Using Nonlinear Normal Modes for Execution of Efficient Cyclic Motions in Articulated Soft Robots ⋆

Cosimo Della Santina[1,2,3], Dominic Lakatos[1],
Antonio Bicchi[4,5], Alin Albu-Schaeffer[1,2]

[1] Institute of Robotics and Mechatronics, German Aerospace Center (DLR), Oberpfaffenhofen, Germany, alin.Albu-Schaeffer@dlr.de,
[2] Informatics and Mathematics Department, Technical University of Munich (TUM), Garching bei Mnchen, Germany
[3] Cognitive Robotics Department, Delft University of Technology, 2628 CD Delft, The Netherlands, cosimodellasantina@gmail.com,
[4] "Enrico Piaggio", University of Pisa, Largo Lucio Lazzarino 1, 56126 Pisa, Italy,
[5] Department of Advanced Robotics, Istituto Italiano di Tecnologia, via Morego, 30, 16163 Genova, Italy

Abstract. Thanks to their body elasticity, articulated soft robots promise to produce effective and robust oscillations with low energy consumption. This in turn is an important feature which can be exploited in the execution of many tasks, as for example locomotion. Yet, an established theory and general techniques allowing to excite and sustain these nonlinear oscillations are still lacking. A possible solution to this problem comes from nonlinear modal theory, which defines curved extensions of linear Eigenspaces called Eigenmanifolds. Stabilizing these surfaces is equivalent to exciting regular hyper-efficient oscillations in the robotic system. This paper proposes a first experimental validation of the Eigenmanifold stabilization technique. It also proposes a simple yet effective means of injecting energy into the system, so to sustain the oscillations in presence of damping. We consider as experimental setups a single robotic leg, and a full soft quadruped. Preliminary locomotion results are provided with both systems.

Keywords: Articulated Soft Robotics, Locomotion, Nonlinear Oscillations, Feedback Control

1 Introduction

Inspired by the vertebrate branch of the animal kingdom, articulated soft robots are robotic systems embedding compliant elements into a classic rigid (skeleton-like) structure [9,2]. Soft robots promise to push their limits far beyond the barriers that affect their rigid counterparts. However, existing control strategies aiming at achieving this goal are either tailored on specific examples [11,14], or rely on model cancellations [5,10] - thus defeating the purpose of introducing elasticity in the first place [8].

In a series of recent works [3,1,7,6], we have proposed a framework for exciting efficient oscillatory motions in robots immersed in a potential field. The long term aim

⋆ This work is supported by the European Union ERC project 835284 M-Runners.

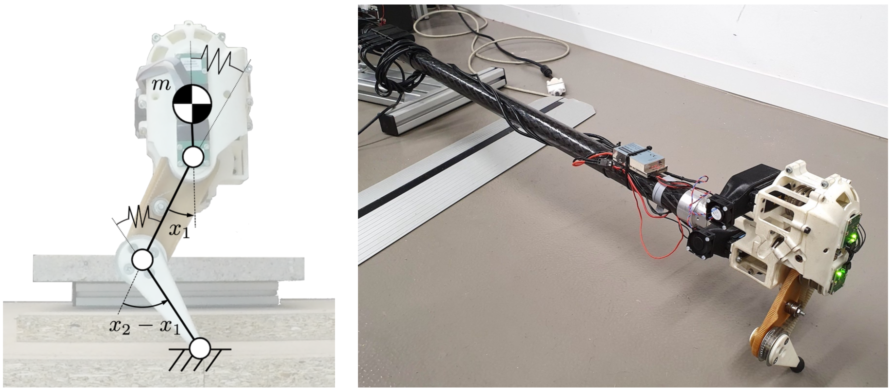


Fig. 1. Experimental setup: a 2-DoF (the upper part is constrained to stay vertical) segmented leg. The left panel shows also a sketch of the robot scheme with main quantities highlighted. The right panel shows the pole system constraining the upper part of the leg.

