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Abstract:
This paper presents a multi-body dynamics model of hypoid gear pairs, showing the interactions between gear pair dynamics, NVH and friction in the thermo-elastohydrodynamic teeth pair conjunctions during meshing. The multi-body model consists of a two-degree of freedom torsional model developed in the ADAMS environment. The coefficient of friction is calculated using an analytical formula for non-Newtonian behaviour of a thin lubricant film. Additionally, road data and aerodynamic effects are used in the form of resisting torque applied to the output side of the gear pair. Sinusoidal engine torque variation is also included to represent engine torsional response. Results are presented for a light truck in both low speed city driving condition in 2nd gear and steady state cruising in 4th gear. Transmission efficiency is obtained under both conditions. Transmission performance under both cold and hot steady state cycles within the New European Driving Cycle (NEDC) is assessed.
Keywords—Multi-body dynamics, Differential hypoid gears,Transmission efficiency, NVH

Nomenclature

a	: Vehicle acceleration
A_p	: Instantaneous pinion angle
A_f	:Vehicle frontal area
b	:Half gear backlash
C_D	: Drag coefficient
f_{rl}	: Rolling resistance coefficient
f_r	:Totalflank friction
f_b	:Boundary friction contribution
f_v	:Viscous friction contribution
g	: Denotes the gear wheel
m	: Vehicle mass
$nDOF$	: Number of degrees of freedom
p	: Denotes the pinion
P_f	: Frictional power loss
R_a	: Aerodynamic resistance
R_{rl}	: Rolling resistance
R_g	: Gravitational resistance

R_t	: Transmission ratio
t	: Time
T_p , T_g	: Externally applied torques to the pinion and gear
T_{frp} , T_{frg}	: Frictional moments at pinion and gear
V	: Vehicle speed [mph]
W	: Vehicle weight
μ	: Coefficient of friction
ρ	: Density of air

1-Introduction

The high load carrying capacity usually required of the final drive constitutes partially conforming meshing teeth pairs at relatively high loads. This requirement brings about the hypoid gear pair geometry, which presents gradual changes in geometry of an elliptical contact footprint. Therefore, since the inception of the automobile, the differential hypoid gear pairs with their orthogonal axes have become the final drive feature in all vehicles. They are one of the most important elements of drive train system, particularly in the current trend towards better fuel efficiency, enhanced power and improved NVH refinement.

Most research work on gearing systems are dedicated to the dynamics of parallel axis transmissions, with only limited investigations reported for the dynamics of non-parallel axes gears such as hypoid and bevel gears (Kolivand*et al* [1] and Xu *et al*[2]). This dearth of analysis has been due to the complexity of their contact kinematics and meshing characteristics.

The hypoid gear teeth pairs form elliptical contact footprints and are often subjected to high loads of the order of several kN, particularly in the case of commercial vehicles. The regime of lubrication is usually elastohydrodynamic with a thin film of lubricant being crucial for reducing friction, thus providing enhanced transmission efficiency and reduced Noise, Vibration and Harshness (NVH) (Karagiannis*et al* [3] and Mohammadpour*et al* [4]). Thus far, most reported dynamic models consider dry contact analysis, which is an unrealistic assumption with regard to the estimation of friction. A recent work by Karagiannis *et al* [3] presented a dynamics model of hypoid gears, focusing on the torsional vibrations of differential hypoid gear pair under realistic loading conditions. They included an analytical, quasi-static elastohydrodynamic lubrication (EHL) analysis, taking into account the non-Newtonian shear of thin lubricant films and generated heat, thus estimating the contact friction. They also included the effect of dynamic transmission error (DTE) as a concern in their dynamic study of the system. A more detailed numerical solution for the lubricated contact with the resultant transmission efficiency is provided by Mohammadpour*et al* [4].

The main challenge in the study of EHL of hypoid gears is their complex meshing geometry, which is obtained in both [3] and [4] using tooth contact analysis (described by Litvin and Fuentes [5]). In order to take into account the dynamic behaviour of the gear pair, realistic data, particularly estimation of a dynamic load is required (Karagiannis*et al* [3] and Mohammadpour*et al* [4]).

