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# Periodic Replenish and Recount Policy to Address Record Inaccuracy from Stock Loss

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To the Graduate Council:

I am submitting herewith a thesis written by Colton K. Ku entitled "Periodic Replenish and Recount Policy to Address Record Inaccuracy from Stock Loss." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Industrial Engineering.

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# Periodic Replenish and Recount Policy to Address Record Inaccuracy from Stock Loss

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## ABSTRACT

Inventory record inaccuracy (IRI) often arises in retail environments due to unaccounted stock loss. Theft, misplacement, spoilage, and transaction errors will reduce the true inventory values without changing the inventory record. As previous inventory replenishment policies assume perfect record accuracy, increasing IRI can cause unexpected stockout events, mistimed reorders and replenishment freezes. Solutions to rectifying IRI vary from the use of improved tracking technologies to prevent it initially occurring at all to recounting programs which estimate true inventory value. Unfortunately, in retail environments, high-tracking technology is unsuitable and continuous counting programs are too costly. To address the limitations of current solutions, we offer a Periodic Replenish and Recount Policy (PRRP) which accounts for stochastic stock loss and minimizes total costs including recounting. The theoretical foundation of PRRP allows for the discovery of both an optimal order quantity as well as optimal count frequency for a given inventory system. We find that in instances of stochastic stock loss, PRRP balances the trade-offs between shortage, surplus and counting costs.

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# CHAPTER ONE

# INTRODUCTION AND GENERAL INFORMATION

## Inventory Inaccuracy from Stock Loss

The ability to accurately track inventory levels is critical for an organization to keep holding costs low and to avoid overstock and stockouts. Inventory managers rely upon accurate data to inform decisions on replenishment strategies, warehouse layouts, and even staffing requirements [1]. Yet, true inventory values often mismatch what they are purported to be within ERP systems. Item misplacement, incorrect counting, theft, and restocking errors are a few of the many ways in which an inventory record can deviate from its true value in real-case scenarios. As seen in Figure 1-1, uncorrected record inaccuracies can become worse over time. As stock loss compounds, the discrepancy can become so large that stockouts and replenishment freezes may occur. Therefore, significant effort has been placed on understanding the nature of inventory record inaccuracy (IRI) as well as methods for its correction [2, 3]. We suggest a gap exists for simple inventories in which expensive tracking technologies are impractical and cycle counting programs ineffective. Our work bridges this gap through the development of a replenishment policy which does not rely upon an accurate inventory record. The policy recommends periodic replenishment of inventory with subsequent recounting after a set number of reorder cycles (Figure 1-2).

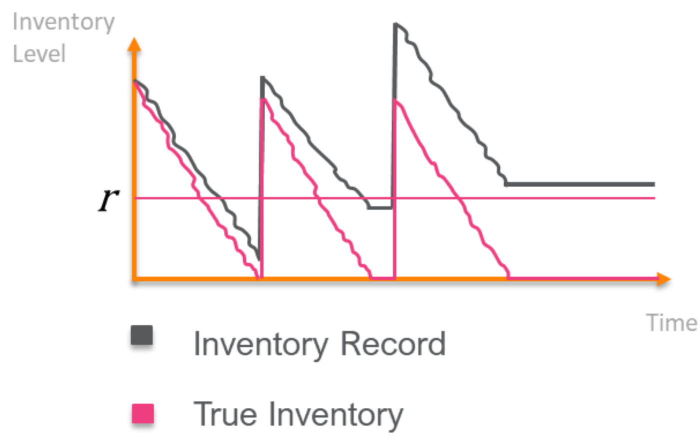



Figure 1-1. Inventory inaccuracy increasing over time due to compounding stock loss

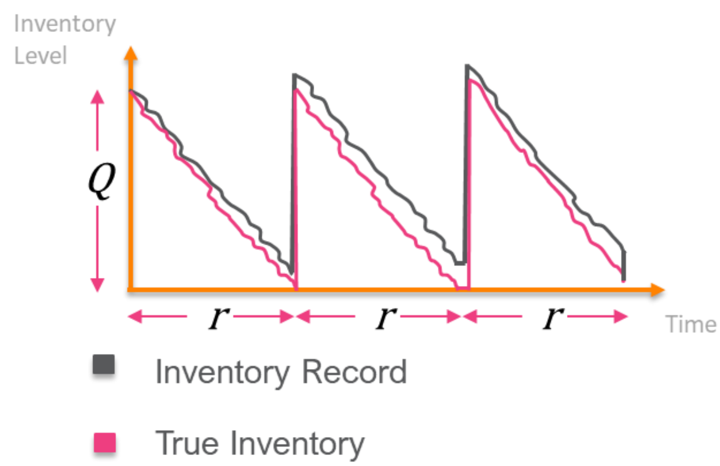



Figure 1-2. A replenishment policy that recommends recounting after three reorders to correct IRI

# CHAPTER TWO

# LITERATURE REVIEW

## Previous Methods Addressing Inventory Inaccuracy

This brief review documents notable discoveries in the causes of IRI as well as the two majority prescribed solutions: RFID and inventory cycle-counting. In 2001, Raman *et al*. reported that over 65% of inventory records from the 370,000 stock keeping units (SKUs) stored at a major retailer were incorrect [4]. Broader studies by Kang *et al*. indicated that for a given company's 500 stores, the highest performance was an accuracy of 75–80%. On average, the inventory accuracy of the company was a little over 50% [5]. Similar reports reflect the prominence of inventory inaccuracy among various industries [6-8]. Our research provides an additional tool for inventory managers to practically rectify these inventory inaccuracies.

