Quasiclassical Negative Magnetoresistance of a 2D Electron Gas: Interplay of Strong Scatterers and Smooth Disorder

A. D. Mirlin,[1,2,*] D. G. Polyakov,[1,†] F. Evers,[1] and P. Wölfle[1,2]

[1]*Institut für Nanotechnologie, Forschungszentrum Karlsruhe, 76021 Karlsruhe, Germany*

[2]*Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, 76128 Karlsruhe, Germany*

(Received 5 April 2001; published 4 September 2001)

We study the quasiclassical magnetotransport of noninteracting fermions in two dimensions moving in a random array of strong scatterers (antidots, impurities, or defects) on the background of a smooth random potential. We demonstrate that the combination of the two types of disorder induces a novel mechanism leading to a strong negative magnetoresistance, followed by the saturation of the magnetoresistivity $\rho_{xx}(B)$ at a value determined solely by the smooth disorder. Experimental relevance to the transport in semiconductor heterostructures is discussed.

DOI: 10.1103/PhysRevLett.87.126805 PACS numbers: 73.40.–c, 05.60.Cd, 73.50.Jt

Magnetotransport in a two-dimensional electron gas (2DEG) has been the subject of intensive research during the last two decades. This interest has been motivated by the progress in preparing high-quality semiconductor heterostructures, opening up new areas in both fundamental physics and applications; see [1] for a review. Within the quasiclassical approach (valid for not too strong magnetic fields B), impurity scattering is commonly described by a collision integral in the Boltzmann equation. This leads, for an isotropic system, to the B-independent Drude value of the longitudinal resistivity, $\rho_{xx}(B) = \rho_0 \equiv m/e^2 n_e \tau$, where n_e is the carrier density, m the effective mass, and τ the transport scattering time.

It has become clear, however, that this description is not always valid. In particular, in the case of smooth disorder memory effects induce a strong positive magnetoresistance (MR) [2] followed by an exponential falloff of $\rho_{xx}(B)$ due to adiabatic localization of drifting electrons [3,4]. To our knowledge, these effects have not been experimentally observed in the electron transport in low magnetic fields, since the Shubnikov–de Haas oscillations develop at lower B. On the other hand, the memory effects do show up in the composite fermion transport, explaining the peculiar shape of the MR around half filling of the lowest Landau level [2,4].

In the present paper we study the quasiclassical MR of a 2DEG moving in a random array of rare strong scatterers (modeled by hard disks) and subject additionally to a smooth random potential. Apart from the purely theoretical interest, our work has been motivated by two types of experimental realizations of this problem. The first one is random antidot arrays. Experiments on this kind of structure [5–8] show a strong negative MR which has not been analyzed theoretically. Less obviously, our model is relevant to transport in the unstructured high-mobility 2DEG. To clarify this point, we recall that in order to increase the 2DEG mobility the donors in currently fabricated heterostructures are separated by a large distance $d \gg k_F^{-1}$ (with k_F the Fermi wave number) from the 2DEG plane. It is usually assumed that these remote donors constitute the main source of disorder, inducing small-angle scattering of electrons. It is known, however, that in samples with a wide spacer ($d \gtrsim 70$ nm) large-angle scattering on residual impurities [9–11] and interface roughness [10] becomes important, limiting the mobility with further increasing width of the spacer. In particular, Ref. [10] concludes that about 50% of the resistivity is due to such scattering processes, while Ref. [11] finds that for samples with very high mobility this value is as high as 90%.

