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DETERMINATION OF TRANSFORMATION MATRIX IN A HYBRID MULTI-AXIS LASER-AIDED MANUFACTURING SYSTEM AND ITS PRACTICAL IMPLEMENTATION

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Abstract

The Laser Aided Manufacturing Process (LAMP) is a multi-axis hybrid manufacturing process comprised of both an additive process, laser deposition, and a subtractive process, CNC machining. Determination of transformation matrix is one of the most important tasks to bridge the gap between process planning (software) and real deposition/machining process. The first part of the paper discusses an algorithm for computing the position of point/points in three-dimensional space, using homogenous transformation matrices. The second part of the paper discusses about how the algorithm was used in practice to build 3-D parts and part-repair using hybrid manufacturing process.

Introduction

Laser Aided Manufacturing Process (LAMP) part repair uses laser deposition process and machining to restore a damaged metal part to near-original condition. The LAMP repair process is a hybrid repair process. The damaged portion of the work-piece is first machined, both to remove damage and to make a surface suitable for laser deposition. Then metal powder is laser deposited at the damaged location. Finally, the work-piece is finish machined back to its original condition.

Alignment of the work-piece becomes highly critical in this application. Poor alignment might result in deposition or machining at the wrong location or even damage to the deposition system itself. The strategy for alignment is as follows. A Renishaw touch probe is used to get the point cloud data from the work-piece. This data is then used to orient the work-piece in a direction we want before machining away the damaged portion. After machining, the touch probe is used again to calculate the orientation of the work-piece. Then, The LAMP system is used to repair the part. The work-piece is again probed to make sure that the finished part has the required accuracy.

It is important in the repair process to be able to predict the position and orientation of the work-piece accurately to get a succesful deposition. Also, this is very important for process planning for fabrication of complex 3D parts.

Equipment Overview

Laser Metal Deposition (LMD) is a layered manufacturing process where metal powder is focused into a melt pool created by a laser incident on a substrate. The advantage of this process is that complex geometries can be constructed with near net shape. The LAMP system at the University of Missouri Rolla is comprised of a 1.4 KW Nuvonyx diode laser (808 nm) with integrated 5-Axis FADAL CNC with a maximum spindle speed of 7500 RPM. Powder is delivered by a Bay State thermal spray powder feeder. Figure 1 shows the tool holder for the touch probe and the laser deposition nozzle. A Renishaw MP11 touch probe is used to obtain point cloud data from the work-piece. It is shown in Figure 2.

Figure 1. Machining Axis and Laser Axis   Figure 2. Renishaw Touch Probe in Action

<u>Reference Point</u>

The FADAL 5 axis CNC consists of three linear axes: X, Y, and Z, and two rotary axes: A and B. The two rotary axes were added on later to the three-axis CNC. To find the coordinates of a point on the work-piece after it has been rotated known angles in A and B, transformation matrices are used. For the transformation calculations, it is necessary to have a reference point in world coordinates. This has to be a point that is physically accessible to the touch probe in several orientations of the rotary axes. Also, the geometry of the reference point should allow for repeatable measurements. Due to the physical limitations in accessing a suitable location on the CNC as a reference point, it was decided to mount a fixed datum on the CNC. Two options were considered – a rectangular gage block (Figure 3) and a spherical gage ball (Figure 4).

<u>Rectangular Gage Block</u>

A corner point of the rectangular block set as reference point. Three orthogonal planes are to be probed to set the zero at a corner. If the X, Y and Z planes of the block are in the same

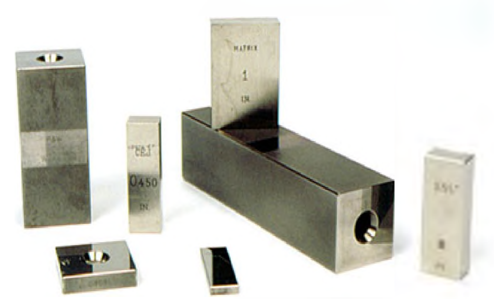


Figure 3. Rectangular Gage Block

direction as that of the CNC X,Y and Z directions, it is easy to compensate for the probe radius. This is not always thecase and hence probe radius compensation is an issue.

