Universality of the Edge-Tunneling Exponent of Fractional Quantum Hall Liquids

Xin Wan,[1] F. Evers,[1] and E. H. Rezayi[2]

[1]*Institut für Nanotechnologie, Forschungszentrum Karlsruhe, 76021 Karlsruhe, Germany*
[2]*Department of Physics, California State University, Los Angeles, California 90032, USA*
(Received 22 September 2004; published 29 April 2005)

In a microscopic model of fractional quantum Hall liquids with electron-electron interactions and confinement, we calculate the edge Green's function via exact diagonalization. Our results for $\nu = 1/3$ and $2/3$ suggest that, in the presence of Coulomb interaction, "external" parameters such as the sharpness of the edge and the strength of the edge confining potential, which can lead to edge reconstruction, may cause deviations from universality in the edge-tunneling $I - V$ exponent. In particular, we do not find any direct dependence of this exponent on the range of the interaction potential as suggested by recent calculations in contradiction to the topological nature of the edge.

DOI: 10.1103/PhysRevLett.94.166804 PACS numbers: 73.43.Jn, 73.43.Lp

The fractional quantum Hall (FQH) effect [1] occurs in a clean two-dimensional electron system in a strong magnetic field. It is formed at certain rational filling factors ν: the ratio of the electron density to the magnetic flux density. It is a striking manifestation of strongly correlated physics that leads to a novel incompressible fluid phase of matter with fractionalized quantum numbers [2]. Wen [3] has argued that Hall fluids, which lack any kind of local order parameter associated with a broken symmetry, nonetheless possess topological order. This new order is more subtle and hence less amenable to a direct experimental probe. It can manifest itself in the ground state degeneracies (when the system obeys toroidal boundary conditions) that survive disorder or other symmetry breaking perturbations. Topological order has a wider generality and may appear in other highly correlated systems [4]. The nature of and the transitions among such phases of matter are subjects of considerable current interest.

Another manifestation of the topological order is the edge structure of the quantum Hall fluids, which is determined by the gapped state in the bulk. In particular, for Abelian Hall states the gapless edge deformations form chiral Luttinger liquids [5] (CLLs). Thus differences in the topological order can be discerned from the edge physics. Tunneling characteristic at the edge has long been regarded as an experimental means of measuring the topological order. For tunneling from a three-dimensional (3D) Fermi liquid, CLL theory leads to a non-Ohmic tunneling I-V relation $I \sim V^{\alpha}$, in sharp contrast to the Ohmic prediction of a Fermi-liquid-dominated edge.

For the Hall states at $\nu = n/(np + 1)$ (where n is a nonzero integer and p is a positive even integer), the edge for $n > 0$ does not contain counter propagating modes and the exponent is $\alpha = p + 1$, independent of n. The situation is more complicated for $n < 0$ where counter propagating modes can be backscattered. However, in the presence of strong disorder, the exponent takes on the universal value $\alpha = p + 1 - 2/|n|$ [6]. While experiments [7–11] have confirmed the nontrivial power law behavior, they do not completely agree with CLL values [5]. In particular, Grayson *et al.* found a continuous exponent $\alpha \simeq 1.16/\nu - 0.58$ without plateaus for $1/\nu > 1.4$ regardless of electron mobility, carrier density, and tunneling barrier thickness. However, more recently, Hilke *et al.* observed $\alpha \simeq 2.0/\nu - 0.55$ for $0.75 < 1/\nu < 1.75$ in samples with low 3D Fermi energies. These results suggest that the subtleties of the edge may be relevant to the edge-tunneling exponent, in addition to the bulk filling fraction.

Earlier attempts [12–16] to resolve the apparent discrepancy between experiment and theory have been summarized in [17]. Many of these approaches have invoked additional physics within the standard theory to address this shortcoming rather than abandoning the basic CLL picture. One such addition arises from the presence of a positive background charge. On purely electrostatic grounds the electron density near the edge may become quite different from that of an ideal edge [9]. This effect can even lead to the reconstruction of the edge [18–21] provided the background charge is sufficiently far from the electron layer (which is usually the case in cleaved-edge samples). As a consequence, the tunneling characteristics could become sensitive to the edge profile and the universal tunneling exponents may not necessarily be observed.