We thus consider the following two-component model of disorder: (i) randomly distributed hard-core scatterers (which we will term "antidots" or "impurities" below) with density n_S and radius a (where $n_S^{-1/2} \gg a \gg k_F^{-1}$), and (ii) smooth random potential (correlation radius d, momentum relaxation rate τ_L^{-1}, transport mean free path $l_L = v_F \tau_L$). The mean free path for the scattering on antidots is $l_S^{(0)} \equiv v_F \tau_S^{(0)} = 1/2n_S a$, while the corresponding transport mean free path may be somewhat different, $l_S \equiv v_F \tau_S = \gamma l_S^{(0)}$ with $\gamma \sim 1$ ($\gamma = 3/4$ in the model of specularly reflecting disks). We will set $\gamma = 1$ for qualitative estimates. We will further assume that $\tau_L \gg \tau_S$, so that the zero-B resistivity ρ_0 is determined by the hard scatterers, $\tau^{-1} = \tau_L^{-1} + \tau_S^{-1} \simeq \tau_S^{-1}$. Finally, we will assume that the motion in the smooth disorder is not adiabatic, i.e., has the form of the guiding center diffusion (rather than drift). The condition for this is $\delta \gg d$, where $\boldsymbol{\delta}$ is the guiding center shift after one cyclotron revolution [see Eq. (2)]. For currently fabricated samples this assumption is usually satisfied in the whole range of applicability of the quasiclassical theory.

We start the analysis of the problem by recalling the results [12,13] for the case when only hard scatterers are present ($\tau_L = \infty$), known as the Lorentz gas. In this limit the resistivity $\rho_{xx}(B)$ reads

$$\rho_{xx}(B)/\rho_0 = (1 - e^{-2\pi/\omega_c \tau_S^{(0)}}) \mathcal{F}(\omega_c \tau_S), \qquad (1)$$

 0031-9007/01/87(12)/126805(4)$15.00

where $\omega_c = eB/mc$ is the cyclotron frequency and $\mathcal{F}(x)$ is a function of order unity with the asymptotics $\mathcal{F}(x \ll 1) = 1$ and $\mathcal{F}(x \gg 1) = \gamma$. The first factor in Eq. (1) is nothing else but the fraction of particles moving in rosettelike trajectories around the impurities and hitting a new impurity with the mean free time $\tau_S^{(0)}$. The rest of the particles do not hit scatterers at all. In the sequel, we will consider only classically strong magnetic fields, $\omega_c \gg \omega_\tau \equiv 2\pi/\tau_S^{(0)}$, where the resistivity (1) shows a $1/B$ falloff. Equation (1) is valid below the percolation threshold, $\omega_c < \omega_{\rm perc} = 1.67 v_F n_S^{1/2}$ (note that $\omega_{\rm perc}\tau_S \sim n_S^{-1/2}a^{-1} \gg 1$). For larger magnetic fields, $\omega_c \geq \omega_{\rm perc}$, the resistivity is exactly zero, since the rosettelike families of cyclotron orbits fail to form an infinite cluster.

Clearly, adding the long-range disorder will increase the diffusion constant D_{xx} and thus, in the limit $\omega_c\tau_S \gg 1$, the longitudinal resistivity, by setting free those particles which are localized in cyclotron orbits not hitting impurities. We will be interested in the case of a sufficiently strong smooth disorder modifying the result (1) in an essential way. Specifically, we will see below that new physics emerges in the regime $\delta \gg a$, where

$$\delta^2 \equiv \langle \boldsymbol{\delta}^2 \rangle = 4\pi l_L^2/(\omega_c\tau_L)^3. \tag{2}$$

Let us first outline this new physics on a qualitative level. Naively, one could think that for $\delta \gg a$ the resistivity takes its Drude value. Indeed, let us associate with the particle trajectory a strip of width $2a$ surrounding it. The particle will hit an impurity if the center of the latter is located within this strip. Clearly, in one cyclotron revolution the particle "explores" in this way the area $2v_F a \times 2\pi/\omega_c$, the same as it would explore in the same time for $B = 0$. For $\delta \gg a$ the area explored in the second revolution will overlap only weakly with that explored in the first one, so that one could think that the exploration rate is essentially the same as for $B = 0$, leading to the mean time $\simeq \tau_S^{(0)}$ between the collisions, and thus to $\rho_{xx} \simeq \rho_0$. This consideration is, however, incorrect, since it neglects memory effects. Specifically, there is a probability $P_1 \sim a/\delta$ that the strip after the first revolution covers again the starting point (in other words, there is typically a small relative overlap $\sim a/\delta$). In view of the diffusive dynamics of the guiding center, its rms shift after n revolutions is $\delta_n = \delta\sqrt{n}$, so that the return probability decreases with the number of revolutions as $P_n = P_1/\sqrt{n}$. This formula is valid as long as δ_n is smaller than the cyclotron radius $R_c = v_F/\omega_c$, i.e., for $n \ll \omega_c\tau_L/2\pi$; we will see that the relevant values of n satisfy this condition in view of $\tau_L \gg \tau_S$. Therefore, the total return probability $P = \sum_{n=1}^{N} P_n$ is determined by the upper cutoff N, so that the memory effect is much stronger than one might expect, $P \sim (a/\delta)N^{1/2} \gg a/\delta$. A further indication of the importance of the memory effects comes from the observation that Eq. (1) does not match the Drude formula at $\delta \sim a$.