This paper presents a multi-body dynamics' model of hypoid gear pairs, showing the interactions between gear pair dynamics and NVH and friction in the elastohydrodynamic teeth pair conjunctions during meshing. The multi-body model comprises a two-degrees-of-freedom torsional model developed in the ADAMS multi-body environment. The coefficient of friction is calculated using available analytical formulae for thin non-Newtonian lubricated conjunctions. Additionally, road data and aerodynamic effects are included in the form of resistance applied to the output side of the gear pair (i.e. on the road wheels).The usual sinusoidal variation in engine torque (as the result of engine order vibrations) is also included in the model (Rahnejat[6]).

A thin lubricant film is formed during most of the meshing cycle. Thus, mixed regime of lubrication is prevalent. Greenwood and Tripp [7] model is used to take into account the effect of any interactions of ubiquitous asperities on the contiguous contacting meshing teeth surfaces. The film thickness and inefficiency have been calculated in conjunction with gear dynamics and the NVH behaviour of the gear pair.

2-Model Description

The multi-body model comprises a two-degree of freedom torsional model, developed in the ADAMS multi-body environment (figure 1). The inertial properties of the mating gear pair are listed in table 1. The values of inertia include different parts of transmission system, such as the retaining shafts; driveline and rear axle in addition to the gears themselves. The list of constraints, used in the multi-body model, is given in table 2.

Based on the Chebychev-Grüebler-Kutzbach expression, the number of degree of freedom can be obtained as follows:

$$nDOF = 6(parts - 1) - \sum \text{constraints} \qquad (1)$$

This expression yields two degrees of freedom for the devised model, which represents the torsional motions of the pinion and the gear. The governing equations of motion are automatically generated by ADAMS in constrained Lagrangian dynamics of the form [6]:

$$[J]\begin{Bmatrix} \delta q^j \\ \delta \gamma_j \end{Bmatrix} = \{F_a\} \qquad (2)$$

where, $\{q^j\} = \{x, y, z, \psi, \theta, \phi\}^T$ and $[J]$ is the Jacobian matrix of the form:

$[J] = \begin{bmatrix} \left[\dfrac{\partial K}{\partial \dot{q}^j} + \dfrac{\partial U}{\partial q^j}\right] & \left[\dfrac{\partial C_{ik}}{\partial \gamma_j}\right] \\ \left[\dfrac{\partial C_{ik}}{\partial q^j}\right] & [0] \end{bmatrix}$, where j refers to co-ordinates $i, k \equiv p, g$, K is kinetic energy and

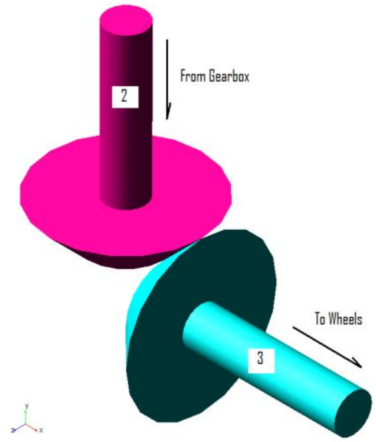

Figure 1: Overview of the multi-body dynamics model

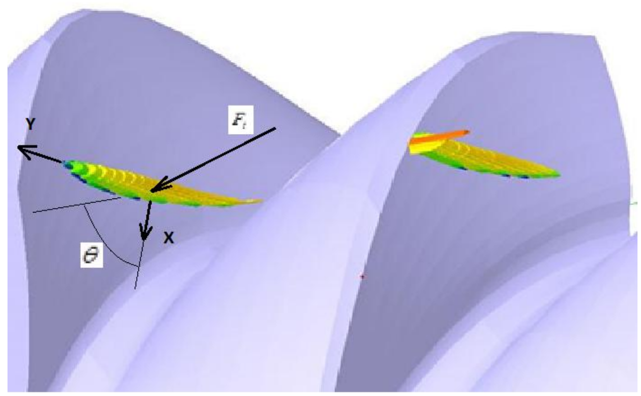

Figure 2: Flank load on a specific tooth

U the potential energy. Thus, the generalised Eulerian forces are: $F_{q^j} = -\frac{\partial U}{\partial q^j}$ which in cases of bodies i,k in this example are:

$$F_{\psi i} = R_i k_m f(l) + c_m \dot{\psi}_i \tag{3}$$

where: $i,k \in p,g$, k_m is the dynamic meshing stiffness obtained through tooth contact analysis [3], c_m is the structural damping factor and:

$$f(l) = \begin{cases} l-b & l \geq b \\ 0 & -b < l < b \\ l+b & l \leq -b \end{cases} \tag{4}$$