<u>Spherical Gage Ball</u>

Probe radius compensation is not needed for a sphere. When a sphere is probed, the data attained will be that of a bigger sphere on account of the probe radius. But the center point will still be the same. The orientation of the vise is not an issue for the sphere. Whereas, for the rectangluar block the orientation of the vise makes a difference due to the probe radius compensation issue. Also the procedure to find the center of a sphere is more repeatable than the procedure to find the corner of a rectangular block.

Figure 4. Spherical Gage Ball

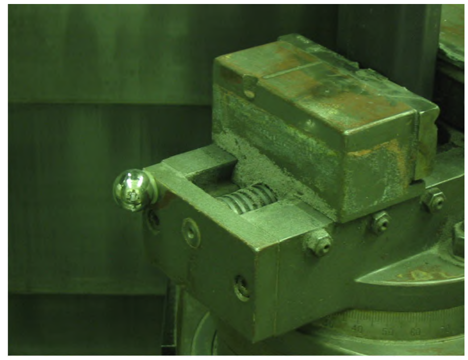


Figure 5. Gage Ball Mounted on the Vise

Based on the above observations, it was decided to mount a sphere as the reference point. The sphere gage ball used for the purpose is made of a chrome steel alloy, hardened to R/C 63, ground and lapped to a surface finish of 1.5 micro-inch. It has a sphericity within 0.000025 inch. The gage ball mounted on the vise is shown in Figure 5.

Method to Find Sphere Center

Four Points Method

There is a unique sphere that passes through four non-coplanar points if, and only if, they are not on the same plane. If they are on the same plane, either there are no spheres through the 4 points, or an infinite number of them if the 4 points are on a circle.

Given 4 points, $\{x_1, y_1, z_1\}$, $\{x_2, y_2, z_2\}$, $\{x_3, y_3, z_3\}$, $\{x_4, y_4, z_4\}$ the equation of the sphere with those points on the surface is determined by solving the following determinant [1].

$$\begin{vmatrix} x^2+y^2+z^2 & x & y & z \\ x_1^2+y_1^2+z_1^2 & x_1 & y_1 & z_1 \\ x_2^2+y_2^2+z_2^2 & x_2 & y_2 & z_2 \\ x_3^2+y_3^2+z_3^2 & x_3 & y_3 & z_3 \\ x_4^2+y_4^2+z_4^2 & x_4 & y4 & z_4 \end{vmatrix} = 0 \qquad (1)$$

There are conditions on the 4 points. They are listed below and correspond to the determinant above being undefined (no solutions, multiple solutions, or infinite solutions).

- No three combinations of the 4 points can be colinear.
- All 4 points cannot lie on the same plane (coplanar).

Evaluating the cofactors for the first row of the determinant gives the solution. The determinant equation can be written as an equation of these cofactors:

$$(x^2 + y^2 + z^2)\cdot M_{11} - x\cdot M_{12} + y\cdot M_{13} - z\cdot M_{14} + M_{15} = 0 \tag{2}$$

This can be converted to the canonical form of the equation of a sphere:

$$x^2 + y^2 + z^2 - (M_{12}/M_{11})\cdot x + (M_{13}/M_{11})\cdot y - (M_{14}/M_{11})\cdot z + M_{15}/M_{11} = 0 \tag{3}$$

Completing the squares in x and y and z gives:

$$x_0 = 0.5\cdot M_{12}/M_{11} \tag{4}$$

$$y_0 = -0.5\cdot M_{13}/M_{11} \tag{5}$$

$$z_0 = 0.5\cdot M_{14}/M_{11} \tag{6}$$

$$r_0^2 = x_0^2 + y_0^2 + z_0^2 - M_{15}/M_{11} \tag{7}$$

It is to be noted that there is no solution when M_{11} is equal to zero. In this case, the points are not on a sphere; they may all be on a plane or three points may be on a straight line.