The cutoff N is given by the number of cyclotron revolutions it takes for the particle to hit the next impurity. When the memory effect leads only to a small correction to the Drude value (the condition will be specified below), the characteristic value of N is $N = \omega_c\tau_S^{(0)}/2\pi$, so that the total return probability is $P \sim (a/\delta)(\omega_c\tau_S)^{1/2}$. This determines the fraction of the area explored twice, implying an effective reduction of the exploration rate and thus a negative correction to the resistivity,

$$\Delta\rho_{xx}/\rho_0 \sim -(a/\delta)(\omega_c\tau_S)^{1/2} \sim -(\omega_c/\omega_0)^2, \tag{3}$$

where $\omega_0 \sim v_F(a^2 l_S l_L)^{-1/4} \sim \omega_{\rm perc}(l_S/l_L)^{1/4} \ll \omega_{\rm perc}$. Note that the sign of MR is opposite to the case of one-scale smooth disorder [2]: the returns *increase* the rate of scattering in [2], whereas they make the time between collisions with different strong scatterers longer, thus *decreasing* the scattering rate in the present case.

We turn now to a more rigorous and quantitative derivation. Generalizing the formalism of [2], we start from the Liouville-Boltzmann equation for the distribution function $g(\mathbf{r}, \phi)$ of electrons on the Fermi surface,

$$(L_0 + \delta L)g(\mathbf{r}, \phi) = \cos(\phi - \phi_E), \tag{4}$$

$$L_0 = v_F\mathbf{n}\nabla + \omega_c\frac{\partial}{\partial\phi} - \frac{1}{\tau_L}\frac{\partial^2}{\partial\phi^2}, \qquad \delta L = -\sum_i I_{\mathbf{R}_i}, \tag{5}$$

where $\mathbf{n} = (\cos\phi, \sin\phi)$ is the unit vector determining the direction of velocity, $\mathbf{v} = v_F\mathbf{n}$, and ϕ_E is the polar angle of the electric field. The operator L_0 describes the motion in the smooth random potential, while δL corresponds to the scattering on antidots with (random) positions $\mathbf{R}_i$. The explicit form of the collision operator $I_{\mathbf{R}}$ (for a hard-wall scatterer) can be found in [14], but we will not need it. We will only use the following properties of $I_{\mathbf{R}}$: (i) it is nonzero only within the distance a from the impurity location $\mathbf{R}$, and (ii) its Fourier-transform $\tilde{I}_{\mathbf{q}}$ satisfies $\tilde{I}_0\mathbf{n} = (-1/n_S\tau_S)\mathbf{n}$.

Expanding in δL, averaging over the positions $\mathbf{R}_i$ of scatterers, and resumming the series, one finds the Green's function of the Liouville operator, $\langle(L_0 + \delta L)^{-1}\rangle = (L_0 + M)^{-1}$, where M is the "self-energy" operator. The resistivity is expressed in terms of M as follows [2]:

$$\rho_{xx} = (m/e^2 n_e)(\tau_L^{-1} + M_{xx}), \tag{6}$$

where $M_{xx} = \int(d\phi/\pi)\cos\phi M\cos\phi$. One may use a diagrammatic technique analogous to that developed for the Lorentz gas [14,15] to calculate M. The leading term corresponds to a single scattering process, yielding $M^{(1)} = -n_S\tilde{I}_0$ and thus the Drude contribution, $M_{xx}^{(1)} = \tau_S^{-1}$, to the resistivity (6). The next-order contribution represents the correction induced by the return process,

$$M_{xx}^{(2)} = -n_S\int\frac{d\phi}{\pi}\cos\phi I_{\mathbf{R}}D^{(1)}I_{\mathbf{R}}\cos\phi, \tag{7}$$

where $D^{(1)} = (L_0 + M^{(1)})^{-1}$ is the electron propagator with the leading-order self-energy included.