l denotes the spatial line of approach between meshing teeth pairs ($l \propto q_i \in x,y,z$). This is the dynamic transmission error, hence:

$$l = \int_0^t R_p \psi_p dt - \int_0^t R_g \psi_g dt \tag{5}$$

γ_j are unknown as Lagrange multipliers and C_{ik} are constraint functions for joints in the multi-body system for the parts; pinion and gear. These are revolute joints to the ground for parts i, resulting in the constraint functions:

$$x_i = 0, y_i = 0, z_i = 0, \sin\theta_i \sin\varphi_i = 0, \sin\theta_i \cos\varphi_i = 0 \tag{6}$$

The applied forces, F_{ai} are the torques resident on the pinion and the gear and contribution due to flank friction as:

$$F_{ai} = T_i + T_{fri} \tag{7}$$

The resisting torque applied to the wheels is due to traction, which comprises vehicle inertia (motive force), rolling resistance, aerodynamic interaction and grading (Gillespie [8]):

$$T_g = r_w \sum F \tag{8}$$

where r_w is the laden wheel radius and $\sum F$ is obtained from vehicle longitudinal dynamics as:

$$\sum F = R_a + R_{rl} + R_g \tag{9}$$

where:

$$R_a = \frac{\rho}{2} C_D A_f V^2 \tag{10}$$

$$R_{rl} = f_{rl} W \tag{11}$$

and:

$$f_{rl} = 0.01\left(1 + \frac{V}{147}\right) \tag{12}$$

The demanded instantaneous input torque (on the pinion) is obtained as:

$$T_p = \frac{R_p}{R_g} T_g \left(1 + 0.1\cos\left(R_t A_p\right)\right) \tag{13}$$

Flank friction between pairs of meshing gear teeth contribute to the applied forcing (equation (7)). A thin elastohydrodynamic lubricant film is usually formed in conjunctions of the meshing teeth pairs of differential hypoid gears. These thin lubricant films are subject to non-Newtonian viscous shear, supplemented by any asperity interactions (boundary friction as the result of direct contact of surfaces).

$$T_{fri} = R_i f_r \tag{14}$$

where the flank friction is obtained as:

$$f_r = f_v + f_b \tag{15}$$

Viscous friction is calculated using:

$$f_r = \mu F_i \tag{16}$$

Evans and Johnson [9] presented an analytical expression for the coefficient of friction, based on the regime of lubrication. The expression takes into account the thermal properties and pressure-dependence of lubricant properties.

To obtain boundary friction, the Greenwood and Tripp [7] model is used. The film thickness h is obtained from the extrapolated film thickness expression obtained numerically by Chittenden et al[10] for an elliptical point contact with angled flow entrainment.

It is necessary to calculate the contact load F_i for all the meshing teeth pairs, which is required for both equation (16) and film thickness equation and also the friction formulae. This is obtained through tooth contact analysis (TCA). The method is outlined in detail by Litvin and Fuentes [5]. lf is a load distribution factor calculated as a function of the pinion angle for simultaneously contacting teeth pairs. This is the ratio of the applied load F_i on a given flank to the total transmitted load F_t (Xu and Kahraman [2]). In fact it represents the ratio of the normal load on a specific flank during the meshing cycle to the total transmitted load. This is shown in figure 2:

$$lf = \frac{F_i}{F_t} \tag{17}$$

The specifications for the face-hobbed and lapped hypoid gear pair in this study are listed in table 3.

3-Results and Discussion

Transmission efficiency (thus reduced parasitic losses) and NVH refinement arekey concerns in the design of differentials. The current analysis investigates these performance characteristics for a pair of hypoid gears of a front wheel drive transaxle light van. The related input parameters for the analyses are presented in table 4.Two driving conditions at extreme ends of the differential's performance are considered. Firstly, typical city driving in line with NEDC (New European Drive Cycle) is considered. This drive cycle is concerned with low speed steady state vehicle motion in traffic (in this case 20 mph is used), typically when the transmission is at second gear (gear ratio of 1.5:1 in the case considered). Two steady state cycles are described in the NEDC; "cold" and "hot" vehicle states (with bulk transmission fluid at $40^{\circ}C$ and $100^{\circ}C$,respectively).At the other extreme, a high speed manoeuvre in 4th gear of transmission overdrive (with a gear ratio of 0.73:1) is examined. The latter condition is potentially related to whine phenomenon of hypoid gears that usually take place whena higher transmission speed is engaged. The vehicle speed is approximately 70 mph (this being the maximum legal speed in motorway driving in the UK). The pinion rotational speed is 91.23 rad/s in the 2nd gear and 322.86 rad/s in the 4th gear, respectively.

