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An action research investigation into the implementation of the New National Curriculum (2014) in mathematics, in one primary school in the South West of England, and the changes and effects it provokes on children's learning, with a focus on the operation of division

by

Rachel Hardwidge

A dissertation submitted to the University of Bristol in accordance with the requirements for award of the degree of Doctor of Philosophy in the Faculty of Social Sciences and Law.

July 2021

61,596 word count

Synopsis

In this action research study, I investigate and explore the changes and effects of the implementation of a New National Curriculum (2014) with respect to the primary mathematics curriculum. I focus on the operation of division and investigate both informal and formal methods. The aim is to gain a better understanding of learning and teaching strategies so that they can be enhanced in future practices. The investigation was conducted with teachers and children from my current workplace, a primary school (children aged 7-11 years).

I look at the work of Anghileri (2005), Thompson (2012) and Richards (2014). They suggest that division, in the primary arithmetic curriculum, faces the most challenges and criticism, needing a progressive structuring of children's methods. I review literature concerning the operation of division to see where the confusion and problems lie, sorting strategies used by teachers and children into categories, using an adapted framework (Chick and Baker, 2005a) of procedural, conceptual or mixed approaches.

Key research questions, responses to which are gathered through interviews, questionnaires, test analysis and documents are:

> Q1. How do teachers perceive the changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics?
>
> Q2. What is an effective progression of concepts and processes in the teaching of division?

Key findings are that the implementation of the National Curriculum has provoked significant changes in children's approaches to division, with far fewer purely procedural methods in the last years of the study compared to the first. There is evidence that children move from idiosyncratic methods, which show an understanding of the concepts, to methods which continue to display understanding but also use of an efficient procedure. A key output from the project is a progression chart for teaching division across the primary school years, which has been implemented across my school.

Acknowledgements

I would like to acknowledge the help and support given to me in the School of Education: I would particularly like to thank my mentors and supervisors, Laurinda Brown and Alf Coles for all their help, guidance and expertise and for always being there.

I am thankful to the staff and children at my workplace for their co-operation with and support for my research. I would particularly like to thank KC and WA for their encouragement and support and for giving me time when needed.

Finally, I would like to thank my family for giving me the confidence and support to complete the task: To Mum for always believing in me; to James for giving me space when I needed to focus; and to Arti and my other four-legged friends for helping me to relax and think in between study periods.

Declaration

I declare that the work in this dissertation was carried out in accordance with the requirements of the University's Regulations and Code of Practice for Research Degree Programmes and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, the work is the candidate's own work. Work done in collaboration with, or with the assistance of, others, is indicated as such. Any views expressed in the dissertation are those of the author.

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Acronyms used in this thesis

BSRLM	British Society for Research into Learning Mathematics
C.P.A.	Concrete, Pictorial, Abstract techniques
DfE	Department for Education
KS1	Key Stage One – Years One and Two
KS2	Key Stage Two – Years Three, Four, Five and Six
LKS2	Lower Key Stage Two – Years Three and Four
UKS2	Upper Key Stage Two – Years Five and Six
M ed.	Master of Education
NC	National Curriculum
NCETM	National Centre for Excellence in the Teaching of Mathematics
NNS	National Numeracy Strategy
PCK	Pedagogical Content Knowledge
QCA	Qualifications and Curriculum Agency
QCDA	Qualifications and Curriculum Development Agency
QTS	Qualified Teacher Status
SATs	Standard Assessment Tests
SEND	Special Educational Needs

Glossary

Specific terms used in this thesis

Efficient	To be able to choose and use a range of strategies appropriately and to make connections between different approaches, e.g., having a secure knowledge of the multiplication tables in order to solve division problems.
Junior School	A school for children aged between seven and eleven years old.
NCETM spines	The National Centre for Excellence in the Teaching of Mathematics split the curriculum up into a small number of areas –these are called 'spines': *Spine 1: Number, Addition and Subtraction*, *Spine 2: Multiplication and Division* and *Spine 3: Fractions*.
NCETM segments	Each National Centre for Excellence in the Teaching of Mathematics spine is composed of a number of segments. These provide a recommended teaching sequence across the three spines.
Primary School	A school for children aged between four and eleven years old.

1 Introduction

1.1 My background and personal interest in the research problem

In this section I present the beginnings of the journey I have been on and how I developed my interest in a particular research field. Throughout this thesis, alongside my "researcher" voice, I present a parallel strand of personal reflections, which I mark by right-justified text in italics, these thoughts are questions motivating the thesis development as well as the development of the research questions.

I begin with some relevant biographical details, which I offer chronologically, taking me up to the start of embarking on a PhD and deciding on the precise focus of this study. As a person who has always enjoyed mathematics, both teaching and learning, it has always puzzled me as to why others do not share my thoughts.

I wonder, what do some people not like about mathematics?;
What happened to them in their upbringing and education to form their negative opinion?;
Why do they shut down at the mention of the subject?

Ever since I can remember, I have been passionate about the subject of mathematics. I love the processes involved with the finding of the answer, the manipulation of number, its links to real life and so much more. My Mum animatedly tells stories of me, as a seven-year-old, teaching my younger sister her times tables whilst she was lying asleep in her cot. I also remember having copies of all the times tables up to 12x12 plastered across the wall in our bedroom.

These were written on old computer paper and produced in poster form in my neatest handwriting in the hope that I could develop and instil the passion I had in my younger sister.

At primary school (aged 4 -11 years) I remember always taking part in the subject with enthusiasm, zest and curiosity. I ate up all that the teachers had to offer and often finished all the given tasks well within the time given and was then asking for more.

(I am sure this probably irritated my teachers slightly, especially knowing what I know now about the work that goes in to planning and preparing for a lesson!).

I was competitive, wanting to be the best and getting things wrong was not an option. I would work at the calculations set until I was sure that I had them correct and could prove my work using strategies such as the inverse operation. I ploughed through numerous textbooks and always asked for extra homework. I wanted to do well. I found mathematics came naturally and in a way to me it was easy to work out. Mathematics made complete sense to me whereas other subjects were dull and did not have the same attraction and appeal. I do not know where this love and passion came from, I just remember always being passionate about anything related to mathematics or anything that needed to be solved through numbers.

In contrast to the passion I had, Boaler (2015, p1) wrote that she believes that “far too many students hate mathematics”. In fact, she goes as far as to say that there are also “adults all over the world that fear it” (p1). Boaler also stated that she believes that mathematics has “the power to crush a child’s confidence” and she qualifies this further by saying she believes that quite often children are made to feel failure and inadequacy in mathematics from an early age and this in turn results in them forming negative opinions about it. Boaler continued by saying that she believes mathematics receives this negativity because of the way it is taught and represented in schools as she believes mathematics in the classroom does not reflect everyday mathematics and the mathematics of real life. Hersh and John-Steiner (2011, p301) would agree with this idea as they wrote that they believe it is not actually mathematics that people dislike. They continued to qualify this by saying many people enjoy life mathematics such as “puzzles, playing chess and recreational problem solving”. Hersh and John-Steiner (2011, p305) said that it is actually just school mathematics that people dislike. They commented upon this further by saying “people are not born disliking mathematics - they learn to dislike it in school”.

In support of this, and further to it, I understand Haylock (2006) to believe that the image of mathematics as being a difficult subject is often picked up from parents, friends and even teachers. He put across the idea about there being a background of anxiety and confusion and the fact that he sees the problem as being related to the language that is used in mathematics that is not used in everyday life. It seems Devlin (2000, p128) would agree with this as he wrote about “a barrage of instructions written in a language that learners cannot make sense of”. Further to this, I understand his writing to reflect that his belief is that it is not necessarily the mathematics that is being misunderstood as learners never really get beyond a certain point to access what is considered to be the real mathematics.

Eastaway and Askew (2010, p9) wrote that, “of all the subjects covered in school”, it is mathematics that creates the most fear in parents and guardians. They add to this by giving the possible reason that it is because parents believe that mathematics is done in a different way these days. In their writing, they stated that nowadays mathematics teaching helps the children to create visual representations, maps and a deeper understanding rather than them learning and just remembering a list of rules to complete a calculation. Eastaway and Askew stated that the children are now being taught how the mathematics works and how it can be broken down rather than just doing pages of calculations. As a practising primary practitioner, what Boaler, Haylock, Eastaway and Askew say resonates deeply with me. I have similar thoughts, experiences and beliefs in my own classroom and school.

Back to my own journey, I think I lost my own way a little in secondary school (aged 11-16 years). Although always in the top set, I seemed to lose the love of mathematics. The want in me to do well dissipated. I knew I could do it but it did not seem so natural anymore. It seemed to be just another subject that had an end result - a test. My creativity and my risk taking gone, my passion subsided. It was not fun anymore. Lessons in mathematics seemed to be a chore and did not really seem very realistic - perhaps reflective of the thoughts of Boaler and Hersh and John-Steiner noted earlier I wonder whether that was in part due to the teachers as well as the subject material as I did not enjoy my years at secondary school. However, on reflection, when I look back at my results from secondary school, it is perhaps noteworthy that mathematics was one of only two subjects that I passed with a good grade. Perhaps this was reflective of a love that was once there.

After secondary school, I moved to a further education college where I studied Accountancy at A level - perhaps my love of number beginning to show through once again. I began to develop that fondness of number, a love of the creativity that it had and a belief again that there were endless possibilities in order to get to an answer. My accountancy teacher seemed to have the same passions and beliefs. I thrived. My passion for mathematics and numbers was reignited. Again, one of my best grades was achieved through my use and love of number.

My next move was to university – the degree path I followed was a Bachelor of Arts in Sports Science with Qualified Teacher Status (QTS). Another step away from mathematics perhaps but a step towards my other love, sport. Throughout this degree, I predominantly learnt about things related to physical education, but the mathematics element was always something I thought about. The other part of my degree was the Q.T.S. and, when in lectures or in the classroom, it was always in the mathematics lessons that I thrived. I seemed to have a flair for the subject and the passion that was needed to do well. I seemed to thrive when teaching others the basics but also when pushing the higher attainers in the subject. I loved it and I came to life when I was talking about anything mathematical - I was animated and happy. This was both in university lectures and also in the classroom when on teaching placement.

Four years after I started my course, in 1998, I qualified and got my first teaching post in a primary school teaching in a Year 5 class (children aged from 9-10 years). After my first year, I took on the role of physical education coordinator but, in a way, I remember that I looked at the mathematics coordinator and aspired to be them. I felt in my mind that I could do what they were doing but I also knew it would come in the years to follow so I was not too worried at the time. I felt that I was destined to do something in the future that followed a mathematical path, but it was obviously not the time.

After just two years of teaching, I took a seemingly sideways step toward my dream of mathematics coordination and away from the sporting lead that I had. I was observed and met the expectations of and qualified as an Advanced Skills Teacher (AST) with a speciality in Mathematics and Computing. My passion was fully reignited and I was supporting other teachers with their mathematics and trying to instil a love of the subject in them. I stayed in this role for the next 4 years before I made a decision to move home to Somerset to be closer to family. When home, it felt that, in a way, I would have to start again in the teaching field, as the Local Authority did not know me and so job hunting started.

In my new role as a supply teacher, I was soon employed in a permanent position, not through my teaching of mathematics, but through my behaviour management skills. However, in this position I worked closely with the local secondary schools as I was teaching Year 6 (children aged from 10-11 years). Teachers and school leaders would come in and observe lessons and children. One observer, noticing my flair and passion in mathematics actually asked whether I would consider a move to the mathematics department at their secondary school. Although my response was a negative one, I was extremely pleased to see that my passion was once again showing through and it made me wonder once again what I needed to take my passion further in order to lead others around me.

A few years down the line I still had not done anything about it as life, both in and out of work, became too hectic but the passions and thoughts were still there. I moved, yet again, to another school where I became the Assistant Headteacher. I had again been brought into this school because of my behaviour management skills and not my mathematical skills. I suppose I was drifting as the role took me rather than following my passion. I was actually approached by the school and asked if I was available to apply for this particular job. I had taught at a neighbouring school and they had been told that I would be the perfect person for the role.

At this school, we had an Ofsted inspection in my first month of employment. During the inspection, I taught and was observed in what I would call a real-life mathematics lesson. I started the session using an individual whiteboard question and answer activity. The lesson had three different levels and in it the children were given a number to start with and then I fired questions at them which used each of their answers in the following question - we worked on doubling and halving. The lesson then moved on and I gave the children a mathematical problem related to real-life. The children were given an imaginary £100 and also a take-away menu. Their task was to plan a party for a set number of friends. The meal had to contain a starter, a main and a dessert. The children were challenged to get as close to the £100 as possible. The lower attainers used simple calculations such as addition and subtraction and the higher attainers used all operations. The children were not allowed to go a penny over budget.

A real buzz ensued as the children got to work. They spoke clearly about the mathematics that they were doing and why they were using the processes they were. The problem was appealing to the children as it dealt with food and money. The children did not relate the task to learning and as they were having so much fun, they were quite surprised that this task was their mathematics lesson. The children were focused and wanted to try lots of different selections. They were keen to use different methods within the task.

The feedback given by the inspector, an outstanding, which is the highest category that can be given, made me once again think about mathematics and my love of it. It made me think about the different ways in which I could create opportunities for those in my class to take responsibility for their own learning and develop their skills. It made me realise that I wanted to listen to and work with others, have an impact on them, to be able to encourage them to talk about their mathematics and most importantly to ensure they enjoy it as much as I do. I believe that learning in mathematics should be practical, hands on, lively, fun and interactive. Those who are learning mathematics should be able to see connections, learn key skills and it should be accessible to all learners.

However, once again I was seemingly stuck on a different pathway and once again the mathematical side of things faded away as other responsibilities took over. The mathematical passion still dug away in the back of my mind.

In 2010, twelve years into my career as a teacher, due to an unforeseen incident, my teaching and education pathway changed once again. However, it was a positive change because, as a result of this incident, I registered on the Masters in Mathematics Education at the University of Bristol. I had only ever dreamt of doing this but, due to circumstances in my life at the time, it was the perfect opportunity to take the plunge, to develop my love of mathematics again and to perhaps become an academic researcher in the area. I found myself beginning to ask numerous questions with regard to mathematics in primary schools.

Is it an area that is taught well?
Do we instil a love of learning in the subject?
Do we create a stimulating, purposeful and enriching environment where children are ready to take a risk and make marvellous mistakes in order to learn and get the desired results?
Are all learners receiving opportunities in a way that encourages them to embrace their learning and to develop a passion for the subject?

I was becoming proactive in an area in which I felt I was at home.

The course I chose was perfect for me and I achieved my best ever grades in my assessed writings. It was as if I had arrived, this was what I wanted to do and what I should be doing. I wanted to look at the subject of mathematics, how it is taught, how it affects children and perhaps how barriers to its learning and understanding are formed. There were so many possibilities opening up to me and although only a part-time student, I finished the course in two years with a distinction. I thoroughly enjoyed every minute of the course and wanted more.

In my Master's dissertation (Tutcher, 2012), I explored the relationship between Pedagogical Content Knowledge (PCK) and the approaches and strategies used by primary teachers delivering the primary mathematics curriculum. This was an area that really interested me and saw me looking mainly at the teachers and their skills. I looked at procedural and conceptual strategies and developed a real interest in the area. Through the study, I aimed to gain a better understanding of mathematical teaching practices in order to try and develop and enhance these in the future. I looked at misconceptions and errors, strategies used to probe thinking and practices in the classroom. I asked whether there was just one approach or multiple approaches and which of these should be encouraged. In the study, I focused on the mathematical area of word problems. In short, my study found that teachers perceived conceptual strategies to be the best to develop a child's understanding, however, in practice, the procedural strategies were the ones that were most commonly used. From this study, my interest in the teaching and learning of mathematics grew further still. I wanted to look more at the usage of procedural and conceptual strategies, I will discuss these in chapter three, and also the different strategies imposed by the impending introduction and delivery of the New National Curriculum (2014). These strategies will be discussed in later chapters.

Whilst studying, I was also in full-time employment again and the school in which I was working was pleased with the parent courses and other mathematical provisions I was offering. They encouraged me to take my learning forward and supported me in all that I did. In 2013, I again registered at the University of Bristol but this time under the PhD pathway and hence this study was born. The endless possibilities, the love of mathematics and being surrounded by others with the same passion really ignited the love that once was, the belief that anything is possible and also the belief that this was what I was supposed to be doing.

At the time of beginning this study, in 2013, there was the introduction of a new National Curriculum. With the presentation of this new curriculum came teacher unrest and uncertainty of what would actually follow. The new curriculum proposed that children would have to learn certain concepts earlier in their school lives. In fact, Adams (2013, n.p.) wrote that the Prime Minister at the time, David Cameron, had described the new curriculum as 'tough and rigorous'. Cameron had also said that 'some people are worried about that but it is the right thing to do'. In the same piece of writing, Adams (2013, n.p) stated that Kevin Courtney, the deputy general secretary of the National Union of Teachers, had said that the 'timescale of implementation was ridiculously short' and that this would lead to 'confusion and chaos'. In support of Courtney's thoughts, Hanson (2013, p5) wrote that an 'issue that stands out at first glance is the very substantial increase in the amount and difficulty of the content'. The new curriculum's introduction led to concerns that children may have to move on to topics before they had developed the mathematical grounding necessary. Hanson (2013, p5) stated that the new curriculum 'demands six-year-olds are taught abstract mathematics'. She states this is a concern as 'it is widely recognised that some students will need to work with concrete and visual mathematics'. Adams (2013, n.p.) also wrote that Kevin Courtney had stated that teachers would 'have concerns whether this curriculum is right for children with special educational needs'. In addition to this, Hanson (2013, p3) suggested 'schools are struggling to cope with this new curriculum'. Handal and Herrington (2003) and Cavanagh (2006) wrote about teachers' beliefs in teaching and learning mathematics being critical in determining the pace and interpretation of curriculum reform and so in my mind this is a necessary factor to research if this study is to contribute to future practice.

With such a shift, I wanted to look at how the New National Curriculum (2014) would affect our school, our teachers and our children. Would they embrace the reform or, like so many other changes, would it be a change that was soon forgotten? I also wanted to look at how this change would affect the relationship between home and school as we are all one team - the child, parent and the teacher. Due to this triangulation, I became interested in the work of the late Martin Hughes who dedicated his life to understanding the social context of children's learning. He worked on children's informal and formal understanding of number. Through reading his work, *Children and number* and *Improving primary mathematics: Linking home and school (1986),* I became interested in and wanted to look at similar research and discover if the new curriculum implementation would hinder or improve home and school relationships with regard to the teaching of number.

Due to the timing of my registration on the PhD course (2013) and the impending changes in the curriculum, it was a perfect time for me to look further at strategies used in teaching and learning and perhaps how these are, or can be, affected by a reform in curriculum. Before progression, (part of the doctoral process, which comprises of a written report and a viva to present my ideas) I wanted to develop an investigation using both home and school elements looking into the implementation of the New National Curriculum (2014) in mathematics in one primary school and the recent changes and effects this provokes on children's learning. However, even before progression, it soon became clear that with this was far too ambitious and the study far too wide. I decided to narrow the study and chose to look at only division rather than the whole of the mathematics curriculum at primary level.

My decision to follow division was mainly because it is what I would call a 'Marmite subject', you either love it or you hate it! Personally, I love it, it makes sense to me but I find that others are not of the same opinion. It is one area where you see people physically shudder at the mere mention of it - teachers, parents and children. In support of this, Back (2011, np) states that 'a lot of teachers struggle with teaching division to children.' I began to wonder why it is an area that is feared and hated by so many when in reality it is just one of the four basic operations that we use on a daily basis. Even at a teacher conference, I heard the speaker ask if anyone actually liked long division - I smiled but kept quiet and observed. Apparently, I, as a person that likes long division, am in the minority, sparking my interest further.

I wonder: Why is this? What is it about division that people do not like?
What is it that they find difficult?
How can we as teachers make division easier for children when they are learning it so as to suppress their hatred toward the operation?
What are the procedural and conceptual strategies used, understood and preferred and perhaps which are the strategies that are hated, avoided and talked negatively about?

Another reason for wanting to research division was due to the different strategies that have been used over the years in the different curriculums that teachers have been given. In 1999 the National Numeracy Strategy (NNS) highlighted the use of the chunking method in order for the children to achieve a better understanding of division – this method is presented in chapter 4, figures 7 and 8. It led teachers away from formal written methods such as short division (chapter 4, figures 12, 13 and 14) and long division (chapter 4, figures 16 and 17) and encouraged the conceptual approach from Year 4 onwards. Chunking was basically seen as an approach that took chunks away from a number in order to find how many of another number went into it. At the time, I remember that many teachers found it confusing (in my opinion) and actually over the next 20 years, in my experience, this apparent confusion did not change much. In 2006, Julia Anghileri, of the University of Cambridge (U.K.) completed a study of the National Numeracy Strategy (NNS) focusing on the impact of reform on students' written calculation methods. Her study focused on division strategies and on p366 of her report she wrote that 'division has traditionally been seen as the capstone of the primary arithmetic curriculum'. Anghileri's study found that since the introduction of the NNS there had been a move away from the traditional methods to ones which were more informal particularly in problems where a number was being divided by a two-digit divisor. This led me to think about the strategies being used in the classroom and whether there would once again be a shift in the way division was taught and learnt.

In 2015, for a British Society for Research into Learning Mathematics (BSRLM) day conference, I presented a paper, entitled 'To chunk or not to chunk', of a small-scale study which looked at methods used as a division algorithm. I looked at methods children used to solve division problems, which they preferred and why. In general, the procedures chosen by the children were the chunking method or the short division method. Within the research, I wanted to look at progression, which methods needed to be taught first and why in order to gain a better understanding and mastery of the area. The findings showed that actually there could be many other considerations rather than teaching in a specified order, for example, the ordering depends on the individuals that are involved. The research showed that children need a plethora of ways to develop a concrete understanding. If the children need to do the how before the why or vice versa then teachers should act accordingly and perhaps not follow a suggested prescribed path.

This work then led to writing being published in the summer 2017 edition of the journal, *Primary Mathematics*. My article, titled ‘The great divide’, explored progression in division and in a way reiterates the findings of my original BSRLM study. Within the readings I wrote about, I note research by Richards (2014) who states that primary age children find division the most difficult of any area in the mathematics curriculum, bringing me back to one of my reasons for this study.

As part of my BSRLM work and also the work above, I reflected upon times where I have taught division in my classroom. In that work, I wrote about a time when I taught mathematics to a set of lower achieving Year 6 children. My plans at the time, following the guidelines set out in the school’s progression chart, stated that the class needed to divide using the chunking method. We looked carefully at the steps we had to take and the understanding we needed. After a while, I was faced with a class where only 5% of the children were happy that they thought they knew what they were doing. The rest were feeling depressed and unsuccessful. Many children believed the chunking algorithm to be unachievable and practically impossible. These beliefs seem to be reflected in Thompson’s work (2012, p45) where he wrote that in the Ofsted report *Good practice in primary mathematics: evidence from 20 successful schools* that chunking is the algorithm that receives ‘the worst press’ and is ‘never seen in a positive light.’ Comments in my class were made that it was too long-winded and the children said that they found they made numerous errors on the way to completion of the problem. What was really frustrating for me, as their teacher, was that they lacked confidence even when they were getting it right. This all led to me asking questions about fluency, understanding and mastery.

Is there another way that division could be taught to help them feel successful?

How can we expect children to learn if they, the children, just simply do not get it?

In response to my class's anxiety, I changed the approach we were using and showed them the standard short method. After only a few minutes, the children were engaged and feeling successful. I remember one child even shouting out, 'Why do we not use this all the time, it is so much easier!' It was this statement that got me thinking and wondering about division and the progression of it. Through background reading, I found that Ofsted (2011, p8) wrote that 'schools were confident that the large majority of their pupils become proficient in using the formal algorithms for addition, subtraction and multiplication but most said that division is a different story'. Ofsted (2011, p16) also wrote that an issue is that the 'mathematical thinking behind the method of short division, which most people master, is different to the thinking behind chunking.'

Is it necessary for all learners to learn all strategies?
What is the role of our professional judgement to determine what is right for our learners at the time in order to gain an understanding and be successful?

In a way, I want my research to seek to understand human behaviour toward division and I want to build future practices from these findings.

At progression, I realised that my study was still too wide and too broad. I decided that I needed to condense it further and perhaps drop another area so that I could focus more in depth on a more defined subject matter. Being a practising teacher, one of my main concerns and personal interests is the development of the child and so my new focus was formed around the teacher and child only and the idea of the triangulation of home as well was dropped with the idea of picking it up again in a possible future study. Over time, I found my PhD study was beginning to take shape and was forming around the following broad areas of interest:

How does the new curriculum impact on a child's learning with regard to division?
What changes will occur in teaching and learning with regards to the new curriculum reforms that are being introduced?
What effects will there be?
Will understanding and attainment improve?
How will the different levels of attainment of learner be affected?
How will teachers be affected?
What different strategies will be put in place to aid the learner and will the fear and hatred of division ever change?

1.2 Aims for this thesis

The information gained from my research for my MEd, and also in smaller studies leading up to this thesis, confirmed that this research area was real and that there was indeed a problem with regards to the teaching and learning of division and all the strategies that are used. It also demonstrated that there was a need to try to find a way to help and improve the attainment of *all* children. My aim to both investigate and understand the teaching and learning of division in order to improve practice in school as well as the knowledge gained from my role as a teacher, influenced my decision to choose action research for the design of this thesis. So, after narrowing the study into a more manageable topic, the following aims came into being:

1) To gain a thorough understanding of what mathematics actually is considered to be in a whole sense but also then more specifically in the primary curriculum and classroom.
2) To develop a deeper understanding of division, covering ideas about what division is and what other researchers write about it.
3) To investigate how division is taught with regards to progression and methods used (conceptual, procedural and mixed) and how this is possibly affected, if at all, by the introduction of the new primary curriculum.
4) To gain a better understanding as to why division is seen as such a difficult concept.

With these aims I hoped to add to an understanding of children's learning in mathematics, starting by clarifying what mathematics is with regard to primary aged children and more specifically, the operation of division – the learning of it and the challenges faced. These aims in turn also aided the development and structuring of this thesis. An overall aim of this action research study was to suggest possible ways forward for the school in the future so that all learners can access the division curriculum and be successful.

With these aims in mind, and after completing some literature research, I used an idea developed by Brownhill (2015, 2017) and framed/developed my research questions through the use of the Ice Cream Cone Model. Brownhill (2015, np) states that it is "widely recognised that educational research is both fascinating and problematic". He notes in his online tutorial *Research aims and objectives* that research questions are "a critical part of the research" and that they are the "spine which holds the whole research process together". Another point Brownhill (2015, np) noted is that "when you look at different cycles or explanations of research, you can clearly see that the question is a pivotal point in the research process". This is why I used Brownhill's Ice Cream Cone Model (ICCM) to help frame my questions.

1.3 Following action research

To me it was important to remember that the whole motivation behind this study was based on my work as a teacher. Feldman (2007, p251) stated "Teachers have goals and objectives." He also said that "Teachers have a responsibility to shape situations for their children and give them freedom to choose" (ibid). This study lies in my ambition to improve the quality of professional practice and provision in primary mathematics. An influential guide to conducting action research is a book by Altrichter, Feldman, Posch and Somekh (2008) which is based around teachers investigating their work. According to this book, it is our own professional knowledge that informs our practices. My study aimed at developing professional knowledge, contributing to academic debate but also informing my practice and the practice of others, hence, I recognised and realised that I needed my study to be action research. Kock (2004, p267) wrote that action research "has its roots in studies of social and workplace issues." He also noted that a goal of action research is to improve practices within schools as well as contributing to "academic knowledge."

> Action research happens when people research their own practice in order to improve it and to come to a better understanding of their practice situations. It is action because they act within the systems that they are trying to improve and understand. It is research because it is systematic, critical inquiry made public. (Feldman, 2007, p242)

In my study, I looked at the practice of the teaching and learning of division within the whole school in which I work and through this knowledge and understanding developed future practices. I looked at the strengths and weaknesses of the school and reflected upon them and translated them into actions that would improve what we do and in turn improve the outcomes of our children. Burns (2010) stated that the main aim of action research is to identify an area that is a problem and look at it systematically. Division is certainly an area with split views.

I also reflected upon Altrichter et al.'s (2008) idea of the circle of action and reflection (p9), a picture of which can be seen in appendix 4, which is how they see action research taking place. They noted that data collection leads to interpretation, which in turn leads to consequences and ideas and this then leads to action. This is how I wanted my research to develop although I also felt that in my work it was not so straight forward. What I mean is that sometimes the research that I did redirected itself and took steps backwards and then again forward rather than just being on a simple circle – more like a collection of circles that interlinked. Kemmis, McTaggart and Nixon (2014, p19) created a similar idea and called it the "action research spiral". This spiral begins with planning, moves onto an acting stage, then an observation stage, next comes a reflecting stage and then a re-planning phase. Their spiral continues for as long as the study lasts. This study follows both Altrichter et al.'s "circle of action and reflection" and Kemmis et al.'s spiral. Figure 1 is an adapted version of both Altrichter et al.'s (2008) idea of the circle of action and Kemmis et al.'s action spiral research and shows how my action research developed. Although the cycles developed over time, for ease of your reading, Figure 1 shows the complete set of cycles for the whole study.

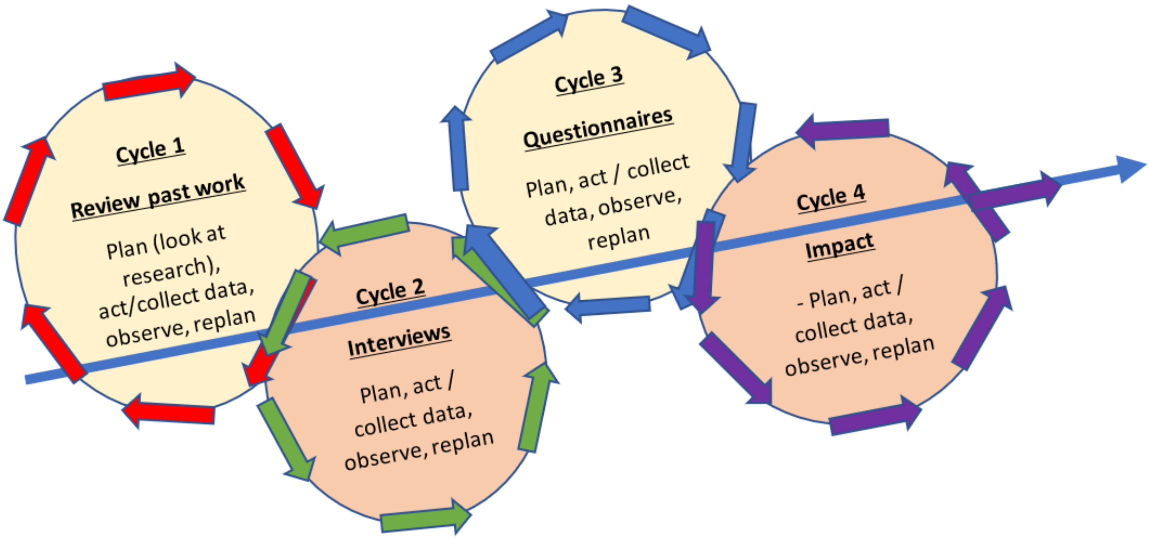


Figure 1: The process of action research in this study – a collection of circles that interlink (adapted from Altrichter et al.'s (2008) idea of the circle of action and Kemmis et al.'s action spiral research).

Each cycle involved a process of planning, action and reflection, leading to further planning. I believed that by being in situ I was well placed to take immediate action so data collection sometimes led straight to action or after interpretation of data I realised that other data needed collecting. Being in situ also meant that I could collect data over a long period. This study took place over a seven-year period (2013-2020). I could gradually develop ideas rather than having to dip in and out with a set pathway. Altrichter et al (2008, p7) mention the idea that action researchers will not accept problems day to day but will reflect upon them and seek "solutions and improvements" and this is how I saw my work.

I am mindful that there are disadvantages of teacher researchers, for instance, that my involvement may get in the way of effective research as I may bring a "personal bias" (Kock, 2004, p269). This is a danger I kept in mind whilst carrying out my research. Kock (2004, p269) also stated that "the personal involvement of the researcher is likely to push him or her into interpreting the research in particular ways." He is of the opinion that some of these interpretations may be wrong and therefore I also kept this thought in my mind. To help counteract this, I aimed to collect a wide range of data and share my interpretations with participants and also with the wider academic community through meetings and conference presentations. The final note to make is that Altrichter et al. (2008, p6) comment that action research can achieve “remarkable results”.

Cycle 1 of this action research study began with the literature review. Altrichter et al. (2008, p13) state that “action research starts from practical questions arising from everyday professional practice” and, as this study is based around mathematics in the primary curriculum it made sense to gain clarity and understanding in these areas. Cycle 1 of this action research study developed further by looking at how division is taught with regards to progression and methods used (aim 3) and also gaining an understanding as to why division is seen as such a difficult concept (aim 4). Altrichter et al. (2008, p7) wrote that “doing action research involves acquiring some new skills, which is not easy until you have sense of the whole process” and this is why I feel it is important to gather data that develops an understanding of the primary mathematics curriculum and more specifically the operation of division before gathering data related to division in the researched school.

In the following chapter, I build a picture of what mathematics is perceived to be and I look at the foundations of mathematics in order to try to make sense of it and understand it.

2 Toward an understanding of what mathematics is

The literature review is divided into three chapters. These chapters are further divided into sections that consider my aims and follow my typically structured and methodical approach. My intention, in this part of my action research study was to build up a picture to enable an understanding of the different ideas and elements which are involved in the teaching and learning of division in mathematics.

This chapter, chapter 2, starts by examining the question “What is mathematics?” as I wanted to look into the subject of mathematics and find out what it is perceived to be. Moving on from this, the foundations of mathematics are presented in order to give more of a background into the subject in relation to various ‘isms’, providing other perspectives on what the subject is. The chapter then looked at the philosophical underpinnings of the study. In order to conclude this chapter, I presented an explanation of action research and clarified how the literature review makes up cycle 1 of the study.

I wonder: Will I understand what I am researching?

Will the working definition I construct actually relate to the mathematics studied at primary level?

The researchers and educators used throughout my literature review at first were chosen either because they were used in my previous studies, they were current/had published their work in recent years or they had been referenced or talked about by others whom I had read, followed or heard about in meetings in conferences. However, as this is an action research study, the reading I did continued to inform my actions hence the reason for presenting my work in a chronological manner, with respect to my study. As in chapter 1, throughout this thesis, alongside my "researcher" voice, I have presented a parallel strand of personal reflections, which I mark by right-justified text in italics, these thoughts are questions motivating the thesis development as well as the development of the research questions. I am aware as a researcher that I have many questions and as I work and read, these questions develop and change, as do the authors whom I read, and this is reflected in the development of my study. Through the study I have focused on key authors, those of who came up consistently in the reference lists of articles that I read.

2.1 What is mathematics?

Cockcroft (1982, p1) wrote in his report that "It would be very difficult – perhaps impossible – to live a normal life in very many parts of the world in the twentieth century without making use of mathematics of some kind". In support of this and 17 years later, the Department for Education and Employment (DfEE) and the Qualification and Curriculum Development Agency (QCDA) (1999, p60) stated that "Mathematics equips pupils with a uniquely powerful set of tools to understand and change the world". Kilpatrick, Swafford and Findell (2001, p15) are of the opinion that mathematics is a "universal, utilitarian subject – so much a part of modern life that anyone who wishes to be a fully participating member of society must know basic mathematics". These authors agree about the significance of mathematics as a part of education, adding support to need to consider what is mathematics.

There are, unsurprisingly, many ways in which mathematics can be defined. Table 1 illustrates some of these differences.

Davis and Hersch (1981, p6)	"the science of quantity and shape"
Flato (1990, p8)	"a key to explaining the world"
Colyvan (2012, p1)	"mathematics is a study of entities such as number, sets and functions as well as the structural relationships between them".
Hersh (2012, p1)	"Mathematics is a science, like physics or astronomy; it constitutes a body of established facts, achieved by a reliable method, verified by practice, and agreed on by a consensus of qualified experts. But its subject matter is not visible or ponderable, not empirical; its subject matter is ideas, concepts, which exist only in the shared consciousness of human beings. Thus, it is both a science and a "humanity." It is about mental objects with reproducible properties.
Boaler (2015, p16)	"a study of patterns or a set of connected ideas."
Askew (2016, pix)	"mathematics can perform magic on the world; letting us see patterns and regularity where previously we might have seen chaos and confusion"

Table 1: Definitions of mathematics

Some of the common features from the definitions in table 1 are the ideas/ views that mathematics is a study of entities such as number, pattern or shape. Another common feature is that mathematics is viewed as a science that can explain the world in which we live.

Closely related to definitions of what mathematics is, are descriptions of its use. Tao (2007, p2) described mathematics as being "a complex area that can evolve in unexpected and adaptive ways". He listed the different types of mathematics and stated that all the various types represent different ways in which we improve our understanding and usage of the subject. Tao's work shows how diverse and vast the topic is and how difficult it is to define. For the purposes of this study, I am not expecting a definitive definition.

And, naturally, one of the uses of mathematics is in the context of education. Boaler (2015, p17), suggested that "mathematics is a human activity, a social phenomenon, and it is part of our culture" Montague-Smith, Cotton, Hansen and Price (2018, p2) wrote that "mathematics is one way in which we describe and make sense of the world". Both these views reflect upon the idea that mathematics, as also suggested by Cockcroft (2012), noted earlier, that mathematics is integral to our lives. Mathematics is everywhere – at home, at work and at school.

Boaler (2015) believes that children who do have positive experiences in mathematics are actually very lucky as it can fashion their lives. However, Boaler also talked about the wonder of mathematics and her beliefs that many are not given an opportunity to experience real mathematics. She continued on this point by saying that people without any concept and understanding of real-life mathematics will tell you that mathematics is just "a list of rules and procedures that need to be remembered" (p20). Kilpatrick et al. (2001, p16) also believe that "it is vital that young people understand the mathematics they are learning" as they state that "innumeracy deprives them not only of opportunity but also of competence in everyday tasks".

Other authors compartmentalise the uses of mathematics into different fields. For instance, taking an historical source, Courant and Robbins (1941) in the introduction of their book entitled *What is mathematics?* paragraph 1, wrote that they believe "mathematics, as an expression of the human mind, reflects the active will, the contemplative reason and the desire for aesthetic perfection". They said that the "basic elements [of mathematics] are linked to logic, analysis, generalisation and individuality" (np), noting also that different beliefs and traditions may give different values to the different areas of the topic.

Ziegler and Loos (2017) agree with Courant and Robbins (1941) when they said the only way to answer the question, "What is mathematics?", is through active experience rather than through philosophy. They believe that the answer to "What is mathematics?" shows us more about the individual giving the definition rather than about the subject itself. This leads me to question any definition given.

I wonder: Which are the ones that I should use?;

Are they complete definitions if they do not cover the whole spectrum that is mathematics?;

Ziegler (2010) wonders whether it is indeed impossible to give a good definition of mathematics in just a sentence or two.

This thought makes me wonder the same and makes me ask if I could just be defining mathematics with regard to the primary curriculum as it is a much narrower field; Primary mathematics is at the beginning of the vast world of mathematics.

Also concerned with the different fields of mathematics, Devlin (2000, pp5-6) wrote more about the historical development of a mathematical definition. He suggested that mathematical definitions such as “It is about number” or “It is about the science of numbers”, which are perhaps the responses you would get from someone on the street or at primary level, are “out of date by 2500 years”. Devlin, in his research, suggested that mathematics has changed many times and he noted that in 500 BC mathematics was indeed about number. He continued in his writing to say that, between 500 BC and 300 AD, mathematics went beyond just number as the ancient Greeks became concerned with elements such as geometry. He believes that to the Greeks, mathematics would have been considered as numbers and shape.

Devlin (2000, p7) noted that after the Greeks, the next major change in mathematics occurred in the middle of the seventeenth century when calculus came about. He said that this allowed for the study of motion so "mathematics then became the study of number, shape, motion, change and space". Devlin stated that in the last century there have again been significant growths and changes due to an "explosion in knowledge" and he wrote about branches and sub-branches of mathematics, stating he believes that "there are around sixty and seventy distinct categories".

This leaves me questioning whether I am indeed actually studying mathematics or just a category of it. Although my own knowledge is expanding, I find my study shrinking perhaps to be a study of arithmetical strategies.

As a final thought on what is mathematics, a quite different kind of definition comes from the mathematician Halmos, quoted in the work entitled *Loving and Hating Mathematics* by Hersh and John-Steiner (2011, p46), noted that he believed mathematics to be "Security. Certainty. Truth. Beauty. Insight. Structure. Architecture". This is a more poetic sense of the subject than the previous sense of science, pattern, or categories.

This made me think of mathematics in a completely different way again.
I wonder: what do other individuals / researchers say if they were asked the same question?
How in fact would I define it? Would my definition of mathematics relate to how I feel, the thoughts I have and the mathematics that I work with rather than the whole area?
Will the definition I create relate solely to the mathematics of this study?

Having explored a range of definitions of the subject matter of mathematics, it is clear that different definitions rest on different philosophies of mathematics. In the next section, I consider, what are the foundations of the subject?

2.2 Foundations of mathematics

With all these different definitions and explanations of what mathematics is, I looked again at Colyvan (2012, p2) who said that looking at the philosophy of mathematics “sheds light on what mathematics is all about”. Therefore, in order to get a clearer explanation and clarification of the subject, I find myself needing to look again. Colyvan looked at the foundations of mathematics with regard to various ‘isms.’ He wrote about “Formalism”, “Logicism” and “Intuitionism”. Discussions about the foundations of mathematics were hotly debated in the early decades of the twentieth century and these three ‘isms’ were the major schools of thought from that time. Table 2 shows similarities and differences in thinking with regard to formalism, logicism and intuitionism.

	Formalism	Logicism	Intuitionism
Colyvan (2012)	“nothing more than the manipulation of meaningless symbols” (p4). “mathematics is like a game of chess where symbols can be manipulated according to rules that are primarily about notation” (p4).	mathematics is regarded as being all about “logic” (p3)	“proof and construction” (p6) “mathematical objects need to be constructed before one can sensibly speak about them” (p6).
Stanford Encyclopedia of Philosophy online	“being concerned with symbols and perhaps higher mathematics” (np).	reducing mathematics to logic	“activity of construction” (np).

Table 2: Similarities and differences in thinking with regard to formalism, logicism and intuitionism.

Looking at these three schools of thought with regard to mathematics, I still find myself a little confused and in need a clarification.

My first reaction to looking across the different foundations is that there seems to be a place for all of them in the study of mathematics. There is surely a place in mathematics for logic, for construction that builds on intuition and for learning about symbolic rules. Each of these programmes seem to speak to part of what I know is the curriculum.

Snapper (1979, p207) noted the purpose of logicism as being “to show classical mathematics to be part of logic”. With regard to intuitionism, he noted that he believes all human beings have a “primordial intuition for the natural numbers within them”. He said we have an immediate certainty as to what is meant by the number one and that the mental process that goes into formulating a number can be repeated and, therefore, we can construct numbers. Snapper continued by proposing that the purpose of intuitionism is not to give a valid definition of mathematics but to wait and see what mathematics evolves.

Continuing on, Snapper (1979) stated that formalism is perhaps the best known of the three schools of thought. He writes about formalism as stressing axiomatic proof through theorems. I understand this to mean following the rules and having a self-evident theory. Snapper also noted, at the time of writing, that the influence of the three areas still remained strong.

It is extremely interesting to me to note that he finishes his work by saying that he believes that the key foundations to mathematics lie hidden somewhere among the roots of all three. I would agree with his final thoughts as I feel that each school of thought has its place. I also wonder what mathematics would be like if all three of these views were in place.

So, looking forward to the next chapter, I continue on a structured path which builds progressively from one section/cycle stage to another. I progress my literature review further by looking at an understanding of mathematics and how children learn. I look at the methods and strategies that are used in order to develop an understanding. Building on my BSRLM study mentioned in chapter one, I wanted to find out whether one method or strategy, formal or informal, is better for division than another and how a child's thinking in relation to division, including different strategies, builds and progresses throughout their schooling.

3 Mathematics in the Primary Curriculum

In the last chapter, I looked into a definition of what mathematics is and realised that defining it was not quite as easy as it first seemed due to its vastness. In the current chapter, I developed cycle 1 further and looked at various factors which are involved in mathematical learning. My plan was to start by examining mathematics with regard to the primary curriculum and determine what mathematics actually is with respect to primary children. I then planned to consider National curriculum changes and expectations over the last three decades in order to build a picture of curriculum format, content, methods and approaches used over time. I next wanted to look at views relating to developing a mathematical understanding of the mathematics the children are involved with at Primary School.

Finally, building on my Master's work, I knew I wanted to explore further the areas of procedural and conceptual knowledge and approaches to learning mathematics. These approaches were examined in terms of how other researchers define them and then a definition for the purpose of this study was created. The actions in Cycle 1 were the conduct of the literature review, which led to reflection on what is known about the division and where my study could contribute to new thinking. Of course, it would also be possible to see this whole Cycle as a planning phase, however, my conceptualisation of what I was doing was that there were deliberate actions I was taking, outside of my usual work in school, to explore academic literature, to reflect on past findings and which led to the plan for Cycle 2.

3.1 What is Mathematics in the Primary Curriculum?

It is commonly acknowledged that mathematics is a necessary ingredient for success in life and, as quoted earlier, the 1982 Cockcroft report (p1) stated that "it would perhaps be very difficult to live a normal life in the 20th century without using mathematics of some variety". In agreement with this, Kilpatrick et al. (2001, p15) noted that "children today are growing up in a world permeated by mathematics" and they believed that "anyone who wishes to be a fully participating member of society must know basic mathematics".

Askew, Brown, Rhodes, Wiliam and Johnson (1997, p6) in their work suggested that "mathematics in the primary curriculum is the ability to process, communicate and interpret numerical information in a variety of contexts". Similarly, the National Council for Curriculum and Assessment (NCCA) (n.d., p3) noted that "mathematics gives students a language through which they can interpret, analyse, describe, make predictions and solve problems in everyday life".

More recently, *The National Curriculum in England* (2013) defines mathematics as

> Mathematics is a creative and highly interconnected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject. (p90)

(*The National Curriculum in England* (2013) is the same as the 2014 National Curriculum. It is the framework document published by the Department for Education).

To me, this definition correlates with the ideas of Ziegler et al. (2017), mentioned in the last chapter, in that mathematics relates to lots of areas and subjects and it is also essential in real-life. It also connects to the notes offered by Flato (1990, p8) in that the "Greek mathematicians saw it as being the key to explaining the world as it states it is essential to everyday life, science and technology".

I wonder, though, if it lacks a simplicity in its definition in order to see the relationship of mathematics with basic number, patterns and shape. What I mean by this is that I wonder if a child looked at this definition, would they really see it as the mathematics they know and learn in the classroom or would they see mathematics as being something completely different?

Is there a difference between mathematics and school mathematics?

Boaler (2015) believes that there is a difference between the work of mathematicians and the mathematics that is learnt in schools. I was intrigued by this idea and so read further to find out that what she meant by it. My understanding is that Boaler believes that mathematicians generally work on and solve complicated problems using different aspects of mathematics whereas in many schools, children are set short and narrow questions and these questions, which are not particularly real, fill hours of mathematics classes.

My intrigue led me to look at other authors too. Lockhart (2009) illustrated a belief that school mathematics actually cheats us out of a fascinating art form. In his work he showed that he had a belief that school mathematics sucks the life out of real mathematics. Lockhart intimated that he believes that children, in school, just sit like trained chimpanzees working through the mathematics set. I understand his words to express that real mathematics, to him, is wandering, playing and imagining. I believe that he is of the opinion that school mathematics, in its current form, is easier to teach than real mathematics as he puts forward the idea that rules are just followed rather than working through a path of creativity and problems.

Beswick (2011) is of a similar opinion she believes that the difference between school mathematics and real mathematics actually lies in the purpose of the mathematics. Similar to Lockhart's thoughts, Beswick (2011, p129) stated that she believes that teachers need to have more "appreciation of the nature of mathematics" that is the mathematics of mathematicians. What I understand by this is that Beswick (2011) believes, like Lockhart (2009), that teachers need to foster the creativity and the flexibility of problem solving in order to bring the mathematics to life, make it fun and keep it real. I believe that they think teachers need to stop simplifying tasks as this takes away from the sort of complex decision making that mathematicians engage in. This makes me reflect on Boaler's (2015) words again as she believes those that are good at mathematics have learnt to solve problems. Her words make me believe that, in her opinion, this problem-solving skill is the key to mathematics and that it all starts with estimating, making guesses and then proving an answer.

Ernest (1999) also wrote about the issue of school mathematics and the real mathematics of mathematicians. In his work he suggested that, in school mathematics, children construct the required knowledge and that teachers assess the learning rather than assessing the mathematics which is what he says mathematicians do. Lockhart (2009), once again, put this in a clearer way as he said that the problem with school mathematics is that there are actually no real problems to solve. I understand this to mean that Ernest and Lockhart are of the belief that school mathematics is where children solve learned strategies rather than being set a problem to properly grapple with.

Similarly, it seems that Burton (2002) believes that we, teachers and researchers, need to recognise that what we expect children to learn in schools is not mathematics as such but is actually just a form of mathematics. Burton's (2002) work suggested that there is little connection between the mathematics in schools and the mathematics that is studied at university and beyond. In fact, I find in her writing that she is of the belief there are very few links between the two.

This makes me question whether it is actually mathematics that I love or just a form of it. It also makes me question whether all schools are being put into one category as I know in my own classroom, we grapple with problems rather than just simply copying and answering a page of algorithms.

Reflecting on these thoughts, I wonder whether a definition of mathematics, in terms of the primary curriculum, should be related to what is taught and learnt to make it have more value for our children. So, in terms of this study, I refer to primary mathematics as a subject through which children can analyse, describe, predict, grapple and solve problems in numbers, pattern and shapes.