The sphere surface was probed and coordinates of four non co-planar points were obtained. They were used to find the coordinates of the center point. The procedure was repeated several times and the results were compared. It was found that the deviation between the results is large. Hence it was decided to probe the surface of the sphere several times and use a fitting algorithm to find the center accurately.

Fitting Method

Using the touch probe 30 points on the sphere is probed. Then, a fitting algorithm is used to find the center of the sphere. The fitting algorithm is shown in Figure 6. The algorithm works as follows:

1. The algorithm first takes the first three points from the data for 30 probed points.
2. Then it checks for a fourth point that is most orthogonal to the three points.
3. Using the four points, the center of the sphere is computed.
4. The distances of the probed points from the computed center is calculated.
5. Adjust the center based on the distances and directions of the vector from the computed center to the probed points.
6. Repeat steps 4 through 5 until an acceptable value of the error is obtained.

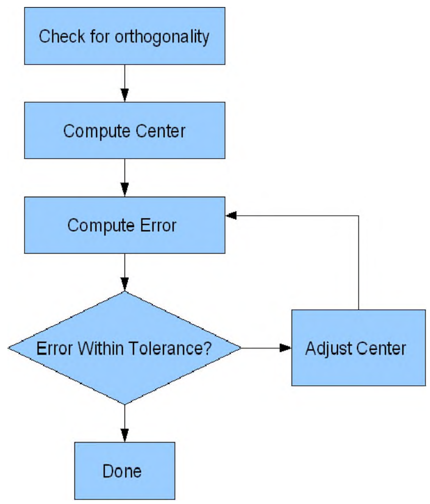


Figure 6. Fitting Algorithm for Sphere Center Computation

Homogeneous Transformation Matrices

When the A and B axis are rotated, the position of any point on the work-piece in space can be determined using homogenous transformation matrices. Homogenous transformations combine the operations of rotations and translation into one single matrix multiplication. It is explained below [2].

The rotation of a rigid body around the x axis at an angle Θ_X is given as

$$R_X(\Theta_X)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\Theta_X) & -\sin(\Theta_X) & 0 \\ 0 & \sin(\Theta_X) & \cos(\Theta_X) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

The rotation of a rigid body around the y axis at an angle Θ_Y is given as

$$R_Y(\Theta_Y)=\begin{bmatrix} \cos(\Theta_Y) & 0 & \sin(\Theta_Y) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\Theta_Y) & 0 & \cos(\Theta_Y) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

The rotation of a rigid body around the z axis at an angle Θ is given as

$$R_Z(\Theta_Z)=\begin{bmatrix} \cos(\Theta_Z) & -\sin(\Theta_Z) & 0 & 0 \\ \sin(\Theta_Z) & \cos(\Theta_Z) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

The translation of a rigid body, dx in x direction, dy in y direction and dz in z direction is given as

$$T(dx,dy,dz)=\begin{bmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

When a combination of rotations and translation is done, the order of rotations is important. Also, it is to be noted that rotations are performed first and then translation. For the FADAL CNC in LAMP Lab, the axis for A rotation is positive Z axis. This is based on the right hand thumb rule. Similarly, the axis for B rotation is negative Y axis. If the work-piece is rotated 'a' degrees and 'b' degrees respectively in A and B axes,

$$\phi = a\frac{\pi}{180} \tag{12}$$

$$\theta = -b\frac{\pi}{180} \tag{13}$$

The transformation matrices can be formulated as follows

$$R_A=\begin{bmatrix} \mathrm{Cos}(\O) & -\sin(\O) & 0 & 0 \\ \sin(\O) & \cos(\O) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

$$R_B=\begin{bmatrix} \mathrm{Cos}(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \mathrm{Cos}(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{15}$$

When the work-piece is translated x, y and z inches in the X,Y and Z directions respectively, the translation matrix is given by

$$T_{XYZ}=\begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

The part coordinates after the work-piece has been rotated and translated, P is given by

$$P=R_BR_AT_{XYZ} \tag{17}$$

The next step is to find out translation transformation matrix. For this the vector from the AB rotation center to the reference point(center of spherical gage ball) has to be found out. For this three methods were tried.

