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Pulse train generation in a distributed gain fibre amplifier

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ABSTRACT

This work shows the generation of an array of chirped compressed periodic waves with distributed gain in a nonlinear fibre. In particular, with suitable tailoring of the gain to vary along the longitudinal distance while the dispersion and nonlinearity parameters are kept constant. Exact analytical equations using the self-similar analysis technique were obtained to describe this process. In addition, the stability of these generated periodic waves is studied under finite perturbations.

1. Introduction

Dispersion and signal losses are some of the factors that limit optical communication. Whereas, dispersion leads to pulse broadening as different frequency components of the wave arrives at different timing, fibre losses on the other hand leads to reduction in signal intensity. It has been shown that these two factors can be reduced by propagating optical signals at certain wavelengths (Agrawal, 2000, 2012). Specifically, the dispersion effect can be controlled by propagating optical waves through specially designed fibres (Mori et al., 2001). Furthermore, optical pulses can travel a distance without experiencing broadening when there is a proper balance between the group velocity dispersion (GVD) and nonlinearity as a result of self-phase modulation (SPM). This phenomenon is referred to as soliton communication (Kivshar and Agrawal, 2003). On the other hand, to control the effect of fibre losses in optical communication, signals need to be amplified periodically. Two of such schemes have been utilised to achieve this purpose; distributed and lumped amplification schemes (Agrawal, 2000). One distinct advantage the distributed scheme has over the other is that, within the fibre link, there exist a lower build-up of noise and an improvement of the signal to noise ratio (Hasegawa, 1984; Mollenauer et al., 1986). Gain tailoring can be achieved in distributed amplifiers by using Raman amplifiers which is one of the available distributed amplification systems. This system comprise of a device that has been doped with phosphorus or germanium to enhance its gain. In Raman distributed system, the fibre used for transmission is also used for amplification, the input signal is often pumped in the reverse direction and provides gain over long distances (>20 km). One associated challenge with this system is that lasers with very high power are needed for pumping. Another alternative distributed amplification system for long haul light-wave communication is Erbium-doped fibre amplifiers (EDFA). Here, the fibre core is doped during the manufacturing process with rare-earth elements as a gain medium. In EDFA systems, the amplifier properties such as gain bandwidth and the operating wavelength are determined by the dopants rather than the silica fibre which only functions as a host conveying medium. Most EDFAs provide 20–25 dB amplification over a distance of ~10 m through a high-density dopants of ~500 parts per million (Agrawal, 2012).

As a result of an exponential increase in the need for fast speed signal processing and also, the eventual approach to the limits of conventional lasers is gradually been reached. The development of an alternative approach which relies on the beating of a dual-frequency signal in a nonlinear fibre has been achieved (Chernikov et al., 1993; Trillo et al., 1994; Pitois et al., 2002). Under the influence of nonlinearity, the propagated continuous wave signal degenerates into very short array of optical pulses (Hasegawa, 1984; Tai et al., 1986). Using this scheme, pulse train with very high frequencies can be obtained. In this work, I analytically demonstrate the generation of a train of optical pulses that get periodically amplified and their pulse widths compressed during evolution. The profile of these array of periodic pulses are described in the form of Jacobian elliptic function and possess a linear chirp. By proper transformation, the results obtained in this distributed gain amplification case can be shown to be equivalent to the evolution of an optical pulse in a dispersion decreasing fibre with a constant gain (Moores, 1996). Periodic trains can also be generated by Akhmediev breathers and super-regulated breathers (Liu et al., 2017, 2018).