of this theory is to equip generic soft robots (of both articulated and continuum kind) to execute periodic tasks efficiently and effectively. A main component of this theory are Eigenmanifolds, that we defined in [1] as nonlinear continuations of the classic linear Eigenspaces. When the soft robot is initialized on one of these manifolds, it evolves autonomously while presenting regular - and thus practically useful - evolutions. We call these evolution normal modes. In addition to that, we proposed in [7] a control strategy making modal manifolds attractors for the system, and acting on the total energy of the soft robot to move it from a modal oscillation to the other. In this way, a large class of autonomous behaviors can be excited, which are direct expression of the embodied intelligence of the articulated soft robot. Finally, [1,7] describe a specialization of the theory to systems with strong (yet implementable) kinetic and potential symmetries.

Despite the core ideas behind our work coming from physical intuition and preliminary experimental validations [12], the formulation that we have provided so far is rather theoretical, and very much in need of an experimental validation. The aim of this paper is to provide such an experimental validation using as testbed the articulated soft leg in Fig. 1. We will introduce a simplified control strategy, and we will test its effectiveness on this system to implement swing-like oscillations. Preliminary application to the articulated soft robotic quadruped Bert [13] is also provided, showing that these techniques can be employed to implement locomotion.

2 Technical Approach

2.1 Eigenmanifold: a very concise definition

Building upon a theory laid down by more than one century of research in mathematics, physics, and engineering, in [1] we propose an extension of linear modes to robotics, which is then summarized in a coordinate dependent framework in [7]. We must give

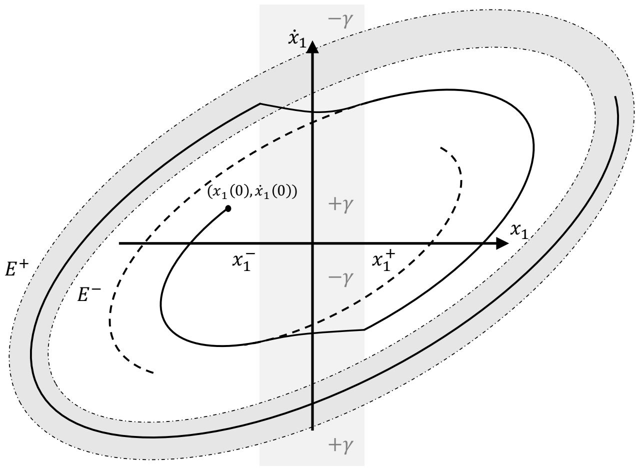


Fig. 2. The state evolves under the control action (6). When the system is in a neighborhood of equilibrium configuration $x_{\mathrm{m}} = 0$ (i.e. when it crosses the gray area), energy is injected by the controller, moving the system to another of its autonomous orbits. Eventually this brings the robot in the region of state space with the desired amount of energy.

those definitions for granted here, due to space limitations, and only provide an intuitive introduction to the concept. Consider a mechanical system in the standard form

$$M(x)\ddot{x} + C(x,\dot{x})\dot{x} + G(x) = \tau, \tag{1}$$

where $x \in \mathbb{R}^n$ are the configuration coordinates of the robot. $M(x), C(x,\dot{x}) \in \mathbb{R}^{n\times n}$ are the usual inertia and Coriolis matrices, and $G(x) \in \mathbb{R}^n$ is the potential field. $\tau \in \mathbb{R}^n$ are the control inputs. The total energy is

$$E(x,\dot{x}) = \frac{1}{2} x^{\mathrm{T}} M(x) x + V(x), \tag{2}$$

where $V(x)$ is the potential associated to $G(x)$.