Method I

This method is similar to the fitting method used for finding the center of the spherical gage ball. To find the center of the gage ball, the ball was probed for 30 different points on the sphere surface and a fitting algorithm was used to find the center. In the case of the AB rotation center, the center of the gage ball is found in different positions of A and B axis rotations. Each of these centers make up the surface of a larger sphere with the center as the AB rotation center. Using the same fitting algorithm used in finding the center of the gage ball, the AB rotation center is determined. Figure 7 is a graphical representation of Method I.

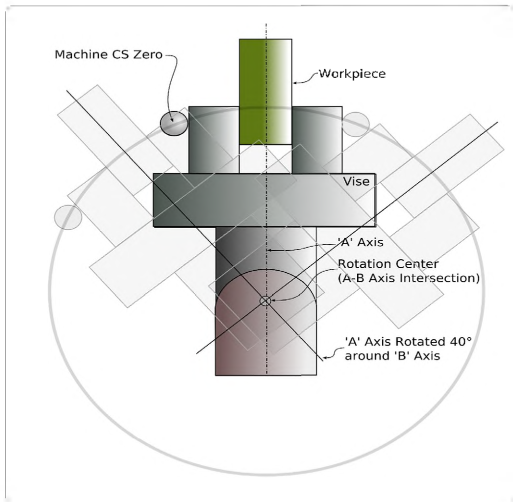


Figure 7. Method I

A test was conducted to test the validity of the value thus found out. A laser burn-paper was pasted onto a plate and using the laser a rectangular box was traced on the paper. The plate was then rotated in a known angle of A and B. Using the transformation matrix, the coordinates of the four corners of the box were calculated. The objective of the test was to trace the same outline as the earlier box after the plate has been rotated. The result is shown below.

Figure 8 shows how the two boxes (before and after rotation) are misaligned when the AB rotation center value used in the transformation matrix is erroneous. Figure 9 is the actual result of the experiment descibed above. The result passes a visual inspection, but it is impossible to determine the accuracy of the transformation to any degree of precision from this test. The value for the rotation center that was attained by Method I was used in some actual deposition experiments. It was found the value is not as accurate as it is needed to be. Its

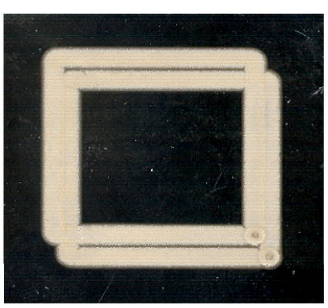


Figure 8. Erroneous Rotation Center Value

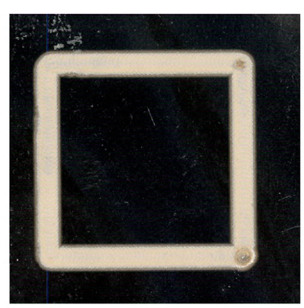


Figure 9. Method I Visual Validation

resolution was less than the resolution of the CNC. Hence it is necessary to find out a second method for finding the rotation center.

Method II

The second method that was used is based on factorial experiments. From Method I, an approximate value for the rotation center was obtained. The objective of this second experiment is to obtain a value that is close enough to the resolution of the CNC. A full factorial experiment with three factors and two levels was done. The three factors in the experiment are x, y and z : the X, Y , and Z co-ordinates of the rotation center. The experiment design is given in Table 1 [3].The procedure for the experiment is as follows:

The value for the rotation center found out from Method I is used as the intial value for the X, Y and Z coordinates. Then, using two levels, eight new centers are calculated using the factorial design. A wax block is mounted on the vise. At A=0 and B=0, two holes are drilled to get the orientation of X axis. Eight holes are planned to be drilled at a known distance from these holes at a known value of A and B. It is to be noted that for the calculation of the rotated position of each of these eight holes, a different value for the rotation center is used. The values that are used for the rotation center comes from the factorial design. A image of the drilled holes is then obtained. The image is analyzed using ImageJ and the distance between the actual hole position and the position where the hole was intended to be is calculated. The hole with the least variation from the intended position is found out and the rotation center value corresponding to that hole is noted. This center point is the initial center value for the next set of experiments. The data about the variation in the hole positions is used to find a regression equation.