The work of Serkin and Hasegawa (2000, 2002) showed that the nonlinear Schrödinger wave equation (NLSE) with varying dispersion, nonlinearity and gain parameters are integrable using the inverse scattering transform (IST) technique among the list of system equations investigated for different applications. One of such applications which

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is of interest to this study is that of a chirped soliton for dark and bright solitons. For the first time, chirped soliton solution was obtained using the self-similar transformation method by Moores (1996), Kruglov et al. (2003, 2002) and the numerical analysis of this compression has been studied (Mostofi et al., 1997). The generalise form of the NLSE is used in the study of optical rogue waves (Solli et al., 2007). In our previous work, we showed the generation and propagation of these pulses with a fixed gain by using the Hirota bilinear transformation approach (Atuba et al., 2016). The uniqueness of these pulses is that they possess a linear chirp, also, during propagation, the pulses get amplified while their pulse widths get compressed and that at long wave limits, these so-called Jacobian elliptic functions degenerate to the case of chirped solitary waves. Here, exact chirped periodic wave equations were generated using self-similar analysis technique for the given nonlinear Schrödinger wave equation (NLSE). I further placed a constraint that the GVD and SPM parameters should be constant while the gain changes along the longitudinal distance. The generated optical pulses is shown to be stable, maintain their shapes and are robust.

2. Periodic wave solutions

Firstly, I start off from the generic inhomogeneous NLSE and then simplify the equation to admit the case of a constant dispersion and nonlinearity condition. The generic NLSE has the form;

$$i\frac{\partial A}{\partial z} + p(z)\frac{\partial^2 A}{\partial t^2} + q(z)|A|^2A + ig(z)A = 0. \tag{1}$$

where A is the axial electric field. $p(z)$, $q(z)$ and $g(z)$ are dispersion, nonlinearity and distributed gain of the amplifier, respectively.

The work of Kruglov et al. (2003) showed that Eq. (1) have solutions as long as there exist a certain relationship between $g(z)$, $q(z)$ and $p(z)$. The solution is realisable by noting that a similarity transformation changes the system equation to a standard homogeneous form. Eq. (1) has been shown to be integrable by the inverse scattering approach (Serkin and Hasegawa, 2000, 2002), in addition, the governing equation can be mapped into a constant coefficient nonlinear Schrödinger type equation (Özemir and Güngör, 2012).

According to Kruglov et al. (2003), $A(z,t)$ admits wave as,

$$A(z,t) = U(z,t)\exp[i\phi(z,t)], \tag{2}$$

where U is the amplitude and has the form.

$$U(z,t) = \frac{1}{\sqrt{1+c_0D(z)}}M\left[\frac{t-t_c}{1+c_0D(z)}\right]\exp(G(z)). \tag{3}$$

and a quadratic phase which is,

$$\phi(z,t) = a(z) + c(z)(t-t_c)^2 \tag{4}$$

and this corresponds to a linear chirp. Here t_c, $D(z)$ and $G(z)$ corresponds to the pulse centre position, cumulative dispersion and cumulative gain accumulated over the length of fibre z. The constant phase and chirp along the fibre length are denoted as $a(z)$ and $c(z)$ respectively. M is a function to be identified and in this work, periodic wave functions in the form of a Jacobian elliptic function. The longitudinal varying functions of $a(z)$, $c(z)$, $D(z)$ and $G(z)$ are defined by the expressions,

$$c(z) = \frac{c_0}{1+c_0D(z)} \tag{5}$$

$$D(z) = 4\int_0^z p(z')dz' \tag{6}$$

$$a(z) = a_0 + \lambda\int_0^z \frac{p(z')dz'}{[1+c_0D(z')]^2} \tag{7}$$

$$G(z) = -\int_0^z g(z')dz'. \tag{8}$$

Here c_0 is an input chirp parameter and a_0 is an integration constant. The self-similar transformation realised from the inhomogeneous NLSE is only possible if the distributed parameters satisfy the condition in Eq. (9) (Kruglov et al., 2003, 2005),

$$g(z) = \frac{2c_0p(z)}{[1+c_0D(z)]} - \frac{1}{2}\left[\frac{q(z)}{p(z)}\right]\frac{d}{dz}\left(\frac{p(z)}{q(z)}\right) \tag{9}$$

