting agents disagreed. This dynamic concept was previously considered in [19, 53, 71, 96, 100]. These models could be considered in the context of people attending a large workshop or conference. The random model represents people wandering aimlessly, and interacting with anyone they meet, whereas the random repel captures the idea that individuals will leave locations where others are voicing opinions they disagree with.

In the section below we discuss both random mobility models.

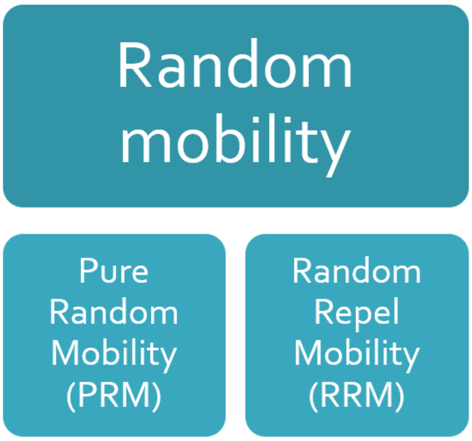


Figure 3.2: Different models of random mobility

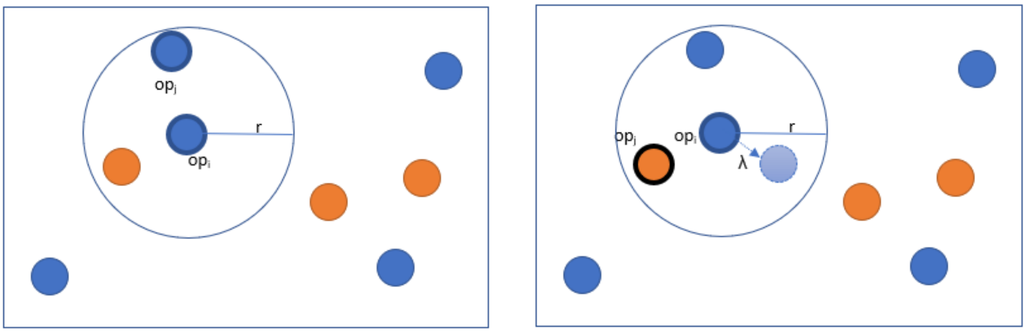
Pure Random Mobility (PRM) At each time step, the inviting agent a_i in an interaction moves to a random location in the neighbourhood regardless of their agreement (Algorithm 3).

Random Repel Mobility (RRM) Following Algorithm 4, agent a_i relocates to a random location within the neighbourhood if an interaction with a peer is unsuccessful

Algorithm 3 Pure Random Mobility - PRM

function PRM(a_i, λ)
  $r \leftarrow r_s \lambda \sqrt{U([0,1])}$
  $\theta \leftarrow 2\pi U([0,1])$ ▷ a_i moves randomly
  $xy_i' \leftarrow (x_i + r\cos\theta, y_i + r\sin\theta)$
  return xy_i'
end function

(Figure 3.3b). However if they agree, both agents remain in their current locations (Figure 3.3a).



(a) Agreement: don't move (b) Disagreement: move randomly

Figure 3.3: RRM diagram that demonstrates disagreement triggered via random mobility.

Algorithm 4 Random Repel Mobility - RRM

function RRM($a_i, a_j, \epsilon, \lambda$)
  if $|op_i - op_j| > \epsilon$ **then**
    $r \leftarrow r_s \lambda \sqrt{U([0,1])}$ ▷ a_i moves randomly encountering different peers
    $\theta \leftarrow 2\pi U([0,1])$
    $xy_i' \leftarrow (x_i + r\cos\theta, y_i + r\sin\theta)$
  else
    $xy_i' \leftarrow xy_i$ ▷ a_i stays in original location with similar peer
  end if
  return xy_i'
end function

3.2.2.2 Directed mobility

In this section we propose a directed mobility model for the co-evolution of opinion and location by extending the DW opinion model to incorporate mobility. Mobility in this section is decisive with a direction toward the new location- instead of random direction.

A relatable scenario to directed mobility is if you consider people attending a conference. Attendees choose their interactions based on similarity of research topics and would avoid others whom they don't share interest towards. They have the option to choose their direction, to either approach or avoid the other. At the last day of the conference we see groups forming with people of similar interest.

We present two mobility models that are inspired by important psychological theories. The first mobility model is Attractive Mobility (AM) inspired by *homophily* [84], where agents are more *attracted* towards similar peers. The second model is Repulsive Mobility (RM) inspired by theory of *cognitive dissonance* ([35]), where agents avoid those that are different. To fulfil a complete and thorough investigation we continue to investigate a Hybrid Mobility (HM) that combines both mobility components of attraction and repulsion.

Unlike the random mobility, when performing directed mobility for an agent the direction of mobility becomes important. The movement scale parameter $\lambda = 0$ indeed leads to no distance to be moved, but $\lambda = 1$ denoting that a_i moves *toward* the same exact location as a_j following a successful interaction (and in the *opposite* direction for unsuccessful).

Attractive Mobility (AM) This models homophily, which illustrates mobility that is only triggered by attraction feature. More specifically, when the inviting agent a_i interacts with a random neighbour a_j, it will move closer to a_j if their opinions are similar (Algorithm 5).

Repulsive Mobility (RM) This models cognitive dissonance, which demonstrates mobility that is triggered by disagreement, and as a consequence the inviting agent a_i will physically move away in exactly the opposite direction (Algorithm 6).

Algorithm 5 Attractive Mobility - AM

function AM$(a_i, a_j, \epsilon, \lambda)$
    if $|op_i - op_j| \leq \epsilon$ **then**
        $xy'_i \leftarrow xy_i + \lambda(xy_j - xy_i)$ ▷ a_i attracted to similar peer
    else
        $xy'_i \leftarrow xy_i$ ▷ a_i retain location
    end if
    return xy'_i
end function

Algorithm 6 Repulsive Mobility - RM

function RM$(a_i, a_j, \epsilon, \lambda)$
    if $|op_i - op_j| \leq \epsilon$ **then**
        $xy'_i \leftarrow xy_i$ ▷ a_i retain location
    else
        $xy'_i \leftarrow xy_i - \lambda(xy_j - xy_i)$ ▷ a_i repelled from different peer
    end if
    return xy'_i
end function

Hybrid Mobility (HM) This models a hybrid model that combines both the attract and repel mobility (Figure 3.4). Following an interaction (Algorithm 7), agent a_i moves closer to their peer a_j (Figure 3.5a) if they are close in opinion, and further away in the opposite direction if they differ (Figure 3.5b).

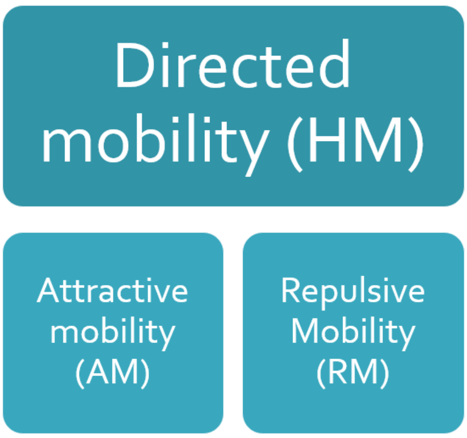
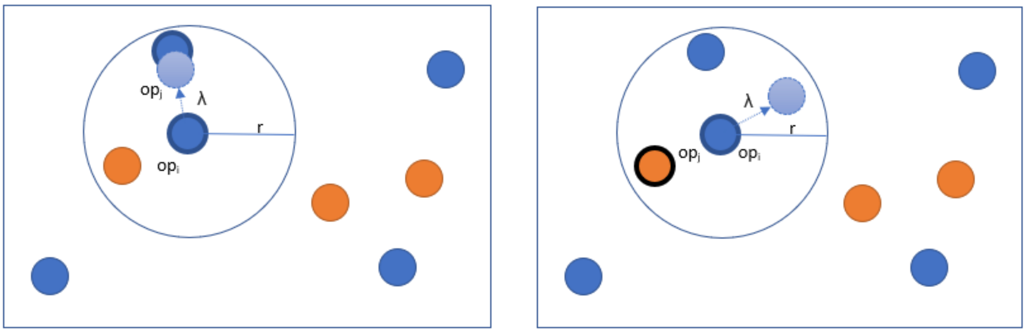


Figure 3.4: Directed mobility of Hybrid Mobility - HM

Algorithm 7 Hybrid Mobility - HM

function HM($a_i, a_j, \epsilon, \lambda$)
  if $|op_i - op_j| \leq \epsilon$ **then**
    $xy_i' \leftarrow xy_i + \lambda(xy_j - xy_i)$ ▷ a_i attracted to similar peer
  else
    $xy_i' \leftarrow xy_i - \lambda(xy_j - xy_i)$ ▷ a_i repelled from different peer
  end if
  return xy_i'
end function



(a) Attractive movement: op_i and op_j are similar opinions where a_i is moving toward a_j

(b) Repulsive movement: op_i and op_j are different opinions where a_i is moving away from a_j

Figure 3.5: HM diagram that demonstrates the attract and repel mobility

3.3 General methodology

In the next chapters we will conduct simulation-based experiments, where we will identify the independent variables to test over a number of simulations and analyse the results.

3.4 Evaluation metrics

We focus on this section to provide quantifiable metrics to assess the wide experimentation conducted in this thesis. It must be noted, the literature does not have a conviction for measuring the models output. Most papers provide interesting results, however, with many missing details, one of them is the their evaluation approach.

In this thesis, we are primarily interested in studying the models dynamics in both macro and micro levels. Where *macro-levels* describe the status of the entire population and *micro-level* demonstrate an individual's inner circle or local area.

We are interested in the nature of shared opinions that emerge as agents interact. Since we are dealing with continuous opinions (rather than discrete values), we say that two agents a_i and a_j hold the *same opinion* when the absolute difference between their opinions op_i and op_j is below a small threshold, i.e. when:

$$|op_i - op_j| < \delta_{op}.$$

Note that this value is different from the opinion threshold for a pair of agents to successfully interact (ϵ). For the remainder of this thesis, we set $\delta_{op} = 0.01$.

Similarly, we say that agents share the *same local area* if their Euclidean distance is below a threshold δ_{mov}:

$$d(xy_i, xy_j) < \delta_{mov}.$$

with $\delta_{mov} = 1$.

The impact of the choices for these thresholds is explored later on, in Chapter 4.

We summarise the input parameters for the models in Table 3.1.

3.4.1 Convergence time

One of the main metrics reported in research into opinion models is the convergence time, which compares the time taken to reach a stable configuration between different simulations. However, across the relevant literature there isn't a clear consensus on precisely how convergence time is measured, with many works simply showing the opinion distribution against time, demonstrating the opinions dynamics have (approximately) come to a stop.

Parameter	Description	Value
$L \times L$	Region size	10×10
$seeds$	Number of averaged simulation runs	20
n	Number of agents	100
$limit$	Maximum number of time steps per simulation	40000-70000
ϵ	Opinion threshold for influence (see Definition 3)	$[0.1, 0.2 \ldots, 1]$
μ	Convergence rate	0.5
r_s	Interactive radius	$[1, 2 \ldots, 10]$
p	Probability of movement	$[0.1, 0.2, 0.4, 0.6, 1]$
λ	Movement scale factor	$[0.1, 0.2 \ldots, 1]$
δ_{op}	Opinion change threshold	0.01
δ_{mov}	Movement distance change threshold	1
N_F	Number of time steps without opinion change	10000

Table 3.1: Input parameters

In addition, several different terms have been used to describe the concept of convergence. For example, it has been termed *equilibrium* [93, 96, 100] or *relaxation time* [105], while [64] defined *convergence time* as the number of updates per vertex needed to reach consensus, with [71, 29, 120] used the same terminology of time of convergence as well. There is also variation in these definitions about when convergence is found, either requiring complete consensus [105] or only local convergence [93]. Regardless of the precise definition of exactly *when* a configuration is considered to be stable, a review of the literature shows the importance of tracking when a stable opinion has emerged among agent. In this thesis we propose the following definition of convergence:

Definition 8. ***Convergence*** *is the time when either the opinion dynamics or mobility are stable with minimum change.*

To the best of our knowledge, research in opinion dynamics has only considered convergence of opinion, and not convergence through mobility. However, studying the co-evolution of agents opinions and locations is of interest. For example, to investigate whether agents behaviour is predominantly driven by one of the two features. That is, do geographic clusters of agents occur who then converge in opinion, or vice versa? To address this limitation, we define separate measures for convergence time in opinion and location. In both cases we measure the changes in opinion/location in each time step, and if a specified number of time steps (N_F) passes without significant change, the simulation is considered *converged*. The changes required to track convergence are highlighted in red in Algorithm 8.

We define the *convergence time in opinion* to be the earliest time step t, such that no agent changes their opinion by more than δ_{op} in the following N_F time steps. Similarly, the *convergence time in movement* is the earliest time step t, such that no agent changes their location by more than δ_{mov} in the following N_F time steps. Algorithm 8 returns the final time step t, together with the number of stable time steps that have elapsed in terms of opinion (t_{op}) and location (t_{mov}), allowing the determination of the following states:

No convergence. Both $t_{op}, t_{mov} < N_F$ and the final time step $t = limit$, indicating that the simulation has not reached stability for either opinion or movement.

Convergence in opinion but not movement. $t < limit$ with $t_{op} = N_F; t_{mov} < N_F$, giving a time of convergence in opinion of $t - t_{op}$. This can occur either be because the agents opinions can no longer be influenced (have found agreement or they are too distanced in opinion for a successful interaction to occur) or because they have become isolated with no local peers to interact with.

Convergence in movement but not opinion. $t < limit$ with $t_{op} < N_F; t_{mov} = N_F$, giving a time of convergence in movement of $t - t_{mov}$. This can occur either because agents are “happy” with their surrounding local peers or because they have become isolated with no local peers to interact with.

Algorithm 8 Simulation framework with convergence time

Require: Input parameters $(n, limit, r_s, \epsilon, \mu, p, \lambda, N_F, \delta_{op}, \delta_{mov})$
Require: Initial population A of n agents
for $limit$ time steps **do**
  $a_i \leftarrow U(A)$ ▷ Select random *inviting agent*
  $a_j \leftarrow U(N(xy_i, r_s))$ ▷ Select random *invited agent* from neighbourhood
  if $|op_i - op_j| \leq \epsilon$ **then** ▷ DW model ([29])
    $op'_i \leftarrow op_i + \mu(op_j - op_i)$ ▷ Successful interaction: Opinion influenced
    $op'_j \leftarrow op_j + \mu(op_i - op_j)$
  else
    $op'_i \leftarrow op_i$ ▷ Unsuccessful interaction: opinion unchanged
    $op'_j \leftarrow op_j$
  end if
  if $U([0, 1]) < p$ **then** ▷ Apply mobility rate
    $xy'_i \leftarrow mobility(a_i, a_j)$ ▷ Apply mobility model
  else
    $xy'_i \leftarrow xy_i$ ▷ No movement
  end if
  $\Delta_{op} \leftarrow |op'_i - op_i|$ ▷ Change in opinion
  $\Delta_{mov} \leftarrow d(xy'_i, xy_i)$ ▷ Change in location
  $op_i \leftarrow op'_i$; $op_j \leftarrow op'_j$ ▷ Update opinions
  $xy_i \leftarrow xy'_i$ ▷ Update location
  if $\Delta_{op} < \delta_{op}$ **then**
    $t_{op} \leftarrow t_{op} + 1$ ▷ Insignificant opinion change
  else
    $t_{op} \leftarrow 0$ ▷ Significant opinion change - reset
  end if
  if $\Delta_{mov} < \delta_{mov}$ **then**
    $t_{mov} \leftarrow t_{mov} + 1$ ▷ Insignificant location change
  else
    $t_{mov} \leftarrow 0$ ▷ Significant location change - reset
  end if
  if $t_{op} == N_F$ or $t_{mov} == N_F$ **then**
    return A, t, t_{op}, t_{mov}
  end if
end for
return A, t, t_{op}, t_{mov}

Convergence in opinion and movement. On rare occasions or for extreme parameters, we may have $t < limit$ and $t_{op} = t_{mov} = N_F$, where both opinion and movement have converged on exactly the same time step.

3.4.2 Clusters and communities

To analyse the data, we consider the clustering of the outcome, in terms of agent's common opinions and locations. Clustering the opinion can return a quantitative value that can make assessment between different experiments applicable. Also, we are especially interested in the geographical distribution of these opinions. Our aim is to detect how many opinion groups are produced in geographical space, therefore we'll focus on clustering methods based on distance. Mainly, we'll explore two clustering algorithms.

One of the most popular clustering algorithms is the K-means method [38]. Technically, it is an algorithm that clusters the data based on distance and takes the number of groups as an input. Initially, it detects a centroid to classify the members of that cluster. Thereby making one of its main limitations is its inability to detect non-spherical clusters and it's incapable to handle noisy data and outliers. For this model, this will be an issue, because clusters are in arbitrary shapes and it's likely to have outliers since agents have the freedom to move (not following a specific/single leader). Another issue is the requirement to assign the number of clusters at the start. This is not useful for this model because mainly that is the actual output under investigation. Furthermore, this is not possible because different experiments produce different numbers of opinion clusters.

Another clustering algorithm that is less commonly used but relatively well-known is the Density-based Spatial Clustering of Applications with Noise (DBSCAN) [33]. This method clusters based on density and it groups together those whom are near each other and marks any isolated agents as outliers. Also, it requires minimal previous knowledge of the data (no need to pre-assign the number of clusters). These features serves our wide investigation between different experiments.

Our evaluation will consider the nature of clusters that form as the system evolves, by applying Algorithm 9 of the DBSCAN to find groups of agents that are similar in opinion and locations. This method involves joining similar data points together into one cluster depending on two key input parameters. Firstly, the $minsamples$ it defines the minimum number of members in a cluster to be able to label those agents to a *cluster*, otherwise classified as a *loner*. The second parameter is δ, which essentially governs how close two agents must be to be considered as part of the same cluster. An agent isn't classified in a cluster if either rule is violated: not satisfying the cluster's minimum size $\leq minsamples$ or outside the threshold $\geq \delta$. We let the distance between two sets of points S_1 and S_2 be defined as $d(S_1, S_2) = mind(p, q)|p \in S_1, q \in S_2$ [33].

Algorithm 9 DBSCAN algorithm [101]

```
Require: Population A, δ, minsamples
  for a_i ∈ A do
      if label(a_i) ≠ undefined then                  ▷ Skip agents already assigned
          Continue
      end if
      if |N(xy_i, δ)| < minsamples − 1 then           ▷ Consider neighbours
          label(a_i) ← loner                 ▷ Not enough neighbours to be in a cluster
          Continue
      end if
      c ← next cluster label                          ▷ Start new cluster
      label(a_i) ← c
      S ← N(xy_i, δ)                                  ▷ Create seed set
      for a_j ∈ S do
          if label(a_j) = loner then
              label(a_j) ← c
          end if
          if label(a_j) ≠ undefined then
              Continue
          end if
          label(a_j) ← c
          if |N(xy_j, δ)| < minsamples − 1 then
              Continue
          end if
          S ← S ∪ N(xy_j, δ)
      end for
  end for
```

Initially, the DBSCAN labels the agents individually as core, border or noise points depending on the density of their local peers within a cluster. As an example we will use Figure 3.6 with distributed agents in 2D space. *Core points* are the agents with a dense population surrounding them $\geq minsamples$ (such as points x and x_1 and y), and therefore capable to extend the cluster size. For example, agent y is a core point because it has enough agents within δ and therefore is able to extend the membership of the clusters for b to join.

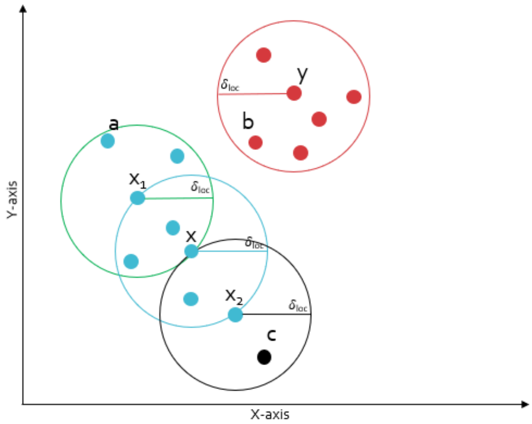


Figure 3.6: Example of DBSCAN clustering

Another example is agent x_1 which is a member of the blue cluster located at the edge of the cluster. This agent is considered a core agent as well and not a border point because the number of local agents satisfies $>= minsamples$. As a result, x_1 is able to extend the blue cluster's size with another agent a. Now agent x_2 and agent a are considered in the same blue cluster even though they exceed the actual δ. In this research we refer to this as the *chaining effect*. More specifically, it's when a pair of agents within the same cluster are more than δ apart, however they might be closer $(< \delta)$ to another member in the cluster and therefore will join.

Then we consider the *border points*, these type of points are the agents located at the edge of the cluster and surrounded by at least one core point and therefore they are not able to extend the cluster size such as, x_2. As a result the nearby agent c cannot join the cluster and it's considered as noise or *loners* (N_{loners} is a set of agents classified as loners) as we refer to them in this thesis. Loners were mentioned in [71] as separated nodes that are not a member of an opinion cluster.

3.4.2.1 Clustering evaluation

For the purpose of this thesis, the DBSCAN clustering [33] is used to identify *clusters* of agents that are close in opinion and/or location once the system has stopped, otherwise the agents are considered as *loners*. We introduce two different uses of clustering, one focusing on the opinion only (opinion clusters) at a macro-level and the second focusing on the geographical distributions of these opinions (communities) at a micro-level.

In the literature some research have studied the emergence of communities when *internal consensus* was reached [64, 48], where members of a segregated group share the same opinion [64]. The communities were measured based on the distribution of community size. However, not quantifying the number of communities as in [93]. However, they all don't consider loners that don't belong to a community, mostly because their algorithm re-links whenever the link is broken. This does not allow any loners to emerge.

While opinion clusters are already extensively studied, the way opinions are quantified is not clear or not sufficient. Some researchers have focused on the largest opinion group [57, 71, 53], ignoring minor opinion clusters and others provide numbers of final opinions without the process of measurement [70, 48]. Others average out the opinions in the population [57, 120]. However, in this thesis we explore a wide range of experiments against many runs, therefore, we develop a measurement that can quantify the opinions clusters independently, without relying on the largest opinion cluster or the average of the population.

Different terms are used to describe the state of opinions. Usually if multiple opinions exist this is described as non-equilibrium [64], meta-stable state [98, 102, 83]. While a polarised opinion is called, incoherent state [96]. As for a single dominating opinion is termed as complete consensus [105] or ordered state [57]. In this thesis will refer to opinions as multiple opinions if they are more than one and complete consensus otherwise. We will discuss in detail how the evaluation is performed.

Definition 9. *An **opinion cluster** is a cluster of agents containing at least 5 members that share close opinions* [2]*, otherwise considered as a **loner outside opinion clusters**. Opinion clusters are found by applying the DBSCAN algorithm with the distance between a_i and a_j defined as $|op_i - op_j|$, and a threshold of δ_{op}.*

Definition 10. *A **community** is defined to be a cluster of agents of at least 5 members that share close opinions and locations. Otherwise considered as a **loner outside communities**. Communities are identified by applying the DBSCAN algorithm with the distance between agents a_i and a_j as*

$$\begin{array}{rl} d(xy_i, xy_j) & \textit{if } |op_i - op_j| < \delta_{op} \\ \infty & \textit{otherwise} \end{array}$$

A threshold of δ_{mov} is then used within the DBSCAN algorithm to identify adjacent agents.

With the DBSCAN, two key parameters are set in the algorithm, the cluster's minimum size ($minsamples = 5$) and distance threshold (δ). An average degree of four/five members of a group is commonly seen in simulations (eg. [71, 64]), also Von Neumann's neighbourhood settings are widely used, it represents 4 neighbours around an agent. For clustering the data the results can vary from $[0 - 10]$ clusters. Detecting 0 clusters means the agents are structure-less and all the agent's are classified as *loners* while detecting any higher clusters ≤ 1 means that there is self-organisation between the agents.

In general, when there are large numbers of 'loners' outside communities, this can either mean that there aren't any close enough local agents that can be detected or they are indeed not enough supporters for the opinions in the local area, as the number of agents in the cluster doesn't comprise at least five members. However, when this

[2] The opinions are close but not exactly the same due to the chaining effect discussed previously

number is low it means that most of the agents are classified in communities of uniform opinion in geographical space.

In this research we will include the investigation of outliers (i.e., loners). Other research has investigated the DW model and found that loners are frequently reported. [78] reports that outliers exist due to structural reasons, and they identify that [29, 115] has chosen to ignore clusters with one agent. [78] further highlights that [61] (group-based interactions) in comparison to the DW model (peer-based interactions) have no minor clusters. [48] demonstrates the DW model while also implementing group-based interactions while permitting rewiring and these outliers disappear, similar to the static [61] model. [71] as well have mentioned that isolated agents disappear under a rewiring structure. In the literature, to the extent of our knowledge, isolated agents are either only reported or ignored but have not considered a quantifiable measurement of the isolated agents occurrence. This suggests that the investigation of loners will be more interesting under the implementation of mobility.

3.4.3 Local opinion diversity

The final metrics study the opinion distribution at a micro-level. We measure the diversity of opinions within the immediate *local area* of each agent, from different aspects: the percentage of agents that hold different opinions and the mean opinion in the local area. Figure 3.7 illustrates the radius of each geographical variable in the thesis.

Definition 11. *Agents holding opinions within δ_{op} are considered to hold the* ***same opinion*** *and be in* ***agreement****, while the others (outside δ_{op}) are considered to hold* ***different opinions****.*

Definition 12. *Agents within δ_{mov} are considered in the same* ***local area****, and δ_{mov} is specifically used to evaluate the geographical structure between the agents. For an agent a_i, the set of agents sharing the same local area is given by $N(xy_i, \delta_{mov})$, and*

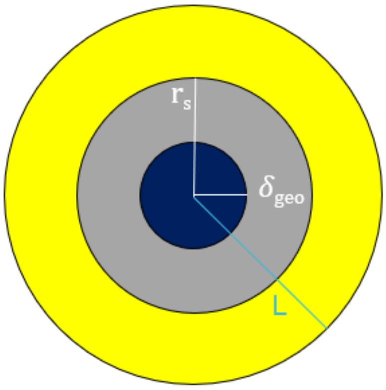


Figure 3.7: Overall diagram of geographical distance representation: agents within δ_{mov} are local agents, within r_s are interactive neighbours, all other agents within L are global.

we define N' to denote the subset of these agents that hold a different opinion, where:

$$N'(a_i, \delta_{mov}) = \{j \in N(xy_i, \delta_{mov}) : |op_j - op_i| > \delta_{op}\}$$

Definition 13. *The **tolerance** of an agent a_i, denoted by $tol(a_i)$, is defined as the proportion of their local peers that hold a different opinion (at the end of a simulation), where:*

$$tol(a_i) = \begin{cases} \dfrac{|N'(xy_i, \delta_{mov})|}{|N(xy_i, \delta_{mov})|} & \textit{if } |N(xy_i, \delta_{mov})| > 0 \\ 0 & \textit{otherwise} \end{cases}$$

The tolerance of a population A is denoted $tol(A)$ and defined as the mean tolerance of all agents $a_i \in A$:

$$tol(A) = \frac{1}{|A|} \sum_{a_i \in A} tol(a_i)$$

Tolerance is a value in the range $[0, 1]$, where 0 shows that all local peers agree on a single opinion. However, anywhere in the middle shows the ratio of the agents holding a different opinion, with 1 showing that all local peers hold a different opinion. This is common at the start of the simulation when the opinions are first randomly distributed.

Definition 14. *The **local diversity** of an agent a_i measures the mean difference of opin-*

ion among its local peers, and is denoted by $div(a_i)$, where:

$$div(a_i) = \begin{cases} \dfrac{1}{|N(xy_i, \delta_{mov})|} \displaystyle\sum_{j \in N(xy_i, \delta_{mov})} |op_i - op_j| & \textit{if } |N(xy_i, \delta_{mov})| > 0 \\ 0 & \textit{otherwise} \end{cases}$$

The local diversity of a population A is denoted $div(A)$ and defined as the mean local diversity of all agents $a_i \in A$:

$$div(A) = \frac{1}{|A|} \sum_{a_i \in A} div(a_i)$$

Although theoretically $div(a_i) \in [0, 1]$, results from this thesis are normally in the range $[0, 0.3]$.

3.5 Conclusions

In this chapter we have introduced an extended model of the DW model by applying different mechanisms of mobility that was not considered in the literature before. These mechanisms bring psychologically inspired mechanisms into the mobility space, based around the concepts of homophily and cognition dissonance which can motivate human behaviour.

Our approach is to use ABM to investigate the model and we have discussed the input parameters and evaluation metrics to assess the model. Mobility has rarely contributed to the literature, in particular freely distributed agents (2D continuous space). Evaluation metrics proposed in literature either ignore the details of the some outcomes or behaviours are only reported without metrics. Therefore, we develop evaluation functions to track the model's behaviour and asses both opinions and their geographic locations simultaneously, these metrics have been investigated carefully and thoroughly. These evaluation study the model from different aspects.

A first important aspect is to study the convergence where the opinion dynamics become stable, this has been widely considered in the literature. However, convergence for agents movements have not been considered, this will lead to inaccurate suggestion of the communities structure, if the agents are still moving. A second important evaluation is the opinion evolution, which assesses the opinions in opinion space and distinguish those of whom form clusters of close opinions or stay isolated with their own opinions. Also, the opinions structure in geographical space is equally important, to study the nearby agents, if they are uniform or different in opinion. Therefore, we consider communities assessment, which provides both a quantitative analysis and a visual distribution of the agents distribution to asses if either agents cluster together into communities or become outliers.

After the introduction to the model and demonstrating the evaluation metrics, we will present an extended analysis on the evaluation parameters that will validated our assessment in Chapter 4.

Chapter 4

Baseline Parameter Settings

In Chapter 3 we introduced a model of opinion dynamics with mobility and proposed metrics that we will use to evaluate the outcomes. As identified in Chapter 2, many papers in the literature draw conclusions from the results of simulations without clearly defining the metrics used or justification for the subset of parameters that they vary. In later chapters, we will experiment across a range of parameters that control the level and scope of interaction within the population, but here we first undertake some initial investigations to set appropriate values for other parameters. In particular, we consider δ_{op}, δ_{mov} and $minsamples$ to cluster the agents sufficiently. Followed by a demonstration of the DW model while applying the proposed metrics in Section 3.4, chapter 3.

4.1 Methodology

In this section we will present an investigation into the DW static model as defined in Algorithm 1, Section 3.2.1, to set a basis for the investigations in the rest of the chapters. Through this investigation the fixed variables are listed in Table 4.1 and the independent variables are set as shown in Table 4.2.

Aim The aim of this chapter is to test and choose the most suitable evaluation parameter values that best fit our model. Also, we perform an initial investigation to apply

Parameter	Description	Value
$limit$	Maximum number of time steps per simulation	$40,000$
r_s	Interactive radius	10
μ	Convergence rate	0.5
p	Probability of movement	0
λ	Movement scale factor	0
N_F	Number of time steps without opinion change for convergence	$10,000$

Table 4.1: Fixed variables

Parameter	Description	Value
Mobility models	Static model	Static model
$L \times L$	Region size	$[10, 20, 30]$
$seeds$	Number of averaged simulation runs	$[10, 20, 100]$
n	Number of agents	$[100, 500, 1000]$
ϵ	Opinion threshold for influence (see Definition 3)	$[0.1, 0.2, 0.3]$
$minsamples$	Number of agents that form a cluster	$[1, 5]$
δ_{op}	Opinion change threshold	$[0.01, 0.05, 0.1]$
δ_{mov}	Movement distance change threshold	$[0.25, 0.5, 1]$

Table 4.2: Independent variables

the proposed evaluation metrics (defined in Section 3.4) while highlighting how other work in the literature performs their evaluation.

Experiments As a basis for this thesis, we focus on evaluating the static model, which is defined to mimic the original DW model (Algorithm 1, Section 3.2.1).

Evaluation There are two main sections in this chapter. The first one explores the evaluation parameters (δ_{op}, δ_{mov} and $minsamples$) used to cluster the agents. This is performed at the start of the simulation on the static model when $t = 0$ before organisation start to form. The second section evaluates the static model after the

Parameter	Description	Value
Convergence in opinion (see Algorithm 8)	Time step when opinion change is settled	0 - 40,000 time steps
Opinion clusters (see Definition 9)	Mean number of different opinion clusters	0 - 10 clusters
Communities (see Definition 10)	Mean number of clusters that share opinion and location	0 - 10 clusters

Table 4.3: Dependent variables

simulation comes to an end, to apply the evaluation metric (in Table 4.3) and interpret the results while comparing it to the main model in [29].