An Eigenmanifold is a direct extension of an Eigenspace to this kind of nonlinear mechanical systems. It is defined by imposing to a curved surface many of the properties that define an eigenspace in the linear case. It is a two dimensional invariant submanifold of the state space $(x,\dot{x})$. It contains an equilibrium configuration of the robot. Also, it is such that any evolution contained in it is periodic, it has a trajectory which is line-shaped, and it is identified by its energy level. Consider an eigenspace of the linearized system at an equilibrium

$$ES = \mathrm{Span}\{(c,0),(0,c)\}, \tag{3}$$

with $c \in \mathbb{R}^n$. In [1] we show that we can always describe the Eigenmanifold prolonging this linear eigenspace as the set of states such that

$$X(x_{\mathrm{m}},\dot{x}_{\mathrm{m}}) = x, \quad \dot{X}(x_{\mathrm{m}},\dot{x}_{\mathrm{m}}) = \dot{x}, \tag{4}$$

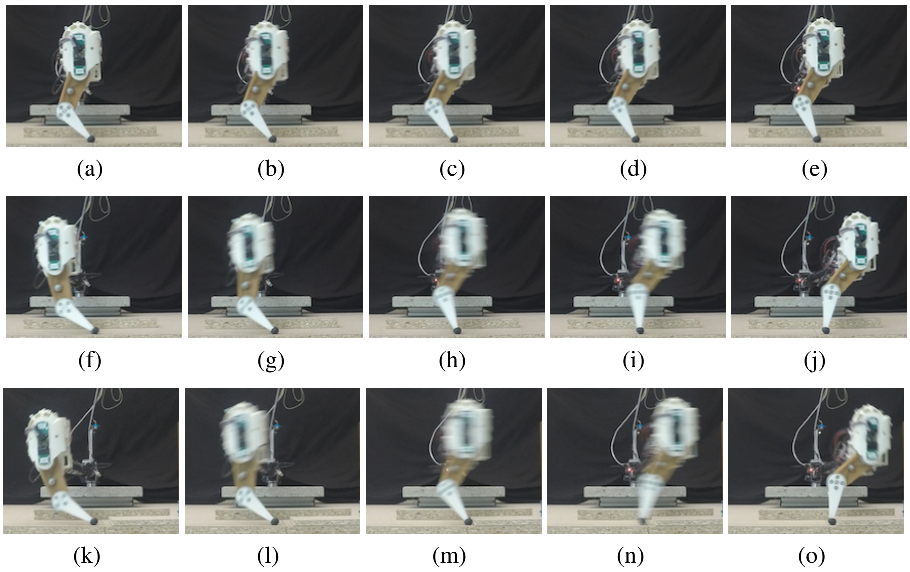


Fig. 3. Nonlinear oscillations induced by the proposed algorithm on a segmented soft leg. Panels (a-e) present one oscillation for $\alpha = 0.3$Nm, panels (f-j) for $\alpha = 0.7$Nm, and panels (k-0) for $\alpha = 0.9$Nm. The oscillations amplitude increase with the increase of the gain α.

where X and $\dot{X}$ are two functions from ES to $\mathbb{R}^n$ describing the manifold geometry - called coordinate embedding - and $(x_{\mathrm{m}}, \dot{x}_{\mathrm{m}}) = (c^{\mathrm{T}}x, c^{\mathrm{T}}\dot{x})$. In [1] we discuss how to extract $(X, \dot{X})$ from ES.

2.2 Exciting nonlinear oscillations with a simple feedback

Let $(X, \dot{X})$ be a coordinate embedding of an eigenmanifold. In [7] we proposed the following feedback control to excite nonlinear oscillations in a robot subject to a potential field

$$\tau(x,\dot{x}) = M(x)\left(\kappa_{\mathrm{p}}\left(X(x_{\mathrm{m}},\dot{x}_{\mathrm{m}}) - x\right) + \kappa_{\mathrm{d}}\left(\dot{X}(x_{\mathrm{m}},\dot{x}_{\mathrm{m}}) - \dot{x}\right) + \alpha\tau_{\mathrm{E}}(x,\dot{x},\bar{E})\right). \tag{5}$$