Inventory inaccuracy can theoretically occur in either the positive or negative direction, meaning inventory records may either exceed or be less than its true value. However, researchers have more often discovered inventory records to fall below its true value, rather than exceed it [9]. Most presume this fact is due to the more numerous ways an item may be lost or removed from a system when compared to the ways it may be added. Four scenarios in which inventory inaccuracy may arise are:

(i) the true value stays the same and the inventory record increases,
(ii) the true value stays the same and the inventory record decreases,
(iii) the true value increases and the inventory record stays the same, and
(iv) the true value decreases and the inventory record stays the same.

Kang *et al.* rationalized that most causes of inventory inaccuracy may be attributed to stock loss, transaction error, inaccessible inventory, or incorrect product identification [5]. Stock loss occurs through internal theft, external theft, unauthorized consumption, product expiration, spoilage, or damage. Transaction errors may occur at either the inbound receipt or outbound checkout of a good. During the inbound phase, receiving clerks may incorrectly track the receipt amount of a shipment. On the other hand, checkout clerks may decide to scan a single item multiple times instead of each individual item when tasked to scan a batch of similar looking goods. While the same result occurs if the multiple items share the same SKU, similar looking goods may hold different SKUs, resulting in incorrect tracking of outbound inventory. Inaccessible inventory is that which is misplaced due to either a customer or employee moving it from its originally assigned location. Finally, incorrect product identification may occur when a wrong tag or label is placed on an item.

In simulating inventory record errors over time, typical models follow either a Poisson [5] or normal distribution [10]. Additionally, error sources have been found additive with little interaction effects [11]. Our assumptions agree with previous work that stock loss follows a Poisson distribution. We suggest that stock loss is memoryless and that both the loss in a single day and the loss in a full reorder cycle follow a Poisson distribution. This assumption encourages a certain frequency of inventory record rectification to prevent unaccounted stockouts or inventory buildup.

Effects of IRI have been studied in retail supply chains [12], retail outlets [10], and even three-echelon supply chains [11]. Stock loss was shown to predictably cause IRI and stockouts [5], while IRI was also shown to negatively impact service levels [13]. IRI has also been extensively studied to quantify its overall costs. Lee and Özer quantified the value of inventory accuracy through comparison of a perfect inventory record scenario with one of the downstream effects of IRI [14]. Our work is consistent with these findings that the major cost of IRI is attributed to stockouts of SKUs that are recorded as a greater inventory value than its true value.

RFID technology was first invented in 1946 and expanded to commercial use by the 1980s. In the 1990s, standards for its use were implemented across a wide variety of industries. More recently, the technology's affordability has improved and many more companies have adopted its use [15]. Since 2005, emphasis on RFID's impact on supply chain improvements have radiated throughout academia and industry. Specifically, RFID has been proposed to serve as solutions for inventory inaccuracy, bullwhip effects, and optimized replenishment policies [16, 17]. The promise of RFID as a silver-bullet hinges upon its successful implementation. That is, to reap the benefits of RFID, an inventory management system must first correctly add either passive [18] or active RFID tags to each item or pallet of inventory which needs to be tracked. We find the assumption that this task is viable, or even possible in many situations, unlikely. While collaborating with a typical small-sized organization with less than 10,000 SKUs, the capital costs of an RFID system were found inhibitory. Therefore, our work recognizes the benefits of RFID, but chooses to focus on a practical method of IRI correction which can be implemented with little to no cost to a firm.

Besides RFID technology which promises to automatically sync the inventory record with its true value, a second option inventory managers have available to correct IRI is a physical inspection. Physical inspections are costly in both time and capital and are typically used as sparingly as possible. For most small and mid-sized uninformed firms, this translates to an annual inspection for auditing and accounting purposes. On the other hand, large-sized firms have recently implemented cycle-count programs which verify inventory accuracy through an on-going basis of counting only a portion of randomly selected items [3]. Through statistical methods, reasonable levels of inventory accuracy can be achieved with moderate confidence levels without the inhibitory costs associated with a full physical count. Kök and Shang further improved upon these methods through their development of an inspection adjusted base-stock (IABS) policy and cycle-count policy with state-dependent base-stock levels (CCABS) [19, 20]. While both the IABS and CCABS solutions for IRI are massive improvements over non-inspection policies, their implementation is still predicated upon the use of ABC classification systems. These systems introduce bias within the policy, as ABC classification inherently requires subjective weights to be applied to each grouping [21, 22]. Thus, a gap is identified in which a non-biased inspection policy would bridge. Our inspection policy allows practitioners with little to no experience in statistical methods to ensure the highest inventory accuracy without the inhibitory costs of RFID or the complex set-up of cycle-count programs. Additionally, our non-biased approach uses easily accessible parameters to determine optimal count frequencies.