3.2 What should be taught / learnt? – the curriculum and its reform

> Reform across the world nurtures the intention to involve students in constructing personal understandings consonant with accepted mathematical ideas. The common objective is to prepare students for a technological society in which the emphasis has changed from routine, process orientated calculating to the application of calculation in a wide range of contexts and situations through the development of more strategic thinking. (Anghileri, 2006, p364)

In England, over the last thirty years, since the late 1990s and the Education Reform Act of 1988, that was introduced by the Thatcher government, there have been significant changes in the way mathematics has been specified to be taught in schools. The National Curriculum for mathematics was introduced as a nationwide curriculum in England, Wales and Northern Ireland. One of the reasons for its introduction was to standardise the content that was taught across schools in order for all children to be given a core knowledge and also the same standard of education. The National Curriculum was also introduced as a way to raise attainment and standards countrywide.

At its inception, the National Curriculum required children to develop and use a variety of methods for calculating including mental, written and electronic. However, since this time, there have been a number of attempts at reforming the mathematics curriculum with regards to format and content to improve learning for all. My personal recollections of the first National Curriculum and its content was that it was criticised and met with some hostility from teachers across the country. I believe that some of these struggles and hostilities were related to coverage and this, I find, is partly echoed in the writing of Brown and Johnson (1996, p116) where they stated that in primary schools teachers were dissatisfied because they "were struggling to make sure all subjects were covered and, and were under pressure to keep detailed 'checklist' records and prepare for national tests".

Looking more closely at the idea of reform, Winter, Andrews, Greenhough, Hughes, Salway and Yee (2009) are of the opinion that politicians and teachers often talk of the need to improve children's attainment in mathematics at school, hence the need to change and reform the curriculum. Murphy (2004, p3) indicates that she believes reforms are used as "a policy lever to alter practices" whereas Wang, Liu, Du and Liu (2018, p53) state that "Curriculum reform is a fundamental factor in pushing forward educational development and reform".

Table 3 (p38) demonstrates reforms in the National Curriculum in England from 1989-2014. It shows the changes in format and content of the mathematics curriculum in order to improve the mathematical experiences and achievements of the learner. The National Curriculum, as mentioned previously, was introduced to raise attainment and standards and over time the curriculum, as demonstrated in the table, has been revised and developed in order to make it more manageable and to make the teaching of mathematics easier. Strategies and methods used have been revised and adapted to include different formal and informal approaches.

1989	The Mathematics Curriculum set out guidelines for mathematical content to be grouped. *These groups were divided into fourteen sections - Attainment Targets (A.T.). Each of these targets had 10 levels: Programmes of study were also given to each Attainment Target and the levels covered depended on the age of the children. An example of this would be that at Key Stage Two the majority of children would be taught Attainment Targets 1- 14 but at levels 2 – 6, as these were deemed appropriate for the differing abilities and maturities of the children of that age group.*
1991/1992	In 1991, the National Curriculum for Mathematics was revised and subsequently put into action in 1992. The content was unchanged. *The revisions meant that there would be five Attainment Targets instead of the original fourteen. The five new Attainment Targets would be made up from the original fourteen. In the 1991 revision, the Attainment Targets became: 1) Using and applying; 2) Number; 3) Algebra; 4) Shape and Space; and 5) Handling data. The new Attainment Targets were set out in strands in order to help teachers work out the level to be taught. This revised version was also intended to make the order of teaching easier.*
1994/1995	Review of the National Curriculum was launched by Ron Dearing. to reduce the curriculum and make it more manageable. In 1995, a new curriculum was introduced with a condition that it could not be changed for 5 years. *In this version of the curriculum, Programmes of Study set out what the children should be taught and Attainment Targets stipulated the expected standards. 10 level descriptors were reduced from 8 and a suggested range of achievement was given.*
1997- 2000	The Secretary of State, David Blunkett, introduced the intention that the National Curriculum would be overhauled once more. A decision was made to spend more time teaching English and Mathematics. In 1999, this new curriculum was published for teaching from September 2000. *This new curriculum relating to mathematics, The National Numeracy Strategy (NNS), recommended a daily lesson in all primary schools of between 45 minutes and 1 hour. Within this lesson there would be more of a push on understanding number, oral and mental work. The NNS was put in place as part of the government's commitment to raising standards. According to Ofsted (2011) the aim of the NNS calculation element was to develop a series of strategies, both mental and informal, to help a child's achievement in number and this would enable children to call upon a range of methods and approaches before they moved on to the more standard and traditional methods.*
2010-2014	The new Conservative government appointed Michael Gove as Education secretary in 2010 and he commissioned a panel of experts to report on a framework for a new National Curriculum and so in 2013 a new Curriculum was published. This New National Curriculum in England (2014) was introduced with a stronger emphasis on modelling in mathematics and also on problem solving in order for children to gain a deeper understanding of the mathematics being taught.

Table 3: An overview of the changes to the National Curriculum in England from 1989-2014.

In the most recent reform, 2010 - 2014, the curriculum has moved toward one of mastery and thinking in order for learners to gain a deeper understanding. However, it is necessary, before looking at this, to look at the idea of why the curriculum has been reformed in this way. Reflecting on the following statement, even though it was written almost ten years ago, could be considered as one of these reasons.

> Our overall impression of the National Curriculum is that it has far outgrown the initial concept and has become overly prescriptive. It has been interfered with and micro-managed by central government which has reduced the scope for teachers to innovate and take control of learning. The Department needs to accept that it must move away from a culture of imposition to a culture of trust and support, otherwise the National Curriculum, for all its virtues, will continue to be perceived by many to be an instrument of central control rather than a facilitator of excellent learning.
>
> (House of Commons Children, Schools and Families Committee, 2009: p60)

In support of this quote, Handal and Herrington (2003, p59) shared some worries concerning curriculum reform as they commented that "Curriculum implementation may only occur through sufferance as many teachers are suspicious of reform". Curriculum reform, in my mind, is generally introduced as a way to help and support teachers in their work. However, as Handal and Herrington also note, the intended curriculum, the implemented curriculum and the attained curriculum do not always match. They reflect on this as being due to the beliefs and strengths of the teachers delivering it.

The 2011 review of the National Curriculum stated that the new curriculum would set out only the essential knowledge that children should acquire hence giving schools and teachers more freedom to decide how to teach most effectively. The review also stated that schools would be able to design a wider school curriculum that best meets the needs of their children. Anghileri

(2006, p363) stated that "Many reform movements in school mathematics are beginning to change the emphasis from the disciplined application of standard algorithms to a more thinking approach to match the needs of today's society". She also noted that "Changes are proposed from the dominant use of traditional algorithms, that lead to overly mechanical calculating, to more flexible approaches with strategies that are appropriate for the context of the calculation and the numbers involved". In addition to this and on similar lines, Wang et al. (2018, p56) noted that through mathematical reforms in China "students were to experience the process of mathematical modelling, which would allow for the interpretation and application of the problem-solving process". They also commented "as was the hope of mathematics education reformers elsewhere in the world, students would be enabled to grow in mathematics understanding, mathematics thinking ability, attitudes toward mathematics, and appreciation of mathematics" (p57).

In support of this and also in order to support the need for reform, Ofsted (2009, p3) wrote that "primary teachers need to develop stronger subject knowledge in order to provide a challenge for their children". Ofsted also indicated that it is of "vital importance, for teachers, to shift teaching and learning in mathematics away from a narrow emphasis on disparate skills, a procedural approach, towards a focus on pupils' mathematical understanding", a conceptual approach. In order for all learners to be successful, Ofsted noted that the fundamental issue to be considered by teachers is how better to develop their children's understanding.

In line with this, Drijvers, Kodde-Buitenhuis and Doorman (2019, p438) wrote about "the importance of mathematical thinking as a key higher order learning goal in mathematics education" and Katz (2014, pxii) wrote about the idea of "mathematical thinking" being an important part of inquiry-based mathematics in schools. Considering the point made earlier about teachers and their delivery of the curriculum, Handal and Herrington (2003, p65) noted that "the current trends in mathematics education towards constructivist learning environments and assessment of learning based on demonstrable outcomes will only succeed if teachers' beliefs about these reforms are considered and confronted".

Thinking about these teacher beliefs, with regard to the introduction of this New National Curriculum (2014) there was a concern that it was tough and certain elements were going to be taught at an earlier age than before. The newly reformed *National Curriculum in England* (2013, p90) stated that "mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas". The new curriculum continues by noting that children will need to be "fluent in the fundamentals of mathematics", meaning developing an efficiency, accuracy and flexibility within the subject area. McClure (2014, np) said the key to fluency is 1) ensuring children are able to make the "connections" and 2) doing so at the "right time in their learning".

The new curriculum also stated that children would need to develop their skills in mathematical reasoning and competence. I take this to mean that children will need to be able to confidently and competently work forwards, backwards and any other way through their mathematics work, hence mastering the subject area. The National Centre for Excellence in the Teaching of Mathematics (2014) refers to mastery as having a competence and confidence within a subject area and they also note that the aim of the 2014 curriculum is that a large majority of children will achieve mastery. For this aim to be realised, children will need to acquire a deep knowledge of mathematics and a capacity to use effective strategies. With these changes through curriculum reform and also the ideas of Handal and Herrington (2013), Katz (2014) and Drijvers et al. (2019) presented earlier, I look at the statement by Wang et al. (2018, p70) where they note that "the success of the curriculum reform demands rigorous academic attitudes, national responsibility, and steady work".

I begin to wonder and ask; when thinking about the 2014 primary mathematics curriculum, can/will teachers develop and reform their strategies in order to enhance their understanding of children's thinking and do they know how to make these changes?

What are the most effective approaches and strategies for learning and whether they are the same for all?

In the following section of this chapter, I will look at an understanding in the primary mathematics curriculum leading to a discussion on conceptual and procedural methods and approaches.

3.3 Understanding in mathematics

Eisenhart, Borko, Underhill, Brown, Jones and Agard (1993, p9) stated that teachers are now being asked to spend "more and more time and attention to developing a child's understanding in mathematics". As a practitioner, although this quotation is almost thirty years old, I see it as still being relevant and linked to the idea and practices that we are now trying to embed in order to get children to a level of *mastery* in mathematics. The National Centre for Excellence in the Teaching of Mathematics (NCETM) stated that the essence of mathematics teaching for mastery is to gain understanding alongside fluency because each supports the development of the other. They say that mastery is concerned with competence and confidence. However, Hiebert and Lefevre (1986) stated that teaching mathematics for understanding is a difficult activity and process. But, I ask, what is understanding? Is it actually possible to understand something?

Michener (1978, p361) wrote a paper entitled *Understanding Understanding Mathematics* where she looks at some of the "ingredients and processes" she believes are involved in the understanding of mathematics. She wrote about mathematicians possessing a skill to sense "what to use, when to use it and what is worth remembering" when understanding mathematics. I see this as being able to select a strategy correctly and knowing what to do with it and when to use it again – to be able to do things forwards, backwards and any other which way. Michener (1978, p373) also wrote that she believes that "understanding mathematics is a very active process that must be explored and manipulated". She stated that to understand the mathematics one must travel freely within it. Importantly, she also states that understanding is never truly finished as it has so many levels.

Further to this, and perhaps for a more simplistic definition, the *Cambridge English Dictionary* (2020) defines the meaning of "Understanding" as "knowledge about a subject or situation or about how something works". As an alternative, the *Collins English Dictionary* (2020) states it is to "know how it works or know what it means". So, with regards to this study, understanding will be knowing how something works and being fluent within it as well as having knowledge about it. It is being able to move freely within an area and having the skills to select the correct manipulatives and strategies needed.

With all this in mind, I looked again at the work of Hiebert and Lefevre (1986) where they said teaching for understanding is a difficult process. I wanted to find reasons as to why they suggest understanding is complex and indeed whether others agree with them or not. In recent years, there have been many studies with regards to the approaches that can be used when teaching mathematics for understanding in the primary classroom. Haylock (2006) is of the opinion that children hardly ever get any explanation or teaching that resembles anything other than a procedural one. He also suggested that he believes that teachers are simply just not aware of any other approach. It seems, furthermore, that other researchers such as McGowan and Davies (2001) and Wees (2012) would agree with this opinion. Wees referred to the work of Skemp on relational and instrumental learning. He thinks our education system tends to work with and prefer instrumental understanding. His work also suggested that he believes that instrumental learning is useful if you want to do a something quickly and are not too worried about how it actually fits into other things that are related to it.

However, through my literature search and also talking to colleagues, I am not sure I agree with this idea of teaching being predominantly of a procedural nature, especially as Ofsted (2009, p7) stated that "conceptual approaches and practical activities promote understanding" which allow for misconceptions to surface and to be solved. In my experience, just the mere mention of Ofsted has teachers in a frenzy trying to jump the hurdles they suggest. In addition to this, I note again the work of Merttens (2012), who wrote that she is of the opinion that children, in English schools, practise with conceptual elements daily. Furthermore, Star (2005, p404) stated that the mathematics education community actually have "animated conversations" about procedural and conceptual knowledge in children's learning. Working in the field, I know about these discussions and recently have had many talks with colleagues about reasoning and mastery of skills in mathematics that goes far beyond what is considered to be a procedural approach. Therefore, as it is an important part of this study, I find it is necessary to define both procedural and conceptual approaches and knowledge and outline the uses, reasons for and benefits of both. It is necessary to ascertain how other researchers define the terms and then to determine how I will use them for the purposes of this study.

3.4 Procedural and conceptual approaches

A good teacher, when teaching, uses a plethora of styles and techniques to introduce a new topic to learners. They are trying to reach out to all individuals and their unique learning styles in order to achieve success with a topic. The styles and techniques, that teachers use, are generally based around approaches that use procedural methods, conceptual methods or a mixture of both. Rittle-Johnson and Schneider (2014, p1118) stated that "conceptual and procedural knowledge and understanding cannot always be separated". However, they do say that it is useful to distinguish between the two. Hiebert and Lefevre (1986) note that the terms procedural and conceptual, when used, show a contrast that is often made between two forms of mathematics. This, again, links to of Skemp's (1976) theory when he suggested that there were two kinds of learning in mathematics. These two kinds of learning in Skemp's mind are known as *relational* and *instrumental*: Relational being linked to conceptual and instrumental being linked to procedural.

Although Haapasalo and Kadijevich (2000, p139), in their study, noted that "procedural and conceptual knowledge are not easy to define precisely", numerous researchers such as Eisenhart et al. (1993); Chick and Baker (2005a); Star (2005); and Rittle-Johnson and

Schneider (2014) cite definitions by Hiebert and Lefevre (1986). I have synthesised into the table below a range of the key and most cited definitions I have found, in historical order..

Authors	Procedural	Conceptual
Skemp (1976)	Instrumental - a mechanical or rote style of learning where there is a rule, method or an algorithm that is known. Gives quicker results for the teacher in the short term.	Relational- more meaningful learning in which a child is able to understand links and make relationships that actually give the mathematics a structure. More beneficial in the long term as it aids motivation.
Hiebert & Lefevre, 1986,	Made up of two distinct parts. One part is composed of the formal language, or symbol representation system, of mathematics. The other part consists of the algorithms or rules for completing mathematical tasks. (p6) Used to solve tasks by following step-by-step instructions and that these instructions are "executed in a predetermined linear sequence The *how* of mathematics.	Knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network. (pp3-4) The *why* of mathematics.
Eisenhart et al. (1993)	The mastery of the computational skills and knowledge of procedures. (p9)	An underlying structure of mathematics. (p9) Relationships between ideas and interconnections that explain and give meaning to procedures.
Graeber (1999)		Able to be applied to various contexts. Gives children a form of mathematical power.
Haapasalo & Kadijevich (2000)	calls for automated and unconscious steps. (p141)	requires conscious thinking. (p141)
Kilpatrick et al. (2001)	Knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. (p121)	An integrated and functional grasp of mathematical ideas. Students with conceptual understanding know more than isolated facts and methods (p118)
Star (2005)	rote learning.	can have different levels such as a superficial level or a deep quality level.
Baroody, Feil and Johnson (2007)		Knowledge about facts and principles.
Star & Stylianides (2013)	a knowledge of action sequences or algorithms.	knowledge of concepts, principles and definitions. (p6) a deep knowledge.
Rittle-Johnson & Schneider (2014)	A series of steps or actions completed in order to accomplish a goal.	knowledge of concepts. (p2) importance in rich connections.

Table 4: Different definitions for the terms procedural and conceptual.

Common features shown in the procedural column of table 4 note it as steps, skills or actions in order to solve a problem. Common conceptual features noted are that it is considered to be a rich, deep and underlying knowledge of the mathematics.

Hiebert and Lefevre (1986), shown in table 4, suggest that conceptual knowledge and understanding is the *why* of mathematics. An example of this might be:

? + ? = 4

Within this mathematics, a child/learner with conceptual knowledge and understanding will know about the operation addition and its inverse subtraction in order to solve the problem. They will also understand the value of four. They will know that there are in fact numerous possibilities to achieve the answer and they will be able to explain why. A learner without conceptual knowledge and deeper understanding may just use trial and error in order to try to achieve the correct outcome rather than selecting it automatically or they might see the question as meaning the same number so 2 + 2 = 4 rather than seeing there are other possibilities.

Hiebert and Lefevre (1986), as shown in table 4, suggested that procedural knowledge and understanding is best described as the *how* of mathematics. I understand this to mean quite simply that children follow a set procedure to complete the mathematics they are doing – this is how it must be done in order to complete the question, such as, 2 + 2 = 4 or ○ ○ + ○ ○ = ○○○○. Graeber (1999) agreed with this but describes it as a skill that can be memorised, recited or performed to gain the necessary outcome.

Haapasalo and Kadijevich (2000) suggest definitions are difficult to define precisely as researchers have different frameworks as well as different interpretations for the same thing. And I believe this is visible in table 4, in the range of definitions that have been used. However, Star (2005) believes we need to be careful with our usage of the term procedural and conceptual as they entangle ideas of knowledge-type and quality. He defined knowledge type as referring to what is known and, quality as a way something is known and how well it is understood. Star noted that procedural and conceptual approaches have both deep and superficial layers and so we need to be careful as the terms are wider than first seen. Star and Stylianides (2013) defined this deep-level knowledge as being linked to understanding, flexibility, evaluation and critical judgement whereas superficial or surface level knowledge is linked to rote learning, the reproduction and inflexibility of mathematics.

Star (2005, p404) noted that there is “a perception that procedural knowledge acquisition has been de-emphasised and actually deemed less important than conceptual knowledge”. This possibly puts into doubt the notion of Haylock (2006), that teachers are unaware of anything other than a procedural level. Star (2005, p404) continued by stating that some educators believe procedural knowledge, in the form of algorithm knowledge, “should play a secondary supporting role to conceptual knowledge” if a deeper learning and understanding is to be gained. He also noted that if the focus is actually on a procedural rather than with a conceptual approach this could lead “to the development of isolated skills and rote knowledge”.

Star and Stylianides (2013) are actually of the opinion that the terms procedural and conceptual should be abandoned, new words being selected to describe the knowledge. They say that we should raise awareness of the difficulties in the terms and we must make it clear what we are focussing on. Star and Stylianides (2013, p18) wrote that "we could try to raise awareness of how and why mathematics educators and psychologists use conceptual and procedural knowledge in the way that they do". While I take on board the entanglement of these concepts, the fact they are so widely used, and also recognised by teachers, leads me to want to work with them and to bring some clarity to their use. In the next section I consider how the concepts might be related.

3.5 Relations between procedural and conceptual knowledge

Haapasalo and Kadijevich (2000, p145) looked at the two approaches (procedural and conceptual) and came up with what they called the four relations between them:

> Inactivation view: Procedural and conceptual knowledge are not related.
>
> Simultaneous activation view: Procedural knowledge is a necessary and sufficient condition for conceptual knowledge.
>
> Dynamic interaction view: Conceptual knowledge is a necessary but not sufficient condition for procedural knowledge.
>
> Genetic view: Procedural knowledge is a necessary but not sufficient condition for conceptual knowledge.

In this, they tried to categorise how researchers see the relationship between conceptual and procedural. Haapasalo and Kadijevich suggest many of the researchers I have looked at in table 4 fit into the category of a "simultaneous activation view". This is where they believe that procedural understanding helps conceptual understanding. They also believe that conceptual knowledge can inform procedural knowledge. The inactivation view seems implausible. My own experience suggests that some children can develop procedural knowledge with little conceptual understanding of the process they are performing, which would suggest a genetic view of their relation. Rittle-Johnson and Schneider (2015), Kilpatrick et al. (2001) and Katz (2014) are in agreement that if all children/students are to learn efficiently and achieve a mathematical competence then a mixture of approaches, both procedural and conceptual, need to be promoted.

> Procedural fluency and conceptual understanding are often seen as competing for attention in school mathematics. But pitting skill against understanding creates a false dichotomy. (Kilpatrick et al., 2001, p122).

In fact, although a dated citation, Eisenhart et al. (1993, p35) suggested that "both procedural and conceptual knowledge" and approaches are necessary in order to gain good mathematical understanding. In support of this, Mason, Stephens and Watson (2009, p11) state that "mastering procedures is an important component of taking advantage of opportunities to make mathematical sense, but that it is of little value to learners if it is simply a procedure". They explain further by noting that they believe as the number of procedures increases then it becomes more difficult to remember or retrieve them. They continue by noting that when these procedures are joined by even a little understanding then "learning shifts to focusing on re-construction" rather than relying on rote memory.

Rittle-Johnson and Schneider (2014) wrote about procedural and conceptual understanding as being iterative, in other words, gains in one, lead to gains in another. They talked about this as being the well-accepted perspective historically with regards to the relations between conceptual and procedural knowledge. However, Rittle-Johnson and Schneider (2014, p1126) also stated that they believe “conceptual instruction has a stronger influence on procedural knowledge” than the other way around. In their work, Rittle-Johnson and Schneider (2014, p1124) actually talked about “four different theoretical viewpoints on the causal relations between conceptual and procedural knowledge”. They looked at work by different researchers such as Baroody (2003), and Haapasalo and Kadijevich (2000) and put forward ideas about: concepts-first views, procedures-first views, inactivation views and iterative views.

> Concept-first view: “children initially acquire conceptual knowledge, for example, through parent explanations” and then they “derive and build procedural knowledge from it through repeated practice solving problems”. (2014, p1124)
>
> Procedures-first view: “children first learn procedures, for example, by means of explorative behaviour, and then gradually derive conceptual knowledge from them by abstraction processes, such as representational re-description”. (2014, p1124)
>
> Inactivation view: “conceptual and procedural knowledge develop independently”. (2014, p1124)
>
> Iterative view: “the causal relations are said to be bi-directional, with increases in conceptual knowledge leading to subsequent increases in procedural knowledge and vice versa”. (2014, p1124)

Rather than the logical relations posed by Haapasalo and Kadijevich (2000), who looked at necessary and sufficient conditions, Rittle-Johnson and Schneider consider relations in terms of child development – something more directly linked to my concerns in this project. If there are patterns in how children develop different forms of knowledge, this clearly has significant implications for teaching.

In keeping with the iterative approach, Baroody, Feil and Johnson (2007, p127) talked about a recommendation from the National Research Council that said "strands of mathematical proficiency be taught in an intertwined manner". They make a case that "conceptual understanding is a key basis for all other aspects of mathematical proficiency, including procedural fluency". At this point, I would agree that the iterative approach, where knowledge is constructed from all different directions, and it also fits with my experience of child development.

Kilpatrick et al. (2001, p116) wrote about a "mathematical proficiency" as having five components. These components were: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. They noted that the components were seen as "aspects of a whole" rather than being independent. In fact, they state that the components are "interwoven and interdependent in the development of proficiency in mathematics", giving further support for iterative relation between the procedural and conceptual and also for the view that these words pick out helpful aspects of mathematical competence. Kilpatrick et al. also explained that they believe that "mathematical proficiency is not a one-dimensional trait, and it cannot be achieved by focusing on just one or two of these strands". In order to achieve a clearer picture of their idea they created a diagram, shown in figure 2, which depicts how the components are interwoven.

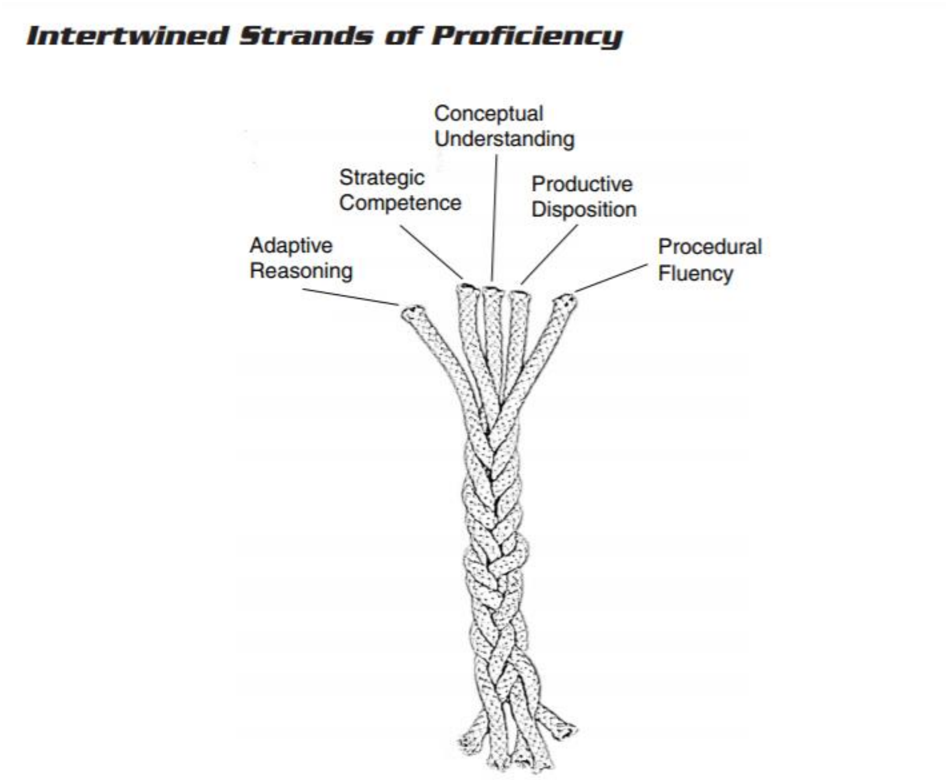


Figure 2: Intertwined components of proficiency as developed by Kilpatrick et al. (2001, p117)

Conceptual and procedural appear here as two of the five strands. While I acknowledge the potential significance of the other three components, I am concerned this would widen my study too broadly. The key point, as mentioned just above, is that an iterative view of procedural and conceptual knowledge is emerging as one that has support from a range of other researchers and fits with my own professional experience. Questions remain however, about just how these forms of knowledge relate and interact.

3.5.1 Relating models / terms to this study

Rittle-Johnson and Schneider's (2014) iterative view and Kilpatrick et al.'s (2014) model of intertwined components of proficiency can be applied to the informal method of division called chunking in that the mathematical proficiency of the calculation, identifying multiples of the dividend and then subtracting them until finding the quotient, is dependent on the interwoven elements of the strategy. The conceptual understanding and procedural fluency of chunking are bi-directional and with an increase in conceptual understanding comes the ability to carry out the procedure and vice versa. With less conceptual knowledge and understanding the bi-directional component is affected and hence the proficiency of using the strategy or carrying out the procedure is lowered.

With formal written methods of division this is also the case. The interwoven strands of the conceptual understanding of place value and the multiplication tables are interdependent and gains in one area lead to gains in another and also enable proficient usage. Both of these areas are needed to conduct the long division procedure and success in this reinforces the other. However, use of the short-written method can, in some cases, be iterative but in other cases can be seen as Kilpatrick et al.'s (2001) inactivation view - this is dependent on the learner carrying out the calculation. This is because, as Richards (2014, p5) suggested (see chapter 4.2.3), the short-written calculation can be carried out as a series of steps procedurally, without an understanding of place value. The interwoven strands are not needed to gain an answer but in order to gain a mathematical understanding and proficiency the iterative and interwoven components are necessary.

3.6 Definitions in terms of this study

So, having looked at the viewpoints and definitions written by other researchers, the following definitions have been produced taking into consideration all of the noted points. For the purpose of this study:

Procedural knowledge will be a series of actions or steps in order to achieve a goal.

Conceptual knowledge will be to make connections (at any level), reason the mathematics and create a deep understanding.

However, it is also important to note that for this study it is important to reflect the belief that procedural and conceptual knowledge work collectively in order to gain a deep understanding. They will intertwine and build on each other to scaffold and develop knowledge and understanding of mathematics. It is also important to understand that informal and formal calculations can have both procedural approaches and conceptual approaches and due to the nature of mathematics and the intertwined aspect, noted earlier, it is possible to argue that there are aspects of both approaches within calculations. Table 5 illustrates examples of this.

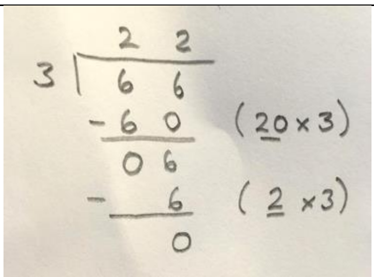
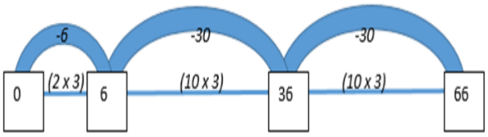
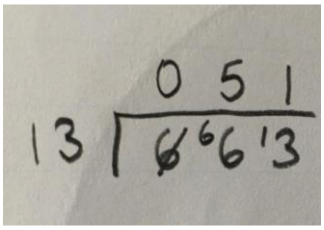
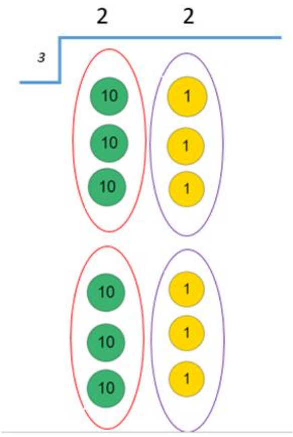
	Procedural approach	Conceptual approach
Informal method *In each case an informal chunking method has been followed for division.*	 *The subject has followed a prescribed step-by-step sequence / procedure. (using a set number of steps).*	 *The subject has used a number line to remove chunks from the gin number to solve the problem.*
Formal method *Short division*	 The example demonstrates that a step- by-step procedure has been followed to solve the formal calculation.	 This example shows the same formal approach but with the support of place value counters.

Table 5: Informal and formal approaches shown using both conceptual and procedural approaches.

Having developed a working definition in terms of this study for procedural and conceptual approaches, I ask;

How does this affect the learning and teaching of division?

What is division and how is it taught and learnt?

What strategies do children learn and in what order?

What other researchers have written about division and what are the challenges faced by children and teachers?

3.7 Research questions

In research cycle 1, I looked more closely at mathematics in the primary curriculum. Within this area, I have considered educational reforms over the past 30 years since the late 1990's before looking at the approaches they use in mathematics. Following this literature review, and as part of my reflections on the actions of cycle 1, using Brownhill's (2015) and Brownhill, Ungarova and Bipazhanova's (2017) Ice Cream Cone Model, I formulated the research questions for this action research study. Brownhill's model is a framing device that helps to structure a question that can lead to broad explanations. It looks simply like an Ice Cream with a cone. The Ice cream section, as stated by Brownhill (2015, np) in his online tutorial is considered to be "the broad area that the research will focus on". The cone is then divided into five more sections which detail "the aspect of the focus". These sections are: an aspect of the main focus purpose; context; participants; and the question stem.

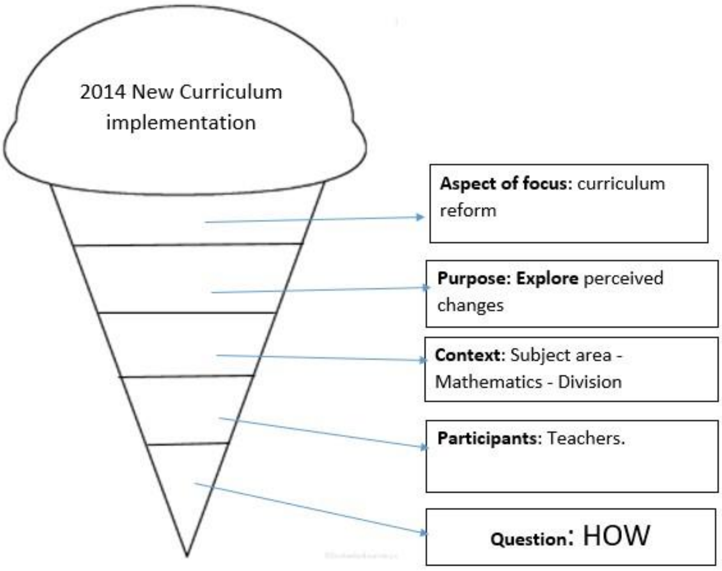


Figure 3: Ice Cream Cone Model used to formulate research question one.

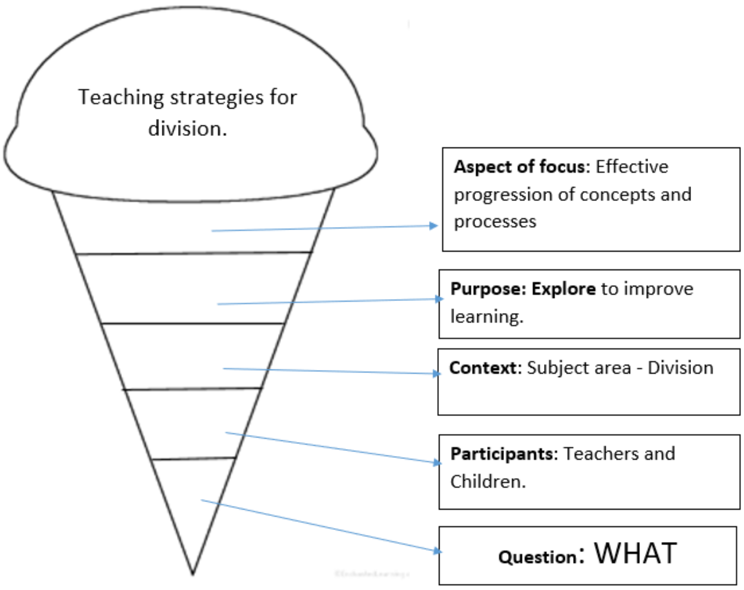


Figure 4: Ice Cream Cone Model used to formulate research question two

Figures 3 and 4 show the Ice Cream Cone Models, for this study, which were used to formulate the research questions that follow.

Q1. How do teachers perceive the changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics?

Q2. What is an effective progression of concepts and processes in the teaching of division?

In the next chapter, I continue cycle 1, as part of the replanning stage and develop the literature review so as to cover the mathematical operation of division. This then end the first of my research cycles, the planning, the action of reviewing past work, the reflecting and replanning. I consider the strategies used in the teaching and learning of division in the primary National Curriculum. I reflect on the New National Curriculum (2014) and the NCETM documents in order to gain an understanding for the research questions as it is necessary to develop a better understanding of the operation division and to get an insight into the strategies used currently in schools.

4 What is division in the primary mathematics curriculum?

In the last chapter, after presenting the reasons behind curriculum reform and attempting to define what mathematics is at primary level, different approaches (procedural, conceptual and mixed method) with regard to teaching mathematics were considered and discussed. In this chapter, which is still part of action research cycle 1 (planning, reviewing past work, reflecting and replanning), I look more closely at the mathematical operation of division with regard to the primary mathematics curriculum. Within this, I considered what division is and discuss literature including research by Thompson, Anghileri, Downton, Benson and Richards, all of whom have clear views on the subject of division in schools. The challenges faced by teachers are also considered in this chapter and cover areas such as a lack of times-table knowledge and an apparent plethora and overload of too many strategies for the same type of problems.

The New National Curriculum (2014) objectives related to division are then presented alongside suggestions from the National Centre for Excellence in the Teaching of Mathematics (NCETM). Within this discussion, I present the strategies used to tackle division problems set, how the teaching of division is developed and how a child's strategies/approaches are progressed through their primary school years.

4.1 Toward a definition and understanding of division

Anghileri (2006, p366) noted that division is traditionally seen as "the capstone of the primary arithmetic curriculum". This summit or final challenge of the arithmetic curriculum has Thompson (2012) and Richards (2014) both noting their beliefs in the fact that division is not easy and if you talk to a primary aged child about aspects they find difficult in mathematics then they will, more often than not, say division. In my opinion and experience, it is not just the children who say this but also the parents and sometimes even the teachers (especially when the long division algorithm is mentioned).

Further to this, Thompson (2003, p21) commented on himself thinking he had a good understanding of division. He noted that he had taught division to children, trainees, teachers and consultants over the years “without any apparent problems” and then when he came to study division, he decided he had understood a lot less than he had originally thought. In fact, at the end of his document, he referred to himself as confused and said he may start to limit himself to researching multiplication as he sees it as possibly more straightforward.

These thoughts once again make me wonder why this is?
Why do learners shy away from division more than other mathematical operations?
What is it that they find so complicated?

Richards (2014, p17) commented that with regard to division, there were a number of problems that children encountered. The first of these problems is times table knowledge. Richards (2014) noted that the link between multiplication and division is clear cut. Benson (2014, p31), in agreement with Richards, noted that in order to be efficient in a division calculation, “a range of strategies and secure knowledge of multiplication tables knowledge are required”. Benson is of the opinion that there is a real need to promote and strengthen multiplication table knowledge. He states that a weakness in this area inhibits a child's attainment when carrying out division problems. A knowledge of these facts develops a child’s fluency in the area and in turn aids both informal and formal calculations. In his work, Benson (2014) also noted that a knowledge of the multiplication tables helps to solve problems quickly, flexibly and efficiently.

Anghileri (2001) suggested that division needs a secure understanding in order to move from mental methods through to standard algorithms. Leferve and Morris (1999) also commented upon times table knowledge in their work. They wrote that a large number of their trial participants reported using multiplication facts in order to solve the division problems that they were faced with. Similarly, Downton (2013, p249) reported that "the majority of students consistently used multiplication to solve division problems". All these researchers noted related facts as being necessary in order to find a solution and understand the work. In other words, without this knowledge of the multiplication tables, children would struggle with understanding and making connections in the mathematical operation of division. They would lack the fluency needed to solve problems and reason their work.

Ofsted (2011) commented upon children finding chunking, an informal division strategy (see section 4.2.1), and the efficiency of it as difficult if they do not have a secure knowledge of the multiplication tables and the Department for Education (2019) stated that a multiplication tables knowledge is essential for future success in mathematics. The DfE say that it is critical for everyday life especially when tackling more complex problems.

Richards (2014) noted that another problem encountered by children, within the operation of division, is the numerous methods or strategies that can be selected, taught and used. These range from mental methods, to informal strategies and then move onto more structured approaches, which are possibly more beneficial when used in more complex problems. Anghileri (2001) wrote about there being a rich selection of strategies and approaches to choose from when working with division, from informal to formal, mental to written. However, she also noted that children can often select and use strategies that are encouraged by teachers rather than those that they may prefer.

The issue of strategies which children might prefer is considered by Benson (2014), who wrote that it is necessary for children to be in control of and choose their own approach if they are to be successful and gain an understanding. He believed, however, this decision and skill to choose is something that children can struggle with. Benson's ideas suggest that children might sometimes be unable to select the appropriate and most efficient method because they have so many to choose from and perhaps because they lack a deep knowledge of the area, and lack confidence. The implication from Anghileri (2001), is that children's reasons for not choosing the most appropriate strategy might also be due to using the ones their teachers have taught and prefer and that children may simply have not encountered the most efficient ones for them, or have not been encouraged to consider such a question.

Ofsted (2009, p8) wrote about the effect of only using one method at a time. They believe in the case that if only one strategy is developed then the children involved would not "gain confidence and intellectual flexibility" and perhaps this supports Benson's (2014) ideas because he suggested that children need to know numerous strategies in order to develop their understanding. Similarly, Van Putten, Van den Brom-Snijders and Beishuizen (2005, p1) state that "building on students' informal strategies has been recognised as a new teaching approach that contributes to understanding of formal strategies."

In contrast, in my own work (Tutcher, 2015), I considered the experiences of lower attaining children and whether it is indeed an idea to teach them lots of strategies or whether it is more helpful to teach them to be successful with one. I reflect again upon the words of Anghileri

(2001), where she suggested that teachers need to teach carefully. With all these strategies available to teach, we perhaps need to choose ones that suit the individuals in our classes and also encourage their use of supporting strategies to help the children prove and reason their mathematics. Plunkett (1979, p4) suggested the reasons for teaching the standard algorithm were “out of date” in 1979 and that they lead to “frustration, unhappiness and a deteriorating attitude to maths”. There is clearly still a need for further work on what methods are indeed the most beneficial in order to avoid overloading and confusing children and in turn aiding them to develop their understanding. The issue of the progression of methods is also one that it noted by several researchers.

A further problem related to division is inadequate mathematical skill (Richards, 2014, p15). Richards stated that in “addition, subtraction and multiplication there is a gradual progression towards more formal written methods”, but that with division this is not the case. Thompson (2012, p47) would perhaps agree with this as he stated that chunking and short division are not related and therefore one does not lead to the other; they are not progressive. He explained this further by saying that children are unable to progress to chunking from short division because the “thinking behind it is different”; and, because the “thinking behind long division is also different”, children are “just as likely to be unsuccessful using this algorithm”. In support of this, Anghileri (2006) stated in her work that it is important that strategies are progressive in order for children to become efficient (see glossary).

Van Putten et al. (2005, p1) talked about a need to ask questions with regard to how a learner’s development, the teaching they receive and their “mathematical knowledge” can affect how they move from informal strategies to formal strategies. This is why there is a need to further investigate, in this thesis, children’s learning in division so that a way forward can be found that might work against the destructive belief that division is a complicated and difficult operation. In the next section, I consider the role of informal and formal strategies for division and the influential idea of a progression from concrete to pictorial to abstract techniques.

4.2 Using Concrete, Pictorial and Abstract techniques to support formal and informal strategies

Van Putten et al.'s (2005) whole research programme is based on the premise that informal strategies contribute to understanding and can lead to more formal strategies. Richards (2014) proposed adapting and personalising informal written methods so as to help learners access calculations that they may find difficult. Benson's (2014) suggestion that a learner needs to be in control of decisions with regard to what method or approach, would imply a mix of formal and informal strategies is used so that learning suits learners' own needs. With these ideas in mind, I consider a particular approach to informal strategies and building towards the formal and abstract from the informal and concrete.

In recent curriculum reforms there has been an emphasis on the idea of "concrete, pictorial and abstract" (C.P.A) approaches to learning, in all areas of primary mathematics including division. C.P.A. developed from Bruner's 1966 concept of the enactive, iconic and symbolic modes of representation. The technique of C.P.A. has been used in countries such as Singapore since the early 1980s. The concrete aspect is where the children use manipulatives or resources such as multi-link cubes, *Dienes blocks* or *Numicon,* see figure 5, among other things, to structurally build and solve their mathematics problems. This is an activity-based approach, which is generally about learning by doing. By choosing a suitable manipulative a child can build meaning and understanding through exploration.

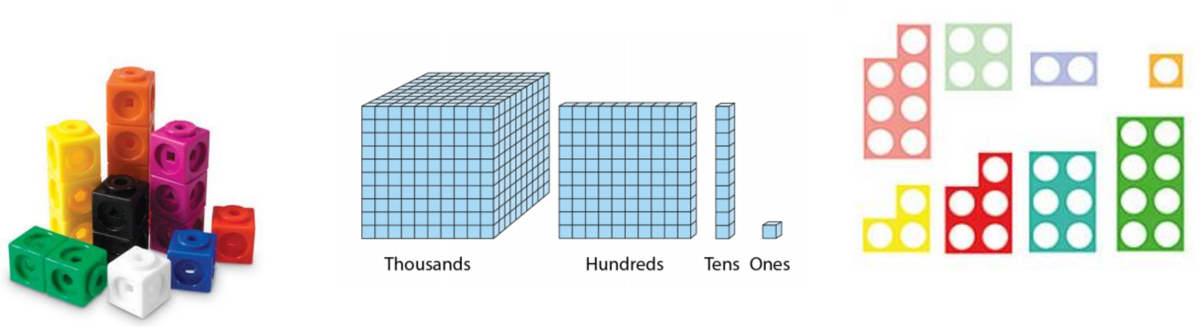


Figure 5: Manipulatives: multi-link cubes, *Dienes blocks* and *Numicon*.

The pictorial aspect involves drawing pictures in order to see and solve the mathematics, see figure 6 for an example.

21 ÷ 7 =

21						
3	3	3	3	3	3	3

Figure 6: Example of pictorial representation – The bar method. 21 ÷ 7. The top bar represents the whole (21). The divisor (7) tells us how many equivalent pieces to divide this whole bar into and then we label each piece with its value.

Figure 6 is an example of a bar model which is a pictorial representation of a problem. In this case it is being used to solve a division problem but bar models can be used for any of the four number operations. The bars within the pictorial representation represent known and unknown quantities. In the case of figure 6, the whole length of the bar is 21. The smaller bars are the '21' divided equally into 7 sections. Each of these smaller sections has a value of '3'. The pictorial representation helps children with their understanding of the problem. However, children might have a range of ways of making sense of such images and so teachers need to be sensitive to these varying ideas.

Finally, the abstract aspect is where children write using symbols and digits in order to solve calculations. Although I will explore other perspectives on this below, the whole idea is to bring mathematics to life. It is perhaps important to note, although I have described these and write about them in the order, concrete, pictorial and abstract, they can be used at any stage of mathematics and they can also be used in any order as they complement each other.

Leong, Ho and Cheng (2015) describe concrete, pictorial and abstract learning as being at the heart of the mathematics education in Singapore. However, they are surprised by the lack of writing about it with regards to its theoretical roots. They say that C.P.A is an activity-based tool where children learn by doing, by exploring. Through this exploration the children construct meaning and understanding. Merttens (2012) states that in order for C.P.A tools to be useful, that the concrete stage really must be hands-on and not just another form of pictorial representation. Merttens draws on Piaget and his view of the importance of physical actions leading to learning.

Merttens (2012, p35) also linked to the work of Hughes (1986) in *Children and Number* that showed children, of 3 years old, were able to answer mathematical problems that were presented in a way that made sense to them, for example counting elephants – "What is two elephants and one more?". The work of Hughes shows that there is no inevitability about children progressing through the stages of C.P.A.. Hughes' research showed that children could learn through an imaginary context where there were no concrete or pictorial elements to assist them.

Hughes (1986) believed there was a place for visual aspects of learning mathematics. Indeed, he suggested that visual images are amongst the most powerful in providing meaning. Hughes commented that it is important for children to relate to and use concrete items because if they cannot picture 9 bricks and 5 bricks then they will not understand the abstract digits of 9 + 5. Hughes also believed that a child's own scribbles are far more appropriate ways for them to be recording their thinking, than trying to force them to use standard notation in the early part of their learning, as these scribbles are personal to the individual. In Donaldson's work (1978, pp77-78), she presented results similar to those of Hughes (1986) and suggested that children are capable of "disembedded thought" if a problem that is given actually makes sense to them, in other words, thought that would classify as abstract on the C.P.A. model.

A further issue, relating to C.P.A., is the question of whether concrete approaches are particularly relevant to particular groups of learning. For example, Leong et al. (2015), suggest using C.P.A. is particularly effective for the lower attaining children, those that are having difficulties with mathematics as it helps them see and be involved with the mathematics. However, researchers Willingham (2017) and McNeil and Jarvin (2007) suggest concrete approaches can actually hinder learning. Manipulatives, the concrete resources, can contain irrelevant features, such as colour and shape, that distract the children from the mathematics.

Willingham (2017, p25) wrote that he believes when manipulatives are "not handled in the right way" they "can actually make it harder for children to learn". In order for children to utilise the concrete resource effectively, the child must be able to see it as just an object or a symbol. McNeil and Jarvin (2007, p313) talked about this as manipulatives having "dual representation". In other words, children have to potentially grapple with not only how these structured materials fit together but also how they relate to the mathematics they are doing. Willingham (2017, p3) states that sometimes there is a problem transferring the learning when manipulatives are used. McNeil and Jarvin (2007) suggested that this is because the “manipulatives are not transparent” meaning that children cannot “immediately see the mathematical concepts just by interacting with them”. I understand this to mean that sometimes when moving from concrete to abstract working that an understanding can be hindered. Thinking about the abstract form, the written form, of mathematics, I will now present and discuss the informal written method of chunking as well as the formal written methods of short division and long division as these are common division strategies.

4.2.1 The informal approach: Chunking

The chunking method, although most would probably express thoughts about it being relatively new at the introduction of the National Numeracy Strategy in 1999, has actually been around, as noted by Thompson (2005), since circa 1865BC in the Kahun papyrus. Another date Thompson notes for the emergence of chunking was 1965, where he said it appeared in a School Council’s Curriculum Bulletin. However, despite these occurrences, Anghileri (2006) noted in her research that an apparent shift/change in the use of strategies, concerning division, happened in 1998 and in my mind, this would coincide with the introduction of the National Numeracy Strategy. She stated that up until 1998, the standard formal written methods were traditionally used but when researching and looking again in 2003 she discovered that there had been a shift toward using the informal methods. I now see this as shifting once again in more recent years as *The New National Curriculum in England* (2013) guidance gets children moving toward formal written methods by Year 4 (aged 8 - 9 years). Part of my study aims to explore the impact such changes have on the strategies children use.

In neither the QCA document (1999) nor the NNS document (1999) could I find the term chunking, however, I found what I believe to be chunking within these documents, noted as formal written methods. Figure 7 shows an example from page 69 of the NNS document and figure 8 shows the example from page 54 of the QCA documents.

