Reconfigurable Linear Antenna Arrays for Beam-pattern Matching in Collocated MIMO Radars

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Abstract

Beam-pattern matching plays an important role in multiple-input multiple-output (MIMO) radars. In the vast majority of research done in this area, the aim is to find the covariance matrix of the waveforms fed into the transmit array. Also, reconfiguring a preset array of antennas (antenna selection) which means turning off some of the antennas in the array, is an effective technique to reach the desired beam-patterns, dynamically. In this paper, we introduce a novel multi-step method to implement this reconfiguration technique to a uniform linear array (ULA). In each step, by exploiting the relation between the diagonal elements of a covariance matrix resulted from solving a beam-pattern matching problem and the transmitted power of the antennas, we find the least important antenna of the array and turn it off accordingly. Then, we repeat this process until a predefined number of antennas remains. Our proposed method outperforms its counterparts in the literature in terms of beam-pattern matching as well as computational complexity, which makes it an appropriate method for real-time applications. Simulations used to show the validity and superiority of the proposed method.

Index Terms

MIMO radar, beam-pattern matching, reconfigurable antenna array, antenna selection.

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I. INTRODUCTION

Multiple-input multiple-output (MIMO) radar is an emerging technology in radar systems. This technology has gained considerable attention in recent years after the results of utilizing MIMO antennas in wireless communication systems showed considerable improvements [1], [2]. MIMO radars are generally classified into two types based on their antenna layouts: widely separated (or distributed) and collocated MIMO radars. In widely separated MIMO radars, the antennas are placed far enough to view different target's radar cross-sections (RCSs) and achieve spatial diversity to improve detection and localization accuracy [3], [4]. In collocated MIMO radars [5] the antennas are closely spaced and may provide more flexibility in beam-pattern design, improved spacial resolution [6], [7], better parameter identifiability and increased maximum number of detectable targets [8], and higher detection probability in the presence of interferences [9]–[11]. In contrast to the phased array radars in which antennas are excited by phase-shifted prescriptions of a specific waveform, MIMO radar antennas transmit a wide range of waveforms. These waveforms can be orthogonal, can have some correlation, or can be fully correlated. This property which is called "waveform diversity", provides more degrees of freedom to improve the performance of radar systems in terms of the above-mentioned properties [1], [6], [12].

In MIMO radar design process, the beam-pattern matching problem, as an important part, is defined as follows: matching or rather approximating the transmit beam-pattern (in mean square error (MSE) sense) to a desired beam-pattern over a region of interest and also minimizing the cross-correlation beam-pattern (a beam-pattern that is composed of the signals from different directions) for some specific directions in the region of interest. The goal for solving such a problem is to find the appropriate probing waveforms or the waveforms covariance matrix which is realized by the probing waveforms [5], [9], [13]–[23]. The design of waveforms covariance matrix for ULAs has been studied in various frameworks, including beam-pattern matching in the literature. In some of the researches such as [9], [17]–[20] iterative methods are used to design the covariance matrix. To this end, authors in [17] numerically optimize two various cost functions using a so-called barrier method. Authors in [18] propose a semi-definite quadratic programming (SQP) method to solve the beam-pattern matching problem. A method to control the ripple and transition band of beam-pattern has been proposed in [19] and in order to design a covariance matrix that enhances the signal to interference plus noise ratio (SINR) in the receiver, authors of [20] solve a semi-definite programming (SDP) problem. Also, some closed-form methods [21],

[22] are presented to reduce the complexity of the iterative methods for designing the covariance matrix, although they can not achieve the accuracy of the iterative methods. Moreover, sparse or thinned receiver array design has been of great interest to researchers in the last half-century [24]. Reducing the number of antennas (and thus reducing the cost) which is needed to synthesize the desired spatial receiving beam-patterns has been the purpose of sparse array design [24]–[26]. Extending the ideas behind sparse receive array methodologies to that of sparse MIMO array design with new terminologies such as array optimization, antenna placement, antenna selection, etc. has been done in studies such as [24], [27]–[32].

In the recent studies [30]–[32], the problem of joint design of waveform covariance matrix and antenna position vector (a binary vector that indicates the position of active and inactive antennas in the array with one and zero values, respectively) has been investigated. Solving this problem has two goals: a) to select a specific number of antennas N from a set of candidate antennas M which are initially configured in a ULA array (the number of candidate antennas is larger than the selecting antennas, i.e., $M > N$); and b) to design the waveform covariance matrix which matches the transmit beam-pattern to the desired one, using the reconfigured array. This joint design can outperform the traditional covariance matrix design with the same number of active antennas. Intuitively, due to the more degrees of freedom that the additional antennas provide and the relatively larger aperture that is formed, this problem provides an improved result compared with the ULA counterpart with an equal number of antennas (i.e., N antennas). In [30] and [31] the MSE criterion for beam-pattern matching is considered and to tackle the joint design of covariance matrix and antenna “position vector” problem, a cyclic approach is utilized. In each cycle, first, for a fixed position vector the covariance matrix is designed, then the position vector is updated with the given covariance matrix. However, the optimization of the position vector is a quartic Boolean combinatorial nonconvex problem [30] which is hard to solve directly. Therefore, authors in [30] proposed a method based on alternating direction method of multipliers (ADMM). Also in [31] an iterative greedy local binary search approach based on dynamic programming and evolutionary algorithms is devised to handle the position vector optimization. In [32], for the joint design problem, authors considered two optimization criteria, i.e., the MSE for beam-pattern matching as well as a defined criterion for controlling mainlobe ripple and sidelobe levels. In both formulations, first, the couple between the covariance matrix and the position vector is eliminated, and then a successive convex approximation (SCA) algorithm is proposed to solve the resultant problems.

Despite the high computational complexity of the proposed methods in [30]–[32] due to the cyclic and iterative methods for optimizing position vector utilized in these works, none of them guarantee the optimality of the results. The reason is in one hand, the problem of joint design of covariance matrix and antenna position vector is not convex and can have several local minimums, and on the other hand, all of these methods have a random parameter in their process which results in falling in these local minimums rather than necessarily reach the global optimum point. Hence in practice, several optimization runs are usually carried out and then the minimal objective value is chosen [32]. Also in some cases, the optimization process will not converge at all because of the inaccurate choice of optimization parameters.

In this paper, we propose a fast and at the same time accurate method to reconfigure a preset ULA antenna array for improving beam-pattern matching performance in collocated MIMO radar systems. Our proposed method is based on the relation of the covariance matrix and the transmitted power from the antennas. We first find the best reconfiguration (antenna positions) of a preset ULA by a multi-step method; starting from the full array, in each step we decide to deactivate one of the antennas to finally reach the predetermined number of antennas we aim to remain active. The problem we tackle in each step is solving a simplified form of the SQP beam-pattering matching problem and then finding the minimum value of the diagonal of the resultant covariance matrix. After the selection of the desired number of antennas, the beam-pattern matching is carried out by solving the SQP problem [18] to design the covariance matrix.

The prominent features of our proposed method can be summarized as follows:

- Given a set of candidate antennas, the proposed method matches the transmit beam-pattern to a desired one with improved performance compared to the ULA array with the same number of active antennas and the other recent reconfiguration methods in the literature. Furthermore, the MSE of our proposed method is close to that of the case that all candidate antennas are active. This means that our proposed method can design a desired beam-pattern with a less number of antennas compared to counterpart methods.

- As the proposed method only needs to solve a fixed number of well-known SQP optimizations and simple minimum searches, the computational complexity (or equivalently the consumed time) of the method is significantly lower than that of recent methods (except for the case of small number of antennas for the method in [32] which are the same). This makes the proposed method a serious candidate to be exploited in real-time applications.

The remainder of this paper is organized as follows. In section II, we introduce the signal model. Then, the proposed method is described in section III. Simulation results are discussed in section IV, followed by the conclusions in Section V.

Notation: In this paper bold upper case letters $\mathbf{X}$ and lower case letters $\mathbf{x}$, denote matrices and vectors, respectively. Conjugate transposition of a matrix is denoted by $(.)^H$ and $\mathrm{E}\{.\}$ denotes the statistical expectation. Also, the *(m,n)*th element of a matrix is denoted by $\mathbf{X}_{mn}$. We use $\mathbf{X} \succeq 0$ to show a matrix $\mathbf{X}$ is positive semi-definite, and, $\odot$ represents Hadamard product.

II. Signal model

In this section, the signal model for a ULA in a collocated MIMO radar system is described. In this model, we consider M possible antenna positions uniformly as the array grid points along with a straight line with the grid spacing of d, and N transmit antennas selected out of M grid points. We define the binary antenna position vector $\mathbf{p}$ as follows

$$\mathbf{p} = [p_1, p_2, ..., p_M]^T, \tag{1}$$

where each p_i is considered to take binary values ($p_i \in \{0, 1\}$). p_i is one, when the corresponding grid point is selected for antenna placement; otherwise p_i is zero. As the number of active antennas is N, the number of non-zero elements of vector $\mathbf{p}$ is N. An example of such a configuration with $M = 10$, $N = 6$ and $\mathbf{p} = [1, 1, 0, 1, 0, 1, 0, 0, 1, 1]$ has been presented in Fig. 1.