# CHAPTER THREE

# MATERIALS AND METHODS

## Business Assumptions and Sequential Events

Consider a single-item inventory in which the following occurs sequentially:

0. Inventory begins at $Q$
1. Each day, inventory decreases by the sum of a deterministic amount $D$ and a random amount $V$
    a. $D$ represents demand which is accurately accounted through traditional inventory tracking technologies
    b. $V$ follows a Poisson distribution and represents stochastic stock loss which reduces inventory levels without updating the inventory record
2. If the sum of $D$ and $X$ is greater than the current inventory level, a stockout event occurs and no backorder is created (lost sale)
    a. In the case of a stockout, the inventory level will be reset to $Q$ upon the next reorder
3. After a predetermined period of days $r$, $Q$ is added to the inventory level
4. After a predetermined number of reorder periods $n$, the inventory is counted and reset to $Q$

Because the reorder period is predetermined, lead time is not considered. Additionally, opportunities to count the inventory are restricted to the ends of reorder periods to minimize the required labor. Only three costs are considered in this study: shortage cost, holding cost, and counting cost. At the end of a reorder period, shortage cost is incurred for every stockout which occurred in the reorder cycle. Similarly, if the ending inventory is positive, a holding cost is incurred for every positive unit of the inventory level. Counting costs are only incurred if the inventory is counted. The benefit to counting is to correct any record inaccuracies and reduce excess inventory. Because both ordering costs and holding costs for cycle inventory are considered sunk costs, they are not considered in the analysis of this model.

The purpose of this study is to evaluate how changes to the reorder quantity $Q$ and reorder cycles until recount $n$ affects total costs. It was found that due to the higher sensitivity of the model to holding costs rather than shortage costs, frequent counting to reduce excess inventory buildup is generally rewarded.

## PRRP Under Deterministic Stock Loss

First, we can determine the optimal reorder period $r$ for a theoretical scenario in which both demand $D$ and stock loss $V$ are deterministic and known. In this case, the sum of $D$ and $V$ may be considered an aggregate demand. After the exact length of $\frac{Q}{D+V}$ days, inventory will be completely exhausted (Figure 3-1). Once the ordering and cycle inventory holding costs are known, $Q$ may be set to the economic order quantity and the reorder period $r$ set to $\frac{Q}{D+V}$.

If we consider that stock loss $V$ is 'known but invisible,' i.e., our estimation of stock loss is perfect, yet the inventory record only takes account $D$, we can determine that every reorder cycle adds the unaccounted stock loss as 'ghost inventory' to the baseline (Figure 3.2). We can also determine how much earlier $r$ should be in comparison to a standard EOQ model which does not consider stock loss by taking the difference between reorder periods (Figure 3.3). Intuitively, the reorder period is much shorter since more inventory is removed from the system every day. Since in the deterministic case we can perfectly predict inventory level, there is no need to implement any recounting.

While this deterministic scenario does not accurately represent a real-life system, it is important to first consider it to gain a baseline understanding of how stock loss compounds and creates a larger and larger discrepancy between the true inventory and the inventory record. In the stochastic case, stock loss is random such that within a reorder period there may be more or less than the expected average. If less stock loss occurs, then at the end of a reorder cycle there will be remaining inventory in the system. On the other hand, if there is more stock loss than the average, stockout events will occur.

## PRRP Under Stochastic Stock Loss

We next extend our model to the case in which stock loss $V$ is random and follows a Poisson distribution with parameter $\lambda$. Because the replenishment system is automated, stockouts are not immediately apparent to inventory managers. Instead, the stockouts simply reduce sales to zero for that and any subsequent days. Once the reorder cycle completes, then the next order quantity $Q$ is added to the system such that the new inventory level is $Q$. For the case in which less stock loss than the average occurs, surplus inventory is held. Once the reorder cycle completes and the next order quantity is added to the system, the next reorder cycle will begin with an inventory level as the sum of $Q$ and the surplus inventory from the previous cycle.

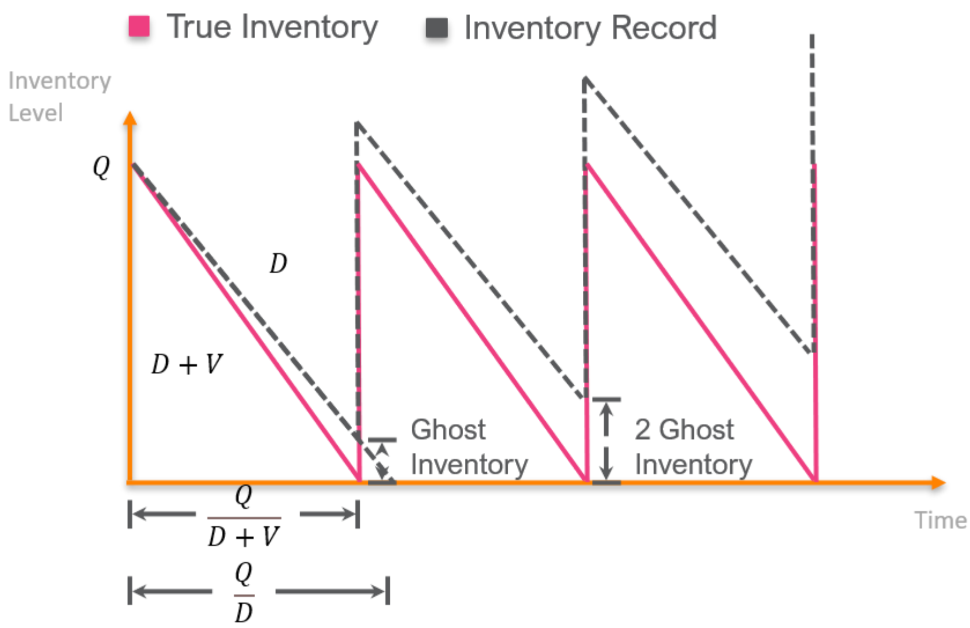



Figure 3-1. True inventory and inventory record under deterministic demand and stock loss