Since $\delta \gg a$, the propagator $D^{(1)}(\mathbf{r} - \mathbf{r}', \phi, \phi')$ describing propagation from the point $\mathbf{r}', \phi'$ to the point $\mathbf{r}, \phi$ can be replaced in (7) by $D^{(1)}(0, \phi, \phi')$. Furthermore, we note that once the particle hits an impurity, its guiding center is shifted by an amount of order of R_c. As a result, the contribution of such trajectories to $D^{(1)}$ can be neglected and only noncolliding orbits should be taken into account. Since the motion without collisions is limited by the times $\sim\tau_S \ll \tau_L$, the particle will return with almost the same direction of velocity, i.e., $D^{(1)}(0, \phi, \phi')$ will be peaked at $\phi \approx \phi'$. We can thus approximate $D^{(1)}(0, \phi, \phi')$ by $D_0^{(1)}\delta(\phi - \phi')$, where $D_0^{(1)} = \int d\phi D^{(1)}(0, \phi, \phi')$. This quantity is easily found to be

$$D_0^{(1)} = \sum_{n=1}^{\infty} \frac{e^{-2\pi n/\omega_c \tau_S^{(0)}}}{(\pi n)^{1/2} v_F \delta} \simeq \frac{(\omega_c \tau_S^{(0)})^{1/2}}{(2\pi)^{1/2} v_F \delta}. \tag{8}$$

Substituting this in (7) and using that $\tilde{I}_0 \mathbf{n} = -(1/n_S \tau_S)\mathbf{n}$, we find $M_{xx}^{(2)} = -(1/n_S \tau_S^2) D_0^{(1)}$. This implies, according to (6), the negative MR,

$$\Delta\rho_{xx}/\rho_0 = M_{xx}^{(2)}/\tau_S^{-1} = -\omega_c^2/\omega_0^2, \qquad \omega_c \ll \omega_0, \tag{9}$$

$$\omega_0 = (2\pi n_S)^{1/2} v_F (2\gamma l_S/l_L)^{1/4}, \tag{10}$$

in agreement with the above qualitative considerations.

This derivation is valid as long as the correction remains small, i.e., for $\omega_c \ll \omega_0$. For stronger magnetic fields, when the quantity P defined above becomes large, $P \gg 1$, it acquires the meaning of the number of returns. It should then be found self-consistently. Specifically, the sum over n is now cut off at the time $\tau_S' \sim P\tau_S$, since multiple returns to the same area lead to a corresponding increase of the time needed to hit a new impurity. We thus get a self-consistency equation $P \sim (a/\delta)(\omega_c P \tau_S)^{1/2}$, yielding a $1/B^4$ drop of the resistivity,

$$\rho_{xx}/\rho_0 \sim \tau_S/\tau_S' \sim 1/P \sim (\omega_0/\omega_c)^4, \qquad \omega_c \gg \omega_0. \tag{11}$$

