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Numerical Assessment on Fatigue Failure of Castellated Steel Beams under Sinusoidal Vibration

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Abstract: Increase cost of material in the construction industry has led to the adoption of castellated steel beam as an alternative to substitute conventional steel rolled beam. However, the presence of web opening has resulted to various structural behaviour under static action and uncertainties under dynamic loading, especially fatigue failure. Therefore, this paper presents the numerical assessment on fatigue failure of castellated steel beams. The design of castellated steel beam was based on the parent steel beam of UKB 254 x 102 x 28. Meanwhile, various shapes of web opening (hexagonal, circular and rectangular) with size of 0.75*D* were considered. The finite-discrete element method program was used as a platform of numerical modelling. The stress range was analysed at 3 Hz of different load amplitudes of sinusoidal vibration. The fatigue life was compared among each shape of web opening at detail categories 90 and 160. At the initial load, the stress range reaches 65 MPa to 150 MPa. When the load increased, the stress range changes diminutively around 240 MPa to 280 MPa. The fatigue life attains at plateau value of 10^8 cycles, where circular castellated steel beams showed the best performance.

Keywords: Castellated steel beam, finite-discrete element method, stress range, fatigue life, displacement

1. Introduction

Castellated steel beam or widely known as steel beam with web opening was introduced back in mid-1930 to provide construction alternatives due to the increasing cost of steel material. Castellated steel beam is formed by cutting the conventional steel rolled beam along the centreline, according to desired shape and re-joining the two halves by welding. Subsequently, the overall beam depth is increased by approximately 50%. By increasing the total beam depth, the vertical stiffness, moment of inertia and section modulus can also be increased without any additional of weight. Hence, castellated steel beam yields higher strength to weight ratio as compared to the conventional steel rolled beam and capable to provide the passage for mechanical and electrical conduit services. Due to the benefits it provided, castellated steel beam remains as sustainable structures.

Basically, there are few different shapes of web opening such as circular, hexagonal, diamond and rectangular. Erdal et al. (2011) and Rodrigues et al. (2014) accentuated that the shape and size of web opening have apparent effects on the bearing capacity and performance of castellated steel beam. Complexity of shear distribution across the web

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opening has caused enormous failure in different patterns. In the construction field, castellated steel beam is utilized for the commercial and industrial buildings as an economic solution on the time constraint, cost and durability. However, adaptation of castellated steel beam as main structural component remains passive due to various consideration on structural behaviour and lack of understanding on the performance under dynamic loading. Therefore, this paper presents the fatigue failure of castellated steel beam, where special attention was given for sinusoidal vibration.

Early experimental study by Hosain & Speirs (1093) indicated that the length of weld section is important factor in determining optimum geometry of web opening. Meanwhile, a series of investigations by Kerdal & Nethercot (1984) revealed that castellated steel beam possesses three distinct of failure behaviour, which included Vierendeel mechanism, web weld rupture and web post-buckling due to the excessive shear. Other failures such as lateral torsional buckling and bending are similar as conventional steel rolled beam. The governing factor that contribute to failure is mainly due to the size and shape of web opening, slenderness and type of loading. Wakchaure & Sagade (2012) proved that web opening of castellated steel beam has altered the stress distribution within the member and influence the collapse phenomenon.

It was observed by Mohan & Prabjakaran (2015) that hexagonal castellated steel beam has the best performance in term of flexural deflection and bearing capacity. Meanwhile, Soltani et al. (2012) performed a non-linear analysis on hexagonal and octagonal castellated steel beams. The finding identified that octagonal castellated steel beam was more likely to possess web post-buckling. In numerical modelling, Lie & Chung (2003) conducted finite element analysis under the influence of various sizes of web opening. It was observed that Vierendeel mechanism is critical at web opening and failure behaviour also affected by type of support. A comparison of deflection and shear handling was investigated by Jamadar & Kumbhar (2015) using ANSYS software, where diamond shape of web opening performed better than circular castellated steel beam.

Unlike conventional steel rolled beam, understanding on fatigue failure of castellated steel beam is still not well established. Few studies such as conducted by Vahid et al. (2013), Mara et al. (2014), Sarvestani (2017), Khosravi et al. (2015) and Hoseini & Nateghollahi (2003) were concentrated on structural behaviour under cyclic loading and earthquake. The assessment on fatigue failure indicated that castellated steel beams experience collapses due to initiation of crack that happen around the web opening. The local yielding was also observed at the lower flange. Other failure modes are flexural failure, lateral and torsion buckling and web connection.