The idea is to make the modal manifold an attractor by means of a PD-like action (first two terms), and then pick the right oscillation among all the available ones through energy regulation (implemented by τ_{E}). The control gains are $\kappa_{\mathrm{p}}, \kappa_{\mathrm{d}}, \alpha \in \mathbb{R}$. The PD regulation is quite simple to implement, since X and $\dot{X}$ are available in closed form (e.g. as polynomial). Due to this simplicity, we experienced a high level of robustness when testing the technique in simulation. This has been very consistent across several kinds of uncertainties and many type of systems. With this paper, we want to double-check this insight experimentally.

The possibility of injecting or removing energy from the system allows to select the desired mode within the modal manifold, therefore increasing or decreasing the amplitude of oscillation (and/or its frequency). In [7], $\tau_{\mathrm{E}}(x,\dot{x},\bar{E})$ realizes energy regulation through the feedback loop $\dot{x}(\bar{E} - E(x,\dot{x}))$. This is a more complex component to implement since the energy is a transcendental function of the state, and fundamentally

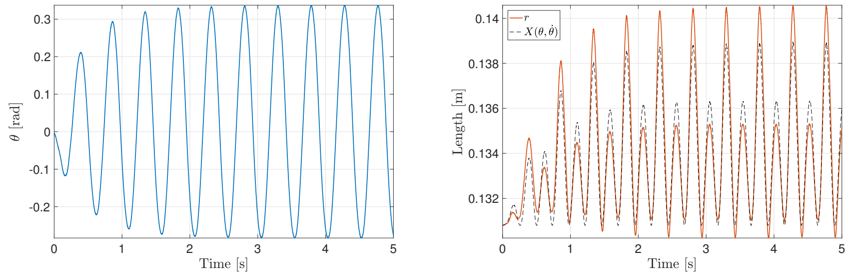
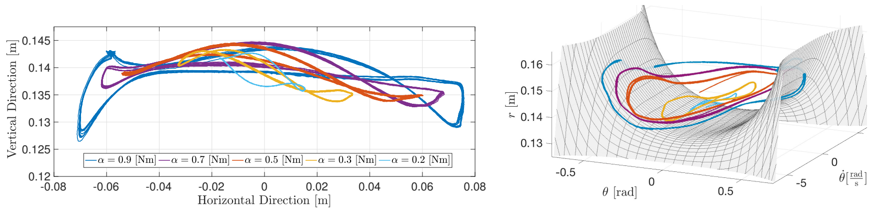


Fig. 4. Experimental evolutions of (θ, r), for $\alpha = 0.5$Nm. The right panel reports also the ideal evolution on the manifold $X(\theta, \dot{\theta})$ for the measured evolution of θ, as a dashed gray line.



(a) (b)

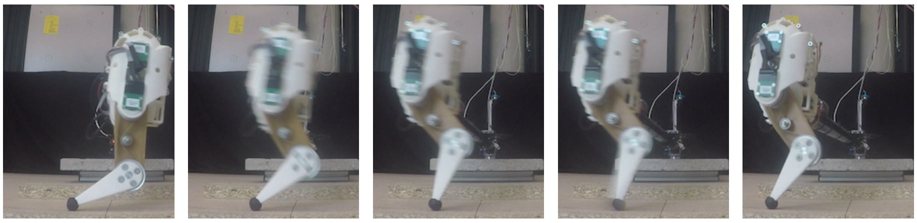
Fig. 5. Experimental trajectories Panel (a) shows the evolutions in Cartesian coordinates of the leg's center of mass. Panel (b) presents the same evolutions in the space $(\theta, \dot{\theta}, r)$. 15s of oscillations are considered. The ideal modal manifold $r = X(\theta, \dot{\theta})$ is superimposed.