977 ÷ 36 is approximately 1000 ÷ 40 = 25.

```
977 ÷ 36      977
            - 360      10 × 36
              617
            - 360      10 × 36
              257
            - 180       5 × 36
               77
               72       2 × 36
                5
      Answer:          27 5/36
```

Figure 7: An example of ‘chunking’ using multiples of the divisor (977 ÷ 36) from the NNS (1999, p69).

```
              432
10 buses     −150
              282
10 buses     −150
              132
 4 buses      −60
               72
 4 buses      −60
               12
```

Answer 28 remainder 12. Add 1 more bus = 29

Figure 8: An example of ‘chunking’ in the subtractive form (432 ÷ 15) from the QCA (1999, p54).

In both of these examples, you can see that chunks of multiples are being subtracted from the original number hence why I have chosen them to be examples of chunking even though the term is not actually used. *The New National Curriculum in England* (2013) also does not refer to chunking as such. From Year 3 it is suggested that children develop reliable written methods and progress onto standard written methods of short division as soon as possible.

Thompson, who has written numerous pieces of work on division, wrote the paper *To chunk or not to chunk* in 2012. It focused on another document which was written by Ofsted in 2011 called *Good practice in primary mathematics: evidence from 20 successful schools*. Thompson (2012, p45) focused on the parts of the document related to the algorithm of chunking as it was this method, he believed, that "receives the worst press". However, Thompson noted that he saw the Ofsted document as one that has a bias. He commented upon the work of other researchers and stated that all evidence should be referred to before an actual judgement is made, perhaps implying that he believes that not all areas have been covered in the document.

Van Putten et al. (2005, p2), on the other hand, in their research note that an informal chunking method is actually seen as a positive and an alternative approach to long division which they comment has been "a stumbling block in the area of algorithmic procedures for many students". In their research, they noted the importance of "schematic notation" and "concrete context". The example they give, shown in figure 9 shows three different chunking solutions. The first shows, a less efficient, subtraction of multiples of 10. The other two examples show larger chunks being subtracted. Van Putten et al. (2005, p3) described this as "gradual strategy development". They noted the importance of discussions regarding "final solution" and "solution methods" hence eliminating the sense of a standard algorithm and stressing the importance of the development of the informal strategy.

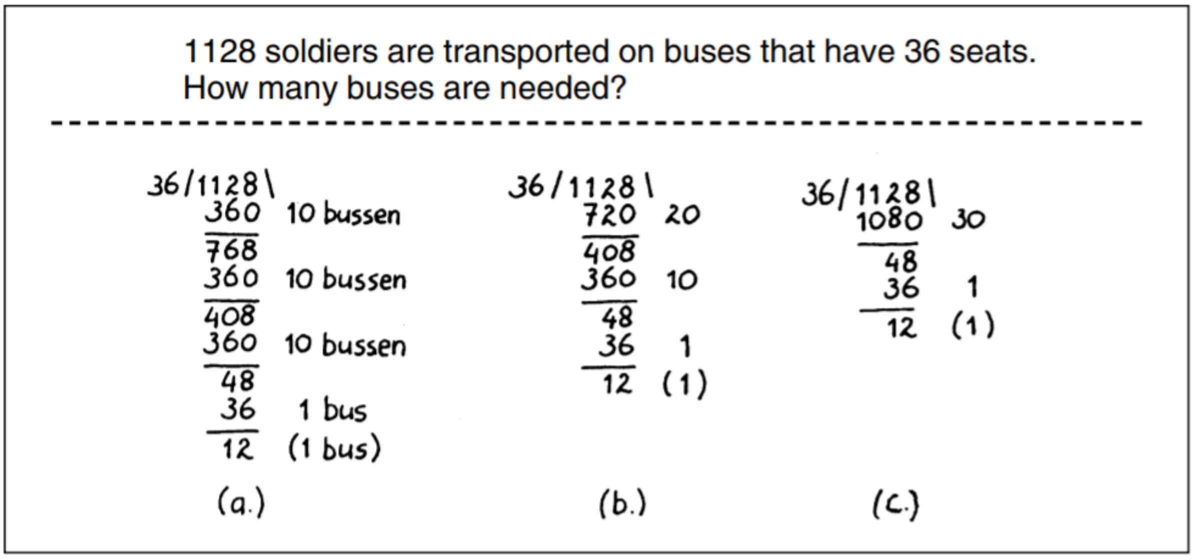


Figure 9: Levels of progressive schematisation of informal long division strategies in Dutch realistic mathematics education (RME). From Van Putten et al. (2005, p2)

Further to these thoughts and building on them, in my work Tutcher (2015), *To chunk or not to chunk: learning division, the why before the how or vice versa*, I looked at the possible need to put one method before the other, the procedural before the conceptual or vice versa. I found that the informal chunking method, which is generally seen as a conceptual approach, can in fact help a learner stay on track therefore knowing their next steps. It is a method that supports the understanding of place value and it is easy for the teacher to locate errors and misconceptions. The short division method is seen as quick, easy and there are fewer opportunities to go wrong.

It is perhaps important to clarify further that when I teach chunking, I see it as a method that teaches the learner to understand the operation of division rather than being just a procedure, although this actually depends on how it is used. I therefore label chunking, loosely, as a conceptual approach, *the why*. To me this is in line with what Hiebert and Lefevre (1986) define conceptual knowledge as being and I, repeating myself from the previous chapter, say that a conceptual approach is rich in relationships. Chunking is exactly that – a method that relates and connects multiplication to division and it is this relationship between the inverse strategies that Downton (2013) stated as being important for mathematical development in terms of informal and formal strategies.

Richards (2014, p15) pointed to one positive aspect to chunking, which is that it allows for differentiation with respect to the size of chunks that can be taken away from the divisor. This also relates to Van Putten et al.'s (2005) idea of "gradual strategy development" shown in figure 9. However, Richards (2014) then goes on to say that as a result of these different sized chunks there is a possible problem that arises in that it means some learners have to do "more calculations" because they generally take smaller chunks away each time and this can lead to possible problems in the subsequent workings of the algorithm. I believe this is possibly where the frustration and the apparent difficulties arise.

4.2.2 An alternative approach – additive chunking

Richards (2014, p15) suggested that when people visualise division, they would "chunk towards rather than away" from the number. He also noted that chunking, which is a subtractive method, is actually "counterintuitive to many children". Further to this, he noted that chunking works on multiples and when there is a remainder due to there not being a multiple of the divisor, this adds yet another problem to the situation. The challenges he mentioned just seem to snowball from one to another.

In response to these possible problems that could arise, as a modified approach to the chunking approach of division, Richards (2014) suggested a strategy that uses additive chunks. This means that instead of the usual taking away/subtracting from the divisor, the learner would actually be working towards it and, therefore, Richards believes that this fits more with the mental methods the learner would have been learning, hence, being more progressive from the work already covered. He writes about additive division being able to be deconstructed for different levels of understanding and also states that this can then be personalised by the learner using it.

There is more research, that supports the idea of Richards. For example, Thompson (2005) believed that errors can creep in when using the subtractive form of chunking and so he has also looked at this idea of additive chunking. He calls it division by complementary multiplication - again using the relationship between the two operations. Thompson (2005, p6) wrote that he believed that “subtraction is more difficult than addition” and so “it would seem sensible to try an algorithm for division that is dependent on addition and multiplication rather than subtraction”. He also talked about a need of having skills in doubling, halving, multiplying by 10 and having accurate addition abilities. Again, I can relate to his ideas and below, in figure 10, is an example of how Thompson would begin to solve a problem using complementary multiplication. In this figure, multiples of 26 are demonstrated – they are constructed through the use of doubling, multiplying by 10 and also halving.

1 = 26
2 = 52
4 = 104
10 = 260
5 = 130

Figure 10: A list of multiplications to assist division strategies in chunking.

Thompson continued this method by building up to the desired number, (he took the example of nine hundred and forty-six). He said that a child would do this by adding selected chunks to an appropriate starting number shown in figure 11. Thompson also noted that the size of the chunk used would depend on the child's achievement or confidence but it is clear that in this scenario he believed also that the children would be using the less difficult operation of addition in order to build up to the number and so would in essence find it easier.

260	10
260	10
260	10
780	30
~~260~~	~~10~~
~~1040~~	too big try x5
130	5
910	
26	1
936	so the answer is 36 remainder 10

Figure 11: An example of complementary multiplication / additive chunking.

Thompson also stated in his work that remainders can also be found by complementary addition. He says that children would begin to realise, know and ensure that the remainder is never larger than the divisor.

So, in terms of a definition of chunking for this study;

> **Chunking** is an informal computation strategy used for dividing larger numbers that cannot be divided mentally. Chunking, in this study, is repeated addition or subtraction of multiples of the divisor. In other words, working out how many groups of a number fit into another number.

4.2.3 The formal written method of 'short' division

Thompson (2003, pp21-22) looked at definitions of division terms in his work *United we stand; divided we fall*. He referred to the *National Numeracy Strategy* (NNS, 1999) and the *Qualifications and Curriculum Agency* (QCA, 1999) and how they defined division terms. Thompson (2003) wrote that the NNS framework for teaching mathematics interprets short division to be where a number is divided by a single digit number. He stated that, in these situations, times table facts can be used to help solve the calculation. His work, using the example in figure 12, actually seems to conflict with my own understanding, definition or interpretation of what I would teach as short division. Figure 12 shows the same example that appears on page 69 of the NNS Framework.

```
   ______
 6) 196
  - 180   30 × 6
     16
   - 12    2 × 6
      4

Answer:  32 R 4
```

Figure 12: The example of short division (196 ÷ 6) from the NNS (p69).

In my opinion, this looks more like the informal written method known as chunking as there are extra jottings noted to the side of the algorithm which indicate to me conceptual workings. These workings are then subtracted from the original number in chunks of the multiple.

This made me think carefully about how I would define short division for this study?

Thompson (2003) showed the following calculation which he referred to as ‘compact short division’:

$$\begin{array}{r} 209 \text{ rem } 2 \\ 6\overline{)1^{1}25^{5}6} \end{array}$$

Figure 13: The example of compact short division (1256 ÷ 6) from the QCA document (p53).

The example in figure 13 looks more familiar to me and would be described by teachers in my school as short division or the bus stop method.

Benson (2014) argues that the short division method emerged as a preferred method for many when in a test situation; Richards (2014, p15) pointed out that children can use the short division method “without understanding how it works”. Richards believes it is difficult to give meaning to the procedure and so children just have to learn the steps to succeed. He believes that to really understand short division a child’s knowledge of place value has to be complete and “water-tight”. I see this as being related to the fact that children often forget that the different digits stand for thousands, hundreds, tens and units. My understanding of what Richards is trying to say is that children would just follow the procedure and look at sixes in six (thousand) then sixes in nine and so on.

Perhaps this preference for using the short division strategy is why Thompson (2012) stated that by teaching the ‘bus-shelter’ short division method alone you end up with children unable to use it for dividing by a 2-digit number. This makes me reflect on Benson’s (2014) work again as he talked about the dangers of a learner having an over-reliance on procedure over understanding.

With the New National Curriculum (2014) in mind, and the fact that teachers are told to teach efficient (see glossary) and effective written methods, the definition that I will follow for this study is as follows:

> **Short division,** which sometimes is otherwise known as the bus stop method, is an arithmetical strategy in which the answer is written directly without a succession of intermediate workings. The divisor tends to be a single digit number although larger numbers can be used. It is represented as the example from the QCA document, as in figure 13, and also as shown in *The National Curriculum in England* (2013, p127) as shown in figure 14.

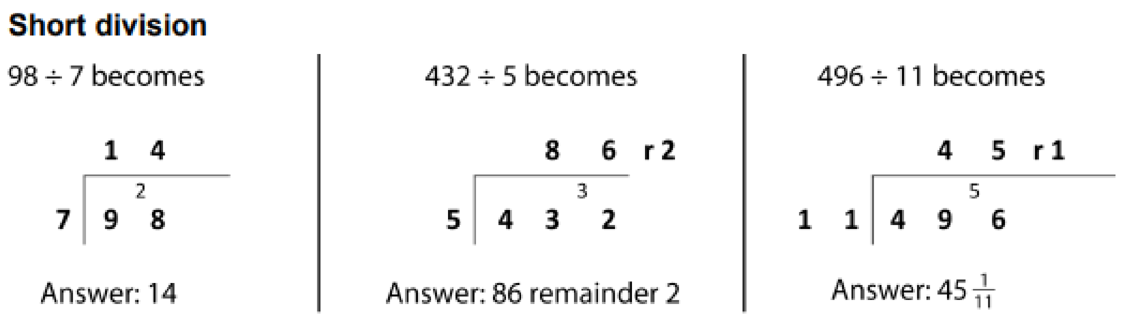


Figure 14: The examples of short division as shown in *The New National Curriculum in England* (2013, p127).

4.2.4 The formal written method of long division

Following the same line of enquiry, I first looked at Thompson (2003) and the NNS framework to determine a definition for long division for the purpose of this study.

```
                                          27
36) 972                            36) 972
  - 720     20 × 36                  - 72
    252                                252
  - 252      7 × 36                  - 252
      0                                  0
Answer:     27
```

Figure 15: The example of long division (972 ÷ 36) from the NNS document (1999, p69).

Once again, I found this example to be conflicting with my own views. I see the example in figure 15 as something that I would relate to and refer to as chunking in subtractive form. It appears from his writing that Thompson is of the same opinion. I therefore also looked at the QCA (1999) document to see what they referred to as long division. They wrote about a standard long division algorithm and the example in figure 16 is what I found.

```
      28
  15)432
     300     x 20
     132
     120     x  8
      12     x 28
Answer 28 remainder 12.
```

Figure 16: The example of long division (432 ÷ 15) from the QCA document (1999, p69).

In order to solve the division problem in figure 16, the document notes that division is emphasised in this method through its inverse operation to multiplication – this link between the two operations is what Downton's (2013) research notes as important. The QCA document shows that long division involves working successively through the hundreds, tens and one's (units in the old NC) columns. So, in terms of their example, the largest tens multiple of 15 that can be used is 20 as 15 x 20 is 300. They then need to find out what is left of the number and so subtract this 300 from the original 432 to find that they have 132 left. They then show that the largest single digit multiple of 15 is 8 leaving 12 remaining. The alignment of digits is critical in order for this method to be successful.

I still see figure 16 as subtractive chunking; the action of reviewing past work. Haylock (2006, p101) referred to the method shown in figure 16 as being something he calls an “ad hoc subtraction approach”. He commented that long division is complicated and so he would encourage the use of this alternative method as he says it builds on “mental and informal approaches”. This is something Van Putten et al, (2005) would agree with. Both figure 16 and the middle calculation in figure 17 are similar to his ad hoc method. Haylock commented that this method works well and can build on an individual’s personal confidence.

Moving on from this and still in my search for a definition of long division, I looked at *The New National Curriculum in England* (2013, p127) document and found figure 17 in their examples of written calculations. The third example with the decimal notation is what I was taught and understand to resemble the standard written method of long division. I should note that the first two methods still look like the ‘chunking’ form of division in that they subtract multiples of the number in order to arrive at the answer.

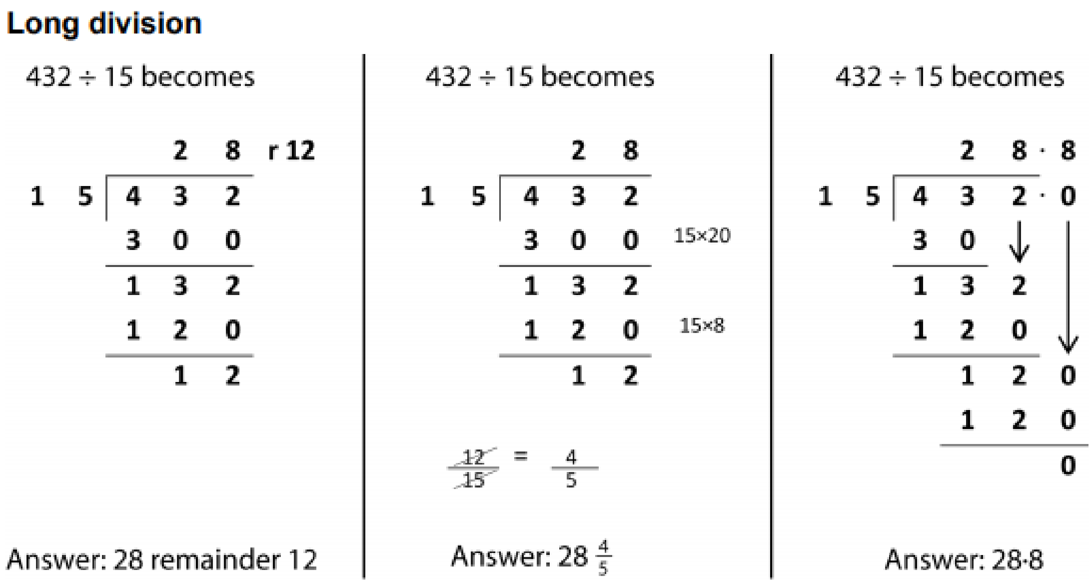


Figure 17: The examples of long division (432 ÷ 15) from *The New National Curriculum in England* (2013, p127).

So, after researching multiple documents for a definition, in terms of this study:

> **Long division** will be an arithmetical strategy in which the divisor has two or more digits and the dividend is generally a three-digit number or bigger. It has a similar layout to short division and is simple enough to perform by hand as it breaks down the division steps into a series of easier steps. Its format is as seen in the last, third example in figure 17.

Having looked at informal and formal methods of division and looking back at chapter 3.5 regarding Rittle-Johnson and Schneider's (2014) iterative view and Kilpatrick et al.'s (2001) model of interwoven components of proficiency, it is possible that the iterative and interwoven views of connection and relationships could have an impact on progression depending on how the informal and formal methods are taught. Informal methods used, such as chunking, are based around a conceptual understanding interwoven with a procedural fluency. Formal methods are based around following a set structure with a procedural fluency. However, in order for progression to take place and for it to be purposeful and be built upon logically for mathematical proficiency then connections and relationships between the informal and formal methods need to be clear.

To illustrate this point further, Van Putten et al.'s model (figure 9), shows the levels of progressive schematisation of informal long division strategies. It demonstrates different levels of conceptual understanding based on the multiplication tables. However, without the connections and conceptual understanding of multiplication, its inverse operation, and the other processes needed to solve a calculation, then the learning and the ability to carry out the method falters. In my work, Tutcher (2015), I wrote about the possible need of putting one method before the other in order to be successful. With the iterative views and interwoven methods in mind it is clear there is a need for children to see and make connections between the informal and formal methods they use in order to develop a mathematical proficiency, rather than just doing one method or the other.

In opposition to Van Putten et al.'s work, Thompson (2005) and Richards (2014) both wrote about the difficulties with subtractive chunking, putting forward the idea of additive chunking as being more relatable to the mental methods that children have been taught and have learnt hence aiding progression and perhaps even enabling children to make the connections they need for interwoven strands and understanding. By relating procedural fluency and conceptual understanding, children are able to adapt more readily to the different strategies and problems presented to them.

4.3 The progression of strategies in division in the primary phase

Having presented others researcher’s views on division, I moved on to discuss some of the problems encountered with the operation. I looked at the different ways it can be taught/learnt with regard to concrete, pictorial and abstract techniques before I considered and presented the most common informal and formal written strategies. In this section, I note how the New National Curriculum (2014) suggests that the teaching of division should progress through a child’s primary education. I contrast this with the National Centre for Excellence in the Teaching of Mathematics (NCETM) ideas for the same Key Stages. I then analyse the differences and present them with suggested images and manipulatives and how these relate to the key ideas.

Before moving on, I clarify the difference between the National Curriculum and the NCETM:

> The National Curriculum is a document set out by the government, which gives a set of standards used by schools. These standards are in place so that all children in the country learn the same things and reach a certain level.
>
> The NCETM is a team of people who work alongside teachers supporting them in embedding the National Curriculum in respect of mathematics. The team builds on the National Curriculum to produce materials that will enable teachers to break concepts down into logical steps. The NCETM is a professional development team. Their work splits the curriculum into three separate spines, which cover all elements of the mathematics curriculum. Spine 2 is concerned mainly with multiplication and division but there are links to other areas of the mathematics curriculum too. The work of the NCETM is there to support and improve teacher subject knowledge. Their material is there to stimulate thinking, provide activities and suggest representations to aid the delivery of the New National Curriculum (2014) and the mastery approach in a coherent way and so children can make connections.

4.3.1 The progression of strategies in division in Key Stage One and Two

In Appendix 5 there is a detailed breakdown of both the National Curriculum and NCETM approaches to division. In this and the next section I summarise from this appendix, to offer the key progression of strategies envisaged in each document. A key similarity in the approaches to division in both the NC and the NCETM documentation is that both follow the statutory requirements presented in the NC. However, the apparent differences in planned progression routes through division as seen in the National Curriculum and the NCETM (which is a DfE funded body) are stark when laid out next to each other. By Year 3 in the NC, children are meant to have made steps and moved onto formal strategies, whereas informal strategies are emphasised in the NCETM document all the way up until Year 6. The NCETM focus is on a teaching for mastery approach and so they split the curriculum areas into spines and segments (see glossary). Within these spines and segments they develop smaller steps of learning to aid mathematical thinking and understanding. The NCETM do not rule out the use of formal strategies, what they suggest is the use of the informal strategies to support and develop a deeper conceptual understanding in the area. The NCETM note that it is important to look at these algorithms (formal and informal) together so similarities and differences can be seen. It is also important for children to see that the underlying mathematics is the same.

	The National Curriculum (2014)	NCETM
Year 1	Children should have access to and be using concrete objects, pictorial representations and arrays to solve one-step problems.	Children should count efficiently in groups using skip counting (2, 4, 6, …; 5, 10, 15, …) this must be done in groups of a given number (2, 5 or 10).
Year 2	Children should be able to solve problems involving division, using materials, arrays, repeated addition, mental methods, and division facts, including problems in contexts. Children should link division and multiplication.	Quotative and partitive division are distinguished. Children should use arrays and a number lines to solve problems. Children should link division and multiplication.
Year 3	Children should progress to using formal written methods in order to solve division problems.	Children should continue to use informal methods to solve division problems, making use of the number line and place value counters.
Year 4	Children should recall division facts for all the times tables up to 12 x 12. Children should be introduced to informal methods but also encouraged to move onto standard formal written methods of division such as short division.	Answers to a division calculation should be interpreted carefully in order to make sense of the answer and the remainder; continued use of informal methods. Children should work on an understanding of the structure of the division algorithm. Language plays an important part in the understanding of the operation.
Year 5	Children should be able to divide numbers mentally drawing on facts that are known to them.	Continued use of informal strategies, partitioning, leading to short division and then long division. Enable children to gain a deep understanding of the underlying mathematics; children should be able to make sensible choices about strategies and representations that they use.
Year 6	Children are required to divide numbers of up to four digits by a two-digit whole number using the formal written method of long division.	Continued use of informal strategies, partitioning, short division and long division.

Table 6: Progression of division strategies in Primary School.

The NCETM note that a key focus of their progression is to enable children to gain a deep understanding of the underlying mathematics rather than learning by rote or just procedure.

Another difference that stands out in table 6 is the NCETM's inclusion of the importance of using mathematical language and appropriate representations. The NCETM (2019, p3) note that within their work "there is a strong focus on careful use of language to accurately describe division" and the different structures. At Key Stage One, the NCETM, in addition to the New National Curriculum (2014) terminology of grouping and sharing, begin to use the terms 'quotitive' (grouping) and 'partitive' (sharing) for division. The NCETM (2015, p17) explain that they believe that "the quality of children's mathematical reasoning and conceptual understanding is significantly enhanced if they are consistently expected to use correct mathematical terminology".

The disparity between guidance from two branches of government signal the need for further research into the progression of strategies of division. The first stage of my action research also highlighted a clear need for this further work.

4.4 Action Research cycle 1

I began this action research study with a plan of looking at how the introduction of a New Curriculum would affect teachers in their delivery of primary mathematics. Through the actions associated with analysing relevant literature, outcomes of this cycle were: the definitions of key terms (procedural and conceptual); an elaboration of different aspects of division (partitive and quotative); the key strategies for division (short division, long division, chunking); and, key distinctions relevant to the teaching of division (use of formal/informal strategies, use of C.P.A., progression of strategies). Cycle 1 highlighted contradictions in advice to teachers about the progression of strategies for division and a need for further research in this area. These were outcomes that meant I was ready to plan my own data collection.

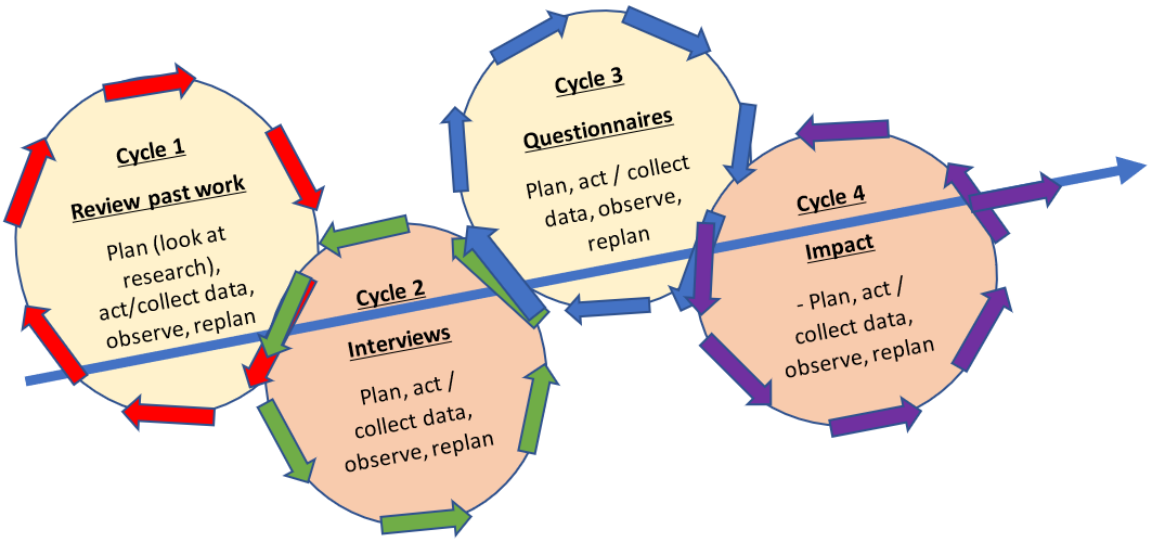


Figure 1 (repeated): The process of action research in this study – a collection of circles that interlink. (adapted from Altrichter et al.'s (2008) idea of the circle of action and Kemmis et al.'s action spiral research).

In the next chapter, I move onto cycles 2 and 3 and set out the methodology that was followed. I planned, acted, observed, collected data and then replanned. Cycles 2 and 3 overlapped, as they were both concerned with gaining an understanding of what is happened in the studied school but the data is collected in different ways (interviews and questionnaires). These cycles could be viewed as part of a single cycle, but for purposes of presentation I have found it helpful to separate them initially and then present the findings together.

5 Methodology

In chapter one, I reflected upon reading *Teachers investigate their work* written by Altrichter et al. (2008). Through this reading, I identified this study as action research as it enabled me to implement changes in my own school's practice. Tripp (2003, p4) states that action research is a form of "action inquiry". He noted in his work that action inquiry has many forms such as "reflective practice" and "action research" and that it is the characteristics of each that determine what type of inquiry is being followed. However, saying this, Tripp (2003, p6) also commented that a "crucial defining characteristic" of both these forms is "strategic action". He defines strategic action as an "action which is based upon an understanding achieved through the rational analysis of deliberately sought information, in contrast to action which is as a result of habit, instinct or opinion". I see this study as action research rather than reflective practice because as Tripp explains;

> In the example of action research I suggested that I might interview some students; were I engaging in reflective practice I might also do that, but I would not code their responses, try to explain them in theoretical terms and verify my explanation; rather I would simply listen carefully to their ideas and opinions and take them into account in deciding what to do next time. (Tripp, 2003, p7)

Looking at Tripp's example, I saw that my research was more action research than reflective practice as I always intended to code, explain and back up my explanations. Tripp wrote that when a researcher thinks about what has happened then some sort of description happens. Tripp (2003, p7) noted that if this description is "simply a privately recalled memory" then it is deemed reflective practice. He continued by noting that by simply writing it down "moves the practice more towards research as it increases both the formality and the opportunity for public scrutiny" of the work. One further point to note is that Tripp states that "there comes a point at which the systematic recording of recollections in writing is recognised as a valid research procedure, which would clearly shift the practice from reflection to research”. This is the justification of why my research is action research.

The overall aim of this, UK-based, study was centred around my belief that children should receive positive experiences in mathematics from all those that support and encourage them. In order for this overall aim to be broken down and investigated, and as a result of the literature researched in cycle 1, the research questions were formulated and presented at the end of chapter 3. The research questions were:

> Q1. How do teachers perceive the changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics?
>
> Q2. What is an effective progression of concepts and processes in the teaching of division?

Chapters two to four were concerned with the first stage of my action research as they considered what mathematics is and covered teaching approaches used. After the formulation of the research questions and a narrowing down of the topic, the research in chapter four focused on literature surrounding the operation of division. The strategies used in the teaching and learning of division were discussed and as the study used the NCETM and the New National Curriculum (2014) documents it had a political dimension. Following the planning, reflecting and replanning of research cycle 1, it was confirmed that division in schools was an area that needed further investigation. As well as this, within cycle 1, terminologies were clarified and definitions given in relation to this study.

In this chapter, I discuss the participants and ethical considerations for the study. I then show how my design was influenced and created in order to answer my research questions and to provide results that informed future practices. Within the school, in which the research took place, all staff and children had their own thoughts, beliefs and experiences in mathematics. They had their own level of prior attainment and perceived skill. This action research study drew on the thoughts and experiences of these staff and children, at the researched school, and created a picture of the attainment and knowledge at the school and therefore helped increase success rates when carrying out division problems. The design of the study drew upon interpretative theories of learning and in this chapter, I clarify my researcher positioning as well as reasoning the use of an interpretative framework. .

5.1 Participants

This action research study, in part, was concerned with the different strategies and procedures used by children and teachers when solving division problems. I looked at a rounded picture – the teaching, the learning and also the progression. I looked at how curriculum reforms affect how things are taught and why this was. I wanted to see how this affected the strategies children used. Following on from this, I looked at the advantages and disadvantages in the strategies that have been taught and used in order to think about an appropriate progression in the future.

The school in which this study was undertaken was a larger than average Primary School in a rural setting. It had 230 children in the Junior part. The vast majority of the children were of White British heritage and nearly all lived close to the school. The proportion of children who were known to be eligible for free school meals was below average and a below average proportion of children had a special educational need. The junior children were divided into eight different classes with approximately 60 children, split into two classes, in each year band or phase. The aim for this study was to involve all junior teachers (11) and junior children (230) at the school. Participation, however, was on a voluntary basis.

5.2 Ethical considerations

Hammersley and Atkinson (1995, p209) stated “there are ethical issues surrounding social research just as there are in any form of human activity”. They are also of the opinion that there is a need for researchers to be careful in their goal of producing knowledge as their research can throw light on other issues. Clough and Nutbrown (2012) state that requirements in the area of ethics are designed to provide protection for researchers and their participants and so the University of Bristol Ethics of Research Policy and Procedure (2015) and the BERA Ethical Guidelines for Educational Research (2018) were reflected upon throughout this action research study.

BERA (2018, p21) state that “the confidential and anonymous treatment of participants data is considered the norm for the conduct of research”. With this in mind, within this study, participants were assured that no names, titles or other traceable links would be used and care was taken with voice recordings so that readers would not be able to infer participants identities. Data was stored in a locked cupboard or on an encrypted memory stick until being destroyed securely or deleted.

It was not intended that anyone would come to any harm in this action research study. Ensuring practices suit individuals and all levels of skill (differentiation) without singling any individuals out was paramount. This participants in this study were recruited on a voluntary basis and the BERA (2018, p9) guideline “researchers should do everything they can to ensure that all potential participants understand, as well as they can, what is involved in a study. They should be told why their participation is necessary, what they will be asked to do, what will happen to the information they provide, how that information will be used and how and to whom it will be reported” was followed.

In addition to this, BERA (2018, p18) noted that “researchers should recognise the right of all participants to withdraw from the research for any reason, and at any time, and participants should be informed of this right”. Participants were therefore told before starting of this right but, in order to minimise the risk of participants withdrawing, the whole process was unobtrusive so that those who took part did not feel stressed. Hammersley and Atkinson’s (1995) thoughts about being part of research in itself causing fear and anxiety were kept in mind throughout the study.

As children were involved in this study, it was also necessary to be aware of age-related ethical considerations. BERA (2018, p15) stated that “the best interests of the child are the primary consideration, and children who are capable of forming their own views should be granted the right to express those views freely in all matters affecting them, commensurate with their age and maturity”. Children who were considered unable to take part, for example those with special educational needs that affect maturity rather than academic levels, were withdrawn from taking part by their class teachers. In this study, children were told about the researcher’s interest and told that their results and thoughts would help with future practices at the school. The children were told that they did not have to participate if they did not want to and that they could withdraw or stop at any point. As the study was also part of the researcher's role and used for school improvement practices then consent was also gained from the Headteacher, as the appropriate adult, to use confidential test data as long as names and other revealing data were removed.

One ethical dimension within this study was seen as my dual role as researcher and teacher. BERA (2018, p13) write that "An important consideration is the extent to which a researcher's reflective research into their own practice impinges upon others". Due to my appointment to mathematics co-ordinator at the school, I was able to navigate problems that arose. I remained professional throughout the collection of data and I remained open and honest with all participants. Everyone who volunteered to take part in this action research study was treated with respect and valued. Any impact from pressure placed upon participants, in terms of time given or taken, was kept as low as possible and this was also the case for the level or quality of completion of the different data collections. There was a danger in relation to the dual role, of acting as researcher in some contexts, when what would have been best for my pupils would have been acting as teacher. For this reason, I chose not to collect any data about my own teaching, or how pupils responded in the usual course of lessons.

I worked closely with the Senior Leaders at the school and kept appropriate rapports with colleagues and children, in each role. I felt that the dual role actually enhanced my roles in each – the researcher in me developed my teacher practice and the teacher in me informed the researcher side as to what would be possible and what would be too much for colleagues to take on board. An example of this was when as a researcher I got a little frustrated at the little time that teachers could give me for interviews but the teacher in me sympathised – I navigated this by keeping teacher wishes and well-being at the forefront. The dual role also made me aware of the barriers faced with regard to the understanding and vocabulary used by a mathematics academic compared to that of a non-academic. I also kept in mind, throughout the study, BERA's (2018, p29) point where they stated "educational researchers should not criticise their peers in a defamatory or unprofessional manner, in any medium".

Another important factor of the study was to keep all participants informed of the aims and developments throughout. BERA (2018, p32) wrote that "educational researchers should communicate their findings, and the practical significance of their research, in a clear, straightforward fashion, and in language judged appropriate to the intended audience". Throughout the timeline of the study, I presented staff meetings and had informal discussions with all participants with regard to findings and developments. Children were informed through discussions with their class teacher and changing practices in the classroom. The ethics forms related to this study, see Appendix 1, were submitted to the Graduate School of Education in 2014 (pre-progression) and then, due to the General Data Protection Regulations introduced, in 2018 (post progression). E-message approval notes can also be found in Appendix 1.

5.3 Philosophical underpinnings and an Interpretative framework

Research is governed by the author and their particular beliefs. Bailey, Barrow, Carr and McCarthy (2010, p30) noted that "the paradigm 'isms' might be more accurately described as particular viewpoints on particular issues". They said that to subscribe to them is to have a view about a matter. A paradigm itself can be interpreted as a belief that guides the way we do things and this can range from thought patterns to actions. Paradigms, in research, govern how decisions are made, how research is carried out and determine what techniques are used. As a researcher, it is important to know to which discipline you belong as there are different ways of viewing the world and one's own approach to knowledge is indeed one of many.

My journey through the 'isms', in my PhD taught units (positivist, post-modernist, critical theory and interpretivist), brought about much confusion. My initial learning led me to the work of Clough and Nutbrown (2012). They wrote about the 4P's and the fact that they believe they are a good way of studying and evaluating educational research: "purpose, position, persuasion and politics" (p4). They indicated that purpose is to do with a clear vision and a reason to study. They continued with position and said that this is to do with the viewpoint and the stance of the author. Further to this, Carson, Gilmore, Perry and Gronhaug (2001, p1) state that "a research position will have implications for what, how and why research is carried out". No paradigm is right or wrong, we just have to justify our position, views and work. In support of this, Taylor and Medina (2011, p1) write that "no research paradigm is superior, but each has a purpose in providing a unique knowledge".

I follow an interpretivist paradigm as I believe that I construct meaning from human thoughts and beliefs. However, I question this as I wonder if I am a positivist as I believe in 'truth'.

According to Guba (1990), paradigms can be characterised through their ontology, epistemology and methodology. So, through looking at work by Crotty (1998), Carson et al. (2001), Mackenzie and Knipe (2006), Bailey, Barrow, Carr, and McCarthy (2010) and Patel

(2015), I have developed and produced table 7. It gives an outline of what each paradigm is considered to be and how it can be broken down with regards to ontology, epistemology and methodology. The idea of table 7 is to show how one paradigm differs to the other in terms of beliefs and methodologies.

Paradigm	**Ontology** *The study of being. What exists and what is reality?*	**Epistemology** *The theory of knowledge. What and how can I know reality / knowledge?*	**Methodology** *How it is carried out – a system of methods. A procedure to acquire the knowledge.*
Positivism "Offers assurance of unambiguous and accurate knowledge of the world. Based on observation and experience" Crotty (1998, p18)	"There is a single truth or reality". (Patel, 2015, np)	Reality can be measured and known (Patel, 2015, np)	"quantitative methods tend to be predominant. Experiments. Quasi-experiments. Tests Scales." (Mackenzie and Knipe, 2006, p7) "Concentrates on description and explanation." "Detached, external observer." (Carson et al., 2001, p6)
Interpretivism. "All knowledge and reality is dependent on human practice – meaning is not discovered it is constructed." Crotty (1998, p42)	"There is no single reality or truth". (Patel, 2015, np) Each person builds their own reality so there are multiple interpretations. This reality needs to be interpreted.	"Reality needs to be interpreted. It is used to construct meaning of activities and events". (Patel, 2015, np)	Usually qualitative – qualitative interview, observation, case study, life history. (Patel, 2015, np) "Qualitative is predominant but quantitative can be utilised. Interviews Observations. Document reviews. Visual data analysis." (Mackenzie and Knipe, 2006, p7) "Concentrates on understanding and interpretation." "Researchers want to experience what they are studying." (Carson et al., 2001, p6)

Table 7: How two research paradigms, positivism and interpretivism, are defined and broken down.

Due to the nature of this research and also my beliefs following on from the deconstruction and clarification of the paradigms, this study follows the interpretivist paradigm as it is a study which sought to build on people's practices, thoughts and ideas. I do not believe there is a single truth to be found in the research I carried out and I believe that the knowledge gained has been interpreted and constructed to give meaning and direction. Within mathematics there may be one correct answer but, in my opinion, and experience there are multiple ways, formal and informal, straightforward and complex, in which any answer can be arrived at and therefore it is the interpretation of these methods/approaches that will form this study.

An interpretivist position feels most natural for me to follow especially when I read the analogies presented by Taylor and Medina (2011) where they said that "a positivist fisherman is one who stands on a riverbank and describes (without getting his or her feet wet) the social properties of a species of fish by observing the general tendency of their group behaviour as they swim around" (p2). In comparison, they wrote that they believe "the interpretive fisherman enters the water, establishes rapport with the fish and swims with them, striving to understand their experience of being in the water" (p4). As a current education practitioner and researcher, I relate to the interpretive fishermen as I do indeed swim with my colleagues. In support of this, Carson et al. (2001, p9) wrote that "in positivism the researcher is independent but in interpretivist research the researcher is involved". I intend for my research to make a difference to my colleagues and other practitioners and I want to listen to them, interpret what they say and construct meaning from it to act accordingly. Black (2006, p319) states that "the strength and power of the interpretivist approach lies in its ability to address the complexity and meaning of situations. I want my study to be used and understood by my colleagues and other practitioners and also to add to the knowledge of the reader.

Action research sits well in an interpretative approach, which Hammond and Wellington (2020, pp158-159) propose “argues that human behaviour can only be explained by referring to the subjective states of people acting in it”. Interpretivism focuses on social practices and “it assumes that all human action is meaningful and hence has to be interpreted and understood within the context of social practices” (Usher, 1996, p18). For this study, my interpretivist approach meant it was necessary to gain a picture and understand from the practices of the researched participants what was happening in schools in order to support them and suggest ways forward in their mathematics practice. I repeat the analogy of Taylor and Medina’s (2011) fisherman where they stated that an interpretative fisherman enters the water, builds a relationship with the fish as he/she works with them and shares their experiences.

This study, also fits well within the viewpoints and techniques of social constructivists and, as defined by Creswell (2014, p8), this is typically an approach used in qualitative research. Creswell (2014, p8) stated that “social constructivists believe that individuals seek to understand the world in which they live”. Creswell, in his work, suggested that social constructivists read and translate what they find in respect of their own thoughts and experiences. He believes that rather than starting with a proposal or idea, social constructivists devise and build them as they see patterns appearing. I view interpretivism and social constructivist, as closely connected (Glesne 2006; Creswell 2014) as they both consider the social world and an understanding of it. I follow Carson et al. (2001, p15) in taking social constructivism to be “under the umbrella term of Interpretivism”. My philosophical positioning is reflected in the aims of the research and the research questions because as Carson et al. (2001, p5) wrote “the interpretative approach allows the focus of the research to be on understanding what is happening in a given context". This is the exact premise of the study – to gain an understanding of what was happening in the studied school with regards to the implementation of a new curriculum. And, within an overall interpretivist philosophical stance, I have adopted a social constructivist epistemology and ontology.

A social constructivist builds understanding and knowledge as a result of social interaction and language use, for instance, Carson et al. (2001, p16) state that the aim of social constructivism is to "achieve an understanding of the similarities and differences of constructions that both the researchers and the respondent" hold.

Qualitative research designs were used in this study as they sit well in the interpretivist/social constructivist paradigms. The reason that these research designs were adopted in this study instead of quantitative ones are because, as Mason (2002, p1) stated, "through qualitative research we can explore a wide array of dimensions of the social world, including the texture and weave of everyday life, the understandings and experiences". Neuman (2007, p43) noted that "interpretive researchers tend to trust and favour qualitative data" as they believe it "can more accurately capture the fluid processes of social reality".

Given my research aims, a fully quantitative approach does not suit the purpose of this study. To consider this further, Glesne (2006) explained that quantitative approaches tend to be where the researcher is detached from the study, takes an experimental approach and seeks causal explanations. Glesne, in contrast, described the qualitative approach as being one where the researcher is involved personally, carried out in a natural setting, hence fitting to an action research study. Understanding and interpreting behaviour is inevitably a qualitative undertaking.

Brown and Dowling (1998, p82) also offer an idea or distinction between qualitative and quantitative approaches by saying that "qualitative approaches are often concerned with research that sits in an interpretative framework in studies which are concerned with the production of meaning", which fits well in terms of this study. They continue by stating that "the quantitative methods are more often than not associated with a positivist enquiry where there is a need to search for facts". The quotation below also offers further support as to why qualitative approaches will be better for this study:

> Qualitative research is an approach for exploring and understanding the meaning individuals or groups ascribe to a social or human problem.
>
> Quantitative research is an approach for testing objective theories by examining the relationship among variables. (Creswell, 2014, p4)

Objective theories would appear to have little relevance to the day-to-day life of a teacher. I was searching for the meaning ascribed by pupils and teachers to their work and aspired to the aim that "qualitative research gives us compelling descriptions of the human world" (Brinkman and Kvale, 2009, p47). In addition to this, Ritchie and Lewis (2003) were of the opinion that qualitative research is a form of research that informs us of the what, how and why instead of the how many. So, in this study, I used a qualitative approach in order to try and understand and make sense of human behaviours and practices in the area of mathematics. I looked at the why, the how and the what elements in order to make sense of what was happening and how it could be improved. However, I did have the opportunity of accessing assessment data across the whole school and so there was a quantitative element to the study, which was used to triangulate the findings from the more qualitative data.

Methods associated with this approach, focus groups (both formal and informal) and questionnaires, were followed in cycles two and three of this action research study. I deemed focus groups to be useful and appropriate given that they allowed for flexibility of answers as well as additional questioning if the situation arose. The questionnaires were used in order to gain a knowledge and understanding of how teachers and children perceived their own attainment in mathematics and also to ascertain what strategies they fall back on in certain situations. They were used as they have the capacity to extract meaningful and rich data from a participant.

Following the interpretivist and social constructivist paradigms, this action research study sought to understand human behaviour and also aimed to build future practice from it. It used approaches similar to those in other studies in the same field (Hughes, 1986; Cavanagh, 2006; West, Noden, Edge, & David, 1998; Chick & Baker, 2005a). I believed these methods had been tried and tested by others and so were appropriate to gain the data needed.

Table 8 shows how the cycles of this action research study were split, for clarity, in this study. The cycles were split into study parts so that teacher and child responses were kept separate.

Action research cycle	Qualitative approach	Study parts
2	Focus group - teacher	A
	Focus group - child	B
3	Questionnaire – teacher	C
	Questionnaire – child	D

Table 8: Overview of methods to be used in cycles 2 and 3.

5.4 Focus groups (Study parts A and B, action research cycle 2.)

Oatey (1999), described interviews as giving flexibility and freedom to an interviewee and Frey and Oishi (1995, p1) noted that interviews allow for “purposeful communication”. Interviews are also seen as allowing a researcher to be actively involved in the data collection where they can explore and examine someone else’s life. Brinkman and Kvale (2009, p47) described “research interviewing as a knowledge - producing activity”. In line with a social constructivist view, Brinkman and Kvale (2009, p54) commented that “knowledge is not merely found, mined, or given, but is actively created through questions and answers”. Adams and Cox (2008, p21) wrote that they believed “interviews are used when a researcher wants to obtain more detailed and thorough information than can be gained from a questionnaire”.

There are many types of interviews: structured; unstructured; semi-structured; and focus groups. It was therefore important to select the most appropriate for the research being carried out. Focus groups were selected for this study, in preference to the others, because set questions were asked and then, as the interview unfolded, extra questions were asked when needed to aid understanding of the answers given. Vaughn, Schumm and Sinagub (1996, p5) described the "goal of focus group interviews is to create a candid, normal conversation that covers, in depth the selected topic". They also noted that focus groups draw out "perceptions, feelings, attitudes and ideas of participants". Bogdan and Biklen (2007) are of the opinion that this approach also allows those who are being interviewed to give their own views or stories as the questions are not too structured. In their writing they also suggested that a richer understanding of the beliefs and thoughts of those being interviewed can be gained from this sort of talk.

Focus groups were used, for both teachers and children, instead of individual interviews as these enabled participants to hear each other's answers, add to them and it also gave them the security of others being around. This gave me the opportunity, as the researcher, to gather more rich data by guiding participants with prompts and nods to keep them focused on the questions so they could get their views and opinions across. I believed the participants might discuss things in a manner or way that I, the researcher, may not have thought about.

Bogdan and Biklen (2007) suggested that the most successful interviews are ones which settle the participants and allow them to be at ease. They are of the opinion that, in this type of interview, where participants are relaxed and settled, they will talk openly and freely about their points of view. They argue that this approach produces rich data that is full of words that reveal the participants' thoughts and ideas. Mason (2002, p67) noted that focus groups are just like having a "conversation with a purpose". It was my intention, in this study, to develop an interview where all participants felt relaxed and at ease to discuss positives as well as concerns. Adam and Cox (2008, p23) wrote that the researcher should "talk for at most for 5-15 percent of the time" and should ensure that individuals who are usually dominant do not steal the limelight by giving all the responses.

In this action research study, I talked to teachers and children in order to gain an understanding of what was happening in our school and also to identify how things could be adjusted in order to avoid confusion and conflict in the future. However, I kept Brinkman and Kvale's (2009) words in the back of my mind as they suggest that the amount and type of knowledge collected can be affected by what and how much the interviewee wants to reveal.

5.4.1 Study part A teacher focus groups

My first research question sought to find out how teachers perceived the New National Curriculum (2014) reform would affect their teaching. In order to gain the necessary data on this, I carried out three separate focus groups interviews. With Adams and Cox's (2008) work in mind, they suggested that focus group should not exceed half a dozen participants and should be equal to or more than three, I decided to split the teacher participants into three groups. Lower Key Stage teachers from Year 3 and 4 (4 participants), Year 5 teachers (3 participants) and Year 6 teachers (4 participants). In all three groups, I began by talking through what I intended to be doing and pointing out that hopefully this meant that the focus groups would not take up too much of their valuable time. I then told them that there were no right or wrong answers as I wanted their opinions and ideas. I told them this in order to ease any worries they may have.

I piloted the use of audio recording devices in a previous qualitative study (Tutcher, 2015) and so, before starting this study, I asked teachers if they minded the use of two recording devices. I explained to the teachers that the use of this device would save a lot of time as I did not have to write as we went through. It was necessary to ask them as I had to obtain their consent, see section 5.2 and Appendix 1 for further details. I told the teachers that they could listen to the recording afterwards, and also see the transcript when it has been written up, in order to check that they were happy with it. They were also allowed to do this as it enabled them to see that any personal or sensitive data had been removed. In order to collect the data effectively, I used two audio recording devices: a Dictaphone and also the voice recorder on my laptop. This was done to ensure that the data was captured on at least one instrument. Denscombe (2007) wrote that this method of audio-recording is standard in the process of capturing interview data. However, it is important to note that Bogdan and Biklen (2007, p112) stated that it is important to ask participants if they mind the use of such devices and that a researcher should "never record without permission". As a group we sat in an informal environment, an empty classroom, at the end of the school day, which allowed for professionalism and comfort. Each focus group lasted between 20 and 30 minutes. The teacher focus groups were not piloted as I wanted responses to the questions to be the teacher's first thoughts rather than thought out responses.