Figure 3 shows the Dynamic Transmission Error (DTE) of the hypoid gear pair for the case of cold steady state NEDC. The corresponding spectral content is shown in Figure 4. Two peaks dominate the spectrum; one at the meshing frequency, f_m, and the other at the forcing frequency (engine torsional vibration resident on the transmission output shaft; $f_e = \frac{1}{2\pi} R_t A_p$, see equation (13)). Also, note the modulation effect between these frequencies; f_m-f_e and f_m+f_e. Higher harmonics of the meshing frequency with similar modulation effects are also present. Idealised meshing(representing no transmission error)corresponds to a spectrum with no other contributions than the meshing frequency.

Figures5 and 6showresultsunderthe high speed driving condition for the 4thgear engaged. The carrier wave in figure 5 is at the forcing frequency, whilst short wavelength oscillations occur at the meshing frequency and its harmonics. Figure 6 shows the corresponding spectrum of vibration. An important feature is the significant increase in engine order vibrations (forcing frequency) that could lead to impacts of meshing teeth pairs. This effect can cause loss of contact and improper meshing, a phenomenon known as axle whine in industry (Koronias *et al* [11]). The results indicate that this phenomenon does not occur for the differential hypoid gear pair studied here. Nevertheless, increased levels of NVH are noted, compared with the city cycle driving as would be expected.

Aside from the NVH issues, prediction of transmission efficiency is important, because any underlying mechanical/frictional losses can lead to increased fuel consumption and consequently higher emissions. Differential hypoid gear pair frictional losses account for 1-2% of all powertrain losses, but nevertheless quite significant. Adverse contact conditions can also lead to wear due to thinness of the lubricant film and/or fatigue spalling of surfaces of meshing teeth due to high generated contact pressures.

For the hypoid gears of the differential studied, 3 pairs of teeth interact during a typical meshing cycle. A typical meshing cycle is marked in figures 7 and 8. Meshing teeth pairs are referred to as the trailing, middle or the leading pair in transition from entering into contact, proceeding through, and gradually separating. The transition points between 3 pairs of teeth in this routine in a meshing cycle are also shown in figures 7 and 8. Figure 7 shows the film thickness formed between any pair of teeth during the aforementioned meshing transition. The film thickness is obtained for all teeth pairs from Chittenden equation [10] for an elliptical point contact with an angled flow lubricant entrainment into the contact on the account of rolling and sliding nature of the contact [4, 12]. The lubricant used is Shell Vitrea 220, whose rheological information is listed in table 5.

The results correspond to bulk oil temperature of 100 ˚C during normal steady state vehicle motion in 4th gear and at a speed of 70 mph. The regime of lubrication is non-Newtonian thermo-elastohydrodynamics with the film thickness being very thin (average of 0.125 μm) and quite insensitive to large variations in contact load for any given teeth pair throughout mesh (the inset to figure 7). Thus, significant boundary contribution to friction occurs. Power loss occurs as the result of generated teeth pair friction in the form: $P_{fj} = f_{rj}\Delta u_j$, where P_{fj} is the frictional power loss, Δu_j is the sliding velocity of teeth pairs, j. This is a small percentage of the engine power. The ratio of frictional power per meshing teeth pair to the supplied power is shown in figure 8.This represents the instantaneous inefficiency caused per any of the simultaneous meshing teeth pairs. Overall, in the case studied, the maximum inefficiency for any teeth pair through mesh is around 1.2% for the case of high speed steady state vehicle motion in 4th gear. However, efficiency is usually an issue of concern in low speed city driving condition, thus the reason for the NEDC. Figure 9 illustrates this when percentage power loss (inefficiency) of a single meshing teeth pair is compared for the cases of high speed 4th gear vehicle steady state motion with steady state "cold" (40˚C) and "warm" (100˚C) differential conditions, corresponding to the NEDC at 20 mph. An increase of 25% in power loss is noted between city driving and high speed vehicle motion (at warm bulk oil temperature).This is because the lower speed of entraining motion results in a thinner lubricant film under the prevailing elastohydrodynamic conditions, hence increasing the boundary and viscous friction contributions. Note that increased loading has insignificant effect on the film thickness under the elastohydrodynamic regime of lubrication. Another interesting feature is the slight increase (about 5%) in power loss under warm lubricant condition, because a thinner film increases the contribution due to boundary friction more than the increased viscosity under cold steady state NEDC.