realizes a form of strongly model dependent feedback. Since we want to bring this abstract theory on an experimental ground, we consider here an even simpler and more robust strategy. This is effectively the energy regulation introduced in [7], reduced to its most essential components

$$\tau_{\mathrm{E}} = \begin{cases} 0 \ \text{ if } x_{\mathrm{m}} \notin [x_{\mathrm{m}}^-, x_{\mathrm{m}}^+] \ \vee \ E(X, \dot{X}) \in [E^-, E^+] \\ 1 \ \text{ if } x_{\mathrm{m}} \in [x_{\mathrm{m}}^-, x_{\mathrm{m}}^+] \ \wedge \ ((E(X, \dot{X}) < E^- \wedge \dot{x}_{\mathrm{m}} > 0) \vee (E(X, \dot{X}) > E^+ \wedge \dot{x}_{\mathrm{m}} < 0)) \\ -1 \ \text{ otherwise} \end{cases}, \quad (6)$$

where $E^+ > E^- > 0$, and $x_{\mathrm{m}}^+ > 0 > x_{\mathrm{m}}^-$ are scalar constants. This controller idea is shown in Fig. 2. Similar to a swing which is kept in persistent oscillations by an occasion push in the right direction, the idea is to inject or remove energy in small chunks until $E(x(t))$ reaches $[E^-, E^+]$. Note indeed that the conditions selecting the sign of the torque are such that the change of energy $\dot{E} = \dot{x}^T \tau_{\mathrm{E}}$ is always positive when $E(X, \dot{X}) < E^-$ and negative viceversa. Note that this strategy is akin to the well-known swing up controller proposed in [4].

Although designed with the goal of being intuitive and robust, this control action makes the closed loop system hybrid. Therefore the actual proof of convergence will require some effort which is beyond the scope of the present paper. Our aim here is instead to give experimental substantiation to the whole idea of exciting complex nonlinear oscillations by means of simple feedback control actions stabilizing Eigenmanifolds, and see which kind of lessons we can learn from this validation.



(a) (b) (c) (d) (e)

Fig. 6. Stills of an articulated soft leg jumping forward by exploiting a nonlinear mode excited through a simple controller. Panels (a,b) show the stance phase, Panel (c) the take off, Panel (d) the flight phase, and Panel (e) the landing.

3 Experimental validation

3.1 Experimental setup

As a first experimental validation of the proposed strategy, we consider here the soft segmented leg in Fig. 1. It is made of two links with same length b - considered here massless - and a main body - with mass m. Linear revolute springs act on both joints (mechanism not shown in figure). Please refer to [12] for more details on this system. The leg is mechanically constrained to evolve on the Sagittal plane and the main body to remain vertical, by the pole shown in the right part of Fig. 1. We hypothesize infinite friction between the foot and the environment, so that the ground contact behavior is approximated with a revolute joint. Thus the configuration of the robot can be described by the two angles x_1 and x_2 in Fig. 1.

3.2 The model, the mode, and the control algorithm

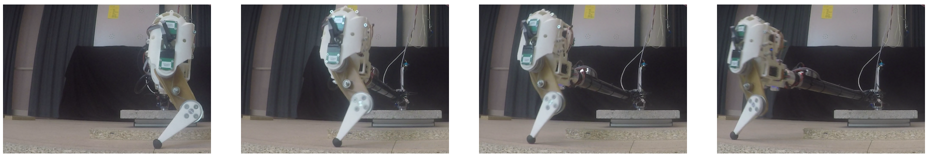
We are interested here in generating swing oscillations of the center of mass. We describe the system configuration through polar coordinates of the center of mass expressed w.r.t. the foot frame

$$\theta = x_1/2 + x_2/2, \quad r = b\sqrt{2\,(1+\cos(x_1 - x_2))}. \tag{7}$$

These two novel coordinates represent respectively the distance between hip and foot, and its angle w.r.t. the vertical. The resulting dynamics has the following form (see [12] for the detailed derivation)