4.2 Clustering parameters

As defined in Section 3.4, we are interested in the *opinion clusters* and *communities* that emerge within the population, which will be calculated by applying the DBSCAN algorithm. Our implementation of the DBSCAN approach, along with our definitions of convergence, tolerance and local diversity, are dependent on three parameters:

- δ_{op}, a threshold in opinion to add a new candidate agent to an opinion cluster/community;
- δ_{mov}, a threshold in distance to add a new candidate agent to a community;
- $minsamples$, the number of agents required to form a non-trivial opinion cluster or community (agents in smaller clusters are considered loners).

We use an empirical approach to justify our choice of δ_{op}. In Table 4.4, we consider the number of opinion clusters that are found by DBSCAN at the beginning of a simulation, where each agent receives a random opinion from $[0, 1]$, taking the mean over 20 simulations with different random seeds. Note that threshold values larger than $\delta_{op} = 0.1$ allocate all agents to a single opinion cluster.

$minsamples$	δ_{op}	Opinion clusters		$\lvert N_{loners}\rvert$	
		Mean	Standard deviation	Mean	Standard deviation
1	0.01	35.35	2.6	0	0
1	0.05	1.5	0.5	0	0
1	0.1	1	0	0	0
5	0.01	4.2	1.3	74.6	8.8
5	0.05	1.5	0.5	0.4	1.2
5	0.1	1	0	0	0

Table 4.4: Initial opinion clusters found via DBSCAN

Note that DBSCAN does not strictly ensure that all pairs of agents a_i, a_j in an opinion cluster have opinions such that $|op_i - op_j| \leq \delta_{op}$. Instead, due to the *chaining effect* (discussed in Section 3.4.2) agents may be merged into other clusters rather than being classified independently. For that reason the number of opinion clusters found via DBSCAN may be smaller than first expected.

After inspecting the results in Table 4.4, we rule out cases with $minsamples = 1$, as opinion clusters containing only one agent are of little interest. From the remaining results for $minsamples = 5$, we suggest linking the threshold to the separation resulting from uniformly distributing agents across the opinion space ($[0, 1]$), i.e. $\delta_{op} = \frac{1}{|A|} = 0.01$ gives a reasonable balance between non-trivial opinion clusters and the number of loners.

In addition to δ_{op}, the definition of communities is based on a threshold δ_{mov} that defines when agents should be considered "near" geographically. To identify a suitable value for δ_{mov}, we investigate the initial distribution of agents in the region. First we show in Table 4.5 the average distance between an agent a_i and members of its neighbourhood $N(xy_i, r_s)$ for the range of interaction radii that we will consider in our experiments. Also, we show the size of the neighbourhood within r_s. Note, $r_s = 0.5$ shows that from 20 runs there is a probability not to find a neighbour. However, under $r_s = 1$ finding 2-3 neighbours is reasonable to evaluate a community, considering the fact this is at the initial distribution before organisation starts to form.

r_s	$\lvert N(xy_i, r_s)\rvert$	Average distance between neighbours
0.5	0.78	0.09
1	2.84	0.43
2	10.44	1.16
3	21.34	1.82
5	46.41	3.00
10	96.39	5.10

Table 4.5: Mean distance between neighbours over 20 simulations with different random seeds.

$minsamples$	δ_{op}	δ_{mov}	Geographic clusters		Communities	
			Mean	$\lvert N_{loners}\rvert$	Mean	$\lvert N_{loners}\rvert$
5	0.1	1	6.2	42.3	0	100
5	0.1	0.5	0.5	97.15	0	100
5	0.1	0.25	0	100	0	100

Table 4.6: Geographical clusters found via DBSCAN

As for δ_{mov}, we also consider initial random populations. Instead of focusing on communities, we instead consider purely *geographic clusters* and loners based on applying DBSCAN with Euclidean distances, which are shown in Table 4.6. Applying the same logic as for δ_{op}, we consider all agents uniformly distributed across the region, suggesting $\delta_{mov} = \frac{|A|}{L^2} = 1$. Table 4.6 shows that this is a reasonable assumption, yielding approximately 6 geographic clusters with 40 loners. Given that a value of 1 results in at least a couple neighbours (2.8) based on distance (shown in Table 4.5), its reasonable to find some of the population clustering into a few clusters and others as loners.

4.3 Exploratory analysis of the Deffuant-Weisbuch (DW) model

In this section we explore the features of the DW model in the context of our proposed evaluation measures. Recall that we use the term *static model* to mimic the DW model with its global interaction approach. We set the probability of movement as $p = 0$

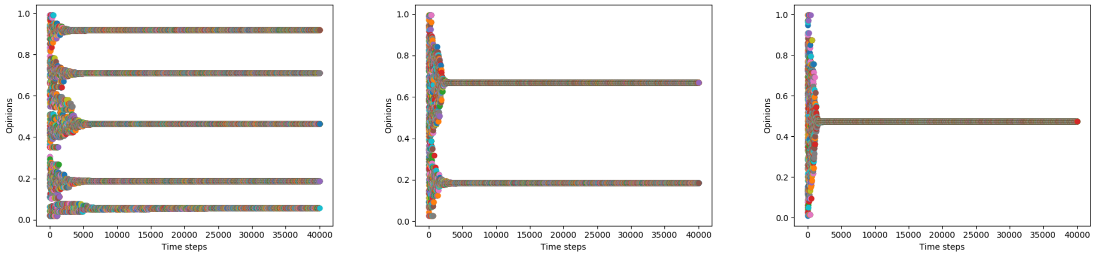
and the interaction range as $r_s = 10$ so that any agent may interact with any other (Algorithm 1, Section 3.2.2.2).

Firstly, we demonstrate and visualise the number of opinion clusters that result from simulations as the opinion threshold ϵ is varied. Then, we discuss the distribution of opinion across the region from the static model by showing visual examples of individual simulation runs. Then we discuss averaging the results under multiple runs to set the suitable number of runs that will generate outcomes with more rigour. Finally, in order to restrict the range of parameters to be considered in future chapters, we discuss the density and scalability as the size of simulation increases.

4.3.1 Opinions at a macro-level

The main feature of interest in the field is the evolution of opinions and emergence of consensus. We demonstrate the opinion evolution in different ways to capture the distribution patterns. One of the most common approaches adopted to present the development of opinion in the literature is by plotting the opinion values of the interacting pair of agents for each time step [70, 115, 29, 22]. As an example, Figure 4.1 shows the outcome of representative simulations from the static model highlighting the effect of ϵ, we will show the relevant results and figures for these experiments through this chapter. The impact of ϵ typically leads to three distinct opinion distributions: consensus around a dominating single opinion (*complete consensus*), a polarised population centred on two opinions (*polarised groups*), or small groups of agents centred on a larger number of opinions (*fragmented*). When the system reaches stability, this usually occurs relatively quickly, with Figure 4.2 showing a more detailed snapshot of agent behaviour at the start of the simulation.

The extension to the static model described in Section 3.2.2 adds locations, hence it is useful to visualise the geographic spread of opinion over the region. Figure 4.3 shows each agent's opinion and their distribution around the region. Each different

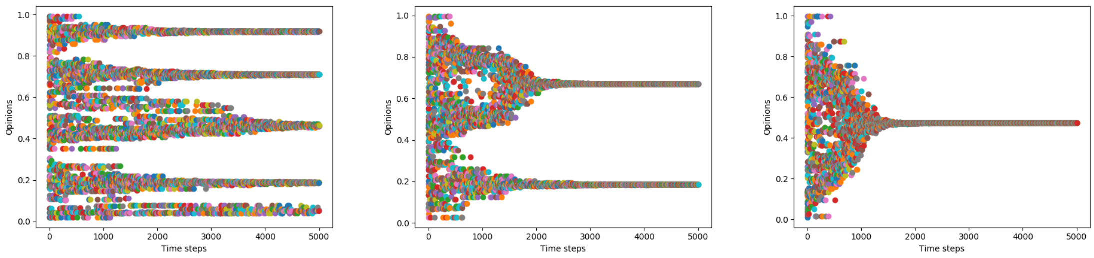


(a) Fragmented opinions under the static model ($\epsilon = 0.1$)

(b) Polarised groups under the static model ($\epsilon = 0.2$)

(c) Complete consensus under the static model ($\epsilon = 0.3$)

Figure 4.1: Opinion distribution example of a single run of a total of 40,000 time steps.



(a) Fragmented opinions under the static model ($\epsilon = 0.1$)

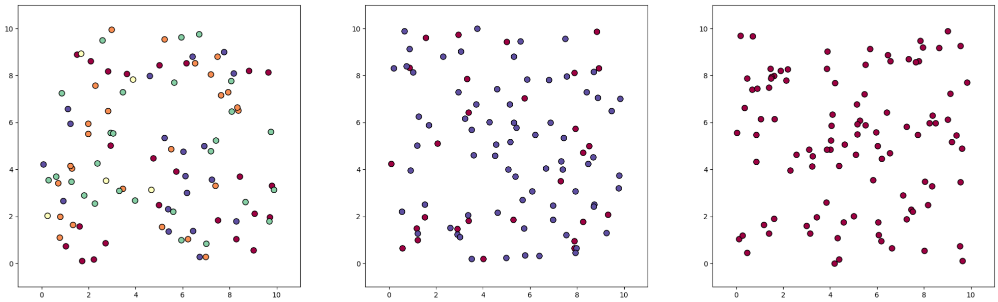
(b) Polarised groups under the static model ($\epsilon = 0.2$)

(c) Complete consensus under the static model ($\epsilon = 0.3$)

Figure 4.2: Closer snapshot of the opinion distribution of a single run of a total of 5000 time steps.

colour represents a different opinion cluster. In Section 3.4.2 we specify our use of the DBSCAN clustering algorithm which produces these figures, joining close opinions into a single cluster and assigning each cluster a different colour.

In the literature opinion clusters are demonstrated in different ways. For example, [39] measured the probability when a complete consensus appears to detect the critical ϵ. However, the most common approaches for considering multiple opinion clusters, focus either on the size of the largest opinion group [53, 57, 71] or average out the opinions in the system [121, 57, 52] to capture the opinion distribution at a macro-level. The first method focuses on the size of the largest dominant opinion clusters, ignoring any other opinion groups that are formed. The second method will not highlight fragmentation in the opinion distribution, all the opinions will be treated as one value. All in all, both methods ignore the distribution of different opinion clusters in



(a) Five (fragmented) opinion clusters ($\epsilon = 0.1$)

(b) Two (polarised) opinion clusters ($\epsilon = 0.2$)

(c) One opinion cluster (complete consensus) ($\epsilon = 0.3$)

Figure 4.3: Opinion clusters under the static model from applying DBSCAN to individual simulation runs. Colours denote agents belonging to the same opinion cluster.

opinion space. We provide an example below in how these methods are not enough to demonstrate the opinion evolution.

Example We demonstrate an experiment on the AM as an example to implement these different methods to assess the final opinions under AM ($r_s = 5, p = 1, \epsilon = 0.2$) and show how different approaches impact the interpretation of the outcomes. Firstly, the average of all opinions in the population is 0.48, which unfortunately doesn't show the appearance of the other two opinions (Figure 4.4). The size of the largest opinion group is 53 agents giving $53/100 = 0.53$, which despite covering over half of the population, still ignores the two other opinions that persist. However, we are more interested in the opinion distribution in more detail. Using the metrics defined in Section 3.4.2.1, we find three opinion clusters and the three opinions are clearly plotted in Figure 4.4.

4.3.2 Opinion distribution in geographical space

In this research we are equally interested in the opinion distribution in geographical space across the region as in opinion space. For a deeper investigation we plot the

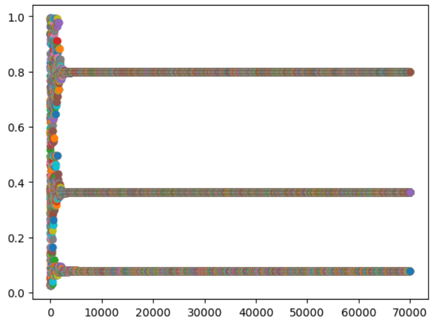
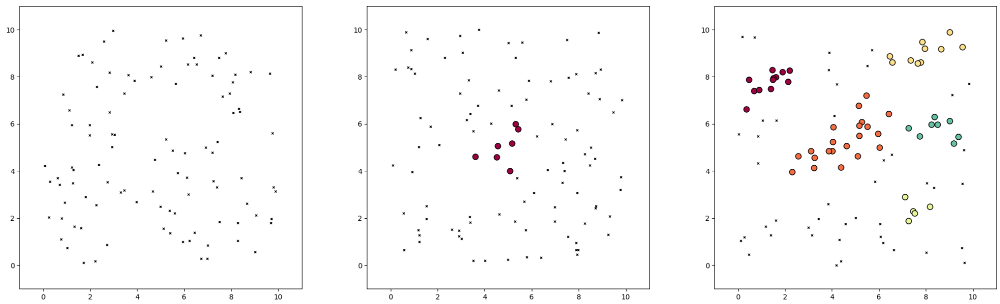


Figure 4.4: Opinion spread across time under AM $(r_s = 5, p = 1, \epsilon = 0.2)$ **for an individual simulation run.**

agents that have the same opinion and close in location to show numbers of communities (e.g. Figure 4.5). With the static model (Figure 4.5a), in most cases patterns of community formation can't be found, due to the lack of movement, and as a result, the DBSCAN algorithm returns a plot with many loners (small dots) outside communities. With larger ϵ some communities are found since all the agents hold the same opinion and therefore the distance factor becomes the only driver to cluster them, increasing the chances to find some clusters (Figure 4.5c), shown by coloured large dots.



(a) No community formation ($\epsilon = 0.1$)

(b) One community ($\epsilon = 0.2$)

(c) Six communities ($\epsilon = 0.3$)

Figure 4.5: Communities formed in individual runs under the static model. Colours denote agents belonging to the same community.

4.3.3 Multiple runs

After exploring the individual seeds we calculate the mean of the results for multiple runs and average the outcome of the different evaluation metrics. In this thesis, we demonstrate the mean results in either line graphs or heat maps. The line graphs are used to compare between different mobility models as will be demonstrated in Chapters 5 and 6. The heat maps show the collected data in a more visual and dimensional diagram. For example, Figure 4.6 demonstrates the mean number of opinion clusters under the static model with a single interaction range ($r_s = 10$ to demonstrate a global interaction). This allows us to visualise what happens as ϵ gets larger, it shows a single opinion is found of complete consensus. Later in Chapter 7 and 8, we expand the heat map with more values of ϵ and r_s.

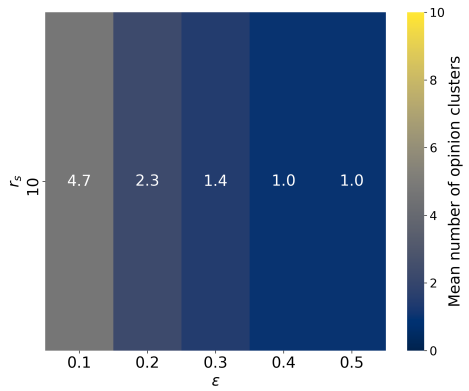


Figure 4.6: Mean number of opinion clusters under the static model for ($\epsilon \leq 0.5$)

One of the most important factors when running a simulation is to average out the results of multiple runs, and balancing the rigour of multiple simulations with the computational cost. The majority of results in this thesis were generated using the computing facilities of Supercomputing Wales. This enables to decrease the execution time, individual runs are executed in parallel, more details are provided in the next Chapter 5 Section 5.1.

These simulation runs are named differently in the literature, some refer to realisations [71, 48, 94] others call them samples [39, 29] and other refer to them as experiments [120] or simulations [111]. For this thesis we will refer to them as the the number of runs. We must note that the number of runs in the literature vary, for example, [93, 111] use 30, [96] use 50, others vary from 100 to many hundreds [29, 48] or even thousands

[39, 120] of averaged runs.

In our experiments, many runs are required to satisfy all different parameters. Locations are constantly computed and updated and consequently the related neighbourhoods are updated as well. Therefore, we need to balance the trade-off between the number of different experiments with the number of averaged runs. We have experimented with a various accumulated numbers of independent runs as we will discuss further down below.

Next we explore the parameters sensitivity in our model in terms of the number of simulation runs with different seeds that are averaged to draw conclusions. For this, we first set an appropriate value of ϵ in order to continue the investigation. In the literature, the threshold value of ϵ that leads to complete consensus appears to fall between 0.27 and 0.5, (for example, [29, 39, 40]). The variation appears to arise due to subtle differences in the models applied and definition used for complete consensus. For example, [29] discusses this in the context of opinions showing a single "peak", whereas [39] defines complete consensus more formally as all agents having the same opinion (within a small threshold). Therefore, we select the case $\epsilon = 0.3$, since the consensus in the literature varies if a single opinion cluster is reached, to confirm what happens under our model. Also, in Figure 4.7 the largest variation decreases after $\epsilon = 0.3$, hence, it will be beneficial to understand the dynamics for that ϵ. We will continue the rest of the investigation with $\epsilon = 0.3$.

Each run of the simulation has an independent random seed. Table 4.7 shows the results while increasing the numbers of runs. The mean of runs over 20 seeds is close to the mean opinion clusters from 100 seeds, this is similar to the opinion convergence time as well. Figure 4.8 shows the cumulative mean opinion convergence time as more simulations are performed. As the number of averaged out experiments increase, the line gets more stable and straighter toward the larger number of runs. Therefore to save more computational time to allow experimentation over a wider range of parameters, in the remainder of this thesis, we will present results over 20 simulation runs.

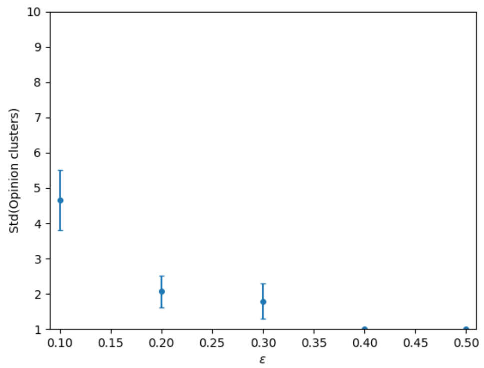


Figure 4.7: Standard deviation (opinion clusters) for 20 runs

L	$seeds$	n	ϵ	Mean number of opinion clusters	Mean opinion convergence time
10	10	100	0.3	1.5	2119
10	20	100	0.3	1.45	1997
10	100	100	0.3	1.51	2841

Table 4.7: Static model over multiple simulations

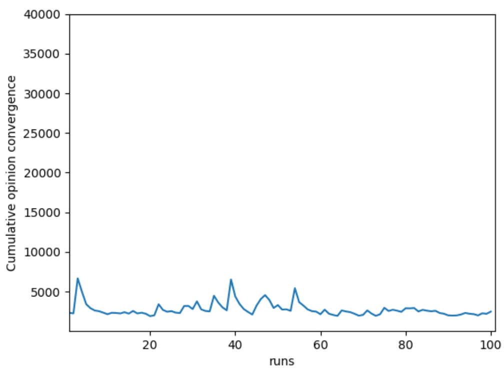


Figure 4.8: Cumulative convergence in opinion $\epsilon = 0.1$ for 100 runs

It must be noted that we have also tried some experiments while changing μ, however it required high computation power and time. There were some difficulties with the experiments not ending nor capable to save the output due to the long run times. Therefore, in this research we have left μ fixed and experiment with the other parameters.

L	$seeds$	n	ϵ	Mean number of opinion clusters
10	20	100	0.3	1.45
10	20	500	0.3	1.2
10	20	1000	0.3	1

Table 4.8: Mean number of opinion clusters while varying the density

L	$seeds$	n	ϵ	Mean number of opinion clusters
10	20	100	0.3	1.45
20	20	400	0.3	1.15
30	20	900	0.3	1.05

Table 4.9: Mean number of opinion clusters while varying the region size

4.3.4 Region size and density

In this section we investigate the sensitivity of the static model to increases in the size of the region and density of agents. In the literature the size of the population varies from 100 [121], to a range between 25 and 625 [93], a range between 100 and 500, up to a population thousand or over [121, 29, 39, 71].

First we will increase the density (agents per unit area) and observe the impact on the evolution of opinions. In Table 4.8 we see that the higher the density, the more likely that complete consensus emerges, in line with other work [39, 71, 48].

We further investigate the consistency of the static model as the size of the region scales. Similarly as increasing density, the larger the scale, the more likely we get one complete consensus (Table 4.9). It appears that the static model is sensitive to changes in both density and scale.

In conclusion, we can replicate the DW model results in the literature. For our experiments we assume one agent per square meter as the default. This helps us collect more data and utilise the computational power we have provided more effectively.

4.4 Conclusions

In this chapter we have explored the static model to provide a basis of the original model. Also, we conducted extensive experiments on the parameters with smaller experiments to draw out the fittest value to evaluate the outcomes. Finally, we found a meaningful set of parameters that allows the evolution of the network to opinion clusters of either a single or islands of opinion clusters from a situation of random spread out opinions. In Chapter 5 we introduce mobility to the opinion model and we present and analyse the impact of mobility.

Chapter 5

Hybrid Model

In this chapter we consider the hybrid mobility model (HM) introduced in Chapter 3, Section 3.2.2.2. This is inspired by the psychological theories of homophily and cognitive dissonance, where agents are attracted/repelled by their neighbours based on their agreement/disagreement. We synthesise and discuss results between HM and the random mobility model (RRM), introduced in Chapter 3 Section 3.2.2.1. The primary aim of this chapter is to investigate the effect of homophily and dissonance on the speed and structure of opinion convergence among agents that are free to move within a two dimensional region in comparison to random mobility triggered by disagreement. Note, part of the work in this chapter was published in [5].

5.1 Methodology

It is relatively rare that mobility schemes are introduced to opinion dynamics models in the literature (see Chapter 2), with those that have been proposed usually adopting a random mobility scheme, typically inspired by statistical physics. This chapter introduces mobility to the static opinion model described in Chapter 3, Algorithm 2 and studies it's impact on the opinion evolution.

A general property [39] of the DW model is that when opinion threshold $\epsilon \geq 0.5$ then the system reaches complete opinion consensus (only one opinion exists). Further extensions of the model, using rewiring dynamics [48, 71] or scale free networks [96]

Parameter	Description	Value
$L \times L$	Region size	10×10
$seeds$	Number of averaged simulation runs	20
n	Number of agents	100
$limit$	Maximum number of time steps per simulation	$40,000$
r_s	Interactive radius	2
δ_{op}	Opinion change threshold	0.01
δ_{mov}	Movement distance change threshold	1
N_F	Number of time steps without opinion change for convergence	$10,000$

Table 5.1: Fixed variables

show that a critical opinion threshold smaller than 0.5 leads to a complete consensus. [61] has stated that modelling disagreement is just as important as consensus and continued with the following statement from [2]: *'Since universal ultimate agreement is an ubiquitous outcome of a very broad class of mathematical models, we are naturally led to inquire what on earth one must assume in order to generate the bimodal outcome of community cleavage studies.'*.

In this chapter we are more interested of when multiple opinions emerge, therefore we restrict our attention to the cases $\epsilon = 0.1, 0.2$ and 0.3, and the scale of movement ($\lambda \in [0.1, .., 1]$). For each case we vary the probability of movement ($p \in [0.1, 0.2, 0.4, 0.6, 1]$), and investigate the effect on convergence, clustering and tolerance. We focus our investigation with small probability of movement to catch the movement's impact under small chances of mobility. All simulations are run for a maximum of $40,000$ time steps. Other simulation variables, both fixed and independent are listed in Tables 5.1 and 5.2.

Early experiments were conducted on a standard desktop PC (Intel Core i7-6700, 32GB, 512 GB SSD), however, the majority of experiments relied on the Supercomputing Wales high performance computing facilities. This reduced the run time from

Parameter	Description	Value
Mobility models	Static and directed mobility models	Static, local static, HM
ϵ	Opinion threshold for influence (see Definition 3)	$[0.1, 0.2, 0.3]$
p	Probability of movement	$[0.1, 0.2, 0.4, 0.6, 1]$
λ	Movement scale factor	$[0.1, 0.2 \ldots, 1]$

Table 5.2: Independent variables

the order of weeks down to days. The main time on the supercomputer was the waiting queue of other jobs, but the actual execution itself would not take more than 18 hours for one mobility model. One mobility model would consist of three variations of ϵ and 6 variation of p and 10 variation of λ. This results in conducting 180 experiment for 20 runs that are executed in parallel. Note, this time only considers the execution and recording the data, however, it does not include plotting the agent's distribution in the region for individual experiments. Also, the execution time varies from a mobility model to the other depending on how frequently are the agents location computed and updated. This facility has enabled us to explore the model more widely and efficiently.

Aim In this chapter, our *aim* is to investigate mobility models that are inspired by psychological theories, where movement is triggered according to the interaction feedback. Initially in this chapter we study two different mechanics: random (RRM) and a directed hybrid mobility model (HM). Our aim is to investigate the impact of mobility mechanisms in the co-evolution of opinions and location, and in particular, the extent to which the nature of mobility affects the end result.

Hypothesis Realistic models of opinion are likely to lead to more realistic outcomes, and in particular, less likely to result in complete consensus, which rarely occurs in real-world scenarios. In this chapter's investigation, our hypothesis is that mobility models that include attraction and repulsion will allow larger number of opinions to persist.

Experiments We investigate two different mechanisms for mobility (random and directed), driven by psychological theories. We compare the effect between the main HM model (Algorithm 7) with the random mobility RRM (Algorithm 4).

As a *baseline* for comparison, all figures show results for the static model with global interactions as a constant dashed blue line (as this does not depend on λ). In addition, we investigate the local static opinion model while applying restricted interaction as a straight orange line. Results are shown for both RRM and HM movement and a range of sets of parameters. The following will present the models that will be studied in this chapter.

1. Static models
    - Static: the original DW model (Algorithm 1, Section 3.2.1)
    - Local static: a modified version of the DW model, with interaction restricted to peers within the interaction range r_s.
2. Random mobility
    - RRM: is a random mobility model, where agents are only triggered to move encountering disagreement (Algorithm 4, Section 3.2.2.1).
3. Directed mobility
    - HM: is inspired by the psychological theories of homophily [84] and cognitive dissonance [35], where agents are more attracted toward similar peers, and repelled when they seek to minimise cognitive dissonance based on alternative opinions (Algorithm 7, Section 3.2.2.2).

Evaluation Firstly, we *evaluate* each model's convergence time. Initially, we only evaluate convergence in opinion but not in movement, however, as the chapters progress we will investigate the convergence in movement. Following this, we explore

Parameter	Description	Value
Convergence in opinion (see Algorithm 8)	Time step when opinion change is settled	0 - 40,000 time steps
Opinion clusters (see Definition 9)	Mean number of different opinion clusters	0 - 10 clusters
Tolerance (see Definition 13)	Ratio of different opinions	$tol(A) \in [0, 1]$
Local diversity (see Definition 14)	Mean difference of opinion	$div(A) \in [0, 1]$
Communities (see Definition 10)	Mean number of clusters that share opinion and location	0 - 10 clusters

Table 5.3: Dependent variables

the number of opinion clusters at a macro-level emerging in the population, while also considering the agent's geographical structure at a micro-level to specifically study the opinion similarity in the agent's local area. Finally, we examine the tolerance showing the diversity of opinions that can be sustained in the local area. A description of these metrics are listed in Table 5.3.

5.2 Results

In this section, we report results for each evaluation measure in turn, beginning with the fundamental property of convergence, before exploring the nature of consensus that emerges. In all experiments (except *static*), the interaction range r_s is set to 2.

5.2.1 Convergence

Figure 5.1 shows the convergence in opinion for both models over a range of opinion thresholds, probability of movement (p), and distance moved (λ). In the plots, the dashed lines denote results for the random model, with solid lines representing HM. The colour of each line denotes the probability that mobility is applied after each interaction. "Static" denotes the static DW model (with global interactions and no movement $p = 0$), while "Local static" restricts the interactions in DW to $r_s = 2$.

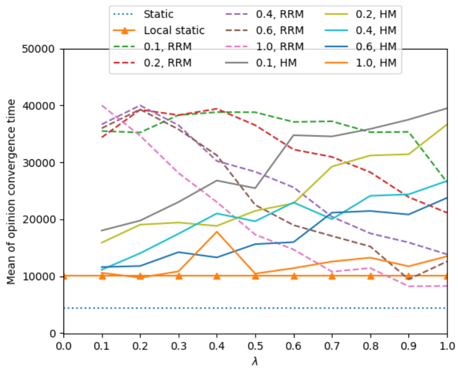
For both models, convergence is quicker when agents are more mobile (i.e. as the probability of movement p increases), with the highest mobility ($p = 1$) under HM approaching the convergence time of the standard static model with global interactions. As may be expected, convergence in opinion is quickest under the static model, however, restricting interactions under the local static model takes longer time to converge in opinion. As with the static model, there appears to be a step change in behaviour when moving from an opinion threshold of 0.1 to 0.2 or 0.3.

The impact of λ as a control on the distance moved is more pronounced for lower opinion thresholds ($\epsilon = 0.1, 0.2$), but, interestingly, it shows an increasing correlation for RRM, but a decreasing relationship for HM. For RRM, higher λ means further distance moved with more opportunities for interaction and therefore less time steps for convergence. However under HM, opinions are more structured in geographical space, therefore, further distance means more potentially influential interactions that will slow the convergence.

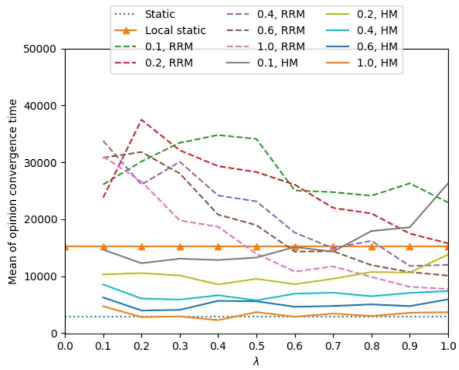
5.2.2 Opinion clusters

We investigate how agents are clustered with respect to opinion following convergence, firstly considering the number of clusters detected (Figure 5.2). In the local static model, consensus forms around a larger number of opinions than the static model [71, 16].

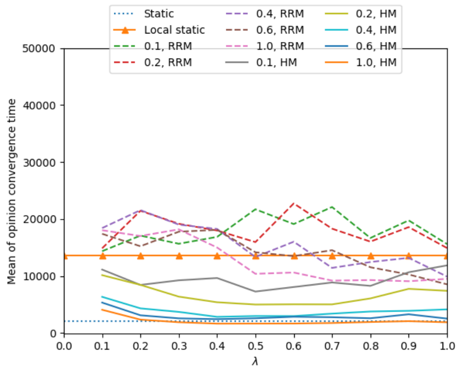
When allowing mobility ($p > 0$), larger numbers of opinion clusters are identified under HM (solid lines in Figure 5.2) compared to the local static case. In contrast to convergence time, the probability of movement has little impact on the number of opinion clusters formed, except where $p = 0$. Fewer clusters are formed under RRM, with the numbers being very close to the static model. In HM, agents have higher



(a) $\epsilon = 0.1$



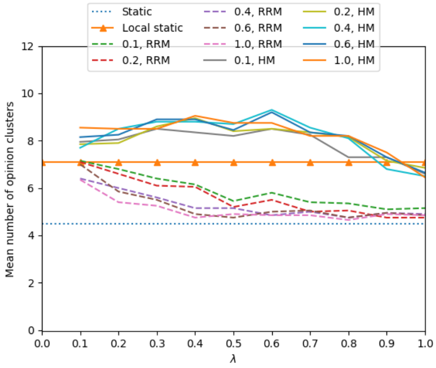
(b) $\epsilon = 0.2$



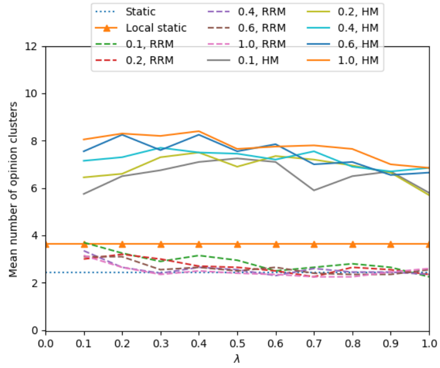
(c) $\epsilon = 0.3$

Figure 5.1: Mean opinion convergence time when varying p under HM and RRM, convergence is not found when $t = 40,000$.

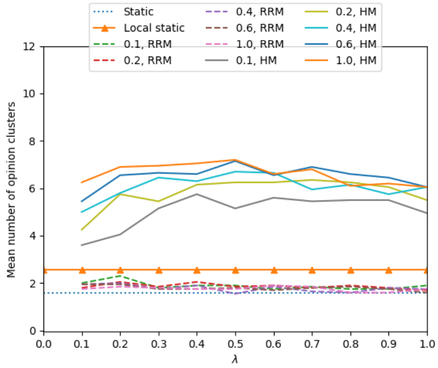
chance to attract and move towards agents with similar opinions, leaving a substantial distance to those holding different opinions that would otherwise influence them. So when movement is directed they are able to recruit more similar opinionated agents that can be detectable. However this also leads to a larger numbers of loners (Figure 5.3) for the lowest values of λ, particularly for small ϵ due to the restricted range of movement. For $\epsilon = 0.1$, it is of interest that increasing the size of movement λ causes the number of loners to reduce, although it doesn't increase the total number of clusters. In essence, the slower evolution allows greater structure to occur. In comparison to RRM, lower numbers of loners emerge under HM. In fact, the number of loners under RRM is close to that of the static model, behaving as a fully connected network. Similar behaviour is observed for the number of identified opinion clusters.