$$Ghost\ Inventory = \ Q - D\left(\frac{Q}{D+V}\right)$$

$$Ghost\ Inventory = \frac{QV}{D+V}$$

Figure 3-2. Formulation of Ghost Inventory per reorder cycle

$$Reorder\ Time\ Difference = \frac{Q}{D} - \frac{Q}{D+V}$$

$$Reorder\ Time\ Difference = \frac{QV}{D(D+V)}$$

Figure 3-3. Formulation of difference between EOQ $r$ and PRRP $r$

Due to this unique property of the inventory system resetting to $Q$ upon stockout but accumulating inventory for periods without stockout, it is expected that the first reorder cycle would always have the highest probability of stockout given no additional safety stock was added. Figure 3-4 illustrates a single reorder cycle operating under stochastic stock loss in which two different scenarios may arise: the inventory ends with a shortage, and the inventory ending with a surplus.

### *Formulation of $R_n$*

$R_n$ is defined as the probability of surplus through $n$ reorder cycles.

The probability of the system ending with a surplus may be calculated using the probability mass function of a Poisson distribution (Figure 3-5). If the stock loss is less than or equal to $Q - Dr$, no stockout will occur. However, the probability of surplus of subsequent reorder cycles are dependent upon previous cycles. For example, if there was very little stock loss within the first reorder cycle, then the probability of surplus in the next reorder cycle would be greatly increased. To calculate the exact probability of a surplus in any given reorder cycle, a recursive summation is used in which increasing bounds represent the increased chance of surplus given a previous surplus. This recursive relationship is stated in the following lemma.

**Lemma 1**:
Assuming stock loss follows a Poisson distribution, when ending inventory of the first reorder cycle is $y$, the **probability of surplus** through $n$ reorder cycles is

$$R(n,\, y) = \begin{cases} \displaystyle\sum_{i=0}^{y} \frac{\lambda r^i}{i!} e^{-\lambda r} & n = 1; \\ \displaystyle\sum_{i=0}^{y} \frac{\lambda r^i}{i!} e^{-\lambda r} \cdot R(n-1, y + Q - Dr - i) & n \geq 2. \end{cases}$$

### *Formulation of $S_n$*

$S_n$ is defined as the probability of stockout during the $n^{\text{th}}$ reorder cycle.

Once the probability of surplus is known, the probability of a stockout event occurring can be easily calculated considering the overall sample space (Figure 3-6). In a single cycle, either a stockout event occurs or a surplus event occurs so the stockout probability is the difference between one and the

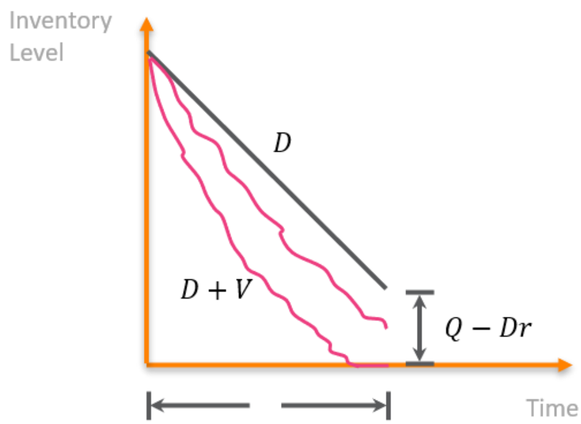



Figure 3-4. Single reorder cycle with possible scenarios shown in pink

$$R_1 = \sum_{W=0}^{Q-Dr} \frac{(\lambda r)^W e^{-\lambda r}}{W!}$$

Figure 3-5. Probability of surplus through the first reorder cycle

$$S_1 + R_1 = 1$$
$$S_1 + S_2 + R_2 = 1$$
$$S_1 + S_2 + S_3 + R_3 = 1$$
$$\ldots$$
$$S_n = R_{n-1} - R_n$$

Figure 3-6. Relationship between $R_n$ and $S_n$

probability of surplus. In higher reorder cycles, the sample space is composed of stockout events occurring at any one of the cycles or a surplus event occurring throughout. The probability of stockout can thus be calculated according to the following lemma.

Additionally, the probability of stockout in the $n^{\text{th}}$ reorder cycle may be fundamentally calculated through use of the probability mass function in an analogous method to calculating the probability of surplus through $n$ reorder cycles. If the stock loss in the first reorder cycle is greater than the maximum ending inventory $Q - Dr$, then a stockout event will occur. Due to the resetting behavior of the system and the way $S_n$ is defined, $S_2$ means that the first reorder cycle ended in surplus, then the stockout occurred in the second reorder cycle. Similarly, $S_3$ means a surplus occurs in the first two reorder cycles.