It remains to analyze the conditions of validity of Eqs. (9) and (11). First of all, we assumed that the particle finds a new scatterer by exploring the new area in the course of the diffusive motion of the guiding center. There exists, however, another mechanism, namely that of the rosettelike motion around a scatterer, which determines the transport in the pure Lorentz gas ($l_L = \infty$). Comparing (11) with the Lorentz-gas result $\rho_{xx}/\rho_0 = 2\pi\gamma^2/\omega_c\tau_S$ [see Eq. (1)], we find that the two formulas match at $\delta \sim a$. The corresponding crossover frequency is $\omega_{\text{cross}} = v_F(4\pi n_S^2 l_S^2 l_L^{-1})^{1/3}$. Second, we assumed $n_S R_c^2 \gg 1$, or, equivalently, $\omega_c \ll \omega_{\text{perc}}$. It is easy to see that in the opposite limit, $n_S R_c^2 \ll 1$, the resistivity will be determined by the smooth disorder, with scattering on antidots giving a small $[\sim(\omega_{\text{perc}}/\omega_c)^2]$ correction only, so that $\rho_{xx}(B)$ will have a plateau with the value $\rho_{xx}(\omega_c \gg \omega_{\text{perc}}) = m/e^2 n_e \tau_L$.

Comparing the characteristic frequencies, ω_τ, ω_0, ω_{cross}, and ω_{perc}, we conclude that the following three situations can be distinguished, depending on the strength of the smooth disorder (Fig. 1): (A) $\omega_0 \ll \omega_\tau$, or, equivalently, $l_L/l_S \gg (1/2\pi^2)(n_S l_S^2)^2$. In this case the smooth disorder hardly affects the Lorentz-gas result (1). (B) $\omega_\tau \ll \omega_0 \ll \omega_{\text{cross}} \ll \omega_{\text{perc}}$, or, equivalently, $2.7(n_S l_S^2)^{1/2} \ll l_L/l_S \ll (1/2\pi^2)(n_S l_S^2)^2$. This is an intermediate situation; the resistivity drops first according to Eqs. (9) and (11), and then crosses over at $\omega_c \sim \omega_{\text{cross}}$ to the Lorentz-gas behavior (1). (C) $\omega_\tau \ll \omega_0 \ll \omega_{\text{perc}} \ll \omega_{\text{cross}}$, or, equivalently, $l_L/l_S \ll 2.7(n_S l_S^2)^{1/2}$. In this case the Lorentz-gas behavior (1) is completely destroyed, and the results (9), (11) hold in the whole range of ω_c below ω_{perc}.

For $\omega_c > \omega_{\text{perc}}$ the resistivity shows in all the cases a plateau, as explained above. [For curve B on Fig. 1 the saturation value $\rho_{xx}/\rho_0 \simeq 0.1(\omega_0^{(B)}/\omega_{\text{perc}})^4 \sim 10^{-3}$ is too small to be seen on the scale of the figure.] On the side of strong magnetic fields this plateau will be modified either by entering into the adiabatic regime (preceded by the positive MR studied in [2]) at $\delta \sim d$, meaning $\omega_c \sim \omega_{\text{ad}} = v_F(4\pi/d^2 l_L)^{1/3}$ (its implications for the present problem will be considered elsewhere [16]) or by the development of Shubnikov–de Haas oscillations.

It is worth mentioning that while we used the condition $\tau_L \gg \tau_S$ for the derivation of our main results, a pronounced negative MR will also be observed for $\tau_L \sim \tau_S$ (which seems to be frequently the relevant situation for high-mobility structures [10]). In this case there is a crossover from $\rho_{xx}(0) = (m/e^2 n_e)(\tau_L^{-1} + \tau_S^{-1})$ to $\rho_{xx}(\omega_c \gg \omega_{\text{perc}}) = m/e^2 n_e \tau_L$ which takes place around $\omega_c \sim \omega_{\text{perc}}$ (note that $\omega_0 \sim \omega_{\text{perc}}$ for $\tau_L \sim \tau_S$).

We have performed numerical simulations of the MR by fixing parameters of the Lorentz gas ($\omega_{\text{perc}}/\omega_\tau = 5.3$) and the correlation length of the smooth disorder ($d/a \sim 2.5$)