(a) $\epsilon = 0.1$



(b) $\epsilon = 0.2$

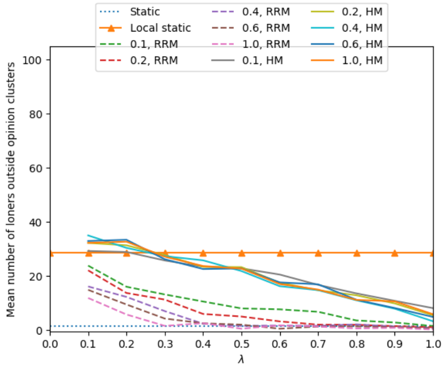


(c) $\epsilon = 0.3$

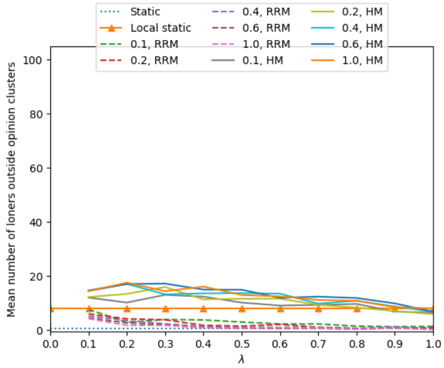
Figure 5.2: Mean number of opinion clusters when varying p under HM and RRM.

5.2.3 Local opinion diversity

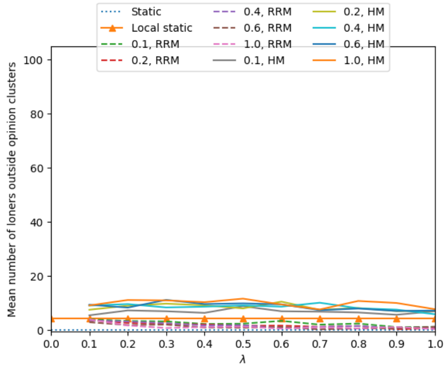
Figure 5.4 highlights that the faster convergence in opinion for HM is also associated with lower tolerance levels. In particular, for $\epsilon \geq 0.2$, HM results in each agent being surrounded by peers that entirely agree with their opinion (i.e. with opinion differences below δ_{op}), while RRM allows high tolerance to persist. In contrast, the behaviour of RRM is extremely consistent across all values of p and λ, which is similar to the case when agents are static. Naturally the tolerance decreases as ϵ increases for all models, due to the increased opinion similarity across all agents. Similar differences between RRM and HM are also seen in the mean local diversity, as shown in Figure 5.5



(a) $\epsilon = 0.1$



(b) $\epsilon = 0.2$



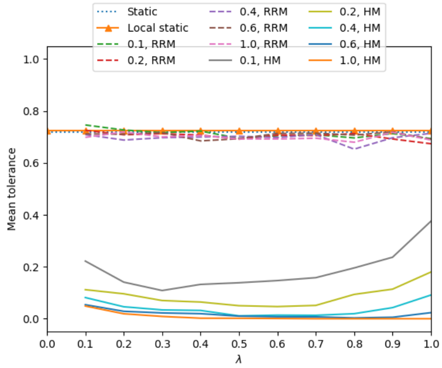
(c) $\epsilon = 0.3$

Figure 5.3: Mean number of loners outside opinion clusters when varying p under HM and RRM.

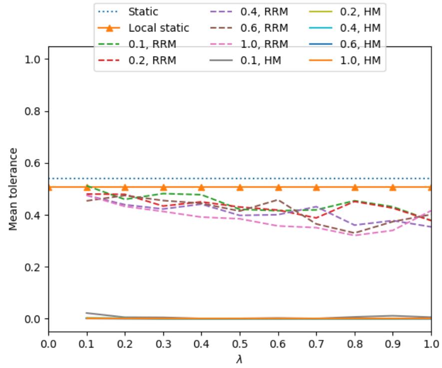
5.2.4 Communities

The differences between the two alternative mobility models are most evident in their effect on community formation (Figure 5.6). HM results in the formation of a large number of communities in comparison to the other models, which remain relatively stable as ϵ is increased. As for previous measures, results for RRM are close to the static model.

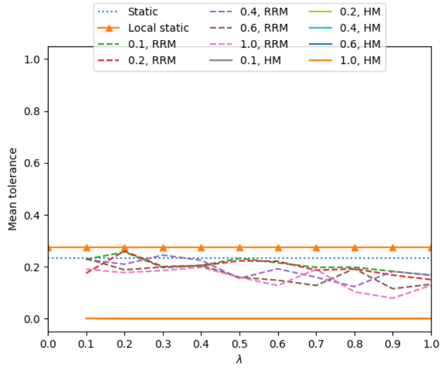
The number of communities needs to be considered in tandem with the number of loners. As ϵ increases for HM, the number of communities stays constant, however the number of loners reduces significantly, showing that these communities are becoming more substantial. Similarly, although the number of communities for RRM increases dramatically, this is in the context of high numbers of loners ($> 50\%$). Recall that



(a) $\epsilon = 0.1$



(b) $\epsilon = 0.2$

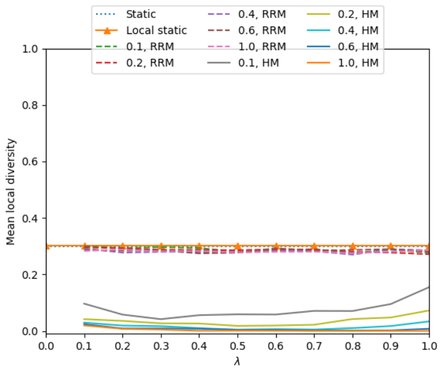


(c) $\epsilon = 0.3$

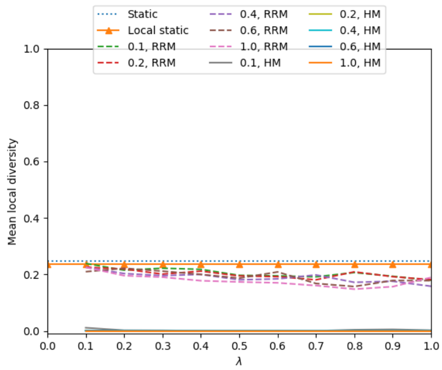
Figure 5.4: Mean tolerance when varying p under HM and RRM

the static model normally results in complete consensus for $\epsilon \geq 0.3$, hence this lack of structure is likely to be related to the geographical separation of agents rather than their opinion.

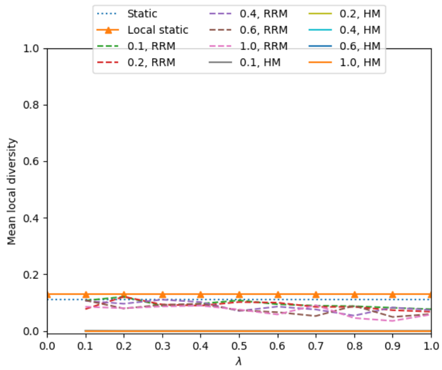
For both static models (static and local static), barely any communities are detected (Figure 5.6) and typically all agents are classified as loners (Figure 5.7), similar to the RRM. The restrictive interaction did not help form communities. However, as ϵ increases the number of communities increase, this is due to the fact that less opinions exist and therefore communities of similar agents nearby each other are easier to be detected.



(a) $\epsilon = 0.1$



(b) $\epsilon = 0.2$

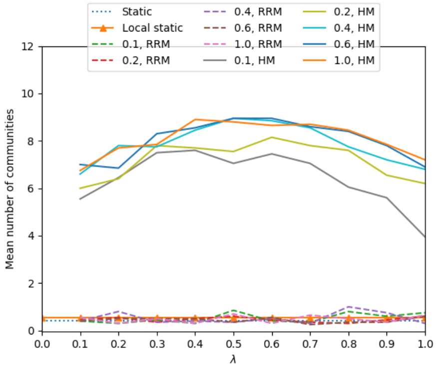


(c) $\epsilon = 0.3$

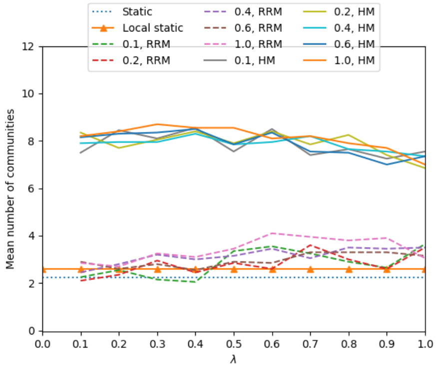
Figure 5.5: Mean local diversity when varying p under HM and RRM

5.3 Discussion

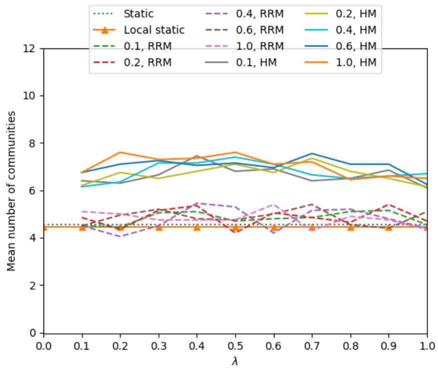
A marked difference can be seen between the random and directed mobility models. The results consistently show that RRM produces very similar effects to the original static opinion model with global interactions, with very little variation due to the probability p or scale λ of movement. Differences are only evident for the speed of convergence in opinion. Inline with other studies that consider opinion formation with models of random mobility (e.g. [121, 105]), we find that increased mobility (either manifested through higher probability of relocating or larger range of movement) leads to faster convergence in opinion. The random mobility is not showing significant difference in the opinion dynamics that could explain why mobility is infrequently considered in the literature.



(a) $\epsilon = 0.1$



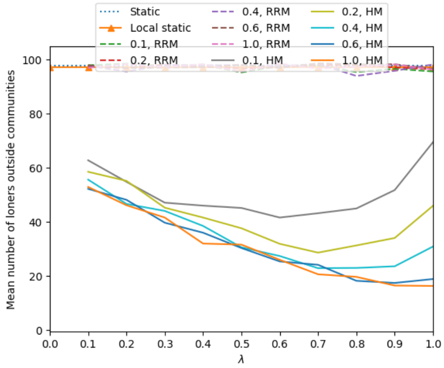
(b) $\epsilon = 0.2$



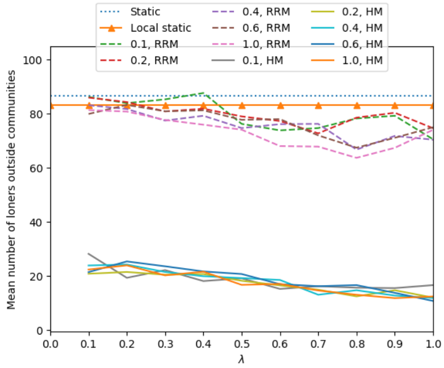
(c) $\epsilon = 0.3$

Figure 5.6: Mean number of communities when varying p under HM and RRM

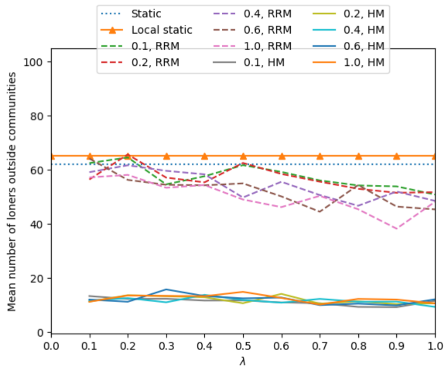
The model we present in this chapter highlights the importance of considering mobility and our psychological behaviour in modelling opinion, with implications for scenarios where individuals have control over their social structures. Our proposed hybrid model is inspired by homophily and cognitive dissonance and results in radically different and results across all evaluation measures, demonstrating a greater propensity for clusters of distinct opinions to survive, with groups separating geographically to avoid conflict. We have shown that incorporating mobility with an incentive associated with a preferred direction (instead of random), produces multiple opinion clusters.



(a) $\epsilon = 0.1$



(b) $\epsilon = 0.2$



(c) $\epsilon = 0.3$

Figure 5.7: Mean number of loners outside communities when varying p under HM and RRM.

5.4 Conclusions

We have investigated two different mechanism for mobility driven by psychological theories, one uses a random incentive for re-location triggered by opinion differences (RRM) and the other is more driven via similarity/difference (HM). We investigate mobility by the two parameters, p and λ, the probability of movement and scale of movement. We found significant difference between the two models which triggers more exploration for this research. Firstly, random mobility has shown no significant impact from the static model, neither can it form structure for the population. Structure and communities were formed under HM where tolerance levels were drastically reduced in comparison to RRM. In addition, we found that more movement (higher p) shows faster convergence in opinion. The most interesting finding is under HM more opinion clusters are found than in the random mobility model and therefore this triggers the question how robust are these results when introduced in a noisy environment, with uncertain agent behaviour. This will be investigated in Chapter 6.

Chapter 6

Impact of noise

In this chapter we investigate the influence of thermal noise during interactions, in order to study the model's robustness under uncertainty. Part of this work in this chapter was published in [6]. A brief introduction to the literature surrounding noise in opinion modelling was presented in Chapter 2, and in this section we will explore this in more depth, to provide comparisons between different approaches that motivate our model.

Generally, the DW opinion model represents a balanced compromise between a pair of agents that are already "similar" in opinion (governed by the parameter ϵ). However, when they are in disagreement their opinions do not diverge. As a result, opinion clusters slowly disappear as ϵ is increased, leading to a central consensus of one single opinion. In our model, this result can be avoided when the agents are mobile, allowing distinct opinion clusters to maintain their existence (as shown in Chapter 5). So, following the DW's model natural effect to complete consensus, the question is how stable is our mobile opinion formation model while adding some noise to the agents behaviour. Will a single opinion dominate over the population even when noise in mobility is taken into consideration?

6.1 Relevant studies of noise

One of the ingredients used in opinion modelling is to incorporate noise to demonstrate fluctuation in behaviour. Models without noise are usually referred to as *deterministic* *models*.

Definition 15. *A **deterministic model** follows strict rules that don't apply noise or uncertainty behaviour.*

Research under both static and *dynamic networks* have been studied while applying noise (some times referred as adaptive networks but in this thesis we'll use dynamic).

Definition 16. *A network is defined as **dynamic** when it allows neighbour changes over time. This can either be represented in a network structure or a 2D lattice.*

It has been agreed that applying noise to the DW on a locally *static network* leads to complete consensus [54, 71]. In the first studies these networks restricts agents interaction within a fixed neighbourhood size and they are unable to change their links but adapt as their opinion is evolving.

Subsequently, [54, 71] have continued to investigate the effect of noise for locally connected *dynamic networks* where agents are able to rewire according to the opinion dynamics. The dynamic model in [71] was shown to be robust in the sense that it produces scenarios with both consensus and polarised opinions. However, [54] raised the objection of whether this polarisation exists while applying a *probability method* to the applied noise. It continues explaining that the noise's nature is *non-symmetrical*, thus agents don't behave randomly when they are similar in opinion but only when they're different. Consequently, [54] claims that when *thermal noise* is applied on dynamic networks, complete consensus is obtained. They claim that this is due to *symmetrical* noise that is applied on both situations of agreeing and disagreeing agents.

Thermal noise originally refers to an analogy in thermodynamics, specifically with the way that metals cool and anneal. The higher the temperature the more the hardness of the material is reduced and able to be re-shaped. This concept is used as analogy to demonstrate the unexpected behaviour of humans and to show if the expected outcome persists or becomes volatile.

Research	Opinion model	Noise type	Opinion type	Static network	Dynamic network
Centola et al. [19]	Axelrod model	Symmetrical [69]	Discrete (vector)	Complete consensus	Diversity
Grabowski [52]	Social Impact Theory	Symmetrical	Discrete	Both opinions survive	N/A
Grauwin and Jensen [54]	DW	Symmetrical	Continuous	Complete consensus	Complete consensus
Guo et al. [57]	Majority rule	Symmetrical [54]	Discrete	N/A	Both opinions survive
Kozma and Barrat [71]	DW	Non-symmetrical	Continuous	Complete consensus	Polarisation
Pineda et al. [94]	DW	Symmetrical	Continuous	Fragmentation	N/A

Table 6.1: Incorporating noise in opinion models

In Table 6.1, we review a number of related works by classifying them as symmetrical or non-symmetrical noise as defined below. We also categorise the different opinion models with the type of opinion representation. In addition, we highlight the results under either static or dynamic networks.

Definition 17. ***Non-symmetrical noise*** *demonstrates unexpected behaviour of not following the rules only when disagreement is encountered.*

Definition 18. ***Symmetrical noise*** *demonstrates unexpected behaviour of not following the rules when both agreement and disagreement is encountered.*

Other opinion models have introduced noise, for example, [52] has preceded [54] by applying thermal temperature, although only on discrete opinion models. Because in the model proposed in [54] the opinions are continuous, the authors are able to estimate the peer's opinion change as an analogy to energy. On the contrary, [52] uses the average of the local opinions connected to the agent to represent energy levels, referencing concepts from the Social Impact Theory model [73] for the opinion modelling. Another application is shown in [57], where the authors applied the thermal noise mechanism to the majority rule model, again a discrete opinion model. These discrete opinion models are simplified in many aspects, one being the fact that all agents have the same chance of interactions. As stated in [29] *'clustering is reinforced when agents diversity is introduced, for instance diversity of influence'*. Furthermore, the end state of the opinion distribution is very limited to either one or two opinions, making it harder to find any patterns in the agents behaviour.

The thermal noise produces different outcomes when applied to different opinion models; one might lead to a disordered state while another leads to ordered states. [16] defines 'order' as a translation in the language of physics of what is denoted in social sciences complete consensus, while a 'disordered' state is when opinions are fragmented without structure. When noise is applied to discrete opinion models, random behaviour is expressed by forcefully following the opposite opinion of the one actually expected [52, 57]. This type of behaviour shows divergence in opinion and as a result the other opinion is never diminished. Therefore, in such models, higher temperature levels lead to disorder, with both opinions surviving without any structure or pattern, in a similar fashion to the random 'turnover' proposed in [94].

As for the continuous opinions under the BC models, the rules depend on thresholds of disagreement and the noise would raise the the probability of convergence regardless of the agreement status (since the rules naturally don't diverge). As a consequence the rules will naturally stimulate consensus and with higher temperatures this leads to an ordered state, holding a single opinion cluster. This is due to the key ingredient of forces of attraction and convergence (based on homophily), thus raising the question of whether the HM model is robust enough to show a more realistic scenario of diverse opinions.

In this Chapter we apply noise to represent the unexpected or imprecise behaviour that might occur in communication between peers. Based on this review of the literature, the key features that should be captured in modelling noise relate to a type of noise that impacts equally on decisions without any bias. In particular, the noise should not be biased towards only disagreeing peers but it should instead occur equally to both agreeing and disagreeing agents. These features are largely present in the *thermal noise* model [54], which has been widely used in social computational models, hence we include this into our model with the goal of investigating its *symmetrical* effect on opinion structures under mobility.

6.2 Methodology

In Chapter 5 random mobility (RRM) did not show any interesting behaviour, therefore, we limit our attention to the HM model, and extend Algorithm 7 by adding thermal noise [54], as described in Algorithm 10.

Attention is restricted to the case where $\epsilon = 0.1$, since the earlier results (in Chapter5) show that this value gives the widest range of opinion clusters and tolerance, in contrast to higher values that lead to a rapid convergence to complete consensus. Also, we decrease the number of experiments with λ values, to save more computation time for more experimentation with values of T. The set of parameters used in the experiments below are shown in Table 6.2 and 6.3.

Figure 6.1 shows the probability of opinion changing with opinion threshold $\epsilon = 0.1$ under different values of temperatures T. The dotted line shows a deterministic model that follows the rules strictly without noise. Showing that if the opinion difference exceeds ϵ the probability of changing the opinion is definitely not going to happen. However, as we add noise ($T = 0.1$) this probability increases, meaning that even if different opinions interact there is a slight probability to actually influence each other. Also, this noise is applied when similar opinions interact, so there is a slight probability they will not influence each other. It is shown as the temperature increase ($T = 10$) the randomness in behaviour increase and therefore come closer to a $50 : 50$ chance of

Parameter	Description	Value
$L \times L$	Region size	10×10
$seeds$	Number of averaged simulation runs	20
n	Number of agents	100
$limit$	Maximum number of time steps per simulation	$40,000$
ϵ	Opinion threshold for influence (see Definition 3)	0.1
r_s	Interactive radius	2
δ_{op}	Opinion change threshold	0.01
δ_{mov}	Movement distance change threshold	1
N_F	Number of time steps without opinion change for convergence	$10,000$

Table 6.2: Fixed variables

Parameter	Description	Value
Mobility models	Static, random and directed mobility models	Static, local static, RRM, HM
p	Probability of movement	$[0.1, 0.2, 0.4, 0.6, 1]$
λ	Movement scale factor	$[0.2, 0.4, 0.6, 0.8, 1]$
T	Noise	$[0.1, 1, 10]$

Table 6.3: Independent variables

changing opinion, where ϵ has less impact on the opinion dynamics.

For this chapter results are presented for three cases. The label *'No noise'* represents the deterministic HM model, with no uncertainty in the decision to interact (figures from Chapter 5). Noise is then applied at low ($T = 0.1$) and high ($T = 1$) temperatures. Further experiments with higher temperature ($T = 10$) showed similar results to $T = 1$, therefore are not presented in this chapter but attached in Appendix A.

Aim In this section, we are interested in whether the addition of noise results in fewer opinions, as widely reported in existing studies [54, 71]. Therefore, the *aim* of this chapter is to study the effect of noise on the general evolution of opinions for agents

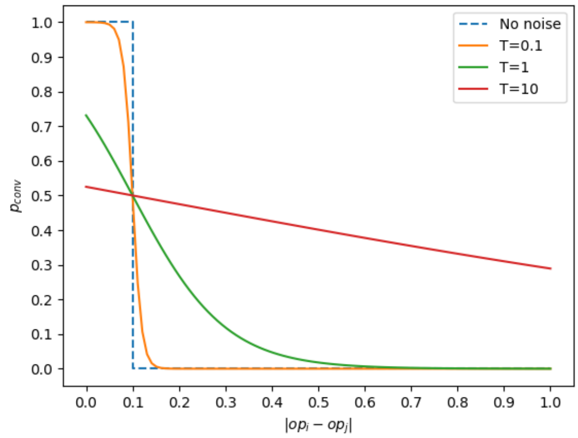


Figure 6.1: Probability of changing opinion under the impact of thermal noise

that have the opportunity to move while interacting within a restricted area around them.

Hypothesis Following the DW's model natural tendency to result in complete consensus, under the impact of noise, complete consensus is an end result even under small ϵ. The question is can our mobile opinion formation model preserve multiple opinions while adding some noise to the agents behaviour. Our hypothesis is that directed mobility will resist the vulnerability of opinion changing to a single dominant opinion.

Experiments We focus our investigation on the directed mobility model (HM) under the impact of noise, considering the static model as a *benchmark* (dashed line in blue). In addition, we show the results for the local static (straight line in orange). The models under investigation are as follows.

1. Static models

    - Static: a demonstration of the DW original model (Algorithm 1, Section 3.2.1)

- Local static: a modified version of the DW model, but restricting the interaction range.

2. Directed mobility

- HM: to move toward/away depending on the agreement between the peers (Algorithm 7, Section 3.2.2.2).

Evaluation We *evaluate* the effect of adding noise to the HM model (denoted HM noise) while varying p and λ using similar structure evaluating the results as in Chapter 5, investigating the listed dependant variables in Table 5.3.

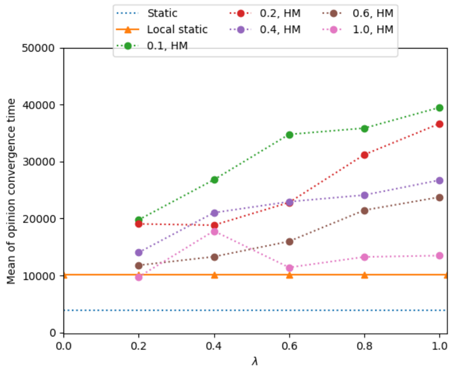
6.3 Results

In this section we present the results following the same structure in Chapter 5, conducting an investigation of the convergence followed by the emergence of opinions and their geographical distribution.

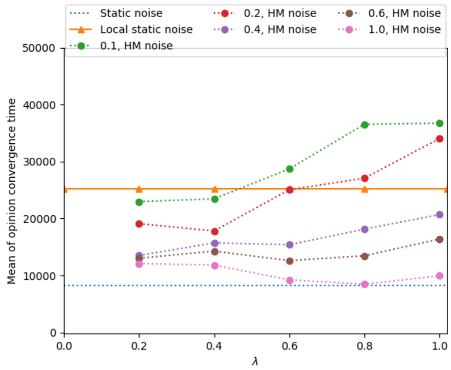
6.3.1 Convergence

Figure 6.2 shows the convergence in opinion following a pattern of faster convergence with higher temperature. We first note that the convergence in opinion for low temperatures (Figure 6.2b) are similar to the deterministic case (Figure 6.2a). However, increasing temperature $T = 1$ results in the fastest convergence, similar to the larger value of $\epsilon = 0.3$ in the deterministic case (Figure 5.1). The static model also follows the same pattern of faster convergence as HM.

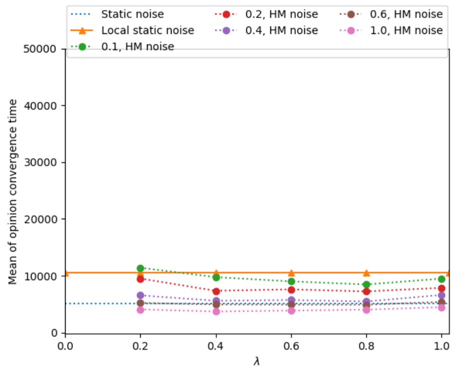
In addition, in a deterministic setting (Figure 6.2a), the highest p is the slowest in convergence compared to lower p values, this pattern is followed as T increase. However, higher λ cause slower convergence but that is not found when $T = 1$. The λ generally cause slower convergence because of the larger distance of movement and the ability to explore new opinions, although this is mitigated when noise is high. The noise naturally decreases the number of opinion clusters and the tolerance (as we'll see in the next sections). Therefore, it decreases the probability of interacting with someone new, making the convergence faster.



(a) No noise



(b) $T = 0.1$



(c) $T = 1$

Figure 6.2: Mean opinion convergence time for $\epsilon = 0.1$, convergence is not found when $t = 40,000$.

6.3.2 Opinion clusters

Figure 6.3 shows the opinion clusters in two columns, one demonstrating the actual number of opinion clusters and the other showing the number of loners outside the opinion clusters.

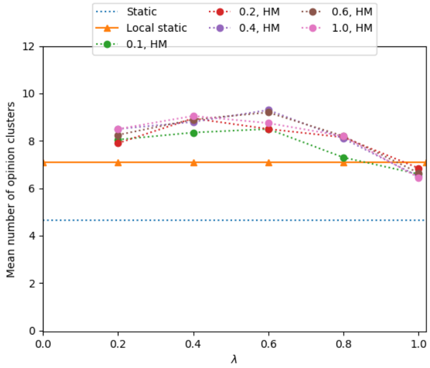
One of the conclusions in Chapter 5 is that HM maintains a large number of opinion clusters (see Figures 5.2). Figures 6.3c and 6.3e shows that high numbers of opinion clusters are still maintained under high values of noise compared to no noise in Figure 6.3a. The number of opinion clusters does indeed decrease as T gets higher but not to the extent of reaching a single opinion. In particular, the more p decreases, the less are the number of opinions that survive. The higher p the more opinion clusters survive under high levels of noise. In addition, increasing T results in almost all agents being classified in a cluster with minimum numbers of loners (Figures 6.3d and 6.3f) compared to no noise in Figure 6.3b, similarly to the deterministic with high ϵ (see Figure 5.3).

For both static models (static and local static), with the highest temperature we observe the formation of a single opinion ($T = 1$, Figure 6.3e), which is in line with the literature when noise has been considered in static networks [54, 71]. One of the findings in Chapter 5 was that the local static model produced more opinion clusters than the static (see Figures 5.2). However, high noise levels reduce the effect of interacting locally in the local static model, which is now behaving similarly to the static model with global interaction. This contrasts with the deterministic model that produces considerably more opinion clusters in the local static model than the static.

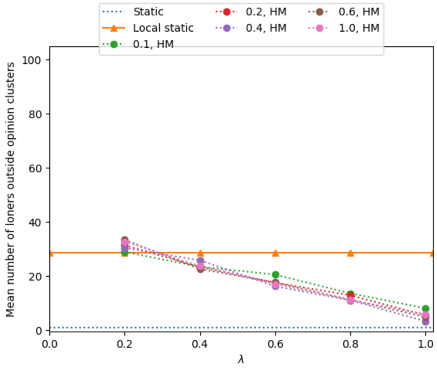
6.3.3 Local opinion diversity

The same pattern of decrease is repeated as T increases when considering the local distribution of opinions. For both tolerance (Figure 6.4) and local diversity (Figure 6.5), low noise ($T = 0.1$) results in only a slight drop in metrics compared to the case with no noise. Conversely, adding high noise ($T = 1$) sees no variation in the opinions of the agents with proximity, in a similar fashion to high values of ϵ for the deterministic model (Figures 5.4 and 5.5). This pattern of behaviour is also observed with both static models.

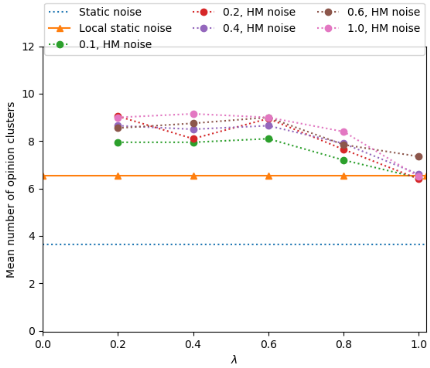
Similar to the convergence in opinion, higher p maintains higher levels for both tol-



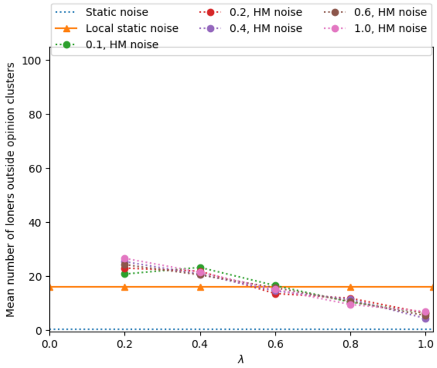
(a) Number of opinion clusters under no noise



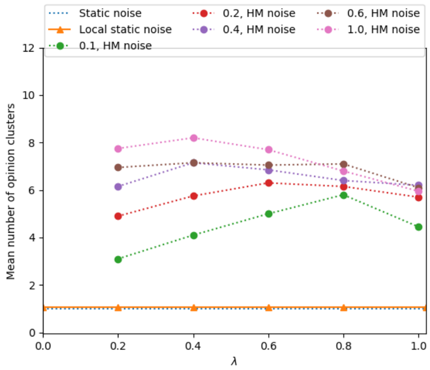
(b) Number of loners outside opinion clusters under no noise



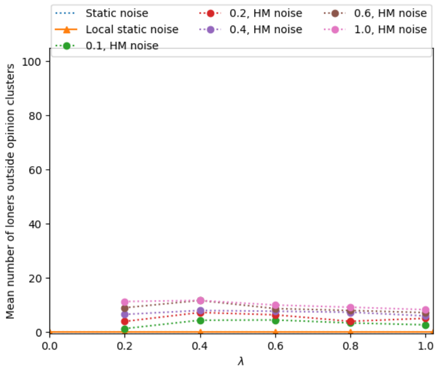
(c) Number of opinion clusters for $T = 0.1$



(d) Number of loners outside opinion clusters for $T = 0.1$



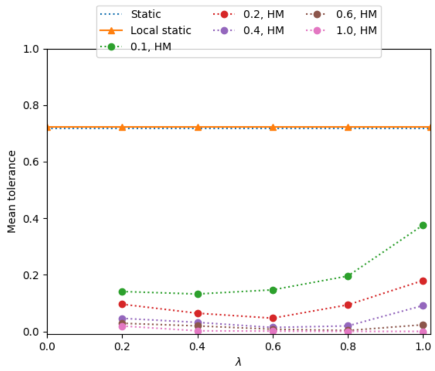
(e) Number of opinion clusters for $T = 1$



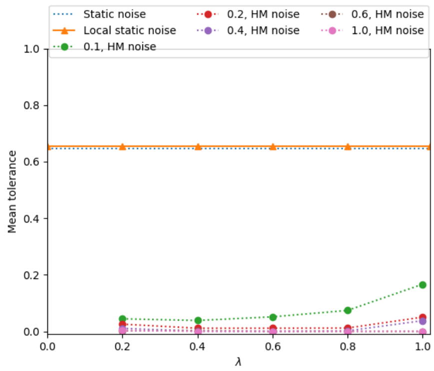
(f) Number of loners outside opinion clusters for $T = 1$

Figure 6.3: Mean number of opinion cluster and mean number of loners outside opinion clusters for $\epsilon = 0.1$.