$$\begin{aligned} \ddot{\theta} &= -2\dot{r}\dot{\theta}/r + g\sin(\theta)/r - 2\gamma\theta/r^2 + \tau_\theta/m, \\ \ddot{r} &= r\dot{\theta}^2 - g\cos(\theta) - \gamma(\Upsilon(r) - \Upsilon(r_0))/\sqrt{4b^2 - r^2} + \tau_\mathrm{r}/m \end{aligned} \tag{8}$$

where $\Upsilon(r) = \arccos\left(1 - r^2/(2b^2)\right)$, γ is the stiffness of both springs divided by the mass m, τ_θ and τ_r are the control actions, g is the gravity acceleartion, and r_0 is the unloaded length of the equivalent spring. The system has an equilibrium in $\theta = 0$ and $\Upsilon(r) = \Upsilon(r_0) - g/\gamma$. Its linearized dynamics is $\Delta\ddot{\theta} \simeq k_\theta \Delta\theta$, and $\Delta\ddot{r} \simeq k_\mathrm{r} \Delta r$, with



(a) Initial (b) 2[nd] jump (c) 4[th] jump (d) Final

Fig. 7. The articulated soft leg jumps forward by exploiting natural oscillations: full length covered. Panel (a) shows the initial configuration, while Panels (b,c,d) show some examples of landings. This picture is meant to give a sense of the total forward motion that it is achieved in 8 seconds, and 6 jumps.

$k_\theta, k_\mathrm{r} \in \mathbb{R}$ being two constant values. The normal modes of the linearized system are therefore two decoupled evolutions: an angular oscillation with fixed radius, and a radial oscillation with fixed angle. The nonlinear extension of the latter is trivial, since for $\theta \equiv 0$ and $\dot{\theta} \equiv 0$ the dynamics collapses into a quasi-linear one, that we studied in [12]. The other mode instead turns into a more complex oscillation, that we investigate here. In this case $c = (1,0)$, i.e. $x_\mathrm{m} = \theta$. We approximate $(X, \dot{X})$ as fourth order polynomials, solving in the Galerkin sense the tangency constraints introduced in [7]. To this end, we approximate the system dynamics 3^{rd} order Taylor expansion around the equilibrium.

3.3 Experiments with a single leg

We performed experiments for five different values of the orbit excitation gain α; 0.2Nm, 0.3Nm, 0.5Nm, 0.7Nm, 0.9Nm. Target energy levels are $E^- = 21$J, $E^+ = 22$J. Due to energy losses, the desired level of energy could not be reached. Instead energy injected through τ_E is compensated by dissipation. A different steady state oscillation is reached for each value of α. Fig. 3 shows oscillations resulting from three of the considered gains. Fig. 4 shows the evolution of θ and r for $\alpha = 0.5$Nm. Control action is turned on at 0s. The transient lasts for about 2 seconds, in which the algorithm pumps energy into the system. After that, the segmented leg starts evolving according to a stable nonlinear oscillation. Measured and ideal trajectories - i.e. $(\theta, \dot{\theta}, r, \dot{r})$ and $(\theta, \dot{\theta}, X(\theta, \dot{\theta}), \dot{X}(\theta, \dot{\theta}))$ respectively - are quite close to each other, as shown by the right panel in the same figure. Fig. 5(a) shows the evolutions of the center of mass in Cartesian coordinates, for all the considered values of α, and for a period of 15s. The bigger is the gain, the larger are the oscillations, and the higher is the energy level reached. The resulting oscillations are slightly concave[6] and highly repeatable. Fig. 5(b) illustrates the evolutions superimposed to the ideal modal manifold, i.e. to the surface $(\theta, r) = X(\theta, \dot{\theta})$. The matching is good, with larger discrepancies for high speeds and positive values of θ.