The works of Kavian and Weissler (1994), Kruglov et al. (2002) show that the pulse energy $\int_{-\infty}^{\infty} | A(z,t) |^2 dt$ is preserved under the transformation of Eq. (2). For the unique case of a distributed gain with the dispersion and nonlinearity parameters being constant, Eq. (1) becomes,

$$i\frac{\partial A}{\partial z} + p\frac{\partial^2 A}{\partial t^2} + q|A|^2A + ig(z)A = 0. \tag{10}$$

The gain in Eq. (9) becomes,

$$g(z) = \frac{2c_0p}{1+4c_0pz} \tag{11}$$

This is the compatibility condition for this periodic case. Here, when $c_0 < 0$ in the anomalous case and vice versa, the fibre amplifier gain increases along the propagation distance.

The phase and chirp parameters $a(z)$ and $c(z)$ given by Eqs. (5) and (7) can be expressed as

$$a(z) = a_0 + \lambda\left[\frac{pz}{1+4c_0pz}\right] \tag{12}$$

$$c(z) = \frac{c_0}{1+4c_0pz} \tag{13}$$

For definiteness, the phase factors of the periodic waves are obtained by substituting respectively in Eq. (12). When $pq < 0$, I obtained two periodic solutions,

- when $\lambda = (2-k^2)/t_0^2$

$$U(z,t) = kA_c(z)\,\mathrm{cn}\left(\frac{t-t_c}{W(z),k}\right), \tag{14}$$

- when $\lambda = (2k^2-1)/t_0^2$

$$U(z,t) = A_c(z)\,\mathrm{dn}\left(\frac{t-t_c}{W(z),k}\right), \tag{15}$$

where k is the elliptic modulus constant, the scaling peak amplitude $A_c(z)$ and the pulse width $W(z)$ are

$$A_c(z) = \sqrt{\frac{2p}{q}}\frac{1}{t_0(1+4c_0pz)}, \tag{16}$$

$$W(z) = t_0(1+4c_0pz). \tag{17}$$

for $pq > 0$ and $\lambda = -(1+k^2)/t_0^2$, the amplitude of the wave is

$$U(z,t) = kA_c(z)\,\mathrm{sn}\left(\frac{t-t_c}{W(z),k}\right), \tag{18}$$

As seen in Eq. (17), the pulse width decreases down the fibre for the normal dispersion case when $c_0 < 0$ and also for the anomalous case when $c_0 > 0$. It is important to note that the pulse width for the normal dispersive case tends to zero at a propagation distance $z = -(4c_0|p|)^{-1}$. This provides the condition for optimal pulse compression in the absence of higher order nonlinear terms in Eq. (1). During this process, these pulse train preserve their shape and linear chirp.

It was shown that by transformation, the solution of the solitary wave results to the dynamics of an optical wave having a constant gain with its dispersion decreasing exponentially (Moores, 1996). The exact periodic wave solutions for distributed amplification when dispersion and nonlinearity are both constant for the cn, dn and sn type waves are shown in Eqs. (19), (20) and (21). These equation satisfies the system equation i.e. Eq. (1). Finally, Figs. 1, 2 and 3 show the initial input and