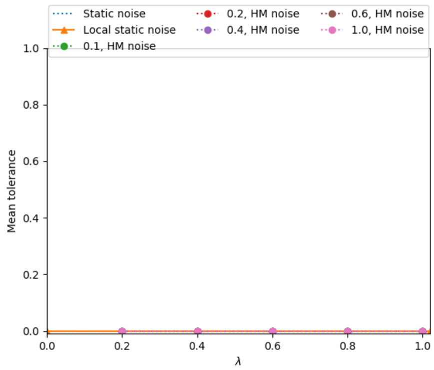
erance and local diversity compared to lower p, due to the restricted movement being able to find an area with absolute uniform opinion, although with higher noise this diversity diminishes as expected.



(a) No noise



(b) $T = 0.1$



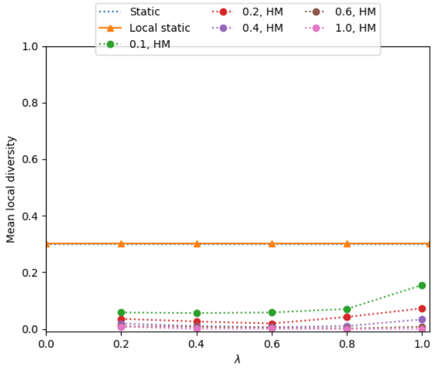
(c) $T = 1$

Figure 6.4: Mean tolerance for $\epsilon = 0.1$

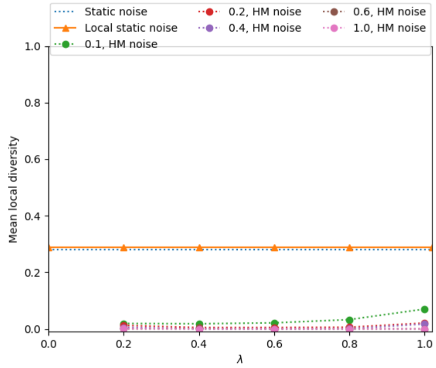
6.3.4 Communities

Figure 6.6 shows the structure of communities in two columns, one demonstrating the actual number of communities and the other showing the number of loners outside communities.

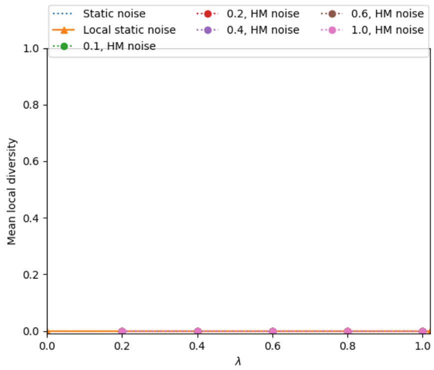
Similarly to opinion clusters, a high number of communities is maintained as T increases (Figures 6.6a,6.6c and 6.6e). Furthermore, p is observed to have a similar impact in the formation of communities to that of opinion clusters, as high p values produce the highest number of communities across all range of T values. Furthermore, with higher T the number of loners decreases as well, forming a more organised population (Figures 6.6d and 6.6f), similar to the deterministic case with high ϵ (see Figure 5.7).



(a) No noise



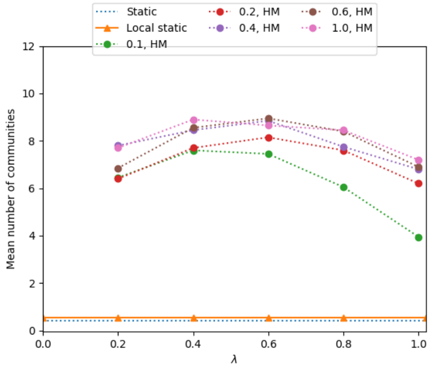
(b) $T = 0.1$



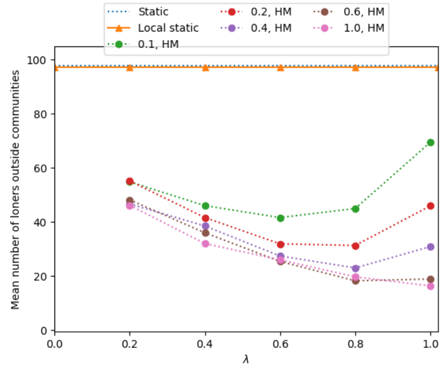
(c) $T = 1$

Figure 6.5: Mean local diversity for $\epsilon = 0.1$

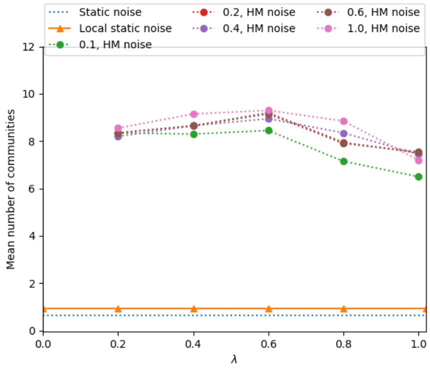
For both static models (static and local static), the case with $T = 0.1$ doesn't behave any different from the case without noise. However, when the temperature rises higher ($T = 1$), we observe the formation of more communities, similar to the deterministic model with high ϵ (see Figure 5.7). In addition, the number of loners decreases from a case with all agents classified as loners (Figures 6.6b 6.6d) to less than half of the population joining a community (Figures 6.6f). This is because there are no more opinion clusters and all the agents have converged to a single opinion (Figure 6.3e), making agents within proximity easier to form communities.



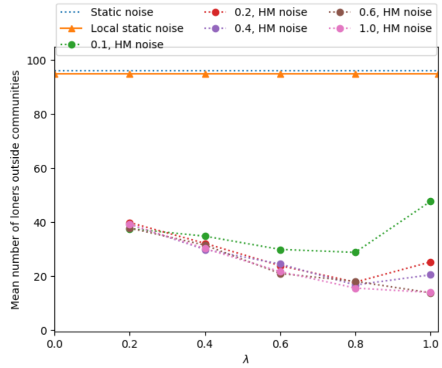
(a) Number of communities under no noise



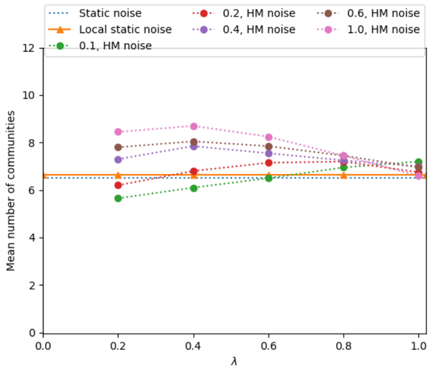
(b) Number of loners outside communities under no noise



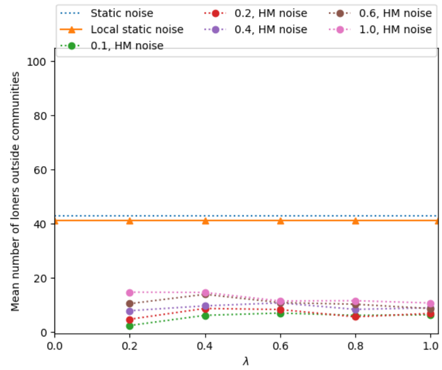
(c) Number of communities for $T = 0.1$



(d) Number of loners outside communities for $T = 0.1$



(e) Number of communities for $T = 1$



(f) Number of loners outside communities for $T = 1$

Figure 6.6: Mean number of communities for and mean number of loners outside communities for $\epsilon = 0.1$.

6.4 Discussion

Previous work considering noise in dynamic networks of opinion (where links can be rewired) suggests two extreme outcomes: either complete consensus [54] or polarisation [71]. This difference is due to the fact that the rewiring algorithm in [71] does not allow bonds between agents in agreement to be broken (asymmetrical noise). In [54], symmetrical noise is implemented, which allows agents to rewire by forming a link at random with any other agent in the network, with no geographical restriction, in either cases of agreement or disagreement.

In our model, the ability to move and react to an interaction is restricted to peers within the local area (i.e. the equivalent of link formation). As a consequence, we do not record high levels of consensus in our results even if we applied substantial level of symmetrical noise, in contrast to [54].

Our model is similar to [54] while implementing the static opinion model but not while implementing the mobility model. More specifically, we don't move away (break links) between agreeing agents. As a consequence, we don't get complete consensus as in [54]. In contrast, we obtain more than a single opinion cluster because, similarly to [71], we don't break links to agreeing agents. This property allows distinct clusters to be maintained both in terms of opinion and location, even in the deterministic scenario without any noise applied. The other effects shown by our model are consistent with the literature [54], speeding up convergence when a form of noise is considered.

6.5 Conclusions

The HM model has shown a robustness to even high levels of noise since both restricting interactions range ($r_s = 2$) as well as allowing the agents to move have a beneficial effect in maintaining multiple opinion clusters. Although convergence in opinion space is always achieved we never achieve complete consensus to one opinion. Furthermore, we neither obtain disorder opinion states, nor have a non-collective structure.

In Chapter 5 the static model without noise naturally tends to converge to complete consensus because of its purely attractive mechanism in opinion influence which doesn't encourage divergence. In contrast when adding mobility which includes attraction and repulsion we have found that opinions can diverge. As a result, with such a method we found that more opinion clusters can be maintained at a macroscopic level. When we apply noise and randomness in behaviour, we note that our model shows strong robustness.

In Chapter 7 we focus our investigation to compare between two random mobility models, the RRM and the widely used random mobility model that demonstrates constant random movement (PRM).

Algorithm 10 Simulation framework with HM and noise

Require: Input parameters $(n, limit, r_s, \epsilon, \mu, p, \lambda, N_F, \delta_{op}, \delta_{mov}, T)$
Require: Initial population A of n agents
for $limit$ time steps **do**
    $a_i \leftarrow U(A)$ ▷ Select random *inviting agent*
    $a_j \leftarrow U(N(xy_i, r_s))$ ▷ Select random *invited agent* from neighbourhood
    $\Delta \leftarrow \frac{|op_i - op_j|}{\epsilon}$
    $p_{conv} \leftarrow \left[1 + e^{\left(\frac{\Delta - 1}{T}\right)}\right]^{-1}$
    if $U([0,1]) < p_{conv}$ **then** ▷ Apply thermal noise
        $op'_i \leftarrow op_i + \mu(op_j - op_i)$ ▷ Successful interaction: Opinion influenced
        $op'_j \leftarrow op_j + \mu(op_i - op_j)$
    else
        $op'_i \leftarrow op_i$ ▷ Unsuccessful interaction: opinion unchanged
        $op'_j \leftarrow op_j$
    end if
    if $U([0,1]) < p$ **then** ▷ Apply mobility rate
        $xy'_i \leftarrow mobility(a_i, a_j)$ ▷ Apply mobility model
    else
        $xy'_i \leftarrow xy_i$ ▷ No movement
    end if
    $op_i \leftarrow op'_i$; $op_j \leftarrow op'_j$ ▷ Update opinions
    $xy_i \leftarrow xy'_i$ ▷ Update location
    if $\Delta_{op} < \delta_{op}$ **then** ▷ Convergence tracking as in Algorithm 8
        $t_{op} \leftarrow t_{op} + 1$ ▷ Insignificant opinion change
    else
        $t_{op} \leftarrow 0$ ▷ Significant opinion change - reset
    end if
    if $\Delta_{mov} < \delta_{mov}$ **then**
        $t_{mov} \leftarrow t_{mov} + 1$ ▷ Insignificant location change
    else
        $t_{mov} \leftarrow 0$ ▷ Significant location change - reset
    end if
    if $t_{op} == N_F$ or $t_{mov} == N_F$ **then**
        return A, t, t_{op}, t_{mov}
    end if
end for
return A, t, t_{op}, t_{mov}

Chapter 7

Random and directed mobility components

Two main conclusions emerged from the initial investigation of the HM model in Chapter 5. Firstly, we showed that directed mobility has a significant impact on the structure of opinions that evolve, allowing more distinct opinions to persist than the static model without mobility (Section 5.2.2). Secondly, we demonstrated that random mobility behaves similarly to the static model (Section 5.2.2, 5.2.3 and 5.2.4). To determine potential reasons why this happened, we now perform a deeper investigation on the random mobility. Also, we explore the individual components (attraction and repulsion) in the directed HM model and investigate their dynamics and stability in terms of parameters.

The aim of this chapter is to investigate which features of these mobility models can form organisation and impact the opinion dynamics. First, we investigate the random mobility (PRM), which is analogous to the constant movement of gas particles, as widely studied in the literature, and compare this to another random mobility (RRM) with a social driver to only move when disagreement is encountered. After this, we study the convergence and stability of the directed mobility (AM, RM, HM) models to conduct a deeper understanding of the parameter space.

Parameter	Description	Value
$L \times L$	Region size	10×10
$seeds$	Number of averaged simulation runs	20
n	Number of agents	100
$limit$	Maximum number of time steps per simulation	$70,000$
p	Probability of movement	1
λ	Movement scale factor	0.6
δ_{op}	Opinion change threshold	0.01
δ_{mov}	Movement distance change threshold	1
N_F	Number of time steps without opinion change for convergence	$10,000$

Table 7.1: Fixed variables

7.1 Methodology

In this chapter we are comparing and exploring the different mechanisms of mobility while widening the parameter space investigation. We widen the investigation to explore $\epsilon \in [0, 1]$ instead of stopping at 0.5. Also we consider a variety of different interaction range $r_s \in [1, 10]$. In Chapter 5, the probability (p) and scale of movement (λ) were varied and they showed an effect on the speed of opinion convergence, although not much difference in the number of opinion clusters is formed. Furthermore, larger p had an impact in forming a more structured population to communities and decreased tolerance levels. Our focus in this chapter is to investigate the full mobility $p = 1$ and focus on comparing the different mobility models and the probability to interact and influence through varying r_s and ϵ. Therefore, we conduct these experiments by having both the probability of movement and the scale of movement fixed ($p = 1, \lambda = 0.6$). The full set of parameters is found in Table 7.1 and 7.2.

Aim In this chapter we explore the different mobility mechanisms defined in Chapter 3 (Section 3.2.2). The *aim* of this chapter is to investigate which mobility mechanism can

Parameter	Description	Value
Mobility models	Static, random and directed mobility models	Static, Local static, PRM, RRM, AM, RM, HM
ϵ	Opinion threshold for influence (see Definition 3)	$[0.1, 0.2 \ldots, 1]$
r_s	Interactive radius	$[1, 2, 3, 5, 10]$

Table 7.2: Independent variables

form organisation and impact the opinion dynamics.

Experiments We consider all the models defined in Chapter 3 (Section 3.2.2). The models in this investigation are as follows:

1. Static models
   (a) Static: a demonstration of the DW original (Algorithm 1, Section 3.2.1)
   (b) Local static: a modified version of the DW model, but restricting the interaction range of r_s
2. Random models
   (a) PRM: An agent moves randomly at each time step (see Algorithm 3, Section 3.2.2.1).
   (b) RRM: An agent moves randomly only when a different opinion is encountered (see Algorithm 4, Section 3.2.2.1).
3. Directed models
   (a) AM: Move toward a similar peer (Algorithm 5, Section 3.2.2.2).
   (b) RM: Move away from a different peer (Algorithm 6, Section 3.2.2.2).
   (c) HM: Move toward/away depending on the agreement between the peers (Algorithm 7, Section 3.2.2.2), in this chapter the *movement toward* is described as the attract component and the *movement away* as the repel component.

Parameter	Description	Value
Convergence in opinion (see Algorithm 8)	Time step when opinion change is settled	0 - 70,000 time steps
Convergence in movement (see Algorithm 8)	Time step when distance moved is settled	0 - 70,000 time steps
Opinion clusters (see Definition 9)	Mean number of different opinion clusters	0 - 10 clusters
Tolerance (see Definition 13)	Ratio of different opinions	$tol(A) \in [0, 1]$
Communities (see Definition 10)	Mean number of clusters that share opinion and location	0 - 10 clusters

Table 7.3: Dependent variables

Evaluation We *evaluate* the random and directed mobility individually. First, we evaluate the impact of random mobility, and we use the static model as the main *benchmark* to asses the two random mobility models (RRM and PRM), considering the opinion clusters, tolerance level, the nature of the geographical/opinion clusters that evolve and the opinion convergence time. These metrics are listed in the dependant variables in Table 7.3. In addition, this chapter investigates the movement convergence time to highlight the stability of the different mobility models. Therefore we extend the simulation time steps to $limit = 70,000$ instead of $40,000$ as in the previous chapters.

Second, we assess the convergence for the directed mobility models (AM, RM, HM) and provide a detailed discussion on their evaluation against other metrics in Chapter 8.

7.2 Random mobility

We will evaluate the opinion and movement convergence; the opinion and community clusters and tolerance. A summary of the results is presented in the figures, with more detailed figures provided in Appendix B to visualise the outcome of the experiments across the different configurations.

In the experiments below we apply the random mobility and change both the interactive radius r_s and ϵ to observe the impact upon the opinion model's dynamics as well as the structure of communities.

7.2.1 Opinion clusters

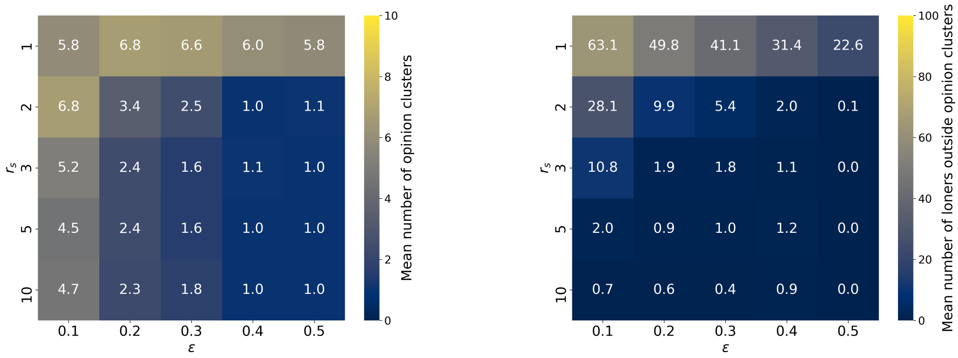
In order to understand the impact of locality, we briefly assess the impact of varying the interaction radius r_s on the local static model, before adding mobility.

7.2.1.1 Effect of the interactive radius on the local static model

In Chapter 5 we found that with local static agents, restricting interaction range r_s is a natural driver to stimulate more opinion clusters (Figure 5.2), specifically for the case of $r_s = 2$. We now conduct a wider range of experiments across different values r_s and ϵ, with results shown in Figure 7.1.

As may be expected, we find that restricting the interaction range to small values ($r_s \leq 2$) increases the number of opinion clusters, especially for lower values of ϵ. This demonstrates that the restricted and fixed neighbourhood substantially limits the global spread of an individual opinion, and as such, a dominant consensus cannot emerge. We also find that more loners emerge without any form of interaction dynamics (Figure 7.1b), which is in line with [71] that found that a static network produces more extensive small clusters than a dynamic network.

When $\epsilon > 0.3$ a single opinion quickly dominates for all values of $r_s > 1$. As a result, opinion similarity is able to spread because of its large opinion threshold ϵ. To conclude, in a local static model, restricting r_s to small values acts as a natural driver to prevent the influence of agents and therefore increase the number of opinion clusters that can persist.



(a) Numbers of opinion clusters

(b) Numbers of loners outside opinion clusters

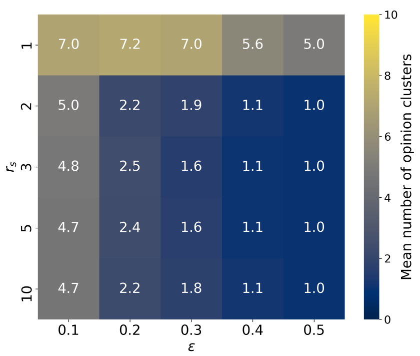
Figure 7.1: Mean number of opinion clusters and mean number of loners outside opinion clusters under local static model.

7.2.1.2 Effect of the interactive radius with random mobility

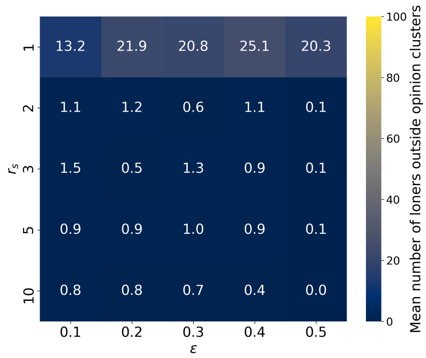
Since restricting r_s in the local static model is able to produce more opinion clusters, we now assess the effect when random mobility is included. Under random mobility models (RRM or PRM), the number of opinion clusters does not increase (Figure 7.2a and 7.2c), but stays similar to the static model under large interaction range ($r_s = 10$) (Figure 7.1a) with the exception of the smallest radius $r_s = 1$. Consequently, adding random mobility tends to *eliminate the effect* raised by the local static model to increase the number of opinion clusters. Figures 7.2b and 7.2d show that loners outside opinion clusters are minimal with the exception of $r_s = 1$ under RRM, as we'll discuss later in Section 7.2.2.1.

7.2.2 Comparison of other metrics under RRM and PRM

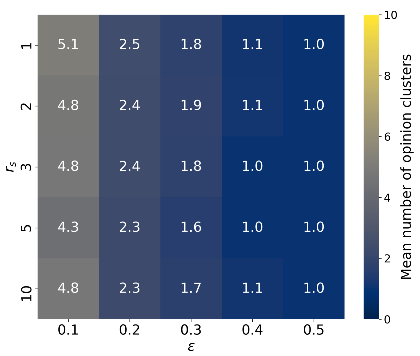
In general, we find most of the metrics showing that both of random mobility models (PRM, RRM) behave similarly to the static model. Furthermore, the time of convergence in opinion is very fast (Figure 7.3). In addition, in presence of multiple opinions (low ϵ) tolerance is always present and uniformity doesn't exist (Figure 7.4), and this



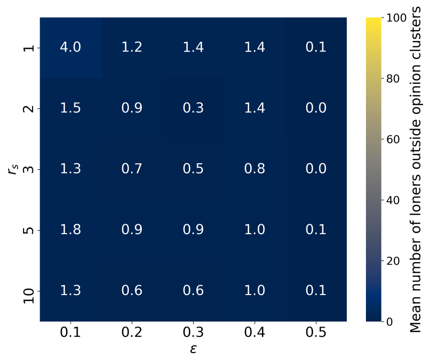
(a) Number of opinion clusters under RRM



(b) Number of loners outside opinion clusters under RRM



(c) Number of opinion clusters under PRM

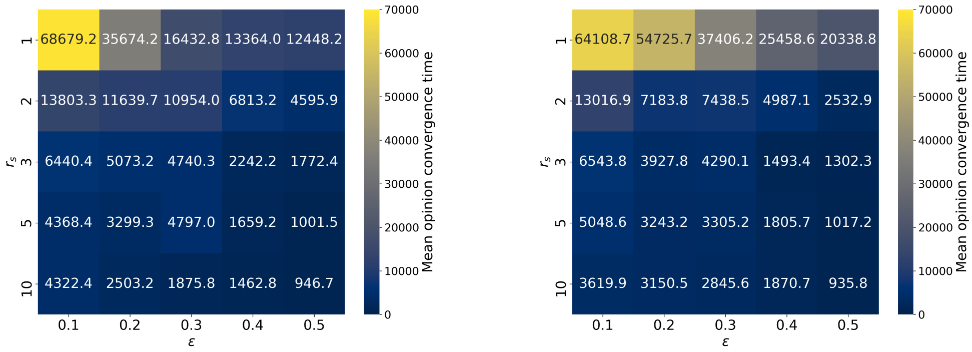


(d) Number of loners outside opinion cluster under PRM

Figure 7.2: Mean number of opinion clusters and loners outside opinion clusters under random mobility.

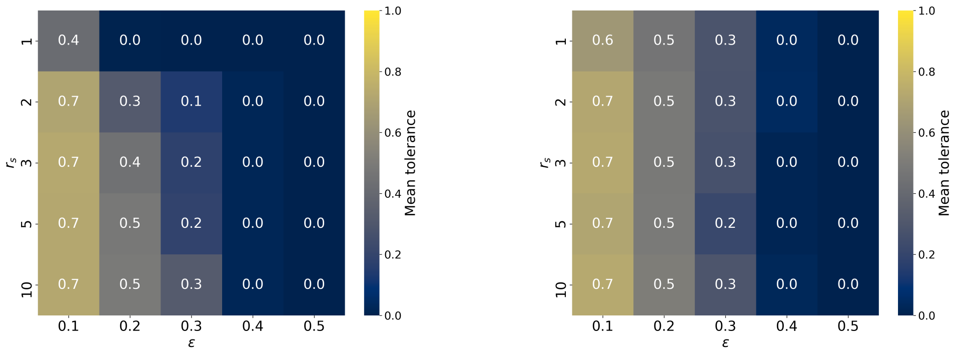
would obviously show no structure in forming communities where most of the population are loners (Figures 7.5b and 7.5d).

With PRM this lack of geographical structure is expected since convergence in movement is hardly ever present (Figure 7.6) due to the agents constant movement. However, with RRM, when complete consensus forms (high ϵ), it is more likely that convergence in movement is found. Note that, when the time steps t run out and convergence is not found, the time step is set to the largest time step, in this case $70,000$. This is because the main factor that triggers movement is disagreement, so as consensus emerges, mobility is reduced. However, as seen in Figures 7.5a and 7.5c some com-



(a) RRM (b) PRM

Figure 7.3: Mean opinion convergence time under random mobility, convergence is not found when $t = 70,000$.



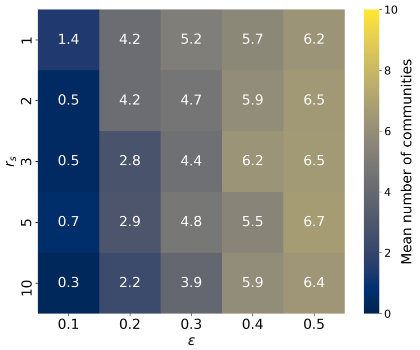
(a) RRM (b) PRM

Figure 7.4: Mean tolerance under random mobility

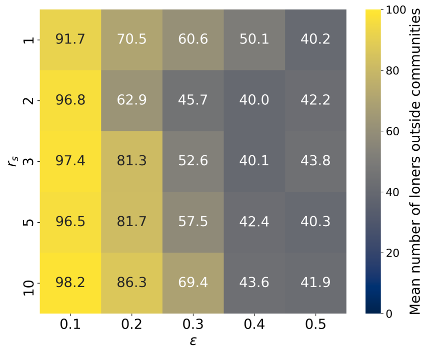
munities emerge and there still emerge a number of loners that become geographically separated, either from peers with similar opinions (that could form clusters) or from peers with dissimilar opinions (that would drive them away to explore the region) (Figures 7.5b and 7.5d).

7.2.2.1 Exception of $r_s = 1$

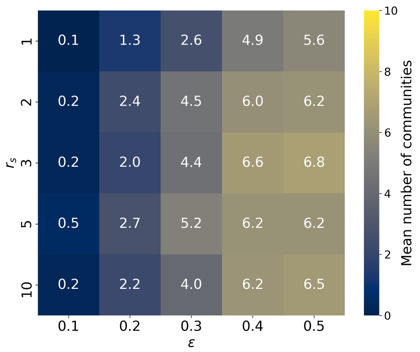
Under both random mobility models (RRM, PRM), when $r_s = 1$ convergence in opinion is rarely obtained. Also, convergence in movement is unusual in showing fast convergence. Not finding convergence in opinion can skew the results of the other



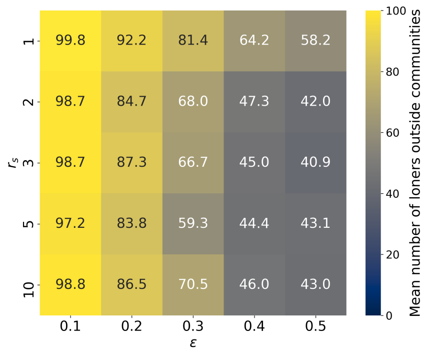
(a) Number of communities under RRM



(b) Number of loners outside communities under RRM



(c) Number of communities under PRM

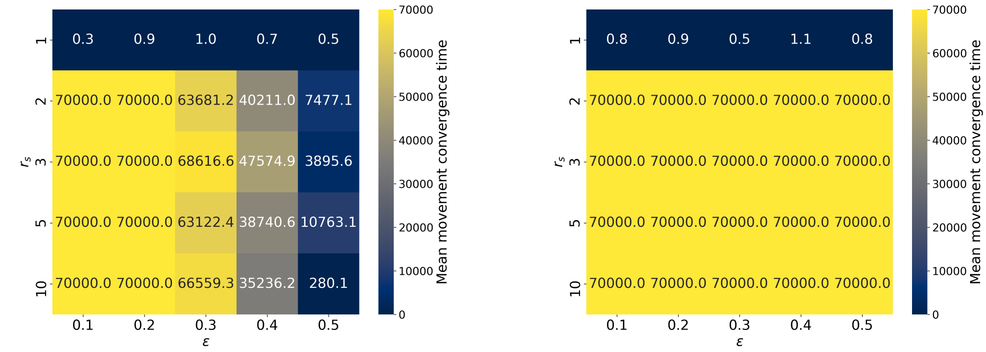


(d) Number of loners outside communities under PRM

Figure 7.5: Mean number of communities and mean number of loners outside communities under random mobility.

metrics. For example, RRM results in larger numbers of opinion clusters (Figure 7.2) than in PRM. However, when convergence in opinion has not occurred, these clusters are not stable and continue to evolve. RRM shows less tolerance (Figure 7.4) when opinions are not stable yet. Due to the very small interactive radius the likelihood of new agents to meet decreases significantly, and therefore the rate at which opinions can fully diffuse through the population decrease too. Therefore, we investigate the convergence for the case under $r_s = 1$.

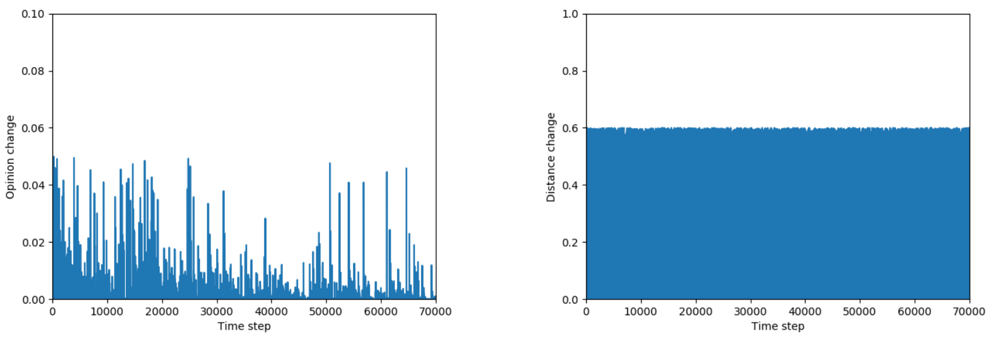
As an example we'll discuss the convergence for RRM when $r_s = 1$, but first we'll discuss the convergence in opinion and later convergence in movement. Figure 7.3 shows



(a) RRM (b) PRM

Figure 7.6: Mean time of convergence in movement under random mobility, convergence is not found when $t = 70,000$.

that *convergence of opinion* for RRM with $r_s = 1$ is not found for $\epsilon = 0.1$ (in yellow). Figure 7.7a is a more detailed figure that shows the opinion change that happens in each time step, which clearly shows that opinion continues to change throughout the entire simulation without stopping. Although, we must note that most of the experiments in this thesis eventually converge in opinion if the simulation is left running for a sufficient time. For example, under RRM for $r_s = 1$ and $\epsilon = 0.1$ the opinions do converge but only after $100,000$ time steps. Therefore, for the remainder of this thesis we extended the time steps to $70,000$ as the maximum, to balance computation time and allow a wide range of experiments.



(a) Opinion change per time step (b) Distance moved per time step

Figure 7.7: RRM convergence dynamics for $\epsilon = 0.1$ and $r_s = 1$

Furthermore, we investigate the *convergence of movement* when $r_s = 1$. Figure 7.7b shows similar results for convergence in movement, highlighting that this is rarely achieved. Note that the case of $r_s = 1$ is anomalous since by default, the distance moved $r_s\lambda$ is always less than the convergence threshold δ_{mov}, giving the impression of convergence while the agents are actually still moving (Figure 7.7b). Therefore, we exclude the cases with $r_s = 1$ from the discussions in the remainder of this chapter and fix $\delta_{mov} = 1$ (see Chapter 4).

7.3 Attract and repel components

In this section we will investigate the individual mobility components that contribute to the hybrid HM model, to quantify the impact of each and the effect of r_s and ϵ. We will discuss the results, both the opinion convergence and movement convergence, in the sections below.