Meanwhile, recent studies [22–24] attribute the nonuniversality of α to the range of the electron-electron interactions. Tsiper and Goldman (TG) studied the edge wave function using exact diagonalization in the presence of Coulomb interaction [22]. They concluded that the tunneling exponent depends on the range of the interactions. Crucial to their conclusion is the assumption that the exponent α may be obtained from the ratio of the electron occupation numbers of the two outermost occupied orbitals for the corresponding Laughlin state in the disk geometry, i.e., $\alpha = \rho(m^L_{\max} - 1)/\rho(m^L_{\max})$. This relation, however, has been derived only in the case of ultrashort-range interactions and its validity for the more generic finite range case is not obvious.

Using composite fermion (CF) theory [25], Mandal and Jain [23,24] (MJ) have arrived at essentially the same conclusion. These authors adopted a hard edge by cutting off angular momentum larger than $m_{\max} = 3(N-1)$ for $\nu = 1/3$ and, as TG, ignored the background charge. They found that for the ultrashort-range potential (which produces the Laughlin state), the asymptotic edge Green's function exponent agreed with CLL theory. On the other hand, for generic potentials, in particular the Coulomb potential, a substantial reduction of the exponent from the CLL value of 3 was observed. MJ attributed this reduction to the residual repulsion (beyond their hard-core) among the composite fermions generated by the long-range Coulomb potential. For $\nu = 1/3$, the exponent is below 2.5 and even larger reductions were found for $\nu = 2/5$ and $3/7$.

These results are at odds with the predictions of CLL and, by implication, with one of the most crucial elements of the FQH physics itself, namely, the concept of topological order. The unusual properties of the chiral edge liquid is understood to be the signature of the topological structure of the bulk and therefore ought to persist as long as the bulk exhibits the FQH effect [3]. Hence, one expects the same exponent irrespective of the range of the interactions so long as the bulk physics remains the same.

In this Letter we show that there are no fundamental contradictions with CLL and/or the topological order of FQH states. In the presence of long-range Coulomb interaction, our findings suggest that the details of the edge confinement is highly relevant to understanding the behavior of the edge-tunneling exponent. We first address the edge exponent in a system with long-range Coulomb interactions in the absence of neutralizing background charge. To this end we evaluate the edge Green's function by exact diagonalization in a microscopic model of the FQH liquids. We impose an edge confining potential by restricting the single-particle angular momentum to be $\leq m_{\max}$. Our results are as follows: We find that, for $\nu = 1/3$, the tunneling exponent remains unchanged with Coulomb interaction for soft edges (large $m_{\max}$). This is in sharp contrast to the reduction of α as found previously by MJ for hard edge confinement (small $m_{\max}$). We then investigate the effect of the edge potential induced by background charge in the presence of long-range interactions. For $\nu = 1/3$ and strong confining potential, we obtain the universal value of α even for hard edges. For weak confining potential, we again observe substantial deviations from the universal value for hard edges, which may be highly relevant to the experimental studies [8–10]. We also find finite-size corrections to α for soft edges, consistent with the edge reconstruction scenario [18–20]. For $\nu = 2/3$, we find behavior consistent with strongly coupled edges for strong confining potential and with a dominant $\nu = 1$ edge for weak confining potential.

We consider a microscopic model of a two-dimensional electron gas (2DEG) confined to a two-dimensional disk with neutralizing background charge distributed uniformly on a parallel disk of radius a, at a distance d above the 2DEG. The choice of $a = \sqrt{2N/\nu}$ guarantees that the disk encloses N electrons and exactly N/ν magnetic flux quanta for the desired filling factor ν. The bare Coulomb interaction between the background charge and the electrons gives rise to the confining potential. We consider electrons confined to the lowest Landau level (LL) in the symmetric gauge, and study the following Hamiltonian:

$$H = \frac{1}{2}\sum_{mnl} V_{mn}^{l} c_{m+l}^{\dagger} c_{n}^{\dagger} c_{n+l} c_{m} + \sum_{m} U_{m} c_{m}^{\dagger} c_{m}, \tag{1}$$

where $c_m^{\dagger}$ is the electron creation operator for the lowest LL single electron state with angular momentum m. V_{mn}^{l} is the matrix element of the Coulomb interaction [26] in the symmetric gauge. U_m is the matrix element of the rotationally invariant confining potential due to the positive background charge [20]. In this Letter the distances are measured in units of the magnetic length $\ell_B = \sqrt{\hbar/eB}$.