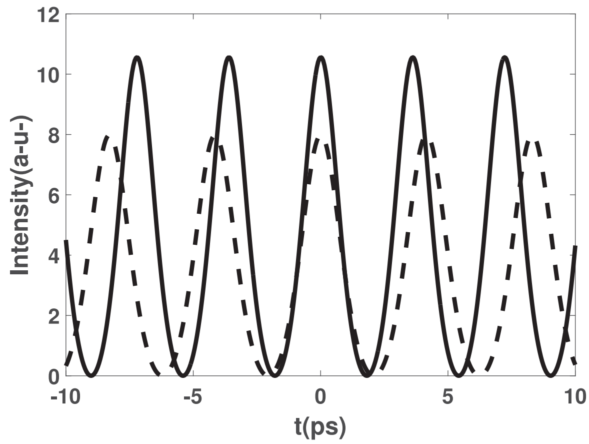

Fig. 1. Intensity profile of initial (dashed lines) and amplified output (solid lines) of the cn-periodic wave through a fibre for Eq. (19). The physical parameters are $p = -0.0553713$ ps^2/m, $q = 0.006794$ W^{-1}m^{-1}, $t_0 = 1$ ps, $tc = 0.01$ ps, $a_0 = 0.03$, $c_0 = 0.03$ Hz/ps, $L_D = 9.822$ m, $z = 2L_D$ and $k = 0.7$.

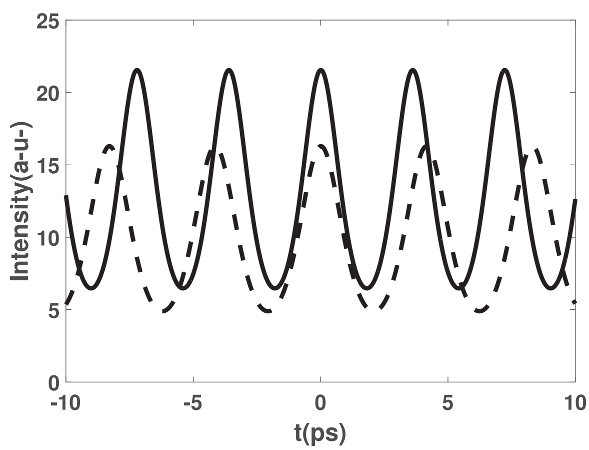

Fig. 2. Intensity profile of initial (dashed lines) and amplified output (solid lines) of the dn-periodic wave through a fibre for Eq. (20). The physical parameters are $p = -0.0553713$ ps^2/m, $q = 0.006794$ W^{-1}m^{-1}, $t_0 = 1$ ps, $tc = 0.01$ ps, $a_0 = 0.03$, $c_0 = 0.03$ Hz/ps, $L_D = 9.822$ m, $z = 2L_D$ and $k = 0.7$.

amplified output of these waves. It is seen from these figures that for a distance of ~20 m, an input pulse width of 1 ps gets compressed to an output value of 0.87 ps.

$$A(z,t) = k\sqrt{\frac{2p}{q}}\frac{1}{t_0(1+4c_0pz)}\,\mathrm{cn}\left(\frac{t-t_c}{t_0(1+4c_0pz)}\right)\times \exp\left\{ia_0 + i\left(\frac{2k^2-1}{t_0^2}\right)\left[\frac{pz}{1+4c_0pz}\right] + \frac{ic_0(t-t_c)^2}{1+4c_0pz}\right\} \tag{19}$$

$$A(z,t) = \sqrt{\frac{2p}{q}}\frac{1}{t_0(1+4c_0pz)}\,\mathrm{dn}\left(\frac{t-t_c}{t_0(1+4c_0pz)}\right)\times \exp\left\{ia_0 + i\left(\frac{2-k^2}{t_0^2}\right)\left[\frac{pz}{1+4c_0pz}\right] + \frac{ic_0(t-t_c)^2}{1+4c_0pz}\right\} \tag{20}$$

$$A(z,t) = k\sqrt{\frac{2p}{q}}\frac{1}{t_0(1+4c_0pz)}\,\mathrm{sn}\left(\frac{t-t_c}{t_0(1+4c_0pz)}\right)\times \exp\left\{ia_0 - i\left(\frac{1+k^2}{t_0^2}\right)\left[\frac{pz}{1+4c_0pz}\right] + \frac{ic_0(t-t_c)^2}{1+4c_0pz}\right\} \tag{21}$$