Table 1: Properties of bodies

Part number	Part name	Inertia [kg m2]
1	Ground	-----
2	Pinion	$1734{\cdot}10^{-6}$
3	Gear	$5.81{\cdot}10^{-2}$

Table 2: Constraints of the multi-body model

Part I	Part J	Constraint type	No. of constraints
Pinion	Ground	Revolute	5
Gear	Ground	Revolute	5

Table 3: Gear pair parameters

Gear pair		
Parameter name	Pinion	Gear
number of teeth	13	36
face-width (mm)	33.851	29.999
face angle	29.056	59.653
pitch angle	29.056	59.653
root angle	29.056	59.653
spiral angle	45.989	27.601
pitch apex (mm)	-9.085	8.987
face apex (mm)	1.368	10.948
outer cone distance (mm)	83.084	95.598
offset (mm)	24.0000028	24
sense (Hand)	Right	Left

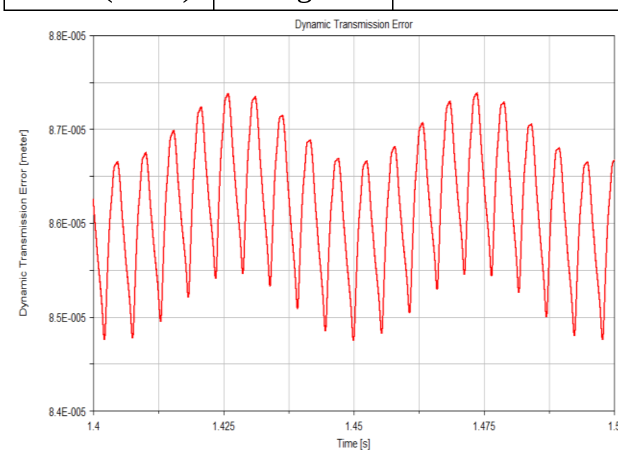

Figure 3:Hypoid gear pairDTE when the 2nd speed of the transmission is engaged under the NEDC steady state cold cycle

Table 4: Analysis conditions

Parameter name	Value
A_f (frontal area)	3.42 m^2
f_{rl} (rolling resistance coefficient)	0.0166
C_D (drag coefficient)	1.15
ρ (air density)	1.22 kg/m^3
W (vehicle weight)	2340 kg
Tyre (type)	P205/65R15 BSW
2nd gear ratio	1.5:1
4th gear ratio	0.73:1
Surface Roughness of solids	0.5 µm

Table 5: Physical properties of the lubricant and solids

Pressure viscosity coefficient (α)	$2.383\text{x}10^{-8}$[Pa^{-1}]
Lubricant atmospheric dynamic viscosity @ 40C (η_0)	0.195[Pa.s]
Atmospheric dynamic viscosity @ 100C (η_0)	0.0171[Pa.s]
Modulus of elasticity of contacting solids	210 [GPa]
Poisson's ratio of contacting solids	0.3 [-]