2. Modelling of Castellated Steel Beam

Design of castellated steel beam was adopted from the parent beam UKB 254 x 102 x 28 with a size of 260.4 mm depth of section (*D*), 102.2 mm width of flange (*W*) and grade S275 as stipulated in BS EN 1993-1-1 (2005). The span of castellated steel beam was erected with the length up to 3000 mm. The sectional properties of castellated steel beams can be referred in Table 1. Three different shapes of web opening (hexagonal, circular and rectangular) were considered in numerical modelling. Size of web opening (d_o) is corresponding to 0.75*D* of the parent beam, while the area of web opening remains fixed at 31400 mm^2. The centre-to-centre spacing (*S*) and spacing of web opening (*e*) were designed at $0.90d_o$ and $0.40d_o$ respectively.

Table 1 - Sectional properties of castellated steel beams.

No.	Property	Value
1	Depth of web opening, d_o (mm)	195.3
2	Thickness of web, h_w (mm)	6.3
3	Centre-to-centre spacing, *S* (mm)	180.0
4	Spacing of web opening, *e* (mm)	80.0
5	Width of web opening, *C* (mm)	
	- Rectangular	157.0
	- Hexagonal	110.0
	- Circular	195.3

Numerical modelling was performed using the finite-discrete element method (FDEM) program. Unstructured solid element of four-noded tetrahedral was defined to provide accurate deformation of castellated steel beam. Basically, size of mesh is determined based on the critical time step and convergency. Throughout mesh sensitivity test, the most compatible size of mesh is lied in the range of 15 mm to 25 mm. Hence, 20 mm was adopted as size of mesh. The solution algorithm was based on the explicit central differential with initial step of 0.2 microsecond and time step of 0.1 millisecond. Fig. 1 illustrates the three-dimensional FDEM model of parent steel beam and castellated steel beams.

Castellated steel beam was assigned with non-linear material model. Therefore, definition of specified material properties was included in the material model as can be seen in Table 2. The Von-Mises criterion was adopted as constitutive law (widely accepted for ductile/homogenous material), previously used by Kumbhar & Jamadar (2015), Taufiq et al. (2018), Frans et al. (2017) and Morkhade & Gupta (2015) in numerical modelling of castellated steel beams. This type of constitutive law was rated as independent model (isotropic) in the form of a non-linear hardening. Piecewise linear hardening data was specified using the relationship of yield stress (275 MPa to 430 MPa) and plastic strain (0.53 % to 2.23 %).

Sinusoidal vibration of 3 Hz, as suggested by Wang et al. (2015), was utilized as dynamic loading. This type of loading, as illustrated in Fig. 2, is a special class of excitation known as forcing function of pure tone with single frequency. Five different load amplitudes of sinusoidal vibration at 1st, 5th, 10th, 15th, 20th and 25th which equivalent with maximum loading of 80 kN, 400 kN, 800 kN, 1200 kN, 1600 kN and 2000 kN were imposed to observe different patterns of fatigue failure. In the finite-discrete element method, the demn properties must be defined to present the geometric parameters and boundary segments associated with discrete elements. There are two types of demn properties that need to be defined, known as discrete element data and contact surface properties, as can be referred in Table 3 and Table 4. It should be noted here that the demn properties were proposed by Bere (2004) and Jaini et al. (2016).

a) Parent steel beam

b) Circular web opening

c) Hexagonal web opening

d) Rectangular web opening

Fig. 1 Three-dimensional FDEM model of castellated steel beams (1/2 span).

Table 2 - Material properties of castellated steel beams.

No.	Property	Value
1	Young's modulus, E (GPa)	210
2	Poisson ratio, υ	0.29
3	Density, ρ (kg/m^3)	7830
4	Yield stress, f_y (MPa)	275
5	Ultimate stress, f_u (MPa)	430

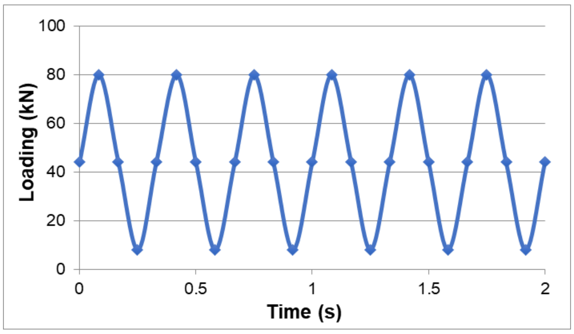


Fig. 2 - Dynamic ultimate stress of castellated steel beam.