We diagonalize the Hamiltonian to obtain the exact many-body ground state ψ using the Lanczos algorithm. We then calculate the equal-time edge Green's function,

$$G_{\rm edge}(\mathbf{r}-\mathbf{r}') = \frac{\langle\psi|\Psi_e^{\dagger}(\mathbf{r})\Psi_e(\mathbf{r}')|\psi\rangle}{\langle\psi|\psi\rangle}, \tag{2}$$

where $\Psi_e^{\dagger}(\mathbf{r})$ and $\Psi_e(\mathbf{r}')$ are field operators, which create and annihilate an electron at $\mathbf{r}$ and $\mathbf{r}'$, respectively, on the edge of the 2DEG disk with a radius of R and $|\mathbf{r}-\mathbf{r}'| = 2R\sin(\theta/2)$. The choice of R is not crucial and will be specified later. In the large $|\mathbf{r}-\mathbf{r}'|$ limit, the edge Green's function is expected to exhibit the asymptotic behavior

$$|G_{\rm edge}(\mathbf{r}-\mathbf{r}')| \sim |\mathbf{r}-\mathbf{r}'|^{-\alpha} \propto |\sin(\theta/2)|^{-\alpha}. \tag{3}$$

Because of the relativistic invariance of CLL, the equal-time and equal-distance exponents of the Green's function are equal; the latter is measured in tunneling experiments.

For comparison, we first consider the ultrashort-range hard-core potential, for which the Laughlin state is the exact ground state. We do not include the background confining potential, but choose the ground state with the appropriate total angular momentum. Figure 1(a) shows the edge Green's function ($R = \sqrt{2N/\nu}$) for the Laughlin state with 6–9 electrons at filling fraction $\nu = 1/3$. We use least-squares fits to match our data to the power law $|G(\theta)| \sim |\sin(\theta/2)|^{-\alpha}$ close to $|\sin(\theta/2)| = 1$, and obtain $\alpha = 3.2 \pm 0.2$. The error bar reflects the dependence of α on system size and range of data to fit. This result is in good agreement with $\alpha = 3$ as predicted by the CLL theory. $|G(\theta)|$ for $N = 6$ shows weak oscillation around the power law fitting curve, but the finite-size effects become very weak for $N \geq 7$. In Fig. 1(b), we replot $|G|$ as a function of $|\mathbf{r}-\mathbf{r}'|$. We observe perfect scaling even for distances $|\mathbf{r}-\mathbf{r}'|$ as small as one magnetic length, which is a strong indication that finite-size effects are indeed negligible.

Next we consider the long-range Coulomb interaction. There is an important difference here with the ultrashort-

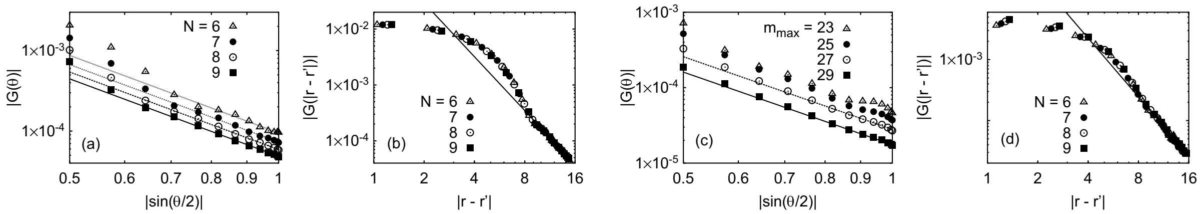

FIG. 1. The edge Green's function $|G|$ for the Laughlin state with 6–9 electrons at filling fraction $\nu = 1/3$ (a) as a function of $|\sin(\theta/2)|$ and (b) as a function of $|\mathbf{r} - \mathbf{r}'|$. (c) $|G(\theta)|$ for 8 electrons with Coulomb interaction confined to orbitals with the largest angular momentum $m_{\max} = 23$–29. (d) $|G(|\mathbf{r} - \mathbf{r}'|)|$ for $N = 6$–9 electrons with Coulomb interaction confined to orbitals with $m_{\max} = (N-1)/\nu + 5$ for $\nu = 1/3$. The lines in the log-log plots (a)–(d) correspond to a power law behavior with $\alpha = 3.2$.