7.3.1 Convergence

We investigate when convergence occurs in both opinion and movement for each simulation in order to determine how convergence in movement and opinion are related, and the conditions that lead to stability.

7.3.1.1 Convergence in opinion

Figure 7.8 shows the convergence in opinion for AM, RM, HM and static model against different values of ϵ and r_s, leading to the following observations.

Higher opinion threshold leads to faster convergence. As ϵ increases, it acts as a driver to reduce the time to converge in opinion because of its nature to increase the number of successful interactions, observed for all the mobility models.

Repel component leads to slower convergence. For low ϵ, the opinion convergence time is a little slower for both the RM and HM mobility models with smaller r_s. When there is mobility, new agents are frequently introduced into the neighbourhood of other peers, giving new opportunities to influence opinion or trigger further movement. This feature results from the repel component (including in both RM and HM, but not in AM), when agents move away they will typically land in the neighbourhood of new peers.

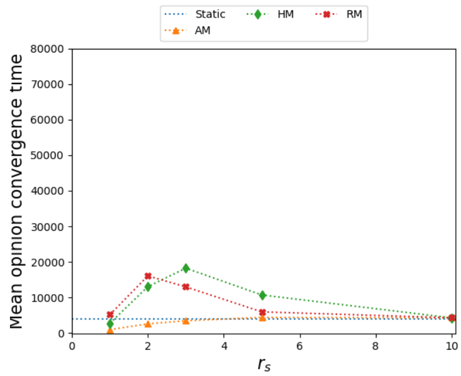
Faster convergence results from attraction mobility. Under AM, disagreeing agents aren't able to explore the area, nor is the neighbourhood exposed to new agents holding diverse opinions, hence agents can become trapped in regions where they disagree with the local majority. Therefore, fast convergence in opinion is expected similarly to the static model.

All mobility models have similar convergence speed to the static model. For low ϵ combined with a larger interactive area ($r_s \geq 5$), all three of the mobility models (AM, RM, HM) have a larger probability to interact with a large proportion of the agents in the population, spreading the influence of opinions widely. As a result, convergence occurs much faster in a similar fashion as the static model.

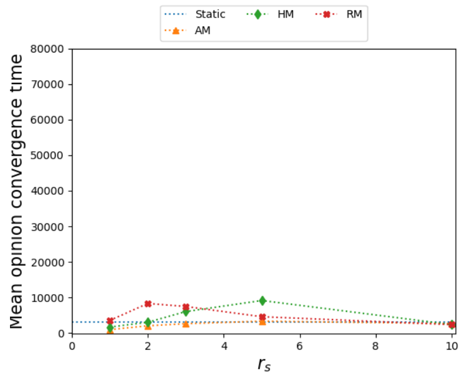
7.3.1.2 Convergence in movement

In this section we compare the convergence in movement between AM, RM and HM (excluding both static models). Figure 7.9 shows the convergence time in movement and the impact of ϵ and r_s, leading to the following observations.

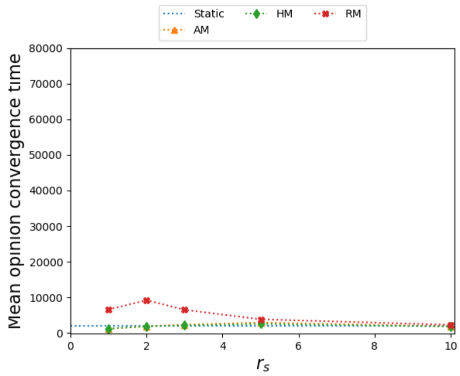
Faster convergence results from a high opinion threshold. For $\epsilon \geq 0.5$, the time of convergence in movement for AM, HM and RM is much faster compared to low ϵ. More specifically, less movement is found, due to the large opinion distance that



(a) $\epsilon = 0.1$



(b) $\epsilon = 0.2$

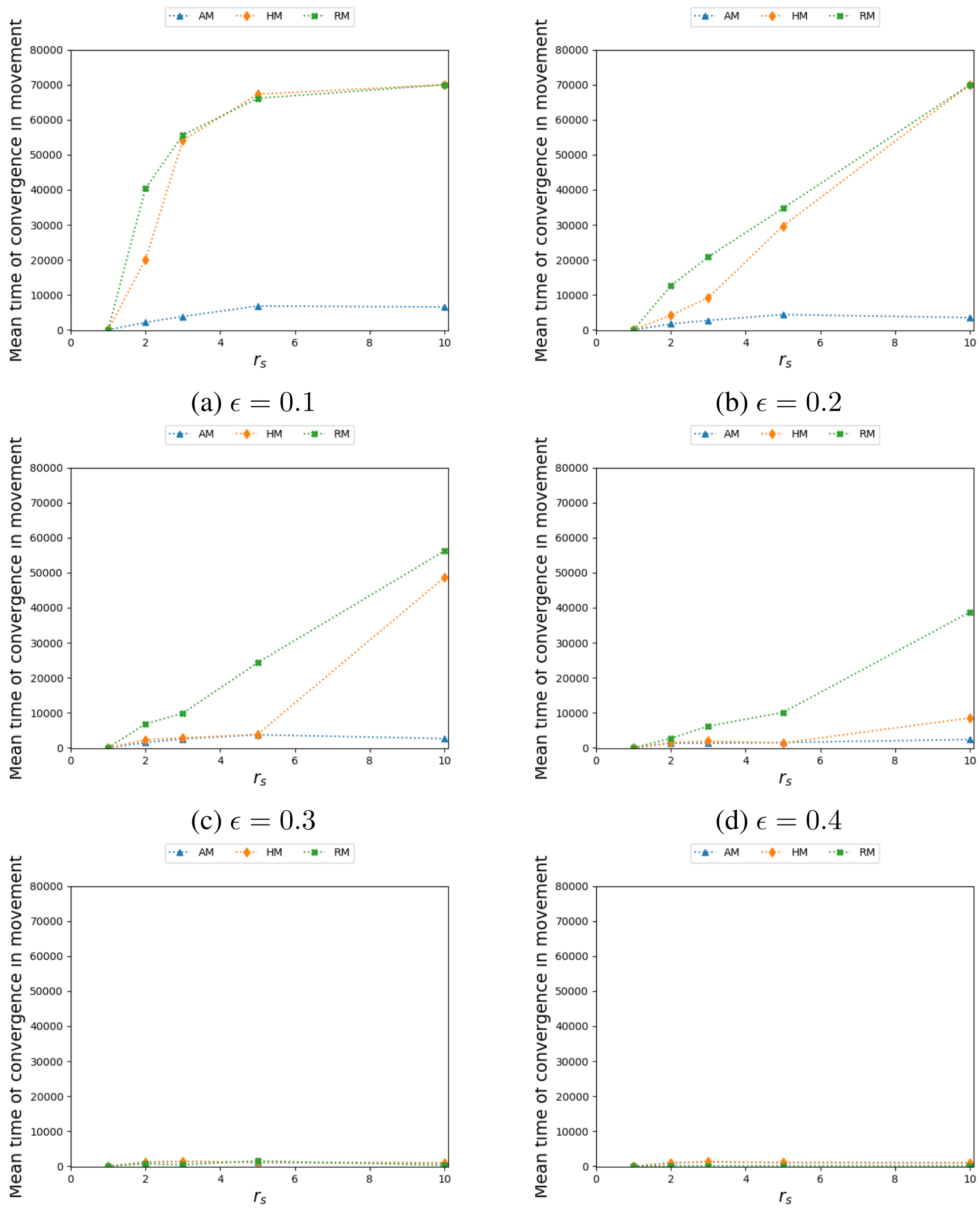


(c) $\epsilon = 0.3$

Figure 7.8: Mean opinion convergence time under directed mobility, convergence is not found when $t = 70,000$.

is acceptable between agents. This results in agents being content in their current locations without incentive to move away.

Repel component causes slower convergence in movement compared to attraction. The convergence time in movement for all three directed mobility (AM, HM, RM) increases as ϵ decreases (Figure 7.9), due to the presence of multiple opinions. This effect is accentuated as r_s increases, particularly for the repel mobility with low ϵ in comparison to the AM. As a result of a large interaction range the agents repel over a large distance. When the moving distance is large, the effect of movement at the boundary begins to have more impact. This behaviour will make the agents structureless similar to the random models. As a result, at the end of the simulation the agents are still moving because they are not content with their current neighbourhood, leading



(a) $\epsilon = 0.1$ (b) $\epsilon = 0.2$ (c) $\epsilon = 0.3$ (d) $\epsilon = 0.4$ (e) $\epsilon = 0.5$ (f) $\epsilon = 0.6$

Figure 7.9: Mean time of convergence in movement under directed mobility, convergence is not found when $t = 70,000$.

to a longer time to find convergence in movement (or no convergence at all).

For low ϵ, HM converges at a similar speed to RM. For HM with low ϵ, even though it includes the attract component, the repel appears to dominate leading to *slower convergence* compared to the AM. HM's convergence for high r_s is less frequently found compared to smaller r_s. With large r_s, interactions that can change opinion are triggered over larger distances and less structure is formed in the population, resulting in longer times for stable structures to emerge. However, the effect of small radius with directed repelling provides stability and structure to the model, resulting in faster convergence in movement. Specifically, this limits the ability of agents to explore new neighborhoods and therefore decreases the number of changes in the simulation, which in turn leads to a faster convergence.

For higher ϵ, HM converges at a similar speed to AM. As ϵ increases, HM behaves more similarly to AM finding *faster convergence in movement*. Naturally, the higher ϵ, the fewer unsuccessful interactions occur that will trigger an agent to move. The drop is especially significant for $\epsilon = 0.4$ and $r_s = 10$. This is because the numbers of opinion clusters decreases to a single opinion, and together with the attract component, this highly supports speeding the convergence in movement.

Convergence in opinion is almost always faster than convergence in movement, suggesting that agents largely settle on opinion before changing their place to find a neighbourhood where they are content. Also, convergence in opinion is always found, although this is not always true for movement. More specifically, we found that the repel component highly stimulates slow or no convergence in movement and therefore that feature is dominating in HM especially via low ϵ.

7.4 Conclusions

From Chapter 2 we have identified that most of the models apply random mobility, consistent with approaches inspired from the movement of gas particles, often with application of techniques from Physics. Therefore, in this chapter we analyse random mobility to quantify it's significance to opinion models.

Before analysing the random mobility we investigated the local static model under a range of restricted interactions (Section 7.2.1.1). We found more opinion clusters emerging, as noted in the literature [71]. However, restricting interaction did not impact random mobility models.

The results presented in Sections 7.2.1.2 and 7.2.2 over a range of parameters show that the two random approaches (purely random and triggered by disagreement) lead to similar outcomes which are largely independent of the interactive radius r_s. In fact, both showed very similar behaviour to the static model where interaction is allowed between any pairs of agents. We believe that this may explain why mobility has received scant attention in the literature, since the most obvious models appear to add little interest to the static case.

Finally, we explored the convergence of the directed mobility models (AM, RM, HM), to assess if these models show stability in their structure across the breadth of parameters. Unlike the random models, these show different behaviour across parameters, although convergence in opinion is always found and occurs faster than convergence in movement (Section 7.3.1.2). Furthermore, convergence in movement is slowed down via repulsion causing for the convergence not to be found with large interaction range (Section 7.3.1.2).

The observations above provide the basis for a deeper investigation on the directed mobility models in the next chapter, where we will attempt to develop a classification that can describe the characteristics of each mobility model.

Chapter 8

Classification of self-organisation

In this chapter our focus is to analyse the different opinion clusters that survive in the system, reflecting the opinion diversity in the population once it has converged. Also, we focus on the structure of the agent's distribution in the geographical space associated to their opinions. In particular, we conduct experiments across relevant parameters in order to determine and classify the potential outcomes into a small number of scenarios. Firstly, we provide a detailed analysis of the results of each mobility model, based on quantitative outcomes. Then, we give an overview of the classification diagrams that we will use to illustrate the results. Finally, we present a complete classification diagram that classifies the different mobility models into a number of scenarios.

8.1 Methodology

In Chapter 7 we have found that random mobility under a broad parameter space behaves the same as the static model. However, in Chapter 5 we found that directed mobility under HM behaved significantly differently from the static model in terms of a significant raise in opinion evolution, more communities forming, decrease intolerance levels and slower convergence in opinion. Also the HM showed robustness under noise (Chapter 6). This triggered the investigation concerning the directed HM model. The HM originally consists of two mobility components: attraction and repul-

Parameter	Description	Value
Mobility models	Random and directed mobility models	PRM, AM, RM, HM
ϵ	Opinion threshold for influence (see Definition 3)	$[0.1, 0.2 \ldots, 1]$
r_s	Interactive radius	$[1, 2, 3, 5, 10]$

Table 8.1: Independent variables

sion. Therefore, to fully understand this model, it is important to ask: Which mobility component has a larger impact? Are they equally important? Does one mobility have more significant impact on HM than the other? In this chapter we conduct a full investigation on the individual components of the directed mobility mechanism. We use the same fixed variables as the previous Chapter 7 (Table 7.1) and focus on the directed mobility models as listed in Table 8.1.

Aim Our *aim* is comparing and exploring specifically the directed mobility mechanisms while widening the parameter space investigation to produce a classification for the different scenarios and self-organisation that emerge.

Hypothesis To fully understand the HM model, considering both the attract and repel components, which mobility component has a larger impact? Our hypothesis is that the mobility forces of attract and repel impact both the opinion and community formation differently, unlike the random mobility.

Experiments In this chapter we will investigate the HM model as well as the AM and RM mobility mechanisms. Since the static model behaves as the random mobility models (as shown in Chapter 7), we use PRM as a *benchmark* to compare to the directed mobility models, excluding the static models. The models we study in this chapter is as follows:

1. Random models

Parameter	Description	Value
Opinion clusters (see Definition 9)	Mean number of different opinion clusters	0 - 10 clusters
Tolerance (see Definition 13)	Ratio of different opinions	$tol(A) \in [0, 1]$
Communities (see Definition 10)	Mean number of clusters that share opinion and location	0 - 10 clusters

Table 8.2: Dependent variables

(a) PRM: An agent moves randomly at each time step (see Algorithm 3, Section 3.2.2.1).

2. Directed models

(a) AM: Move toward a similar peer (Algorithm 5, Section 3.2.2.2).

(b) RM: Move away from a different peer (Algorithm 6, Section 3.2.2.2).

(c) HM: Move toward/away depending on the agreement between the peers (Algorithm 7, Section 3.2.2.2), in this chapter the *movement toward* is described as the attract component and the *movement away* as the repel component.

Evaluation We will *evaluate* the directed mobility models (AM, RM, HM) in terms of opinion cluster, tolerance and communities as listed in Table 8.2. The convergence for the directed models was discussed in Chapter 7, Section 7.3 in both opinion and movement. This has highlighted the spectrum where convergence and stability has been found in the models, this provides a more sufficient assessment against the other metrics presented in this chapter.

To evaluate the results, firstly, we discuss for each metric (in Table 8.2) the observations we find under a large parameter space (in Table 8.1). After that we combine the parameters to identify similar behaviour, to classify into a couple scenarios that describe their behaviour. Finally we combine and synthesise all the metrics to create a classification diagram that describes the ways in which self-organisation of agents occurs.

8.2 Opinion clusters

In this section we present an abstract classification method capturing the final number of opinion clusters ignoring the agent locations, which will be investigated in the Section 8.4. Specifically, this investigation raises questions about the surviving opinion clusters. Usually in the literature the agents reach complete consensus or multiple opinion clusters (if ϵ too low).

We first evaluate the opinion/mobility models across different values of r_s and ϵ, to assess their impact on the emerging opinion clusters from the simulations. We are interested in the numbers of loners outside opinion clusters, which is generally ignored in the literature.

We apply this approach for presentation of the remaining evaluation metrics in Sections 8.3 and 8.4. We first describe the observations that arise from these simulations in the following Section.

8.2.1 Observations

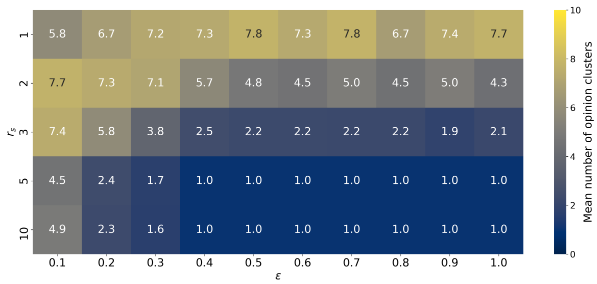
Previously in Chapter 5, (Figure 5.2), we found that where $r_s = 2$ the largest numbers of opinion clusters are found via directed mobility (HM) rather than in both of the static models, either manifested via restrictive interactions (local static, $r_s = 2$) or globalised interactions (static, $r_s = 10$). Therefore, this triggers the question of how much impact the interaction range (r_s) has on different mobility mechanisms. In the discussions below we analyse the directed movement components (attract and repel) and investigate the impact of both ϵ and r_s.

Mobility via high interaction radius behaves as the random mobility In general, for all the mobility models (AM, RM, HM) we have found that when $r_s \geq 5$ (for all ϵ) the resulting number of opinion clusters corresponds closely regardless if mobility was directed or random (Figure 8.1). This is similar to the previous findings in Chapter 7, Section 7.2.1.2 for the random mobility models (RRM and PRM), where opinion clusters again were similar to the static model.

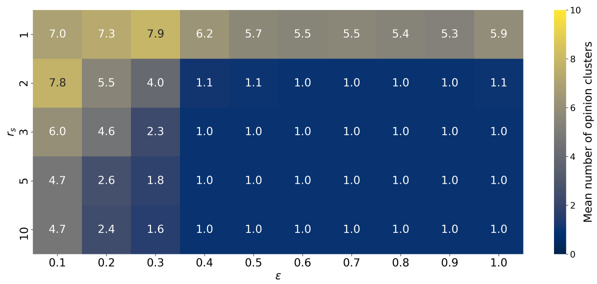
This is because most of the region is within the interaction range and an agent has a higher probability to interact with anyone across the region giving similar results to the static model. More specifically, the agent's coverage area to select another peer is πr_s^2, hence assuming the agent is at the centre of the region. For example, if we consider $r_s = 5$ that gives an agent a probability of 0.78 to interact with any other agent across the population (assuming uniform distribution). This probability can vary if the agents is at one of the corners of the region giving a probability of 0.19. [57] found similar results from their lattice based binary opinion model, which considers mostly local interaction with the concept of "mobility" provided by a small number of global links. As a result of these long links providing influence across the population, they found that increasing mobility (i.e. the number of long links) also increased the size of the largest community relative to the entire population.

Low interaction radius produces more opinion clusters Figure 8.1 shows that more restrictive interactions ($r_s < 5$) stimulate larger numbers of opinion clusters than the random mobility regardless of the type of movement (AM, RM or HM).

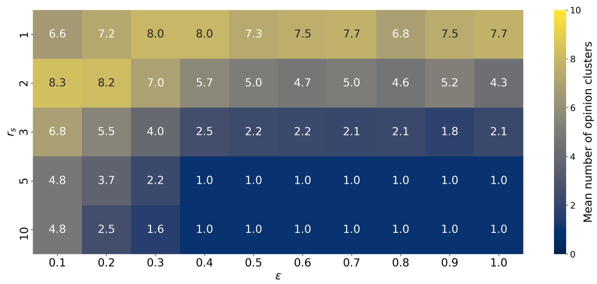
In particular, for both AM and HM (Figure 8.1a and 8.1c) it can be seen that multiple opinion clusters persist even as ϵ increases to reach $\epsilon = 1$. This is due to the presence of the attraction component in both mobility methods, leading to agents co-locating in small isolated groups before they have a chance to spread their opinion by influencing significant numbers of peers.



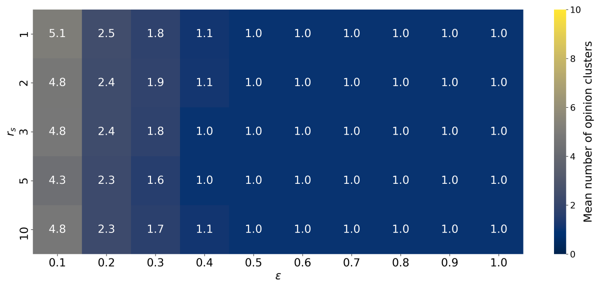
(a) AM



(b) RM



(c) HM



(d) PRM

Figure 8.1: Mean number of opinion clusters under directed mobility

RM behaves as random mobility RM with $\epsilon \geq 0.4$ produces a single opinion cluster regardless of r_s (when $r_s > 1$), a similar behaviour to the PRM model to its equivalent

ϵ. This is due to the nature of the repel mobility, as it triggers the agents to move away when disagreement occurs and therefore land in a new neighbourhood. This mechanism gives the agent a higher chance to explore and meet more new agents further away. Therefore their spread and interactions around the area manifested with large ϵ leads to complete agreement.

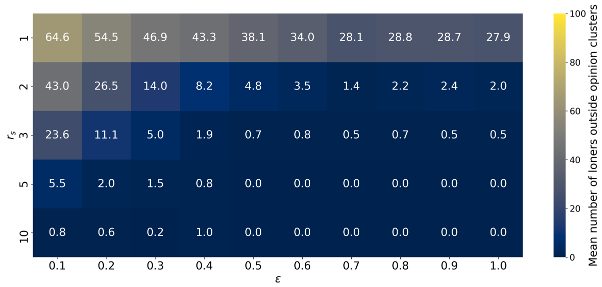
Emergence of loners outside opinion clusters in AM. Figure 8.2 shows that for directed movements with $r_s > 1$, few loners remain outside opinion clusters. However, there is a small exception for AM when $\epsilon < 0.3$ and $r_s < 5$, where more loners appear, showing that where interaction is limited, more loners in opinion will emerge. Under these conditions the stimulus of attraction is not enough to "belong" or find an opinion group.

In contrast, if an agent moves to a new neighbourhood (only possible via the repel component) it has a higher chance of encountering a similar peer and can find where it "belongs", decreasing the number of loners. Note that HM balances these effects to some extent, with the number of loners somewhere between AM and RM in this constrained region.

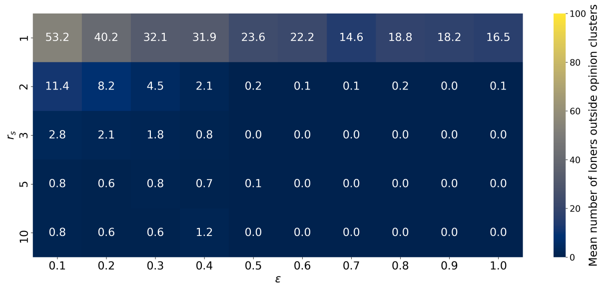
8.2.2 Classification of outcomes

Our findings show that through directed mobility and restricted interactive radius, more opinion clusters survive. This is especially highlighted under the physical attraction component (in AM and HM) since the impact continues until $\epsilon = 1$. These results are interesting especially since the universal property of the DW model always shows that only one opinion survives for $\epsilon \geq 0.5$ ([39]), which is not very realistic to consider an entire population with one opinion.

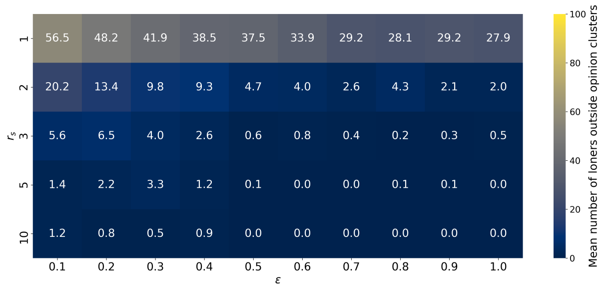
Three clear outcomes are evident in our experiments, with their relationship to ϵ and r_s represented graphically in Figure 8.3:



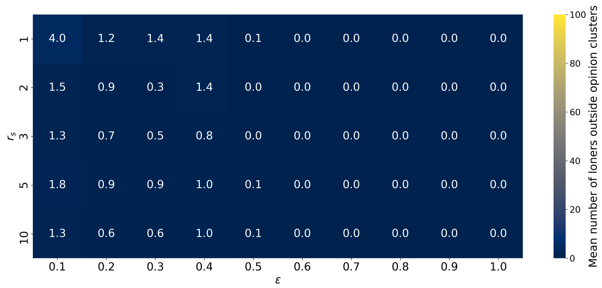
(a) AM



(b) RM



(c) HM



(d) PRM

Figure 8.2: Mean number of loners outside opinion clusters under directed mobility.

- **A single opinion cluster** emerges that is shared by the entire population (e.g. Figure 8.4b).

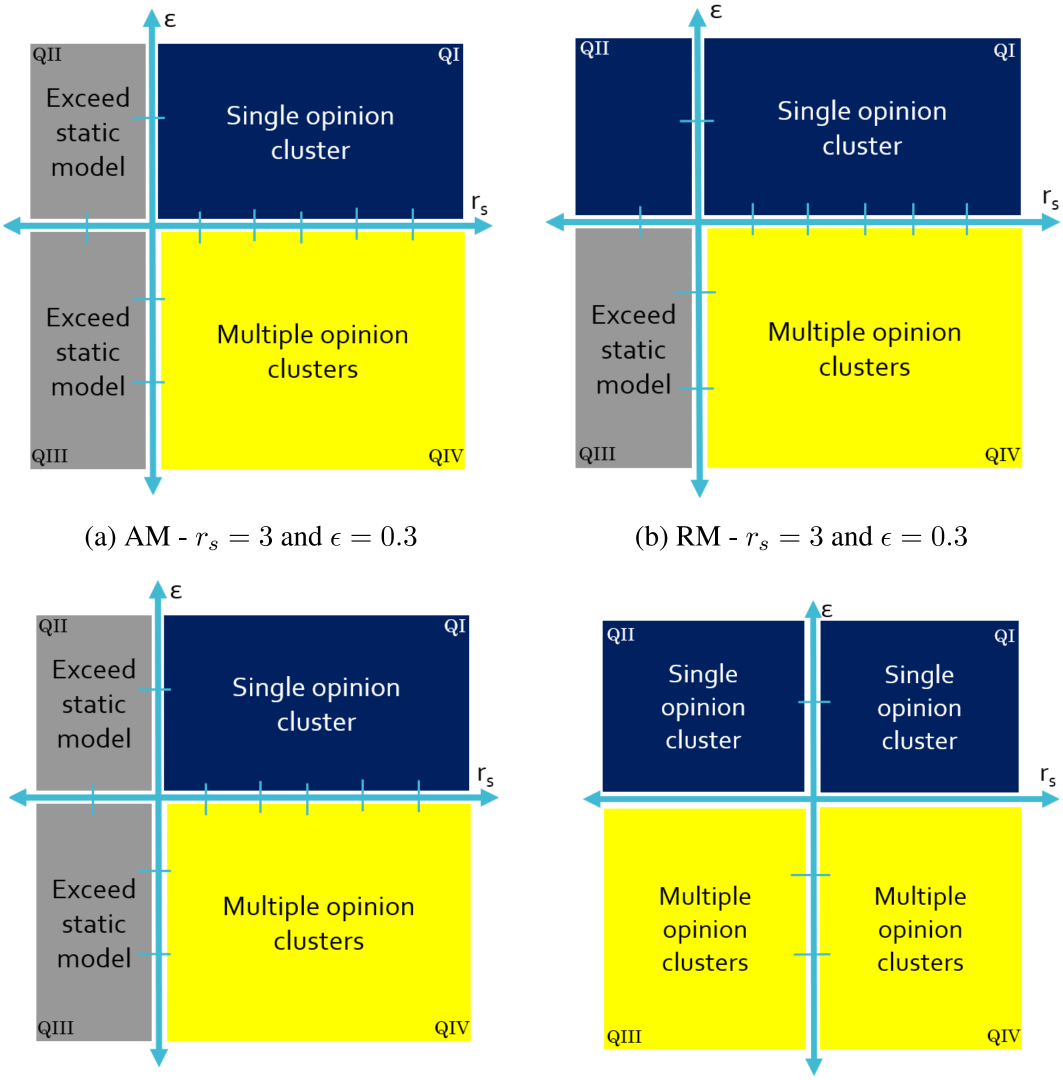
- **Multiple opinion clusters** are formed across the population, at a similar rate to random mobility (e.g. Figure 8.4a).

- **Exceed static model.** More opinion clusters are formed than in the static model (Algorithm 1), this corresponds to more than the maximum number of clusters, which is equal to $\frac{1}{2\,\epsilon}$ [29] (e.g. Figure 8.4c).

The emerging opinion clusters classified in Figure 8.3, show a comparison between AM, RM, HM and random mobility (PRM). For ease of discussion, we describe the classification in terms of four quadrants (Q) labelled as QI, QII, QIII and QIV which represent the quadrants in an anti-clock wise manner starting at the north east. Throughout this chapter, the classification diagram reflects the transition in behaviour across the two axes, the interactive radius r_s (x-axis) against the opinion threshold ϵ (y-axis). This is valid with clearly defined values of ϵ and r_s far from the transition state. With the points being closer to the transition state the values are less clearly defined and have a degree of uncertainty. This can be seen clearly in the previous observation section 8.2.1.

To illustrate the nature of these outcomes, Figure 8.4 shows the result of individual representative simulations, which plots a snapshot at the end of the simulation of the distribution of the surviving opinions across the region, each cluster represented by a different colour. Figure 8.4a shows an example where multiple opinions exist, Figure 8.4b demonstrates when a single opinion cluster emerges and Figure 8.4c shows when more than $1/2\epsilon$ opinion clusters emerge.

As would be expected, as r_s increases (quadrants QI and QIV), it is more likely that any pair of agents may interact, and the outcomes for all mobility models (AM, RM, HM) result in a similar number of opinion clusters to PRM, however consensus is reached quicker.



(a) AM - $r_s = 3$ and $\epsilon = 0.3$

(b) RM - $r_s = 3$ and $\epsilon = 0.3$

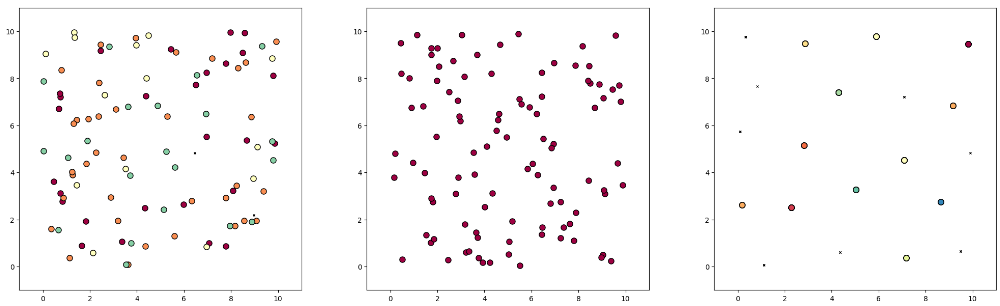
(c) HM - $r_s = 3$ and $\epsilon = 0.3$

(d) PRM - $\epsilon = 0.3$

Figure 8.3: Opinion clusters classification

In quadrant QIII (low ϵ, low r_s), all mobility models (AM, RM, HM) produce more heterogeneity in opinion clusters than PRM. Moreover, for the higher ϵ in quadrant QII, attract forces (AM, HM) are needed to enable multiple opinions to persist, with the increased ϵ leading to complete consensus in RM.

Finally, we note the difference between RM and AM/HM in quadrant QII, highlighting that RM results in global consensus for all high values of ϵ.



(a) Multiple opinion clusters under RM ($\epsilon = 0.1$,$r_s = 10$)

(b) Single opinion clusters under RM ($\epsilon = 0.5$,$r_s = 2$)

(c) Exceed random mobility model in opinion clusters under HM ($\epsilon = 0.1$,$r_s = 2$)

Figure 8.4: Distribution of opinion for representative simulations. Colours denote agents belonging to the same opinion cluster..