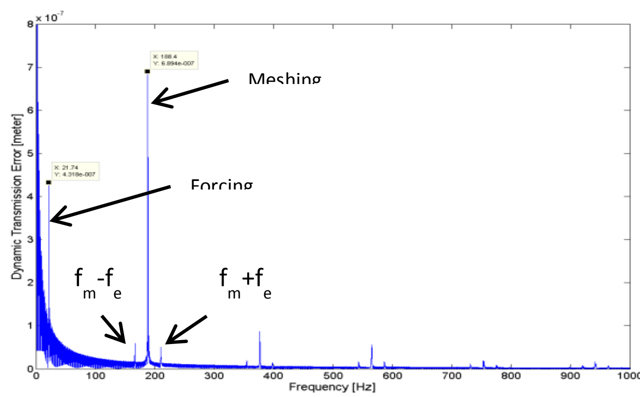

Figure 4: FFT spectra of the DTE of figure 3

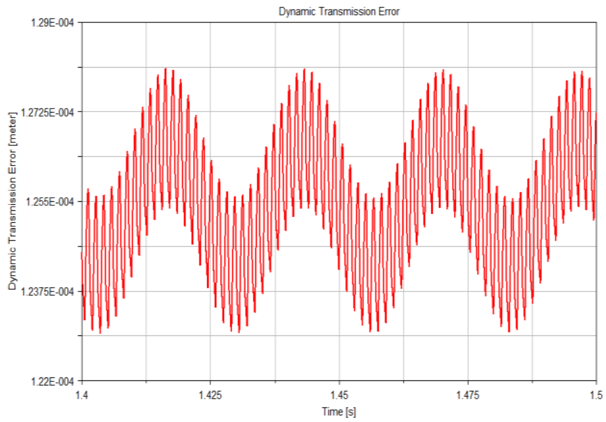

Figure 5: Hypoid gear pair DTE when the 4th speed of the transmission is engaged

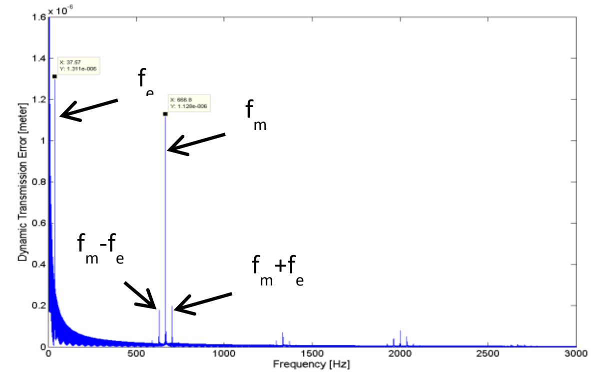

Figure 6: FFT spectrum of the DTE of figure 5

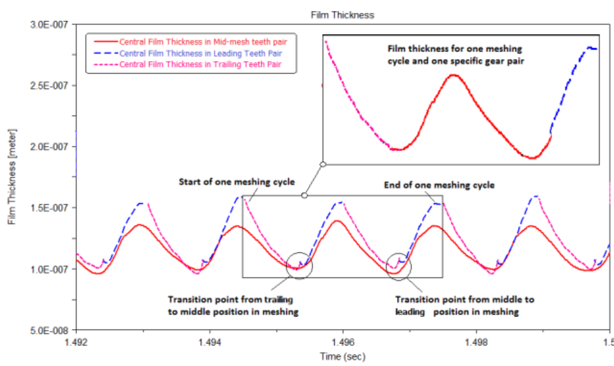

Figure 7: Hypoid gear pair film thickness during meshing when in 4th gear

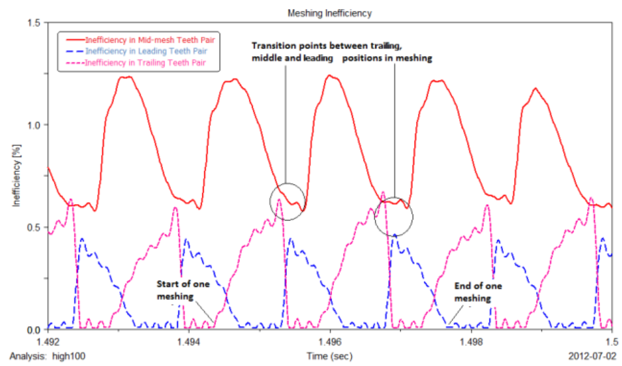

Figure 8: Hypoid gear pair inefficiency during meshing when in 4th gear.

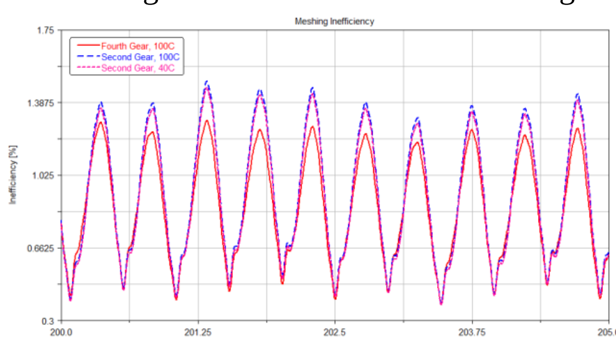

Figure 9: Inefficiency comparison of the hypoid middle teeth meshing pair in cold and hot cycle when 2nd and 4th gears of transmission are engaged respectively.

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