Bogdan and Biklen (2007) suggested that chit chat builds a rapport. They also believe an interview should begin with small talk. However, I also thought about the fact that they noted that when an interviewer is known by the interviewees then it is possible for the interviewer to get straight to business and so this will be the case in my interviews. So, once, after everyone had said they were happy and ready to participate, I began the process by simply asking the first research question using the exact wording. The first section of the focus groups started with the question: How do teachers perceive the changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics? In keeping with my interpretivist views, I do not assume that from an interview I can get to know the effect itself, I am concerned more explicitly with the perception of it.

When all participants had answered and the conversation started to dwindle, I moved on to ask: What are the advantages and disadvantages of teaching conceptual and procedural strategies in relation to division? During the interview I listened and nodded. I stayed quiet in order for participants to develop their answers further. I also stayed quiet as I did not want to influence their answers in any way although if needed, for instance if the dialogue was going off track or if I needed more details to gain an understanding, I prompted the participant who was speaking with questions such as ‘So, how do you think that will affect your teaching?’ Bogdan and Biklen (2007) and Denscombe (2007) believe that effective interviewers nod their heads, use the appropriate facial expressions, are attentive and non-judgemental. I kept this in the back of my mind when I was interviewing so that the teachers and children felt respected and valued. I also kept to the suggestion of Adams and Cox (2008) who stated that a good researcher only talks for 5-15 percent of the time.

Clough and Nutbrown (2012, p148) wrote that “once you have carried out your interview, you have a wealth of data that you must process and analyse”. Silverman (2013) believed that looking at data and analysing it carefully is an essential part of research. So, with this in mind, my first step was to transcribe the data collected in the interviews. Transcription is a necessary, although time-consuming process, that allows for familiarisation of the data and Clough and Nutbrown (2012) talked about it as being important to get a feel of the data in order to build an impression of it. They suggested in their work that throughout the transcription stage, threads can be pulled together and themes can be developed. In my mind, the more you immerse yourself in familiarisation with the data then the more you can get from it. As an extra point to note, it is important to see that Bogdan and Biklen (2007) suggested that a transcript should be filled mainly with participants’ remarks. They also suggest that, in presenting a transcript, for a new speaker there should be a new line, noting who the speaker is on the left and so this was the format adopted in this study.

Following a successful small pilot study, (written up in Tutcher, 2015), I used thematic analysis to interpret my data. In my pilot study, I interviewed a teacher and a parent with regards to their perceptions of mathematics and how they thought it had changed since they were at school. In the pilot study, to identify themes, I looked for key words and synonyms of them. I also looked for antonyms. This method worked in creating themes for the pilot study and so I decided to use the same approach in this study. The pilot study also made me realise that thematic analysis would indeed be appropriate for this action research study as it highlighted the fact that there are no right or wrong ways of doing it and that the analysis of themes can indeed bring a wealth of knowledge.

> Thematic analysis is a method for identifying, analysing and reporting patterns (themes) within data. It minimally organises and describes your data in (rich) detail.
>
> (Braun & Clarke, 2006, p79)

Braun and Clarke (2006) wrote about thematic analysis and its theoretical freedom. I understand this to mean that it is flexible, helping to gain a detailed and rich account of the data collected. In my mind, thematic analysis fits with an interpretative view, such as mine, so fitting with my study. Braun and Clarke (2006) believe that, due to the lack of clear and concise guidelines, care should be taken as there is a chance that anything goes. Further to this, I think about Mason's (2002) work where he infers that interviews can be read in different ways. He actually suggested that they can be read in three ways. The first way they can be read is that they can be interpreted as what readers think they mean. They can also be read literally and so the reader takes on the actual dialogue of the interview. Finally, they can be read reflexively meaning that the reading of the data will be related to the role of the reader. So, with this in mind and to help the analysis of my data, I looked at the work of Braun and Clarke (2006). In their work, they developed a table that suggests a six-step thematic analysis procedure.

Phase	*Description of the process*
1. Familiarizing yourself with your data:	*Transcribing data (if necessary), reading and re-reading the data, noting down initial ideas.*
2. Generating initial codes:	*Coding interesting features of the data in a systematic fashion across the entire data set, collating data relevant to each code.*
3. Searching for themes:	*Collating codes into potential themes, gathering all data relevant to each potential theme.*
4. Reviewing themes:	*Checking if the themes work in relation to the coded extracts (Level 1) and the entire data set (Level 2), generating a thematic 'map' of the analysis.*
5. Defining and naming themes:	*Ongoing analysis to refine the specifics of each theme, and the overall story the analysis tells, generating clear definitions and names for each theme.*
6. Producing the report:	*The final opportunity for analysis. Selection of vivid, compelling extract examples, final analysis of selected extracts, relating back of the analysis to the research question and literature, producing a scholarly report of the analysis.*

Table 9: Phases of thematic analysis, Braun and Clarke (2006, p87).

The framework showing the phases of thematic analysis, in table 9, was adopted for this study. Following phase one of thematic analysis, the transcription process, I read and re-read the data, familiarising myself with it and I came up with ideas for themes. With Silverman's (2013) point in mind, I spent a lot of time familiarising myself with the data. The flexibility of thematic analysis appealed to me as it allowed me to search for patterns and themes related to my research questions.

After the familiarisation, I developed a colour code, I wanted to be inclusive, thorough and comprehensive in finding some interesting patterns and themes. Denscombe (2007) believed that a theme that is repeated across different interviews suggests that an idea is shared. This idea also fits with phase four of Braun and Clark's thematic analysis as it suggests that themes are reviewed and work in relation to other areas of the analysis. Within this area of review, I was interested in looking at the similarities and the differences displayed and so phase five, reviewing and naming the themes came into being. These named themes can be seen in chapter 6.1. In chapter 6, as phase 6 suggests, I looked at and analysed selected extracts relating them to the research questions and also to related literature.

5.4.2 Study part B children focus groups

In order to involve the children of the school, because ultimately the whole study is designed to improve their opportunities and learning in mathematics, I used similar processes to those in the teacher focus group. The children were put in focus groups but this time with eight participants in each. There were three child focus groups - one from lower Key Stage Two, eight children (4 classes) and two groups from Upper Key Stage Two, sixteen children (4 classes). Two children from each class were selected based on random sampling. In the case of this action research study, I requested numbers 16 and 27 from the class register to participate unless the teachers felt that they would not participate fully and then the teacher sent the next child on the register. In the Upper Key Stage, I also asked for numbers 7 and 12. The reason for these extra children was that I wanted a representation of 10% from the whole school. The children’s focus groups took place in an empty classroom which is usually used for group lessons and so was a familiar place to the children. These interviews took approximately 30 minutes and were conducted straight after the lunch break, on separate days, for each group. The children’s focus group questions were piloted on a group of children from my own class at the time. The children used in the pilot group were not used in the actual focus group interviews.

The questions used in the children’s interviews were more simplistic than those of the teacher interviews and were designed to find out how the children felt about the support they receive and the methods they use. The questions were:

1) Do you like division?
2) What is division?
3) What methods do you use?
4) Why do you use those methods? (Upper Key Stage Two only)

The thinking behind these questions was, asking a child whether they like or dislike division, it gave the researcher a quick insight into the thoughts of the children and also allowed the children to settle with a quick “yes” or “no” question. Question two aimed to ascertain what the participants, the children, thought division was and the words in which they described it would enlighten the researcher, me, with regard to their understanding. What I mean by this is, do they use vocabulary such as sharing, grouping or opposite of times. Questions three and four were asked in order to ascertain how the children would reach an answer in their work as it is possible to gain some insight into the method that they use to solve a problem, obtaining a glimpse of their understanding of that mathematical area.

A change from the teacher focus group study was that I recorded the data in the children's groups in a slightly different format. Rather than just record the conversation using a Dictaphone, I also made jottings with pen and paper. Because of this, as well as having a purposeful conversation with the children, I ensured that I gained more direct answers from each and every child. I allowed every child to give their own answer related to the question rather than adding to the answer of the previous child although discussion elements were also possible and useful. In order to delve deeper into the children's answers, thematic analysis was used, to analyse their responses, to look for similarities and differences.

5.5 Questionnaire (study parts C and D, action research cycle 3.)

Bell (2005) is of the opinion that a questionnaire should be carefully designed in order for the researcher to gain all the information they require. She also believes that questionnaires should not give you any problems at the analysis and interpretation stage. Adams and Cox (2008, p18) in their work noted that "it is important to understand that a questionnaire is a tool" which "must be usable" so that the reader can understand, interpret and complete easily as this will increase the accuracy and participation. Oppenheim (1992, p100) called a questionnaire "an important instrument of research - a tool for data collection". He says that the job of a questionnaires is one of measurement although this must be related to the research aims, plans and objectives.

Oppenheim (1992, p103) also wrote about the main purposes of questionnaires and within this he suggested that the use of questionnaires allows for the avoidance of "interview bias". In an interview there is the possibility for the interviewer to lead the subject in a way that they want it to go whereas in a questionnaire the respondent does it completely in his or her own way. My questionnaires were anonymous and so with this came the disadvantage of such an approach in that Oppenheim wrote about there being no possibility of probing further after an answer is gained in this approach. Adams and Cox (2008) and Oppenheim are in agreement that questionnaires should not be too lengthy. To explain this, Adams and Cox (2008, p19) wrote about a participant's attention span being short and also that "long questionnaires are completed less accurately because people rush to finish". With these ideas kept in mind, I designed questionnaires that were short and accessible for all participants, teachers and children. The questionnaires followed the focus group interviews and were designed to ascertain which methods teachers and children use when solving division problems as this related to the research questions.

Brown and Dowling (1998, p66) are of the opinion that researchers need to ensure that their questionnaires are "free from bias and that they do not lead the respondent toward a particular answer". They also argue that questions should also be "standardized" so that the answers are "comparable". Brown and Dowling state that "the researcher has to be confident that each question will be interpreted by each respondent in a similar manner". I understand from this that questions should be free from uncertainty and doubt.

So, when I designed my questionnaire, as well as having the above in mind, it was necessary to consider the purpose of each question. I reflected upon what I expected to get from the answers and also how I would be analysing the data when I got it. Oppenheim (1992) talked about the importance of exploratory pilot work. So, I conducted a small pilot with one class, my class, of children with the division problems I intended to use. I wanted to see how they would answer the questions and whether the questions would be suitable for the main action research study. In the pilot study, I found out that the questions were indeed suitable as the children had used a variety of methods to answer the questions asked. The methods shown in the pilot study answers ranged and were procedural, conceptual and a mix of these methods and so I decided that the questions from the pilot study did not need to be changed. As the pilot study was successful, rather than requestion the class used, the questionnaire sheets for that class were included in the main study rather than repeating them so that the classes first responses were gained.

The following questions aimed at all participants in study parts C and D were designed to gain an understanding of their practices. The questions were used to analyse the variety of ways in which different division questions are solved and attempted. The first few questions, although not supporting and providing evidence toward the research questions, were asked in order to gain a picture of the participant's attitudes toward mathematics as I believe this affects how they teach or complete it. The participants were asked simply if they like mathematics, if they like division and finally if they think they are good at it. To me this was important to build a picture and insight into mathematical thinking and belief.

My view is that there may be many ways for individuals to reach an answer to a problem depending on their understanding. The responses to the following four questions were used to help satisfy part of the second research question, which was to identify and look at the ways participants are taught and solve division problems. Identical questionnaires were given to every child on the school roll who was present on the day that it was given out and it was also given to all class teachers (11 teachers). The questions used, see table 10, were aimed at understanding the practices in the school and the progression through the years.

Question 1	**Question 2**	**Question 3**	**Question 4**
12 ÷ 3 =	**95 ÷ 5 =**	**275 ÷ 25**	**5,542 ÷ 17 = 326 Explain how you can use this fact to help you solve 18 x 326**

Table 10: Division problems posed in questionnaire.

The questions used in table 10 purposefully increased in complexity in order to get the children and teachers to show the use of different strategies. The researcher was aware that some children from each year group would be unable to access certain questions but this was intentional as the idea was to encourage the use of different strategies in answers – the progression of strategies through the school was an important factor being looked at in the questionnaire. Questions 1 and 2 were included in the knowledge that most children would be able to access them as they were related to simple times tables facts (x3 and x5). Question 3 was more complex in order to see if children would simply miss it out or try to solve it through informal or formal strategies. Question 4, although not encouraging a division strategy per se, was used due to its relationship with the operation. In order to alleviate any stress or worries with the questionnaire, participants were told simply to miss a question out or draw a picture to show their feelings about the question if they did not know how to or did not want to answer it.

After the questionnaires were administered, the next step was to collate them and analyse them. The analysis of the questionnaires followed a different approach to that of the thematic analysis used for the focus groups. In my Master's study (Tutcher, 2012), I looked at work by Chick and Baker (2005a) who had investigated teacher responses to child misconceptions and errors which was part of a larger study into teacher Pedagogical Content Knowledge. In their study, they used a questionnaire comprising of seventeen items examining mathematical teaching situations and beliefs. They then analysed the responses in a framework, see Appendix 18, which identified strategies and categorised them into common themes. I adapted this framework from my Master's study, see Appendix 18, in order to look further into the data that I collected. It was hoped that this framework would give an insight into my second research question about the progression of strategies.

Category	Definition
Formal written method	A method used which is written.
Chunking method	Chunking is a method used for dividing larger numbers that cannot be divided mentally. Chunking is generally repeated subtraction of the divisor and multiples of the divisor – in other words, working out how many groups of a number fit into another number.
Singapore bar method	The Singapore-style of maths model, bar modelling allows children to draw and visualise mathematical concepts to solve problems
Sharing	A method where a quantity is shared equally to determine how much each person/ thing gets.
Mixed approached method	The workings show both a procedure and an underlying understanding in order to support the calculation.
Arrays	An arrangement of objects, pictures, or numbers in columns and rows – usually associated with multiplication.
Inverse strategies	Mathematically, inverse operations are opposite operations so if the question is division then the multiplication algorithm would be used as the inverse.
No answer	There has been no response to the question.

Table 11: Framework to be used to analyse the strategies used by teachers and children when solving division problems in questionnaires and in test analysis.

Table 11 is my adapted framework to be used in this study. The categories, instead of being methods used by the teacher in response to student misconceptions in my Master's study, (Tutcher, 2012, see Appendix 19), have been changed and adapted to reflect strategies adopted and used by the teachers and children in their answers to division problems. A definition for each has then been given in order to give clarity to each category. An analysis of these themes allowed for a further insight into the possible progression of division through the school. Answers from the questionnaire were studied further in order to relate to research question 2. To be judged as conceptual the children's responses needed to demonstrate a skill to make connections and give reasons. This was in contrast to procedural strategies where children needed to just carry out the calculation in a set manner showing no links to other areas (see table 12 for further clarification on these judgements):

Procedural method	**Mixed approaches**	**Conceptual method without a procedural method**
The question has been solved using a written method such as bus stop.	The answer is shown using a standard procedure but there is either another set of workings using inverse strategies or other calculations that support the answer (conceptual). For example, the times tables may be written to the side of the page.	No standard written method is shown. The workings show an underlying understanding in order to answer the calculation, for example, using a pictorial representation.

Table 12: Clarification for judgements on methods used.

It is not assumed that the questionnaire suggests anything fixed with regard to child knowledge but it gives an insight or a snapshot of this. A response marked as procedural does not imply that a child has no conceptual knowledge, it is just what they have used in this instance.

Clegg (2005) is of the opinion that we live in a disorganised and messy world where events are never simple. She suggested to look beyond just the answers, to look at the methods used to solve problems as these will show you more about the understanding of a child. I agree with this and that is why when giving out the questionnaire’s children were told to use whatever method they wanted, whatever they were most comfortable with. They were allowed to show different methods for each question and if they did not understand a question or if they thought it was too complicated for them then they could just indicate this and move on.

In this chapter, I presented a brief outline of the participants and then any ethical considerations in respect of the study. I set out the design of this action research study as a qualitative one. I justified the reasons for my choice. I then presented the methods with which I analysed each of the studies and the data collected. In the following chapter, I present my findings from the questionnaires and focus group interviews using the frameworks I developed for this study. This chapter set out the planning for cycles 2 and 3 and the next chapter details the results of the actions taken as a result of this planning.

6 Presentation of findings for cycles 2 and 3

In the last chapter, I presented and discussed the methods I used in order to collect the data for cycles 2 and 3 of my action research study. In this chapter, I will present my findings and attempt to interpret the main points that they illustrate. Any reference to literature and further analysis will then be noted in chapter 8 where I will relate my findings in each area to the research questions. This part of the study, the presentation of findings for cycles 2 and 3, is divided into separate sections where each area of the action research will be reported on individually, before attempting to consider them together and make connections to each other and also, in chapter 8, the literature.

6.1 Study part A: teacher focus groups

This first section, cycle 2, is related to the teacher focus groups. Each teacher focus group was based around the research questions, given at the end of chapter 3.

> Q1. How do teachers perceive the changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics?
>
> Q2. What is an effective progression of concepts and processes in the teaching of division?

In each transcript, see appendices 6, 7 and 8, these are shown in bold. The focus groups were carried out in order to ascertain teacher thoughts and feelings that were spontaneous fitting with my interpretivist views. The first transcript involved myself as the interviewer (I) and five teachers (T1-T5) as participants belonging to Lower Key Stage Two – years 3 and 4. The second transcript is from the Year 5 teacher group and has myself as interviewer (I) and three teachers (T6-T8) as participants. Finally, the Year 6 teacher group transcript involved myself as interviewer (I) and four teachers (T9-T12) as participants. The findings were collated through the use of a Dictaphone and also through the use of the voice recorder on the laptop computer as a back-up. The use of the Dictaphone allowed for natural fluidity in the conversation rather than stopping after each question and answer to note each down and therefore possibly halting the flow. It also meant that no information was lost.

Using Braun and Clarke's (2006, p87) six phases of thematic analysis, see table 9, chapter 5.4.1, at stage one, I familiarised myself with the transcribed data by reading and then rereading the data many times. Throughout this familiarisation process, initial codes and themes were developed. The themes identified were based around the research questions and so I looked for words that were similar or related to the questions in order to create them. An example for this would be for question 1, How do teachers perceive the changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics?, where I looked for words or phrases linked to the perceived effects of the New National Curriculum (2014) implementation such as change, harder or methods. For question 2, What are the advantages and disadvantages of teaching conceptual and procedural strategies in relation to division?, I looked for positive and negative responses related to procedural and conceptual approaches. The codes developed were word and colour related.

6.1.1 Focus groups related to Research question 1

Five themes were developed in relation to research question one. These themes are detailed below.

Theme 1 – Methods / written methods / changes

See appendices 6, 7 and 8 – colour coded – red text.

Throughout the familiarisation phase, it became apparent that there was a similarity in the focus groups in that the teachers talked about changes in the methods that the children would need to be taught. This element appeared in all three teacher focus groups. In the Year 6 focus groups the teachers spoke about it more regularly. Below are some examples of the data that demonstrates the teachers' thoughts related to this theme:

Year 3, 4 focus group: T4: we are under pressure to teach formal methods.

Year 5 focus group: T8: I suppose the biggest difference in my teaching is the written calculation methods.

Year 5 focus group: T7: I don't think it would actually change much to be honest with you – I would still be using the methods that I was using before.

Year 6 focus group: T9: specific written methods.

Year 6 focus group: T12: hopefully our teaching is going to change this way in that we won't have to teach all the written methods as they should come to us already knowing and therefore just needing long division to be taught.

The transcripts show a relationship in the belief of the teachers as they all spoke about changes in content taught and methods taught at specific age groups. However, in the year 6 focus group, the teachers spoke about changes down the line, in years to come, rather than just the imminent changes that the other year bands talk about. Another different point to note is that T7, in the Year 5 focus group, said that they did not think there would be any changes in their teaching, a point that does not seem to be reflected in the voices of the other teachers. In essence, the transcripts show that the teachers are of the belief that there will be a difference in the methods used, teaching will be more specific and also they believe that the higher Year bands, 5 and 6, will be moving away from the teaching of the standard formal written methods as they believe that these should already have been taught and understood. So, in relation to division, as shown in the last extract, they believe they will only need to teach long division as the rest should already be known.

Theme 2 - Different content/ level of difficulty.

See appendices 6, 7 and 8 – colour coded – purple text.

Another element that became apparent during the analysis and familiarisation of themes were the similarities of responses in the area of perceived change that linked to the content of what was to be taught and also the increased difficulty of the mathematics.

Year 3 and 4 focus group: T2: perhaps the biggest impact is that we are now applying things perhaps too high too soon.

Year 3 and 4 focus group: T2: There's a lot of new content, for example finding the effect of a 1- or 2-digit number by 10 and 100 – that used to be in yr4.

Year 5 focus group: T8: the new objectives are more specific and the level of difficulty has risen.

Year 5 focus group: T12: It's a very difficult part of the curriculum now – you can't get away with just not teaching it just because they are not ready for it, you've just got to do it.

From the extracts it is clear that, in the focus groups, the teachers state that they believe that there will be higher expectations with the New National Curriculum (2014). They comment upon the difficulty increasing and the fact that certain year groups will have to teach specific things such as, in the final extract, Year 6 will have to teach long division. The first two extracts from the Year 3 and 4 transcripts show that the teachers also believe that they will be teaching some content sooner than would have been expected before. One extract shows the concerns of the teachers with regard to this perceived change, linking to the comment of T12 above.

Year 6 *focus group*: T11: *even if they are not ready for it, they still have to be taught it.*

Theme 3 - Catch up / gaps.

See appendices 6, 7 and 8 – colour coded – orange text.

During familiarisation, another element that became apparent was the similarity in discussion about the need to catch up with various elements of the mathematics curriculum. Catch up is defined to be when teachers have to fill the gaps that have not been taught in previous year groups and so ensure that parts of the curriculum are not missed by the children making subsequent algorithms to be taught more comprehensible. Below are some extracts from the transcripts that show the teachers' responses related to this theme:

Year 3 and 4 focus group: T3: I think we are still catching up.

Year 3 and 4 focus group: T1: so still catching up now.

Year 6 focus group: T9: hopefully, yes, they should be completely familiar with all styles apart from long division, they are not familiar with all written methods by the time they get to Year 6 at the moment so we are playing catch up.

Although this element was not talked about in the Year 5 focus group, it perhaps shows the concerns of the teachers that work in the year bands near transition stages. What I mean by this is Year 3 who are receiving children from Key Stage One and then also Year 6 teachers who are preparing children for statutory tests and also transition to secondary school.

Theme 4 - Method over understanding.

See appendices 6, 7 and 8 – colour coded – black underlined text.

Continuing on through familiarisation and coding, it became clear to me that the teachers, especially in the older year groups of 5 and 6, were conscious that there was a need to teach a specific method in order to get to an end result and therefore to just use a method rather than worry about the understanding behind it.

Year 5 focus group: T8: Now though, we ignore all other methods and just teach short method – highlighting accuracy and following the method over understanding.

Year 6 focus group: I: so, do you feel you are just doing procedural methods, you must do it this way at the moment?
T9: yes, cos we are desperate.

This was important to note especially as it was another perceived change that the New National Curriculum (2014) would impose on their teaching and so related directly to research question 1, How do teachers perceive the changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics?

Theme 5 - Other.

See appendices 6, 7 and 8 – colour coded – green text.

Finally, I created a theme for other elements that seemed relevant or important or that showed similarities or differences across the focus groups:

Year 5 focus group: T7: I was doing the dreaded chunking, I do not like the chunking because you've got too many different methods to go along and it's too long winded but that's the method I was doing last year and that's as probably as far as I went – I never did the short method with them – which I would prefer as I am more confident in that. I had stages when I've done the chunking method when I've had to get other people in to say what I go with next.

Year 5 focus group: T6: well, I for years – went / stumbled through chunking and the teaching of it without ever feeling that I taught it particularly well and each year I sort of tweaked it – I don't think I was particularly clear in teaching it as they certainly weren't that great at ever doing it.

Year 6 focus group: T9: a number of those in my class that came up didn't have any recognisable method! No chunking nothing.

These extracts give some insights into the perceptions of the teachers with regard to their own skills and thoughts related to division strategies. Although not specifically related to research question 1, the comments flag up issues relating to division that could affect the provision and progression of the children. As the mathematics co-ordinator at the school, these extracts point to future actions for me in relation to decisions about the teaching of chunking and short division. They make me wonder about the whole stigma that surrounds division.

6.1.2 Focus groups related to the question, What are the advantages and disadvantages of teaching conceptual and procedural strategies in relation to division?

The transcripts, shown in appendices 6, 7 and 8, had two clear sections and the latter section of each conference was related to the question, What are the advantages and disadvantages of teaching conceptual and procedural strategies in relation to division? A similar code to the first section was adopted for the second section because of its visual clarity. This time, in order to be visually different from the first codes, highlighting of the text was used. Again, five themes were developed. These themes were:

Theme 6 - Advantages of conceptual strategies.

See appendices 6, 7 and 8 – colour coded – yellow highlight.

During the familiarisation phase, for this section I looked carefully for the actual word conceptual or something that was closely related to the definition, given in chapter 3.5.4., such as an understanding. I also looked for phases that showed conceptual approaches in a positive way. The extracts related to this theme were:

Year 3 and 4 focus group: T2: Conceptual is related to understanding how. Has potential for a deeper understanding – good but some children are not ready to grasp this.

Year 3 and 4 focus group: T3: where they were able to look at the fact that this is all the same thing but there are different ways to access it.

Year 3 and 4 focus group: T5: It's more of a transferable skill I would have said. When they've got it in their head, the why then they can take it to another situation.

Year 3 and 4 focus group: T1: a greater knowledge base and understanding of what they are actually doing rather than just trying to get the answer.

Year 5 focus group: T7: Then for another they might be able to get a deeper understanding.

Year 6 focus group: T12: advantages of conceptual methods are an actual deep understanding of mathematics and how to get from A to B in 7 different ways because you understand how all the routes are linked and that is what we want for all children.

Year 6focus groups: T9: They need the conceptual understanding of division and their division facts – when they don't have it, it's very hard to then do the conceptual understanding of long division because they don't have the basics and the foundations.

The extracts show that the teachers in all focus groups believed a conceptual approach to be advantageous due to its deeper understanding and its transferability. T9 also comments that without a conceptual knowledge a child is unable to access the more difficult elements of the curriculum such as the standard formal written method of long division because the knowledge base is not there.

Theme 7 - Disadvantages of conceptual strategies.

See appendices 6, 7 and 8 – colour coded – beige highlight.

With advantages there also come disadvantages and so this theme seemed a natural progression from theme 6. I looked again for the words conceptual or understanding but this time with a negative perspective. The extracts below are examples of this theme:

Year 3 and 4 focus group: T1: it may take more time though.

Year 5 focus group: T5: Going back to chunking, you need a lot of background information.

Year 6 focus group: T9: we do not have the time for conceptual, so a disadvantage of conceptual approaches is that it takes time.

In all the focus groups there was a similarity in that the teachers thought about the disadvantages of conceptual approaches being linked to time. Teachers also talked about conceptual approaches as needing a lot of background knowledge and understanding.

Theme 8 - Advantages of procedural strategies.

See appendices 6, 7 and 8 – colour coded – light blue highlight.

The second question, What are the advantages and disadvantages of teaching conceptual and procedural strategies in relation to division?, used in the focus group was asked to find out about what teachers thought about procedural approaches. Whilst familiarising myself with the transcript, I looked for the words procedural or not understanding something thoroughly but being able to do it. The extracts below are examples of this:

Year 3 and 4 focus group: T2: procedural is to teach a method to achieve an end result – this can work if practised enough and can build confidence.

Year 3 and 4 focus group: T1: So, for those that technically don't get it, they can still get the same answer without understanding it.

Year 5 focus group: T7: I think the procedure they would get straight away if you didn't go into anything, they would just be able to do it.

Year 5 focus group: T7: for one child it might be better just doing procedure and for the lower ability it would be cos they need to do it.

Year 6 focus group: T12: procedures can help them get the outcome even if they don't understand what they are doing.

Year 6 focus group: T11: the advantage to procedural is that it offers a quick solution which they can get right.

The story that developed in these extracts shows me the similarities are that the teachers spoke about the advantages of procedural approaches as being: just being able to do it; being able to answer the question without the need of understanding it; and the fact that it allows for a quick solution as it is a method to achieve an end result. T7 notes that they believe procedural approaches are good for the lower attainers.

Theme 9 - Disadvantages of procedural strategies.

See appendices 6, 7 and 8 – colour coded –blue highlight.

Once again, with advantages there are also disadvantages and so theme 9 was created. As in theme 8, I looked for words and phrases such as procedural or lack of understanding. The following extracts are examples of this:

Year 3 and 4 focus group: T5: They can't apply it if they've been taught procedurally whereas conceptually, they can. If they are told they can only take two steps then they will always look for those two steps regardless of the problem, and they wouldn't necessarily understand what they were doing it for.

Year 3 and 4 focus group: T4: they can get hung up on the procedure.

Year 5 focus group: T7: their knowledge of number and their understanding of it wouldn't be there.

This theme began to form stories in that the teachers shared a belief that the disadvantages of a procedural approach are that there is no understanding and so if the children were to become confused then there would be no way of them sorting themselves out. This, in turn, could lead to a lack of confidence with the mathematics, especially if the children just get hung up on the procedure as T4 mentions.

Theme 10 - Needs and other.

See appendices 6, 7 and 8 – colour coded – grey highlight.

The final theme was once again developed in order to bring to light other issues that seem relevant to the question:

Year 3 and 4 focus group: T2: There needs to be a balance.

Year 5 focus group: T7: it depends on the child, the class, the set – loads of different things.

Year 3 and 4 focus group: T6: yeah that's the trouble cos almost everything in teaching comes back to well it depends on the child and their needs.

Year 6 focus group: T11: We wouldn't have the constraint that they have to get a certain mark to pass. We would be teaching to their needs.

The story that develops in this theme is that the teachers, in all focus groups, had other thoughts with regard to the advantages and disadvantages of procedural and conceptual approaches. The teachers were in agreement that, as T2 mentions, there needs to be a balance of approach and that ultimately, as T7 suggests, it will depend on the child and the class as to what may be best.

6.2 Study part B: children focus groups – action research cycle 2.

The findings in this next section are concerned with the focus groups between myself and the randomly selected children from across the whole school (24 children – 10% of the school roll – 8 children from Lower Key Stage Two, 2 from each class, and 16 children from Upper Key Stage Two, 4 from each class). As stated in chapter 5, the children were selected randomly for example by just asking for the children who were number 16 and 27 in the register. Gender and prior mathematical attainment were not considered when selecting participants as selection was completely random. The answers, given by the children, were used to gain an insight into research question 2, What is an effective progression of concepts and processes in the teaching of division? In order to keep it relaxed for the children, so that they gave their spontaneous thoughts, a different approach was taken to that in the teachers' focus groups.

In this section, I recorded the answers as handwritten notes as well as using a Dictaphone so the children had time to think and respond and not feel under pressure. I decided to do this as I felt that all the children would want to give answers to the same question and I wanted to value all their contributions even if they were identical. I felt that just having an audio recording, as carried out with the teachers, would have meant that some children would have possibly sat there without saying anything due to nerves and also as some children are more forthright than others. The questions, were simple ones so that the children did not become confused and then disengaged. The data collected was then entered into Microsoft Excel in order to present the results in a clear and useable style.

Question one, Do you like division?, was used to ascertain the children's feelings toward division. The findings are shown in appendix 3. Most children, that were selected to take part, either like or like parts of division. A sixth of those randomly selected from the school (4 out of 24) told me they did not like division at all. All the children from Year 5 commented that they like division. I wonder whether the reasons for this are based around their learning experiences and their feelings towards mathematics in general?

Tables 13 and 14 give responses, from the children sampled, for question 2, What is division? The Year 5 and 6 (Upper Key Stage Two) responses also show what methods the children use when they are faced with a division problem and why. These extra responses from the Year 5 and 6 children also give an insight into the initial stages of research question 2 with regard to concepts and processes taught and used in division..

Year 3 and 4 (8 children)	Year 5 (8 children)	Year 6 (8 children)
Bus stop method. (4) Sharing. (3) Opposite of times. (1)	Division is where you have a number and then you have another number say 100 and 9 you have to see how many 9s there are in 100. (4) 36 divided by 6 – so 6s in 36 – multiplication. (3) Different word for divide is halving – sharing. (1)	It's where you get a number and take a number out of it. (1) You see how many numbers are in a number. (2) Say it was 16 divided by 4 = 4 (1) Divide is a symbol. (1) You use your times tables. (1) Coupling / splitting / sharing out / halving. (2)

Table 13: Findings related to the question: What is division? (Numbers in brackets are the number of children that gave each or similar response.)

Year 3 (4 children)	Year 4 (4 children)	Year 5 (8 children)	Year 6 (8 children)
Inverse of times. (1) Arrays (3)	Bus stop (short-written) (4)	Short –quick and not much space – easier to get it right. (8) Do not like chunking – takes a whole book – don't understand it – waste of time (8)	Short – easier and quicker (5) Chunking – known entity (1) Long – frustrating – takes a long time. Difficult to grasp. (2)

Table 14: Findings related to what methods children know about and use and in years 5 and 6, also including reasoning/preferences behind choices. (Numbers in brackets are the number of children that gave each or similar response.)

Tables 13 and 14 also give an indication into the methods that are known and used by the children. These methods seem to reflect the curriculum, in that in Year 3 the children talk more about arrays and division being the inverse of multiplication. It seems that the answers in Years 4, 5 and 6 reflect the methods that the children have been used to in their learning at that point. Long division is only mentioned in Year 6. Chunking is mentioned in Year 5 and 6 as they would have covered it in Year 4.

The teacher focus groups in cycle 2 have given insight into ideas related to the advantages and disadvantages of procedural and conceptual approaches. The findings have shown that there is a belief that a mix of approaches is needed for a deeper mathematical understanding. The children's focus groups, within cycle 2, have given insight to the methods that the children know and use with regards to the operation of division. These findings link to those in cycle 3 as Bowen (2009, p28) comments that 'by examining information collected through different methods, the researcher can corroborate findings across data sets'.

6.3 Study part C: teacher questionnaire – action research cycle 3.

In order to collate answers from the teachers' questionnaire (see appendix 2), I entered the data into the computer program Microsoft Excel in order to present the findings clearly in tables and graphs. Questions 1 to 3 were used to settle the teachers into the questionnaire and to make them feel comfortable. My findings showed that all teachers like mathematics, like division and that they feel themselves to be good at division. Although all responses are related to division, it is noticeable that 'sharing' is the most commonly associated word used by teachers. In fact, six of the eleven teachers used the term "sharing" in their definition. Three teachers used the term "grouping" and the rest, two teachers, used the phrase "the inverse of multiplication". Examples of the teachers' responses were:

1) Division is to share a larger number
2) Division is to share something equally
3) Division is when an object or number is grouped into equal parts
4) Division is the inverse of multiplication

Table 15 indicates teacher methods used when presented with different division problems. The reason behind presenting teachers with questions in this way was to see what methods teachers use in each situation. Findings show that as the questions increase in difficulty so more complex strategies are used. To explain this further, in the question 12 ÷ 3, the strategies used tended to relate to what was known (73% used an inverse strategy and 18% a number fact = 91%) and methods for solution were not really used; whereas, the question 275 ÷ 25 shows strategies where there was a use of written methods (36%) or a partitioning of facts (45%) in order to answer the question. I wonder whether the strategies used relate to the year band that the teachers currently teach or whether they are due to each teacher's pedagogical content knowledge, mathematical skill or confidence in division. Tables 16, 17, 18 and 19 give examples of the different methods used and the category they were classed as:

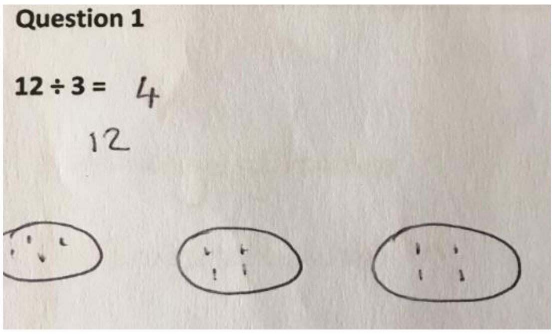
	Formal Written Method	Number fact	Chunking	Sharing	Scaffold/ Partition	Array	Inverse strategy	No answer
12 ÷ 3 =	-	18% (2)		9% (1)	-		73% (8)	
95 ÷ 5 =	36% (4)	18% (2)		-	45% (5)		-	
275 ÷ 25	36% (4)	9% (1)		-	45% (5)		9% (1)	
5,542 ÷ 17 = 326	Explain how you can use this fact to help you solve 18 x 326							
	100% (11)							

Table 15a: Findings related to how teachers answered each division problem set in the questionnaire.

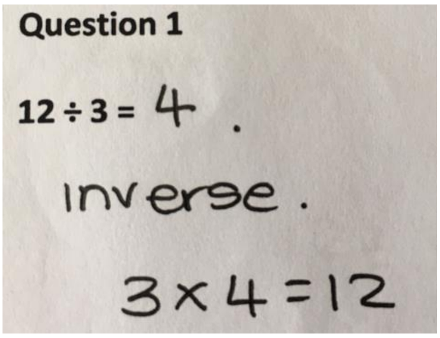
	12÷3	95÷5	275÷25	5542÷17 = 326
Teacher1 -all	Inverse	Scaffold	Scaffold	Number fact
Teacher 2 -year3	Sharing	Scaffold	Inverse	Number fact
Teacher 3 -year3	Inverse	Scaffold	Written	Number fact
Teacher 4 - year3	Number fact	Written	Scaffold	Number fact
Teacher 5 - year4	Inverse	Scaffold	Number fact	Number fact
Teacher 6 -year4	Inverse	Written	Scaffold	Number fact
Teacher 7 -year5	Inverse	Number fact	Scaffold	Number fact
Teacher 8 -year5	Inverse	Written	Written	Number fact
Teacher 9 - year6	Inverse	Written	Written	Number fact
Teacher 10 -year6	Inverse	Scaffold	Written	Number fact
Teacher 11 - year6	Number fact	Number fact	Scaffold	Number fact

Table 15b: Findings related to how teachers answered each division problem set in the questionnaire.

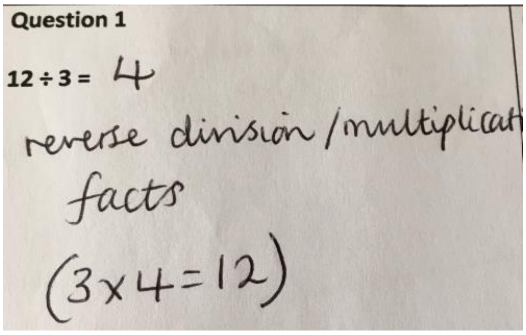
Table 15b breaks table 15a further and illustrates the individual strategies used by each teacher. It seems to show division strategies used progressing according to the year group the teacher works in. Written strategies tend to be used by teachers in Year 4 and above. The strategies that deal with more conceptual approaches seem to be used by teachers who teach in Lower Key Stage Two – this could be reflective of the style and strategies that they are teaching.



Sharing.

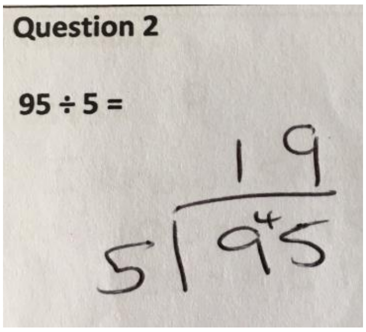


Inverse

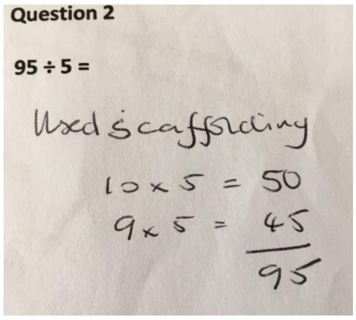


Number fact

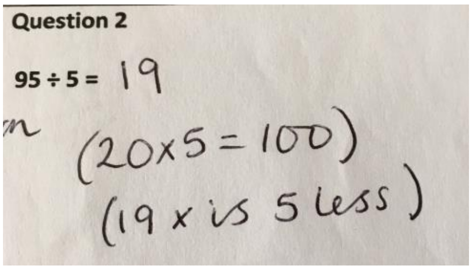
Table 16: Examples of each of the methods used in question 1 of teacher questionnaire.



Formal written method

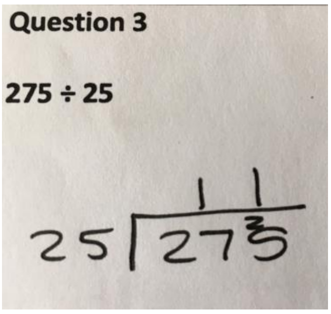


Scaffolding

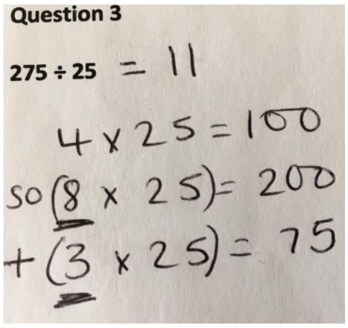


Number fact

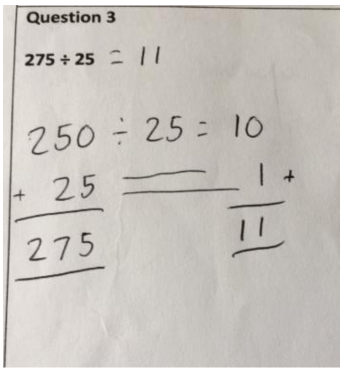
Table 17: Examples of each of the methods used in question 2 of teacher questionnaire.



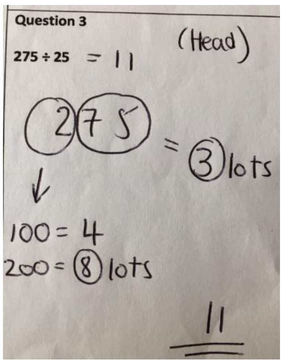
Formal written method



Inverse

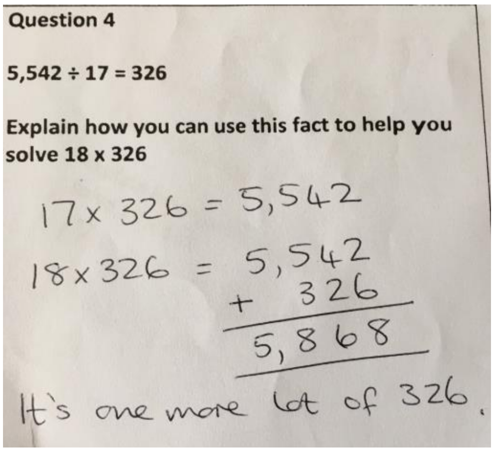


Number facts



Scaffolding / partitioning

Table 18: Examples of each of the methods used in question 3 of teacher questionnaire.



Written method and number facts

Table 19: Example of the methods used in question 4 of teacher questionnaire.

Most of the questions were straightforward with regards to classification, however, question 3 was a little less clear than the others especially with regards to classifying (see table 18). The scaffolding, number fact and inverse categories shown were all quite similar in that they all break down the calculation, however, as shown, the methods used within them are quite different: the inverse strategies using multiplication to aid solution; the number fact method breaks the problem down but then uses prior knowledge of the 10 times table; and the scaffolding solution, although using prior knowledge, partitions the number out into manageable chunks.

6.4 Study part D children's questionnaire -action research cycle 3.

Like the teachers' questionnaire, the first few questions on the children's questionnaire were used aiming to relax the participants. This part of the study involves every child (230 on roll) in the school (see results in appendix 3). The findings shown relate closely to those from the child focus group (Study part B) in that one third of the children said that they did not like division. The results for liking division and being good at division are equal, perhaps indicating that the children like or dislike division dependent on how they perceive their skills in division to be.

Table 20 relates to question 4 of the children's questionnaire where they are asked to present their own definition of division to demonstrate their understanding. Although there are similarities with that of the teacher responses, the most common response, other than no answer given, was that it is the inverse of multiplication. It is not known why some children, 32.75% (75 children), left this blank. Another similarity is that sharing features more commonly than grouping again. On this point, it is also apparent that Year 3 and 4 use the term sharing more than Year 5 and 6 and this could be due to the vocabulary used by their teachers. In Year 6, a new category became apparent in that some children described division as how many of something go into another number. In Years 5 and 6, the children use the term grouping more than sharing.

In the "no answer" column, it seems that, as the children progress up the school, then their capacity to define division is increased; for example, in Year 3, 40% were unable to give a definition of division, whereas, in Year 6, this was only 11%. There were no real problems with coding the answers as the children had written clear sentences and categorisation was completed with regards to the words involved in their sentences such as share, group or inverse. As there was not a lot of space to write their answers, they did not write long answers that could be put in two categories.

Year	share	inverse	hard	sharing	type of maths	No answer	How many in a number
3 (53)	28% (15)	18% (9)	8% (4)	3% (2)	3% (2)	40% (21)	
4 (54)	17% (9)	33% (18)			6% (3)	44% (24)	
5 (64)	3% (2)	49% (31)		6% (4)	6% (4)	36% (23)	
6 (59)	6% (4)	23% (14)		10% (6)	11% (6)	11% (6)	40% (23)
average	13.% (30)	31% (72)	2.00% (4)	5% (12)	7% (15)	32 % (74)	10.00% (23)

Table 20: Findings related to what children perceive division to be.

A few children wrote that division is when you split numbers – it could be argued that this could be categorised as share or group but, in all cases, these answers were put in the share category for consistency. Examples of the sentences can be seen below:

Year 3 – division means to share – like you are sharing sweets.

Year 3 – division is the opposite of times.

Year 4 – division is a type of maths vocabulary that halves numbers.

Year 4 – division is the opposite of multiplication.

Year 5 – division is a number that is the opposite of the times tables and it has remainders.

Year 5 – division is grouping. Like if you have 12 ÷ 3 you put 3 groups of 4.

Year 6 – finds out how many times that number could go into a number you are dividing by.

Year 6 – the inverse of times.

It is perhaps heartening that the oldest class had the fewest who had no answer to the question. It is also striking that 40% of Year 6 children described division as how many times a number goes into another number – an explanation completely absent from all other years. This is a finding that may have implications for what makes an effective progression of strategies.

12 ÷ 3	Year Group	Formal Written Method	Number fact	Chunking	Sharing	Scaffold/ Partition	Array	Inverse strategy	No answer
	3 (53)		4% (2)		47% (25)		39% (21)	2% (1)	8% (4)
	4 (54)	15% (8)		22% (12)	19% (10)			44% (24)	
	5 (64)	9% (6)			11% (7)			78% (50)	2% (1)
	6 (59)	27% (16)						71% (42)	2% (1)
	Overall (230)	13% (30)	1% (2)	5% (12)	18% (42)		9% (21)	51% (117)	3% (6)

Table 21a: Findings related to how children answered 12÷3 in the questionnaire.

95 ÷ 5	Year Group	Formal Written Method	Number fact	Chunking	Sharing	Scaffold/ Partition	Array	Inverse strategy	No answer
	3 (53)		10% (5)		28% (15)		10% (5)	28% (15)	24% (13)
	4 (54)	13% (7)	24% (13)	24% (13)	4% (2)			31% (17)	4% (2)
	5 (64)	38% (24)	12% (8)		2% (1)			45% (29)	3% (2)
	6 (59)	49% (29)	12% (7)					37% (22)	2% (1)
	Overall (230)	26% (60)	14.5% (33)	5.5% (13)	8% (18)		2% (5)	36% (83)	8% (18)

Table 21b: Findings related to how children answered 95÷5 in the questionnaire.

275÷ 25	Year Group	Formal Written Method	Number fact	Chunking	Sharing	Scaffold/ Partition	Array	Inverse strategy	No answer
	3 (53)				11% (6)		4% (2)		85% (45)
	4 (54)	7.5% (4)	7.5% (4)	37% (20)	26% (14)				22% (12)
	5 (64)	23% (15)	2% (1)	16% (10)	23% (15)			22% (14)	14% (9)
	6 (59)	76% (45)			10% (6)				14% (8)
	Overall (230)	28% (64)	2% (5)	13% (30)	18% (41)		1% (2)	6% (14)	32% (74)

Table 21c: Findings related to how children answered 275÷25 in the questionnaire.

5,542 ÷ 17 = 326	Year group	Formal Written Method	Number fact	Chunking	Sharing	Scaffold/ Partition	Array	Inverse strategy	No answer
	3 (53)								100% (53)
	4 (54)		9% (5)						91% (49)
	5 (64)		37% (24)						63% (40)
	6 (59)		76% (45)						24% (14)
	Overall (230)		32% (74)						68 % (156)

Table 21d: Findings related to how children answered 5542÷17=326 in the questionnaire.

The findings in tables 21a, b, c and d represent the children's strategies used to solve different division problems – questions 1 to 4 on the questionnaire. The reason behind this was to see what methods children use in each situation. Examples of how each was categorised are shown in table 22.