An example of the regression equation is

$$\text{Variation} = 0.03 + 0.4x + 0.13y + 0.0001z \quad (18)$$

The objective of the experiment is to minimize the constant term in the regression equation as it is a measure of how much the rotation center value is off. The experiment is repeated till an acceptable value of the constant term is obtained. The drawback of this method is that the method doesn't take into consideration the z-value of the rotation center. The z-value

Table 1. Factorial Experiment Design

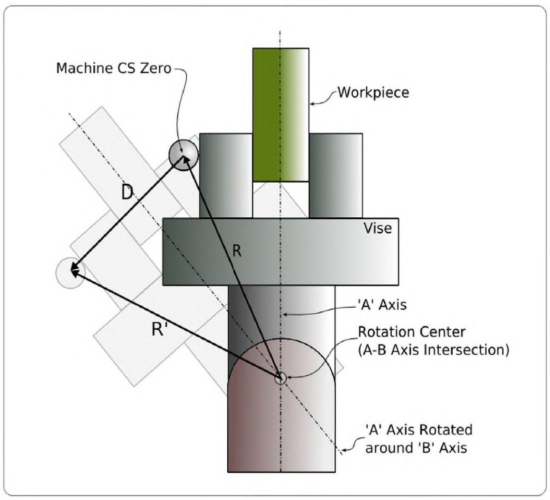
RUN	x	y	z
1	1	1	1
2	1	1	-1
3	1	-1	1
4	1	-1	-1
5	-1	1	1
6	-1	1	-1
7	-1	-1	1
8	-1	-1	-1

will have an effect on the x and y value.When the rotation center value obtained from the method II was tested in actual repair process, this drawback came into play. Hence, an alternate method was investigated.

Method III

Method III is based on vector algebra. The procedure is as follows:
R is the vector from the rotation center to the sphere center when A=0 and B=0. Then, the vise is rotated at a known angle of A and B and the sphere center is found again using the touch probe. R' is the vector from the rotation center to sphere center at this rotated position. D is the vector from the initial position of the sphere center to the rotated position. All the vectors are shown in Figure 10.

R,D and R' form a vector loop which gives rise to Equation (19). Since there in no translation involved, R multiplied with the rotation transformation matrices of A and B gives R' . This is shown in Equation (20).



R = Vector from Rotation center to the sphere center (machine zero)
R' = Vector from Rotation center to the sphere center after rotation
D = Vector from initial position of the sphere center to the rotated position

R_A = Rotation matrix of A axis

R_B = Rotation matrix of B axis

Figure 10. Method III

$$R+D+R' = 0 \tag{19}$$

$$R' = R_B R_A R \tag{20}$$

$$(I- R_B R_A)R = -D \tag{21}$$

Equation (21) is of the form AX=B. It is solved using Matlab to reduce Equation (21) to row-reduced echelon form, obtaining a solution for R, the vector from the rotation center to the reference point. The value for R obtained using this method is (-4.877, 1.4798, 7.0918).

The experiment was repeated several times and it was found that the results are consistent. It was experimentally tested and the results were found to be good.

Machining to Laser Axis Translation

The touch probe used to obtain data from the work-piece is in the machine frame of reference. Repair work has to be done in the laser frame of reference. This give rise to an isssue which is the translation from the machining axis to the laser axis. For this, a burn mark is made on a steel block with the laser and the coordinates are noted. Then, the steel block is moved to the machining frame and the center of the hole is found with the touch probe. The difference between the coordinates in the machining and laser frame of reference gives the transformation. The vector from the machining to the laser axis, L, is found to be (-10.10308, 0.03632, -8.72762).

Applying the vector values obtained above (R and L) in homogeneous transformation matrices, it is possible to predict the position and orientation of the work-piece at arbitrary rotations of the A and B axes in the laser frame of reference. This is used to do repair work using both laser deposition and machining.