**Lemma 2**:
Assuming stock loss follows a Poisson distribution, when ending inventory of the first reorder cycle is $y$, the **probability of stockout** during the $n^{\text{th}}$ reorder cycle is

$$S(n, y) = \begin{cases} \displaystyle\sum_{i=y+1}^{\infty} \frac{\lambda r^i}{i!} e^{-\lambda r} & n = 1; \\ \displaystyle\sum_{i=0}^{y} \frac{\lambda r^i}{i!} e^{-\lambda r} \cdot S(n-1, y + Q - Dr - i) & n \geq 2. \end{cases}$$

Lemma 2 shows how a recursive function may again be used to model the same behavior as $R_n$ for the probability of stockout. Interestingly, the only difference between the two lemmas is the bounds of the most internal summation. The surplus formulation necessitates that stock loss in every reorder cycle be less than the maximum allowable inventory level for that cycle. In contrast, the stockout formulation necessitates that stock loss in every reorder cycle except the last one be less than the maximum allowable inventory level for that cycle. In the last reorder cycle, the stockout necessitates that stock loss be greater than the maximum allowable inventory level.

### *Expected Surplus Inventory*

$P_n$ is defined as the expected surplus in the $n^{\text{th}}$ reorder cycle.

With the probabilities of surplus and stockout in hand, the expected surplus inventory was next formulated. By utilizing the probability of any given stock loss occurring as a weight for the ending inventory values, a weighted average can be calculated. By selectively choosing probabilities and ending inventories that only result in surplus, the specific expected surplus inventory may be calculated for any inventory cycle. This relationship is given in the following lemma.

**Lemma 3**:
Assuming stock loss follows a Poisson distribution, when ending inventory of the first reorder cycle is $y$, the **expected surplus amount** in the $n^{\text{th}}$ reorder cycle is

$$P(n,y)=\begin{cases}\sum_{i=0}^{y}\frac{\lambda r^{i}}{i!}e^{-\lambda r}\cdot(y-i) & n=1;\\ \sum_{i=0}^{y}\frac{\lambda r^{i}}{i!}e^{-\lambda r}\cdot P(n-1,y+Q-Dr-i) & n\geq 2.\end{cases}$$

### *Expected Shortage*

$G_n$ is defined as the expected shortage in the $n^{\text{th}}$ reorder cycle.

In the same manner as the surplus formulation, expected shortage in the $n^{\text{th}}$ reorder cycle may be calculated by utilizing the probability of any given stock loss occurring as a weight for the ending inventory values. The only difference is that the selection of probabilities is limited in the last reorder cycle to be the stock loss which exceeds the maximum allowable inventory level for that cycle. This relationship is explained in the following lemma.

**Lemma 4**:
Assuming stock loss follows a Poisson distribution, when ending inventory of the first reorder cycle is $y$, the **expected shortage amount** in the $n^{\text{th}}$ reorder cycle is

$$G(n,y)=\begin{cases}\sum_{i=y+1}^{\infty}\frac{\lambda r^{i}}{i!}e^{-\lambda r}\cdot(y-i) & n=1;\\ \sum_{i=0}^{y}\frac{\lambda r^{i}}{i!}e^{-\lambda r}\cdot G(n-1,x,y+Q-Dr-i) & n\geq 2.\end{cases}$$

### *Sample Spaces of Various PRRPs*

Using lemmas one through four, the probability of stockouts and surplus as well as the amount of surplus and shortage may be calculated for any reorder cycle. However, in consideration of a PRRP, we must analyze the entire sample space for any set number of reorder cycles. For example, in a one-cycle PRRP, there are two possible outcomes: either the inventory ends in surplus, or the inventory ends in a shortage. This is represented graphically in Figure 3-7. The complexity increases as more reorder cycles are added. In a Two-Cycle PRRP, there are four possible outcomes as shown in Figure 3-8. In a Three-Cycle PRRP, there are eight possible outcomes as shown in Figure 3-9.

The probabilities of any single event may be calculated simply through the product of individual probabilities. For example, the second event shown in Figure 3-13 begins with a stockout in the first reorder cycle followed by two subsequent surpluses. The probability of a stockout occurring during the first reorder cycle is given by $S_1$. The probability of surplus occurring through two reorder cycles is given by $R_2$. Because a stockout resets the inventory to $Q$, the probability of having a stockout in the first cycle followed by two surpluses is given by $S_1R_2$.

In the third event shown in Figure 3-9, a surplus occurs in the first reorder cycle, followed by a stockout in the second and a surplus in the third. Because $S_n$ is defined as the probability of having a stockout during the $n^{\text{th}}$ reorder cycle, $S_2$ represents having a surplus in the first reorder cycle and then a stockout during the second reorder cycle. So, the probability of a surplus occurring in the first reorder cycle, followed by a stockout in the second and a surplus in the third is represented by $S_2R_1$.

### ***Expected Shortage and Surplus of Individual Events for a PRRP***

The expected shortage and surplus for any individual event in a PRRP may be formulated as illustrated in Figure 3-10. In the first example, two surplus reorder cycles occur before a stockout in the third reorder cycle. The expected shortage follows exactly the definition of $G_3$. The expected surplus is the result of only the first two reorder cycles. Thus, the expected surplus for the first event is given by $P_2$. In the second event of Figure 3-10, a surplus occurs in the first reorder cycle followed by stockouts in the subsequent two reorder cycles. The expected surplus is given by $P_1$, the surplus expected in a single reorder cycle. However, the expected shortage is given by $S_2G_1$, indicating a surplus occurring in the first reorder cycle, a stockout in the second, and a final stockout in the third with expected shortage of $S_2G_1$.