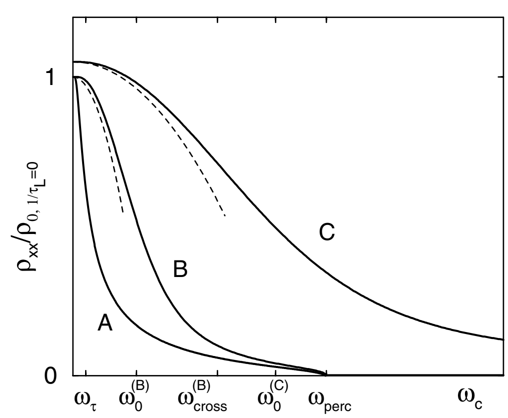

FIG. 1. Magnetoresistivity at fixed τ_S and different τ_L: curve A—Lorentz gas ($\tau_L = \infty$); curves B and C with $\tau_L^{(B)} > \tau_L^{(C)}$ correspond to regimes B and C, respectively (see text). The dotted lines denote the asymptotics (9).

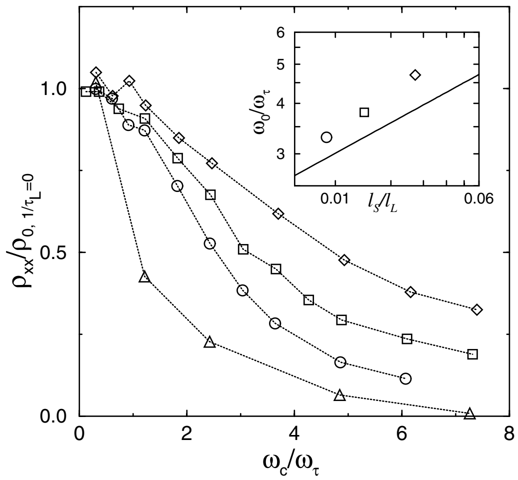

FIG. 2. Magnetoresistivity at fixed τ_S and different τ_L from numerical simulations; $\tau_L/\tau_S = \infty$ (Lorentz gas, △), 111 (○), 70 (□), 37 (◇). Inset: ω_0 determined from the fit to Eq. (9); the full line corresponds to the analytical result (10).

and changing the strength of the latter. The results (Fig. 2) are in good agreement with the analytical predictions. It is seen that a very weak smooth disorder (giving negligible contribution to ρ_0) affects crucially the MR. Some deviations in the values of ω_0 from Eq. (10) (see inset) can be attributed to the conditions $\omega_\tau \ll \omega_0 \ll \omega_{\rm perc}, \omega_{\rm ad}$ being met only marginally in our simulations.

Finally, let us estimate the characteristic values of B for existing experiments. Typical parameters in the experiments [7] on the antidot arrays were $n_e = 5 \times 10^{11}$ cm^{-2}, $n_S = (0.6\ \mu\text{m})^{-2}$, $l_S = 1.3\ \mu$m, $l_L = 16\ \mu$m. This implies the following values for the characteristic magnetic fields [in the obvious notations $B_\tau = (mc/e)\omega_\tau$, etc.]: $B_\tau \simeq 0.5$ T, $B_0 \simeq B_{\rm perc} \simeq 0.3$ T. We see that the condition of a dilute antidot array assumed above, $B_\tau \ll B_{\rm perc}$, is not met. Clearly, the above formulas are not then valid quantitatively. On the qualitative level, we can conclude that there should be a strong falloff of ρ_{xx} around $B \approx 0.3 \div 0.5$ T, in agreement with experimental results [7]. A similar negative MR was observed in other experiments with disordered antidot arrays [5,6,8].