Results

The values (-4.877, 1.4798, 7.0918) for R and (-10.10308, 0.03632, -8.72762) for L were used in building 3D-parts and in the repair of die-casting cores. They are (1) An Arch, (2) Repaired Core, and (3) Bearing Seat.

An arch is shown in Figure 11. It was built using the five-axis transformation as outlined above. The procedure used for building the arch is graphically represented in Figure 12. First, the coordinates of a point on the substrate (Point 1 in Figure12) is probed. A wall of the arch

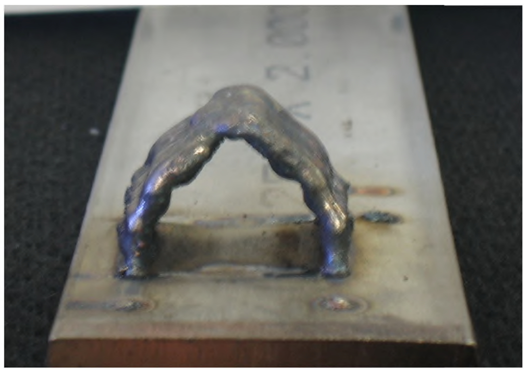


Figure 11. Closed Arch Using 5-axis Transformation

is then built to the required height. Next, the coordinates of point 2 is computed using the transformation matrix. The next section of the arch is built based on that point. Point 4 is then tranformed the required angle and the remaining sections of the arch are built. After each section, a second section, mirrored about the part's plane of symmetry, is deposited, as outlined below in Figure 12. The part is constructed in this order to avoid collision between the part and the nozzle.

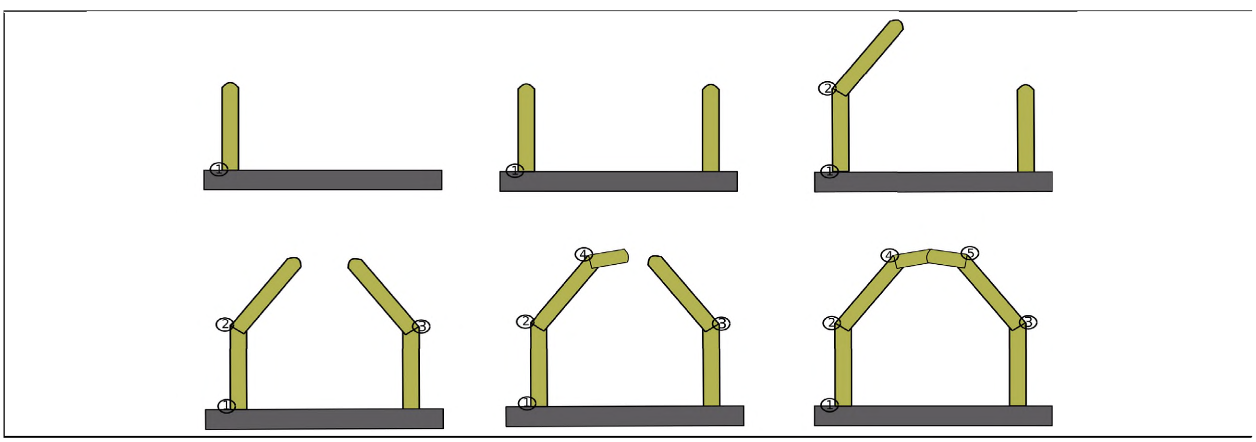


Figure 12. Arch - Stages of Deposition Process

Figure 13 shows a core that has been damaged. The CNC code for the repair were generated using process planning software. For this, the values of the rotation center R and the machine frame to laser frame vector were used. Figure 14 shows the core that was repaired using automated part repair.