Figure 3-7. Graphical representation of a One-Cycle PRRP sample space

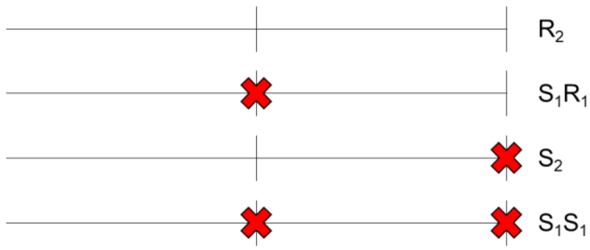



Figure 3-8. Graphical representation of a Two-Cycle PRRP sample space

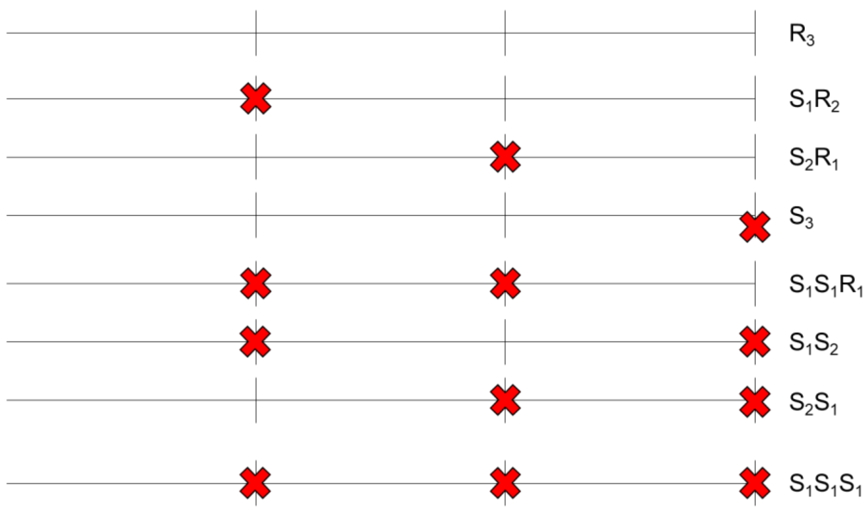
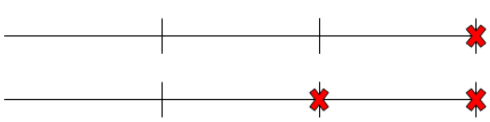



Figure 3-9. Graphical representation of a Three-Cycle PRRP sample space



| Probability | Expected Shortage | Expected Surplus |
|---|---|---|
| $S_3$ | $G_3$ | $P_2$ |
| $S_2S_1$ | $S_2G_1$ | $P_1$ |

Figure 3-10. Analysis of single event in a Three-Cycle PRRP

# CHAPTER FOUR

# RESULTS AND DISCUSSION

## Simulation of Periodic Replenishment Under Stochastic Stock Loss

To validate the probability, stockout, and surplus inventory formulations, a simulation was created and run. The results of which can be seen in Figure 4-1 and Figure 4-2. For each run, once a stockout event occurred, the simulation terminated. By terminating the simulation at the first occurrence of a stockout, the ratio of stockouts in each cycle to total number of simulations represents $S_n$. In the example shown, variables and parameters of $Q = 420$, $D = 13$, $\lambda = 1$, $r = 30$ were chosen. According to the probability formulation, these variables and parameters correspond to $S_1 = 0.45$, $S_2 = 0.13$ and $S_3 = 0.07$. The simulation is thus validated as $4588$ of the $10{,}000$ runs encountered a stockout within the first cycle, while $1346$ of the $10{,}000$ experienced a surplus in the first cycle and stockout in the second, and $665$ of the $10{,}000$ experienced a surplus in both the first and second cycles and stockout in the third.

Additionally, the excess inventory formulation may be validated through further analysis of the same $10{,}000$ simulated runs. The average inventory of the last day before a restock was captured and documented for every run that did not end in a stockout under the fourth column of Table 1. This extra inventory is calculated as the expected surplus inventory shown in column 5 of Table 1. With these various formulations validated, the next step in the study is to conduct a scenario analysis in which the costs are analyzed of various PPRPs to evaluate the benefits and detriments of varying reorder quantities and recount frequencies.

## Cost Function

$TC(n)$ : Expected total cost for PRRP with recount after $n$ reorder cycles
$h$ : Cost of surplus ($/unit) incurred at the end of each reorder cycle
$b$ : Cost of shortage ($/unit) incurred at the end of each reorder cycle
$c$ : Cost of counting incurred at the end of the $n^{\text{th}}$ reorder cycle

Calculating the total cost of a PRRP requires multiplying the total surplus across $n$ reorder cycles by the parameter $h$, multiplying the total shortage across