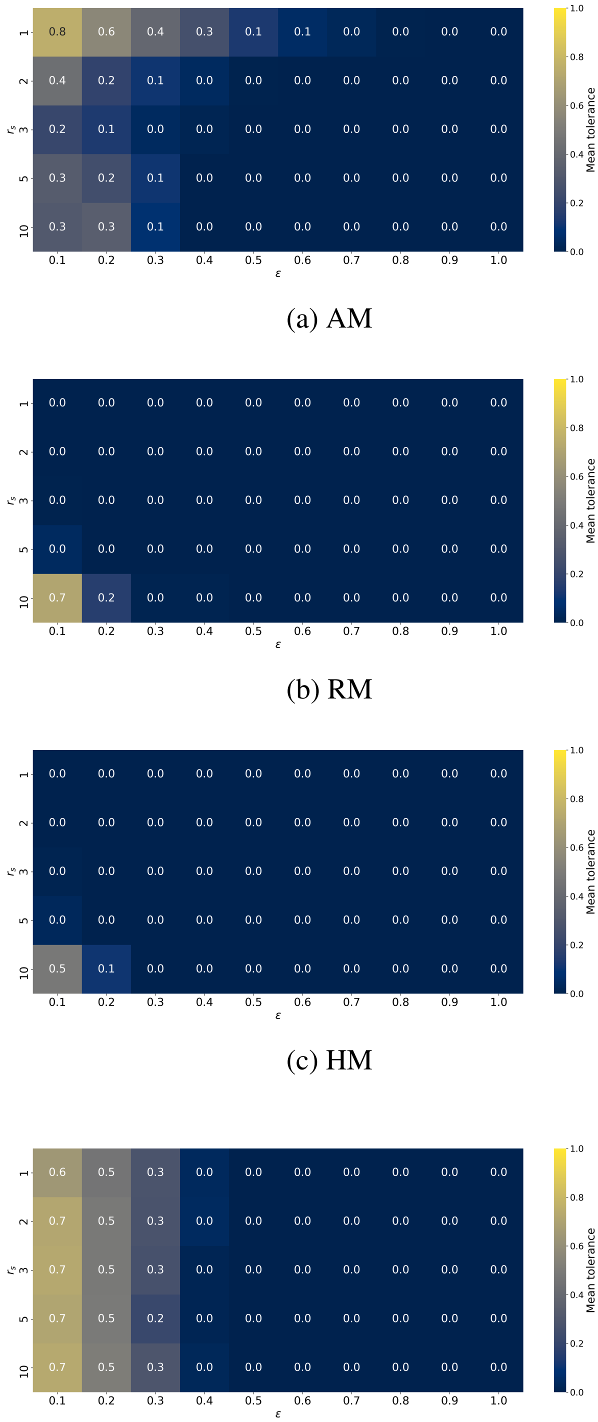
8.3 Tolerance

In this section we classify the distribution of opinions around each agents in terms of tolerance. Recall that if individual agents in a local area typically hold *different* opinions then tolerance will be close to 1 and if the agents have the *same* opinions then tolerance will be close to zero.

8.3.1 Observations

In general, increasing ϵ acts as a driver to decrease tolerance in the population because of its nature in reducing the total number of opinions (Figure 8.1). Ultimately the disagreement between the agents decreases as well (Figure 8.5).

Repel component dominates in the Hybrid Model. Figures 8.5b and 8.5c show that the tolerance of RM and HM are similar, and distinct from AM. Both RM and HM models rarely result in agents sharing a neighbourhood with different opinions. Tolerance is found only in cases where the interaction range is high (leading to high probability that any pair of agents will attempt to interact) and ϵ is low (so a low probability



(a) AM

(b) RM

(c) HM

(d) PRM

Figure 8.5: Mean tolerance under directed mobility

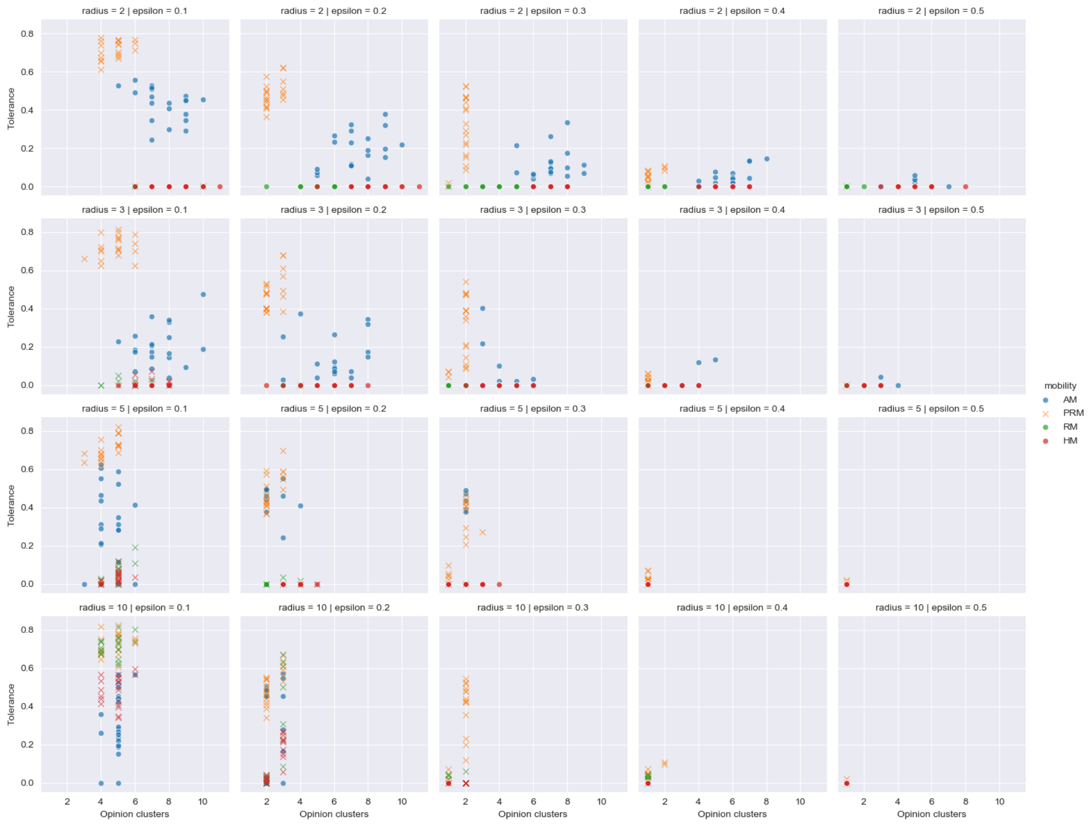


Figure 8.6: Opinion clusters and tolerance. 20 runs shown for each experiment, $\circ$ denotes the simulation converges in movement, and $\times$ no convergence..

that the interaction will be successful). In these cases, multiple opinions survive and their movement struggles to find convergence (Figure 7.9 in previous Section 7.3.1.2). Figure 8.6 shows the number of opinion cluster (x-axis) against the tolerance (y-axis). Each individual square of the grid is an experiment against a set of ϵ and r_s with different mobility models shown in different colours. For each mobility experiment, there are 20 results of individual runs. The figure differentiates between convergence and non-convergence in movement. This Figure 8.6 shows that tolerance in RM and HM is only found when the simulation does not converge.

Attract component finds convergence and tolerance for restricted interactions. As seen in Figure 8.6, in contrast to RM and HM, AM has non-zero tolerance when the interaction range and ϵ are small. This is due to the inability of agents to escape areas of high disagreement or be influenced by these opinions. Note that random mobility also produces high tolerance for small ϵ as well, but without forming organised clusters or achieving convergence.

In conclusion, we find that tolerance is not sustained by RM. Furthermore, we found that using only the attract mobility method (AM) can show tolerance to different neighbouring opinions more than the other models. In the HM model the repel component dominates more strongly against the attract component and therefore eliminates any tolerance.

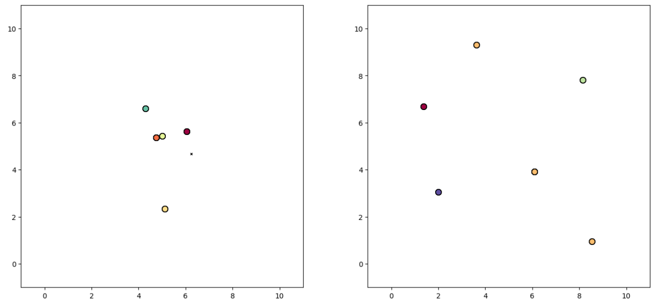
8.3.2 Classification of outcomes

Based on our observations, we propose classifying tolerance into the following two outcomes.

- **Mixed opinion** where geographical neighbourhoods hold a range of opinions ($tol(A) \geq 0.1$, e.g. Figure 8.7)
- **Homogeneous opinion** where neighbourhoods largely hold the same opinion ($tol(A) < 0.1$, e.g. Figure 8.7).

As for the opinion clusters, the parameter space can be divided into rough quadrants based on these outcomes, as shown in Figure 8.8.

For both RM and HM (Figures 8.8b and 8.8c) we find the four quadrants behave similarly, again highlighting the dominance of the repel component with respect to tolerance. Note that although we find *Mixed Opinion* neighbourhoods in quadrant QIV, these are not stable and do not converge.



(a) Mixed opinion clusters neighbourhoods under AM ($\epsilon = 0.1$, $r_s = 10$)

(b) Segregated Homogeneous opinion neighbourhoods under HM ($\epsilon = 0.5$, $r_s = 2$)

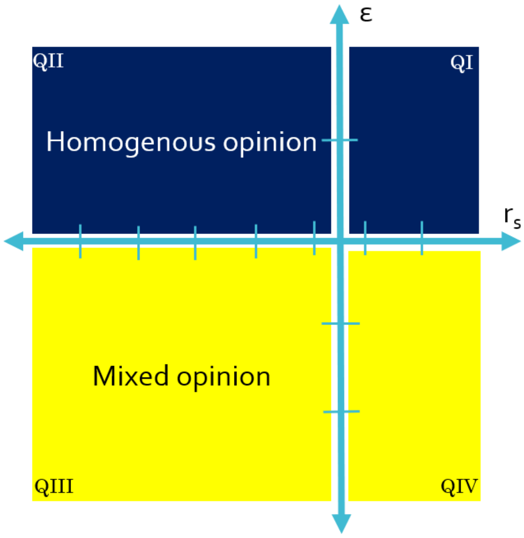
Figure 8.7: Opinion tolerance examples of representative simulations. Colours denote agents belonging to the same opinion cluster..

8.4 Communities

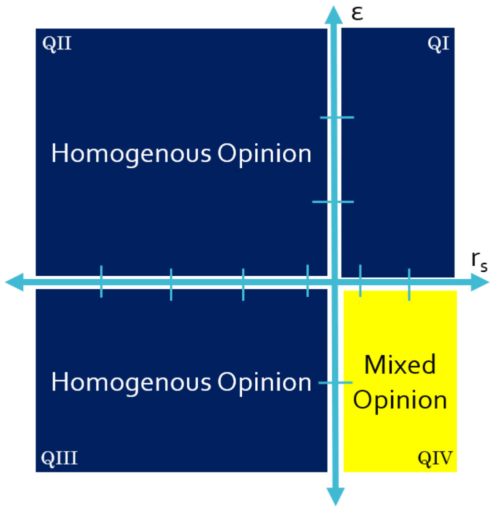
In this section we investigate the structure of agents and evolution of communities by considering the opinion and geographical space simultaneously. Also, We further study under what conditions loner scenarios apply and how much does it impact in the population's overall structure. To recap, this requires three conditions to be valid to define a community: a pair of agents must have close opinions, and be within a local area and the size of the cluster must be five members or more, otherwise they are classified as isolated *loners*. In this section, low loners is used as a measurement to describe self-organisation.

8.4.1 Observations

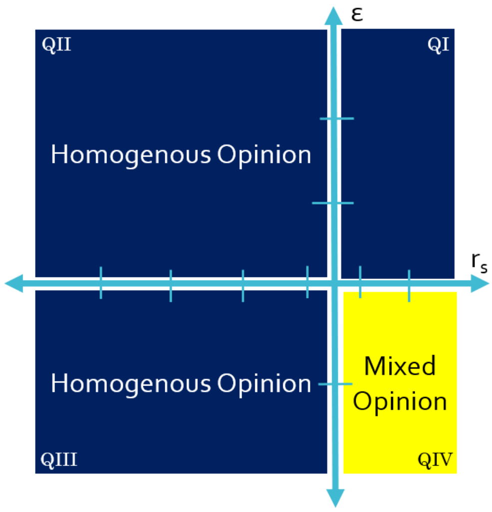
Repel component forms structure-less patterns with a single opinion. In general with RM, as ϵ increases, the population converges in opinion quite quickly to form complete consensus, and therefore the stimulus to move stops. Nevertheless, RM is able to produce a number of communities (Figure 8.9b). However, with high $\epsilon \geq 0.4$ (Figures 8.10b, we find a high number of loners (over 30%) outside communities for all



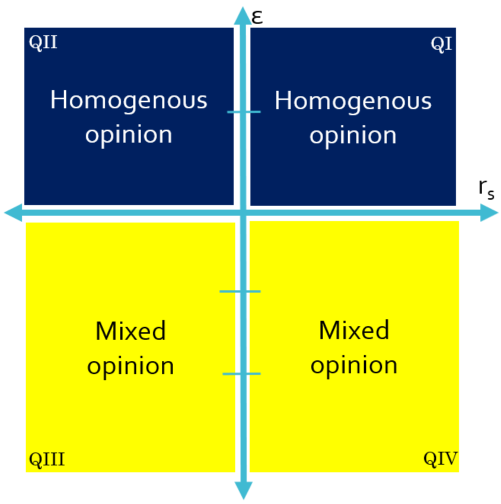
(a) AM - $r_s = 7.5$ and $\epsilon = 0.3$



(b) RM - $r_s = 7.5$ and $\epsilon = 0.2$



(c) HM - $r_s = 7.5$ and $\epsilon = 0.2$

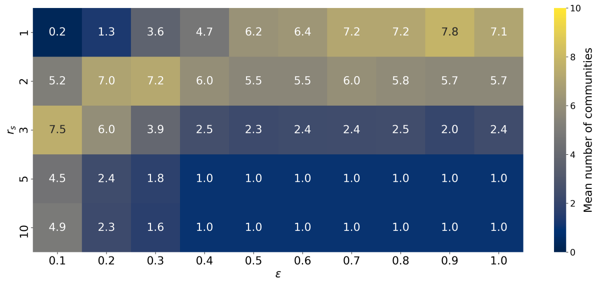


(d) PRM - $\epsilon = 0.3$

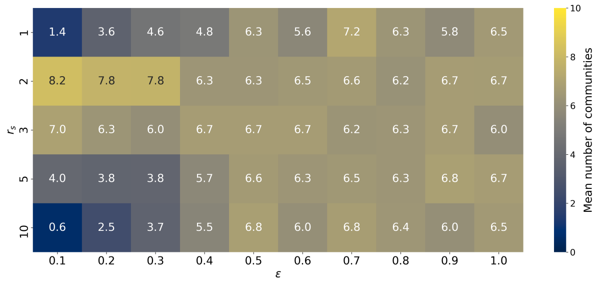
Figure 8.8: Tolerance classification

interaction radius. Even though agents are agreeing in opinion, they lack the impetus to move closer.

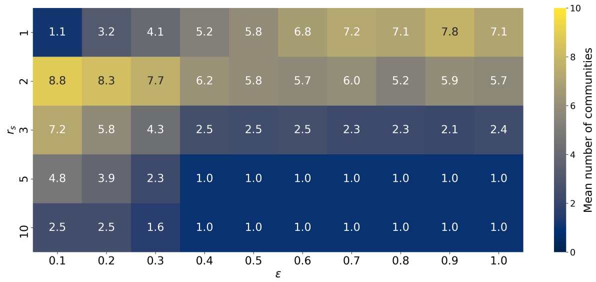
This is a result to be expected as once all of the agents are in agreement, there is no incentive for them to move (in line with the cognitive dissonance theory) and possibly increase the overall structure. The scattered distribution produced is very similar to PRM (Figures 8.10d), and the number of communities are similar as well (Figures 8.9d).



(a) AM



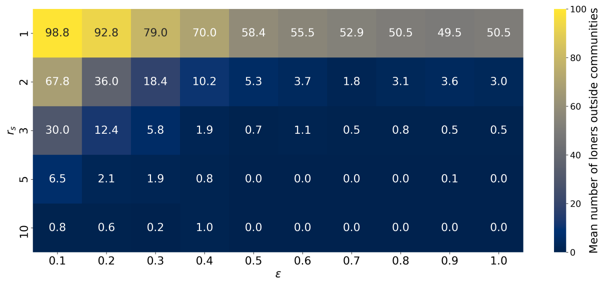
(b) RM



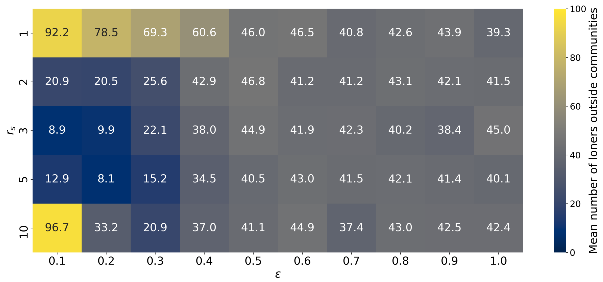
(c) HM

(d) PRM

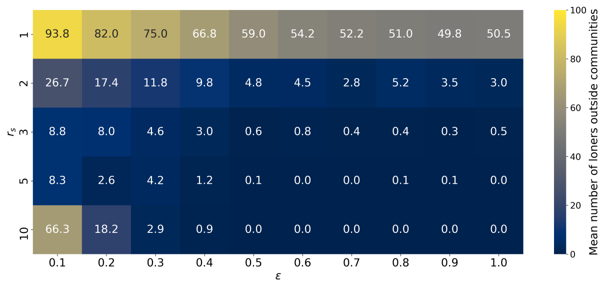
Figure 8.9: Mean number of communities under directed movements



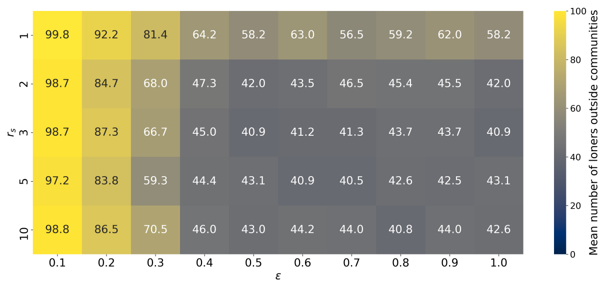
(a) AM



(b) RM



(c) HM



(d) PRM

Figure 8.10: Mean number of loners outside communities under directed movements.

AM component dominates in HM. Results for AM and HM are similar in both the number of communities and number of loners (Figures 8.10 and 8.9), suggesting that the attract component of mobility predominately determines the behaviour. For high levels of interaction ($r_s \geq 5$ and $\epsilon \geq 0.4$), a single community is formed, with almost no loners. For restricted interaction ($r_s < 5$), multiple opinions are able to co-exist as discussed in the previous Section 8.2. These opinion clusters self-organise themselves geographically into separated clusters. As a result we find multiple communities.

The impact of density. Figure 8.9 shows that the number of communities changes depending on the density within r_s. When directed mobility (AM- RM- HM) is enabled more communities are found within small r_s. However if r_s is too small as in $r_s = 1$, a community of at least five agents (based on the Definition 12) is more difficult to form.

Self-organisation is widespread. Figure 8.9 shows that in the majority of cases the system self-organises, in the sense that few agents remain as loners in trivial communities. As expected, heavily restricting interactions (i.e. low values for both r_s and ϵ) leads to high proportions of loners, as agents do not come in range of peers, this is clear in AM, see Figure 8.10a. The loners are mitigated to some extent as ϵ increases. RM (Figure 8.10b) for $\epsilon \geq 0.3$ is also an exception, with around 40% of agents isolated, due to the instability caused by agents being repelled over large distances. This is also the cause of almost all agents being loners for $\epsilon = 0.1$ and $r_s = 10$, where agents are repelled from the majority of peers, and can be clearly observed with HM (Figure 8.10c) in contrast to other combinations of parameters.

Trade off between communities and loners. Figure 8.11 shows the importance of the type of mobility mechanism applied on the balance between loners and communities. This highlights that RM is relatively consistent, with high numbers of communities and loners across the range of parameters, whereas AM and HM tend to move towards a single community with no loners as the level of interaction increases. Importantly, the

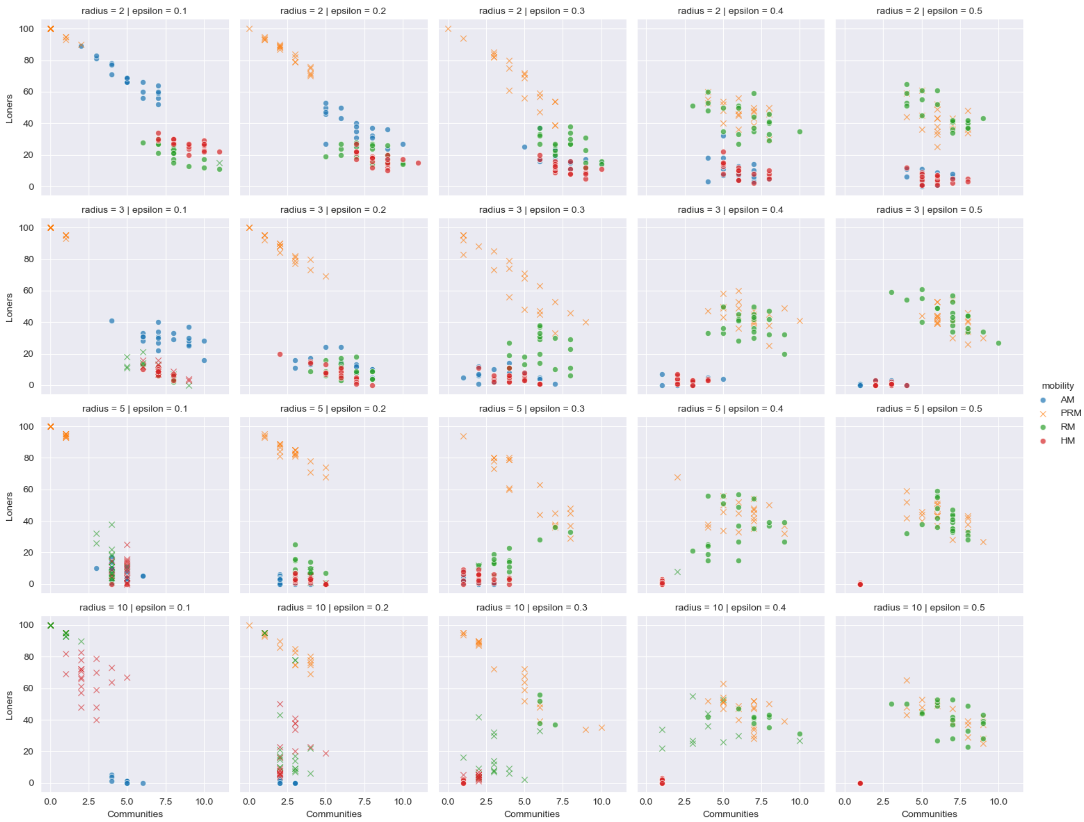


Figure 8.11: Communities and loners. 20 runs shown for each experiment, ○ denotes the simulation converges in movement, and × no convergence..

figure shows the benefit of considering both communities and loners, as for many combinations of parameters (e.g. $r_s = 3, \epsilon = 0.1$), although the number of communities is similar between mobility models, the number of loners is very different. Finally, we note that the relationship between the Hybrid model and individual components varies, being similar to RM for highly restricted interactions, but close to AM elsewhere.

The attract component naturally forms more geographical uniform opinion clusters with minimum noise, with the exception when the model is very restricted in interaction and influence.

The repel component is able to promote self-organisation, only under restrictive interactions. On the contrary, under all the other configurations the model loses it ability to form patterns geographically and results in similar outcome as the static model (and

RRM and PRM).

HM has both the repel and attract components and as a result it produce the most uniform clusters geographically with minimum noise. However, due to the repel impact, when the there are multiple opinions and the interaction range is up to its full potential, a noisy structure geographically results, because of the constant repelling jumps.

8.4.2 Classification of outcomes

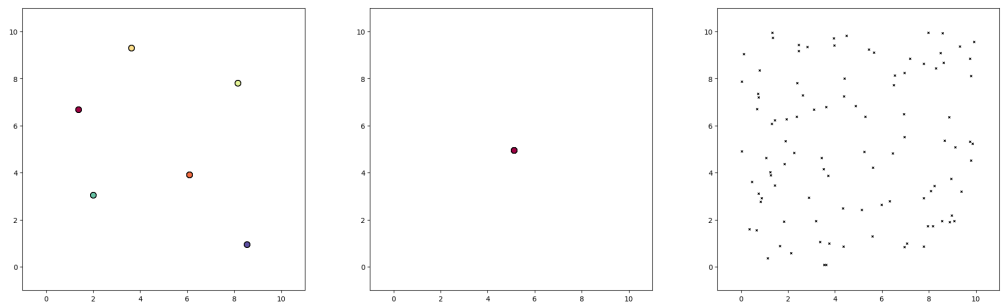
In considering the co-evolution of opinion and location, it is instructive to look at both the communities that emerge by the end of a simulation and the loners that are excluded.

The attract component is naturally able to forms communities with minimum noise (except where interaction is very restricted in interaction and influence and rarely forms any structure). The repel component is only able to self-organise under restricted interactions and in all other configurations results in a similar outcome as random mobility (PRM). As a result of combining both the repel and attract components, HM produces the most uniform clusters geographically with minimum noise.

To demonstrate the organisation of the agents in geographical space, we first classify in Figure 8.14 the potential outcomes of:

- **Multiple communities** of agents that are close in both opinion and distance (e.g. Figure 8.12a).
- **A single community** as shown in Figure 8.12b.
- **Undefined** when no coherent communities (with at least 5 members) are formed e.g. Figure 8.12c.

A second geographical classification (Figure 8.15) is based on the presence of loners ($|N_{loners}|$):



(a) Multiple communities under HM ($\epsilon = 0.5$, $r_s = 2$)

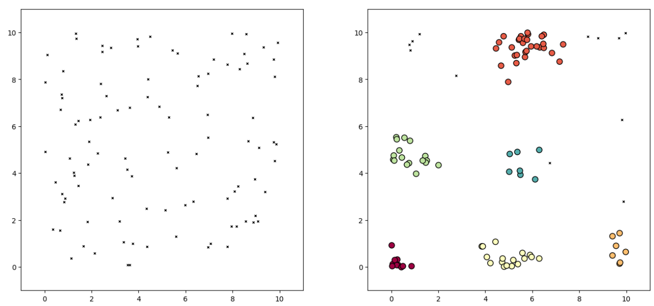
(b) Single community under AM ($\epsilon = 0.5$, $r_s = 10$)

(c) Undefined communities under RM ($\epsilon = 0.1$, $r_s = 10$)

Figure 8.12: Communities in representative simulations. Colours denote agents belonging to the same communities.

- **No structure.** Many agents ($|N_{Loners}| > 30\%$) are isolated outside communities (e.g. Figure 8.13).
- **Organised.** Most of the agents ($|N_{Loners}| \leq 30\%$) are part of a community, located in the same neighbourhood and holding the same opinion (e.g. Figure 8.13).

The agents distribution in geographical space shows that every agent has at least 5 members within the local area holding close opinions.

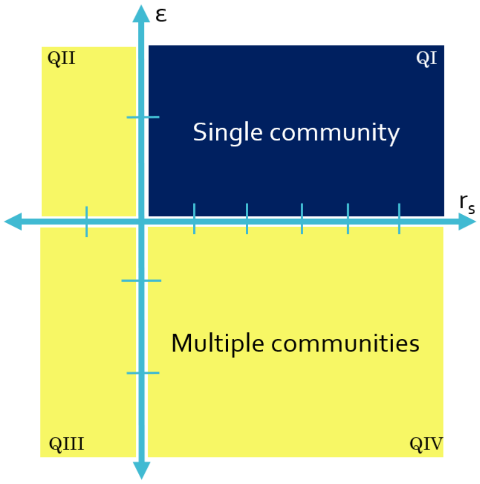


(a) No structure under RM ($\epsilon = 0.1, r_s = 10$) (0 clusters; 100% N_{Loners})

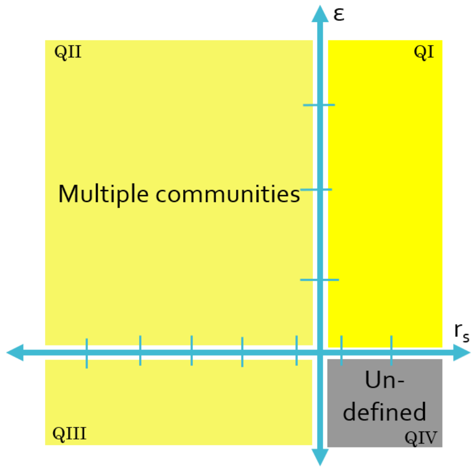
(b) Organisation under RM ($\epsilon = 0.1, r_s = 3$)(6 communities; 12% N_{Loners})

Figure 8.13: Loners in representative simulations. Colours denote communities and dots denote loners.

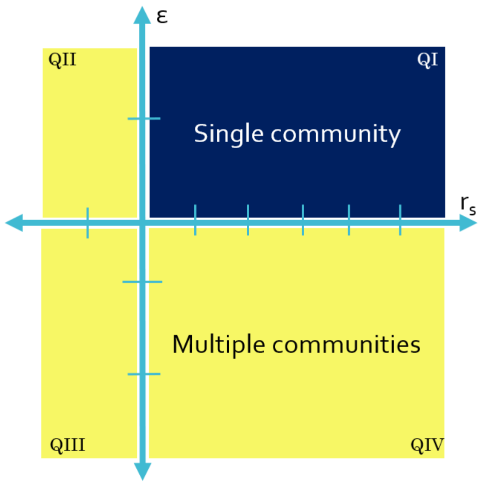
In the previous Section 8.2.2, in Figure 8.3, for low values of ϵ (QIII and QIV), all three models (AM, RM, HM) were similar in terms of opinion clusters, with many



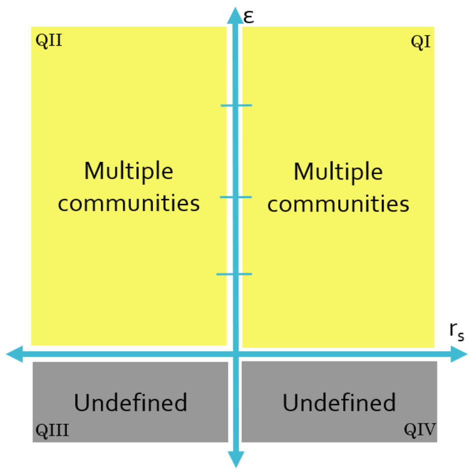
(a) AM - $r_s = 3$ and $\epsilon = 0.3$



(b) RM - $r_s = 7.5$ and $\epsilon = 0.1$



(c) HM - $r_s = 3$ and $\epsilon = 0.3$



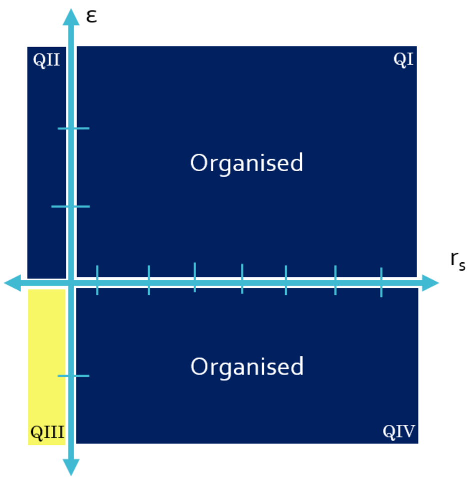
(d) PRM - $\epsilon = 0.1$

Figure 8.14: Classification of community structure

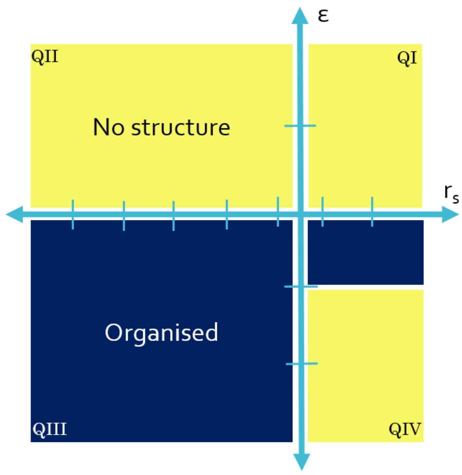
opinions surviving regardless of r_s. However, their outcomes are different when we also consider the geographical structures, as seen in Figures 8.14 and 8.15.

For mobility with a repel component (RM, HM) under large r_s, organisation is difficult to form (Figure 8.15b and Figure 8.15c), due to the fact that convergence in movement isn't found (Figure 7.9). However with restricted interaction organisation is clearly shown. Contrary to AM, a high interaction range is required to obtain the same effect of *Organisation*, however, under small r_s no structure is found (Figure 8.15a).