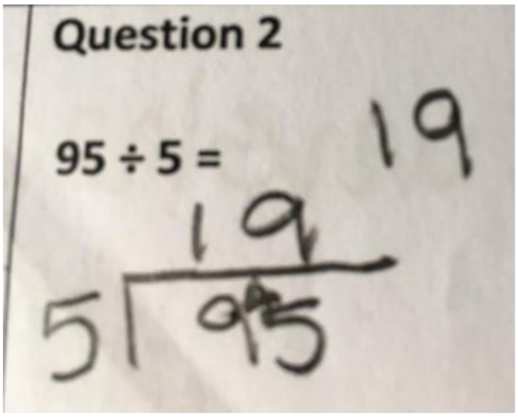
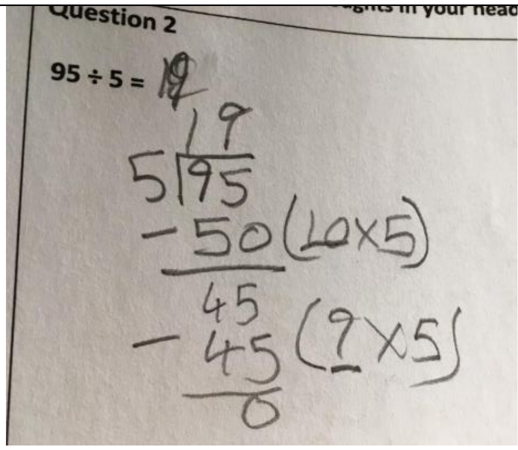
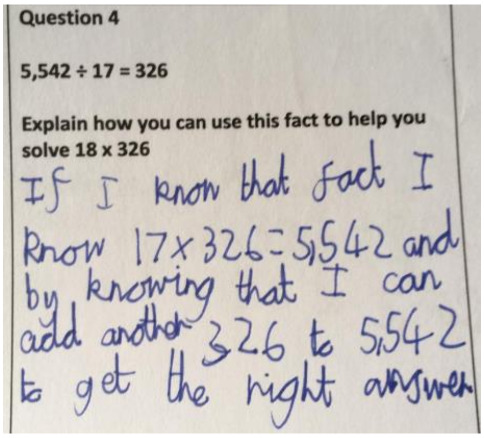
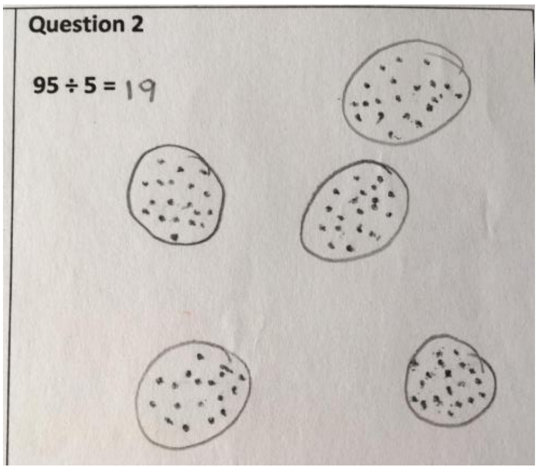
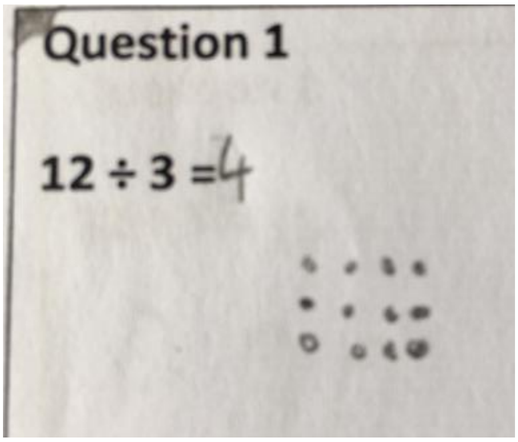
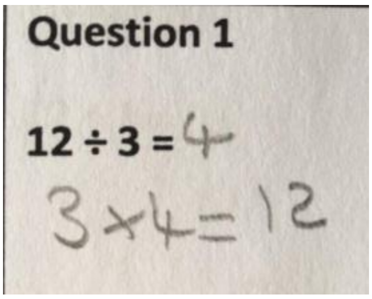
 Example of formal written method	 Example of chunking
 Example of number fact	 Example of sharing
 Example of arrays	 Example of inverse strategy

Table 22: Examples of how each answer in the children's questionnaire was categorised in tables 21a, b, c and d.

The tables show the different strategies children in each year group use/prefer. It gives an insight or snapshot into the progression of strategies being used throughout the school. The tables suggests that as the children progress up the school so the strategies they use change from conceptual type approaches to more known facts and written procedural methods. The findings, similar to those in the teachers' study, show that, as the questions increase in difficulty, so the use of more complex strategies are used. Like in the teachers' responses, in table 15, in the problems that focus on TO ÷ O (tens and ones divided by ones), shown in tables 21a and b, inverse strategies are the preferred method/strategy as if there is a known fact being drawn upon. As the problems increased in difficulty, the findings indicate more varied usage of strategies.

Table 21a suggests a development of using inverse strategies/procedural approaches as the age group of the children increases. Forty-two children (71%) of the Year 6 cohort used inverse strategies whereas in Year 3 only one child (2%) used the inverse to solve the problem. Twenty-five (47%) of the Year 3 children showed a pictorial representation to solve 12÷3. The Year 5 results were similar to those of Year 6. Year 4 however, showed a real mix in strategies chosen to solve the 12÷3 problem and this could be a reflection of their preferred approach as well as their prior attainment in mathematics.

Table 21b suggests that in the lower year groups there is a real diversity in the strategies used to solve 95÷5. Sharing and inverse strategies seem to be favoured at Year 3. The table suggests that in Year 4 once again there are a vast range of strategies being used. In Year 5, 38% of the children used a formal written method and 37% used inverse strategies. In Year 6, there is a similar picture in that 49%, twenty-nine children, of the cohort used a formal written method and 37%, twenty-two children, an inverse strategy. The table suggests that as the children get older then pictorial strategies are used less, although caution is required in making such an interpretation since this data is a snapshot of year groups, not data that tracks the same pupils up the school.

Table 21c suggests that as the complexity of the problem increases so does the need for an increased understanding hence 85% of the Year 3 cohort made no attempt to answer the question. Those, in Year 3, who did answer the question used pictorial strategies to answer the problem – sharing and arrays. In Year 4, the preferred strategy shown in the table is chunking although a vast range of strategies are used. In Year 5, unlike tables 21a and b, like Year 4 for this problem, a vast range of strategies are employed by the children to solve the problem with formal written methods and sharing being the favoured in this case. The table suggests that at Year 6, formal written methods are preferred with 76% of the cohort, forty-five children, using them to solve 275÷25.

Table 21d presents findings for a more difficult division problem. In it the children are asked to use one calculation and solution in order to solve another (5542 ÷ 17 = 326 – Explain how you can use this fact to find the answer to 18 x 326). 68% of the children who took part in the questionnaire did not attempt to answer this question, which perhaps relates to the knowledge involved and their level of learning at the time. The findings show that it was the higher achieving children in the upper end of the school who attempted to answer the question. I had wanted to make sure there was a question which would stretch our highest attaining children in any year group, but I was surprised quite how many did not attempt an answer.

Before moving on to the analysis of these findings, it is necessary to reflect upon the research questions and also think about the aim of this action research study. This chapter has presented the data collected as a result of the actions taken in cycles 2 and 3. The data collected so far gives evidence relating to research question one as it covers the teachers' perceptions with regard to the changes and effects of the implementation of the New National Curriculum (2014) and research question two, which is concerned with procedural and conceptual approaches and what makes for an effective progression.

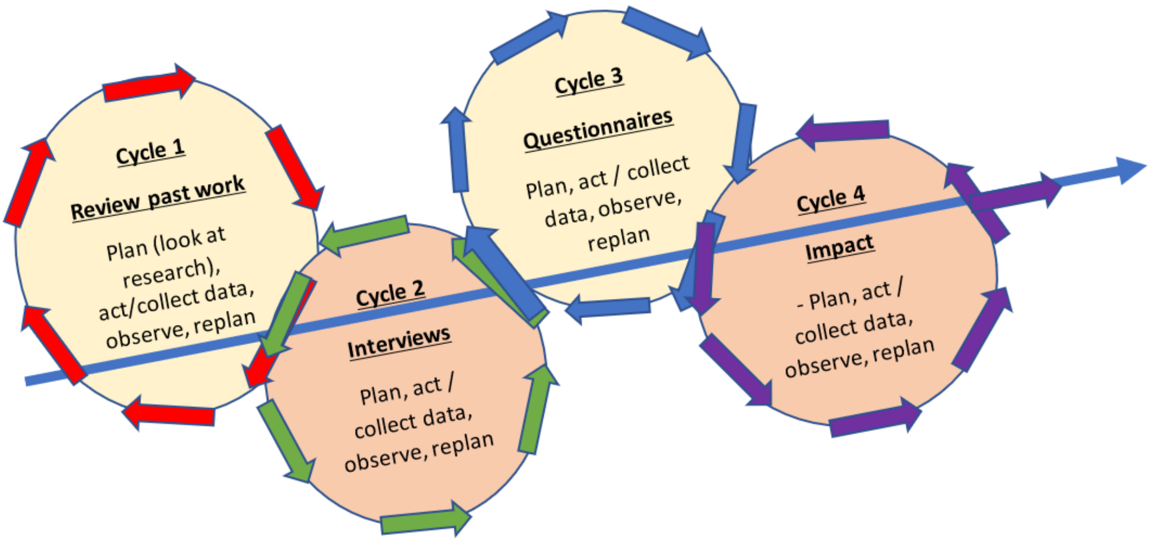


Figure 1: (repeated) The process of action research in this study – a collection of circles that interlink. (adapted from Altrichter et al.'s (2008) idea of the circle of action and Kemmis et al.'s action spiral research).

Reflecting on the actions of cycle 2 and 3, my data has so far shown some possible shifts used by pupils as they get older (from conceptual to procedural). The data collected so far in cycles 2 and 3 covers the strategies that are taught and learnt but it does not show many clear patterns with regard to progression in strategies nor does it show how children work in different situations. As Anghileri (2005) and Thompson (2012) suggest, it is necessary to find out what methods children currently use so that current strategies can be built upon and so it was necessary to create an impact cycle in order to find out what methods children preferred to use in both informal and also in formal situations. One outcome I wanted, in answering research question two was to create a useful progression chart for the future so that experiences in division are enhanced. I needed to cover both formal and informal situations as the children may revert to different strategies accordingly.

A progressive calculation chart could be a clear and coherent document that breaks down the curriculum and suggests a developing sequence for teaching the varying division strategies that the children need to know. The disagreements across past research on division, the differences in government guidance about division, and the proportion of Year 6 pupils unable to answer a question that did not require a division procedure to be carried out, all pointed to the need for such a chart. However, in order to further inform this chart there was a clear need for another cycle of planning and data collection.

So, in the next chapter, I introduce the methodology of my impact cycle. I then present the findings before analysing them with respect to the research questions and against current literature.

7 Impact cycle – action research cycle 4

Having collected and interpreted the data for cycles 2 and 3, in chapter 6, and with the literature review in mind, I found that more data was needed in order to look at the strategies used and preferred by the children. This extra data was needed so that I could develop a calculation progression chart that is reflective of the literature read and also of my results. Cycle 4 represented a new phase of planning, based on the outcomes of the actions completed within cycles 2 and 3. The impact cycle fits with Altrichter et al.'s (2008) circle of action and reflection, see appendix 4, as the research completed to date gave me, the researcher, further ideas for action. One new element of data that I realised I needed, was to find a way to track the work of some pupils over time, in order to be able to analyse their data over time. Further data was also considered important as a means of triangulation. Patton (1999) wrote about the use of multiple data sources being viewed and considered together to develop a better understanding of a phenomenon. Bowen (2009, p28) in agreement stated that 'by examining information collected through different methods, the researcher can corroborate findings across data sets and thus reduce the impact of potential bias that can exist in a single study'.

In the first part of cycle 4 I looked at the strategies' children revert to when in formal environments, tests. In this case the tests used were the Standard Assessment Tasks (SATs) set by the Standards and Testing Agency who are an executive agency sponsored by the Department for Education. The SATs tests are for children aged 10-11 (Year 6). In the second part of cycle 4, I looked at the strategies children use when in a relaxed informal environment, free-time and so I developed a type of document analysis through graffiti walls. Table 23 shows the study parts of action research cycle 4.

Action research cycle	Qualitative approach	Study part
4	SATs test analysis - formal	E
	Graffiti Walls – informal document	F

Table 23: Overview of approaches to be used in cycle 4.

7.1 Test analysis (study part E)

In this section, I looked at the Standard Assessment Tests (SAT) results of children from the Year 6 cohorts in the school over three years from 2017 (71 children), 2018 (59 children) and 2019 (66 children). From these SATs papers, I gathered data from the children's responses to the questions which were related to division or needed division to be used. I used similar types of questions from each year 2017, 2018 and 2019. I recorded the strategy used by each child for each question. This data, although not comparable to children in lower year groups, shows how the strategies being used have changed over the collection period. It also produced a possible triangulation as the children in the 2017 cohort were Year 5 when the questionnaire was collected. The children in the 2018 cohort were Year 4 when the questionnaire was collected and the 2019 cohort were in Year 3. Through this data collection, I was able to look for any patterns/progression of strategies that emerged. This collection of data was possible over this time frame as the records for Mathematics SATs are saved as PDF documents on the school's government portal.

Ethical issues were also considered with regard to the use of this data set and relevant permissions were gained from the headteacher. With the headteacher as gatekeeper consent was given to use the data set as long as participants names were removed, fitting with the school's safeguarding procedures. BERA (2018, p11) note that "when working with secondary or documentary data, the sensitivity of the data, who created it, the intended audience of its creators, its original purpose and its intended uses in the research are all important considerations". Retrospective consent from participants, or their parents, was considered as not needed as consent from the gatekeeper had been gained. BERA (2018, p11) stated that if secondary data is to be used then "ownership of the datasets should be determined" and as the SATs data sets are kept and owned by the school the gatekeeper gave "consent on behalf of the participants".

Although this section draws on some straightforward quantitative techniques, I used the data in a qualitative way as I interpreted what the results showed. I produced a meaningful analysis of what the children had done and how this changed over the years.

Brown and Dowling (1998, p82) stated that "qualitative approaches are often associated with research which is carried out in an interpretative frame in which the concern is the production of meaning" whereas quantitative research is to do with the "search for facts" and is linked to a positivist view. I interpreted the data that was collected in order to produce meaning for future practices.

Another reason for doing this SAT analysis was to see what the children revert to in a test situation when they are under the pressure of a time constraint. Do they use a conceptual method, a procedural method or do they maybe use a mixed approach? See table 12 for definitions of each. Do they adopt the standard written method as prescribed by the New National Curriculum (2014) or do they use the conceptual strategies that were imposed at the time of the National Numeracy Strategy – methods such as chunking? I reflect on and compare the results to the strategies children used in the questionnaires earlier in this study as all the test cohorts took part in that part of the study too - this may show how, over time, the curriculum reform has changed strategies used and preferences that may be emerging. Table 11, my personal development of the Chick and Baker (2005a) framework, see appendix 18, shows how each answer was analysed.

Limitations when analysing methods used in the SATs papers are concerned with the depth of data and the interpretation of the data. The depth of data is limited as the SATs are a snapshot in time and therefore may be considered as not fully reflective of the participant as they are under pressure from a test and the test is on one day of the year. Both these elements could be factors that affect the participant strategy selection and use. With regard to interpretation of the data, this could be seen as a limitation as the researcher may interpret the data differently to how the participant saw it.

Year of paper and questions used	Procedural method	Mixed Approaches	Conceptual method without a procedural method	No workings	No answer
Arithmetic papers in: 2017 2018 2019	The question has been solved using a written method such as bus stop.	The answer is shown using a written calculation (procedural) but there is either another set of workings using inverse strategies or other calculations (conceptual) that support the answer. For example, the times tables may be written to the side of the page.	No written method is shown. The workings show an underlying understanding in order to answer the calculation. For example, using a pictorial representation.	There are no workings at all. An answer is present and shows that the workings have probably been worked out mentally or guessed.	There is no answer present.

Table 24: How strategies used in division problems will be further analysed.

7.2 Graffiti wall (study part F)

One of my reflections on the outcomes of cycles 2 and 3 was that I needed to gather some data on the division methods in informal, as well as formal settings. I designed a different form of data collection that appealed to the children without them worrying about it being part of a study. I formed a data collection method that enabled me to gain further insights into the children's thoughts, methods and approaches associated with division. So, fitting with my interpretivist approach I developed the idea of a graffiti wall where I constructed meaning from their actions.

Bakewell (2008, p1) stated that many "people recognise graffiti as an art form that vandalizes public property". However, she also noted that graffiti has a "rich history in personal expression" and that "designs of mathematics" and "graffiti, if defined as writings on the wall, date back to ancient Rome". There has been little academic research done on graffiti walls. In more professional literature, I did find that Lotriet (2012, p1) writes in her blog *clairelotriet.com* that she believes working in this way "creates a lot of maths talk and energy". She believed that graffiti walls allow children to take part with no fear. Lotriet states that "children seem less scared to make mistakes and they do not seem to mind showing their workings" when creating their walls. In addition to this, Dolling (2017, p1), in her blog called *theteacherideafactory,* said that "children are motivated, engaged and extend their learning when working in this way". One of the methodological contributions of my study is to propose graffiti walls as a method that allows even the shy learner to engage and participate. Thompson and Rubenstein (2000, p571) in their work wrote that a graffiti wall is "a visual tool that encourages children to think".

With this in mind, I researched further and, followed an idea from *Mathsercise,* from Queensland Australia, I developed study part F as part of research cycle 4. The graffiti walls were developed as an art form that children recorded anything they linked to their understanding of division, whether it was words, strategies or anything else. The children completed their graffiti walls in separate year group teams. Similar to the focus group interviews, the children were selected randomly, numbers 4 and 15 in the register and they worked with the researcher in an empty classroom. To keep the selection completely random, mathematical prior attainment, mathematical interest and gender were not considered. The children who participated were given instructions on what the task entailed and were also given the chance to withdraw– none wanted to and they all seemed keen to take part. Consent was given verbally by each participant and no names were given or noted so anonymity was assured. The different year groups worked in an informal setting and were given the simple instruction of producing a poster with everything they know about division. They were told they could do this in any form – text, drawings or mathematical notation. Groups were given an hour to produce their poster. In order to show the walls produced in the study, photographs of the walls are presented in figures 20-23 and Appendices 9-16.

The graffiti wall data sets were collected twice for each year group. The first set were collected in the summer term of 2016 and the second set in the summer term of 2019 – see Appendix 20 timeline of study. Different children were used in each instance. I then analysed and compared the graffiti walls in terms of strategies used and mentioned in each year group, looking at how vocabulary was developed and also marking any changes or progression that became evident.

The depth of data could be seen as a limitation associated with gathering the data in this way without interviewing participants as no supplementary information or explanation has been gained regarding to participants thoughts. The researcher may interpret work differently to how participants meant it to be perceived. With the use of interviews, the researcher may have been able to gain more insight into the participants understanding and also ensured that any forgotten gems of knowledge from the participant had been presented. Nonetheless, analysis of the walls was possible, as will be detailed below.

7.3 Analysis of impact cycle – action research cycle 4

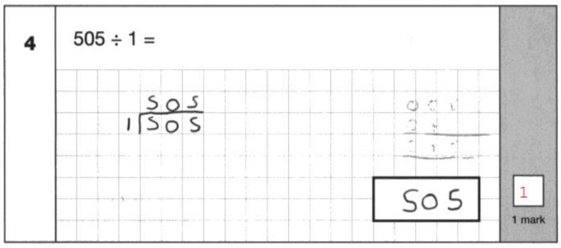
Having introduced these new methods of data collection, in my impact cycle, action research cycle 4, I now present the data found in them. As before, this analysis section is where I interpret the main points that are illustrated in the results. Further analysis and references to literature will follow in chapter 8 where I will discuss my findings.

7.3.1 Study part E: Key Stage Two test analysis

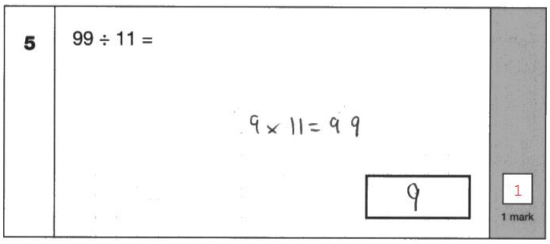
Table 25 shows how Year 6 children, at the school being researched, from the 2017, 2018 and 2019 cohorts, responded with regards to approaches and strategies they used under test conditions to answer division questions. Answers given were looked at and noted as either being procedural, conceptual or mixed methods – see table 24 for examples of how each was categorised. There were no real challenges when categorising the answers given although tables 11 and 12 were kept in mind. For procedural, the answers tended to be a clear formal written method. Conceptual approaches tended to show an understanding of the inverse – see figure 18. Mixed approaches had elements of both a formal written method and a further calculation to aid understanding and working. The workings used could be a sign that the children were possibly used to using certain strategies as directed by their teachers in the classroom. It was not necessary to indicate whether answers were correct or not as the study is interested in strategies used rather than success or failure.

Questions	Year used	Procedural method	Conceptual method	Mixed approaches	No workings	No answer
505÷1 (HTO÷0)	2017	10% (7)			90% (64)	
838÷1 (HTO÷0)	2018	10% (6)			90% (53)	
180 ÷ 3 (HTO÷0)	2019	71% (42)		3% (2)	26% (15)	
72÷9 (times table facts)	2017	34% (24)		8% (6)	58% (41)	
99÷11 (times table facts)	2018	9% (5)	3% (2)	3% (2)	85% (50)	
91÷7 (times table facts)	2019	77% (45)		10% (6)	13% (8)	
714÷17 (HTO÷TO)	2017	51% (36)	4% (3)	30% (21)		15% (11)
645÷43 (HTO÷TO)	2018	15% (9)	2% (1)	73% (43)	7% (4)	3% (2)
888÷37 (HTO÷TO)	2019			93% (55)		7% (4)
2242÷59 (ThHTO÷TO)	2017	41% (29)	7% (5)	27% (19)		25% (18)
8827÷97 (ThHTO÷TO)	2018	12% (7)	2% (1)	59% (35)	2% (1)	25% (15)
8851÷83 (ThHTO÷TO	2019			90% (59)		10% (7)

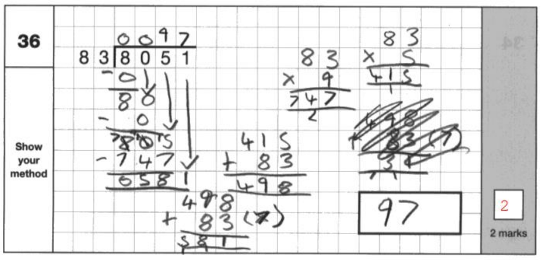
Table 25: Findings related to how children in Year 6 solved division problems under test conditions in 2017, 2018 and 2019.



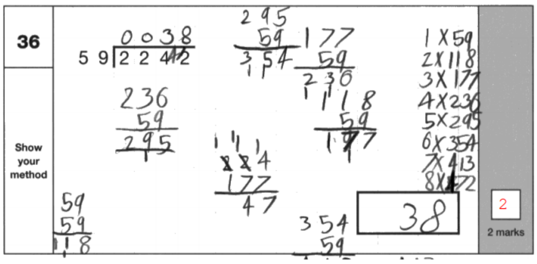
(2017 SATs paper)
Example of a procedural approach – the child has used the short-written format.



(2018 SATs paper)
Example of a conceptual approach – the child has used their understanding of inverse strategies.



(2019 SATs paper)



(2017 SATs paper)

Example of a mixed approach – the children have used a written procedural format but have also used their knowledge of multiplication to solve the problem. The example on the right shows a list of multiples on the right-hand side to aid calculation.

Figure 18: Examples of how strategies used by the children in Year 6 were categorised in table 25.

Figure 18 and Table 25 present findings that will provide further evidence for research question 2, What is an effective progression of concepts and processes in the teaching of division? Findings in table 25 show that the division questions related to times-table facts most commonly were answered with no workings in the years 2017 (58%) and 2018 (85%). Findings also demonstrate that, in the 2018 results, the strategy of no workings was almost 30% higher. This could indicate a better times-table knowledge in that cohort compared to the previous one as the 2017 cohort demonstrate a high percentage of children that used a procedural method to solve a simple inverse times-table fact. However, in 2019, 77% (45 children) of the cohort used a procedural method rather than showing no workings and I wonder if this shows a difference in teaching styles and the need in the current classroom climate to provide proof of an answer.

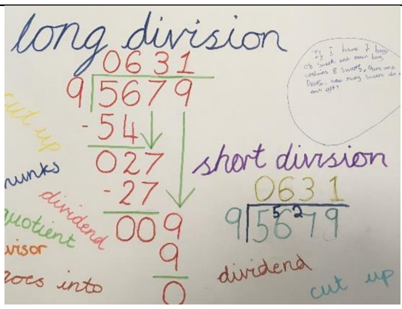
It is also noticeable that conceptual strategies begin to emerge as the questions become more complicated. It is also clear that there is a preference in the use of mixed method approaches to solve the problems presented although this is not the case in the simpler questions such as 12÷3, HTO÷O or times table facts.

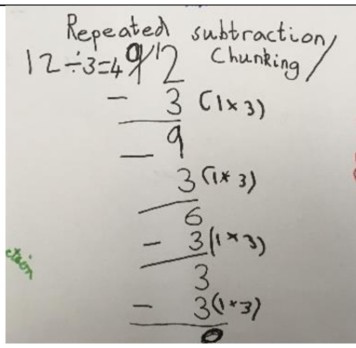
I wonder whether this is due to the changes in teaching due to the implementation of the New National Curriculum (2014)? I wonder whether this trend will continue or whether, as the formal written methods are used more and more in class, that the procedural methods will be adopted further?

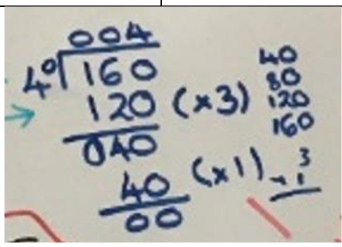
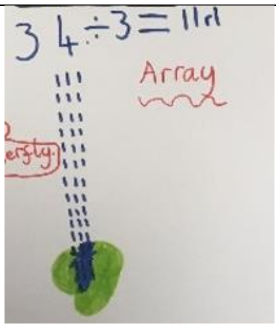
The difference in results of the more complex answers between the years 2018 and 2019 is also noticeable. I will comment upon this further in the next chapter as I wonder if this is due to a change in teaching styles or whether it is related to how long the New National Curriculum (2014) has been being taught.

7.3.2 Study part F: graffiti wall analysis

The graffiti walls were seen as documents that provided an insight into the children's strategy use and understanding with regards to division. Documents can be used as a way of "tracking change and development" (Bowen 2009, p30) and because of this, document analysis was used to analyse the graffiti walls. Bowen (2009, p27) stated that "document analysis requires that data be examined and interpreted in order to elicit meaning, gain understanding, and develop empirical knowledge". He also noted that it is a "systematic procedure for reviewing or evaluating documents". The graffiti walls were analysed using the categorisation of division strategies in chapter 5, shown in table 11, and in order to try and ascertain the children's understanding and learning of division. Figure 19 shows examples of how strategies were categorised.

<table>
<tr>
<td>

Examples of Formal written methods –Y6 - 2019</td>
<td>

Example of Chunking method – Y4-2019</td>
</tr>
<tr>
<td colspan="2">
Example of Mixed Approach – Y6 – 2016</td>
</tr>
<tr>
<td>

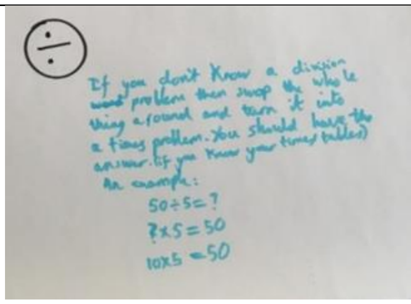
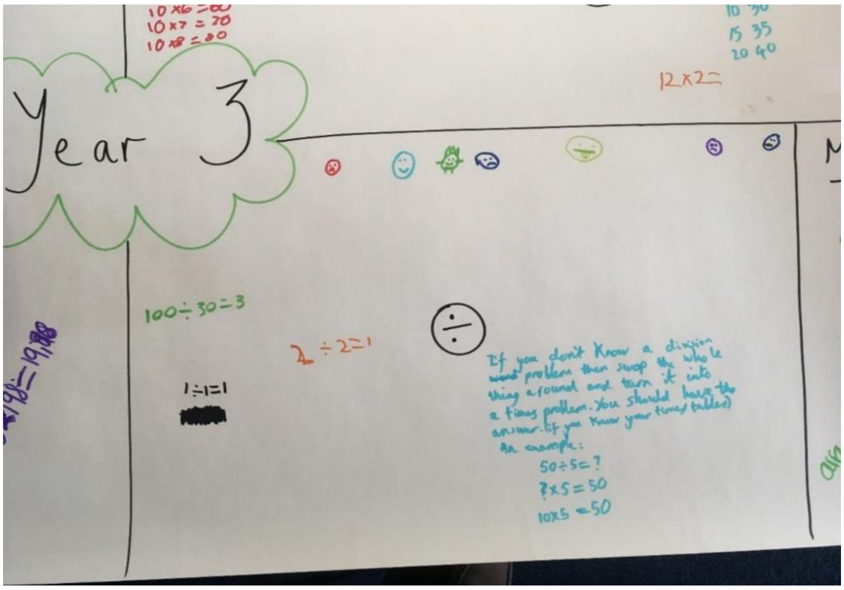
Example of an Array – Y4-2019</td>
<td>
Example of an Inverse strategy – Y3 (2016)</td>
</tr>
</table>

Figure 19: Examples of how strategies used by the children on the Graffiti walls were categorised using table 11.

The walls gave an insight into the question, What is an effective progression of concepts and processes in the teaching of division? The walls highlighted procedural and conceptual strategies used and possibly preferred. There are two graffiti walls presented for each year group. One graffiti wall was collected in July 2016. The second graffiti wall was collected towards the end of the study in July 2019 (see Appendix 20 for timeline of study). The reason for using two per year group was to show how the teaching and learning of division had shifted/changed since the implementation of the New National Curriculum (2014). It was important to gather this data at similar point in the academic year, the summer term, so that variations in teaching input would be minimal.



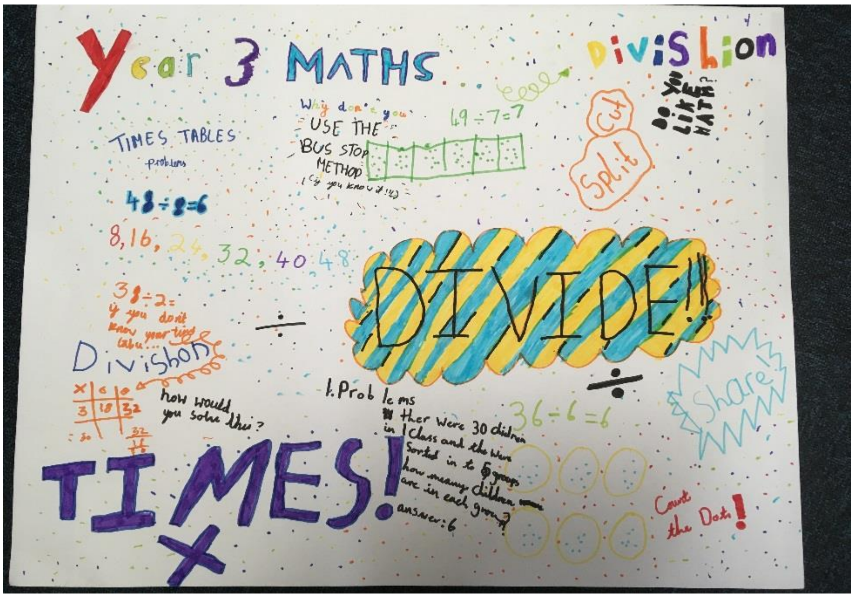
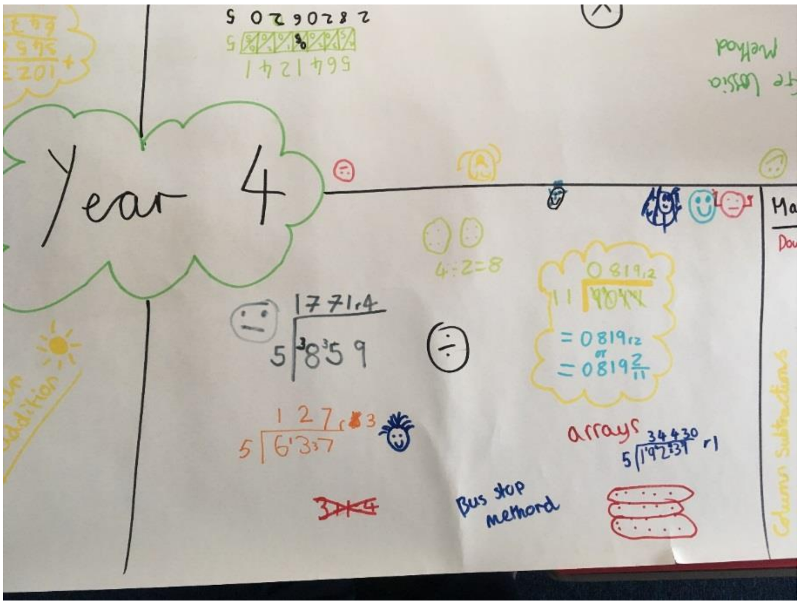


Figure 20: Year 3 graffiti walls – top 2016 and bottom 2019.

Initial findings show that strategies and complexities with the mathematics had definitely shifted/changed as the children moved through the school. Figure 20 shows the year three graffiti walls and the first picture demonstrates the use of inverse strategies to help them in their division. A lack of any true workings in this picture indicates perhaps a low level of procedural knowledge at this age. There are also errors evident in their work. In the second Year 3 graffiti wall, post implementation of the New National Curriculum (2014), there seems to be a development in understanding evident. More vocabulary associated with division is presented and a variety of methods are shown such as arrays, the use of inverse and also a mention of a short formal method (bus stop) although this is not demonstrated.



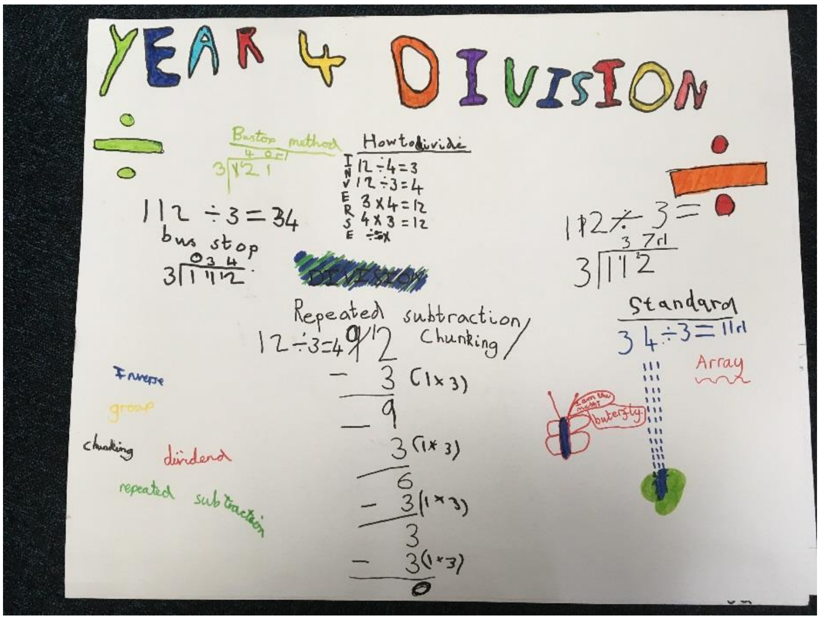
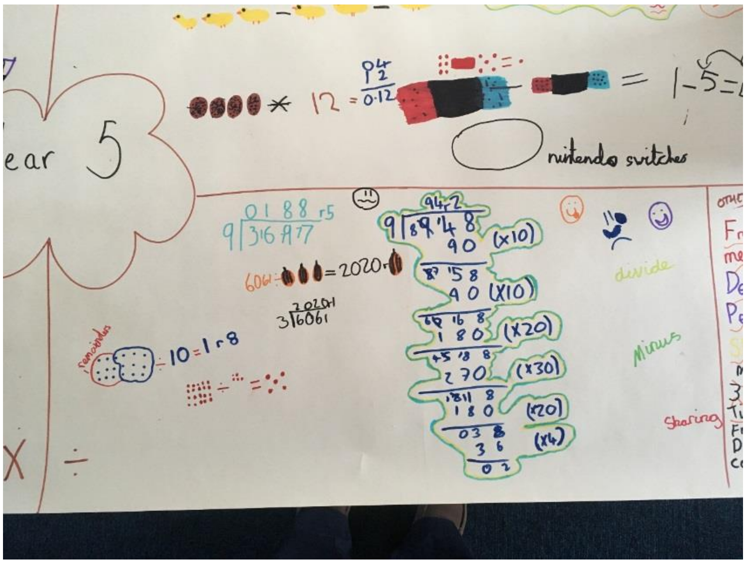


Figure 21: Year 4 graffiti walls – top 2016 and bottom 2019.

Figure 21 shows the Year 4 graffiti walls. The first wall demonstrates a leap from the original Year 3 wall in strategies used and known. Here the children show arrays, known facts and formal short methods (also known as bus stop method) with remainders in different denominations. Once again there are errors evident but, for this part of the study, I am more interested in the strategies they know and use rather than the errors they made.

The second Year 4 graffiti wall shows a development in the children's understanding and more vocabulary is presented. The methods used (inverse, short formal written and arrays) although similar seem to show a better knowledge and evidence of achievement although the children, as in the first graffiti wall, stick to HTO ÷ O (hundreds, tens and ones divided by ones) in their illustrations. The informal chunking method has made an appearance on this wall.



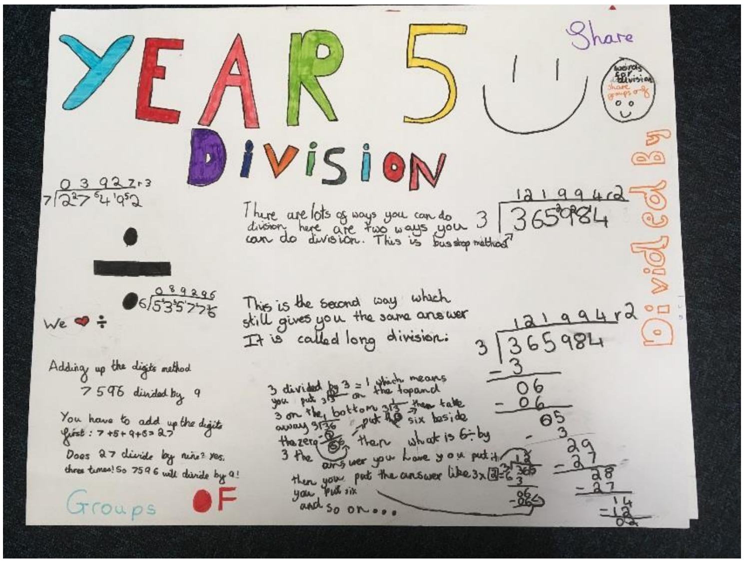


Figure 22: Year 5 graffiti walls – top 2016 and bottom 2019.

The year five graffiti wall again shows an increased understanding and use of strategies from the Year 3 and Year 4 walls. On it, the children have shown a use and knowledge of arrays, formal short methods (HTO ÷ 0) and informal chunking methods (using a conceptual approach). Once again there are errors evident but this, although provoking further study of common misconceptions in my role of mathematics coordinator, is not the focus as I am more concerned with the strategies known and used. However, these errors do perhaps give information and insight into research question 2, What is an effective progression of concepts and processes in the teaching of division?

In the second wall, for Year 5, it is evident that the informal chunking method has not made an appearance. I wonder whether this is because chunking is now taught earlier in the curriculum with the older children being encouraged to use and now more are using the formal written methods. I also wonder whether the use of more formal methods is because some teachers encourage the use of only them. There are no errors on this wall. Perhaps, in a way, these walls give an insight into research question 1, How do teachers perceive the changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics?, as they may be a result of changes in the teaching. At this level, the children are still using a divisor of one digit as an example rather than a divisor with two digits e.g. HTO ÷ TO.

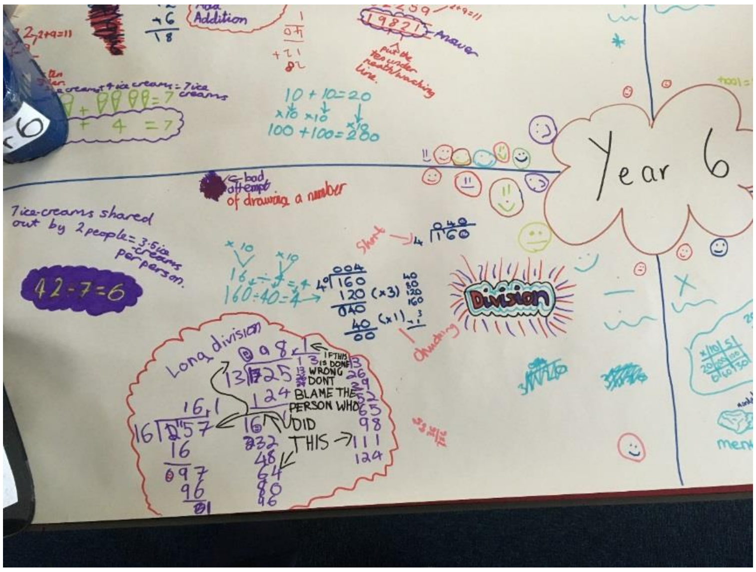


Figure 23: Year 6 graffiti walls – top 2016 and bottom 2019.

The findings in Figure 23 show that there is a change and a development in the strategies used and understood. Like Year 5, there is evidence of simple short division and the informal method of chunking, but the long division formal method has also made an appearance as has evidence of partitioning using known facts to answer questions. It is evident, in the findings, that the level and complexity of the work and the numbers used has also increased. There is, once again, evidence of errors but, as before, the actual errors are not of interest in these findings as such, however, they may give some insights into the disadvantages and advantages of certain strategies and methods and hence relate to possible advantages and disadvantages of teaching conceptual and procedural strategies for division.

The second Year 6 graffiti wall (2019) again shows a shift toward the use of formal written methods both short and long. There is a clear understanding demonstrated and no errors. I am left wondering if this is again related to the implementation of the New National Curriculum (2014) and the need for more complexity to be taught at an earlier age. Another point to note, however, is the fact that the graffiti walls show the dividend of 5679 that has been divided by a single digit divisor (9) rather than a two-digit divisor.

In this chapter, I presented my methodology and then the findings from the impact cycle that arose. In the following chapter, I analyse and discuss what I have presented in chapters 6 and 7 and I relate the findings to the ideas and work of other researchers. I also relate the findings to the research questions that I presented in chapter three. I intend to suggest explanations and list any possible implications of my study in terms of future actions.

8 Analysis and discussion

The design of the study was firmly based around the school environment and was created in such a way that it would be manageable and coherent for those working in primary education. It was hoped that findings from the study would be able to inform future practice in mathematics classrooms and provide implications for policy with regard to the implementation of the New National Curriculum (2014).

Two of the aims of the first cycle of this action research study were to gain an understanding of what mathematics is and then more specifically to gain an understanding of what mathematics is in the primary curriculum and classroom. In order to clarify these aims, for those reading the research, chapter two looked at an understanding of what mathematics is and chapter 3 looked at what mathematics is specifically in the primary curriculum and classroom. The actions undertaken during cycle 1 were to engage in and ponder upon what others think mathematics is so that a working definition could be developed for this study – this is shown in chapter 3. Moving on from this, the next aim, which was still part of cycle 1, was to develop a better understanding of division in the primary curriculum (chapter 4). Initially, in this chapter, it was necessary to conduct a literature review about research into the mathematical operation of division. The review highlighted significant disagreements with regards to progression of division strategies through the primary curriculum, particularly with regards to procedural, conceptual or mixed approaches. Part of the first cycle of my action research compared the New National Curriculum (2014) and NCETM documents for the progression of strategies and concepts regarding division. I found significant differences and hence established that there was a need for research on how a school might navigate the competing and different advice on offer and organise an effective progression of work in division from years 1 to 6.

In chapter 5, I presented the methodology guiding the actions of cycles 2 and 3 of this study. I outlined the participants used in the study and also considered any ethical actions and procedures needed. Following that the design of the study was set out and reasons were justified for choices made. Finally, in chapter 5, I presented the methods that were used to analyse each of the study parts. The beginning of chapter 7, the methodology for the impact cycle, research cycle 4, followed a similar layout.

In chapter 6, which presented the results of the actions from cycles 2 and 3 of the research, and also in the latter part of chapter 7, which was the impact cycle, cycle 4, I presented and interpreted the main points that arose in the findings.

In this chapter, chapter 8, I analyse the findings presented in both chapters 6 and 7 and answer my research questions:

> Q1. How do teachers perceive the changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics?
>
> Q2. What is an effective progression of concepts and processes in the teaching of division?

I explain how the answers relate to and expand upon the existing body of knowledge presented in earlier chapters. I present my results with regards to each of the research questions in turn. Within each of these sections, I will then clarify why each set of data is used as evidence before suggesting what the answers reveal. I will then present my interpretation of the findings.

8.1 Research question 1: How do teachers perceive the changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics?

In order to provide evidence and results for this research question, I will be looking mainly at the focus groups, cycle 2, that took place with the teachers. I believed that this was the best way to pick up on the teachers' perceptions as their answers were in the moment and would provide new thinking/understanding with regard to the implementation of the New National Curriculum (2014). A limitation of my approach was that I would not be accessing teachers' more considered responses. There is also the possibility that, as a group, some might be influenced by the opinions of others.

Whilst reading through the transcripts, the first common theme that emerged was the thoughts of teachers that methods that were taught, and that were then used by children, with regard to division would be changing due to the reformation of the curriculum (see chapter 6.1.1, theme 1). Devlin (2000, p7) wrote about changes in mathematics teaching being due to "an explosion in knowledge" so this research, the thoughts of teachers, reinforces Devlin's thoughts as things would be changing again due to knowledge gained from research.

This evidence suggests that the teachers interviewed in the focus groups are of the belief that the changes in the curriculum would affect their teaching in a way that they are altering the strategies that they teach. The teacher responses support the work of Murphy (2004, p3) where she stated that reforms are used as a "policy lever to alter practices". The teachers discussed the idea that they believe formal written methods, which are possibly regarded as more efficient, will become standard practice. There is a concern with the literature (Benson. 2014, p31) that if children are learning procedures for formal written methods rather than learning for a conceptual understanding then they may possibly begin to have an “over-reliance” on the procedure. As Richards (2014) stated, we need, as teachers, to be aware of the common problems that children face when trying to solve division problems. These problems, as mentioned in chapter four, are a lack of times table knowledge, too many strategies being learnt, the concept of division itself and inadequate mathematical skill. Following my research, as a school, in order to address and action the point of a lack of times-table knowledge we have looked into fun ways to develop times table knowledge through the use of games and interactive applications such as Times Table Rockstars.

The evidence in the transcripts of teachers being interviewed in the focus groups also reveals their thoughts about the chunking method and it still being used in the Lower Key Stages of the school for understanding whilst being replaced in the latter stages of the primary years. This brings to the forefront the worries of Thompson (2012) where he mentioned that chunking and formal written methods are not related and so are not progressive, again bringing into question the coherence of what, as a school, we are offering children in relation to the teaching of division. McClure (2014) said that the key to fluency is ensuring that the children are able to make connections and that they do this at the right time in their learning.

Further to these thoughts, within the teachers’ focus group, a second theme emerged (see chapter 6.1.1. theme 2) when the teachers interviewed mentioned in their responses that there would be a need to teach certain concepts and strategies sooner than they did before. To rephrase this, essentially that mathematical concepts that previously would have been taught in Year 6 are now expected to be taught in Year 5, concepts taught in Year 5 would now be taught in Year 4 and so on. This new theme was titled different content/level of difficulty.

The responses demonstrated that the teachers interviewed in the focus groups were again aware of ways in which their teaching would be affected as they discussed that they would need to teach a higher level of difficulty at an earlier age even if the children did not seem to be ready. These thoughts support Hanson (2013, p5) where she suggested that the "issue that stands out at first glance is the very substantial increase in the amount and the difficulty of the content". The teachers talked about teaching different content and having to fill gaps that were appearing due to the changes that had been implemented in the New National Curriculum (2014).

To complement the perceptions of the teachers with regard to an increased difficulty and concepts being taught at an earlier age, if we were to take a look at the evidence presented on the graffiti walls, in the impact study (study part F, cycle 4, chapter 7.3.2) we can actually see that this change is occurring. The chunking method of division that was originally seen on just the Year 5 graffiti wall (2016), but not in the earlier years, was subsequently seen on the Year 4 graffiti wall (2019) showing that the expectations were higher in the years between the data being gathered and strategies were indeed being taught earlier in a child's primary journey. Neither the Year 5 nor Year 6 (2019) graffiti walls showed the use of the chunking strategy but both showed evidence of the use of formal written methods. In other words, there is corroboration of the finding that strategies are now being taught at least a year earlier than they were.

I am wondering if this is a good thing. Is it necessarily the case that the earlier the better?

With respect to the level of difficulty, Anghileri (2001) suggested that perhaps it is the teaching of division that needs to be questioned rather than the actual operation. What I think she means by this can possibly be seen in the evidence from T7:

Year 5 focus group: T7: dreaded chunking, I do not like the chunking.

Here the teacher actually talks about strategies they used, preferred or disliked. This speaks volumes to me as I believe that the thoughts of the teacher can have a huge impact on the learner and it can affect a learner's understanding and own thoughts. It also reinforces the importance of the thoughts that researchers Handel and Herrington. (2003) and Cavanagh (2006) wrote about in that teachers' beliefs in teaching and learning mathematics are critical. It makes me wonder whether a teacher's thoughts, actions and beliefs, like in this piece of evidence, would affect the teaching and learning of this concept in this classroom, and as Anghileri (2001) stated, shows a problem with the teaching of division and the beliefs around it being a difficult area. An issue raised for me here is the importance of teachers being committed to the progression of concepts and methods that we do adopt. I suspect no progression will be effective if it is not understood and endorsed by the teachers who do the teaching. In contrast to the extensive professional development that followed curriculum change in the 1990s, there was relatively little being offered teachers accompanying the 2014 change. I hope my study will provide a rationale for professional development for teachers in my school and potentially beyond.