range potential as far as the edge is concerned. In the Laughlin state the maxmimum angular momentum of a given electron cannot exceed $m^L_{\max} = (N-1)/\nu$. For more generic interaction potentials there will not be such a hard cutoff and therefore the basis set needs to be enlarged to find the angular momentum cutoff beyond which the properties of the system converge. Figure 1(c) plots $|G_{\text{edge}}|$ for the Coulomb interaction and $N = 8$ electrons at filling fraction $\nu = 1/3$ for an increasing number of orbitals ($m_{\max} + 1$ since we label from $m = 0$). We define the edge by choosing $R = \sqrt{2(m_{\max} + 1)}$ hereafter. For $m_{\max} < 26$ (hard edge), we find a weak oscillation of $|G_{\text{edge}}|$ even near the largest distance of the system. These oscillations are probably induced by the competition between the long-range interaction and the edge confinement. Similar oscillations, existing generically at other filling fractions, can also be observed in the electron density profile in the presence of the Coulomb interaction [18,22]. Therefore, fitting $|G_{\text{edge}}|$ to Eq. (3) to extract α may not produce an accurate exponent. On the other hand, for $m_{\max} > 26$ (soft edge), $|G_{\text{edge}}|$ can be fit very well by the power law with $\alpha = 3.2 \pm 0.2$, which is the same as the ultrashort-range interaction exponent. In Fig. 1(d), we again show a scaling plot of $|G|$ over $|\mathbf{r} - \mathbf{r}'|$ for $N = 6$–9 electrons and $m_{\max} = (N-1)/\nu + 5$ at $\nu = 1/3$, again for the Coulomb interaction. Even with long-range interactions, the data shows good scaling with only small deviations at length scales below 8. We note that the choice of $m_{\max}$ here is the same as in Ref. [22]. The difference in the exponent is caused by the manner in which it was determined. We have verified that the formula used by TG does not agree with the exponent in the Green's function.

So far we have excluded the background confining potential. Without the background charge, electrons tend to move to the edge to reduce their Coulomb repulsion. This seems to induce strong density oscillations near the edge, extending into the bulk rather than forming a roughly uniform droplet. Nor does this model conform to the experiments where a confining potential is always present. Figure 2(a) shows the edge Green's function for 8 electrons with $m_{\max} = 23$ (hard edge) with the corresponding confining potential for $\nu = 1/3$. For $d = 1.0$, where there is no edge reconstruction (strong confining potential), we find that $G(\theta)$ agrees very well with a power law of $G \sim |\sin(\theta/2)|^\alpha$ with $\alpha = 3.2 \pm 0.1$. This is equal to the exponent, for soft edges, in the absence of confining potential. In this case the background has largely mitigated the combined effects of the long-range repulsion and the hard edge confinement. However, this changes for $d > d_c \approx 1.5$, due to edge reconstruction. For $d = 1.8 > d_c$ (weak confining potential), $G(\theta)$ increases its value as a result of electrons moving closer to the edge and changes α to 2.2 ± 0.1. Again, one can see this qualitatively on the electrostatic level; the electron occupation ratio in the lowest Landau level near the edge is larger than $1/3$. We next relax the cutoff in angular momentum space and compare $G(\theta)$ for two different $m_{\max}$ for $d = 1.8$ [Fig. 2(b)]. For $m_{\max} = 29$ (soft edge), $G(\theta)$ shows a crossover from a power law with $\alpha \approx 5.0$ to one with $\alpha \approx 3.0$ near $|\sin(\theta/2)| \approx 0.75$. This suggests that the true asymptotic behavior in the reconstructed case can only be observed at a larger length scale. Such behavior agrees qualitatively with the edge reconstruction corrections to α at short distances: $\delta\alpha \propto v_\phi/v^2$ [19,27], where v_ϕ and v are velocities of neutral and charge modes, respectively.