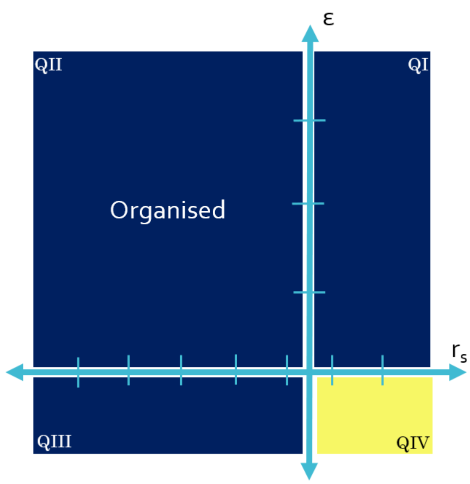
In Figure 8.15b with RM in QIV, we find that around the higher levels of epsilon we



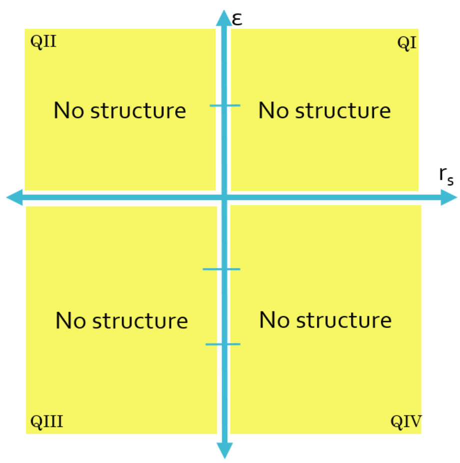
(a) AM - $r_s = 2$ and $\epsilon = 0.2$



(b) RM - $r_s = 7.5$ and $\epsilon = 0.3$



(c) HM - $r_s = 7.5$ and $\epsilon = 0.1$



(d) PRM

Figure 8.15: Classification of loners

can find more structure. This is interesting because the RM is usually unable to produce organisation under large interaction range. However, under this configuration only very few opinions survive (on average $\lessapprox 2$), see Figure 8.1b. This makes disagreement hard to emerge and therefore some structure can be formed.

For high values of ϵ (QI and QII), under mobility with attract component (AM,HM) we found *Organisation* (Figure 8.15a and Figure 8.15c) due to the nature of the mobility mechanism. However, depending on the r_s either a single or multiple communities are formed (Figure 8.14a and 8.14c). On the contrary, whenever the physical attraction is absent (such as in RM and PRM) we obtain only one opinion cluster (Figure 8.1b and 8.1d) although with geographically sparse agents for any value of the interaction radius (Figure 8.15b and 8.15d).

In the next section, we will continue the discussion in more details for each experiment discussing the impact of ϵ and r_s.

8.5 Overall classification of self-organisation

The previous sections of this chapter have investigated and classified the different outcomes that can arise from each component of mobility across different levels of interaction. In this section, we synthesis these to contribute a thorough classification of the different forms of self-organisation that can occur via mobility.

Figure 8.16 provides a classification summary of the co-evolution of agents in opinions and locations, evaluated by the previous metrics under these mobility models (AM, RM, HM). It shows the transition of behaviour as ϵ and r_s move from low to high levels of interaction, by describing how individuals in a population are distributed in space at a given time taking into consideration their opinions as well. We use a labelling approach to categorise each quadrant. Mainly, the labelling first shows the number of opinion clusters in the population followed by a description of the individuals distribution in space. The opinion clusters are described as either ‘single’ or ‘multiple’ opinion clusters. As for the agents distribution in the geographical space, groups can be more or less equally spaced with exact coordinates, clustered in groups or dispersed randomly with no predictable pattern. These are described by these terms: ‘uniform’, ‘clumped’ or ‘scattered’ dispersion. The first two show organisation but in different forms. *Uniform* is when a cluster of agents hold the same opinions and co-located at the exact same coordinates. However, if the group of nearby agents are somewhat spread out, we describe the structure as *clumped*. Lastly, the term *scattered* is used when the agents don’t have any structure in geographical space but eventually converge in movement.

Finally, if there is no convergence in movement, the ‘undefined’ label is applied. Some spectrum of the models have some tolerance in the neighbourhood with agents holding different opinions, therefore, the label ‘mixed’ is added to show the model’s tolerance to different agents.

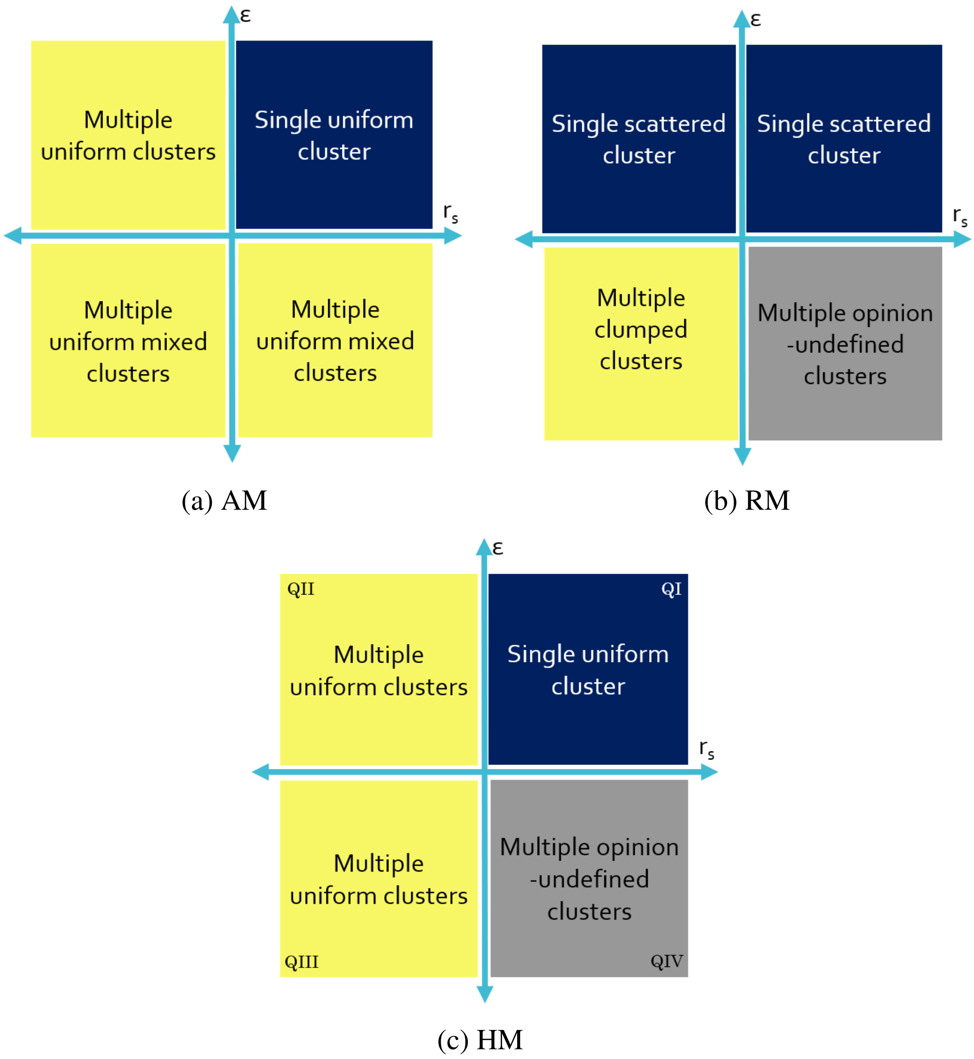
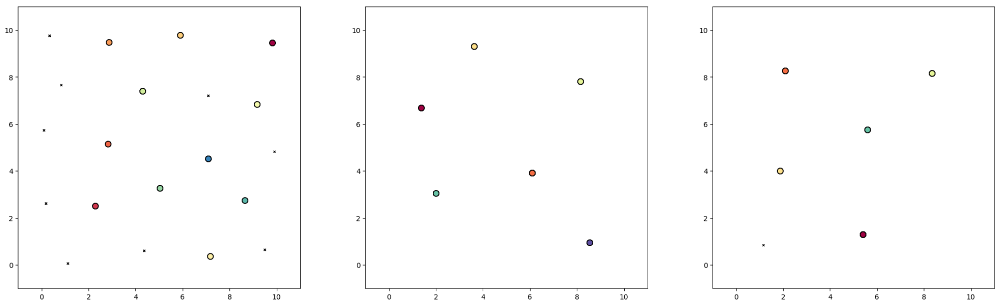


Figure 8.16: Summarised table for mobility models

8.5.1 Potential outcomes

The distinct outcomes from Figure 8.16 are summarised in this section, each illustrated by a representative example selected from the pool of simulations, where colours denote agents belonging to the same communities.

Multiple uniform clusters This describes the outcome when multiple opinion clusters are segregated (more or less evenly) in geographical space with agents co-located at identical locations (Figure 8.17). This behaviour typically emerges as a result of the attract component (AM and HM).

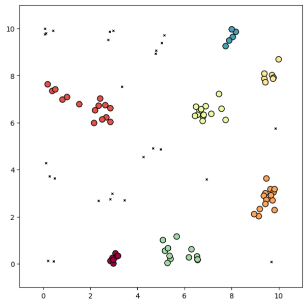


(a) Mobility model HM ($\epsilon = 0.1$, $r_s = 2$) (b) Mobility model HM ($\epsilon = 0.5$, $r_s = 2$) (c) Mobility model AM ($\epsilon = 0.5$, $r_s = 2$)

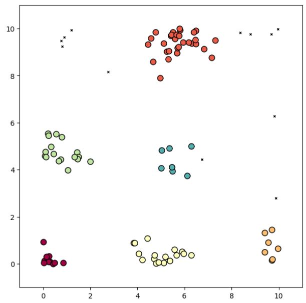
Figure 8.17: Multiple uniform clusters

Multiple clumped clusters The multiple opinion clusters that emerge are segregated in the geographical space (Figure 8.18), although with agents not precisely co-located. This outcome only occurs with repel mobility (low ϵ and r_s), where limited movement and interaction prevents global consensus but allows small pockets to form. The lack of an attractive component prevents agents from co-locating precisely. Self-organisation occurs due to the restricted interactions, since as agents eventually wander into a group where they belong, and consequently stop repelling and, as such, attract forces are not necessary to organise the agents.

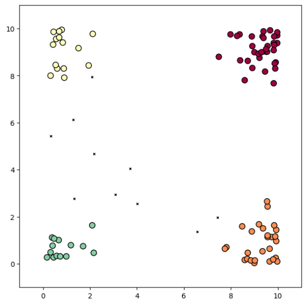
Multiple mix clusters The multiple opinion clusters that emerge overlap within the same neighbourhood but don't interact (Figure 8.19). Multiple mixed clusters are only found with the *attractive* mobility model via low ϵ, regardless of r_s. Clusters with



(a) Mobility model RM ($\epsilon = 0.1, r_s = 2$)



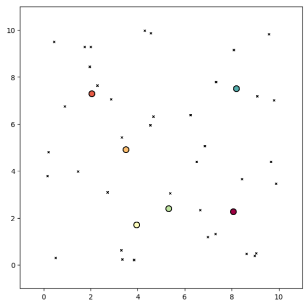
(b) Mobility model RM ($\epsilon = 0.1, r_s = 3$)



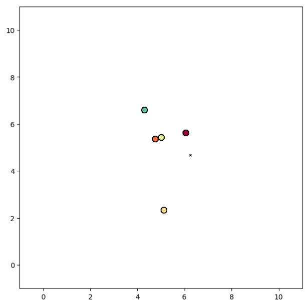
(c) Mobility model RM ($\epsilon = 0.1, r_s = 5$)

Figure 8.18: Multiple clumped clusters

the same opinions converge to a single location while unaffected by different nearby opinions, often resulting in higher tolerance.



(a) Mobility model AM ($\epsilon = 0.1, r_s = 2$)

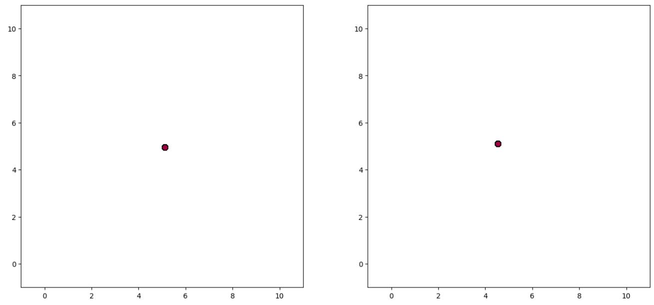


(b) Mobility model AM ($\epsilon = 0.1, r_s = 10$)

Figure 8.19: Multiple mix clusters

Single uniform cluster A single opinion cluster is formed at a single location, which dominates the region, see Figure 8.20. This requires large ϵ and r_s together with an attractive component (AM or HM).

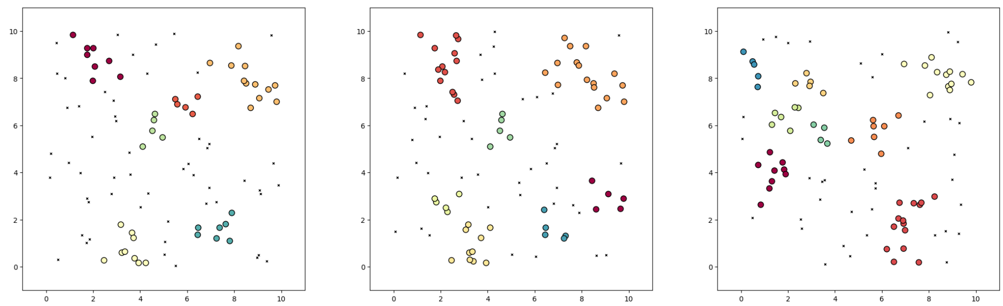
Single scattered clusters A single opinion cluster dominates, however the agents lack geographic structure and are scattered across the region (Figure 8.21). This behaviour is present in the RM model under large ϵ for all r_s. Note that this scattered distribution of a single opinion exhibits a similar behaviour to both the random mobility and the static model.



(a) Mobility model AM ($\epsilon = 0.5$, $r_s = 10$)

(b) Mobility model HM ($\epsilon = 0.5$, $r_s = 10$)

Figure 8.20: Single uniform clusters



(a) Mobility model RM ($\epsilon = 0.5$, $r_s = 2$)

(b) Mobility model RM ($\epsilon = 0.5$, $r_s = 10$)

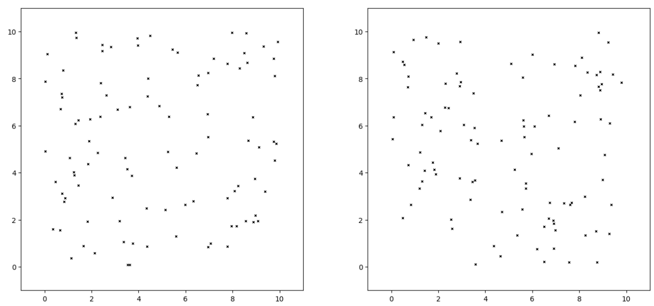
(c) Mobility model PRM ($\epsilon = 0.5$, $r_s = 10$)

Figure 8.21: Single scattered clusters

Undefined clusters This describes the case when multiple opinion clusters are formed (low ϵ), but there isn't any self-organisation or community formed (Figure 8.22). This is observed when r_s is large with a repel component, causing no convergence in movement. The structure is close to random, thus giving a similar outcome to the static model.

8.6 Discussion

In this chapter, we have conducted a deep investigation of the directed mobility models. We examine the different mobility models against the key parameters (ϵ and r_s) while comparing these to the random mobility (PRM).



(a) Mobility model RM ($\epsilon = 0.1, r_s = 10$) (b) Mobility model PRM ($\epsilon = 0.1, r_s = 10$)

Figure 8.22: Undefined clusters

We highlight the main findings and contributions of this chapter in four key observations. We first consider the number of opinion clusters in the population at a macro-level. Then, at a micro-level, we studied the neighbourhood based on two aspects, the number of communities with uniform opinion and the tolerance of different opinions around the neighbourhood. After that, we highlight the significance of the mobility components in the HM: both attraction and repulsion. Finally, we show the classification of self-organisation among mobility models. From these experiments we have found the following:

Opinion evolution is insensitive to certain mobility mechanisms. For example, the number of opinion clusters formed via *random mobility* is highly insensitive to any restriction of interaction range and results in similar outcomes to the static model. Also, the *repulsion model* behaves similar to random models across a large spectrum of the parameter space, only producing more clusters when both the interaction range and opinion threshold (Figure 8.23) are heavily restricted.

This is in contrast to directed mobility (AM, RM, HM), which stimulates significantly more opinion clusters than the random mobility. Such mobility gives the opportunity for agents holding minor opinions to find each other and build communities. More specifically, restricting the interaction range with *attractive mobility* has a larger impact for more opinion clusters to emerge for large opinion thresholds in comparison to the random mobility.

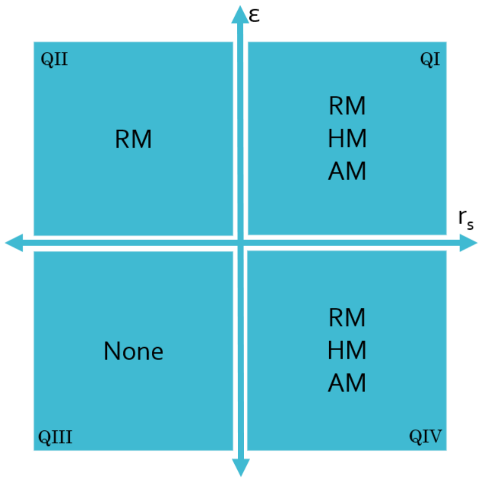


Figure 8.23: Mobility models with similar opinion clusters to PRM in each quadrant.

Emergence of communities via mobility. *Attractive mobility* stimulates uniform communities gathered in the same location. As for the *repulsive mobility* under specific configurations, this is able to form communities where agents are clumped near each other. Otherwise, different opinion agents are structure-less and look similar to the random mobility (Figure 8.24). Furthermore, tolerance of different opinions within an agent's local area can be found in the *attract model* but not in the *repel model*.

HM characteristics emerge from both the attract and repel components. AM and RM show several differences in their evaluation metrics across the parameters. The HM model shares some characteristics with both the attract and repel mobility components. The attract component has a strong impact on the HM to stimulate multiple *opinion clusters* with resistance to extremely high opinion thresholds, simultaneously encouraging multiple uniform *communities*. As for the repel component, it has a greater impact on showing no *tolerance* toward other different opinion and higher convergence time in movement.

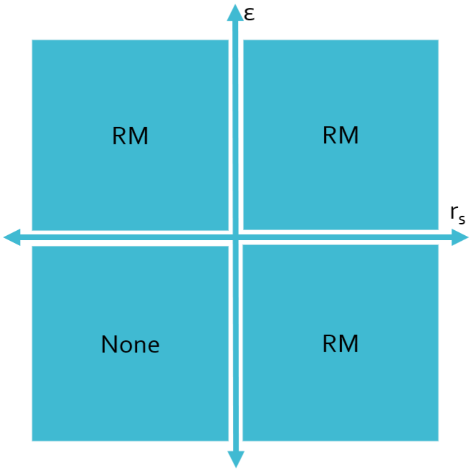


Figure 8.24: Mobility models whose self-organisation is similar to PRM in each quadrant.

Different forms of self-organisation via mobility. By collecting all the findings and synthesising between the different models, we have generated a classification of six distinct structures that emerge among the agents. These different structures can be a mix of a single or multiple opinions and their geographical structure can either be at uniform, clumped, or entirely scattered without any structure.

In conclusion, the HM model abstracts the complicated behaviour of real life agents by capturing some characteristics of opinion evolution in free space dynamic environment. *'The model may partly explain persistent cultural heterogeneity, fractiousness of political parties, and genetic diversity of populations that we see in the world'* [92]. This may shed light on the generic mechanisms observed in opinion formation.

8.7 Conclusions

In Chapters 5 and 7, we found significant impact from mobility on the co-evolution of opinion and location. In this chapter, we have performed detailed simulations across a wide parameter space in order to identify the possible structures that emerge under different mobility schemes, and where possible, classify the parameters that lead to these outcomes. The principal aim was to identify the importance of each mobility component (Sections 8.2.1, 8.3.1 and 8.4.1) and to use this large volume of data to classify the diverse behaviours (Sections 8.2.2, 8.3.2 and 8.4.2) and identify the relationship between metrics to define descriptive scenarios (Section 8.5).

Mobility that is either driven via attraction or repulsion are examples of two scenarios where all people become *similar* or *retain differences* (Section 8.2) depending on the parameters. The mechanism of attractive mobility explains how different opinions *retaining their differences* can be maintained in a population, due to the nature of finding similar peers. In contrast, repel mobility leads to more *similarity* of complete consensus, due to long distance travel that increased the probability for new different interactions that encouraged diffusion.

As a result we found that both the attract and repel have a significant impact on the HM model, each highlighting special characteristic that dominates the HM's behaviour. AM contributes primarily to cluster formation for both opinion and communities and RM to decreasing opinion tolerance within a local area.

Finally, this deep investigation between the different mobility schemes enabled us to create a classification that highlights the different structures of self-organisation that can emerge. This is a valuable contribution to the literature, as the majority of works tend to focus on more adhoc evaluation, reporting a small number of example scenarios, rather than identifying patterns across the breadth of the parameter space. The proposed classification highlights when a community is uniform or diverse, if a population reaches complete consensus or collectively holds multiple opinions (Section 8.5). It mirrors the structure of agents in the geographical space in terms if they are sparse or co-located.

Through this classification (Section 8.5) we can summarise a couple of findings from our mobility models. We found that under attractive forces (AM,HM) uniform clusters (i.e co-located clusters that are also more structured) can be formed geographically but more sparsely via repel. In contrast, under repulsive forces (RM, HM) a structure-less population is dominant under a global interaction. In addition, complete consensus is a characteristic more often found via repulsion.

In conclusion, this contributes to the research field by emphasising the importance of mobility and highlighting how mechanisms inspired by psychological theories make a significant impact on the co-evolution of opinion and location. Also, the proposed classification diagram provides a basis for mobility models to be assessed. This classification sets a benchmark that we hope could be carried forward and followed against models.

Chapter 9

Conclusions & Future Work

In this chapter we will discuss the final conclusions and reflect on some of the limitations of this research as well as the future work.

9.1 Summary

The aim of this thesis has been to consider the effects of mobility upon opinion dynamics, and in particular to understand the co-evolution of both opinion and location. Mobility has a fundamental role related to opinion because human responses to others can guide our preferences for interaction, and exposure to similar/dissimilar opinions. Despite this, the effects of mobility are surprisingly under-represented in the opinion dynamics literature. Models of opinion dynamics have also been criticised for ignoring the fundamental principle of human mobility [103, 102, 16, 116, 53]. This motivates our inclusion of mobility inspired by phenomenon from human psychology - *homophily* [84] and *cognitive dissonance* theory [35].

These theories can govern how humans respond to similarity and difference in opinions. On the one hand a similar opinion gives two human agents something in common, making it easier for them to sustain an interaction or relationship. On the other hand a difference in opinion can be hard for an agent to reconcile, because it may be contradictory to their own reasoning and represent uncertainty. This creates cognitive dissonance, and motivates their behaviour to remove the dissonance. These fundamental psychological concepts can transform themselves into mobility very easily. For example, repelling away from peers that cause cognitive dissonance to an individual is a plausible response behaviour, as is being attracted to individuals where commonality is present. The Hybrid Mobility (HM) model, as introduced in this thesis, translates these concepts into attraction and repulsion forces.

We extend the well-known [29] opinion model to incorporate mobility. Mainly, we have studied the two components of attraction and repulsion individually as well as combined in the HM model. To understand how these models function we have compared them with alternative random models that involve two types of movement, being constant random movement (an analogy to gas particles) and movement triggered by disagreement with peers. We compare these mobility models against the widely studied static model proposed by [29] as a benchmark, leading to a number of significant insights and contributions highlighted below.

The model. We developed a *free space opinion model* that is suitable for our investigation of studying the co-evolution of both opinion and location. We must note this has rarely been studied because most of the models follow the norm in modelling (i.e. lattice, network etc.). We incorporate different mobility to highlight the impact of mobility on the model. To asses the model we developed evaluation metrics that can quantify the behaviour of the model in different aspects, namely convergence, opinion clusters, communities and tolerance. We must note some of this metrics are widely reported in the literature but often lack clear definition in how they function (for example opinion convergence). Therefore, we provided formal definitions of these metrics, expanded where necessary to consider spatial elements. For example, in mobile models, convergence in movement is of similar interest to convergence in opinion. This has never been studied in the literature, due to the fact that the majority of the models are static. Similarly, we quantify opinion clusters to enable comparison between the models, and extend the concept to communities that are close in opinion and location. Finally, in contrast to the majority of the literature, we take into account agents which are outside of communities to reflect on the population's overall self-organisation.

Mobility in the literature. Chapter 2 contributed a detailed literature review about the inclusion of mobility in opinion dynamics. Despite comprising surprisingly few publications, making comparisons within this literature is challenging, as they tend to lack detail of the mechanisms and evaluation, and citations between different approaches are rare. We classified the main attributes of these model, with the most important being the environment where mobility takes place, the *trigger* for mobility to occur, and the *dynamics* to determine the new location. Most commonly, pure random mobility is used without a motive for the decisions taken. For those that followed a different approach (e.g. moving when disagreement was encountered), they still used randomness, either as a trigger or in the *dynamics* (see Table 2.2).

Purely random mobility has minimal impact on opinion formation. Our experiments with random mobility in different forms shows that it behave similarly to the well studied Deffuant-Weisbuch static model in the formation of both opinion and structure, typically causing full diffusion of opinion to form complete consensus and eliminating any structures based on alternative opinions (Section 7.2.2 and 7.2.1.2). We found consistency among the variations of random models that we investigated, each producing faster opinion convergence compared to the static model, in line with the literature [121, 105]. These results may explain why mobility in opinion dynamics has not been widely studied in the literature to date, since the most obvious models to consider produce little effect.

Directed mobility has a significant impact on opinion formation. Static model shows that the number of opinion clusters increases as interaction is restricted [71, 16] (Section 5.2.2). Incorporating directed mobility into our proposed hybrid mobility shows even more growth in opinion clusters (Section 5.2.2), a result which is robust under the addition of random noise (Section 6.3.2). In general, in Section 8.2.1 we found that directed mobility (regardless of the type of forces i.e. attract or repel) compared to the static model, encourages more opinion clusters to form under restricted measures of both low opinion threshold (ϵ) and low interaction range (r_s).

Mobility mechanism behave differently under different settings, some characteristic stand out when the attractive and repulsive mobility models are applied in isolation. Detailed experiments have shown the characteristics of the individual components of mobility in our hybrid model. Attraction encourages more tolerance in a local area, where agents are content in their locations (Section 8.3.1), and when interaction is restricted (Sections 8.2.1 and 8.4.1) this encourages more opinion clusters and communities in the population independent of ϵ when compared to the repulsive mobility. This is somewhat surprising since high ϵ has gained interest in the literature as $\epsilon > 0.5$ usually leads to complete consensus. The main characteristics in repulsion forces is that they promote uniform opinion neighbourhoods (Section 8.3.1) and are the cause of delay in movement convergence(Section 7.3.1.2).

In summary, drivers that are independent of the opinion threshold and show significance impact on the the co-evolution of both opinion and structure involve either i) attraction forces ii) reasonable restricted interaction range.

The hybrid model is shown in Chapter 8 to form structure in a balanced way, between attraction and repulsion. This mobility mechanism allows both opinion and location to co-evolve and organise. It mitigates the occurrence of loners (i.e isolated agents) and can form different communities of homogeneous opinions, while decreasing the diversity in the local area. This is a scenario we see in real life where multiple groups of uniform opinions are segregated. This concludes that the type of mobility mechanism matters considerably and should be studied further.

Classification of self-organisation in opinion formation. Our review of the literature noted high variation in the level of detail and generally limited approaches to understanding the characteristics of opinion formation models Chapter 2. In this thesis we have also presented a rigorous and new approach to classifying how agents self-organise themselves in different ways Section 8.5, conducting a large volume of experiments that enabled us to provide a thorough analysis. We study the models behaviour by analysing each metric (Sections 8.2, 8.3 and 5.6) and categorising similar behaviour dependant on ϵ and r_s. Based on the analysis on the evaluation metrics we divide the outcomes into a small number of scenarios. For example, the random mobility under large r_s and ϵ results in a *single scattered cluster*, with single opinion where agents are structure-less geographically. Finally after synthesising and combining the different outcomes this developed the classification diagram in Section 8.5 which identified a scenarios that describe the agents self-organisation for different mobility models. These different findings highlight the significance of mobility under naturalistic settings and emphasise the importance of applying mobility under the inspiration of psychological theories to resemble more human reaction.

9.2 Limitations

Our research shows that mobility based on psychological human feedback have shown more diverse opinions than applying random mobility. However, the psychological theories used are broadly stated principles that are frequently applied in the field. Connecting researchers from multi-disciplinary fields (such as psychology, computer science etc.) will enhance the knowledge we can gain about opinion dynamics.

Free mobility in continuous space gives more flexibility to new connections. However, as an initial investigation we have implemented this in a simple box. This research has focused on the type and direction of mobility, however in real-life scenarios considering "personal space" is an important feature. In the real world, large numbers of agents can not be precisely co-located at a single point, so adding some measure of personal space between agents would be interesting, where at higher densities this may force individuals to be exposed to conflicting opinions. In future work more studies on the environment's features such as studying crowds should be investigated.

9.3 Future Work

The proposed hybrid mobility has shown that the nature of mobility in opinion dynamics is highly significant. A number of extensions to this model would be interesting to study. For instance, more variability could be introduced to the fundamental attract and repel forces, which are currently applied uniformly across the population. In practice, our individual personality traits may introduce more diversity, for example, where agents that have high "openness" may be more tolerant of conflicting opinions and less likely to repel away.

Another very interesting investigation is applying the same mobility models under different environments. [39] studied the threshold where complete consensus is formed in comparison to a variation of environments that are static. This study would be interesting to add mobility and study the evolution under a lattice, network and free space. Another way that this model can be used is to reflect on the geographical distance in a more abstract way. For example, this could resemble closeness in a online social network. In fact it would be very interesting to study the impact between your virtual community and actual community. [20] provides a survey and discuss how can a network topology present opinion propagation in an online social networks. Adding a graph-based social network on top of free space movement would allow us to study the trade-off between our in-person and virtual interactions in shaping opinions. In each time step an interaction can be either with a node within the area or a node in the network. This may allow the opinion to take effect at long distance. This can highlight the impact of both distances, the physical distance and the online distance.

When studying opinion we can't ignore the fact that an individual holds multiple opinions but one can have a more stronger impact than the other. Influence of other agents might be based on the interplay between various 'interest points'. [40] studied a vector of opinions, specifically opinions that are bi-dimensional vectors and concluded that it was insignificant, but we propose that this may be because of the global interaction scheme applied. Also, it would be interesting if multiple opinions were applied. Multiple opinions can be a vector $[o_1, \ldots, o_n]$ of n different topics, each under an opinion spectrum $o_i \in [0, 1]$. Extending this work to study topics that might have stronger impact than others. Studying how multi-dimensional opinions impact the formation of communities under a mobility model is intriguing, particularly where there are dominant topics (such as political affiliation, pro- or anti-Brexit) where opinions may have a far more significant impact on our movement and social links. Incorporating mobility is very interesting and expanding in the field is compelling. We believe that this field will develop and be more stronger.

9.4 Final Comments

This research has focused to show the impact of different mobility mechanisms on opinion evolution. These mobility mechanisms are triggered by interactions between peers depending on their agreements. In the literature, where agents are immobile, complete consensus is usually the final result, however this is not what is observed in real life. We have found the nature of mobility has a significant role in forming different opinion. Especially when comparing mobility that is triggered by human drivers in comparison to random mobility. Static models and randomly mobile agents resulted in complete consensus, however under directed mobility diverse opinions formed that are different from the literature. Furthermore, mobility have shown different self-organisations in forming communities. Showing different structures of communities, some holding the same opinion others whom are diverse. Comparing between different mobility mechanisms is essential to highlight the nature of self-organisation between communities.

Finally, this research highlights the importance of incorporating mobility that reacts to the interaction between people instead of random mobility. Also, we have proposed a classification diagram that provides a basis for mobility models to be assessed. This classification sets a benchmark that we hope could be carried forward and followed against models.

Appendix A

High noise level

In this Chapter we extend the results from Chapter 6 to show the impact of high noise level on the HM model.

A.1 Directed mobility under high level of noise

In this section we present the results of the different evaluation metrics under very high level of noise ($T = 10$). First, we show the convergence in opinion followed by the emergence of opinions and their local diversity and finally their community formation.

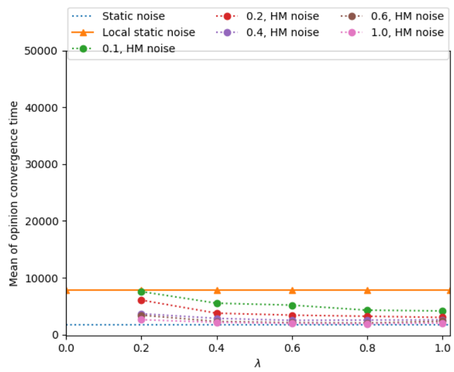
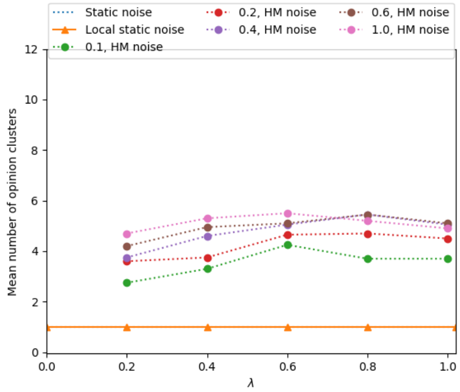
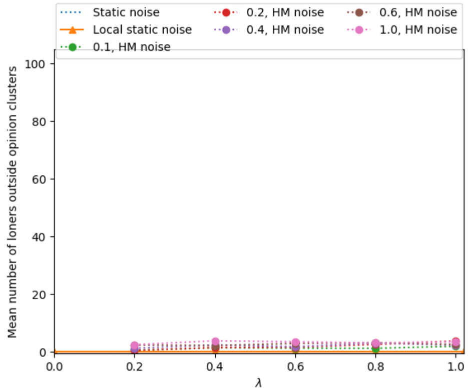


Figure A.1: Mean opinion convergence time for $\epsilon = 0.1$ and $T = 10$



(a) Number of opinion clusters for $T = 10$



(b) Number of loners outside opinion clusters for $T = 10$

Figure A.2: Mean number of opinion cluster and mean number of loners outside opinion clusters for $\epsilon = 0.1$.