Table 3 - Discrete element data.

No.	Parameter	Value
1	Contact damping (%)	60
2	Contact field, l_f (mm)	0.02
3	Dynamic relaxation damping (%)	5.00
4	Contact zone, l_b (mm)	0.75
5	Smallest discrete element, l_s (mm)	0.00

Table 4 - Contact surface properties.

No.	Parameter/Coefficient	Value
1	Normal penalty, α_n (GPa)	300
2	Tangential penalty, α_t (GPa)	30
3	Initial tension cut-off contact, S_o	0
4	Normal contact stress, S_n (GPa)	-10
5	Steel friction coefficient, μ	0.14

Another important data that need to be defined is the dynamic increase factor to control the deformation and damage mechanisms from being excessive than actual behaviour. Since the castellated steel beams were imposed by vibration, hence the dynamic increase factor needs to be incorporate with yield stress and ultimate stress. In this study, the dynamic increase factor was determined based on mathematical equations proposed by Malvar & Crawford (1998) and Naji & Irani (2012), such as:

$$\text{DIF} = \left(\frac{\varepsilon}{10^{-4}}\right)^{\alpha} \tag{1}$$

where, DIF is dynamic increase factor, ε is the strain rate and α is the dynamic coefficient that can be determined based on the yield stress (f_y) and the ultimate stress (f_u).

$$\alpha = 0.0074 - 0.040\left(\frac{f_y}{60}\right) \tag{2}$$

$$\alpha = 0.0019 - 0.009\left(\frac{f_u}{60}\right) \tag{3}$$

Dynamic increase factor was determined at strain rate of 0.0001s^{-1}, 0.001s^{-1}, 0.01s^{-1}, 0.1s^{-1}, 1.0s^{-1}, 10s^{-1}, 100s^{-1} and 1000s^{-1} as can depicted in Fig. 3. By multiplying the dynamic increase factor with ultimate stress, then dynamic ultimate stress of castellated steel beam can be yielded as shown in Fig. 4.

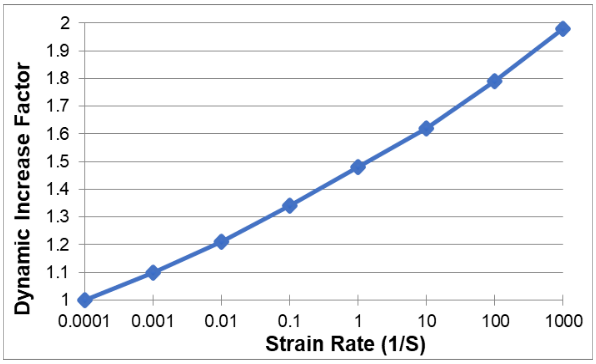


Fig. 3 - Dynamic increase factor of castellated steel beam.