We turn now to unstructured high-mobility samples. Using the parameters of [11] ($n_e = 2 \times 10^{11}$ cm^{-2}, mobility $\mu = 10^7$ cm^2/V $\cdot$ s), we find the mean free path $l \simeq 80\ \mu$m. Let us assume, following the conclusion of [11], that the zero-field mobility is determined by background impurities (i.e., $l \simeq l_S$), while $l_L \simeq 10 l_S$. Using the typical volume concentration of the residual impurities [10], $n_S^{(3D)} \simeq 2.5 \times 10^7$ cm^{-3}, and the value of the Bohr radius in GaAs, $a_B \simeq 10$ nm, we estimate the sheet density of strong scatterers as $n_S \sim a_B n_S^{(3D)} \simeq (2\ \mu\text{m})^{-2}$. With these parameters, we find $B_\tau \simeq 5$ mT, $B_0 \simeq B_{\rm perc} \simeq 60$ mT. The condition of the diluted array of scatterers, $B_\tau \ll B_{\rm perc}$, is now well satisfied. Since $B_0 \simeq B_{\rm perc}$, the Lorentz-gas behavior (1) is fully destroyed [the case (C) in our classification], and the negative MR is determined by the interplay of smooth disorder and strong scatterers, as described above. Note that though parametrically $B_0 \ll B_{\rm perc}$ for $l_L \gg l_S$, in practice their values are very close, since $B_0/B_{\rm perc} \simeq (10 l_S/l_L)^{1/4}$. The predicted negative MR may be used for the experimental determination of the ratio l_L/l_S. Strong negative MR has been observed in very-high-mobility samples [11,17], in qualitative agreement with our theory. A more detailed experimental check of our predictions would be desirable.

In conclusion, we have studied the quasiclassical magnetotransport of a 2DEG with smooth disorder and rare strong scatterers. Interplay of these two types of disorder leads to a novel mechanism of strong negative MR; the latter is shown to saturate with increasing B at a value determined solely by smooth disorder. The results are relevant to experiments on transport in dilute antidot arrays, as well as in high-mobility heterostructures with background impurities or interface imperfections.

We thank M. Heiblum, J. Smet, V. Umansky, and D. Weiss for discussions of experimental results, which motivated this research. This work was supported by the SFB 195 and the Schwerpunktprogramm "Quanten-Hall-Systeme" der Deutschen Forschungsgemeinschaft.

*Also at Petersburg Nuclear Physics Institute, 188350 St. Petersburg, Russia.
†Also at A.F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia.

[1] C.W.J. Beenakker and H. van Houten, Solid State Phys. **44**, 1 (1991); D.K. Ferry and S.M. Goodnick, *Transport in Nanostructures* (Cambridge University Press, Cambridge, United Kingdom, 1997).
[2] A.D. Mirlin *et al.,* Phys. Rev. Lett. **83**, 2801 (1999).
[3] M.M. Fogler *et al.,* Phys. Rev. B **56**, 6823 (1997).
[4] A.D. Mirlin, D.G. Polyakov, and P. Wölfle, Phys. Rev. Lett. **80**, 2429 (1998); F. Evers *et al.,* Phys. Rev. B **60**, 8951 (1999).
[5] G.M. Gusev *et al.,* J. Phys. Condens. Matter **6**, 73 (1994).
[6] K. Tsukagoshi *et al.,* Phys. Rev. B **52**, 8344 (1995).
[7] G. Lütjering, Ph.D. thesis, Stuttgart, 1996.
[8] G. Nachtwei *et al.,* Phys. Rev. B **55**, 6731 (1997).
[9] P.T. Coleridge, Phys. Rev. B **44**, 3793 (1991).
[10] T. Saku, Y. Horikoshi, and Y. Tokura, Jpn. J. Appl. Phys. **35**, 34 (1996).
[11] V. Umansky, R. de Picciotto, and M. Heiblum, Appl. Phys. Lett. **71**, 683 (1997).
[12] E.M. Baskin, L.N. Magarill, and M.V. Entin, Sov. Phys. JETP **48**, 365 (1978); E.M. Baskin and M.V. Entin, Physica (Amsterdam) **249B–251B**, 805 (1998).
[13] A.V. Bobylev *et al.,* Phys. Rev. Lett. **75**, 197 (1995); J. Stat. Phys. **87**, 1205 (1997).
[14] J.M.J. van Leeuwen and A. Weijland, Physica (Amsterdam) **36**, 457 (1967); A. Weijland and J.M.J. van Leeuwen, Physica (Amsterdam) **38**, 35 (1968).
[15] E.H. Hauge in *Transport Phenomena,* edited by G. Kirczenow and J. Marro (Springer, Berlin, 1974), p. 337.
[16] D.G. Polyakov *et al.,* cond-mat/0104417.
[17] J. Smet (private communication); M. Heiblum and V. Umansky (private communication).