We define the steering vector of the array as follows

$$\mathbf{a}(\theta) = [1, e^{-j2\pi\frac{d}{\lambda}\sin(\theta)}, ..., e^{-j2\pi\frac{(M-1)d}{\lambda}\sin(\theta)}]^T, \tag{2}$$

where λ is the carrier signal wavelength, and the baseband transmitted signal vector at each time index k can be written as

$$\mathbf{x}(k) = [x_1(k), x_2(k), ..., x_M(k)]^T. \tag{3}$$

Under the conditions that the transmitted signals are narrow-band and the propagation is non-dispersive the, the received signal by a target located at angle θ can be given by

$$s(k;\theta) = (\mathbf{p} \odot \mathbf{a}(\theta))^H \mathbf{x}(k). \tag{4}$$

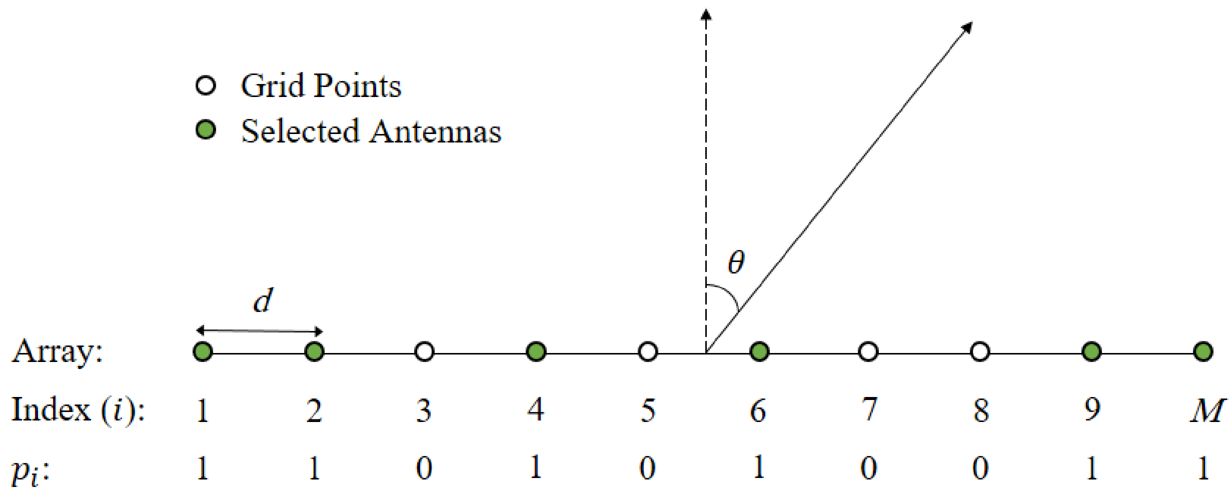

Fig. 1. An example of antennas configuration on a ULA array with $\mathbf{p} = [1, 1, 0, 1, 0, 1, 0, 0, 1, 1]$.

Therefore, the transmit power at location θ can be found as

$$\begin{aligned} P(\theta) &= \mathrm{E}\{(\mathbf{p} \odot \mathbf{a}(\theta))^H \mathbf{x}(k)\mathbf{x}(k)^H (\mathbf{p} \odot \mathbf{a}(\theta))\} \\ &= (\mathbf{p} \odot \mathbf{a}(\theta))^H \mathbf{R} (\mathbf{p} \odot \mathbf{a}(\theta)). \end{aligned} \tag{5}$$

where $\mathbf{R} = \mathrm{E}\{\mathbf{x}(k)\mathbf{x}^H(k)\}$ is the covariance matrix of the transmitted waveforms.

In the literature, the problem of finding R while the transmit array beam-pattern $P(\theta)$ matches or is close enough to a desired beam-pattern $P_d(\theta)$ over $\{\theta_l\}_{l=1}^{L}$, and also the cross-correlation of the signals in directions θ and $\hat{\theta}$ ($\theta \neq \hat{\theta}$) is minimized over $\{\tilde{\theta}_p\}_{p=1}^{\tilde{L}}$, is called "beam-pattern matching". In this definition, L is the total number of scanning directions, and $\tilde{L}$ is the total number of directions in which the cross-correlation term is considered to be minimized. In this paper, we aim to achieve the best configuration of antennas (finding the position vector $\mathbf{p}$) and then convey the beam-pattern matching process (finding covariance matrix $\mathbf{R}$).

Therefore, our objective function is [18]

$$\begin{aligned} &J(\mathbf{p}, \alpha, \mathbf{R}) \\ &= \frac{1}{L}\sum_{l=1}^{L} w_l \left|(\mathbf{p} \odot \mathbf{a}(\theta_l))^H \mathbf{R}(\mathbf{p} \odot \mathbf{a}(\theta_l)) - \alpha P_d(\theta_l)\right|^2 \\ &+ \frac{2w_c}{\tilde{L}^2 - \tilde{L}} \sum_{p=1}^{\tilde{L}-1} \sum_{q=p+1}^{\tilde{L}} \left|(\mathbf{p} \odot \mathbf{a}(\tilde{\theta}_p))^H \mathbf{R}(\mathbf{p} \odot \mathbf{a}(\tilde{\theta}_q))\right|^2, \end{aligned} \tag{6}$$

where α is the scaling factor, $w_l \geq 0$ for $l = 1, 2, ..., L$, is the weight for the lth space grid point and $w_c \geq 0$ is the weight for the cross-correlation term. Then, for a given $\mathbf{p}$ the problem to be solved is in the form of

$$\begin{aligned} \min_{\alpha, \mathbf{R}} \quad & J\left(\mathbf{p}, \alpha, \mathbf{R}\right) \\ s.t. \quad & \mathbf{R}_{mm} = \frac{P_{total}}{N}, \quad m = 1, ..., M, \\ & \mathbf{R} \succeq 0. \end{aligned} \tag{7}$$

In problem (7) the first constraint states that the diagonal elements of covariance matrix must be equal, which is a practical constraint for efficiency of power amplifiers in MIMO radars. P_{total} is the total transmit power of the array. And the second constraint indicates that the designed covariance matrix must be positive semi-definite.

III. Proposed Method

In this section, we devise our method to reconfigure a preset antenna array (finding $\mathbf{p}$) and then designing $\mathbf{R}$ with the reconfigured array. In a specific beam-pattern matching scenario with a determined $P_d(\theta)$ and given a preset array with M possible antenna positions, we aim to select N transmit antennas out of M grid points by a multi-step method.

Starting from the full array, in each step we deactivate one of the antennas until a predefined number of antennas (N antennas) remain. The approach we take in each step is based on the fact that the diagonal elements of covariance matrix are the transmitting power of antennas in general beam-pattern matching process. Hereafter, the phrase "general beam-pattern matching" denotes the beam-pattern matching without the uniform elemental power constraint and the scale parameter α, i.e.,

$$\begin{aligned} \min_{\mathbf{R}} \quad & \frac{1}{L}\sum_{l=1}^{L} w_l \left| (\mathbf{p} \odot \mathbf{a}(\theta_l))^H \mathbf{R} (\mathbf{p} \odot \mathbf{a}(\theta_l)) - P_d(\theta_l) \right|^2 \\ & + \frac{2w_c}{\tilde{L}^2 - \tilde{L}} \sum_{p=1}^{\tilde{L}-1} \sum_{q=p+1}^{\tilde{L}} \left| (\mathbf{p} \odot \mathbf{a}(\tilde{\theta}_p))^H \mathbf{R} (\mathbf{p} \odot \mathbf{a}(\tilde{\theta}_q)) \right|^2, \\ s.t. \quad & \mathbf{R} \succeq 0. \end{aligned} \tag{8}$$

Exploiting the relation between covariance matrix diagonal elements and transmitting power of antennas, we conclude that the minimum covariance matrix diagonal element value corresponds to the antenna with the least impact in the beam-pattern matching process. Based on that, the antenna corresponding to the minimum value of covariance matrix diagonal elements is then selected to be deactivated in each step. In the ith step of configuring array with $\mathbf{p}_{i-1}$, calculated from the $(i-1)$th step which contains $(i-1)$ zeros, solving the optimization problem (8) provides $\mathbf{R}_i$ with diagonal elements given in a vector as

$$\mathbf{e}_i = \mathrm{diag}(\mathbf{R}_i). \tag{9}$$

Then, we find the index of $\mathbf{e}_i$ vector in which the $\mathbf{e}_i$ takes the minimum value, i.e.,

$$\nu_i = \mathrm{find}(\mathbf{e}_\mathrm{i} = \min(\mathbf{e}_\mathrm{i})), \tag{10}$$

where $\mathrm{find}(.)$ denotes the function that finds the index in which the function argument is true.

Then, the minimum value index in vector $\mathbf{e}_i$ (i.e., ν_i) corresponds to the antenna position considered to be zero in the position vector $\mathbf{p}_i$. Therefore, we change the value of this position in $\mathbf{p}_i$ from one to zero as follows

$$\mathbf{p}_i(\nu_i) = 0, \tag{11}$$

meaning that the corresponding antenna is turned off. After $M - N$ steps, the method results in a reconfigured array with a position vector $\mathbf{p}$ containing $M - N$ zeros and N ones.

- *Remark 1:* If the minimum value of $\mathbf{e}_i$ is not unique, the choice of the minimum position, between these candidates, is arbitrary. We mostly face this situation in the first step in which the array for a target point in far-field (which is our case) is symmetric.
- *Remark 2:* The values of $\mathbf{e}_i$ in positions whose values have been selected as the minimum values in previous steps (i.e., steps $< i$) are large enough not to be selected in the current step (i.e., step i).

Then, the beam-pattern matching problem of (7) can be solved to calculate the covariance matrix $\mathbf{R}$ corresponding to the reconfigured array. It should be noted that the optimization problems in (7) and (8) are convex [33] and can be carried out using MATLAB-based optimization toolbox CVX [34]. A summarization of the proposed algorithm is provided in Algorithm 1. To more clarify the proposed algorithm, a flowchart of the proposed algorithm is illustrated in Fig. 2.

On the other hand, our proposed method complexity totally consists of $M - N + 1$ SQP optimizations and $M - N$ local searches as follows: a) $M - N$ general beam-pattern matching problems as in (8) for $M - N$ array reconfiguration steps; b) $M - N$ local searches to find the minimums of the diagonals of the covariance matrices; and c) one beam-pattern matching problem as in (7) for designing covariance matrix corresponding to the reconfigured array. Hence,

Algorithm 1 Proposed algorithm to reconfigure a preset array for beam-pattern matching.

Input: M, N, $P_d(\theta)$ and $\mathbf{p}_0 = \mathbf{1}_{M\times 1}$.
Output: $\mathbf{p}$ and $\mathbf{R}$
for $i = 1$ to $i = M - N$ **do**
    Calculate $\mathbf{R}_i$ via (8).
    Calculate $\mathbf{e}_i$ via (9).
    Find ν_i via (10).
    Set $\mathbf{p}_i(\nu_i) = 0$
end for
Calculate $\mathbf{R}$ via (7).

ignoring the complexity of local searches, the proposed algorithm complexity is $M - N + 1$ times of SQP method [18] complexity which is $O(\log(\frac{1}{\eta})M^{3.5})$ for a prefixed accuracy of η [21].

IV. SIMULATION RESULTS

In this section, to demonstrate the performance of our proposed method, several numerical simulations are presented. In all simulations we assume a ULA array with inter-element spacing of $d = \frac{\lambda}{2}$ and the range of angle to be $[-90, 90]$ with the resolution of 1° which gives $L = 181$. The weights are considered as $w_l = 1$ for $l = 1, 2, ..., L$, and the $P_{total} = 1$.