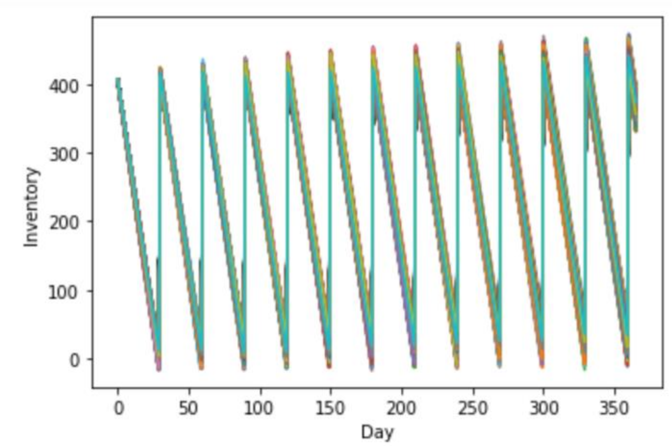
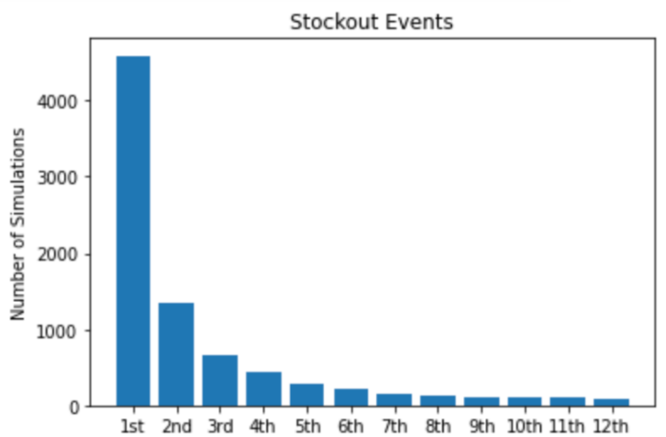



Figure 4-1. Inventory vs time for 10,000 simulated runs

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[4588, 1346, 665, 433, 286, 211, 160, 137, 111, 103, 101, 85]
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Figure 4-2. Stockout events for 10,000 simulated runs

Table 1. Comparison of expected surplus values vs average of $10{,}000$ simulated runs

| **Order cycle $n$** | **Sim $R_n$** | **Exp $R_n$** | **Sim $P_n$** | **Exp $P_n$** |
|---|---|---|---|---|
| 1 | 0.5412 | 0.54835 | 2.1237 | 2.1790 |
| 2 | 0.4066 | 0.41748 | 2.6478 | 2.7378 |
| 3 | 0.3401 | 0.34996 | 2.9345 | 3.0162 |
| 4 | 0.2968 | 0.30718 | 3.1171 | 3.1897 |

$n$ reorder cycles by the parameter $b$, and taking the total sum with parameter $c$. Using the sample space as a guide, $TC(1)$ is therefore calculated as:

$$TC(1) = G_1 b + P_1 h + c$$

$TC(2)$ can similarly be calculated by using its corresponding sample space:

$$TC(2) = (G_1 + G_2 + S_1 G_1)b + (P_1 + P_2 + S_1 P_1)h + c$$

As it becomes quite cumbersome to continually use the sample space to deduce the total cost function, a recursive formulation can be used for the total cost of higher PRRPs. This relationship is illustrated in Theorem 1 below. The relationship arises from how additional reorder cycles double the previous PRRP's sample space. For example, a Three-Cycle PRRP sample space is comprised of eight different possible events. The first four are characterized by a surplus in the first reorder cycle followed by the events of a Two-Cycle PRRP. The remaining four events are characterized by a shortage in the first reorder cycle followed by the events of a Two-Cycle PRRP. The cost function of any PRRP can thus be calculated using the cost function of the previous PRRP.

**Theorem 1**:
The total cost of a PRRP of one recounting cycle with $n$ reorder cycles until recount is

$$TC(n) = \begin{cases} \sum_{i=1}^{n} bG(i) + hP(i) + c & n = 1; \\ \sum_{i=1}^{n} bG(i) + hP(i) + \sum_{j=1}^{n-1} S(j)(TC(n-j) - c) + c & n \geq 2. \end{cases}$$

## Scenario Analysis

To determine the value of counting inventory early, a standard set of parameters was chosen and systematically altered. The effect on total cost per reorder cycle was then documented and compared. First, the order quantity $Q$ was changed while keeping $D = 6$, $\lambda = 1$, and $r = 14$ constant. Parameters $h$, $b$, and $c$ were arbitrarily set to 100. The results can be seen in Table 2. Cells highlighted in green show the lowest cost counting frequency for each order quantity. Reducing the order quantity can achieve a lower total cost than the expected optimal $Q$ of $98$. This illustrates the fact that, given equal cost parameters, the model is much more sensitive to surplus inventory than to shortages. This can be explained because stockouts occur as a lost sale, so negative inventory does not exist. If a shortage were to occur in a reorder cycle, the inventory is reset to $Q$ upon the next restock. On the other hand, if a reorder cycle were to end in surplus, the inventory will carry over to the next reorder cycle. This means that surplus inventory may be penalized by the model multiple times if a stockout does not occur. Counting serves to minimize the buildup of surplus inventory, which explains why increasing the order quantity $Q$ necessitates more frequent counting to achieve the lowest total cost.

Next, parameters $h$ and $b$ were changed to represent a more realistic scenario. Surplus cost $h$ was reduced to $25$ and shortage cost $b$ increased to $200$. As illustrated in Table 3, the optimal order quantity $Q$ and recounting frequency dramatically changes depending on the nature of the stockout, inventory holding, and recounting cost. Due to the higher penalty of stockout, the model rewards increasing the order quantity $Q$ to minimize the probability and severity of shortages. Increasing the order quantity $Q$ to $101$ achieves the lowest cost when paired with a counting frequency of every two reorder cycles. Because surplus inventory will still increase over time, counting is necessary to reset the inventory and ensure surplus holding cost is kept to a minimum. The larger the order quantity is set, the more frequent counting is required.