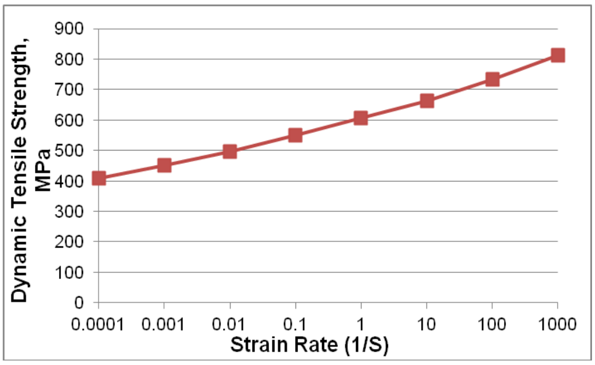
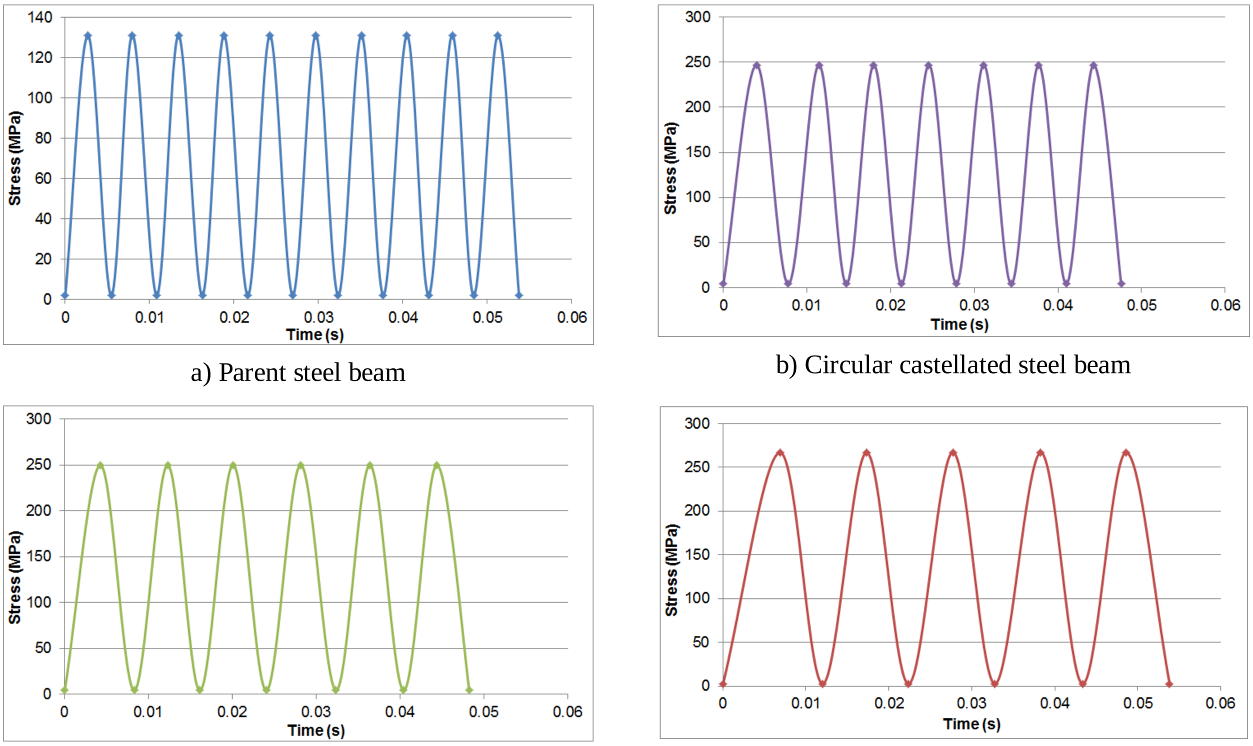


Fig. 4 - Dynamic ultimate stress of castellated steel beam.

3. Results and Discussion

Castellated steel beams subjected to sinusoidal vibration gave a range of fluctuating stresses. This provides the absolute difference between the maximum and minimum values that can be denoted as the stress range. In general, stress range increases steadily with the increment of load amplitude. It was identified that the parent steel beam has the lowest stress range, where the value remains lower than the yield stress. Meanwhile, the rectangular castellated steel beam recorded the highest stress range, followed by the castellated steel beams with hexagonal and circular web openings. Fig. 5 shows the stress time-history that obtained directly from numerical modelling, while Fig. 6 illustrates the contour of stress distribution. The summary of stress range for parent beam and castellated steel beams is tabulated in Table 5.

At the time instant of pre-yielding, castellated steel beams reached the stress value of 250 MPa but the stress distribution shows substantial distinction. In the rectangular castellated steel beam, the highest stress occurs at the corner of web opening, a similar phenomenon observed in the hexagonal castellated steel beam. High stress is distributed around the web opening of fewer corners as it reduced the distribution of shear force across web opening and lead to high stress accumulation. However, circular castellated steel beam has smoother edge in which it even out the shear force distribution hence reduces the stress along the perimeter of web opening. Rectangular shape of web opening has segregated stress distribution at the corners, which proved that this type of castellated steel beam has potentially gain the high value of stress range and lower fatigue life than the other castellated steel beams.



a) Parent steel beam

b) Circular castellated steel beam

c) Hexagonal castellated steel beam

d) Rectangular castellated steel beam

Fig. 5 - Stress time-history at the 5th load amplitude.

a) Parent steel beam

b) Circular castellated steel beam

c) Hexagonal castellated steel beam

d) Rectangular castellated steel beam

Fig. 6 - Stress distribution (along 1/4 span) at the 5th load amplitude.

Table 5 - Summary of stress range, stress mean and stress alternate.