In the first simulation, we consider a beam-pattern matching problem. The desired beam-pattern $P_d^1(\theta)$ is considered as follows

$$P_d^1(\theta) = \begin{cases} 1 & \theta \in \{[-45, -15] \cup [15, 45]\} \\ 0 & \text{otherwise}, \end{cases} \tag{12}$$

which can be considered as two mainlobes centered at $\theta = -30^\circ$ and $\theta = 30^\circ$, and each beamwidth of 31°. Here, we consider two cases of $w_c = 1$ and $w_c = 0$, meaning considering and not considering the cross-correlation term in beam-pattern matching design, respectively. For this desired beam-pattern, we have two mainlobes and therefore $\tilde{L} = 2$. The number of possible antenna locations and the number of active antennas are assumed to be $M = 15$ and $N = 10$, respectively. Fig. 3 depicts the results of the first simulation for the proposed method (Proposed-WC stands for proposed method without cross-correlation term) as well as the methods of ADMM in [30], DP in [31] (DP-WC stands for DP method without cross-correlation term) and SCA in [32]. As can be seen, our proposed method has lower sidelobe levels and the same mainlobe ripple compared to the other methods when $w_c = 0$. In Fig. 4, the synthesized beam-pattern via the proposed method is compared to ULA-M (SQP method in [18] with $M = N = 15$) and ULA-N (SQP method in [18] with $M = N = 10$) in which no reconfiguration is needed. This figure shows that how well the proposed method matches ULA-M in the mainlobe region and in the most part of the sidelobe region. This illustrates the fact that the proposed reconfiguration method, with lower number of antennas, is able to almost achieve the performance of a ULA array in beam-pattern matching design.

In order to compare the beam-pattern synthesis and cross-correlation reduction capability of the proposed method and the DP method [31] while the cross-correlation term is taken into account (i.e., $w_c = 1$), Fig. 5 and Fig. 6 are presented. Fig. 5 shows that although the mainlobe ripple of the synthesized beam-pattern via the proposed method (Proposed-CC stands for proposed method with cross-correlation term) is slightly larger than that of the DP method (DP-CC stands for DP method with cross-correlation term), its sidelobe levels are considerably lower. In Fig. 6, the cross-correlation coefficients between the transmitted signals toward the center of main beams of $P_d^1(\theta)$ as a function of w_c for the proposed and DP methods are plotted. The cross-correlation coefficients for the proposed method, as can be seen, are smaller or almost equal to coefficients for DP method and also are approximately zero for w_c values greater than 0.1.

Note that except for the proposed and DP methods, other ones do not consider the cross-correlation term. Furthermore, all the beam-patterns in this simulation are normalized by α. Table I provides a quantitative comparison of the mentioned methods' performance in terms of the MSE value between the synthesized and desired beam-patterns with the same definition as the cost function in (6). It is obvious that the performance of our proposed method excels the other methods when $w_c = 0$, and is better than that of DP method when $w_c = 1$. Fig. 7 represents the reconfigured arrays corresponding to beam-patterns in the first simulation (ULA-M and ULA-N are drawn for comparison purposes). In this figure, the black circles are possible antenna locations, and the colored ones are selected antennas to be active in the beam-patten synthesis.

In order to gain a better insight on how our algorithm works, the diagonal elements of covariance matrix in each step of reconfiguring the preset array (ULA array with $M = 15$) in simulation one are presented in Table I. The first column of the table presents the element numbers in the main diagonal of covariance matrix and the last column indicates on the selected antennas to remain active in beam-pattern matching. The minimum values in each step have

TABLE I

MSE VALUES FOR DIFFERENT METHODS IN THE FIRST SIMULATION.

Method	MSE
ULA-N	0.0758
DP-CC	0.0633
DP-WC	0.0631
Proposed-CC	0.0615
SCA	0.0609
ADMM	0.0584
Proposed-WC	0.0543
ULA-M	0.0525

been boldfaced. As can be seen, the values of the diagonal of the covariance matrix in positions which have been selected in previous steps are significantly large, so that no repetitive position is selected as a minimum in the next steps.

For the second simulation, we consider synthesizing an asymmetric beam-pattern. The desired beam-pattern is defined as follows

$$P_d^2(\theta) = \begin{cases} 1 \quad \theta \in \{[-60,-50] \cup [-40,-30] \cup [-10,0] \\ \qquad\qquad \cup [5,15] \cup [30,40]\} \\ 0 \qquad\qquad\qquad \text{otherwise}, \end{cases} \tag{13}$$

which consists of five mainlobes centered at $\theta = -55°$, $\theta = -35°$, $\theta = -5°$, $\theta = 10°$ and $\theta = 35°$, and each beamwidth of $11°$. Therefore, for this simulation, we again consider both $w_c = 0$ and $w_c = 1$ cases in which $\tilde{L} = 5$. Here, we assume possible antenna locations and the number of active antennas to be $M = 21$ and $N = 14$, respectively. Similar to the first simulation, all of the beam-patterns are normalized by α. Fig. 8 demonstrates the results of this simulation for $w_c = 0$. As reflected in the figure, the proposed method's beam-pattern has lower ripple in the mainlobe region and comparable sidelobe levels relative to the other methods. In Fig. 9, it is shown that similar to Fig. 4, our method synthesizes a beam-pattern which matches to the desired beam-pattern better than the ULA-N and close to ULA-M. Furthermore, the proposed method, for $w_c = 1$ approximates the desired beam-pattern similar to DP method overall. Similar to the

TABLE II

DIAGONAL ELEMENTS OF COVARIANCE MATRIX IN EACH STEP OF RECONFIGURING THE PRESET ARRAY IN THE FIRST SIMULATION

Element number	Step 1	Step 2	Step 3	Step 4	Step 5	Selected Antennas
1	0.0178	0.0173	0.0135	0.0239	**0.0067**	
2	0.0167	0.0155	0.0090	0.0096	0.0098	✓
3	0.0390	0.0443	0.0475	0.0510	0.1069	✓
4	0.03590	0.0519	0.0499	0.0582	0.0355	✓
5	0.0382	0.0411	0.0466	0.0480	0.1673	✓
6	0.0341	0.0335	0.0532	0.0493	0.0119	✓
7	0.0384	0.0347	0.0431	0.0492	0.0441	✓
8	0.0372	0.0372	0.0266	**0.0086**	42.8600	
9	0.0384	0.0367	0.0544	0.0510	0.0215	✓
10	0.0342	0.0405	**0.0042**	52.1271	42.8600	
11	0.0382	0.0360	0.04250	0.0525	0.0383	✓
12	0.0359	**0.0104**	44.7019	52.1271	42.8600	
13	0.0390	0.0496	0.0518	0.0523	0.0107	✓
14	**0.0167**	47.7062	44.7019	52.1271	42.8600	
15	0.0178	0.0289	0.0352	0.0240	0.0251	✓

Table I, Table III compares the mentioned methods' performance in terms of the MSE value. As can be seen, the proposed method outperforms the other methods in both cases of considering and ignoring the cross-correlation term. Fig. 11 shows the reconfigured arrays corresponding to beam-patterns in the second simulation. For the next simulations and figures, we only consider $w_c = 0$.

Fig. 12 and Fig. 13 investigates the effect of array size on the proposed method performance to synthesize beam-patterns to match to $P_d^1(\theta)$ and $P_d^2(\theta)$, respectively. In these figures, the horizontal axis is the number of grid points (number of possible antenna locations), and the vertical axis is the MSE value between the designed beam-pattern and the desired beam-pattern

TABLE III

MSE VALUES FOR DIFFERENT METHODS IN THE SECOND SIMULATION.

Method	MSE
ULA-N	0.1491
ADMM	0.1372
DP-CC	0.1313
DP-WC	0.1284
SCA	0.1212
Proposed-CC	0.1172
Proposed-WC	0.1100
ULA-M	0.1019

(it is clear that this value equals to the objective value of (7)). For each M, the number of active antennas is considered to be $N = \frac{2}{3}M$. From the two figures, one can verify that the MSE value of our proposed method monotonically decreases with increasing the array size. Also, it achieves lower MSE value for all (M,N) pairs compared to the other methods in various scenarios (e.g., the two mentioned scenarios of symmetric and asymmetric beam-patterns with wide and relatively thin mainlobes). Furthermore, as we expected, the MSE of all methods (except the MSE of SCA method for $P_d^1(\theta)$) lay between the MSE of ULA-N and ULA-M; this means that ADMM, DP, and SCA methods, as well as our proposed method, improve the performance of beam-pattern matching using more degrees of freedom provided by reconfiguring the preset array. Nevertheless, for some specific scenarios (as shown in Fig. 12), the SCA method not only fails to make MSE improvement compared to ULA-N utilizing array reconfiguration, but also worsens it.

In Fig. 14, we explore the effect of increasing degrees of freedom, i.e., increasing the number of grid points M with a fixed N. Here, N is considered to be 15 and M takes the values of 15, 18, 21, 24 and 27. Also, the desired beam-pattern is considered as follows

$$P_d^3(\theta) = \begin{cases} 1 & \theta \in \{[-37, -33] \cup [23, 27]\} \\ 0 & \text{otherwise}, \end{cases} \tag{14}$$

which can be considered as two mainlobes centered at $\theta = -35^\circ$ and $\theta = 25^\circ$, and each beamwidth of $5°$. Fig. 14 illustrates that as the number of possible antenna locations increases, our method shows a better performance in matching the beam-pattern to the desired beam-pattern as well as suppressing the sidelobe levels. The reason is, the larger the M, the higher the degrees of freedom we have to exploit for beam-pattern matching. The reconfigured arrays corresponding to each M are shown in Fig. 15.

Finally, to test the computational complexity of the proposed algorithm, we calculate the run time of our method for various array sizes, i.e., for the M equals to values shown on the horizontal axis of Fig. 16 and $N = \frac{2}{3}M$. For this simulation, the desired beam-pattern is $P_d^1(\theta)$. Besides, our personal computer that the simulations are conducted on has the configurations of 64 bit Intel i5-4200M CPU and 6GB RAM. Fig. 16 shows the calculated results for our proposed method, the ADMM method in [30], DP method in [31] and SCA method in [32]. It's interesting to note that our proposed algorithm has a significantly lower computational complexity compared to the ADMM method and the DP method. Also, our computational performance is similar to that of the SCA method for the case of small number of antennas and outperforms it for large number of antennas. For instance, the run time for the number of grid points $M = 33$, and the number of selected antennas $N = 22$ is 34, 48, 75 and around 12000 seconds for the proposed method, SCA method, DP method, and ADMM method, respectively.