Moving on, I looked at the responses in the teacher focus groups shown in the transcripts with regards to the common theme about gaps appearing and catching up with the mathematics (see chapter 6.1.1. theme 3) These extracts only appeared in focus groups from the lower end of the

Key Stage, year three, and the upper end, Year 6. I wonder if this is because of the expectations of Year 3 after transition from the infant phase and also in Year 6 with the pressures of the end of Key Stage tests. The evidence certainly demonstrates that the interviewed teachers were aware of pressures on their teaching and the effect that the implementation of the New National Curriculum (2014) would have with regards to the possibility of gaps appearing and missed chunks of learning due to the difficulty increasing and the higher level of learning expected. Both T3 and T4 mention gaps in their comments - T3 talks about having to be aware of and filling in gaps ready for future years and T4 talks about marrying the gap that needs closing. In a way, this supports the notion of Hanson (2013, p3) where she stated that "schools are struggling to cope with this new curriculum".

An action taken in response to this theme was that, as a school, we developed a coherent overview of the learning being covered so that teachers were clear where the learning was, where it had been and where it was going in order to fill any gaps that appeared. In support of and extending Hanson's (2013) idea we have encouraged the use of concrete, pictorial and abstract approaches to aid an understanding for *all* in mathematics. Another element we have developed follows Ball's (2000) thoughts of gaining an insight into misconceptions and errors in order to develop teacher understanding and add another level to the learning.

From the focus groups, the next theme to look at and evidence is the common theme of method over understanding (see chapter 6.1.1. theme 4). To explain this further, the teachers interviewed in the focus groups mentioned an awareness of the time constraints that they face and therefore they talk about just teaching a method or a procedure to ensure that 'learning' takes place. It seems here that they are intimating that learning is only superficial as they, the teachers, do not have the time to go deeper and expand the children's understanding. As responses reflected different elements of this theme they are presented separately. One response received and highlighted in relation to this area was:

Year 5 focus group: T8: I use to always encourage the child to use the method in which they are most comfortable and felt chunking was a great method to get an understanding. Now though, we ignore all other methods and just teach short method – highlighting accuracy and following the method over understanding.

The evidence here shows that T8 is aware of the conceptual strategies used and the advantages of them but that they now, in response to implementing the New National Curriculum (2014), are just using the formal written method of short division.

I wonder whether their teaching, using this method only, is due to the time constraints that they are under. I also question whether they are implementing and understanding the new curriculum fully.

I: so, do you feel you are just doing procedural methods, you must do it this way, this way, this way at the moment?

Year 6 focus group: T9: yes, cos we are desperate.

Year 6 focus group: T10: but we feel in time to come this will change and become more conceptual.

This evidence, from the transcript of the Year 6 conversation (see appendix 7 for the full text), shows that the teachers are again aware of the conceptual approaches. It shows that they feel that at Year 6 they have a need to teach toward the Statutory Assessment Tests (SATs) in May and therefore they need to teach a quick fix in order to achieve the necessary results expected by senior leaders and the government. This quick fix is not a feature of the New National Curriculum (2014) but an adjustment in teaching according to need. I ask whether their adjustment in teaching is indeed a fix or whether we are setting learners up to fail in the long term due to a lack of understanding.

As a result of the teachers' thoughts and adjusting their teaching to get a quick fix, and as mathematics co-ordinator, I have actioned the idea of taking more time on the arithmetical operations so that children feel confident. I have also put in place a morning starter activity across the school so that skills can be practised and discussed and in support of findings in the literature from Ernest (1999) and Lockhart (2009) where they wrote about the need to set problems to properly grapple with, I have encouraged the use of new problems where the children need time to think and discuss their ideas with others. We are using and extending the thoughts of Ofsted (2009) where they stated that the fundamental issue to be considered by teachers is how better to develop a child's understanding. Through this, I have promoted the idea of children proving their answers using a variety of methods rather than just finding one solution.

Another action that is currently being discussed at the school is the use of 'hinge' questions. Hinge questions are used to see if the children being taught are ready for the learning to be moved on. They can be used to save time if the children have picked up a concept quicker than expected and can also show whether more time is needed. An example of the hinge questions we use would be: Which is the odd one out? A- 45 ÷ 9, B-125 ÷ 25, C- 72 ÷ 9 or D- 250 ÷ 50. The children would then show on whiteboards their letter choice and the teacher can instantly see whether the learning can move on or whether more input is needed.

A final selection of evidence from T11 is also related to the common theme of method over understanding (see chapter 6.1.1. theme 5).

Year 6 focus group: T11: What I've found with long division, there were at least 3 or 4 children that could solve a division question using the short standard (4289 ÷48) they could do that – so why do they need to do the long division method.

Year 6 focus group: T11: so whichever way they do it they get the mark; this could mean they either have an understanding or they've just learnt the process.

The evidence shows that the teachers in the Year 6 cohort, again due to pressure of the SATs tests, are aware that it does not matter what strategy the children use in order to answer the question as it will still gain a mark if the answer is correct. T11 questions the need to teach the formal written method of Long Division when children are successful and can find the answer using an easier strategy. The complexity of issues around progression of division strategies is apparent here. The National Curriculum and NCETM have their versions, as a school we have a progression mapped (but, at the time of the research, not particularly thought through) in our schemes of work, and then there are the national assessments which provoke a re-working, in different teacher's practices, of those intended progressions.

The responses gathered from the transcripts certainly demonstrate that the interviewed teachers believe they are working within time constraints and highlight issues and that they are simply trying to get things done and covered in order to get results. It seems that although they are working for achievement using the New National Curriculum (2014), they are perhaps doing this to the detriment of the deeper understanding of the operation. Ryan and Williams (2007) talked about a child's needs changing as they get older and the teaching being adjusted accordingly.

Ball (2000), as mentioned in chapter 3, wrote about the need for teaching to be broken down so as to be seen from the child's viewpoint.

I wonder whether just teaching the formal written method of short division as mentioned by T8, and perhaps also T11, really is enough for understanding. If just teaching the procedure of the method, are teachers able to engage in potential misconceptions from the child's perspective with regard to division or do they just see errors related to lack of times table knowledge? Will the children understand what is going on in their mathematics or do they see the procedure as just steps to success?

My research took place in the early days of implementation of the 2014 curriculum. It is possible, once the New National Curriculum (2014) has been embedded longer, as the children would have had this new learning style for longer in their school lives as they move up the school, that method and understanding might come together However, at the time of collating evidence, teaching methods over understanding seems to be the beliefs of T3, T9 and T10 as it is alluded to in their responses shown earlier in chapter 6.1.1. and in appendices 6, 7 and 8. This evidence, the teacher responses, lends itself further to the need for a progressive calculation chart which will be developed in chapter 9 and for teachers to have space to engage in, adapt and endorse the progression. The action of teaching through misconceptions, mentioned earlier, has also been implemented due to these teachers' thoughts.

The last theme that was noted, after analysing the focus groups (chapter 6.1.1. theme 5), was categorised as other. This theme was where I considered there to be an interesting response that did not necessarily fit with any of the other ideas presented. As the category title, other, suggests, the examples for this theme are not all clearly presented in a theme, as such, but individual teachers make interesting and relevant points:

Year 3,4 focus group: T2: I think it's had a relatively small impact, greater focus on fractions, perhaps the biggest impact is that we are now applying things perhaps too high too soon.

I chose this response from the evidence as I was interested in the idea that the teacher (T2) stated that they thought there would be a relatively small impact on their teaching even though they also stated there would be a need to apply things too high too soon. I wonder if this teacher will in fact change their teaching at all or whether they will continue along the path that they have always taught. I suggest that the teacher is actually aware of the changes that are occurring with the implementation of the New National Curriculum (2014) but feels that their teaching will not change, it will just simply adapt to the needs of the children in their class in order for learning to take place. Their response (see appendices 6, 7 and 8), where they say, "I think we will manage better next year", certainly seems to be evidence that they think something is happening/changing.

The next responses that I was interested in looking more closely at from the transcripts were from T7 and T9 see chapter 6.1.1. theme 5. In these responses, both teachers talked about their feelings with regard to using the chunking method. It seems that neither T6 or T7 felt confident when teaching or using the chunking method. Their responses support and extend the thoughts of Thompson (2005) in that chunking can be difficult and there are many opportunities to make mistakes. Thompson noted a belief that errors can creep in when using the subtractive form of chunking. Thompson (2012) also noted about chunking a receiving "the worst press" and I wonder of is these reactions from teachers that fuel /support Thompson's findings.

The final point in this theme that I feel is pertinent is from T9:

Year 6 focus group: T9: just chunking really, and a number of those in my class that came up didn't have any recognisable method! No chunking nothing.

Here, I had asked, in the dialogue, what strategies of division that the children arrived in the latter Key Stages knowing. T9 was worried that at the beginning of the implementation of the New National Curriculum (2014) that the children would have huge gaps in their knowledge especially in the area of division. To have no recognisable method of solution, especially at Year 6, would indeed be a huge concern. The interviewed teachers in the Year 6 group, again supporting the thoughts of Hanson (2013), certainly seemed worried that the New National Curriculum (2014) would cause them a lot of issues and concerns as the expectations were a lot higher than before and the children that had come up into their classes were certainly not appearing to be of the level that they were supposed to be especially with regards to division. This worry is evidenced in T12's response as they spoke about how they thought their teaching would change over the years that follow and that hopefully the children would come to them knowing more and so they might only have to teach long division rather than all the written methods needed.

Before summarising this section of chapter 8, it is necessary to look at the evidence from the impact cycle (study E, table 25, chapter 7.3.1), the Key Stage Two test analysis. These results were collated for multiple reasons looking closely at them at this stage, we can see that the preferred methods and strategies used under test conditions have changed quite noticeably in the three years of the study and data collection. If we look closer at the final question on table 25, which is ThHTO ÷ TO (a four-digit number divided by a two-digit number), we can see that they support the teacher's perceptions of formal methods being taught more at the start of the implementation but then changing to more conceptual as time went on. This was also mentioned by T10 in the transcripts (see appendix 7) of the teacher focus groups. In 2017, the children appeared to favour a procedural method and 25% did not even take a risk to attempt the question. In 2019, 90% of the children used a mixed method approach (10% did not attempt the question).

In summary, in order to answer research question one, how do teachers perceive the recent changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics, teachers believe as suggested, in chapter 1, that the curriculum appears to be much harder. There is a higher expectation in the content presented and the level of difficulty has increased. Further to the reviewed literature, in chapter 1, teachers in the focus groups believe that gaps are appearing, Because of this, teachers stated that they believed there could be a need to teach a specific method rather than teaching for an understanding in order to achieve results in the short term.

The teachers also believed that over time as the children spend longer immersed in the new curriculum then this may change again. The idea regarding time spent immersed in the new curriculum is perhaps supported by the changes seen in SAT result strategies (see table 25, chapter 7.3.1). This teacher belief moves knowledge and skills forward as it suggests changes needed for the initial period of change and the need for an effective progression of concepts and processes so that changes are minimal if there are further curriculum changes. Teachers, in the focus groups, also spoke about a concern that they have regarding children's understanding and having to move on even if they are not secure in their learning.

Having gathered and identified the perceptions of teachers at the school with regard to the implementation of the New National Curriculum (2014) and division, in the next section I will look at the second question used in the teacher focus groups: What are the advantages and disadvantages of teaching conceptual and procedural strategies in relation to division? I will look at the findings gathered in order to support and develop an understanding for research question two.

8.1.1 Focus group question: What are the advantages and disadvantages of teaching conceptual and procedural strategies in relation to division?

If I refer back to the current related literature (presented in chapter 3, cycle 1), we can see that researchers such as Graeber (1999) and Chick (2003) are of the opinion that multiple approaches are generally more effective than just a conceptual or procedural approach. They mentioned that these mixed approaches should be encouraged but through my investigation, and using the evidence collated, I want to extend this thinking to see what teachers perceive the advantages and disadvantages of each approach is and then, in the next chapter, make recommendations for their teaching in respect of what is found.

In this section of the study, I will be looking at the evidence collated in the latter part of the teacher focus groups, (study part A, cycle 2, see appendices 6, 7 and 8). I will also reflect upon the data collected in the focus groups with the children (study part B, cycle 2, chapter 6.2), and then I will attempt to tie in the answers collated from the questionnaires from both the teachers, (study part C, cycle 3, chapter 6.3) and children (study part D, cycle 3, chapter 6.4). Further evidence will also be sought and interpreted from study part E (cycle 4, chapter 7.3.1), the Key Stage Two tests analysis which was collated in the three years of the data collection. Finally, in this section, I will analyse the depictions presented on the graffiti walls (study part F, cycle 4, chapter 7.3.2), relating to the usage of conceptual and procedural strategies and then link them with the thoughts and results from across the research.

Before the teacher focus groups, there were no discussions, previous to the informal discourse, on the meaning of procedural and conceptual with the teachers. However, it is noteworthy that one of the teachers was the mathematics coordinator before I was given the role and also the teachers interviewed in the focus groups had an awareness of the terms due to my M-level study in the same setting.

The evidence in chapter 6.1.2, theme 6, from T1. T2. T4, T7 and T12 reveals that teachers in each of the focus groups mentioned the advantage of a conceptual approach as being linked to understanding. This supports Eisenhart et al.'s (1993) beliefs where they stated that conceptual teaching of a topic is used to help children gain an understanding behind the mathematical procedures they use. Some of the teachers interviewed in the focus groups mentioned the idea of a deep or deeper understanding and this makes me wonder about the work I mentioned by Star (2005) and the idea that each layer of learning has different levels of sophistication.

T12 mentions the idea of a conceptual approach being advantageous as children have the knowledge and skill to join their thinking together and using a conceptual approach correlates and supports the thoughts of Graebar (1999) and Rittle-Johnson and Schneider (2014) that I presented in chapter 3. T1 comments that learning that is conceptual is a transferable skill, which means that learners can use it and apply it in different areas. Skemp (1976) suggested in his theory of relational understanding that a conceptual type of approach enables children to gain a mathematical grounding. He also suggested that this type of learning is beneficial in the long run and can lead to increases in motivation reinforcing the need to extend the thinking of Graebar (1999) and Chick (2003) in the development of an effective progression of concepts and processes.

Theme 7's evolution is discussed in chapter 6.1.2, theme 2. The first selections of evidence from T1 and T9 suggested that possible disadvantages of conceptual teaching were associated with time.

I interpret this evidence to mean that the teachers believe that they are under time constraints and cannot teach conceptual methods effectively in the time available, as they have to move on in order to cover the whole curriculum. Other evidence in respect of the disadvantages of using conceptual approaches were given as relating to a child's mathematical understanding and skill. It seems that T2, T10 and T12 are of the belief that children need to have gained a secure understanding at an early age as they believe picking up conceptual concepts at a later age is difficult. It is evident from the responses that the teachers think that a disadvantage of a conceptual approach is that a lower attaining child may struggle to grasp the many concepts and their complexities and that they just need to be able to get on with it and use a method to solve a problem in a way, challenging the thinking of Graebar (1999) and Chick (2003) that relates to the benefit of multiple approaches. T12 suggests that children just need to learn to cope with a lack of understanding.

Looking back again at the evidence in the transcripts, the next theme to be generated was theme 8 (see chapter 6.1.2). This theme was called advantage of using a procedural approach. The selected sections of the transcripts, from T1, T2, T7, T11 and T12, show that the teachers interviewed in the focus groups perceived the advantage of the procedural method to be the simplicity of it. It seems that they believe that the procedural approach is one that allows all children to achieve the necessary outcome. T11 even notes the procedural approach to be a quick fix. T1 states that they believe that the advantage of this approach is that you can get the answer without understanding it. Another researcher who would agree with the thoughts of T1 is Richards (2014), as mentioned in chapter 4, as he wrote about short division as needing no real understanding to complete.

The perceptions of the teachers also fit with the ideas of Baroody (2003) who described the procedural method as being the how of mathematics. Graeber's (1999) work is also supported in the evidence as T2 suggests that procedural methods can be used purely to gain a desired outcome or an end result. Eisenhart et al. (1993) noted that they believe that the procedural approach is one that is predominantly taught in the schools.

The teachers in the study spoke about the disadvantages of procedural strategies (see chapter 6.1.2 theme 9). The responses selected showed that the teachers believed that a disadvantage of the procedural approach is that the strategy cannot always be transferred and then applied to different situations. T1 even says that a disadvantage of the approach is that the children do not necessarily gain an understanding and are just going through a process. I understand this to mean that they literally just follow a series of rules in order to answer a question with no background comprehension. Richards (2014, p15) wrote about the short division algorithm and children being able to just learn it "without understanding how it works".

One disadvantage as presented by T4 and T1 is that they believe the children can get hung up on a procedure and as T1 suggests if they cannot see another step beyond the two steps, they are told to take, then they are unable to progress further and, possibly sensing failure, give up.

Haapasalo and Kadijevich (2000), as mentioned in chapter 3, wrote about a procedural method being one where children use automatic and unconscious steps. I relate this to the thoughts of T1 and perceive their response to be related to a barrier getting in the way of these unconscious steps.

The final common theme evident in the teachers' conversations with regard to research question two was one I titled needs and other (see chapter 6.1.2 theme 10). This theme was identified because the teachers spoke about other issues that they felt impeded or affected procedural or conceptual approaches. Responses in the transcripts (also see appendices 6, 7 and 8) demonstrated that the teachers interviewed in the focus groups perceived that there is possibly an issue with those children that are lower attainers and have difficulty in learning multiple approaches and elements. T5 seems to agree with this as they talk about, in their opinion, some children only being taught the how as this is what they need in order to be successful in some cases. T2 mentions a need for daily exposure and this is something I also believe in.

In summary, the teacher focus group question: What are the advantages and disadvantages of teaching conceptual and procedural strategies in relation to division? highlights the main issues that the teachers perceive there to be. These issues are related to understanding, time constraints and application. This evidence gives new understanding, with regard to the thoughts of the teachers interviewed, in order to develop an effective progression of concepts and processes that can be implemented with the support of the professionals that it is being developed for.

8.2 Research question 2: What is an effective progression of concepts and processes in the teaching of division?

Having analysed the transcripts from the teachers' focus groups, I will now look at the children's focus groups (study part B, cycle 2, chapter 6.2). This will be completed in order to look at research question two, What is an effective progression of concepts and processes in the teaching of division? as looking at the strategies used and preferred by the children adds another level to the analysis. The evidence, in tables 13 and 14 of chapter 6.2, shows that the children have an understanding of division and that they are aware of the different methods in order to solve problems. The tables also show that most of the children had an awareness of the interwoven strands and connections and relationships associated with the different methods for example, the comments 'opposite of times' and 'use your times tables'. Table 13 shows how the children defined division whereas table 14 actually shows the methods that they prefer using. In the year groups 4 to 6, the children talked about knowing the short formal written method and they described this method as being easier, takes up less space and is quicker.

Within table 14 (chapter 6.2) the children also talked about chunking, which can be considered as both procedural and conceptual depending on how it is used. Some of the children conferenced spoke about chunking as being a waste of time, being difficult to understand and as a process taking up a lot of space. However, it is also evident in table 14 that some children referred to chunking as being a known entity especially as they progressed up the school. The children, in Year 6, talked about long division as being frustrating, difficult to grasp and taking a long time. I suggest that table 14 also reflects the thoughts of teachers regarding the different strategies.

I wonder whether teachers play it safe and just show the children how to do the mathematics needed to solve a problem and whether they only try the more complex strategies with the higher attaining children. I question whether the children are only shown approaches that the teachers are confident in using themselves and this would fit with the comments of T7 about chunking.

Table 14 (chapter 6.2), seems to show that children have a preference to use the formal written methods related to division as these are the ones they comment about positively and more often than other methods; the exception is in Year 3 where they talk about the pictorial concept of arrays and also knowledge of the times tables. This use of pictorial elements is something I am keen to address in the future as I want children to use a multitude of strategies in all year groups and be positive about them all. The focus group responses can be triangulated against the questionnaire from study part D (cycle 3, chapter 6.4), the children's questionnaire. One difference between the focus groups and the children's questionnaires is that the focus groups were with a randomly selected group of children (24) whereas the questionnaire was delivered to every child in the school (230).

I was interested to see if there was a difference in the findings of the children's focus groups and those from the children's questionnaires related to those who like division. The findings from the children's questionnaire (appendix 3) and the results from the focus group analysis (shown in chapter 6.2), show a similar percentage of children not liking the operation of division. Table 20 (chapter 6.4) shows the different ways in which the children described division: 13.5% of the children described it as sharing and 30.75% described it as the opposite of times. Perhaps the most valuable tables in this part of the research though, and the ones that relates most to research question two, What is an effective progression of concepts and processes in the teaching of division? are tables 21a, b, c and d (chapter 6.4). These tables present the findings of how children actually answered division problems in the questionnaire. These tables show the strategies the children use.

The evidence in the children's questionnaires (chapter 6.4) shows that there is a split in the strategies used by the children when answering the division problems presented to them. What I mean by this is that not one approach seems to be favoured. This does not mean that a child who has answered in a procedural manner does not have a conceptual understanding, it just shows the method they have chosen at this time. Table 21a (chapter 6.4) shows that even when a simple problem such as 12 ÷ 3 is presented then children will use a plethora of strategies in order to solve the question. In this case, 51% of the children on the school roll answered the question with an inverse strategy to show their understanding but as the problems increased in difficulty this approach reduced in usage to be favoured by a formal written method or no answer at all.

The question 275 ÷ 25, shown in table 21c, showed that 28% of children answered using a formal written method, 18% used sharing to aid them and 13% used the strategy of chunking. 32% did not attempt the question at all. This to me perhaps is showing that as the children progress up the school then they become more confident with problems that have larger numbers (i.e., HTO ÷ TO, a three-digit number divided by a two-digit number, rather than just TO ÷ O, a two-digit number divided by a one-digit number). Of course, I do not have evidence about children's progress from the questionnaire, as it was a snapshot of the whole school. So, while I cannot discount the possibility that differences may be linked to difference in the year group make up, I note that over the period of the research, our intake statistics as a school did not vary greatly.

If we look back at table 15 (chapter 6.3, study part C, cycle 3), at the teachers' methods for solving division problems, we can see a similar pattern to that of the children. It seems the teachers involved in the questionnaire prefer to use a formal written method unless there is an obvious inverse number fact or strategy that can be used. This could be why the children are happier using these methods too. I have already discussed how the actions of one affect the actions of another and my research provides further evidence for the influence of teachers' preferred strategies, on the preferred strategies of the children they teach- a finding evident in the writing of Anghileri (2006). The evidence in table 15 (chapter 6.3) does not show that the teachers are unable to use a strategy with a conceptual method, it just shows what their preferences are. As I mentioned in chapter 6, I also wonder if the strategies they use actually relate to the year band that they teach for example, teachers of the lower year groups use more pictorial strategies whereas the teachers in the upper year groups use more written methods and inverse strategies reflecting the strategies they teach. I say this as it was noticeable with the grouping and scaffolding strategies used in certain questionnaires and although the questionnaires were anonymous the evidence still raised thoughts in my mind. I also reflect upon the other questionnaires where the usage is mainly procedural methods. I wonder whether this is because they are preferred, quick and known and hence enabling the teachers to get through the task quicker. I look closer here at the problem 5,542 ÷ 17 = 326 and explaining how you can use this fact to help you solve 18 x 326. The method used to solve this in every case was with a use of number facts. Although knowing this problem was possibly too complicated for children in the lower year groups, the question was used to see how and if children would grapple with it and take a risk/have a go using the strategies known to them. If a procedure was not available to the children, would they have any conceptual understanding they could fall back on? Most children noted it was too hard for them without even trying which suggests a limitation to the approach I used. I could have used a question with smaller numbers but as I was also looking at progression, I wanted to find out what we needed to do to improve children's experiences in division problems.

Moving on to the evidence in study part E (the impact cycle, chapter 7.3.1), which was collated to show approaches and strategies used under test conditions, we can see that the children use methods that are quick and easy (see table 14, chapter 6.2). If we look at the questions, over the three-year period, that are HTO ÷ O, a three-digit number divided by one-digit number, we can see a change, over the years, between no workings at all to a use of procedural methods. In both 2017 and 2018, 90% of children simply wrote an answer to the question set and the rest chose the formal written strategy, which was used with no other workings (i.e., classified as procedural for this research). However, in 2019 this changed quite drastically to 26% showing no workings, 3% using a mixed method and 71% using a procedural method (see table 12, chapter 5.5 to see how the strategies used by the children in the division problems were analysed). I can only assume that this change is due to the teaching and the changes in the curriculum being taught as children are now encouraged to check their answers with workings and so to just write the answer would possibly have felt wrong and alien to them even if they already knew the answer. The children are now used to showing their workings and this has become second nature to them. Benson (2014) noted that children need to make their own decisions with regard to the best strategy to be used so that it suits their own style and I see this use of workings as a change and a reflection in what the children have been supported to do in the classroom. I also looked back at my own work, where I stated that short division is seen as quick and easy and that there are less opportunities to go wrong with it compared to the method of chunking (Tutcher, 2015).

The next question, related to times table facts, 72 ÷9, 99 ÷11 and 91 ÷7, (shown in table 25, chapter 7.3.1), demonstrates the same sort of processes. In 2017, 58% showed no workings, in 2018, 85% showed no workings, but in 2019 only 13% showed no workings and 77%, in this year, used a short division method. Benson (2014, p31) in his research wrote about short division emerging as the "preferred method" in a test situation for a HTO ÷ O problem, a three-digit number divided by a one-digit number. The evidence in this research correlates with the ideas in his research but also shows that the New National Curriculum (2014) implementation is showing an advantage in children showing their workings through understanding a concept supporting the thoughts of Graebar (1999) and Chick (2003) with regards to the benefits of using mixed approaches. The change in approach shows also just how influential teaching methods can be on children's strategies. Again, I am conscious that I am reporting on different children here but I feel the extent of the differences are only explainable through the different teaching approaches taken. These findings indicate how significant an effective progression chart for division could be as well as how much influence, as a school, we might have on more general aspects of doing mathematics, such as showing workings, if we take a consistent approach ourselves, across our teaching.

To recap, research question two is concerned with developing an understanding of an effective progression of concepts and processes in the teaching of division. The third problem presented in table 25 (chapter 7.3.1) shows a shift once again in the strategies used and preferred by children when in test situations. The questions posed each year (HTO ÷ TO) show that, in 2017, 51% of children chose to use a procedural method to answer the problem, whereas in 2019 this was actually down to 0%. If we also look at mixed methods used, we can see that in 2017 only 30% used a mixed strategy, using both a procedural method with some other workings indicating an underlying understanding in order to support the calculation (see table 12 in chapter 5.5), whereas in 2019 there were 93% who chose a mixed strategy. I see this as demonstrating a change and a good understanding in 2019 perhaps reflecting the effect of the implementation of the New National Curriculum (2014).

On further investigation, it was apparent that the children used a strategy of writing down the multiples of the divisor to the side of the written method they were using and then using this to aid a procedural strategy, demonstrating a knowledge of and also applying a skill related to the problem – a mixed approach. This relates to and supports the work of Thompson (2005) that I noted in chapter 4. He wrote about using a strategy that uses multiplication facts to support division. Richards (2014, p17) also supported this idea and wrote about showing "partial multiplication facts" next to their work in order to solve their work. Since starting this research, I have preferred to use this method when I teach and perhaps this is why we are seeing this change as I have been promoting it in mathematics meetings held with the teachers at the school. Despite some of the initial concerns and even scepticism of teachers around the 2014 curriculum changes, my research is indicating that the children in our school, taught under the new curriculum for over two years, are showing a greater understanding of division than those in previous years. In terms of my own progression, one important idea I take from this is that the earlier use of methods compared to 2014 may have some advantages, as well as the importance of emphasising showing workings.

The final type of question analysed in study part E, cycle 4 (see table 25 in chapter 7.3.1) was a ThHTO ÷ TO problem, a four-digit number divided by a two-digit number. The findings are similar to those of the previous question, in that in 2017, 41% of those that tackled the problem did so using a procedural method, 7% attempted a conceptual method and 27% used a mixed method. This fits with the teachers' transcripts again as they spoke about having to concentrate on formal written methods, which are considered to be a procedure in order to find an answer over understanding the problem.

In 2018, after about 18 months of the implementation of the new curriculum and perhaps a change in teaching styles, the findings (see chapter 7.3.1) show that the children seem to prefer to use a different strategy for a ThTHO÷TO problem, a four-digit number divided by a two-digit number – 59% using a mixed method and only 12% using a procedural method. Again, in 2019, the findings, in table 25, show another change in that 90% of those that took the test used a mixed method, the other 10% did not attempt the question. This I feel shows a real shift in strategies and approaches that are preferred and a shift which would be welcomed by many academic researchers who advocate the distinct advantage to using a combination of strategies (Askew et al., 1995; Graebar, 1999; Chick, 2003). My own views and also that of T2 in the teacher transcripts bring a question though. To what extent are changes due to the strategies being used and to what extent because children have been immersed in arithmetic almost daily? The children in the 2019 sample certainly had benefitted from a daily dip into the main number operations in order to keep their skills sharp. This was a new strategy tried throughout the school in response to the need for higher expectations in the New National Curriculum (2014) and also in order to increase skills and confidence. Beyond the specific findings about division, my study provides important evidence about the impact of whole school strategies. It is ultimately not possible to say what has caused what, but I can say that changes were made in approach, such as the daily dip into arithmetic and also as commented upon in chapter 9.6, and there have been significant changes in how children approach division problems (chapter 7.3.2, table 25 and the graffiti walls in figures 20 – 23). Study part D in chapter 6.4 (children's questionnaire) and study part E, chapter 7.3.1.(SATs results) can also give comparison/triangulation data as the children involved in each were the same. The SATs results in 2017 were from the Year 5 group in the questionnaire. Unlike the snapshot data, or consideration of children in different years doing similar problems, this data does give me evidence of development.

The SATs results in 2018 were from the Year 4 group in the questionnaire and the SATs results in 2019 were from the Year 3 group in the questionnaire. Tables 26a, b and c show comparisons of this data and how the same children approached similar questions, 1, 2 or 3 years apart.

	2015 questionnaire (Yr5)	2017 SATs analysis (Yr6)
TO ÷ O	Procedural – 38% (24) *Written method – 38% (24)* Conceptual - 59% (38) *Number fact 12% (8)* *Sharing 2% (1)* *Inverse 45% (29)* No answer 3% (2)	Procedural method 34% (24) Mixed 8% (6) No workings 58% (41)
HTO ÷ TO	Procedural – 23% (15) *Written method – 23% (15)* Conceptual – 47% (40) *Number fact 2% (1)* *Chunking 16% (10)* *Sharing 23% (15)* *Inverse 22% (14)* No answer 14% (9)	Procedural method 51% (36) Conceptual 4% (3) Mixed 30% (21) No answer 15% (11)

Table 26a: Findings showing a comparison/triangulation of strategies used by children who were Year 5 when taking the questionnaire and then their Year 6 SATs data in 2017.

	2015 questionnaire (Yr4)	2018 SATs analysis (Yr6)
TO ÷ O	Procedural – 13% (7) *Written method – 13% (7)* Conceptual – 83% (45) *Number fact 24% (13)* *Chunking 24% (13)* *Sharing 4% (2)* *Inverse 31% (17)* No answer 4% (2)	Procedural method 9% (5) Conceptual 3% (2) Mixed 3% (2) No workings 85% (50)
HTO ÷ TO	Procedural 7.5% - (4) *Written method – 7.5% (4)* Conceptual – 60.5% (38) *Number fact 7.5% (4)* *Chunking 37% (20)* *Sharing 26% (14)* No answer 22% (12)	Procedural method 15% (9) Conceptual 2% (1) Mixed 73% (43) No workings 7% (4) No answer 3% (2)

Table 26b: Findings showing a comparison/triangulation of strategies used by children who were Year 4 when taking the questionnaire and then their Year 6 SATs data in 2018.

	2015 questionnaire (Yr3)	2019 SATs analysis (Yr6)
TO ÷ O	Procedural – 0 Conceptual - 76% (40) *Number fact 10% (5)* *Sharing 28% (15)* *Array 10% (5)* *Inverse 28% (15)* No answer 24% (13)	Procedural method 77% (45) Mixed 10% (6) No workings 13% (8)
HTO ÷ TO	Procedural – 0 Conceptual - 15% (8) *Sharing 11% (6)* *Array 4% (2)* No answer 85% (45)	Mixed 93% (55) No answer 7% (4)

Table 26c: Findings showing a comparison/triangulation of strategies used by children who were Year 3 when taking the questionnaire and then their Year 6 SATs data in 2019.

Tables 26a, b and c demonstrate similarities in the 2015 questionnaire columns in that the children used a variety of methods to answer each question demonstrating both procedural (written step-by-step method) and conceptual (no evidence of procedural method) approaches.

However, by the time the children reach Year 6, in each case, the methods used, possibly preferred, have changed.

Table 26a shows the children who were in Year 5 at the time of the questionnaire and then the same children when they were in Year 6 in 2017. A difference, shown in the table, is with the problem HTO÷TO. Initially in 2015, 23% (15) of the children used a procedural method and 47% (40) used a conceptual method. However, when they solved a similar problem in 2017, 51% (36) used a procedural method, 4% (3) used a conceptual method and 30% (21) used a mixed approach. This is evidence of the methods used changing and supports the idea of Ryan and Williams (2007) in that as the needs of the children changed so teaching and provision changed. The drop in conceptual methods that the table shows could reflect the teachers' responses in the focus groups (chapter 6.1.1) where the Year 6 teachers spoke about teaching mainly procedural methods as they are desperate, see appendix 8. However, it also reflects one of the findings of the study in that children, taught under the 2014 Curriculum, in much higher proportions than before, are using a mixed approach (chapter 7.3.1, table 25) showing that they are using a procedural strategy but with support of a conceptual understanding.

Table 26b is similar to 26a but on the TO÷O problem 83% (45) of the children, when in Year 4, answered the problem with a conceptual method, one with no procedural element, but in 2018, when they took their SATs paper, 85% showed no workings at all, 9% (5) children used a procedural method. In the same table, for the HTO÷O problem, 60.5% (38) used a conceptual method. However, in 2018 when they took their SATs test, 73% (43) of the children used a mixed approach perhaps giving evidence of how possible changes in teaching methods were impacting on the methods children used. I assume that the students in Year 6 are unlikely to have forgotten the conceptual, and sometimes individual, methods used. But, having been introduced to procedural methods, the big majority show they prefer them – perhaps linked to their speed in test conditions. The evidence of the shift to the procedural might also relate to the teacher fears of feeling forced to focus on methods for the examination. So, another explanation could be that these children have been drilled in procedural methods and so their earlier conceptual methods do not arise for them as possibilities to use.

Table 26c represents the children who were in Year 3 at the time of the questionnaire and Year 6 in 2019. For the questionnaire TO÷O, 76% (40) of the children used a conceptual method whereas in 2019 for SATs 77% (45) used a procedural method. For the more complex problem, HTO÷TO, in the SATs analysis 93% (55) used a mixed method and the rest made no attempt to answer. The table suggests that the children have retained some of their past conceptual thinking, with workings alongside their procedural method.

The graffiti walls (study part F, cycle 4, chapter 7.3.2) were used as a way to gather more information. Graffiti walls, as Lotriet (2012) noted (mentioned in chapter 7), allow for mathematics talk and risk taking with no fear. Bakewell (2008), in her blog at *education.wfu.edu*, wrote about the use of graffiti as having a "rich history in personal expression", fitting with what I wanted to achieve. The graffiti walls have already been discussed with regard to research question one, How do teachers perceive the changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics?, but they also, when interpreted, show evidence toward the advantages and disadvantages of using the different methods and approaches. It certainly seems that through the implementation of the new curriculum that the skills and understanding of the children has increased. The 2019 walls, in each year group, seem to demonstrate an increase in knowledge but as the children are different in each case this increase is not necessarily clear, so, it is important to just look at the changes in strategies used, which is more likely to be directly linked to the teaching they experienced. The Year 3 walls, from 2016 and 2019, demonstrate an increase in understanding and perhaps indicate a difference in teaching approaches as their walls are littered with different concepts. They mention the formal written method of short division but they do not show it. The 2019 wall shows evidence of links to multiplication and of the use of pictorial strategies.

The Year 4 graffiti walls also give evidence toward the children possibly receiving a different curriculum. It shows a use of conceptual strategies, repeated subtraction/chunking, and it seems that the children have a better grasp of division as their answers are correct whereas the 2016 wall shows a simplistic and incorrect use of the short-written method. The 2019 wall mentions using the inverse strategy of division and so, in my opinion, demonstrates a far superior understanding compared to the 2016 wall.

The picture shown in the Year 5 graffiti wall is similar. The children produced a detailed wall in 2019 that perhaps demonstrates an increased skill from 2016. The first wall shows a use of the chunking method but it also demonstrates the disadvantage in this method in that there are many possibilities for it to go wrong. The 2019 graffiti wall seems to demonstrate a secure understanding and an increased skill but, on closer inspection, shows just a use of formal written methods both short and long. This, in a way, supports the notion of the Year 5 and Year 6 teachers, that is evident in the transcripts of the focus groups (see appendices 6, 7 and 8), where they say that the children are not really thinking mathematically as such, as they are just following a known procedure of steps.

Finally, the Year 6 2019 graffiti wall, again, like the Year 5 2019 wall, demonstrates a use of written methods that are procedural but they are used correctly with no mistakes. To the side of these there is a mention of the use of inverse strategies to help computation and therefore perhaps demonstrating the advantage of using conceptual methods to aid understanding. The Year 6 wall in 2016 is very messy but demonstrates many strategies and approaches be it a little simplistically. The children show the chunking method, the short-written method and an attempt at the long-written method. The difference in the two Year 6 walls perhaps reflects the implementation of the New National Curriculum (2014) in that the formal written methods seem to be at the forefront of the children's minds (chapter 6.2 table 14 and 7.3.1 figure 18 and table 25). In keeping with the SATs paper analysis (table 25), the Year 6 wall in 2019 show a mixed approach, combining procedural methods with some evidence of conceptual understanding.

In summary, this chapter represented a conclusion to cycle 4 and a further reflection on the outcomes of cycles 2 and 3. In the following chapter I conclude my action research study by reflecting on the research completed and presenting the implications of it. I indicate what I have learnt from the study and then I draw conclusions about its strengths and its limitations. Finally, I report upon my personal thoughts about the process of completing this action research study and also about education itself, making possible recommendations for future practice.

9 Concluding comments, recommendations and actions

Having analysed the collected data against the research questions,

> Q1. How do teachers perceive the changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics?
>
> Q2. What is an effective progression of concepts and processes in the teaching of division?

and current related literature in the previous chapter, I now conclude this action research study in this chapter. I reflect upon the various things that I have learnt along the way together with presenting the challenges I have faced. Following that, I look at the strengths and limitations of the study including making comments on how the data collection went. Concluding the study, I attempt to suggest things that I may do differently in the future with regard to this action research study and also propose any possible future research that could stem from this study. I also reflect upon what the action research study has or will contribute to.

This educational action research study, as mentioned in the previous chapters, initially arose from work completed in my Master's research. It also stemmed from theoretical information gained from the reading of academic research literature that surrounds division and its development within primary mathematics, cycle 1. Another element that prompted a need for this action research study were the difficulties faced by children when trying to learn and solve problems related to division and also the difficulties faced by teachers when teaching it. The knowledge of these types of problems was gained from many years of personal and professional experience when teaching children of a primary age, who find it complicated and confusing to comprehend the various elements and the many strategies of the operation that have been suggested in various mathematics curricula over the last thirty years. In this action research study the perceived thoughts of children were that a higher percentage (67%) than expected stated that they like division and think they are good at it. However, knowledge gained through annual test analysis and teaching observations suggested that the area of division is where the school needed to improve.

As it was born out of a desire to help children, an intention that emerged out of this action research study was to create a progression chart which would enable children, who access a mathematics curriculum, to understand, enjoy, satisfy and be in line with the requirements of the New National Curriculum (2014) with regard to the division elements of primary mathematics. A progressive calculation chart, repeated from chapter 6, is seen as a clear and coherent document that breaks down the curriculum and suggests a developing sequence for teaching the varying division strategies that the children need to know. It was also an intention for this action research study to enable teachers, who are not expert or keen mathematicians, to become proficient, confident and happy with the various approaches of the operation as suggested by the New National Curriculum (2014).

9.1 Strengths and limitations of the study

The title of the thesis states that it is an action research investigation into the implementation of the New National Curriculum (2014). Anghileri (2006, p377) noted that "the notion of reform in mathematics teaching is widespread but the actuality of the classroom is that changes are made rather slowly". A strength of this study is that the research and data collection was able to take place over a period of five years (see Appendix 20 for timeline) and therefore a host of valuable insights were able to be collected and represented in respect of the one school being investigated. I was able to collect longitudinal data and compare both different year groups at one point in time and also the same children's work one, two or three years apart. The longitudinal aspect of the study was made possible by researching in the context in which I work. I believe that the time aspect that Anghileri talks about with regards to reform is reflected in this action research study as the changes appear steadily and, in a way, they are still happening. We are almost six years into the new curriculum and we are still developing and adapting to the reforms in mathematics. Of course, there were also limitations that arose from researching a site which is so familiar. There was a danger that I was blind to certain aspects of the research site simply due to its familiarity. The cycles of reflection within the action research process and the triangulation of data both help mitigate this danger but it is impossible to eradicate.

A further limitation of the study would have to be its small scale. As it is only a study of one school, judgements cannot be made with respect to all primary practitioners. Findings cannot be generalised as they are specific to the establishment in which they were collected. However, I must note, it was not my intention for generalisation across establishments to take place at this phase. One of the features of action research is that the outcomes can be of a local nature, but hopefully of strong practical relevance.

Another limitation, in reflection on the conduct of this action research study, was that I did not conduct interviews with specific children or teachers, about their questionnaires or the graffiti walls, to attain a deeper understanding of their answers. Part of my thinking here was to keep anonymity for all participants and so personal information was not kept. In hindsight it would have been possible to retain such personal data for a short period of time in order to conduct interviews and it could have been profitable to interview alongside questionnaires/graffiti walls as then a clearer picture could have been gained with regard to the use of procedural or conceptual approaches as it is sometimes difficult to analyse what strategy has been used. The answers, as they are, do not reflect what someone knows but what they have preferred to use at the time. This does not mean that they only know one way to answer a question but this is the one they have shown. Next time, I would develop my questions further, to take this into consideration, by asking participants to show all possible methods that they know and use to answer a problem.

Another limitation of this study is the fact that within the children's studies, part B and part F, I used random selection. For instance, I did not consider the gender makeup of the groups, nor the mathematical attainment or confidence of the children participating. Differences of attainment across genders have not arisen as a significant issue for the school, hence this may not have been a significant factor, although I cannot know for certain. Given the success of the method of graffiti wall, if I was able to do the study again, I would have got more children doing a wall, in order to cover a range of characteristics.

9.2 Contribution to knowledge

In the 2012 Ofsted publication *Mathematics: made to measure* (p4), Her Majesty's Chief Inspector, Sir Michael Wilshaw was noted as saying that he wants all children to have the best education they can. He commented that he believes mathematics is a fundamental part of this and that it is "essential for everyday life and the understanding of our world". He also commented upon his belief that too many children do not fulfil their potential and that he thinks that those who get off to a poor mathematical start or fall behind in their learning never catch up. I try to reflect this thought in my own teaching so that any gaps are narrowed or filled. This action research study, and in fact the whole reason as to why I actually became a teacher, was to improve the educational experiences for children especially in the subject of mathematics, with specific regard to the operation of division. One of my foci, within this study, is on actions and the empirical improvement for the children and teachers in my school.

With regards to research question 1, how do teachers perceive the changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics?, this study has revealed and confirmed that the teachers interviewed had concerns with regard to the New National Curriculum (2014) implementation and division. In their opinion, through the higher expectations of the New National Curriculum (2014), there were more gaps appearing and misconceptions arising due to the differences in strategies and approaches being used. They also seemed to believe that the lower years would need to develop their curriculum provision further. Despite a higher percentage (67%) of participants than expected, in this study, noting positive thoughts about division, this action research study has confirmed and reinforced the beliefs of researchers Thompson (2003) and Richards (2014) in that division is perceived to be difficult, hard work and frustrating. Anghileri (2006, p366) described division as "the capstone of the primary arithmetic curriculum". I believe the findings of my action research study will contribute significantly to the design of the mathematics curriculum in my school, helping to develop a coherent and consistent experience of division for children, which will lead to an improved and confident understanding of the concept.

With regards to research question 2 and developing an effective progression of concepts and processes in the teaching of division, this study takes further the findings of Thompson (2003) and (2012), Anghileri (2006) and Richards (2014) in that it looks at the numerous strategies and approaches linked to division. My study considers the iterative views of Rittle-Johnson and Schneider (2014) and the model presented by Kilpatrick et al. (2001) of the interwoven strands (see figure 2), looking at the progression of strategies and the connections and relationships between informal and formal methods, and also the links to the other mathematical operations of addition, subtraction and multiplication, so that a greater understanding can be developed.

Anghileri (2006) believed that one of the problems with the teaching of division in schools was the lack of clarity between the informal methods and the standard written methods therefore affecting teacher perceptions of what should be taught and how. This study provided a breakdown of the approaches in order to understand more about how to promote positive views of division. In Anghileri's (2006) research she stated that "Overall, the results show a shift from extensive use of the traditional algorithm in 1998 to more use of informal methods in 2003 and different written methods, particularly in problems with 2-digit divisions where the short division algorithm failed." As a result of my action research study I found that children's methods are now changing again to a mix of approaches. For example, for many children, division of a 2-digit number using the short-written method is now supported by using a partial multiplication list by the side. The analysis of children's SATs questions showed marked changes in typical methods for division over the time of this study. Initial conceptual methods become more efficient and procedural as children move up the school, but there is evidence that recent curriculum approaches have meant that children have maintained both a conceptual understanding of what they are doing as well being able to apply efficient procedures. The marked increase in the use of mixed (conceptual and procedural) approaches in children taught under the National Curriculum for five years (our Year 6 of 2019) is a *new finding* and a hopeful one in relation to the potential for the new curriculum to support children to be successful and confident in what they are doing.

The results of this study build upon the previous research results, by other researchers, to bring to light the possible barriers, such as a lack of times table knowledge and a plethora of strategies to choose from, that children and teachers face when tackling division. This in turn has fed the knowledge of those at the school in order to improve the attainment of children by filling any gaps and offering curriculum development to teachers. This action research study relates ideas in a coherent way and brings the knowledge, through the use of graffiti walls, in an original way. This study also confirms the thoughts of Benson (2014), chapter 4.1, in that it highlights a need for children to become more proficient at the multiplication tables in order to solve division problems as it promotes the use of a mixed method approach when solving division questions and so relating division to its inverse strategy of multiplication.

As a result of this action research study, it was clear there was a need to create a policy/chart that is progressive and builds on the approaches already used by children. My progression chart is a tangible outcome and represents a new distillation of theory and data, drawing from my readings and based on looking at the strategies that children can and do use at different ages, and from teachers' confidence in different approaches, as highlighted by my different forms of data collection. This user-friendly progression document for division fits in with the New National Curriculum (2014) but also suits the needs of individuals. The chart is designed to support the progression of approaches from conceptual to procedural, with the benefits of efficiency this brings, while retaining the all-important conceptual hook or underpinning of what the procedural method is doing. In other words, the chart is designed to support children in moving from conceptual to mixed approaches to division, to have confidence in carrying out procedures, and in what those procedures mean. The chart is in use in my school.

This action research study supports and confirms the views of researchers such as Hiebert and Lefevre (1986), Chick (2003), Rittle-Johnson and Schneider (2014) in that using both procedural and conceptual approaches can be successful in helping children to build an understanding and as a result of this study and the changes made in approaches at the school with regards to division and other mathematical operations, the attainment and confidence has increased. Children now talk positively about mathematics and are able to select an approach which suits the way that they work and enables their understanding. There is evidence, in this study, of this change of approach shown in table 25, chapter 7.3.1., action research cycle 4. In 2017 and 2018 a quarter of the cohort failed to even attempt the question of a 4-digit number divided by a 2-digit number but in 2019 only 10% failed to attempt the question. In 2019, it is also evident that those that did attempt it did so using a mixed approach of a short-written method and a partial multiplication list to support their workings.

9.2.1 Suggested user-friendly progression chart for division after the implementation of New National Curriculum 2014.

When using this suggested progression plan it is important to remember that with some of its elements there can be flexibility. It is necessary to follow through the stages one to two and then three. However, within each stage the informal/formal strategies can be taught in any order to suit the individuals being taught. I view the stages as supporting the parallel and intertwined development of both informal and formal strategies and methods. I think of my own study here, Tutcher (2015) which looked at the need to look at the why before the how or vice versa. The study concluded that it is much better to fit teaching to the individual as what suits one might not suit another. It is important for the children to be able to make connections in their learning so they are able to think deeper and make links across the curriculum. In his work, Benson (2014) confirmed this idea as he commented that there is no one way to be efficient and that what it means to be efficient depends on the learner. The child/learner is the one who must be in control of their learning if they are to be successful. They are the ones who will need to draw on strategies and approaches that they feel comfortable with in order to answer problems when they are alone. So, with this in mind I present a user-friendly progression chart for division that can be used flexibly accordingly to the needs of the child/class. The idea is that concrete, pictorial and abstract approaches can be used at any stage as this takes on the views of Leong et al. (2015) that C.P.A. is effective for those having difficulty with the mathematics. Language use, as stated in NCETM (2019) documents and Department for Education (2020) guidance is also important throughout all of the stages.

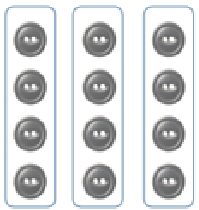
I see this user-friendly progressive chart as being disseminated and used by the school in which the study took place. Then through mathematics network meetings it being rolled out to the rest of our Multi-Academy Trust. Finally, I see it as being shared with other schools through websites and social media platforms.