Interestingly, it is sufficient to slightly tilt the orientation of the leg's main body, to produce forward locomotion. This is shown in Fig. 6 - which depicts a single jump - and in Fig. 7 - which shows the effect of 6 jumps executed in 8 seconds. Note that each

[6] Different choices of the physical parameters (e.g. the spring stiffnesses) could have produced a convex oscillation.

(a) (b) (c)

Fig. 8. Stills of a nonlinear oscillatory behavior induced on a soft quadruped. Each leg follows the same design of the leg used so far. The controller that we developed for the single leg testbed is applied to all four legs. The four oscillators naturally synchronize thanks to the coupling imposed by the closed kinematic chaing. The period of oscillation is about 0.8 seconds.

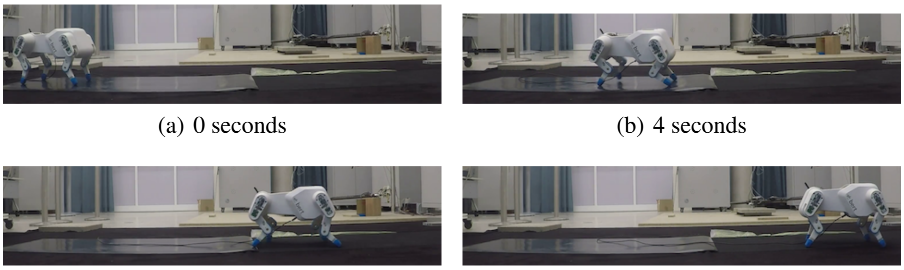
jump is actually produced by two oscillations. The first is only instrumental to inject energy into the system, while the second is when the actual jump happens. Only the latter is shown in figure.

3.4 Experiments with a quadruped

We consider here the application of proposed algorithm to the quadruped Bert [13]. This system is built using four of the above discussed soft segmented legs. We want to test if the proposed control strategy can excite stable oscillations also in this more complex system. We start by applying the algorithm separately to all the four legs. Results are shown in Fig. 8. Without any modification, the algorithm can produce and sustain a regular back-and-forth oscillation, with period 0.8 seconds. We then investigate if such oscillations can be used to implement locomotion patterns. This is done by employing a simple heuristic. The aim is to implement a trot gait. The two legs that at a given time are on the ground are controlled through the proposed algorithm. The other two legs are lifted enough to not interfere with the ground. They are then put forward when the two stance legs are close to stopping. This event signals the beginning of the next phase. Already with such a simple algorithm, the quadruped is able to walk forward, as shown in Fig. 9. The steps are here very small, and therefore the forward speed is relatively small (around 0.15 meters per second). Yet, we believe that this represents a first important proof that the theory that we are developing can be used in locomotion of complex mechanical systems.

4 Experimental Insights

The experiments exhibit a quite different scenario than the one we could have expected by looking at the problem from the pure lenses of theory. For a start, they suggest that the proposed strategy can be used in practice to excite the normal modes of soft robots, generating stable and repeatable nonlinear oscillations also in the presence of many uncertainties and unmodeled dynamics in the controlled system - e.g. the actuators dynamics, the moving contact with the ground, the non zero weight of the legs, inexact identification of system parameters, neglected friction effects. Moreover, $\dot{x}_1, \dot{x}_1$ is not



(a) 0 seconds (b) 4 seconds

(c) 8 seconds (d) 10 seconds

Fig. 9. By implementing a simple variation on the proposed algorithm, the quadruped can move forward by exploiting modal oscillations. The distance covered in 10 seconds is approximately 1.5 meters.