Table 2. Effects of $Q$ and $n$ on total cost per reorder cycle with equal cost parameters

| $Q$ | $n = 1$ | $n = 2$ | $n = 3$ | $n = 4$ | $n = 5$ | $n = 6$ |
|---|---|---|---|---|---|---|
| 96 | 382.19 | 356.29 | 349.74 | 347.94 | 347.79 | 348.30 |
| 97 | 353.88 | **339.42** | 343.53 | 351.64 | 360.69 | 369.76 |
| 98 | 346.77 | 352.81 | 377.09 | 404.99 | 433.49 | 461.67 |
| 99 | 360.86 | 397.01 | 451.78 | 510.52 | 570.19 | 629.84 |
| 100 | 394.73 | 469.51 | 563.71 | 662.62 | 763.13 | 864.24 |
| 101 | 445.91 | 565.68 | 705.49 | 850.60 | 997.82 | 1,146.07 |
| 102 | 511.35 | 680.04 | 868.87 | 1,063.16 | 1,259.71 | 1,457.40 |
| 103 | 587.88 | 807.43 | 1,046.68 | 1,291.21 | 1,537.90 | 1,785.69 |
| 104 | 672.58 | 943.59 | 1,233.59 | 1,528.54 | 1,825.51 | 2,123.49 |

$D = 6 \quad \lambda = 1 \quad r = 14 \quad \boldsymbol{h = 100} \quad \boldsymbol{b = 100} \quad \boldsymbol{c = 100}$

Table 3. Effects of $Q$ and $n$ on total cost per reorder cycle with varying cost parameters.

| $Q$ | $n = 1$ | $n = 2$ | $n = 3$ | $n = 4$ | $n = 5$ | $n = 6$ |
|---|---|---|---|---|---|---|
| 96 | 648.71 | 568.79 | 535.11 | 515.73 | 502.94 | 493.83 |
| 97 | 529.37 | 446.62 | 411.15 | 390.56 | 376.94 | 367.24 |
| 98 | 433.87 | 353.87 | 321.56 | 304.32 | 294.16 | 287.99 |
| 99 | 362.21 | 291.14 | 267.71 | 259.33 | 258.03 | 260.72 |
| 100 | 312.82 | 255.91 | 246.05 | 250.87 | 262.53 | 278.01 |
| 101 | 282.90 | **243.53** | 249.82 | 269.93 | 296.29 | 326.05 |
| 102 | 269.02 | 248.50 | 271.54 | 307.17 | 348.26 | 392.22 |
| 103 | 267.62 | 265.70 | 304.88 | 355.37 | 410.57 | 468.18 |
| 104 | 275.40 | 290.97 | 345.30 | 409.85 | 478.55 | 549.35 |

$D = 6 \quad \lambda = 1 \quad r = 14 \quad \boldsymbol{h = 25} \quad \boldsymbol{b = 200} \quad \boldsymbol{c = 100}$

# CHAPTER FIVE

# CONCLUSIONS AND RECOMMENDATIONS

## Managerial Considerations

In comparison to traditional $(r, Q)$ or $(s, S)$ policies, PRRP brings two distinct advantages. First, the true inventory value is never assumed accurate. This fundamental assumption of traditional policies is the main cause of unexpected stockouts and potential replenishment freezes. With PRRP, after an initial phase of gathering information to estimate average stockout and demand, an ideal frequency of recounting can be discovered. The true inventory value at any time between recounts is not assumed accurate to the inventory record. The second advantage PRRP holds over traditional replenishment policies is the consistent order quantity and period. An often-over-looked aspect of replenishment policies is the willingness of the supplier to maintain flexibility during times of distress. For example, in an $(r, Q)$ policy, the order period fluctuates while the order quantity remains constant. If a period of unaccounted stock loss causes a rush-order from the supplier, the overall supply chain may become stressed. In many cases, the order may not be filled completely or on time. Similarly, an $(s, S)$ policy often requires the supplier to dynamically change shipment quantities depending on how much is needed to order up to the maximum. PRRP may allow for better planning throughout the entire supply chain, as the replenishment policy maintains consistent order quantity and order periods between recounting cycles.

The value of recounting was also shown through the decrease in total cost of PRRPs with higher count-frequencies. While other strategies to address IRI exist, RFID and cycle-count programs are not applicable to all systems. PRRP is particularly suitable for retail environments in which the chance of theft and misplacement is high. In such areas, it is also unlikely that traditional barcodes would be replaced with expensive tracking technologies, such as RFID. PRRP offers a complementary method to address stock loss and inventory record inaccuracy with minimal up-front investment.

# Conclusion

In this study, a new strategy for inventory replenishment was developed that does not assume perfect inventory record accuracy. It was found that for any given PRRP, the first reorder cycle has the highest chance of stockout. Subsequent reorder cycles reduce stockout probability at the cost of holding extra inventory. This continues indefinitely with average extra inventory increasing every reorder cycle. Once a recount is issued, the inventory may be reset to its original level. Surprisingly, this means that initiating a recount will increase the chance of stockout due to depletion of the extra inventory. The optimal trade-off between cost of stockouts, cost of excess inventory and cost of recounting was shown possible to be calculated through use of recursive formulation.

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# VITA

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