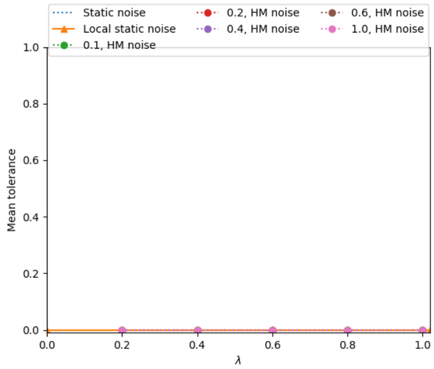


Figure A.3: Mean tolerance for $\epsilon = 0.1$ and $T = 10$

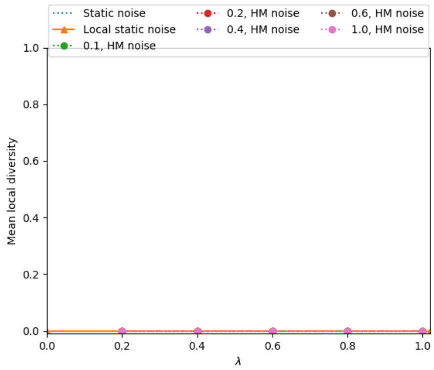
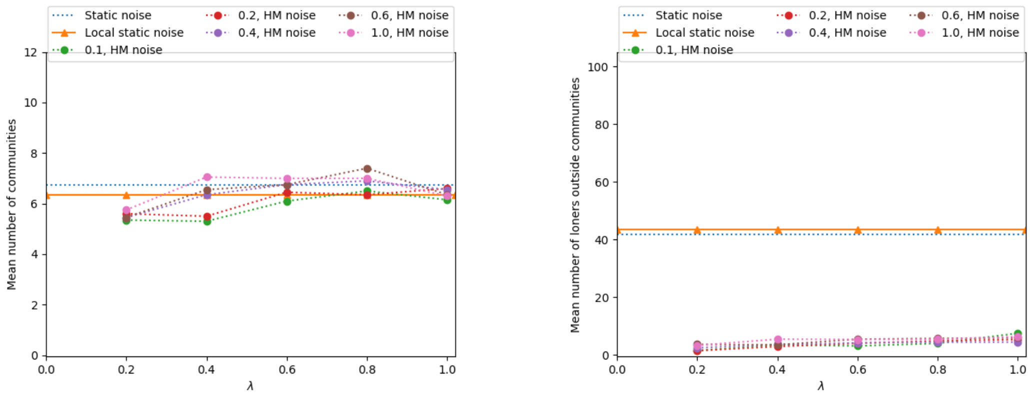


Figure A.4: Mean local diversity for $\epsilon = 0.1$ and $T = 10$



(a) Number of communities for $T = 10$

(b) Number of loners outside communities for $T = 10$

Figure A.5: Mean number of communities and mean number of loners outside communities for $\epsilon = 0.1$.

Appendix B

Outcomes of different models (AM, RM, HM, Static)

In this Chapter we plot some examples of the agents distribution to visualise the outcome of the experiments across the different configurations.

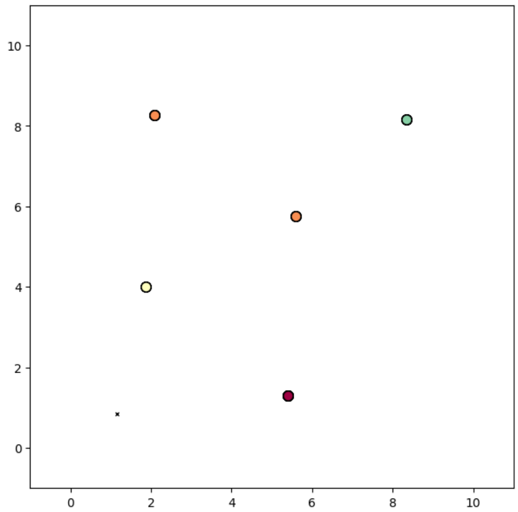
B.1 Display of different outcomes

This section shows a set of figures that demonstrates two type of clustering for each model, the clustering technique is described in Section 3.4.2.1. We specify our use of the DBSCAN clustering algorithm which produces these figures. For example the AM model demonstrate two types of clustering in two independent sets of figures (Figure B.1 and B.2). They present a snapshot taken at the end of the simulation of a single run. For each figure the number of clusters and loners are collected.

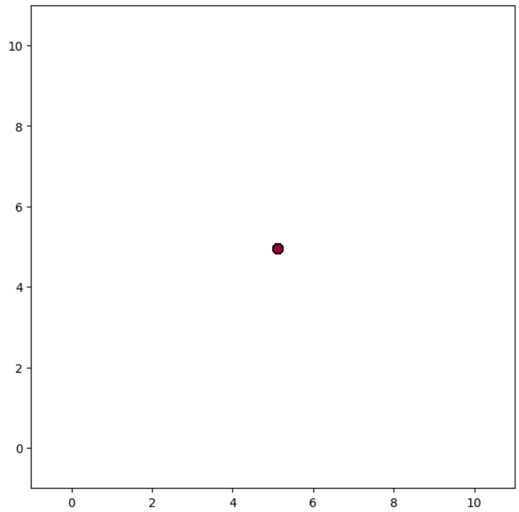
The first one, Figure B.1 shows the opinion clusters, each opinion cluster is assigned a colour and plots their actual locations around the region. The dots in this context are *loners*, meaning, isolated agents with no one sharing their opinion. The four figures are represented in terms of four quadrants (Q) labelled as QI, QII, QIII and QIV to resemble the change in both ϵ and r_s. The top two quadrants (QI and QII) show high values of ϵ and the bottom quadrants with low ϵ (QIII and QIV). As for the quadrants in columns, QII and QIII show low r_s and the right QI and QIV show high r_s.

The second Figure B.2 shows communities, the clustering algorithm joins agents that are similar in their opinion cluster if their nearby in geographical space into a single cluster and assigning each cluster a different colour. The dots are loners that don't belong to a community. The quadrants follow a similar fashion as the previous figure.

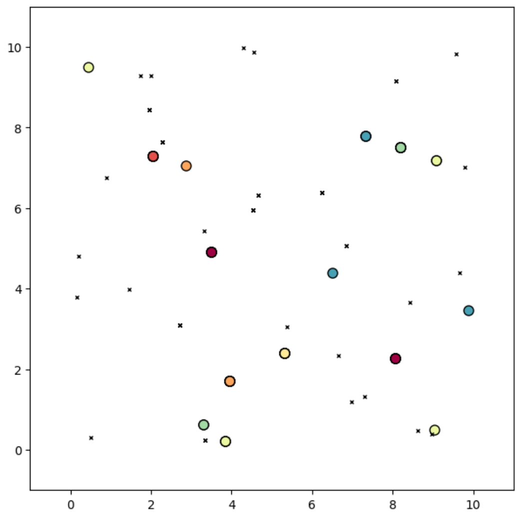
A similar approach is used for all subsequent figures representing a different model.



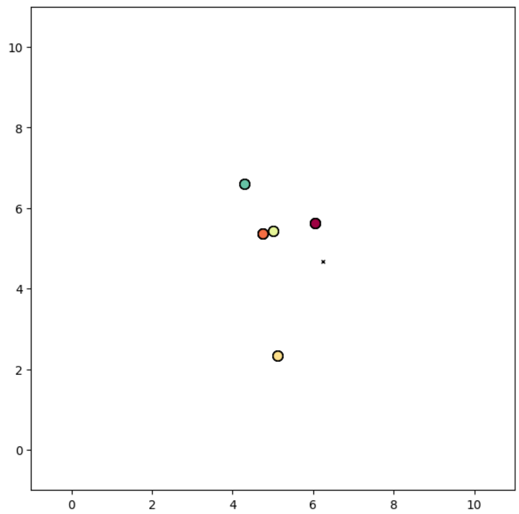
(a) QII: 4 opinion clusters - 1 loner ($\epsilon = 0.5$ and $r_s = 2$)



(b) QI: 1 opinion cluster - 0 loners ($\epsilon = 0.5$ and $r_s = 10$)

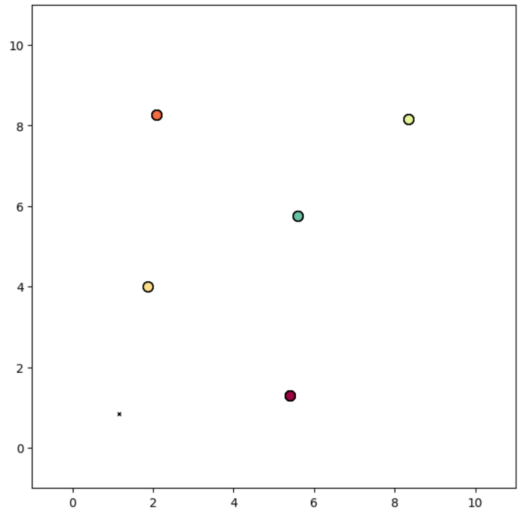


(c) QIII: 7 opinion clusters - 43 loners ($\epsilon = 0.1$ and $r_s = 2$)

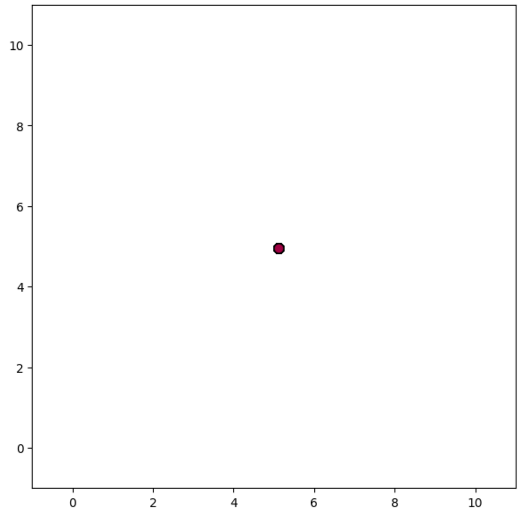


(d) QIV: 5 opinion clusters - 1 loner ($\epsilon = 0.1$ and $r_s = 10$)

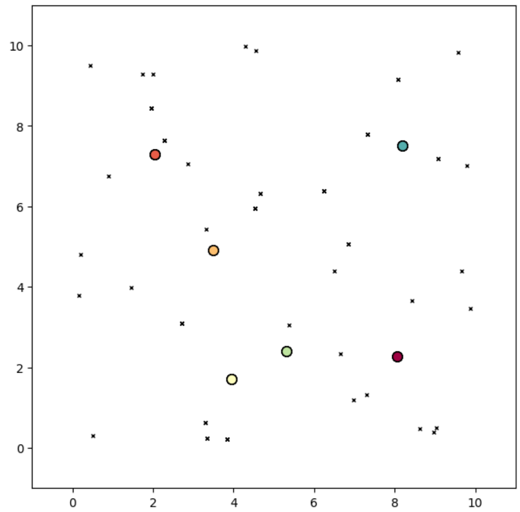
Figure B.1: Opinion clusters under AM of single runs



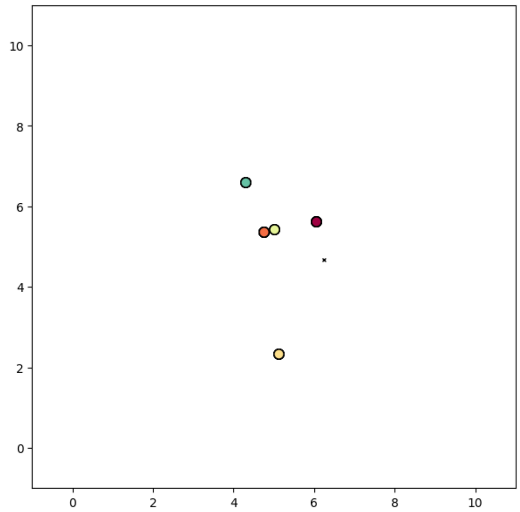
(a) QII: 5 communities - 1 loner ($\epsilon = 0.5$ and $r_s = 2$)



(b) QI: 1 community - 0 loners ($\epsilon = 0.5$ and $r_s = 10$)

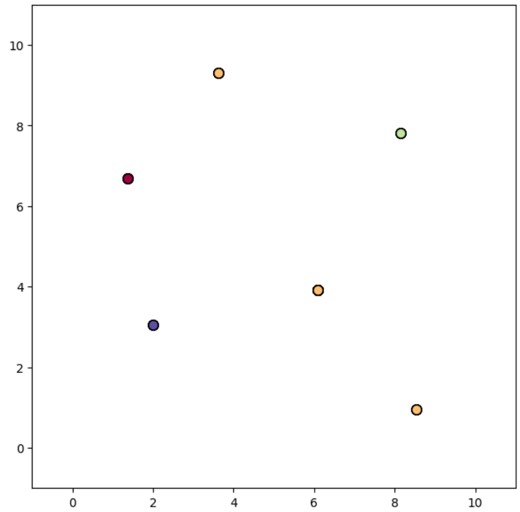


(c) QIII: 6 communities - 58 loners ($\epsilon = 0.1$ and $r_s = 2$)

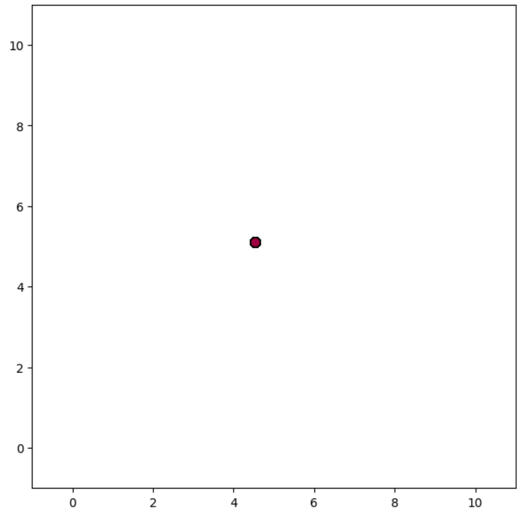


(d) QIV: 5 communities - 1 loner ($\epsilon = 0.1$ and $r_s = 10$)

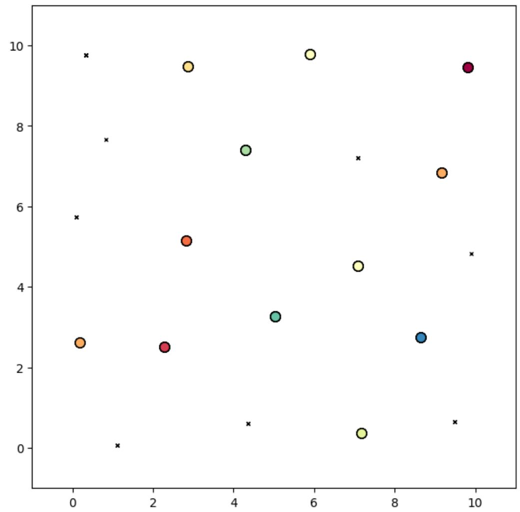
Figure B.2: Communities under AM of single runs



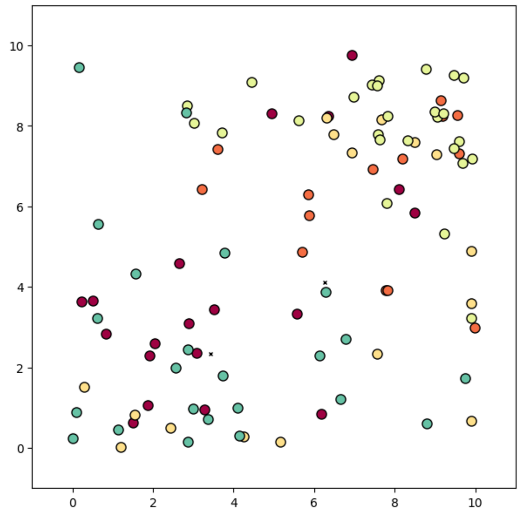
(a) QII: 4 opinion clusters - 0 loners ($\epsilon = 0.5$ and $r_s = 2$)



(b) QI: 4 opinion clusters - 0 loners ($\epsilon = 0.5$ and $r_s = 10$)

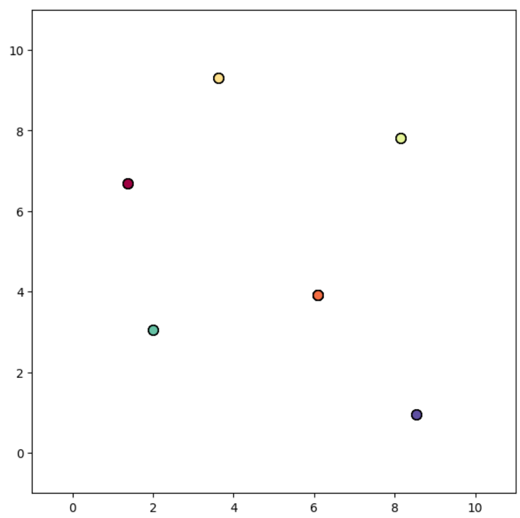


(c) QIII: 10 opinion clusters- 16 loners($\epsilon = 0.1$ and $r_s = 2$)

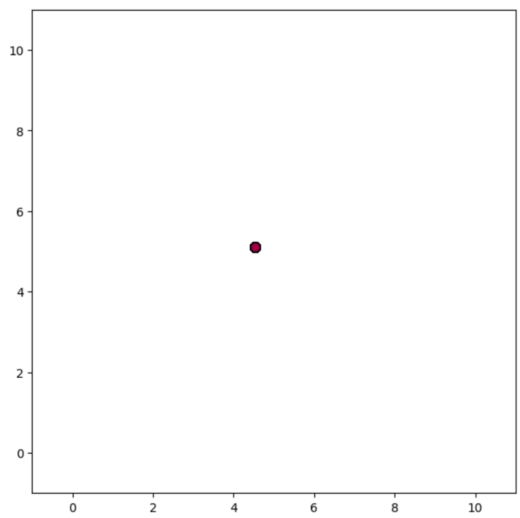


(d) QIV: 5 opinion clusters - 2 loners ($\epsilon = 0.1$ and $r_s = 10$)

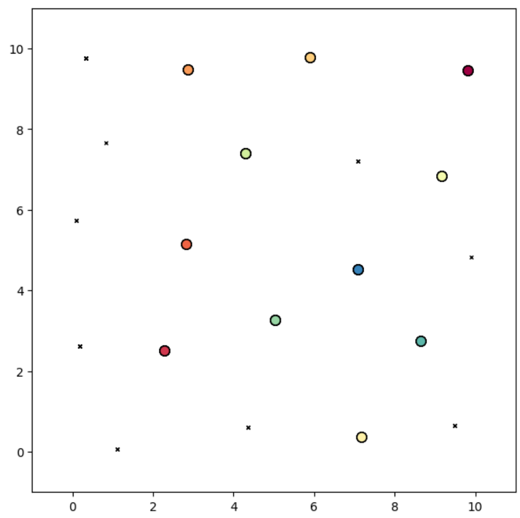
Figure B.3: Opinion clusters under HM of single runs



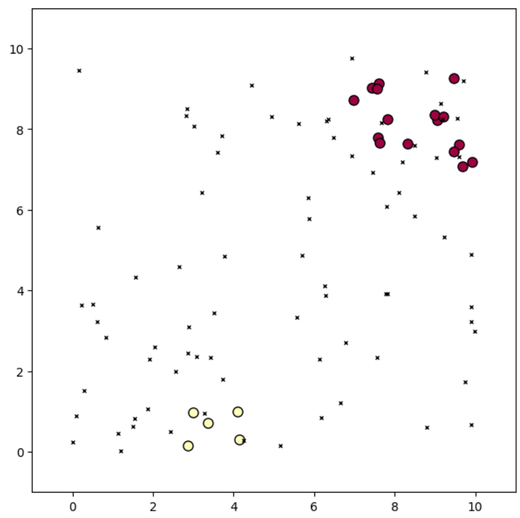
(a) QII: 6 communities - 0 loners ($\epsilon = 0.5$ and $r_s = 2$)



(b) QI: 1 cluster - 0 loners ($\epsilon = 0.5$ and $r_s = 10$)

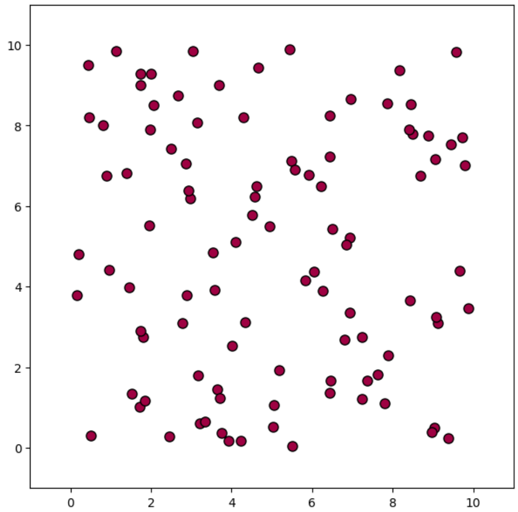


(c) QIII: 11 communities - 20 loners ($\epsilon = 0.1$ and $r_s = 2$)

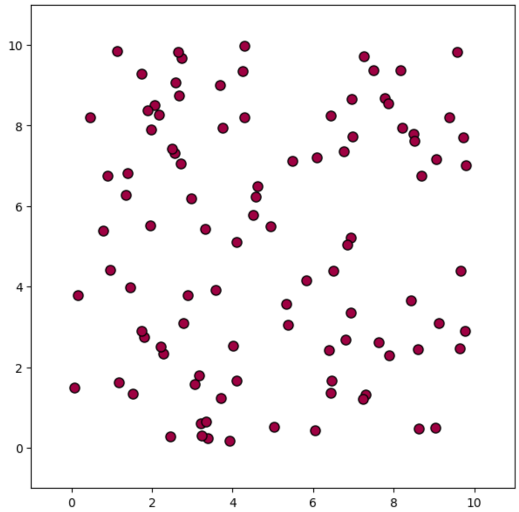


(d) QIV: 2 communities - 79 loners ($\epsilon = 0.1$ and $r_s = 10$)

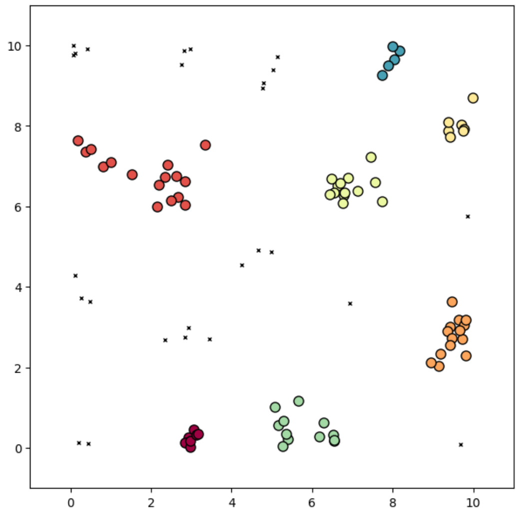
Figure B.4: Communities under HM of single runs



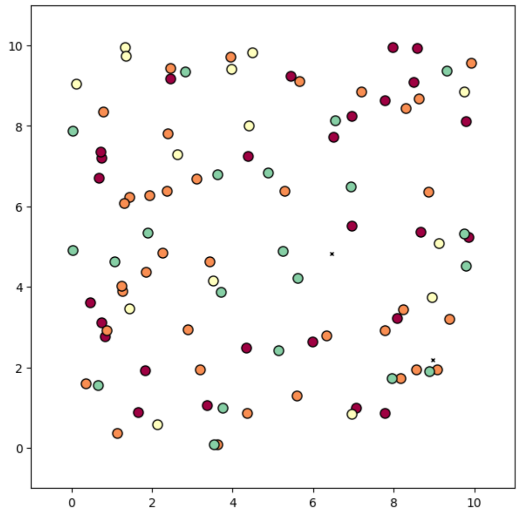
(a) QII: 1 opinion clusters - 0 loners ($\epsilon = 0.5$ and $r_s = 2$)



(b) QI: 1 opinion clusters - 0 loners ($\epsilon = 0.5$ and $r_s = 10$)

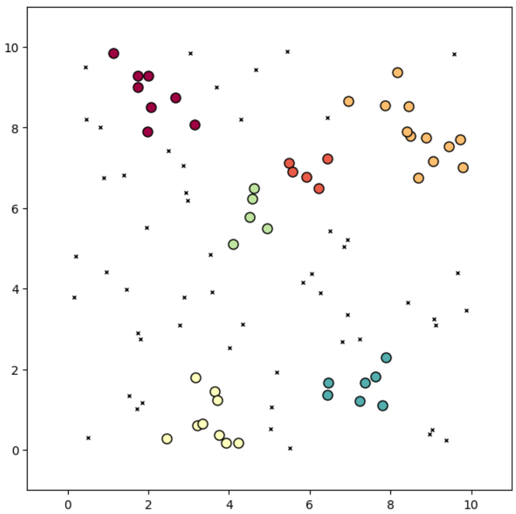


(c) QIII: 7 opinion clusters - 26 loners ($\epsilon = 0.1$ and $r_s = 2$)

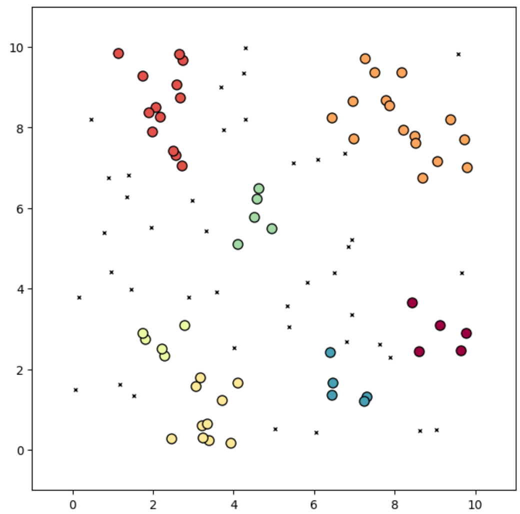


(d) QIV: 4 opinion clusters - 2 loners ($\epsilon = 0.1$ and $r_s = 10$)

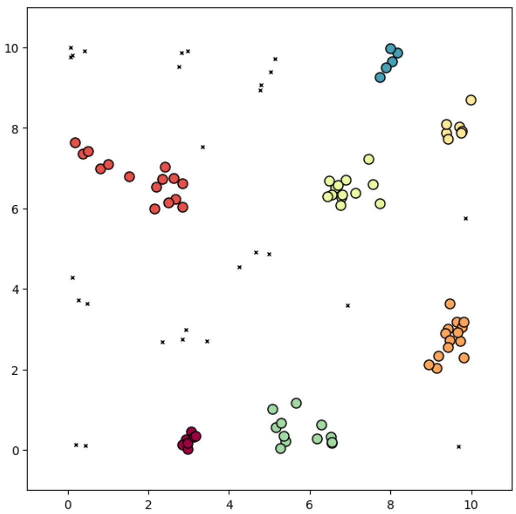
Figure B.5: Opinion clusters under RM of single runs



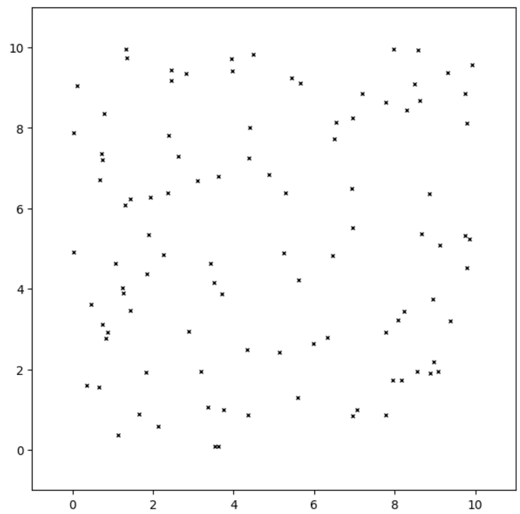
(a) QII: 6 communities - 54 loners ($\epsilon = 0.5$ and $r_s = 2$)



(b) QI: 7 communities-41 loners($\epsilon = 0.5$ and $r_s = 10$)

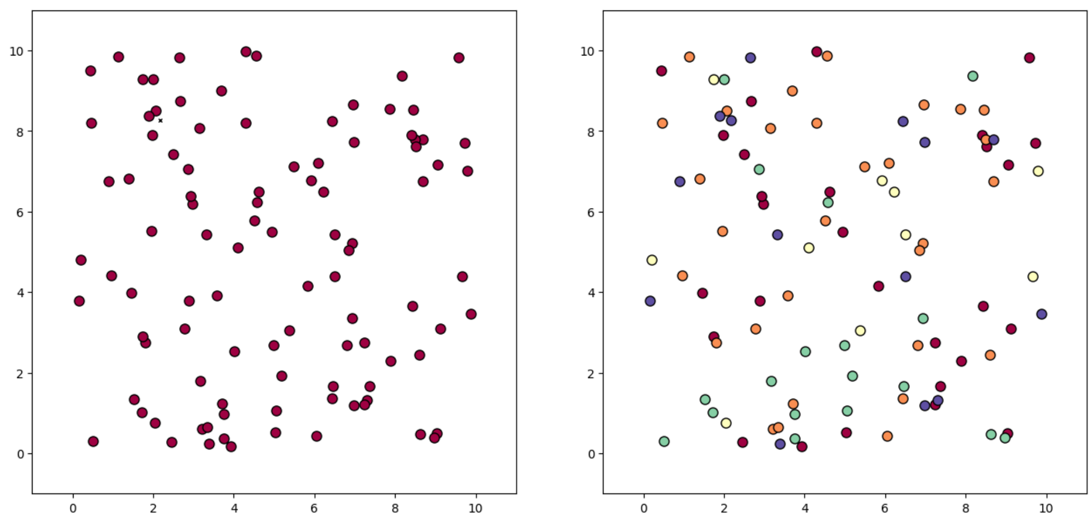


(c) QIII: 7 communities - 27 loners ($\epsilon = 0.1$ and $r_s = 2$)



(d) QIV: 0 communities-100 loners($\epsilon = 0.1$and $r_s = 10$)

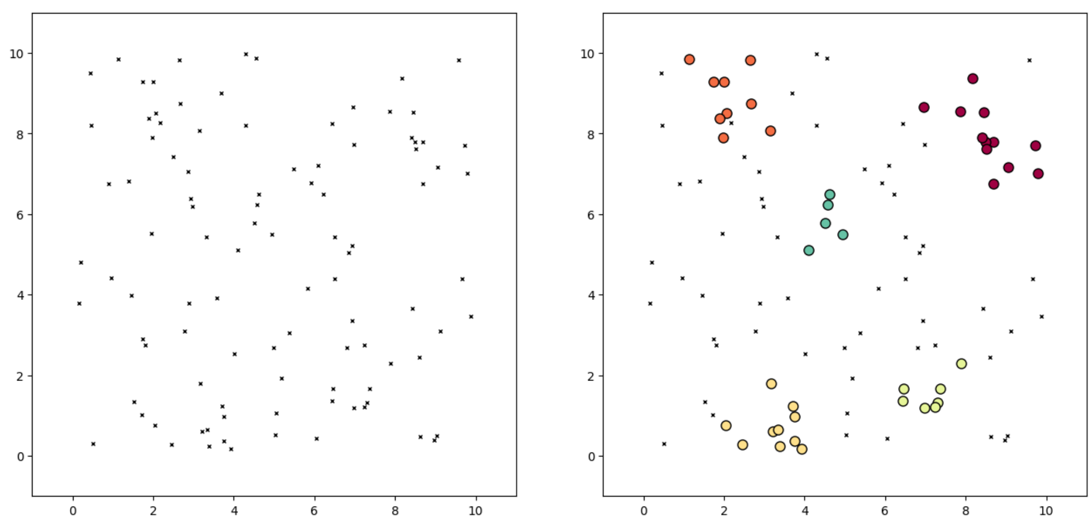
Figure B.6: Communities under RM of single runs



(a) 1 opinion cluster - 1 loner($\epsilon = 0.5$ and $r_s = 10$)

(b) 5 opinion clusters - 0 loners ($\epsilon = 0.1$ and $r_s = 10$)

Figure B.7: Opinion clusters under the static model of single runs



(a) QI: 0 communities - 100 loners($\epsilon = 0.1$ and $r_s = 10$)

(b) QIV: 5 communities - 57 loners ($\epsilon = 0.5$ and $r_s = 10$)

Figure B.8: Communities under the static model of single runs

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