Figure 13. Damaged Core

Figure 14. Repaired Core - via Automated Part Repair

The CAD drawing of a bearing seat is shown in Figure 15. The procedure used for building the part is illustrated in Figure 16. The deposited part is shown in Figure 17 and Figure 18. It was built using five-axis transformation. The direction of the arrows in Figure 16 indicate the build direction during deposition. It can be seen from Figure 16 that the bearing seat is built in three sections. The first section of the bearing seat is built at an angle of A=0 and B=0. This section is built with the coordinates of point 1 as reference. On completion of that section, the second section is built after rotating B axis +90 degrees. For this part of the deposition it is important to get the accurate position

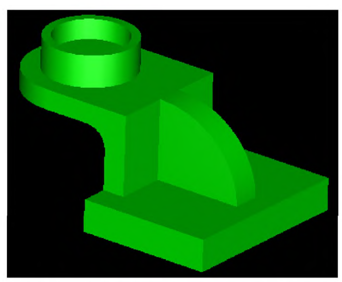


Figure 15. CAD Drawing of Bearing Seat

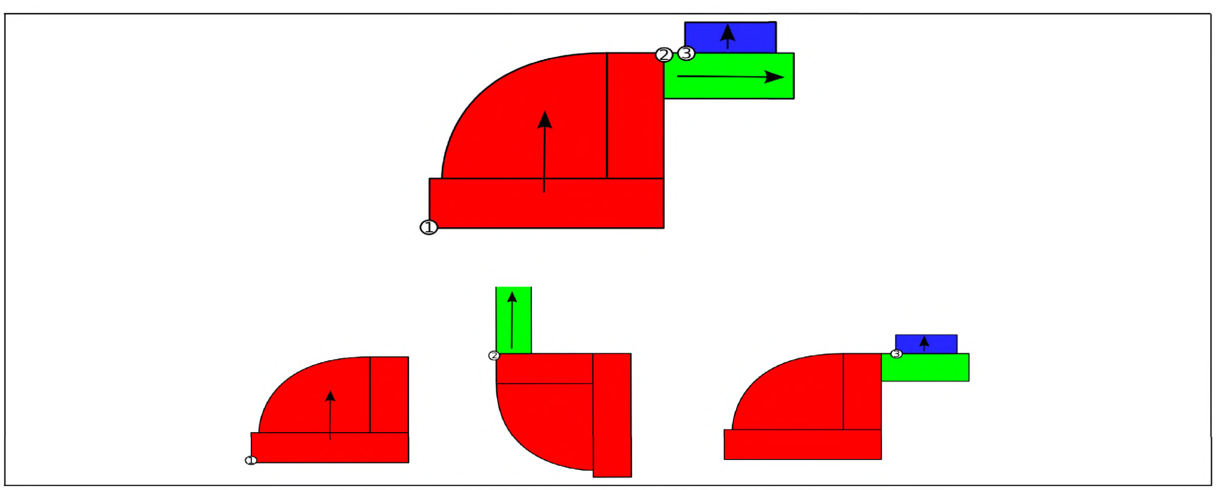


Figure 16. Bearing Seat Deposition Stages

Figure 17. Bearing Seat - View 1

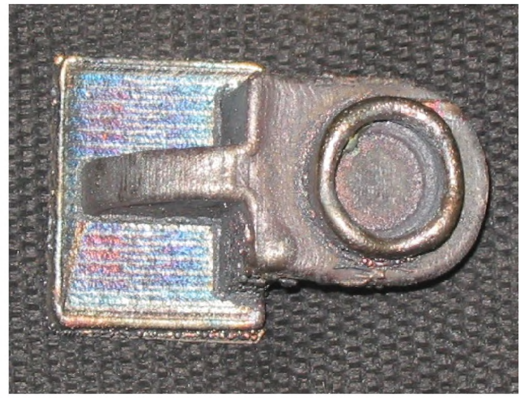


Figure 18. Bearing Seat - View 2

of point 2. This is where the transformation matrix, the value of R, and the machining frame to laser frame vector comes into play. The coordinates of point 2 after rotation in B axis are calculated. The second section is then built based on point 2. After the deposition of section 2, the B axis is rotated back to the original position. The third section is then deposited based on point 3.

Summary

Determination of the tranformation matrix is an important step to bridge the gap between process planning and actual deposition/machining. An efficient method to calculate the vector from the rotation center to reference point/part zero was developed. Also, a method to compute the transformation from machining axis to laser axis was developed. These were applied to process planning software and was using in the repair of damaged die casting cores and in the fabrication of 3D parts.

Acknowledgments

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