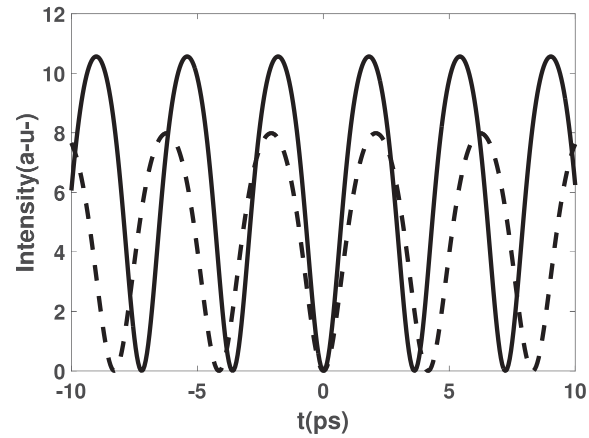

Fig. 3. Intensity profile of initial (dashed lines) and amplified output (solid lines) of the sn-periodic wave through a fibre for Eq. (21). The physical parameters are $p = -0.0553713$ ps^2/m, $q = 0.006794$ W^{-1}m^{-1}, $t_0 = 1$ ps, $tc = 0.01$ ps, $a_0 = 0.03$, $c_0 = 0.03$ Hz/ps, $L_D = 9.822$ m, $z = 2L_D$ and $k = 0.7$.

It is established that optical signals experience reshaping when travelling in a lossy nonlinear optical fibre. A quick way of boosting this propagated signal up is by periodically amplifying the signals as they evolve. Following this, the spacing distance between consecutive amplifiers is key, for this process, the idea of a guiding-centre was initiated (Hasegawa and Kodama, 1991). Practically, this works well when the spacing distance is small in comparison with the soliton period between the reshaped pulse that is produced by the fibre amplifier (Mollenauer et al., 1991). Large spacing is realistic with femtosecond pulses which normally requires a nonlinear crystal or a lasing medium (Malomed, 1994). Recently, intensity of about 1 TW/cm^2 was generated with less than 60 fs having a high amplification gain of more than a 1000 (Vampa et al., 2018).

Optical pulses can experience both amplitude amplification and spectral broadening with the use of dispersion decreasing fibres (Chernikov et al., 1993; Atuba et al., 2016). Distributed fibre losses along the fibre can be overcome with the use of distributed Raman amplifiers, this allows dual flexibility and tunability options in the frequency and pulse width regime. With this, one can realise gain at any wavelength by selecting a suitable pump wavelength (Headley and Agrawal, 2005).

3. Stability analysis

One distinguishing characteristics of periodic wave evolution is its stability against small perturbations, this is because, only stable (or weakly stable) can be experimentally observed and finds useful practical applications (Choudhuri et al., 2016). Therefore, it is pertinent to analyse the stability of the generated nonlinearly periodic waves with respect to finite initial perturbations. This could appear in the form of random noise, slight alteration of the parametric conditions or amplitude perturbations (Choudhuri and Porsezian, 2012). In this work, I investigated the stability of the generated chirped periodic wave solutions using two direct numerical simulations, which are, amplitude perturbation and initial white noise (Zhang et al., 2012; Yang et al., 2005). First, the amplitude of the initial periodic wave is perturbed by a numerical value of 10%. The numerically obtained results are shown in Figs. 4 (a), (c) and (e), this can be compared with the exact unperturbed numerical solutions in Figs. 1, 3 and 2. Second, a 30 dB signal-to-noise ratio (SNR) additive white noise is added to the initial wave and the numerical results are shown in Figs. 4 (b), (d) and (f). From these generated outputs, it can also be observed that the additive white noise did not alter the structure or character of the solutions. More plots were obtained to check the stability of these waves in Figs. 5 with