being directly measured, but estimated through a high pass filter. Finally, the physical system is serially actuated, and the parallel elastic behavior needs to be implemented through an opportune input mapping. First, we map τ_θ and τ_r to torques acting on x_1 and x_2 via pre-multiplication for the transpose Jacobian of the change of variables. These torques are then realized by commanding to motors a displacement equal to the torques divided by the stiffness γ. These experiments also taught us some very important lessons on where to look for improvements. First and foremost, the final energy resulted as an equilibrium between α and dissipative effects, rather than due to some stopping condition connected to the energy level E^- that could never be reached. This is because the energy regulator was built with a conservative system in mind. So experiments suggest that rather than improving the Eigenmanifold stabilization algorithm (e.g. to make it global rather than local), it is probably more practically meaningful to put some effort in developing adaptive algorithms which can dynamically adjust the value of α so to reach a desired amplitude of oscillations. For what concerns the specific leg experiment instead, we saw that mismatches from the manifold start to increase with high velocities, and that the oscillations are typically not perfectly symmetric (higher errors for positive values of δ). This suggests that a model taking into account the legs mass should be considered, therefore breaking the symmetry of $(X,\dot{X})$.

References

1. Albu-Schaeffer, A., Della Santina, C.: A review on nonlinear modes in conservative mechanical systems. Annual Reviews in Control (2020)
2. Albu-Schaeffer, A., Eiberger, O., Grebenstein, M., Haddadin, S., Ott, C., Wimboeck, T., Wolf, S., Hirzinger, G.: Soft robotics. IEEE Robotics & Automation Magazine **15**(3) (2008)
3. Albu-Schaeffer, A., Lakatos, D., Stramigioli, S.: One-dimensional solution families of nonlinear systems characterized by scalar functions on riemannian manifolds. arXiv preprint arXiv:1911.01882 (2019)
4. Åström, K.J., Furuta, K.: Swinging up a pendulum by energy control. Automatica **36**(2), 287–295 (2000)

5. Buondonno, G., De Luca, A.: A recursive newton-euler algorithm for robots with elastic joints and its application to control. In: Intelligent Robots and Systems (IROS), 2015 IEEE/RSJ International Conference on. pp. 5526–5532. IEEE (2015)
6. Calzolari, D., Della Santina, C., Albu-Schäffer, A.: Pd-like regulation of mechanical systems with prescribed bounds of exponential stability: the point-to-point case. IEEE Control Systems Letters (2021)
7. Della Santina, C., Albu Schaeffer, A.: Exciting efficient oscillations in nonlinear mechanical systems through eigenmanifold stabilization. IEEE Control Systems Letters (2021)
8. Della Santina, C., Bianchi, M., Grioli, G., Angelini, F., Catalano, M.G., Garabini, M., Bicchi, A.: Controlling soft robots: Balancing feedback and feedforward elements. IEEE Robotics & Automation Magazine (2017)
9. Della Santina, C., Catalano, M.G., Bicchi, A.: Soft Robots, pp. 1–14. Springer Berlin Heidelberg, Berlin, Heidelberg (2020). https://doi.org/10.1007/978-3-642-41610-1_146-1
10. Garofalo, G., Ott, C.: Passive energy-based control via energy tanks and release valve for limit cycle and compliance control. IFAC-PapersOnLine **51**(22), 73–78 (2018)
11. Hutter, M., Remy, C.D., Höpflinger, M.A., Siegwart, R.: Slip running with an articulated robotic leg. In: Intelligent Robots and Systems (IROS), 2010 IEEE/RSJ International Conference on. pp. 4934–4939. IEEE (2010)
12. Lakatos, D., Friedl, W., Albu-Schäffer, A.: Eigenmodes of nonlinear dynamics: Definition, existence, and embodiment into legged robots with elastic elements. IEEE Robotics and Automation Letters **2**(2), 1062–1069 (2017)
13. Seidel, D., Hermann, M., Gumpert, T., Loeffl, F.C., Albu-Schäffer, A.: Using elastically actuated legged robots in rough terrain: Experiments with dlr quadruped bert. In: 2020 IEEE Aerospace Conference. pp. 1–8. IEEE (2020)
14. Zhakatayev, A., Rubagotti, M., Varol, H.A.: Time-optimal control of variable-stiffness-actuated systems. IEEE/ASME Transactions on Mechatronics **22**(3), 1247–1258 (2017)