It should be noted that in Fig. 12, 13 and 16, due to the enormous amount of time that the ADMM method takes to respond, the results of this method for the number of grid points M greater than 33 are ignored.

V. Conclusion

A novel approach to reconfigure a preset ULA has been proposed for the beam-pattern matching problem. The aim of this approach is to achieve the beam-pattern matching performance of the preset array as close as possible, using a smaller number of antennas. From another point of view, we are going to improve the beam-pattern matching performance of a ULA array, using the same number of active antennas which are not generally on a uniform array. This approach consists of a multi-step method in which the relation between the diagonal elements of covariance matrix and transmitted power of antennas is exploited to deactivate one of the antennas in each step, and in the end, come up with a reconfigured array. Then, using this reconfigured array, the beam-pattern matching is carried out. Thus, by utilizing the proposed approach, a position vector denoting the reconfigured array, and a waveform covariance matrix are designed. The proposed method provides a more accurate beam-pattern matching compared to the current methods in any scenario, with any array sizes, and in both cases of considering and neglecting the cross-correlation term. Also, in the case that this term is taken into account, the proposed method minimizes the cross-correlation coefficients effectively. In addition, the proposed method is a robust algorithm that never fails, and in the worst-case its performance is similar to that of the ULA with the same number of active antennas. Furthermore, it is a straightforward algorithm with stable and predictable computational complexity that is a benefit over current methods. Simulation results approve that our proposed approach outperforms the existing methods in terms of accuracy and consumed time, and hence, is an appropriate candidate for real-time applications such as automated driving systems.

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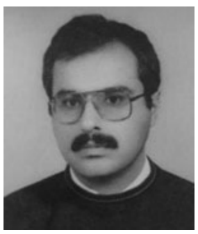

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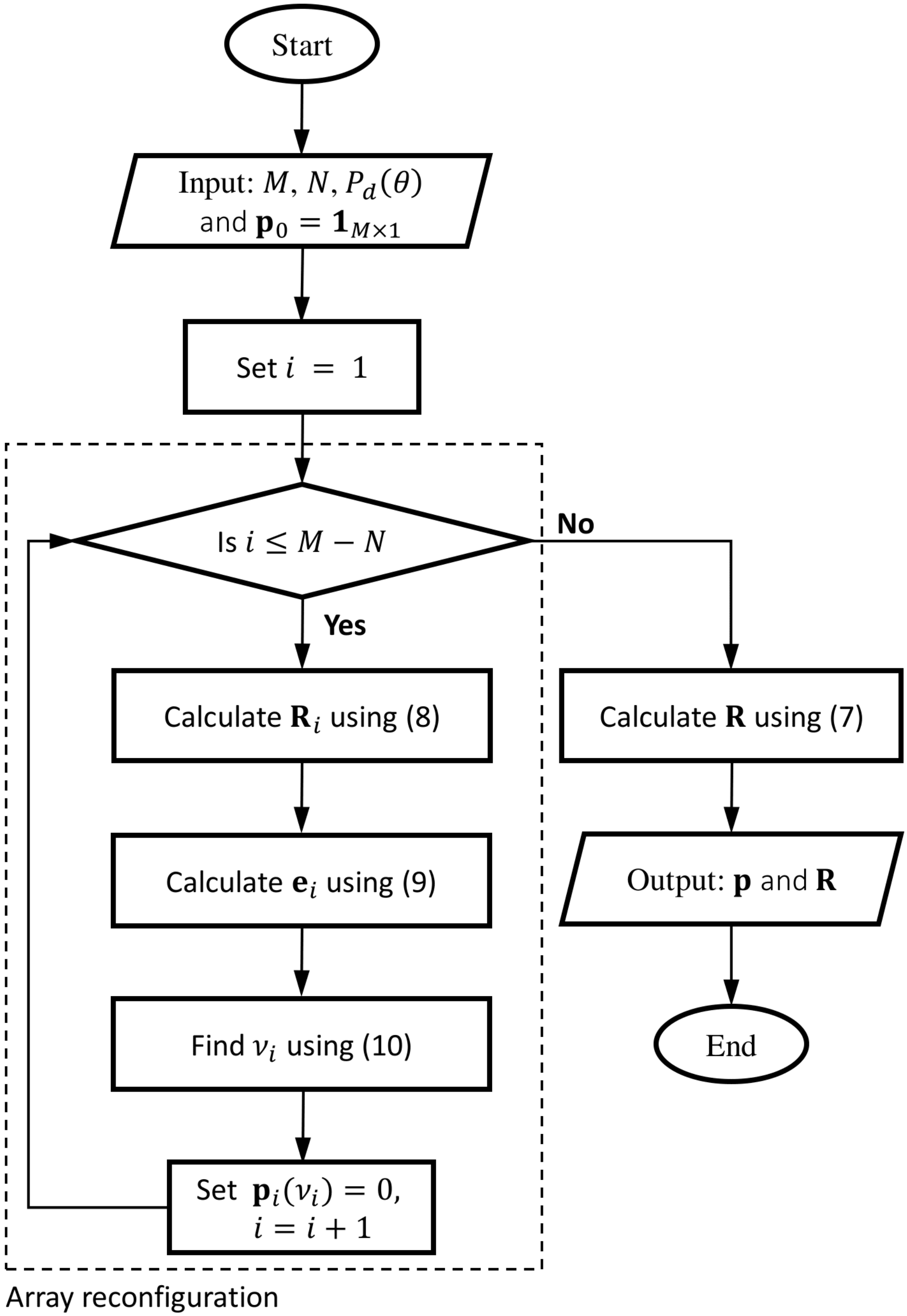

Fig. 2. Flowchart of the proposed algorithm.

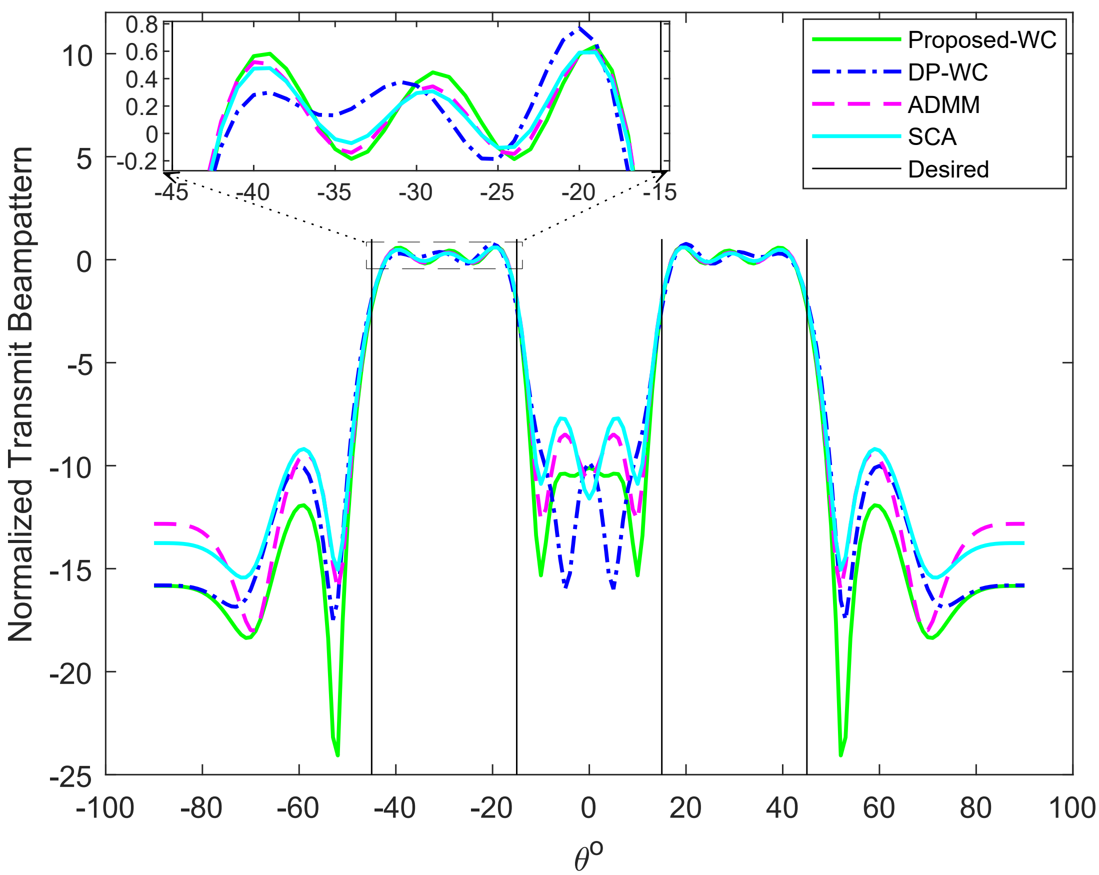

Fig. 3. Synthesized beam-patterns with different methods for desired beam-pattern $P_d^1(\theta)$ with $M = 15$ and $N = 10$, when $w_c = 0$.