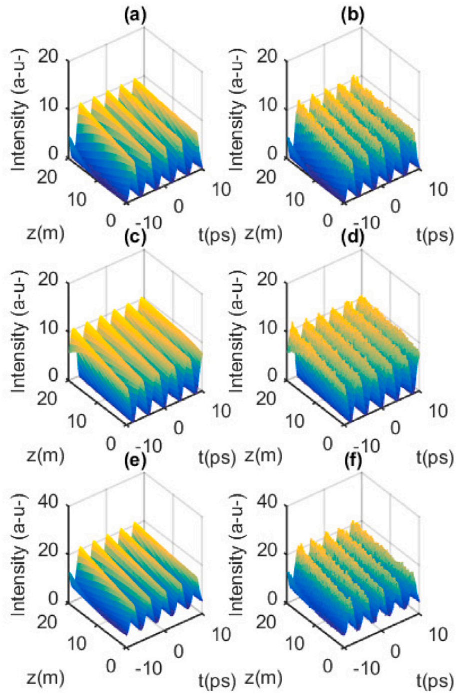

Fig. 4. The numerical evolution of (a) cn-wave, (c) sn-wave, (e) dn-wave under the influence of a 10% amplitude perturbation; (b) cn-wave, (d) sn-wave, (f) dn-wave of exact solutions under the perturbation of 30 dB SNR additive white noise. The physical parameters are $p = -0.0553713$ ps^2/m, $q = 0.006794$ W^{-1}m^{-1}, $t_0 = 1$ ps, $tc = 0.01$ ps, $a_0 = 0.03$, $c_0 = 0.03$ Hz/ps, $L_D = 9.822$ m, $z = 2L_D$ and $k = 0.7$.

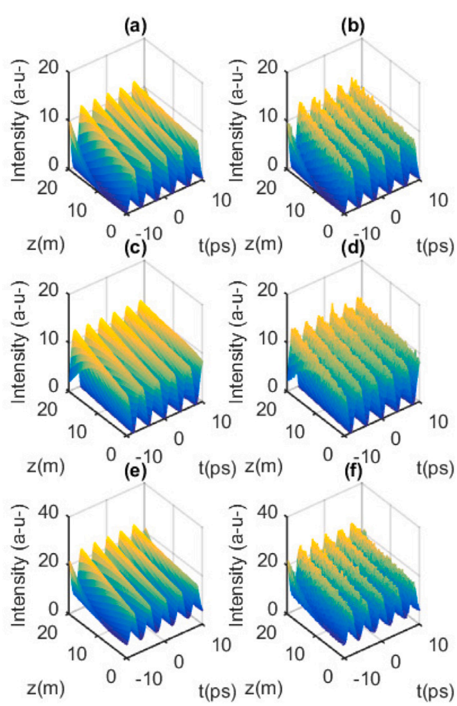

Fig. 5. The numerical evolution of (a) cn-wave, (c) sn-wave, (e) dn-wave under the influence of a 10% amplitude perturbation; (b) cn-wave, (d) sn-wave, (f) dn-wave of exact solutions under the perturbation of 30 dB SNR additive white noise. The physical parameters are $p = -0.07$ ps^2/m, $q = 0.008$ W^{-1}m^{-1}, $t_0 = 1$ ps, $tc = 0.01$ ps, $a_0 = 0.03$, $c_0 = 0.03$ Hz/ps, $L_D = 9.822$ m, $z = 2L_D$ and $k = 0.7$.

different values of dispersion and nonlinearity as compared to Figs. 4. This further shows that the wave can evolve stably under amplitude perturbations and finite initial perturbations of the additive white noise type, therefore, one can deduce that the generated periodic waves of the Jacobian cn, sn and dn-types studied in this work are stable.

4. Conclusion

In this work, I have generated a train of chirped compressed periodic wave and exact periodic wave solutions in the form of Jacobian elliptic profiles of sn, cn and dn for the case of distributed amplification that has a fixed nonlinearity and dispersion parameters. At long wave limits, these chirped periodic waves degenerate to the case of chirped solitary waves. With the help of numerical simulations, I have further investigated their stability under the finite initial perturbations. The obtained results show that the expected character of the solutions are not altered under the finite input perturbations of the amplitude and additive white noise type.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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