This progression map uses, supports and extends the work of Anghileri (2006), Thompson (2003) and Richards (2014). It uses additive chunking as this is progressive whereas subtractive chunking is not. Following the release of the Department for Education (2020) *Mathematics guidance: Key Stages One and Two Non-statutory guidance for the national curriculum in England,* I have added supportive representations from sections of the DfE guidance.

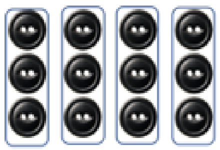
Stage one

Can be carried out with concrete resources and also pictorial representation. Solving one step problems and being aware of the vocabulary grouping and sharing.

Sharing: 12 ÷ 3 =
What is 12 shared between 3?

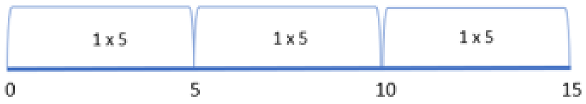


Grouping:
How many groups of 3 are in 12?

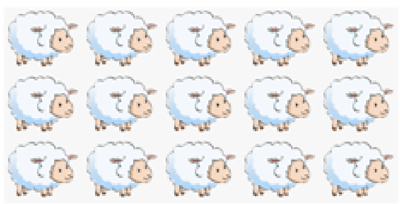


Stage two

Number line
15÷5



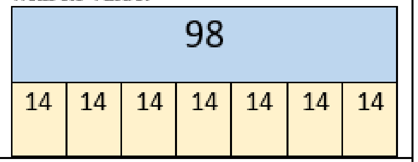
Array 15 ÷ 5



Stage three – informal methods

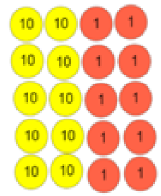
Additive chunking using a number line.
98 ÷ 7

Bar model
98 ÷ 7. The top bar represents the whole (98). The divisor (7) tells us how many equivalent pieces to divide this whole bar into and then we label each piece with its value.



Partitioning
132÷6
136 = 120 + 12
20 groups + 2 groups
Therefore 132 ÷ 6 = 22

132 ÷ 6 place value counters

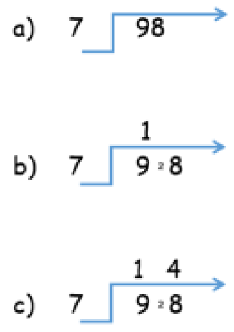
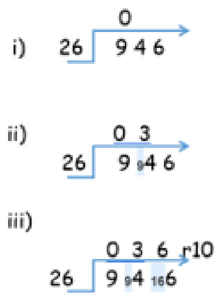
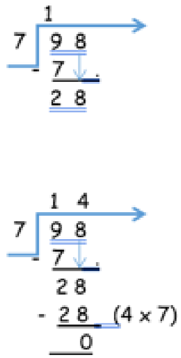
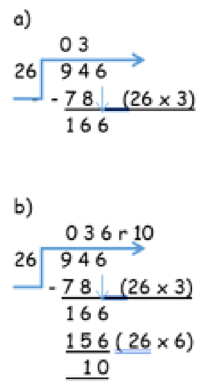


946 ÷ 26
Additive chunking

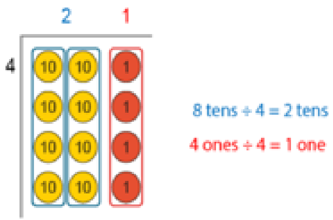
260 x 10
260 x 10
260 x 10
780 = x 30
~~260 10~~
~~1040~~ too big try x5
130 x 5
910
26 x 1
936 = x 36 remainder 10

Stage three

Informal and formal written methods. It is proposed that these strategies/approaches can be completed in any order and at any time to enable the children to be successful but also to enable them to gain a deeper understanding of the mathematics.

Formal written method: short simple / compact 98÷7	Formal written method: short complex For larger numbers to include a mixed approach. 946 ÷ 26	Formal written method: Long 98 ÷ 7 TO ÷ O	Formal written method: Long 946 ÷ 26 HTO ÷ TO
 	 26 x 1 = 26 *Double* 26 x 2 = 52 *Double* 26x 4 = 104 26 x 10 = 260 *Halve* 26 x 5 = 130 26 x 3 = 78 *(x1 + x2)* *Double* 26 x 6 = 156	 	 26 x 1 = 26 *Double* 26 x 2 = 52 *Double* 26x 4 = 104 26 x 10 = 260 *Halve* 26 x 5 = 130

Supportive representations

Short division using place value counters. 84÷4			
 	1 x 26 = 26 2 x 26 = 52 3 x 26 = 78 4 x 26 = 104 5 x 26 = 130 10 x 26 = 260		1 x 26 = 26 2 x 26 = 52 3 x 26 = 78 4 x 26 = 104 5 x 26 = 130 10 x 26 = 260

I am hoping that this action research study will be embraced by teachers and academics and that its contributions will encourage others to be aware of, think about and look at the same issues and concerns. I am pleased with the feeling that this study will aid delivery of a difficult area and will support those that do not have complete confidence in the area. I also feel passionately that this action research study contributes to thinking and knowledge in another way as it is written in a primary teacher friendly manner with vocabulary that seeks to be accessible to all. I want this action research study to be useful to the people with whom I work as it is written in a manner that they will understand and not be overwhelmed by. One of my colleagues commented back in 2013, after reading my Master's dissertation, that it was a pleasure to read as she understood it all. So, as well as making a contribution to academic literature, I want this work to be accessible by those in the spotlight, the teachers. The teachers are the ones who make a difference and if they can understand and use my work then I feel it is a contribution to thinking and knowledge in that way. I see the study as being used as a starting point for discussion and change as well as a resource.

9.2.2 Mini graffiti wall for topic development and reflection

One methodological innovation of this action research study was the development and use of a graffiti wall in the impact cycle, see chapter 7, which allowed for mathematics talk and risk-taking. It can also be seen as similar to or a variation to Haylock's (1984) think-board. Since this study, and as mathematics coordinator, I have developed and implemented the use of a personal graffiti thought wall, see figure 26. These walls/mats allow a child to show their personal thoughts and they allow a mathematical freedom within them in that anything goes. These mats allow for mixed approaches which the study has shown to be valuable for mathematical development and knowledge.

Within the mat/wall in figure 24, which contains a central problem of 12 ÷ 3, a child would be required, in one quarter, to note all their thoughts about the problem. They could write down the vocabulary that comes to mind as well as trying to reflect upon what is being asked of them. The answer is NOT required. In another quarter, the children are asked to show more than one method, such as formal short-written method, chunking or another way, to solve the problem but again no actual answer is needed. In the third quarter, the child is asked to draw what they understand by the question. This could be interpreted in any way that appeals to the child whether it be informal or formal, teddy bears or arrays. The final quarter asks the child to make a connection to a related fact.

On these graffiti thought mats/walls the children could be asked to clarify anything that relates to the central question. The mats are a safe place to demonstrate their thoughts related to the problem and they are encouraged to take risks. The mats allow for teachers to see misconceptions as they arise and they also facilitate mathematics talk in the classroom. They are not a method of assessment and the children are encouraged to show their thoughts in any way that appeals to them. All concrete, pictorial and abstract approaches are encouraged.

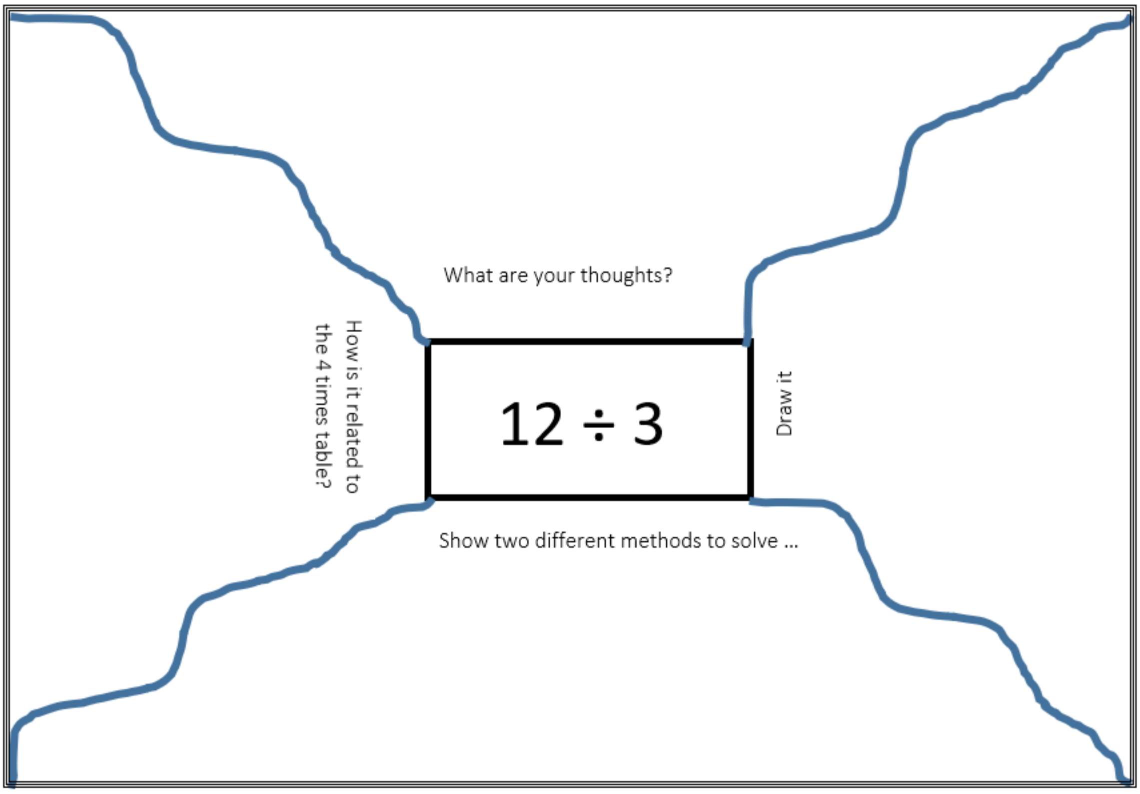


Figure 24: Example of a personal graffiti thought mat.

These graffiti thought mats/walls are currently being developed further to use as an aid/exercise in reading comprehension at the researched school too. They have also been shared with other schools in the Multi-Academy Trust.

9.3 Future practice and research

As an interpretivist/constructivist, the whole point of this action research study has been to understand behaviours and build future practices upon them. This action research study provides a template for the analysis of other concepts in the mathematics curriculum, also aiming to offer a clear and coherent conceptual route of study.

The study has revealed there is a need to ensure that children receive a thorough and varied programme with regards to division. There is a need now to investigate the effects of the use of my progression chart in school. In the future, I am keen to extend this action research study to incorporate and look at parents' views on division. This will enable a triangulation of views across teacher, child and parent so that all are aware of the strategies being taught and feel comfortable with them. Having completed some work, in pilot research, with parents from the school, I am keen to see how a home school exchange can be developed for a more positive experience in mathematics. Currently, I am not an advocate of setting homework as I fear that confusion will arise through parents being unaware of the way strategies and approaches have changed. I want to look into ways of encouraging this triangulation so that everyone is clear and confident with the current practices so that we alleviate any confusion that could arise.

Another possible study, I would be interested in, is to look closer at research question 1 and the perceptions of teachers with regard to curriculum changes. A study using more teachers may give more validity to the results of this study as it would be done on a larger scale. I would be interested to see if the results were replicated when using more teachers. It is quite possible to reproduce the questionnaire used in this study, however, it would not really be pertinent to look at perceived thoughts on the effect of the New National Curriculum (2014) as it is no longer considered new and so a new study would have to take this in to consideration.

Anghileri (2006) in her research noted a gender gap in division. In response to this note, a future study I would be interested in investigating is to see if this gender gap is still a problem or whether it has changed and why. Are there attainment differences across gender? If so, why? This new study could also look more closely at the strategies used by children to see if there is a difference in the choices of method used according to gender. Thompson (2012) spoke about there being a difference between the understanding of an algorithm and the execution of it and this too would give another element to look at in the gender issue.

There are so many possibilities for future research, for me, the aim of my next steps in research in the area of division will be to suggest ways to teach it so that the myth of it being difficult is dispersed and people begin to talk about it like they do addition - in a positive way.

I wonder if the stigma that division has will ever be eradicated? Will people ever admit to liking long division and will the government ever be happy with our teaching of the operation?

9.4 Personal reflections

The design of this action research study was based around my interpretative and constructivist views which build on the feelings and beliefs of the teachers and children involved in the study. It was intended that the design and outcomes would bring to light and develop opportunities for teachers and children to enable them to engage and become more confident in the area of division. My aim was not just to contribute to the body of knowledge relating to the teaching of division in primary schools in England, but to develop my own practice and that of my colleagues.

This action research study confirmed, as literature by Thompson (2003) and Richards (2014) suggests, that some people have negative feelings when teaching and learning all the varying strategies concerned with division. The study highlights that teachers often feel confident and happy with the basic elements of grouping and sharing. It seems, though, for some that as the difficulty increases within the operation of division so the confidence in teaching the area is diminished and lost. I have learnt from the study that using simple procedures and elements of fluency are where teachers felt happy. However, when problem solving and reasoning was required, teachers were not so confident with developing the necessary talk and discussion for the required depth in learning. This in turn meant that the children lost out in this area and became unable to make connections and discuss their learning further. The study confirmed the work of Thompson (2003) and Anghileri (2006) in that methods such as ‘chunking’ and ‘long division’ are where problems arise for some children and teachers.

Whilst working through this action research study, I have found that the children and most teachers involved with the work were keen to engage with discussion and the tasks they were given and they were keen to do well and help. Those who were initially resistant to mathematics and to division began to develop their own passion for the subject.

Through this action research study, I have thought about and developed my own practice with regards to mathematics but especially in the area of division. I have developed the discussion element in my classroom and engaged in a culture where the children feel safe to ask questions. It has been fulfilling to watch previously low attaining and children with SEND achieve and succeed in the division elements in the classroom in general and also in tests. I observed the value of daily practice and talking about the mathematics openly and embracing misconceptions as “Marvellous mistakes”, that we all learnt from. I have embraced the idea of Boaler (2015) when she says all children are born with the capacity to do mathematics but it is the way that it is taught to them that affects their confidence and attainment. This is an idea I have shared with colleagues and, as a school establishment, this is something we are now working with. Although I have not researched/documented my own development, as such, through this study, I know that my approaches have changed as have my skills to draw more information from a child so that they develop their own explanations and knowledge rather than just being told. I now encourage them to find and use ways that suit how they learn rather than dictating a set style.

In this action research study, I have embraced these thoughts and taken on Anghileri’s (2006) idea where she said it is important for a child’s strategies to be progressive so that they can become efficient. In this study, I have presented ideas that are in tune with a child’s already established work patterns and I have furthered the findings from the previous studies of Thompson (2003), Anghileri (2006) and Richards (2014) to create a progressive and user-friendly format for teaching division.

Through completing this action research study, I have found that a mix of approaches, both procedural and conceptual, gives meaning to the mathematics and I reflect upon Haylock’s (2006) words that the best person to teach something is the one who understands and enjoys it. I believe that the teacher is indeed an important factor in the learning of mathematics. The practices they use and the clarity of delivery of them is essential in order to avoid and conquer misconceptions. I believe that teacher knowledge is important because without the knowledge and competence, efficient strategies and understanding cannot be taught effectively and efficiently. From this learning, I now begin to ask questions such as how can effective approaches be taught to those studying to become teachers, as well as those already qualified, so as to ensure children get the best start to their school mathematics? How can we ensure teachers develop approaches that they believe are effective for the children in their class? How can we enable all teachers to feel confident with the numerous procedures and concepts so that they can instantly alter their teaching according to the needs of their class at the time rather than just teaching the basic fluency elements?

9.5 Challenges faced during the study

I was promoted to mathematics coordinator, in the researched school, whilst carrying out this action research study and so, in a way, the study became part of my role. The main challenge, however, was finding time to engage the teachers at the school as time is such a precious commodity in a school environment. There are so many other elements to the curriculum in order for children to be given the rounded schooling that is required. I had to ensure that at every point I asked the teachers to engage with tasks, that they were to the point and also relevant to them at their stage of development. At one stage, a group of teachers who were asked to take part in the focus groups declined to meet with me as they had marking and planning to do. This particular group of teachers, I perceive as focused and work driven. I had to jump through many hurdles and ask for help to actually get them together and onboard. In the end, I promised them that the conference/focus group would take no longer than ten minutes. They agreed to meet me but only with a timer set for exactly ten minutes. This was agreed and followed. The conversation, thinking of the words of Bogdan and Biklen (2007) began by getting right down to the problem and the discussion that followed turned out to be a valuable one for all and they seemed to enjoy it too. Due to the fact that I was recording the conversation and transcribing it afterwards, the actual discourse took just over seven minutes.

This was seen as a positive thing and the teachers in that group were more open to discussion and questions afterwards.

Another challenge that I faced whilst completing this study was being able to juggle my own time. As a full-time teacher and part-time farmer, I had to prioritise and think about the children in my class as well as studying and tending to my animals. It was a challenge in that I had to manipulate the time in order to fulfil the duties of a full-time teacher of marking, planning, meeting parents etc. with farming and also with researching, presenting and attending tutorials. A way I overcame this was to speak to the senior leaders at the school and negotiate study days, also ensuring that my school holidays were used proactively for studying. In term time, I concentrated on my planning, marking and assessment for school as well as obviously teaching a demanding Year 6 curriculum. On a positive note, in order to overcome the challenge of writing up the study, I was also able to negotiate less responsibilities and a change of year group to Year 5 so that I did not have the challenges that come with SATs (Standard Assessment Tests). The farming element carried on as usual as the animals would not have understood the changes.

This time element was also a challenge when it came to attending conferences and presenting my work. Again, to overcome this challenge I was careful with my time and made sure that I attended a minimum of one a year. I also took every opportunity I could to talk about and present my work to colleagues in staff meetings as well as with teachers from across the Multi Academy Trust when attending hub meetings.

The final challenge to be considered was more of a personal one. Nerves! I know that I am good at what I do in my classroom but taking a risk and doing this action research study has made me leap out of my comfort zone. I had to take extra care and time to comprehend and interpret what others had to say in my reading, in my interactions at conferences, and in the conduct of research when gathering and analysing data.

9.6 Addendum – other actions that have taken place as a result of this action research study

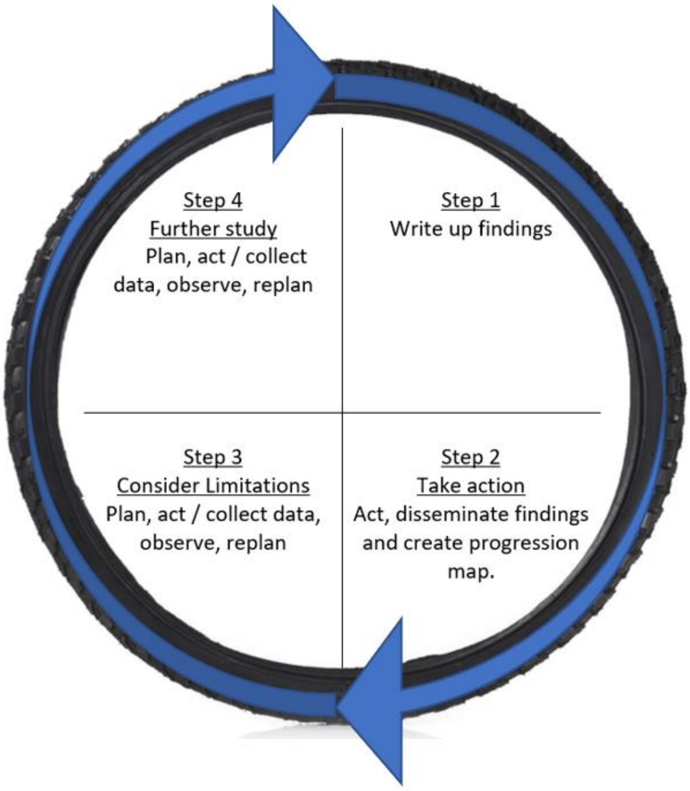


Figure 25: Wheels of action.

Figure 25 shows different steps that have subsequently happened and developed whilst the findings were collated and written up in for this thesis. The following paragraphs outline the actions that have occurred which were beyond the remit of this thesis.

One pleasing and unexpected impact of my research in my school has been some of the other teachers pursuing some of their own data collection. For instance, following the Year 6 focus group, one of the teachers planned a lesson with their class to ask the children in their class what they perceived the advantages and disadvantages of certain methods, the formal short-written method and chunking, used in division to be. The results of which they shared with me too – see figures 26, 27 and 28.

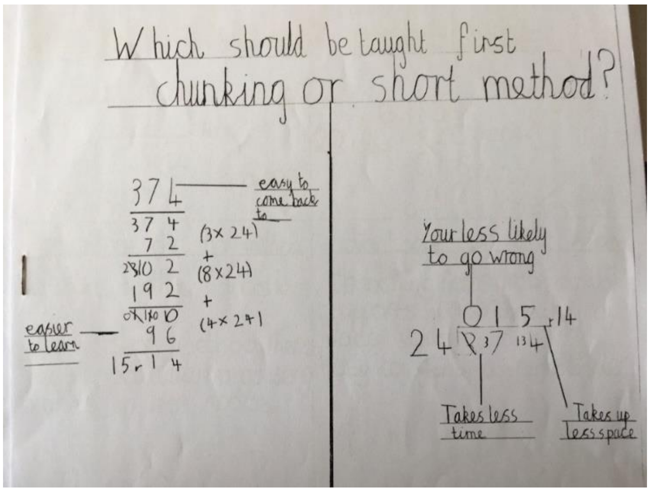
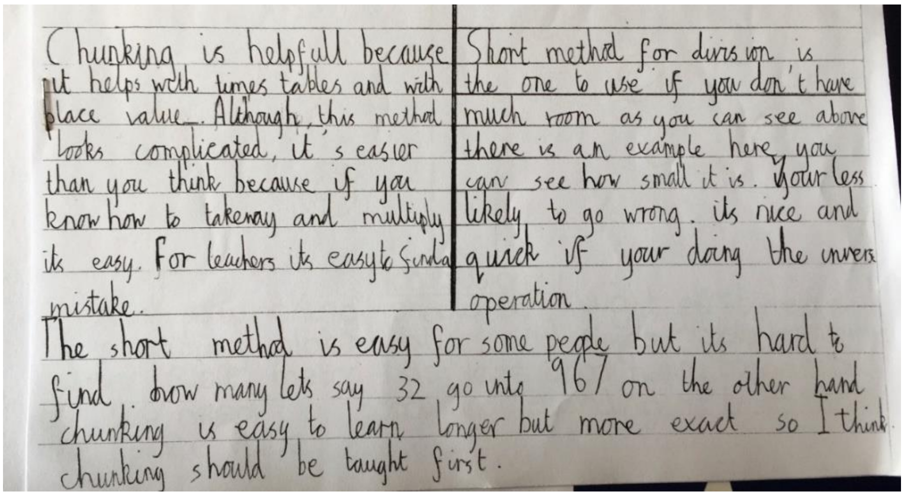


Figure 26: Example 1 of how a Year 6 child compared methods of division.


Chunking is helpfull because it helps with times tables and with place value. Although, this method looks complicated, it's easier than you think because if you know how to takeway and multiply its easy. For teachers its easy to find a mistake.

Short method for division is the one to use if you don't have much room as you can see above there is an example here, you can see how small it is. Your less likely to go wrong. its nice and quick if your doing the invers operation.

The short method is easy for some people but its hard to find. how many lets say 32 go into 967 on the other hand chunking is easy to learn longer but more exact so I think chunking should be taught first.

Figure 27: Example 2 of how a Year 6 child compared methods of division.

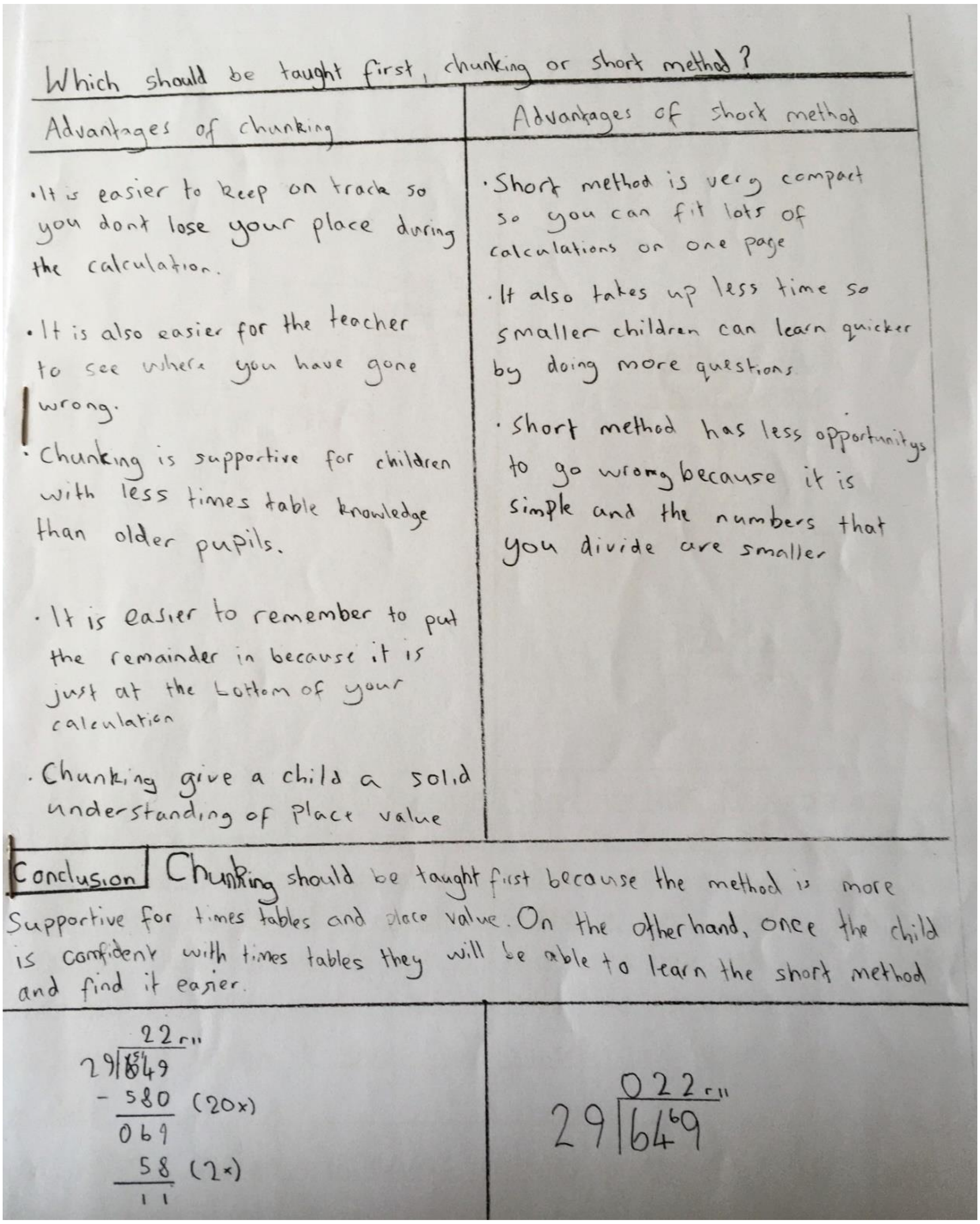


Figure 28: Example 3 of how a Year 6 child compared methods of division.

I find the work demonstrated in figures 26, 27 and 28 adds weight to the work of Thompson (2012) in that it gives a child's view on the different algorithms of chunking and short division. Figure 27, in a way, supports Thompson's belief and also Anghileri's (2006) thoughts that some children may not be able to use short division for dividing by a 2-digit number as one of the children talks about this as being a possible difficulty. The children, in figures 26, 27 and 28, give the view that chunking, although looking complicated, gives a structure of support for the lower attainers and they say that it is easier to keep track of where you are and for the teacher to see errors. Thompson (2003, p21) would agree with the "differentiability" element, however, he sees a negative side to chunking in that it requires an "understanding of place value and the principles of exchange". However, what is most significant to me, is that another teacher was inspired by my action research to undertake actions of their own aimed at developing a deeper understanding of children's views of division.

An action, in response to this action research study, is that the school now uses more mathematical talk in classrooms. We use stem sentences to promote the concepts we teach. I am a firm believer that if you can articulate something then it is more likely to be remembered and understood. Therefore, in response to the teachers' thoughts about conceptual approaches being advantageous and not being worried about time constraints, I have promoted the use of stem sentences that encourage the children to talk about their mathematics, a practice also mentioned in the NCETM spines. The children are given the start of a sentence such as "the main idea in this question is that… and so I know that…". The children then fill in the blanks demonstrating their understanding and showing the teacher where time needs to be spent to support and further the learning. Although trialled in just my classes, this approach is now being implemented in Reception classes all the way through to Year 6. An action that has also been put it place through the mathematics talk and reasoning elements is that children are exposed to these more complex problems in order to talk about solutions and taking a risk – this is still very much a work in progress.

In response to evidence gained and related literature, as a school we have used my progression chart as the basis for looking more closely at the strategies used within division and the misconceptions that arise. Staff meetings have been used in order to address these misconceptions and use them as learning points. We have also discussed, extending the work of Anghileri (2006), Thompson (2012) and Richards (2014), the need to and the benefits of altering our practice so that the structure is indeed more progressive and builds on making connections and seeing relationships between the different methods. In addition to this, we have discussed the need to embrace concrete, pictorial and abstract approaches to support children's learning and understanding and, in my progression chart, I propose the need for *all* children to use concrete, pictorial and abstract approaches.

This research listens to the perceptions of teachers, those at the pivotal point in a child's education, and therefore it moves knowledge and skills forward in the researched school due to its reflective nature and the fact that it is a study of that school and its needs. As mathematics co-ordinator at the school, as an action in response to teachers' worries about strategies and approaches, I have led workshops and supported teachers with regard to the teaching and learning of approaches in division in order to increase their confidence with all the strategies taught. I have also actioned and promoted the idea, supporting the thought of Boaler (2015), that *all* can do mathematics and therefore we as teachers must be positive in our thoughts in front of the children and show them it does not matter if one makes mistakes, since it is completely normal to make these errors and it is another way we learn. It is what we do with these errors that moves our learning forward. And what is true about learning from errors in mathematics also resonates for me as a researcher. I have detailed some of the mistakes I made and the limitations of this study; I have learnt through reflecting on these and am excited to take the next step of my research journey, building further on the findings of this thesis.

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Appendix 1

GSoE research ethics form 2014

gsoe-ethics@bristol.ac.uk

Name(s): Rachel Tutcher

Proposed research project: **An investigation into the implementation of the new National Curriculum in Mathematics and the changes and effects it provokes both at school and in the home.**

Proposed funder(s): Rachel Tutcher

Discussant for the ethics meeting: Laurinda Brown

Name of supervisor: Laurinda Brown / Alf Coles

Has your supervisor seen this submitted draft of your ethics application? Y

Please include an outline of the project or append a short (1 page) summary:

The study is based around the work of the late Martin Hughes.

My research will look at the changes implemented in the Mathematics National Curriculum over the last three decades but will focus mainly on current reforms. The main objectives of the study will be to look at the effects (positives and negatives) that will emerge in the primary school where I work. Finally, it is intended that the research will open lines of communication between teachers, parents and children to ensure a consistency in teaching methods and understanding of the mathematical procedures and concepts that the children need to enhance their attainment during their primary years.

This ethics form is for a pilot study. It is based around parents and their ability to support their children in the basic four number operations of addition, subtraction, multiplication and division. I envisage asking for a group of approximately 12 parents (invited – but voluntary participation) to take part in a relaxed evening of mathematics (two hours). The evening will have two parts- a questionnaire and also teaching of the mathematical strategies used in the school. This is in order for the parents to gain something from the session rather than just answering questions.

Ethical issues discussed and decisions taken (see list of prompts overleaf):

1. Researcher access / exit. RT already works at school where study is to take place and working with parents in this way is also helping toward SDP in mathematics.
2. Participants were invited to take part – voluntary acceptance. They were informed of all procedures and could withdraw at any time or refuse to comment on particular areas.
3. All participants' answers are anonymous and confidentiality kept.
4. Data stored securely in locked cabinet.
5. Research findings will be reported to school and also used for possible journal articles as well as basis for PhD study.
6. Headteacher involved in all steps and permissions granted for this pilot study: Headteacher would like similar groups to be offered to whole parent body of school in future.

If you feel you need to discuss any issue further, or to highlight difficulties, please contact the GSoE's ethics co-ordinators who will suggest possible ways forward.

Signed: RTutcher (Researcher) Signed: L. Brown (Discussant)

Date: February 2014

2014 APPROVAL e-message

- **Gsoe Ethics Mailbox** <gsoe-ethics@bristol.ac.uk>

  [redacted]

  Wed, 26 Feb 2014 at 14:40

  Thank you for submitting your Ethics form. You have completed the GSOE Ethics procedures. If you have any questions or queries please write directly to the GSOE Ethics Co-ordinators Wan (Wan.Yee@bristol.ac.uk) or Frances (Frances.Giampapa@Bristol.ac.uk) Please remember to keep an open discussion with your supervisor.

GSoE RESEARCH ETHICS FORM

Name(s): Rachel Tutcher

Proposed research project: An investigation into the implementation of the new NC in mathematics in one primary school and the recent changes and effects it provokes on children's learning with a focus on the division algorithm.

Proposed funder(s): Self

Discussant for the ethics meeting: Laurinda Brown

Name of supervisors: Laurinda Brown and Alf Coles

Has your supervisor seen this submitted draft of your ethics application? Yes

Please include an outline of the project or append a short (1 page) summary:

The research questions formed are:

Q1. How do teachers perceive the recent changes in the 2014 new National Curriculum as affecting their teaching of division in primary mathematics?

Q2. What are the advantages and disadvantages of teaching conceptual and procedural strategies in relation to the division algorithm?

My research will start by briefly looking at the changes implemented in the Mathematics National Curriculum over the last three decades. It will then focus in more detail on current reforms and how they might affect the way teachers teach – will they need to change their past practices and why might this happen?

The main objective of the study will be to look at the effects (positives and negatives) that will emerge in the primary school where I work. It will then suggest ways that we can move forward effectively so as to ensure a consistency in teaching methods and an understanding of the mathematical procedures and concepts needed so that the children can enhance their attainment during their primary years.

Both staff and children at the school will be involved in the study in the mediums of interview, focus groups and in-depth studies of their work relating to division. All staff (10) in the junior part of the school will be asked to take part but participation is voluntary. All children on roll (254) will participate in the initial questionnaires but focus groups and in-depth studies will be on a random selection of 24 children (approximately 10% / three children from each class) of whom all will remain anonymous. These children will have the option to remove themselves from the study if they do not wish to be a part of it and another child will replace them.

The second part of the study will be based around conceptual and procedural approaches in regards to tackling division algorithms. I will be talking to staff about what methods they use and why they use/prefer these methods. I will then work with the children on the same ideas but look at which strategies they prefer to use and why. I will investigate which they naturally navigate toward and what is their reasoning behind this – I will do this through the use of SATs analysis and also think boards and questioning.

Ethical issues discussed and decisions taken:

BERA Ethical guidelines will be consulted and considered throughout the research.

1. Researcher access / exit. RT already works at school where study is to take place and working with staff and children in this way is also helping toward work with her mathematics lead.

2. Information given to participants: Participants will be fully informed all the way through the research and will know what the purpose of each stage is and what it will entail.

3. Participants right of withdrawal: All those taking part will have been invited to take part – voluntary acceptance. They will be informed of all procedures and could withdraw at any time or refuse to comment on particular areas.

4. Informed consent: Participants will need to confirm that they are willing to take part in the relevant parts of the study. They will be kept informed all the way through all processes so that they can withdraw if they want to – they will know that participation is kept anonymous.

5. Complaints procedure: It is not foreseen that there will be any complaints but in case, participants can inform researcher or the head of school that they are not happy and can withdraw from their part of the research. The tutor's e-address will also be included on written information in case of need.

6. Safety and well-being of participants/ researchers: It is not foreseen that there will be any safety issues with this study as it will be as part of mathematical routines in school and so in safe environments. With regard to well-being, participants will not be overloaded in any of the various parts of the research, which will be dispersed over a large time scale with added time to reflect on participation and also feedback.

7. Anonymity / confidentiality: All participants' answers are anonymous and confidentiality kept. Any work looked into will be nameless and therefore only information needed for the research will be used.

8. Data collection: Interviews / focus groups will take place informally. Questionnaires will be given out by class teachers and remain anonymous – to be completed in class time. Questionnaires to staff will be anonymous, put in pigeon holes and then placed in researcher's pigeon holes – no names – they will be kept minimal and to the point as teachers will complete as part of their school time.

9. Data analysis: Only the researcher will analyse the data. Data may be used for journals and papers presented to BSRLM, Primary Mathematics and PME groups but names of participants will remain anonymous.

10. Data storage: Data stored securely in locked cabinet and on encrypted memory sticks and laptops. Data will be only be kept until research completion.

11. Data Protection act: Only the researcher can access the computer and encrypted memory stick where the research data is held as passwords protect others from accessing. No personal data will be held as this is not necessary for the study.

12. Feedback: Research findings will be reported to school and also used for possible journal articles as well as basis for PhD study.

13. Responsibilities to colleagues / academic community: Participants will be respected and will be free to offer criticism and ideas in staff meetings and informal meetings. All of these ideas/ criticisms will be reflected upon. Any tensions that may arise will be considered as they happen but this is not expected as the research is backed by the SLT and there is an openness regarding the study and no deception.

14. Reporting of research: All participants will have access to the final study and can ask questions at any time along the way. A full thesis will be presented to the school on completion.

If you feel you need to discuss any issue further, or to highlight difficulties, please contact the GSoE's ethics co-ordinators who will suggest possible ways forward.

Signed: [redacted] (Researcher)

Signed: [redacted] (Discussant)

Date: 9th March 2018

Supporting evidence for Ethics committee:

1) **Email from Headteacher. Evidence of Informed Consent** (Information letter for parents/participants; verbal assent for vulnerable populations; where consent is not being acquired, evidence that the institution/school approves the research)

2) **Example of study materials.** A list of questionnaires, sample of questions that will be asked, sample of instructions (as relevant)

Staff:

Staff will be invited to an informal discussion with RH where two questions will be asked. The discussion will be taped on an audio Dictaphone. Staff will attend voluntarily and will remain anonymous – this will all be clear both on invitation note and also before the discussion commences. They will also be made aware of the purpose of the questions i.e. my study. The questions to be discussed are:

How do you perceive the recent changes in the 2014 curriculum will affect your teaching of the division algorithm?

What are the advantages and disadvantages of teaching with conceptual and procedural methods in relation to division?

Children:

Will be given a questionnaire (see appendix 2) that also fits in with RH's role of Math Co-ordinator at the school – RH with co-ordinator head on will be looking at progression through the years, times table practice, homework and child confidence. As a researcher RH will be looking deeper than this – strategies used, procedural and conceptual approaches etc.

The questionnaire will be given to every child in the school. No names will be assigned to the questionnaires and so will remain anonymous. The head teacher and class teachers are aware of the study and will tell the children that RH is looking at ways we solve division problems and so through this they will try to show their best thinking.

2014 APPROVAL e-message

- **From:** Amanda Williams <a.williams@bristol.ac.uk>
  To: Rachel [redacted]

  Sent: Monday, 14 May 2018, 09:33:37 BST

  Subject: RE: Following Up on Ethics

  Hi Rachel, Sorry for delay in response. Since your project has been previously approved, I advise that collecting opt-in from parents and students to use SAT scores in this manner would be the ideal. But since part of an ongoing project, is not absolutely required. Proceed as you see fit.

  Amanda

Appendix 2

Division questionnaire

Year Band 3 4 5 6 (Please circle which one you are in)

1. Do you like maths? Yes No
2. Do you like division? Yes No
3. Do you think you are good at division? Yes No

4. If you could write a dictionary definition about division, what would it say?..

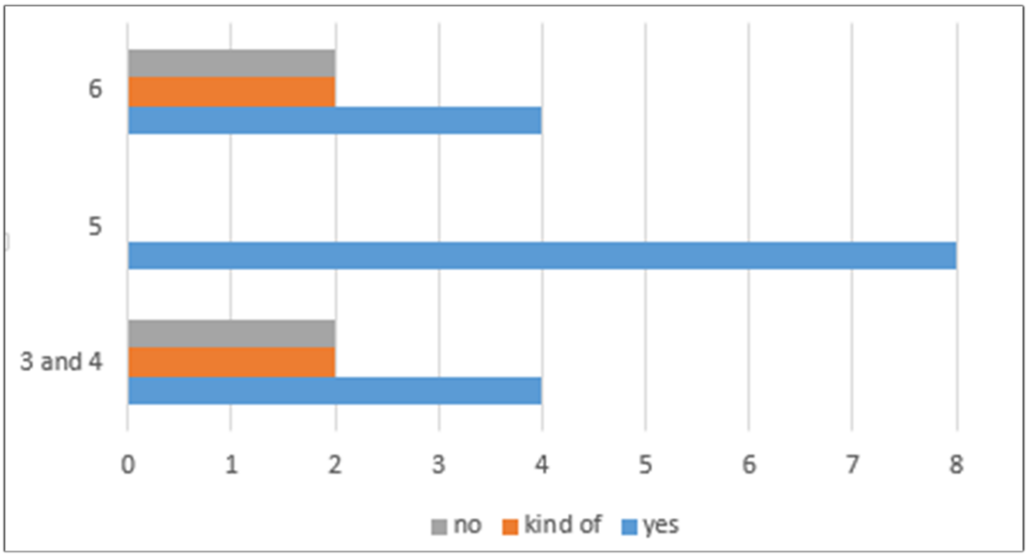
Can you answer the questions below … **show your workings or explain the thoughts in your head**.

Question 1 **12 ÷ 3 =**	**Question 2** **95 ÷ 5 =**
Question 3 **275 ÷ 25**	**Question 4** **5,542 ÷ 17 = 326** **Explain how you can use this fact to help you solve 18 x 326**

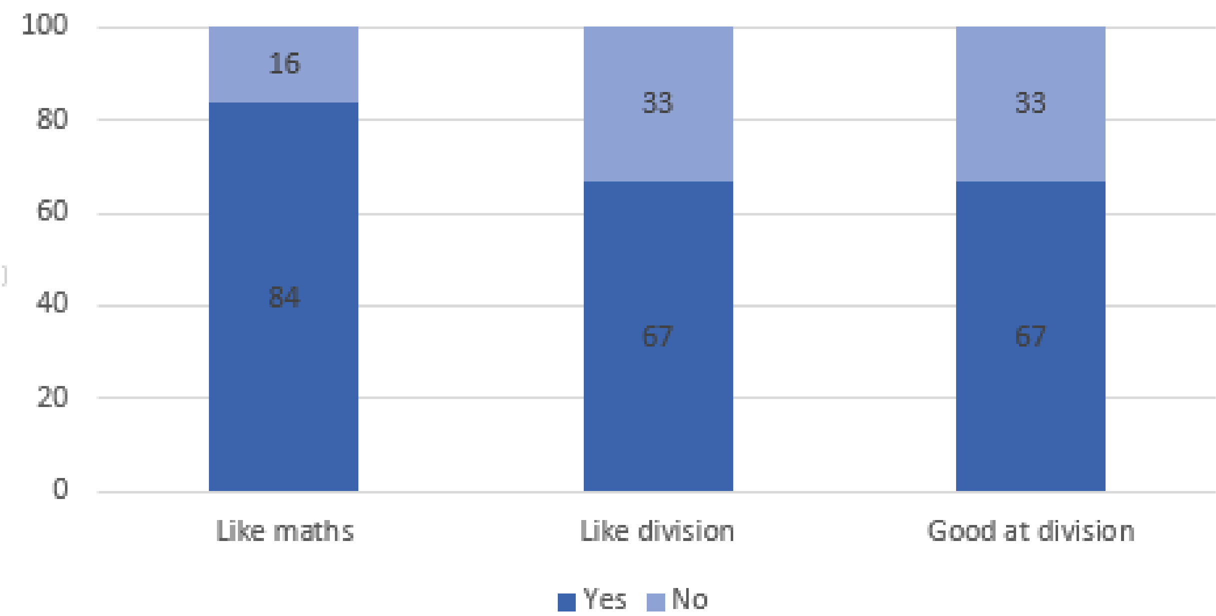
Appendix 3

Children's focus groups and questionnaire

1) Conference results of Question 1 – do you like division?

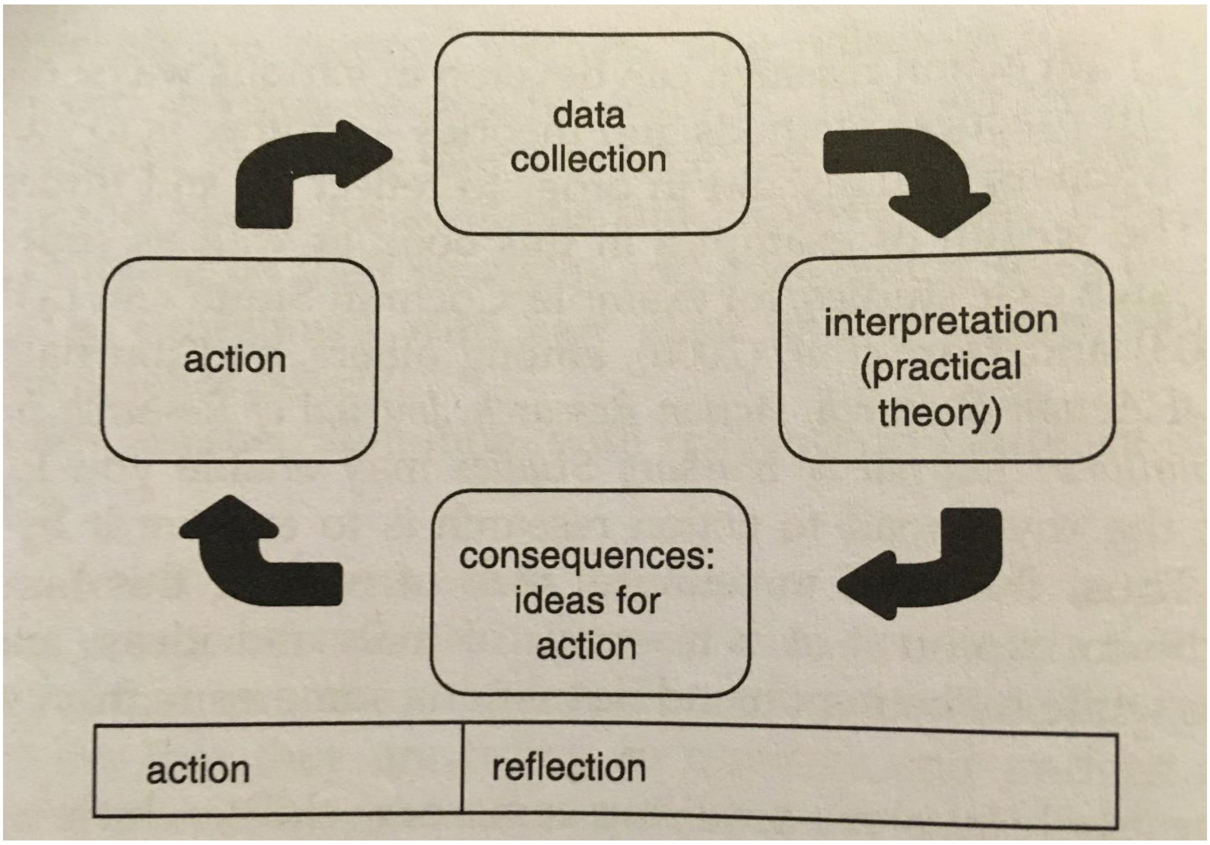


2) Findings related to children's answers to questions 1 – 3 on the questionnaire (230 children). *The numbers on the bars relate to the percentage of response, whether yes or no, not the numbers of participants.*



Appendix 4

Altrichter et al. (2008) idea of a circle of action and reflection (p8)



Appendix 5

The progression of strategies in division at KS1 and KS2

<table>
<tr><th></th><th colspan="3">The New National Curriculum (2014) requirements</th><th>NCETM</th></tr>
<tr><td rowspan="2">KS1</td><td rowspan="2">At Key Stage one, the principle focus of the mathematics curriculum is to make certain that all children develop their confidence and their fluency.

At Key Stage one, which is year one (age 5-6 years) and year two (age 6-7 years), children must make connections between multiplication and division through grouping and sharing.</td><td>Year 1</td><td>Solve one-step problems using division.

A one-step problem is one where you can solve a problem with one calculation only, for example, eight animals went into the ark in pairs, how many pairs were there? (8 ÷ 2 = 4).

Children should have access to and be using concrete objects, pictorial representations and arrays.</td><td>The NCETM also suggest that children should count efficiently in groups using skip counting (2, 4, 6, …; 5, 10, 15, …) - this must be done in groups of a given number (2, 5 or 10).

NCETM (2019, p2), 2.1 Counting, unitising and coins, a teacher guide, they note that their work prepares children for future work in division.</td></tr>
<tr><td>Year 2</td><td>Children should be able to solve problems involving division, using materials, arrays, repeated addition, mental methods, and division facts, including problems in contexts.

Children should begin to show and see more links with the times tables.

Children should begin to make connections to appropriate real-life scenarios.

Children should still have access to and use concrete resources, pictorial representations and abstract number sentences.

The New National Curriculum (2014) stipulates that children should be shown that the multiplication of numbers can be completed in any order but in division this is not the case.

Representations that children can be introduced to are number lines and arrays.</td><td>NCETM note the point as multiplication is commutative (a x b = b x a); division is not commutative.