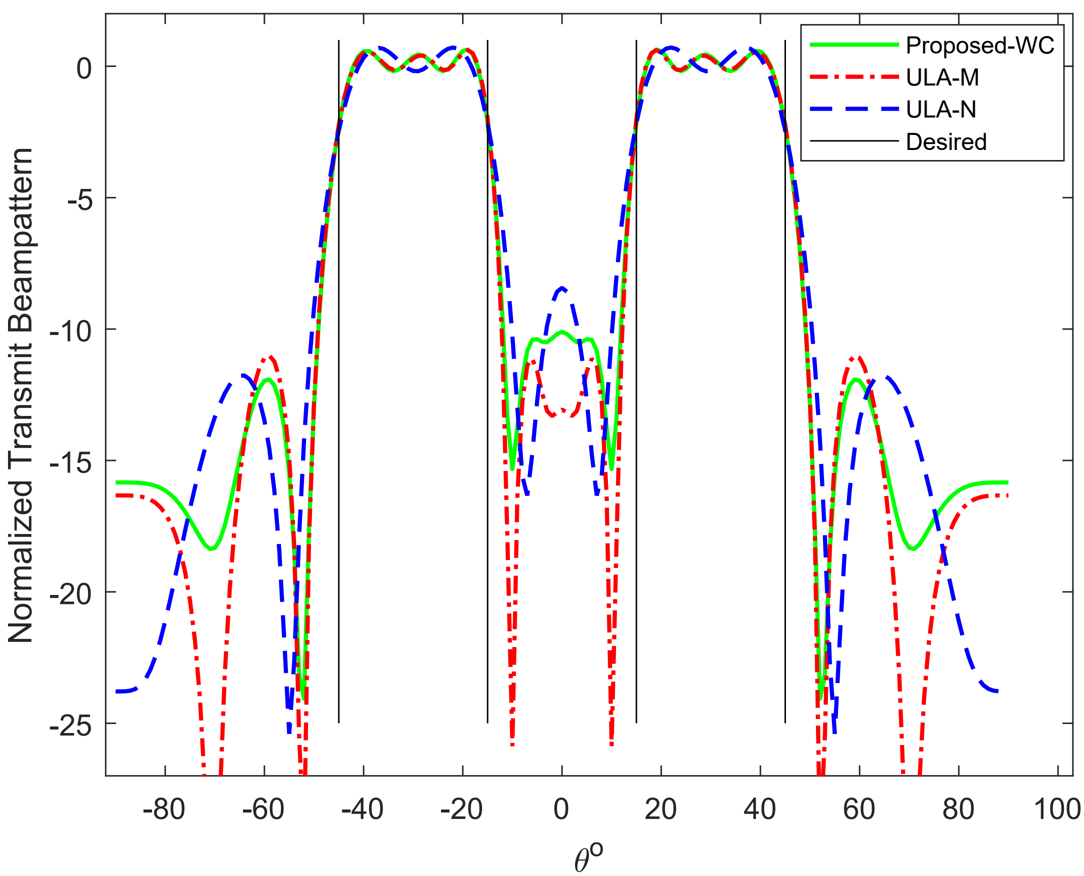

Fig. 4. Comparison of the proposed method with ULA-M and ULA-N for desired beam-pattern $P_d^1(\theta)$ with $M = 15$ and $N = 10$.

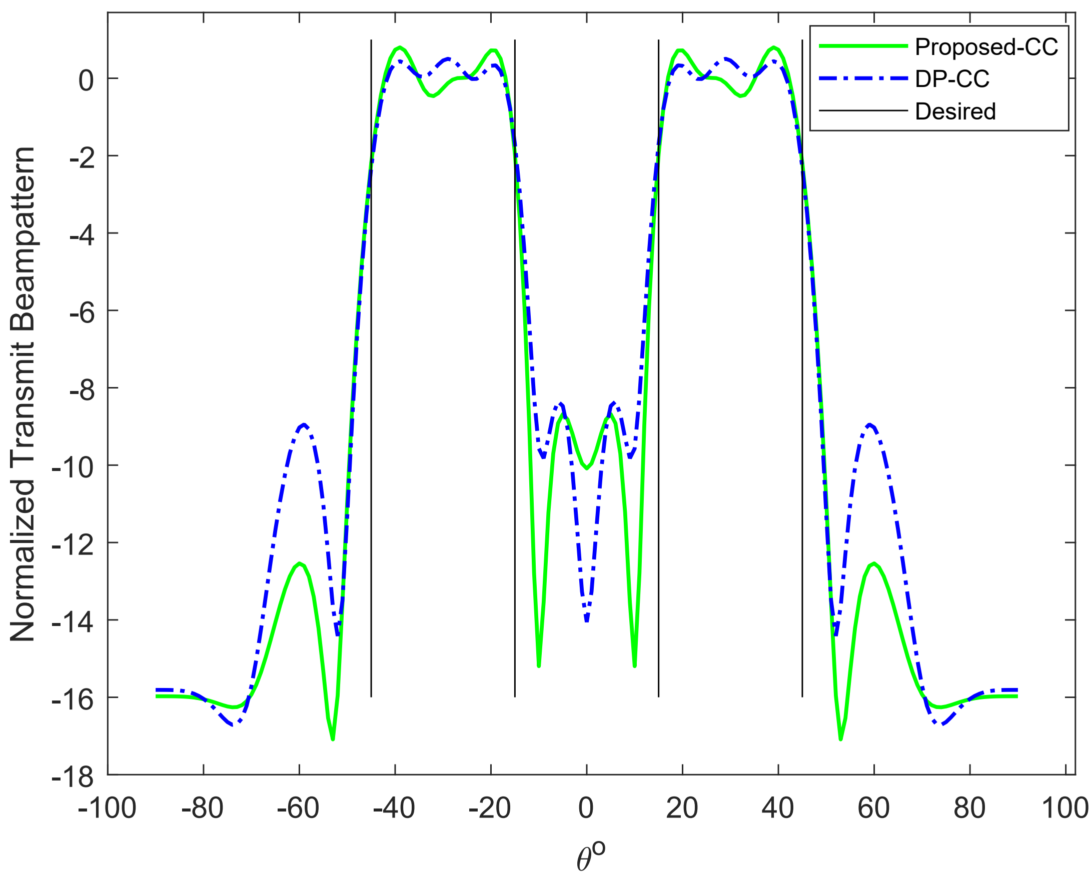

Fig. 5. Comparison of the proposed method with DP method for desired beam-pattern $P_d^1(\theta)$ with $M = 15$ and $N = 10$, when $w_c = 1$.

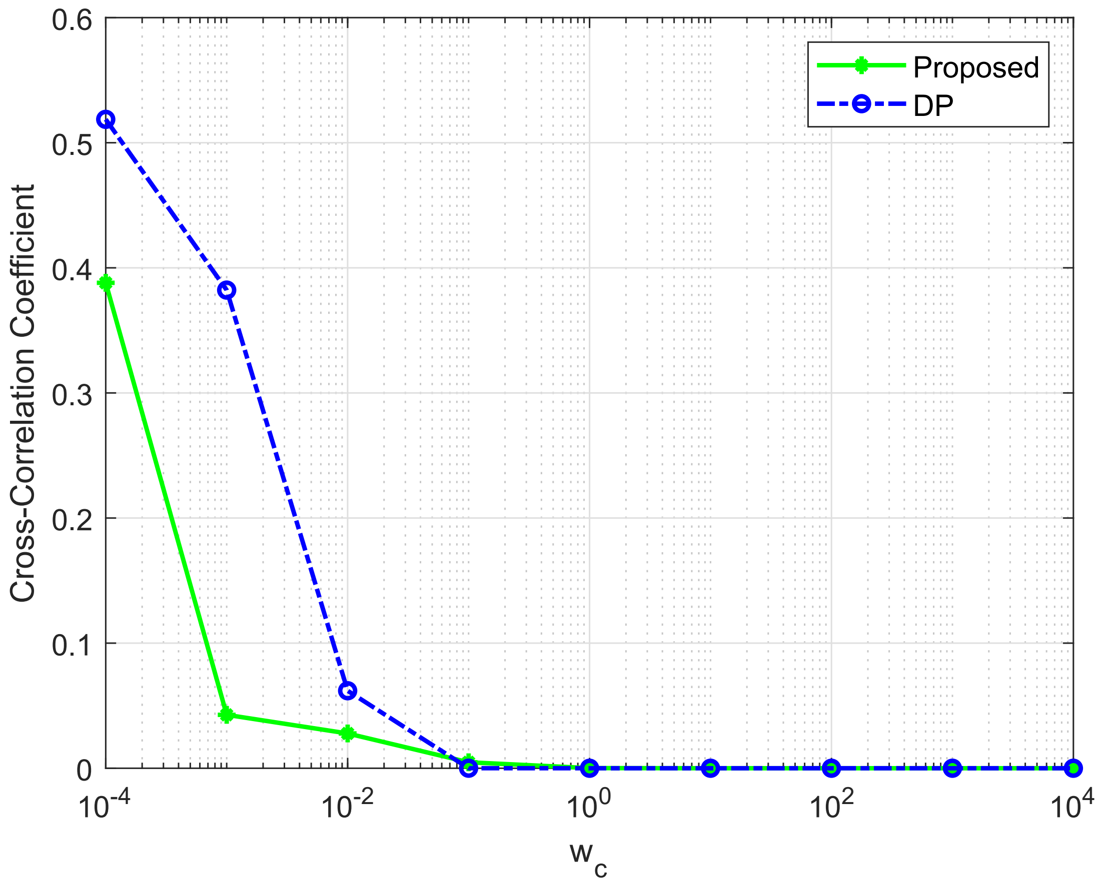

Fig. 6. Cross-correlation coefficients as a function of w_c for proposed and DP methods