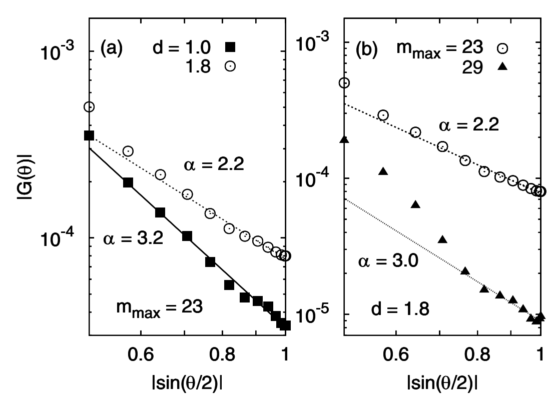

FIG. 2. The edge Green's function $|G|$ of 8 electrons at $\nu = 1/3$ with Coulomb interaction and the background charge confining potential for (a) $m_{\max} = 23$ before ($d = 1.0$) and after the edge reconstruction ($d = 1.8$), and (b) $m_{\max} = 23$ and 29 with $d = 1.8$. The straight lines are power law fits with $\alpha = 2.2$ and 3.2 in (a), and $\alpha = 2.2$ and 3.0 in (b), respectively.

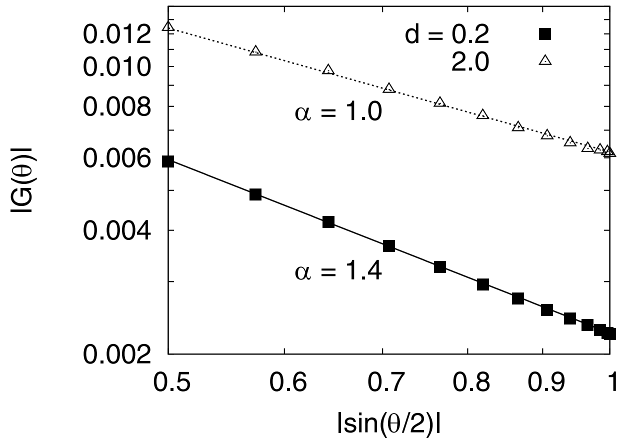

FIG. 3. The edge Green's function $|G|$ for $\nu = 2/3$ with $N = 18$ and $m_{\max} = 27$. The straight lines are power law fits with exponent $\alpha = 1.4$ and 1.0 for $d = 0.2$ and 2.0, respectively.

The significant drop in α for $\nu = 1/3$ in the case of hard edges corroborates the previous results for long-range interactions [22–24]. While it is impossible to determine with certainty what happens in the thermodynamic limit, we agree with the assessment of MJ that these reductions are not finite-size artifacts, notwithstanding the large distance oscillations we find in G. However, our results for soft edges appear to show that the nonuniversal behavior has more to do with the details of edge confinement than the range of the interaction potential. Indeed, Figs. 1 and 2 suggest that the edge confinement, through $m_{\max}$ as well as d, is relevant to α in the presence of long-range interactions. These issues are moot for the ultrashort-range interactions (unless $m_{\max} < (N-1)/\nu$, in which case the Laughlin state cannot even be realized). As pointed out in Ref. [24], the CF ground state with one CF exciton involves only single-particle states with $m \leq m_{\max} = 3(N-1)$, corresponding to the hard edge in our study. It would be interesting to find the precise CF state that would correspond to our soft edge profile.

We have also studied the behavior of the edge Green's function at other filling fractions, such as $\nu = 2/3$, which is not investigated in the CF approach of MJ. The $\nu = 2/3$ droplet can be regarded as a $\nu = 1/3$ hole droplet superimposed on a $\nu = 1$ electron droplet. It therefore supports an inner $\nu = 1/3$ edge and an outer $\nu = 1$ edge [28]. Figure 3 compares the edge Green's function for 18 electrons in 27 orbitals (hard edge) with the corresponding confining potential for $d = 0.2$ and 2.0. For strong confining potential ($d = 0.2$), we find, by fitting $G(\theta)$ to a power law, that $\alpha = 1.4$ regardless of $m_{\max}$. This is close to $1/\nu = 1.5$ and we speculate that the two counter propagating edge modes strongly couple and reconstruct into a dominant charge mode and a negligible neutral mode. On the other hand, for weak confining potential ($d = 2.0$), we find $\alpha = 1.0$, which probably is the fingerprint of the reconstructed outer edge of the $\nu = 1$ fluid.

We thank Matt Grayson, Jainendra Jain, Akakii Melikidze, Xiao-Gang Wen, and Kun Yang for helpful discussions. The work is supported by the Schwerpunkt-programme "Quanten-Hall-Systeme" der DFG and by US DOE under contract no. DE-FG03-02ER-45981.

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