Maximum Load (kN)	Type of Steel Beam	Stress (MPa)		
		σ_R	σ_m	σ_a
80	Parent	25.8	13.3	12.9
	Circular	65.3	32.7	32.7
	Hexagonal	112.2	56.9	56.1
	Rectangular	150.5	75.8	75.2
400	Parent	128.9	66.5	64.5
	Circular	243.0	125.5	121.5
	Hexagonal	246.0	127.0	123.0
	Rectangular	264.0	134.8	132.2
800	Parent	241.9	125.1	120.9
	Circular	269.9	141.5	133.5
	Hexagonal	271.8	144.1	135.9
	Rectangular	274.7	142.7	137.3
1200	Parent	243.8	128.1	121.9
	Circular	273.8	143.1	136.9
	Hexagonal	274.5	142.8	137.3
	Rectangular	278.0	147.0	139.0
1600	Parent	246.7	131.6	123.4
	Circular	274.4	143.8	137.2
	Hexagonal	276.6	144.7	138.3
	Rectangular	278.3	149.9	139.2
2000	Parent	269.6	145.2	134.8
	Circular	275.0	144.5	137.5
	Hexagonal	277.1	146.5	138.6
	Rectangular	281.6	154.2	140.8

Note: σ_R = stress range, σ_m = stress mean and σ_a = stress alternate.

In the numerical assessment, fatigue life under detail categories 90 (for bolted steel beam) and 160 (for rolled steel beam) were specifically investigated. Fatigue life was measured according to the stress range that obtained from the numerical modelling. By plotting the S-N curves, as depicted in Fig. 7, then number of cycles can be determined. It should be emphasized here that the derivation to obtain the S-N curves is mainly governed by the equations provided in BS EN 1993-1-9 (2005). For detail category 90:

$$\log \sigma_R = -0.340 \log N + \log 12299 \tag{4}$$

$$\log \sigma_R = -0.201 \log N + \log 1446 \tag{5}$$

$$\log \sigma_R = \log 35.5 \tag{6}$$

while for detail category 160:

$$\log \sigma_R = -0.340 \log N + \log 21661 \qquad (7)$$

$$\log \sigma_R = -0.201 \log N + \log 2322 \qquad (8)$$

$$\log \sigma_R = \log 64.9 \qquad (9)$$

where, σ_R is the stress range and N is the number of cycles. The limitation and boundary of stress range must be referred accordingly to the code of practise.

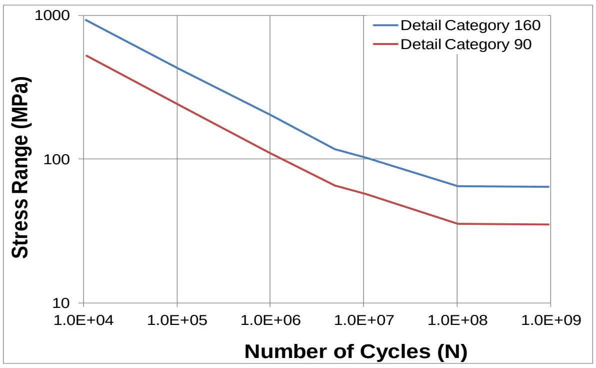
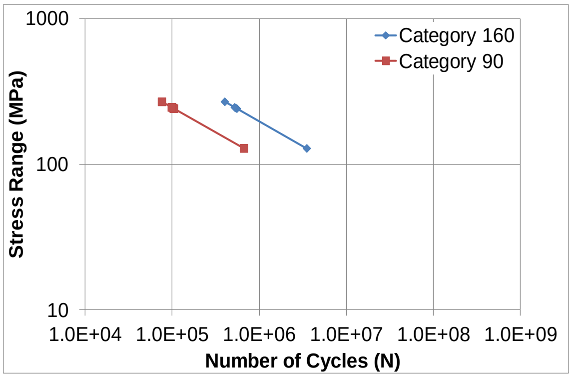


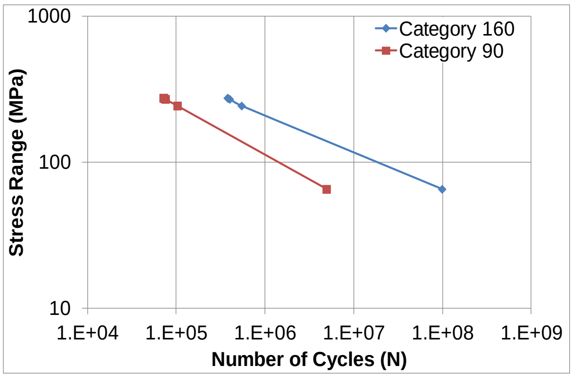
Fig. 7 - S-N curves for castellated steel beams, detail categories 90 and 160.