The NCETM begin to introduce alternative vocabulary at this point.

- quotitive division - where the total quantity, the dividend, and the group size, the divisor, are known but the number of groups, the quotient, is calculated - related to grouping.

- partitive division – the total quantity, the dividend, and the number we are sharing between, the divisor, are known but the size of the shares, the quotient, need to be calculated - related to sharing.</td></tr>
</table>

	New National Curriculum (2014) requirements			NCETM
LKS2	At Lower Key Stage two, which is year three (age 7-8 years) and year four (age 8-9 years), children will become increasingly fluent in division and they will start to develop their use of efficient written approaches and mental strategies.	**Year 3**	**Children of this age group should be taught to write and calculate mathematical statements for division using the multiplication tables that they know.** *This would mean a two-digit number divided by a single digit number, for example 60 ÷ 3 = 20.* **Children should progress to using formal written methods in order to solve division problems.** Further to this, the children are required to be able to solve associated missing number problems involving division. A non-statutory requirement of the New National Curriculum (2014), at this level, is that the children should start to develop reliable written methods for division.	As per New National Curriculum (2014) but stick with informal methods.
		Year 4	**Children should recall division facts for all the times tables up to 12 x 12.** *It is noted that they should be able to do this mentally.* Children should be able to solve division questions in more efficient steps. Although not a statutory requirement, children should be introduced to an informal method at this point. Once skills have been developed and understood, children should move on to calculations that include remainders. Children should also be encouraged to move onto standard formal written methods of division such as short division.	As per New National Curriculum (2014) but stick with informal methods Answers to a division calculation should be interpreted carefully in order to make sense of the answer and the remainder. NCETM suggestions, for progression, work on an understanding of the structure of the algorithm. Language plays an important part in the understanding of the operation.

	New National Curriculum (2014) requirements			NCETM
UKS2	Children in Upper Key Stage two, Year 5 (age 9-10 years) and Year 6 (age 10-11 years) should extend their understanding of mathematical areas and develop connections within the mathematics that they encounter.	**Year 5**	**Children should be able to divide numbers which are up to four digits by a one-digit number using the formal written method of short division.** *Within this requirement, the children need to be able to interpret remainders appropriately for the context of the problem.*	As per New National Curriculum (2014) but stick with informal methods.
			Children should be able to divide numbers mentally drawing on facts that are known to them.	The NCETM suggests that a key focus is to enable children to gain a deep understanding of the underlying mathematics rather than learning by rote or just procedure.
			Children at this level/stage are also required to divide whole numbers and numbers involving decimals by 10, 100 and 1000.	
			A non-statutory requirement for Year 5 is that children should be able to interpret non-whole number answers in division by expressing their results in different ways.	The NCETM also note that at this level children should be able to make sensible choices about strategies and representations that they use.
		Year 6	**Children are required to divide numbers of up to four digits by a two-digit whole number using the formal written method of long division.**	The NCETM are very clear that their progression spines and segments promote a conceptual understanding.
			Children should also be able to interpret any remainders as whole number remainders, fractions, or as a decimal as appropriate for the context.	
			Divide numbers of up to four digits by a two-digit number using the formal written method of short division.	

Appendix 6

Key for thematic analysis of focus groups	
Theme 1	Methods / written methods / changes – *colour coded – red text.*
Theme 2	Different content/ level of difficulty– *colour coded – purple text.*
Theme 3	Catch up / gaps. *colour coded – orange text*
Theme 4	Method over understanding– *colour coded – black underlined text.*
Theme 5	Other– *colour coded – green text.*
Theme 6	Advantages of conceptual strategies– *colour coded – yellow highlight.*
Theme 7	Disadvantages of conceptual strategies– *colour coded – beige highlight.*
Theme 8	Advantages of procedural strategies– *colour coded – light blue highlight.*
Theme 9	Disadvantages of procedural strategies– *colour coded –blue highlight.*
Theme 10	Needs and other– *colour coded – grey highlight.*

Transcript 1: Year 3 and 4 staff focus group

I: The first question is – how do you perceive the recent changes in the 2014 curriculum will affect your teaching of the division algorithm?

T1 – I've only taught this curriculum because I'm an NQT.
T2: I think it's had a relatively small impact, greater focus on fractions, perhaps the biggest impact is that we are now applying things perhaps too high too soon
T3 – um well there's more to it than there had been in previous years, like what was once in higher year groups has now come down um so we have to build in more gaps ready for future years.
T4: Do you mean that we have to go to formal methods quicker? In division quicker – with younger children
T3: There's a lot of new content, for example finding the effect of a 1- or 2-digit number by 10 and 100 – that use to be in yr4.
T5: and are the times tables lower down now
T1: yes, we have to know it by the end of Year 4 and that will have an impact too.
T4: yes, it seems it has come down in years.
T3: yes, so the expectations are higher
I – so how do you think that is going to affect your teaching?
T4: It means that some children that are younger are going to go through the loops that older children had to do. So as teachers, even though we are under pressure to reach formal methods, we have to go through the steps that are age appropriate for their thinking so they have got that concrete before we move on to that sort of stage. So, if we haven't put the steps in before sufficiently then we fail them any way
T3: so there is a wider differentiation, so we have to cover what we normally do and more and also stretch
T1: so surely the first year it came in there must have been a huge gap
T3: I think we are still catching up
T1: ummm , so still catching up now
T2: I think we will manage better next year.
T4: Yes, it's going to take a few years. Does that answer your question?
I: sort of, I wondered if it would change your teaching in any way.
T5: so, do you mean put in more practical
I: possibly
T4: so yes, we'd put in more practical concept steps so we can take them on to that formal method
T1: so yes, instead of just seeing numbers on a board actually handling the objects themselves and separating them out
I: so, making sure that you're perhaps doing more of the Key Stage One elements to get that base but also pushing for the KS2
T5: so, there's more coverage
T4: so, for division we would start it lower down still, so we would do both the concrete and abstract. I think we are under pressure as there are the steps for writing it down as well as the physical steps.

I: Ok, so what are the advantages and disadvantages of teaching with conceptual and procedural methods in relation to division?
T2: procedural is to teach a method to achieve an end result – this can work if practised enough and can build confidence but if a child is unsuccessful it leads you to wonder where to go. Conceptual is related to understanding how. Has potential for a deeper understanding – good but some children are not ready to grasp this. There needs to be a balance – conceptual steps are good but if a child is struggling then they still need to have approach skills in place – this should be used when the child is ready to take it on board. Most children absorb via doing and daily exposure – so no need to necessarily understand in order to do – so we need to explain division or just do it mechanically – I personally like to think we offer the opportunity to understand but provide the skills to succeed.
T4: maybe it allows more opportunities to talk through the conceptual before they go to the procedural written formal method – it's marrying the gap that needs closing.
T3: like we did on that sheet where we had all the different stages – where they were able to look at the fact that this is all the same thing but there are different ways to access it
T1: I think it's different for every single child – some will be able to see 12 divided by 3 and think yes – I know how to do that and can do an array or a sharing pattern whereas some just understand it anyway. They know what it means. So for those that technically don't get it, they can still get the same answer without understanding it.
T4: Division is a tricky one.
I: so let's talk about chunking. Chunking can be taught procedurally but it can also be taught conceptually. Conceptually is where they take numerous steps in order to solve it whereas procedural us where they must follow rules and only take 2 steps. What would be the advantages or disadvantages of that?
T5: They can't apply it if they've been taught procedurally whereas conceptually they can. If they are told they can only take two steps then they will always look for those two steps regardless of the problem, and they wouldn't necessarily understand what they were doing it for.
I: so you're saying the advantage of the conceptual approach is to gain a better understanding?
T5: It's more of a transferable skill I would have said. When they've got it in their head, the why then they can take it to another situation. However, there are some children in my opinion need just to know the how.
T1: yeah
I: when you say some children – what do you mean?
T5: I mean children with perhaps the inability to understand number – they just need to know the fact. They need to know that 12 divided by 4 is 3 and perhaps just stick with that.
T1: I think a disadvantage of procedural is that you're not really teaching them to understand. You're just telling them you do this, this and this rather than giving them an understanding so as they move up the school it would be a greater knowledge base and understanding of what they are actually doing rather than just trying to get the answer – it may take more time though.
T4: I would go as far to say that they should both be taught at the same time, doing the conceptual and showing them how the procedural relates to it – how it transfers.
T1: Quite often children focus on just getting the right answer – they want to see ticks. They don't want to see = can you explain why?
T5 – that's right – they do
T4: but they can get hung up on the procedure
T5: Going back to chunking, you need a lot of background information – you need to know all four operations to get through it and if they don't know their times tables, if they can't subtract then they are at a disadvantage
T3: They have to be able to do more than one thing at once to get the answer – some people lose it as they go through
T1: yes it's quite sophisticated
T4: frightening.

Appendix 7

Key for thematic analysis of focus groups	
Theme 1	Methods / written methods / changes – *colour coded – red text*.
Theme 2	Different content/ level of difficulty– *colour coded – purple text.*
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Theme 6	Advantages of conceptual strategies– *colour coded – yellow highlight.*
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Theme 8	Advantages of procedural strategies– *colour coded – light blue highlight.*
Theme 9	Disadvantages of procedural strategies– *colour coded –blue highlight.*
Theme 10	Needs and other– *colour coded – grey highlight.*

Transcript 2: Year 5 staff focus group

I: How do you perceive the recent changes in the 2014 curriculum will affect your teaching of division? So how had it affected you…?

T8: My teaching hasn't changed mentally for divide by 10,100 or 1000 or in terms of understanding place value. I suppose the biggest difference in my teaching is the written calculation methods. Also, the new objectives are more specific and the level of difficulty has risen.
I: right
T7: um to be honest with you, I haven't actually taught division in yr 5 this year because *** and I have split what we are going to be actually teaching but if I was to be teaching division and obviously I could be using that division method in my own teachings of um shape and space and measure etc … um…I don't think it would actually change much to be honest with you – I would still be using the methods that I was using before
I: which were?
T7: I was doing the dreaded chunking, I do not like the chunking because you've got too many different methods to go along and it's too long winded but that's the method I was doing last year and that's as probably as far as I went – I never did the short method with them – which I would prefer as I am more confident in that. I had stages when I've done the chunking method when I've had to get other people in to say what I go with next.
I: and you?
T6: well I for years – went / stumbled through chunking and the teaching of it without ever feeling that I taught it particularly well and each year I sort of tweaked it – I don't think I was particularly clear in teaching it as they certainly weren't that great at ever doing it. So this year – this is the first year where I have just done the standard method.
I: So that's the way that it's affected your teaching in that instead of doing the chunking you've moved on to standard.
T6: Yeah and we literally just do that and we are all starting from the same point – obviously the high flyers move on more quickly to more complex numbers, remainders and all the rest but it really was a nice way to do it – I call it the bus stop. We talked about 2s into 24 and how to set it out and then 2s into 2 go once and 2s into 4 go 2 – so the answers 12
T8: I use to always encourage the child to use the method in which they are most comfortable and felt chunking was a great method to get an understanding. Now though, we ignore all other methods and just teach short method – highlighting accuracy and following the method over understanding.
I: So basically, your saying that the curriculum changes have meant that your teaching effected in that you're not doing the conceptual chunking method um you've moved onto the standard method as the curriculum dictates – and that's how it's changed your teaching
T6: Yeah
T7: well it would be then
T8: Yes, and I also feel there is also a greater emphasis on interpreting remainders – this links well to FDP.
T6: In a nutshell
T7: yeah well, it's quite good that we've analysed the chunking method to and saying it's not working for some of us

I: What are the advantages and disadvantages of procedural and conceptual approaches when teaching?

T6: is that just with division or as a whole
I: Yeah just with division but you can talk as a whole if you want
T7: I think the procedure they would get straight away if you didn't go into anything, they would just be able to do it but their knowledge of number and their understanding of it wouldn't be there so I think that is why it is important to say, like you were just saying, that it is going into 20 because otherwise they do just think it's going into 2 – well in fact they are not even thinking about it it's just a routine
T6: It's just a procedure
T7: to be honest with you – they probably do better just doing that cos I was taught like that. It's only now that we need to know the importance of understanding
I: but then think about your less able – sometimes they would think it's a hundred and something – would it be best to just teach them the procedure and then come to the concept later or is it best to cover the concept first
T7…mmmmm
T6…yeah
I: what are the advantages and disadvantages of each?
T7: it depends on the child, the class, the set – loads of different things
T6: yeah that's the trouble cos almost everything in teaching comes back to well it depends on the child and their needs
T7: for one child it might be better just doing procedure and for the lower ability it would be cos they need to do it. Then for another they might be able to get a deeper understanding.

Appendix 8

Key for thematic analysis of focus groups	
Theme 1	Methods / written methods / changes – *colour coded – red text*.
Theme 2	Different content/ level of difficulty– *colour coded – purple text.*
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Theme 8	Advantages of procedural strategies– *colour coded – light blue highlight.*
Theme 9	Disadvantages of procedural strategies– *colour coded –blue highlight.*
Theme 10	Needs and other– *colour coded – grey highlight.*

Transcript 3: Year 6 staff focus group

I: How do you perceive the recent changes in the 2014 curriculum will affect your teaching of division? So how had it affected you…?

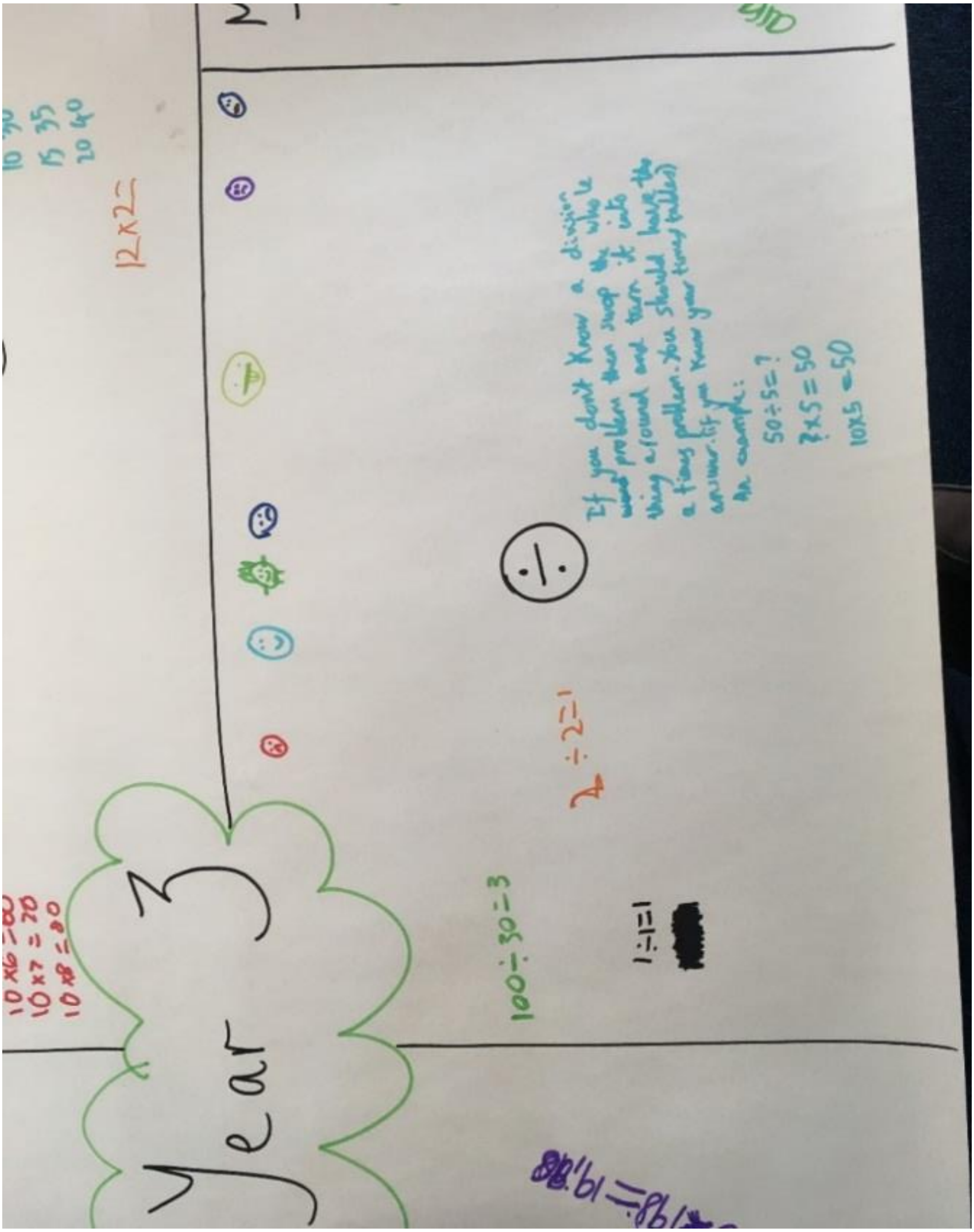
T9: well long division, previously they just needed a written method but now it has to be a specific written method so the introduction of long division
T10: and the arithmetic test
T9: yes, the arithmetic test
T10: they don't just need to know it they need to be able to reason with it
T9: the prevalence of it
T11: and I think as well with the long division for me that appears to be different is that every child has to be taught that method and that to me is the most difficult thing, the change that I have noticed over the past couple of weeks is that even if they are not ready for it they still have to be taught it.
T12: It's a very difficult part of the curriculum now – you can't get away with just not teaching it just because they are not ready for it, you've just got to do it
I: Do you think this is going to change in years to come when Y1 those that have been part of the New style since the start?
T9: hopefully, yes, they should be completely familiar with all styles apart from long division, they are not familiar with all written methods by the time they get to year 6 at the moment so we are playing catch up. It should just be long division that we are teaching.
I: Are you finding they are coming up with any standard methods at the moment?
T9: just chunking really, and a number of those in my class that came up didn't have any recognisable method! No chunking nothing.
I: so do you feel you are just doing procedural methods, you must do it this way, this way, this way at the moment?
T9: yes, <u>cos we are desperate</u>
T10: but we feel in time to come this will change and become more conceptual
T12: I don't think yr6 in five years' time will be teaching any of the written methods in the way that we are teaching them as they will come through with the knowledge. There just isn't time in the curriculum for year6 to teach, column addition, subtraction etc. and so hopefully our teaching is going to change this way in that we won't have to teach all the written methods as they should come to us already knowing and therefore just needing long division to be taught.
T11: What I've found with long division, there were at least 3 or 4 children that could solve a division question using the short standard (4289 ÷48) they could do that – so <u>why do they need to do the long division method.</u>
T12: well if they get the answer right, they get the mark
T11: <u>so whichever way they do it they get the mark; this could mean they either have an understanding or they've just learnt the process.</u>

I: What are the advantages and disadvantages of procedural and conceptual approaches when teaching?

T12: advantages of conceptual methods are an actual deep understanding of mathematics and how to get from A to B in 7 different ways because you understand how all the routes are linked and that is what we want for all children. The other side of things is that not all children are made equal and some children just need to learn how to cope with dyslexia and they need to learn how to cope with a lack of understanding in maths and those procedures can help them get the outcome even if they don't understand what they are doing.
T9: Conceptual is the ideal, it's what I would like to do but particularly this year, we have been put into a position where we do not have the time for conceptual, so a disadvantage of conceptual approaches is that it takes time
T11: the advantage to procedural is that it offers a quick solution which they can get right
T10: but also it's very difficult to pick up conceptual methods now – it's too late – they need to pick up these methods in their earlier learning
T12: there are too many misconceptions to unpick now, whereas if they came pure with no misconceptions
T9: They need the conceptual understanding of division and their division facts – when they don't have it, it's very hard to then do the conceptual understanding of long division because they don't have the basics and the foundations
T12: and children that have learnt their tables by rote but only recently don't understand that you can divide 16 by 5 and have one left because they know they can only divide 15 by 5 or 20 by 5 because they don't, it isn't entrenched enough – they don't have the knowledge to join it altogether – they should be learning it in year 2, not in 5 so they haven't had enough time of understanding what division actually is.
T11: Which wouldn't be a problem if we weren't year 6 and didn't have the constraints of a test because if you think about it as year 6 – we would as teachers be teaching into all the understanding. We wouldn't have the constraint that they have to get a certain mark to pass. We would be teaching to their needs.

Appendix 9

Year 3 graffiti wall 2016

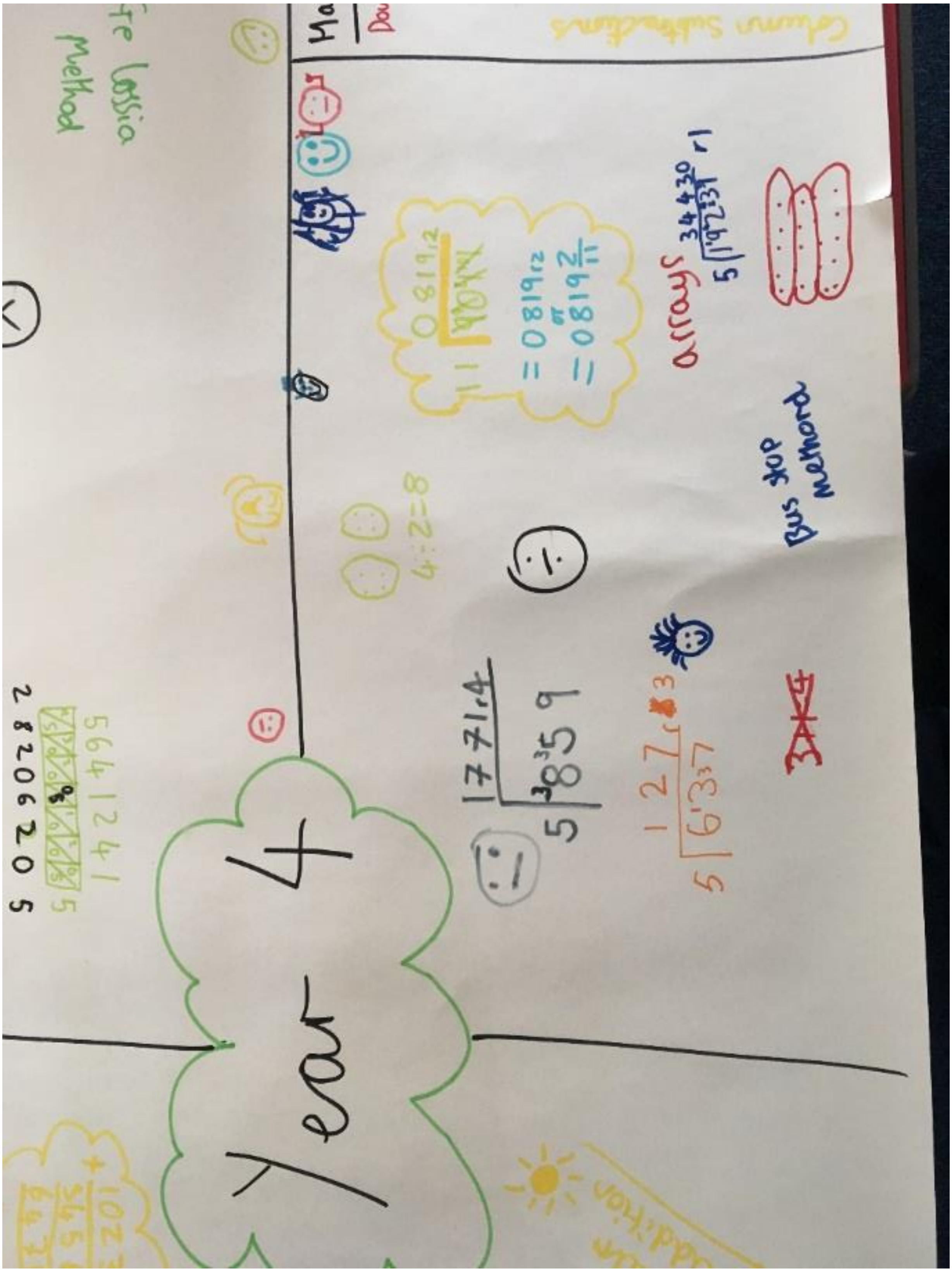


Appendix 10

Year 3 graffiti wall 2019

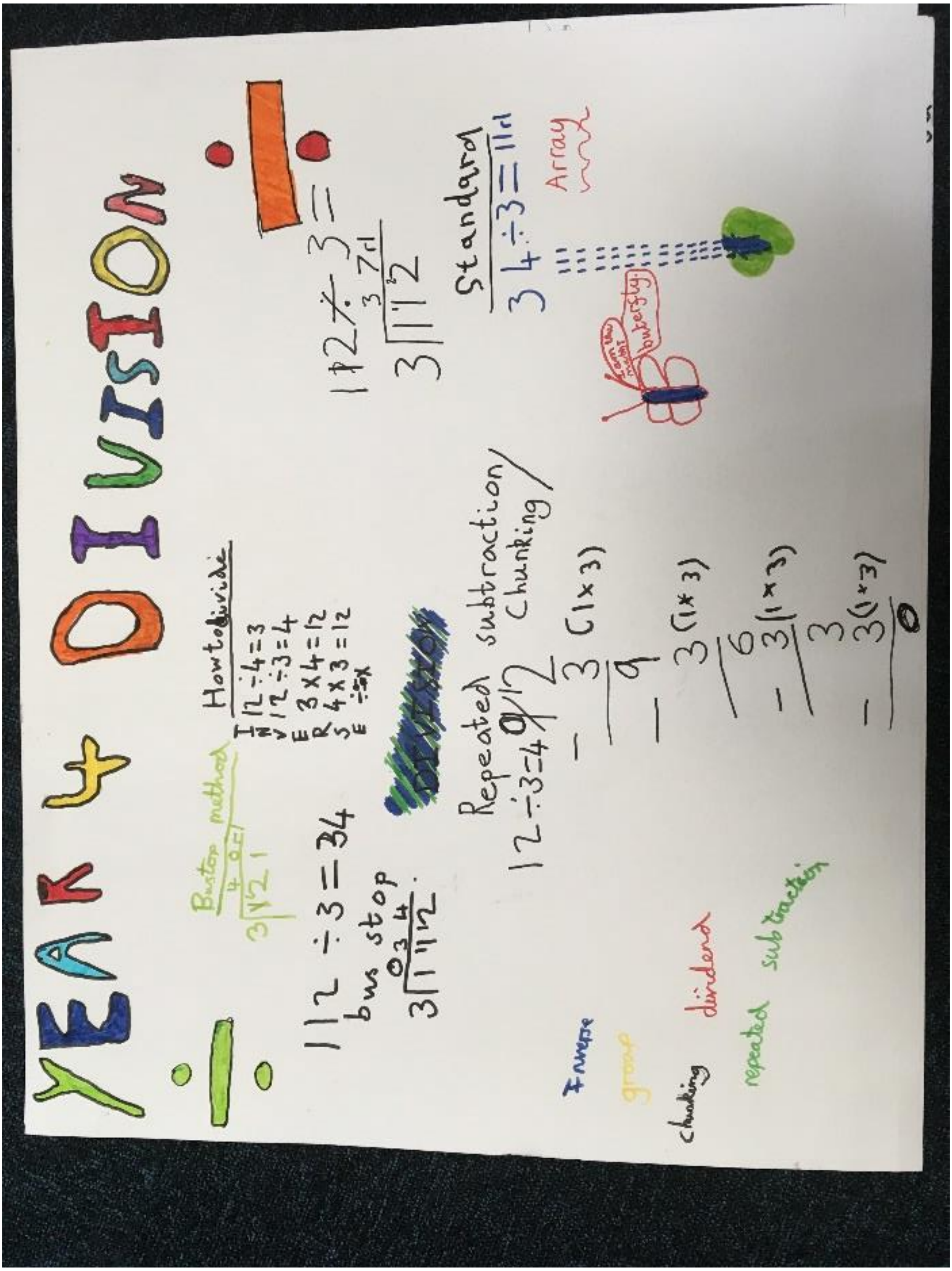
Appendix 11

Year 4 graffiti wall 2016



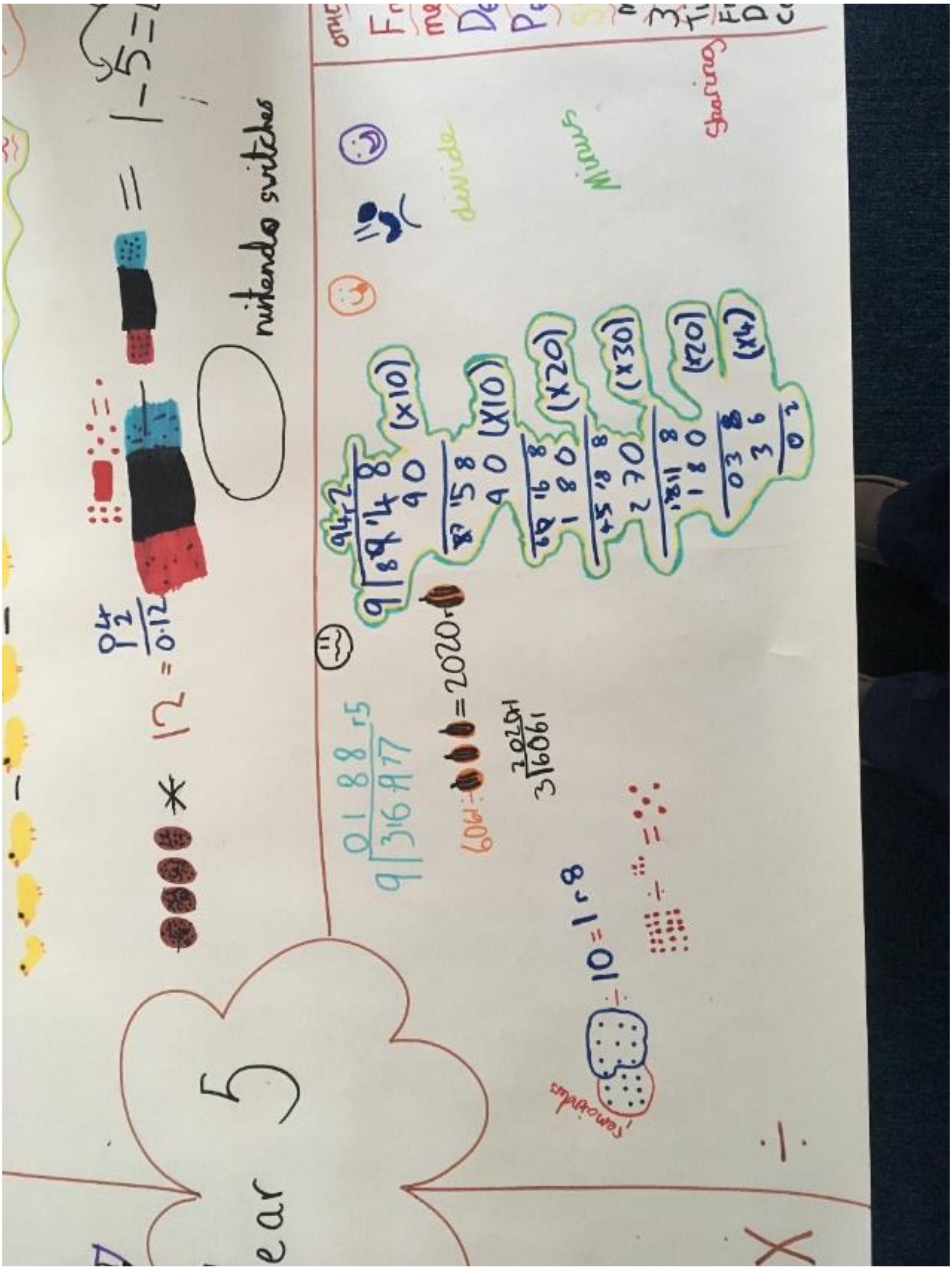
Appendix 12

Year 4 graffiti wall 2019



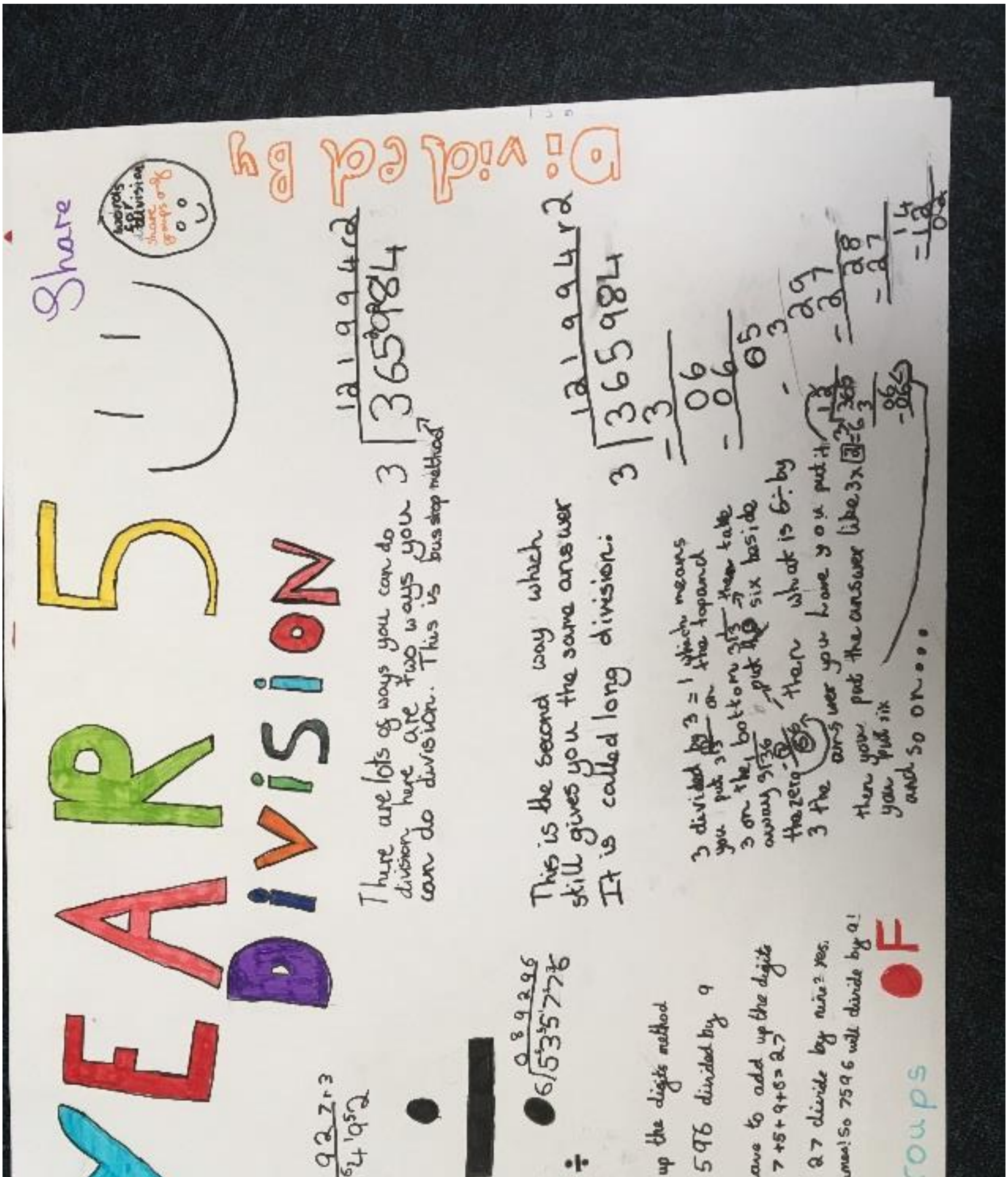
Appendix 13

Year 5 graffiti wall 2016



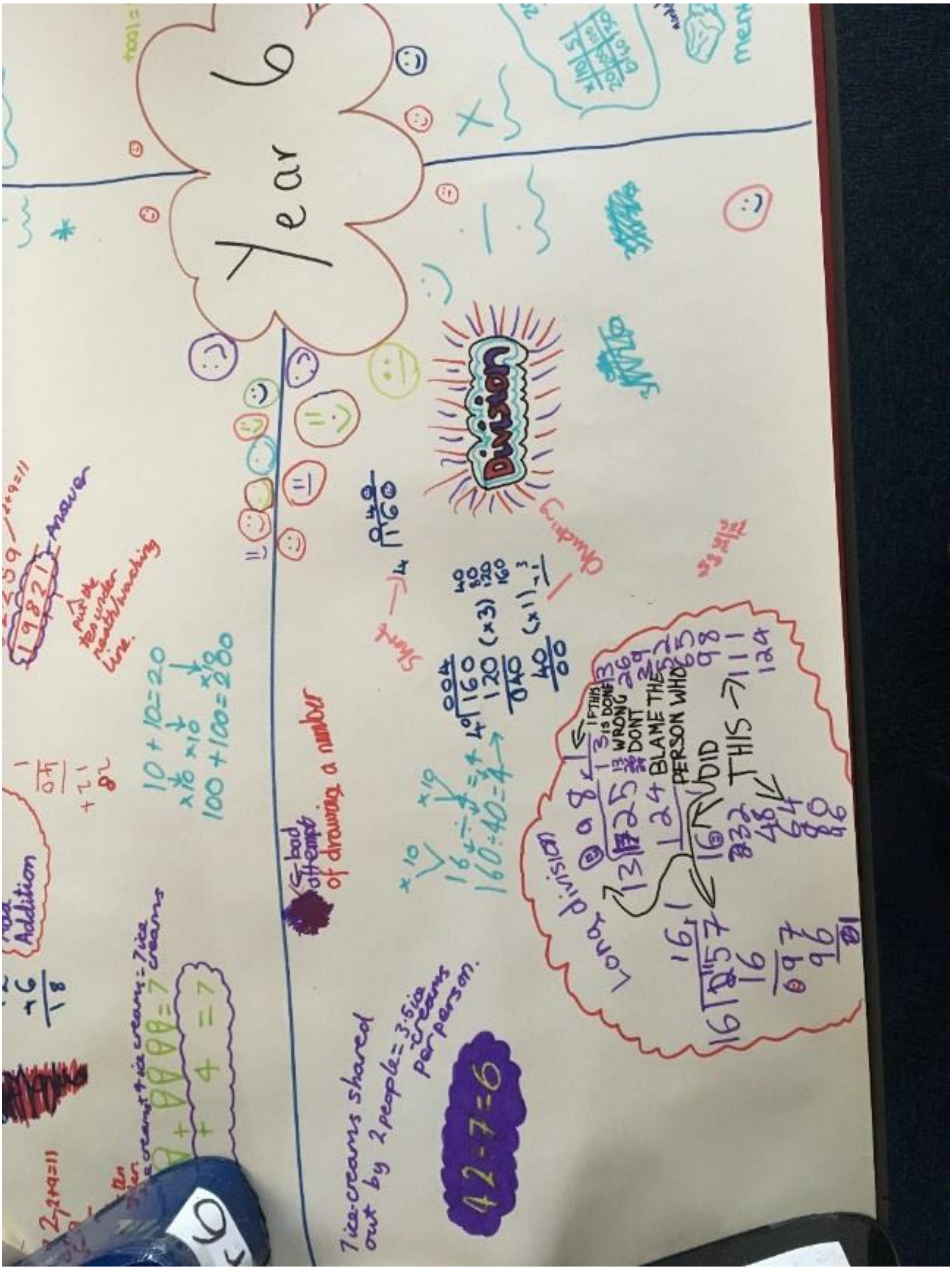
Appendix 14

Year 5 graffiti wall 2019



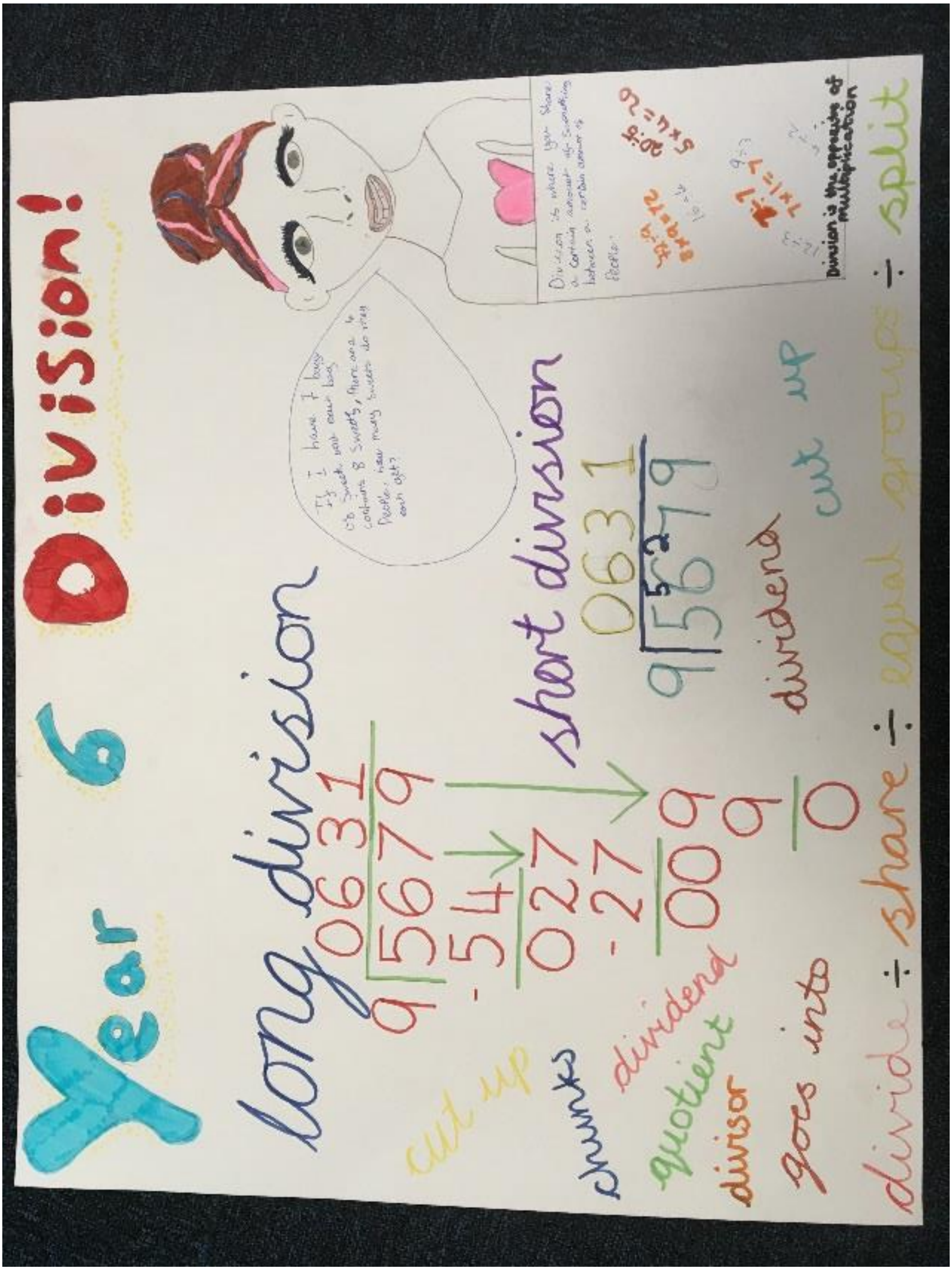
Appendix 15

Year 6 graffiti wall 2016



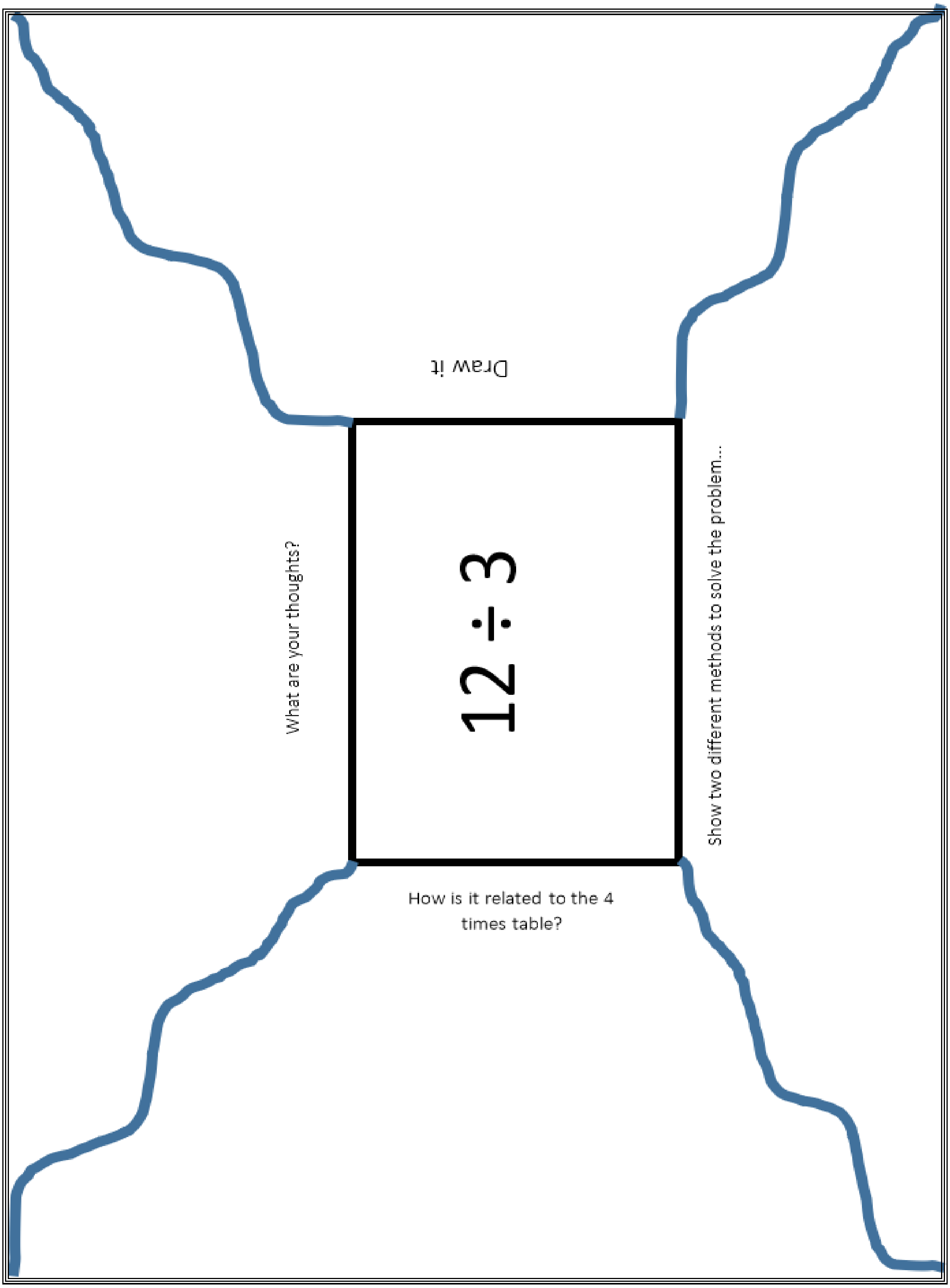
Appendix 16

Year 6 graffiti wall 2019



Appendix 17

Graffiti mat



Appendix 18

Chick and Baker's (2005) framework for categorising teacher responses to child misconceptions

Category	Definition
Re-explain	Explaining or re-explaining any part of either the concept or procedure.
Cognitive Conflict	Setting up a situation in which the student might identify a fundamental mathematical contradiction between the original response and the new situation, thus encouraging the student to re-evaluate the erroneous approach.
Probes student thinking	Asking the student to explain working or thinking, either to discover what the student is thinking to help the teacher decide what to do next, or to get the student to see the error. [It was not always possible from the data to establish which of these the teacher intended, so no distinction was made.]
Other	Any strategy not clearly in the above categories, e.g., "use simpler examples".

Appendix 19

Tutcher's (2012) framework for categorising teacher responses to child misconceptions, adapted from Chick and Baker (2005)

Category	Definition
Re-explain	Explaining or re-explaining any part of either the concept or procedure.
Cognitive conflict	Setting up a situation in which the student might identify a fundamental mathematical contradiction between the original response and the new situation, thus encouraging the student to re-evaluate the erroneous approach.
Probes student thinking	Asking the student to explain working or thinking, either to discover what the student is thinking to help the teacher decide what to do next, or to get the student to see the error. (It was not always possible from the data to establish which of these the teacher intended, so no distinction was made.)
Focus attention	Teacher uses strategy to show child where they need to look.
Kinaesthetic	A learning style in which learning takes place by the student actually carrying out a physical activity, rather than listening- teacher encourages strategies for tactile learning.
Other	Any strategies not clearly in the above categories, e.g., "use simpler examples".

Categories of teacher strategies in response to student misconceptions.

Appendix 20

Timeline of study

	Year 1: term 1	Year 1 : term 2	Year 1: term 3	Year 2: term 1	Year 2 : term 2	Year 2: term 3	Year 3: term 1	Year 3 : term 2	Year 3: term 3	Year 4: term 1	Year 4 : term 2
	2013	2014	2014	2014	2015	2015	2015	2016	2016	2016	2017
Literature review											
Methodology											
GSoE ethical procedures											
Data collection - pupil					conference	questionnaire			Grafitti walls		
Data collection - teacher					conference	questionnaire					
Cycles		1	1	1 and RQ dev	2 and RQ dev	3	2 and 3	2, 3 and 4	4	4	4
Data analysis											
Write up											
UoB units											
Conferences							BSLRM				
Journal papers											
Final write up											

	Year 5: term 1	Year 5 : term 2	Year 5: term 3	Year 6: term 1	Year 6 : term 2	Year 6: term 3	Year 7: term 1	Year 7 : term 2	Year 7: term 3
	2017	2018	2018	2018	2019	2019	2019	2020	2020
Literature review									
Methodology									
GSoE ethical procedures									
Data collection - pupil			SATs data			SATs & GW			
Data collection - teacher									
Cycles			4	4	4	4			
Data analysis									
Write up									
UoB units									
Conferences									
Journal papers									
Final write up									