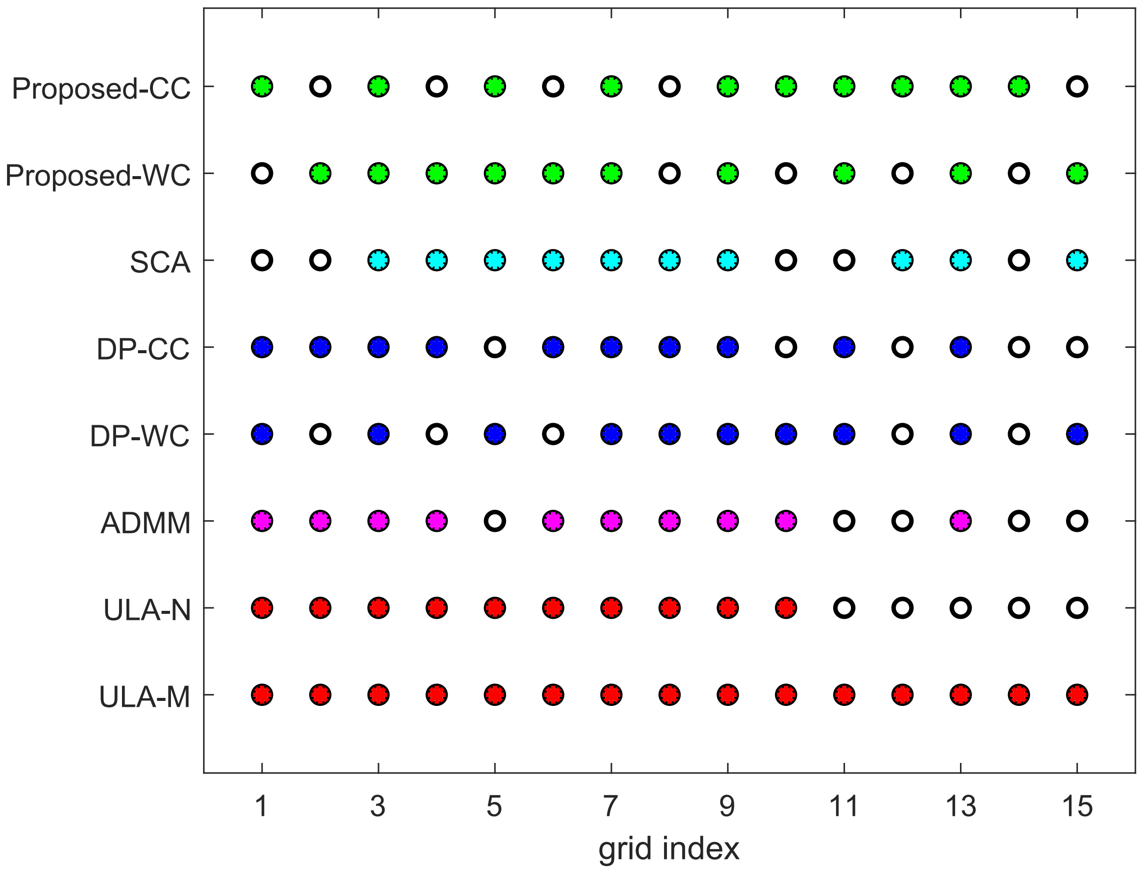

Fig. 7. Reconfigured arrays corresponding to beam-patterns in the first simulation.

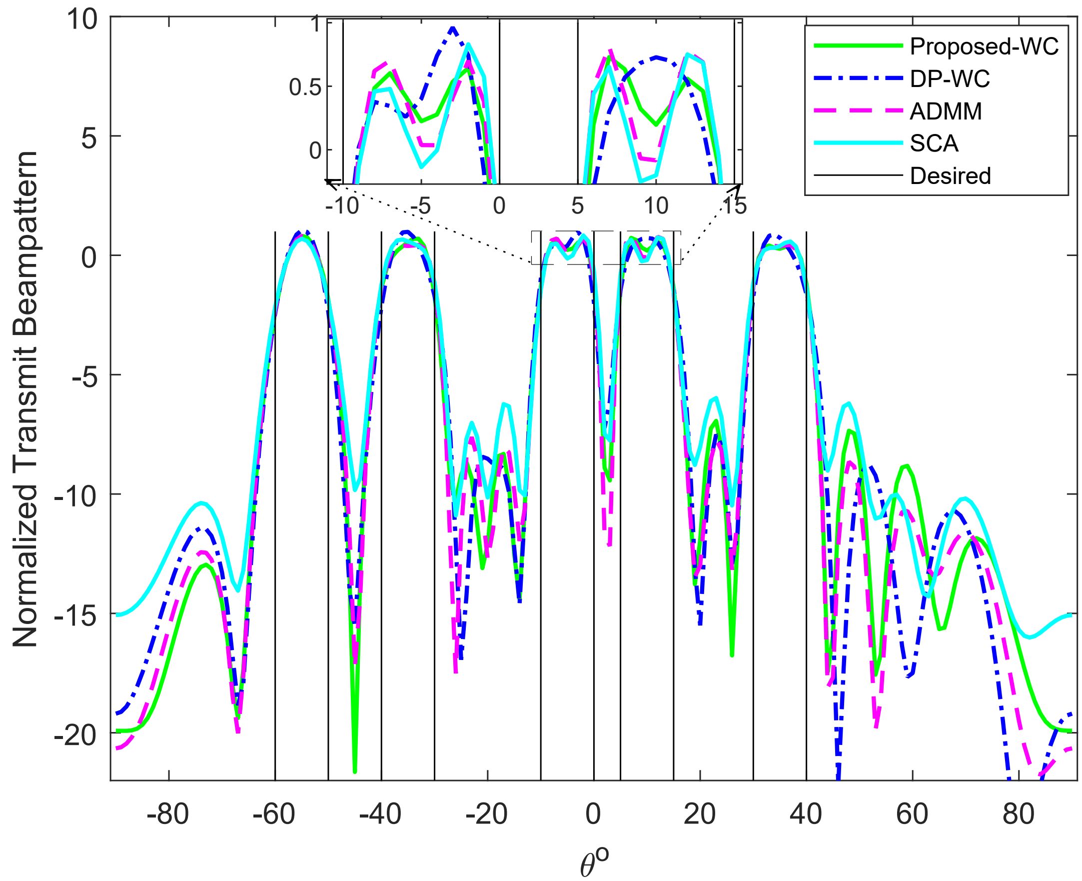

Fig. 8. Synthesized beam-patterns with different methods for desired beam-pattern $P_d^2(\theta)$ with $M = 21$ and $N = 14$, when $w_c = 0$.

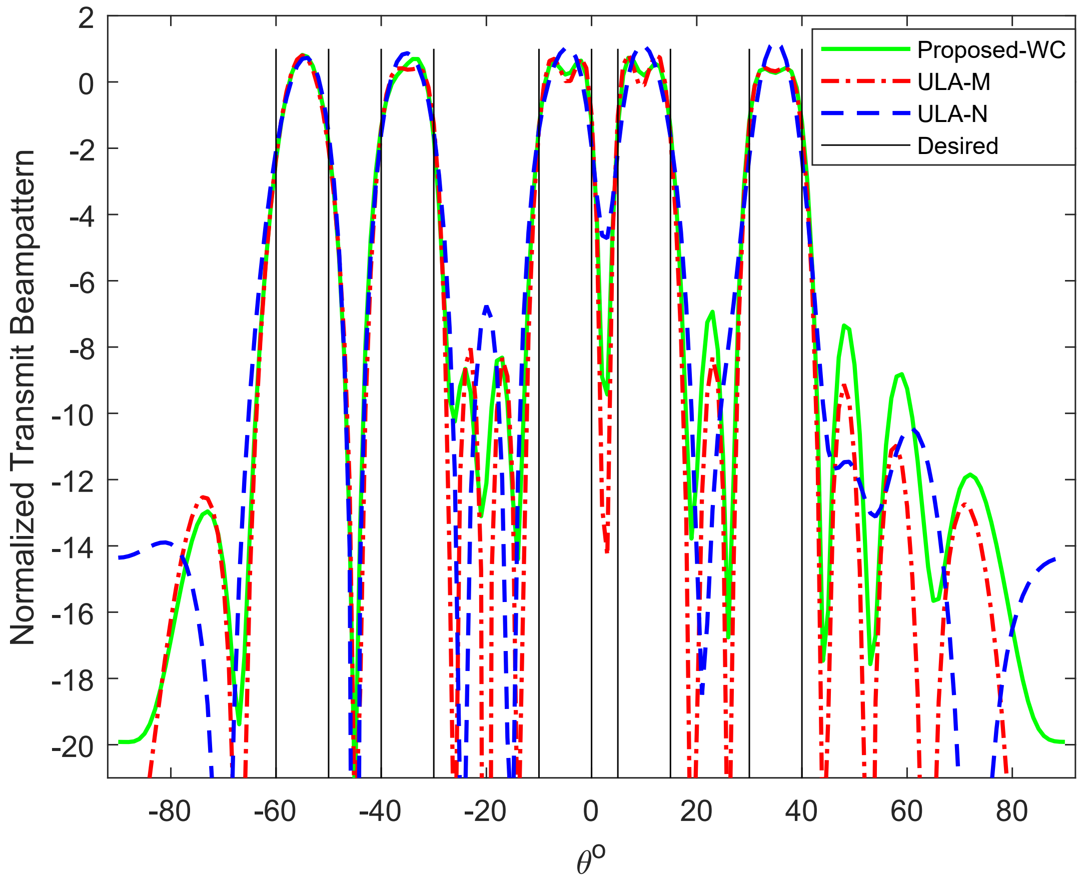

Fig. 9. Comparison of the proposed method with ULA-M and ULA-N for desired beam-pattern $P_d^2(\theta)$ with $M = 21$ and $N = 14$.

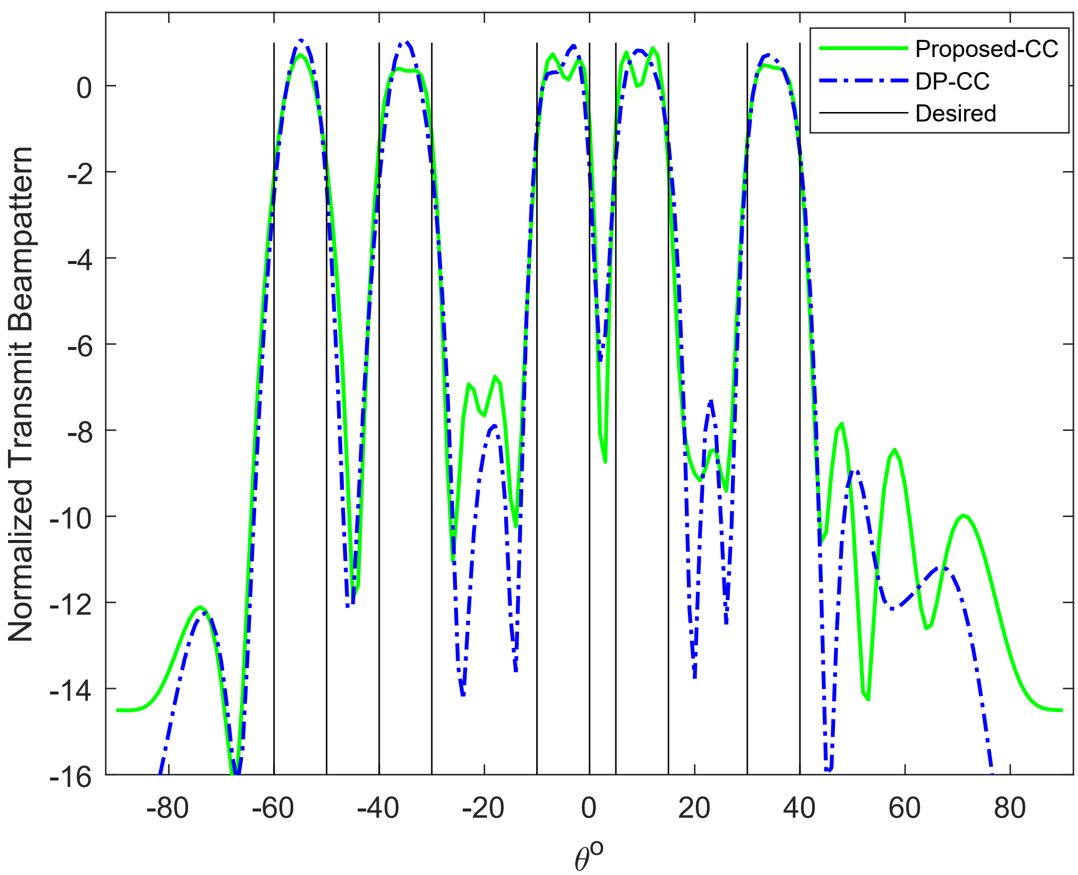

Fig. 10. Comparison of the proposed method with DP method for desired beam-pattern $P_d^2(\theta)$ with $M = 21$ and $N = 14$, when $w_c = 1$.