The observed S-N curve is typical of that for structural material (ideal and accustomed pattern for steel), where fatigue life decreases with increasing the stress range. The data may be fitted by a straight-line on a log-linear plot. The fatigue life attains a plateau value, termed the endurance strength, at about 10^8 cycles. Relationship between stress range and number of cycles for parent steel beam and castellated steel beams can be seen in Fig. 8. A comparison of fatigue life for detail categories 90 and 160 in accordance of the shapes of web opening can also be observed.

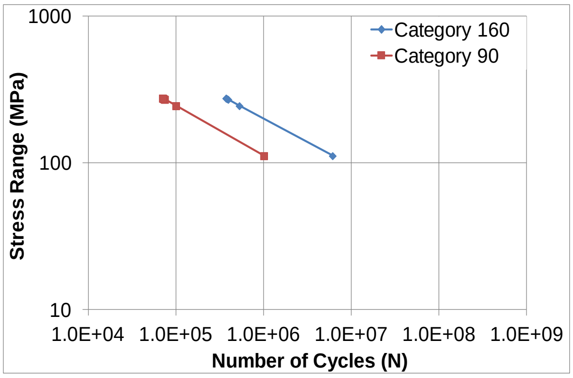
The deflection follows the similar condition of load and unloaded patterns. Therefore, there are maximum and minimum deformations as correspond to the maximum and minimum ranges of sinusoidal vibration. This phenomenon was exceptionally observed within the elastic to the yield stress. Beyond that range, the value of deflection increased significantly until the rupture point. Table 6 shows the deflection of castellated steel beams. It was observed that deflection increases with the increment of load amplitude. Majority of castellated steel beams only survive until 5th load amplitude (400kN). However, circular castellated steel beam accomplishes the serviceability up to 10th load amplitude (800kN) while parent steel beam sustains the sinusoidal vibration until 20th load amplitude (1600kN). Rectangular castellated steel beam shows the highest deflection at all load amplitudes and started to flatten out due to the loading has reached the carrying capacity.



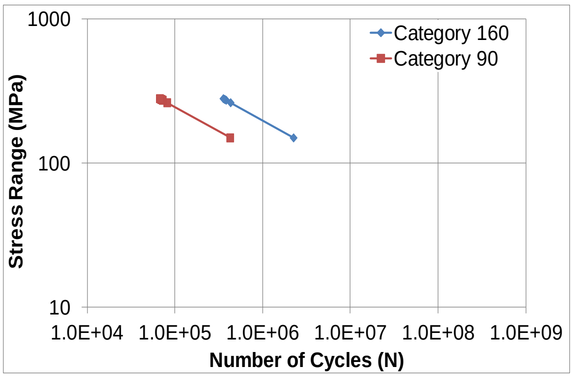
a) Parent steel beam



b) Circular castellated steel beam



c) Hexagonal castellated steel beam



d) Rectangular castellated steel beam

Fig. 8 - Fatigue life under detail categories 90 and 160.

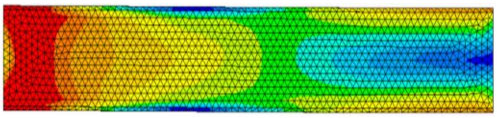
Table 6 - Deflection of castellated steel beams.

Maximum Load (kN)	Type of Steel Beam	*Δ* (mm)	Status
80	Parent	0.4	Passed
	Circular	0.8	Passed
	Hexagonal	1.1	Passed
	Rectangular	1.9	Passed
400	Parent	1.9	Passed
	Circular	3.9	Passed
	Hexagonal	5.1	Passed
	Rectangular	10.3	Passed
800	Parent	3.8	Passed
	Circular	10.0	Passed
	Hexagonal	29.9	Failed