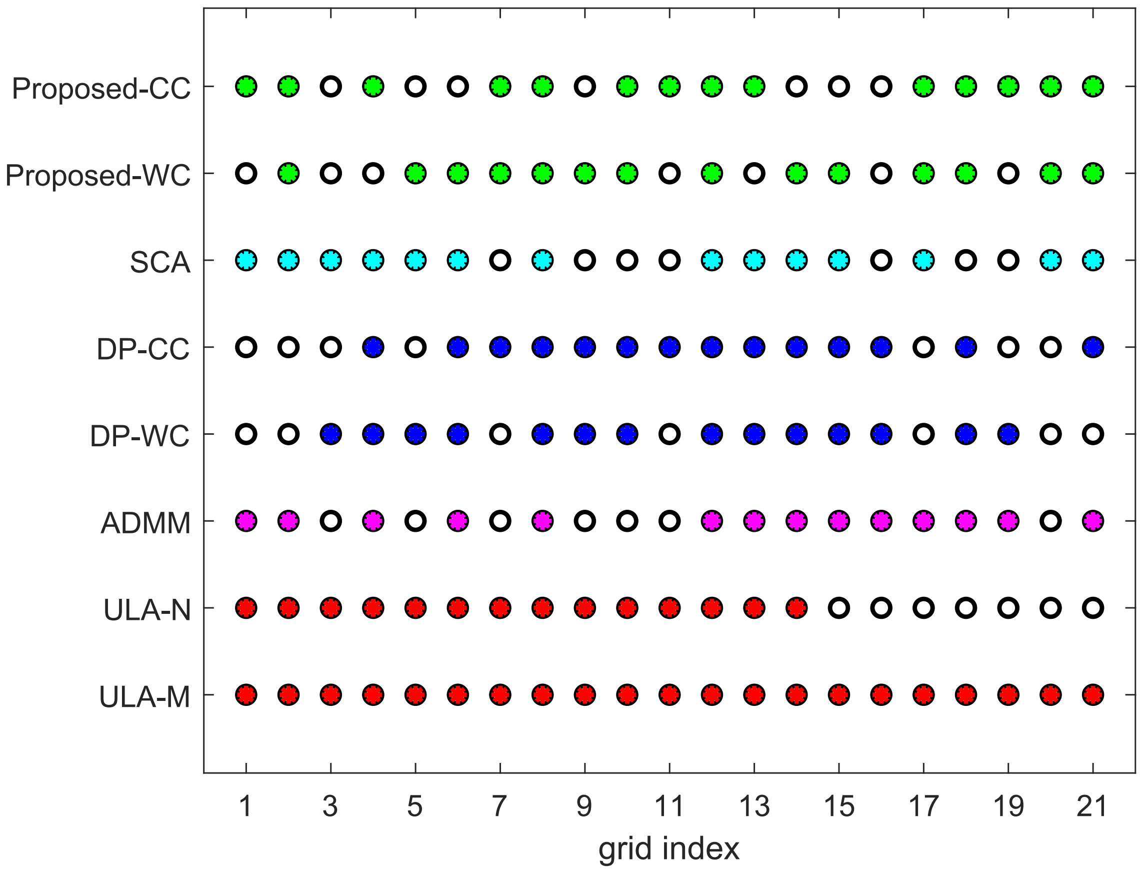

Fig. 11. Reconfigured arrays corresponding to beam-patterns in the second simulation.

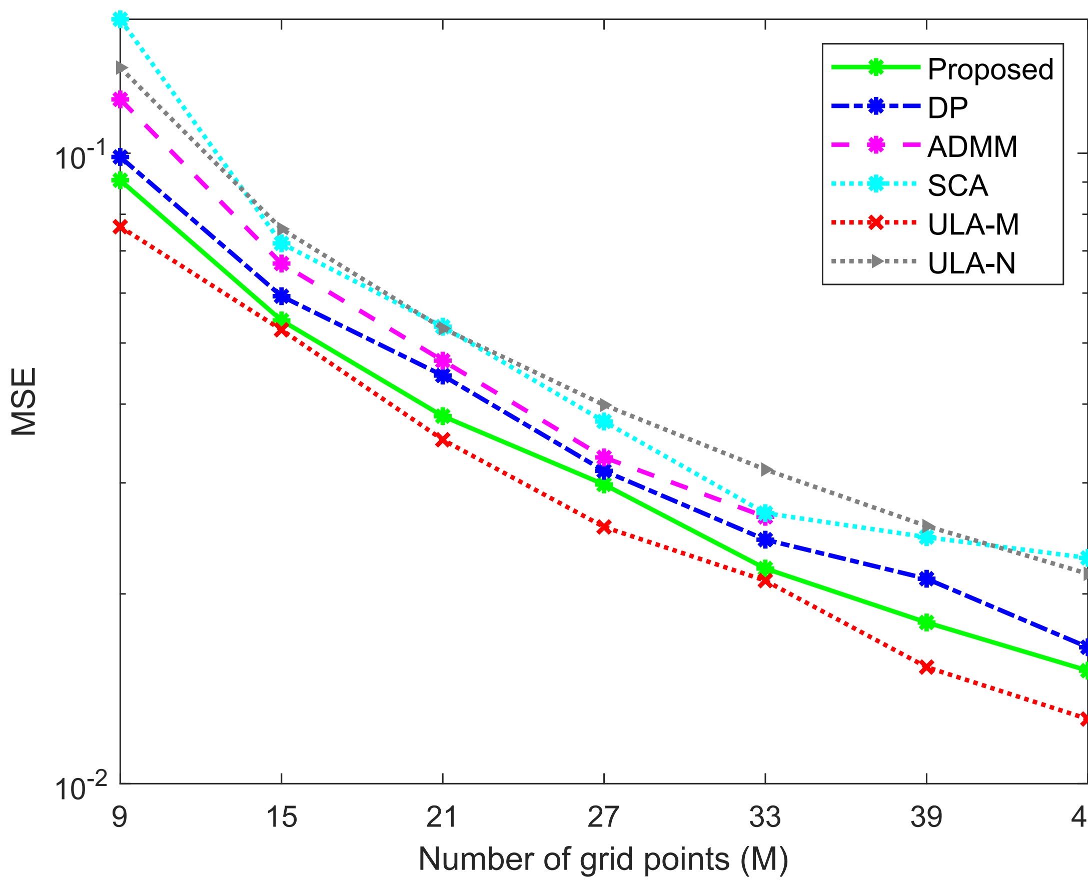

Fig. 12. MSE values for different methods of synthesizing the desired beam-pattern $P_d^1(\theta)$ for different values of M and assuming that $N = \frac{2}{3}M$.

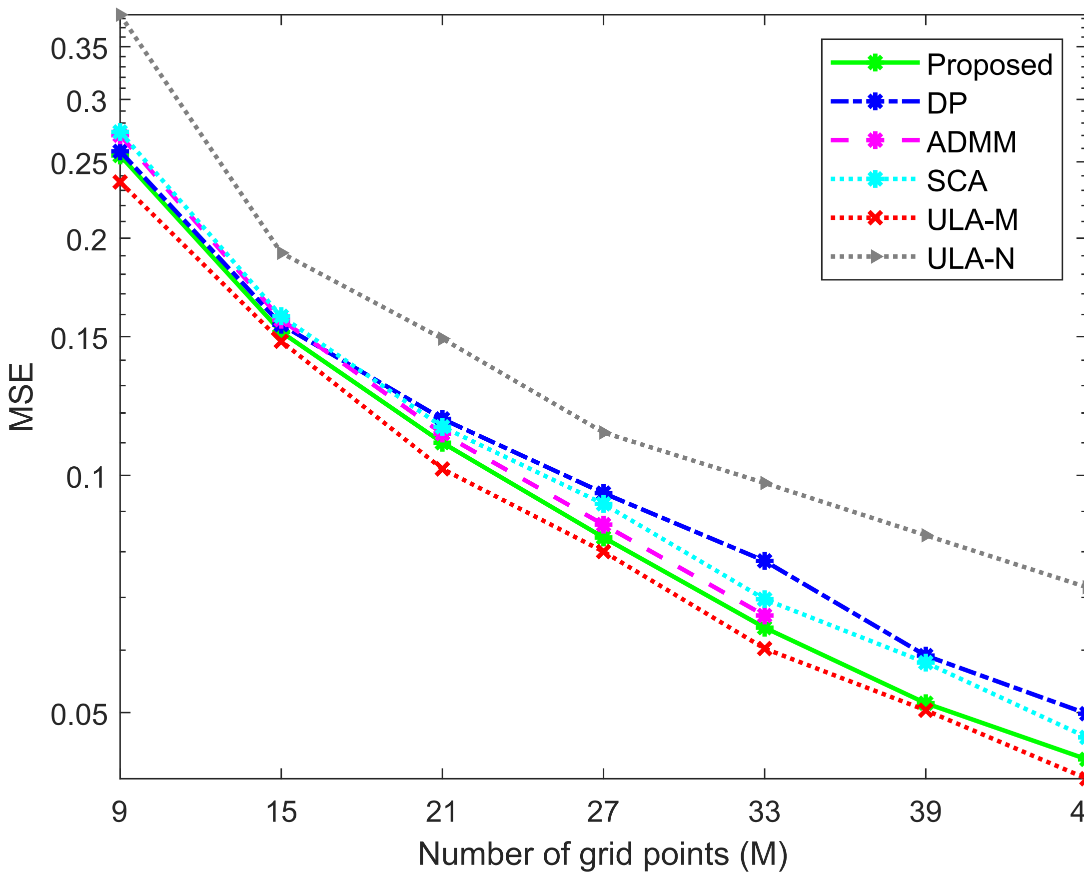

Fig. 13. MSE values for different methods of synthesizing the desired beam-pattern $P_d^2(\theta)$ for different values of M and assuming that $N = \frac{2}{3}M$.