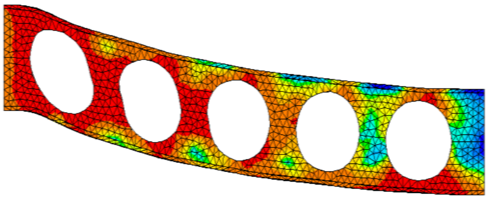
	Rectangular	49.5	Failed
1200	Parent	5.7	Passed
	Circular	52.6	Failed
	Hexagonal	114.5	Failed
	Rectangular	121.7	Failed
1600	Parent	8.2	Passed
	Circular	131.6	Failed
	Hexagonal	310.9	Failed
	Rectangular	228.1	Failed
2000	Parent	20.3	Failed
	Circular	295.0	Failed
	Hexagonal	400.0	Failed
	Rectangular	366.8	Failed

Note: Δ = displacement and status Δ < L/250 = Passed while Δ > L/250 = Failed.

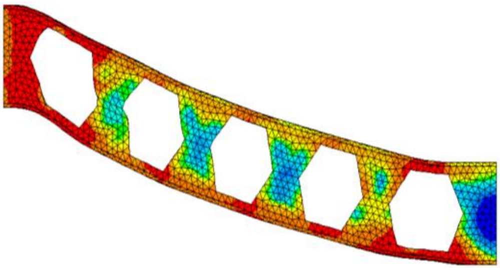
At the yield stress, castellated steel beams remain intact without visible deflection and crack initiation. However, deflection can be observed around time instant after the yield stress. The deflection occurs at the span despite the stress distribution become higher at the supports. Moreover, deflection occurs obviously with the distortion of web opening. At this condition, there is a potential for castellated steel beams to experience Vierendeel mechanism and web weld rupture. Fig. 9 shows the deflection of castellated steel beams that was captured at the 25th load amplitude (2000kN).



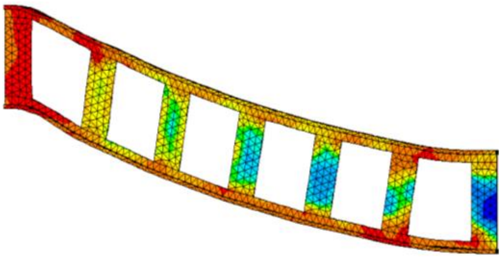
a) Parent steel beam



b) Circular castellated steel beam



c) Hexagonal castellated steel beam



d) Rectangular castellated steel beam

Fig. 9 - Deflection (along 1/2 span) at the 25th load amplitude.

4. Conclusion

Numerical assessment on fatigue failure of castellated steel beams was conducted using the finite-discrete element method program. Various shapes of web opening (circular, hexagonal and rectangular) at different load amplitudes of sinusoidal vibration were considered to investigate the fatigue failure. S-N curve at detail categories 90 and 160 were established, hence fatigue life that denoted by number of cycles is determined based on the stress range. The following are conclusion that can be summarized from this numerical modelling:

- Castellated steel beams under sinusoidal vibration produce fluctuated stress at maximum and minimum ranges, which increase in parallel with the load amplitude.
- Among the castellated steel beams, rectangular web opening recorded highest stress range due to highest stress accumulated at the sharp corner of web opening due to poor shear force distribution. On the other hand, the circular castellated steel beam recorded lower stress range as it has smoother edge, which allows shear force to be distributed evenly across the web opening.
- Therefore, rectangular castellated steel beam was found less suitable to be used as flexural member as it has lower fatigue life. Inversely, the castellated steel beam with circular web opening is likely suitable to be adopted as flexural member as it has a better endure strength under dynamic loading.
- It was justified that fatigue life has significant relationship with the accumulation of stress distribution around the web opening.
- A comparison of fatigue life between detail categories 90 and 160 showed that the presence of weak bolt connection holds lower fatigue life.
- Fatigue life of castellated steel beams can be as high as 9.88×10^7 cycles for detail category 160 and 4.93×10^6 cycles for detail category 90.
- In a nutshell, castellated steel beams imposed by higher load amplitude has lower fatigue life, in which the life cycle of circular castellated steel beam was proved to be better, followed by hexagonal and rectangular.
- Failure mechanism was observed from stress distribution around the web opening. During the yield-stress, stress accumulates around the web opening where crack can be potentially initiated. Castellated steel beams also experience yielding where significant failure mechanism such as Vierendeel mechanism and web post-buckling can be observed.
- A comparison between numerical modelling and limit state indicates that castellated steel beams survive deflection up to 5th load amplitude (400kN), except circular castellated steel beam that has adequate serviceability under 10th load amplitude (800kN).

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