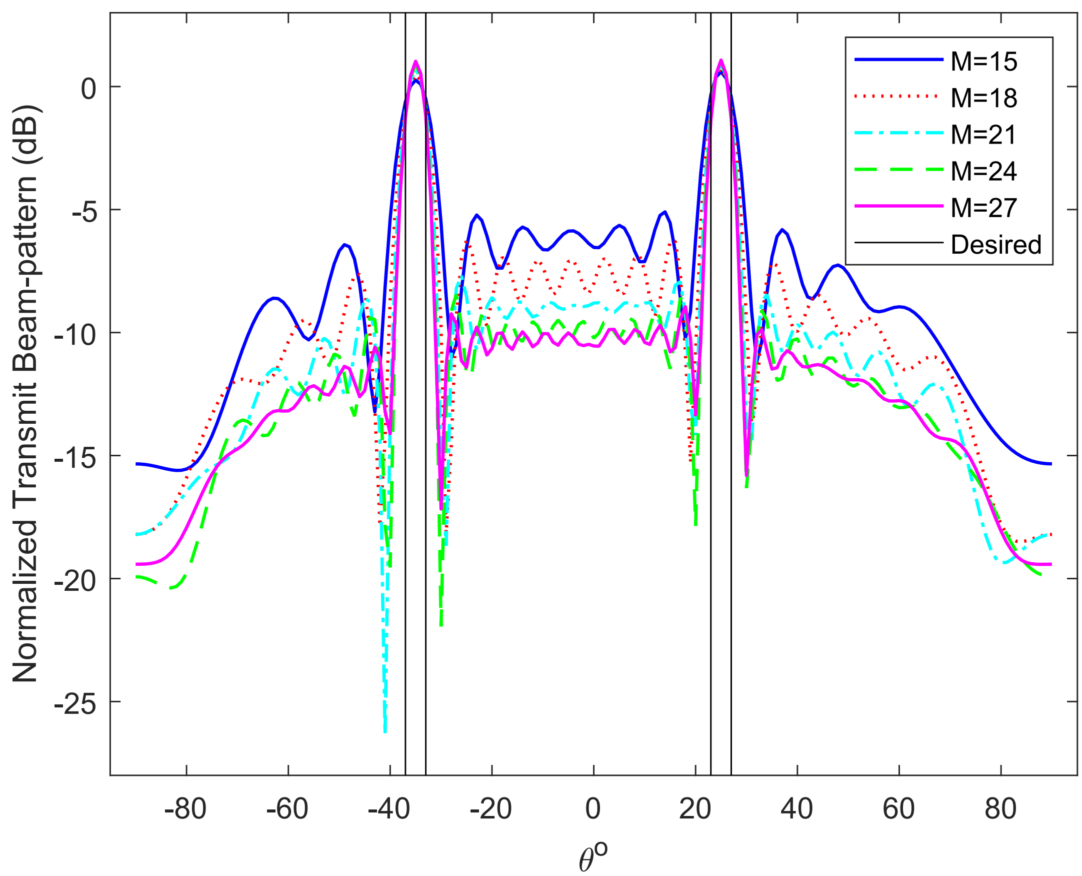

Fig. 14. Proposed method beam-patterns with $P_d^3(\theta)$, for different M values and $N = 15$.

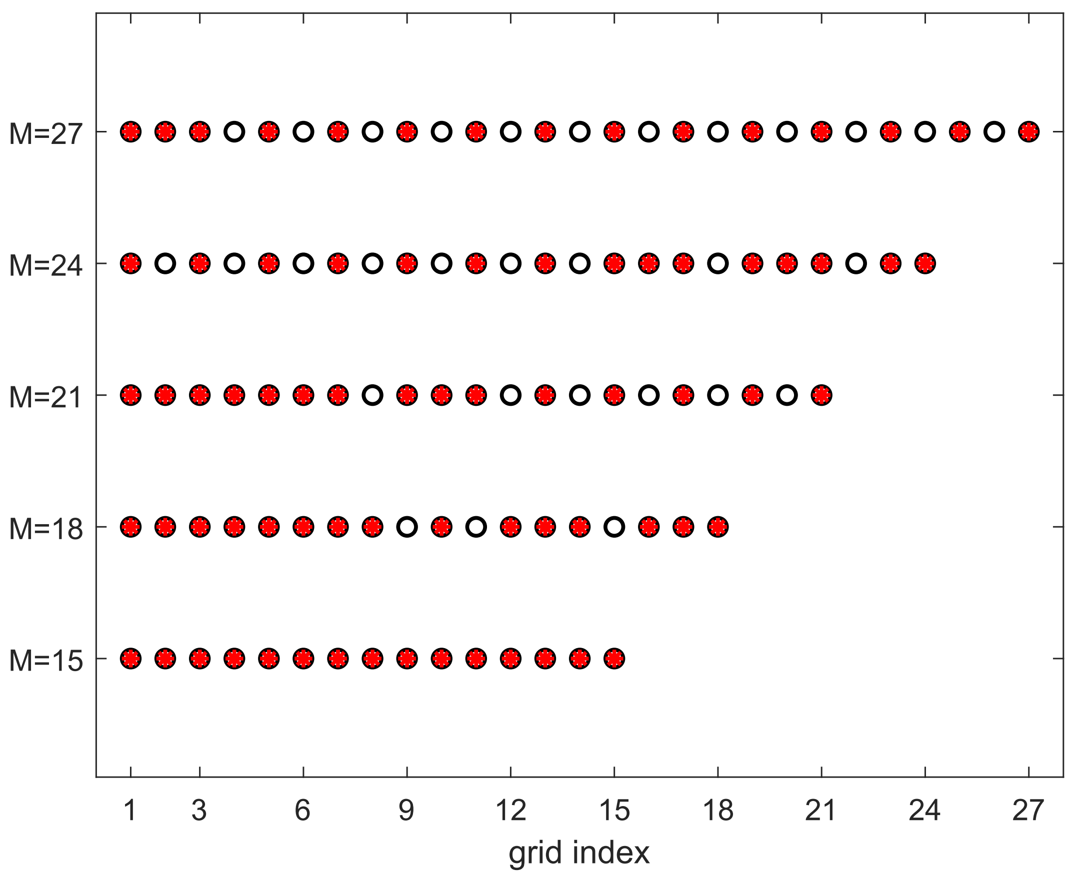

Fig. 15. Reconfigured arrays corresponding to beam-patterns in Fig. 14.

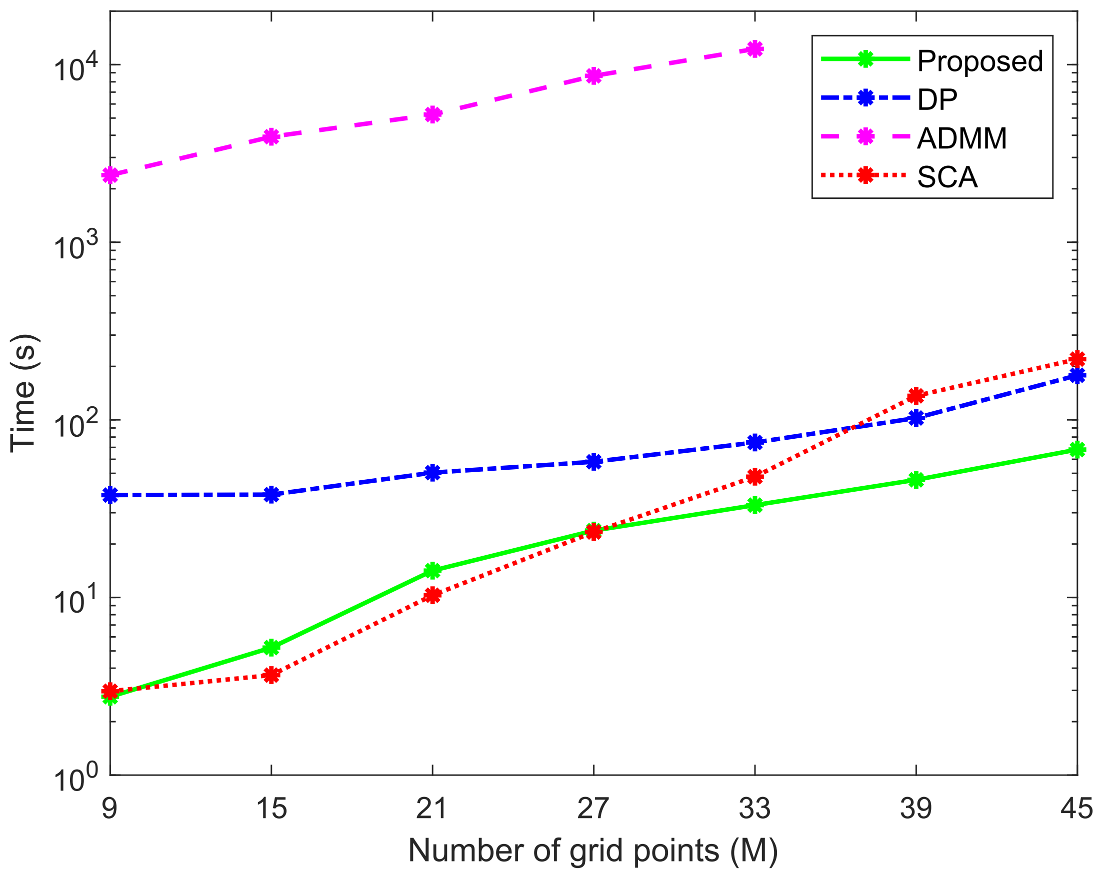

Fig. 16. Time needed to perform different methods.