12-2020

Supplier Ranking System and Its Effect on the Reliability of the Supply Chain

Farshad Rabib
kjalili@vols.utk.edu

To the Graduate Council:

I am submitting herewith a dissertation written by Farshad Rabib entitled "Supplier Ranking System and Its Effect on the Reliability of the Supply Chain." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Industrial Engineering.

Dr. Andrew Yu, Major Professor

We have read this dissertation and recommend its acceptance:

Dr.James Simonton,Dr. John Kobza,Dr. Reza Abedi

Accepted for the Council:

Dixie L. Thompson

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

Supplier Ranking System and Its Effect on the Reliability of the Supply Chain

A Dissertation Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

Farshad Rabib

December 2020

DEDICATION

I dedicate this project to my beautiful daughter Zara, who was born during this century's worst pandemic and political turmoil. Zara is the reason why I was able to complete this endeavor, and I wish for her to know that every time I felt like giving up, it was her smile that gave me energy to continue.

ACKNOWLEDGEMENTS

First, I want to express my deep appreciation to my advisor, Dr. Andrew Yu, who patiently mentored me, and provided his professional expertise so that I could complete this research in a timely fashion.

Also, I want to thank Dr. James Simonton for always being there when I needed support and guidance.

Furthermore, I want to thank Dr. John Kobza and Dr. Reza Abedi, for their critical reviews, guidance and encouragement.

Finally, my gratitude extends to the department's management specialist, Charlotte Henley, who went above and beyond to communicate with me via emails and provide the necessary information and resources needed during my entire graduate study.

ABSTRACT

Today, due to the growing use of social media and an increase in the number of customers sharing their opinions globally, customers can review products and services in many novel ways. However, since most reviewers lack in-depth technical knowledge, the true picture concerning product quality remains unclear. Furthermore, although product defects may come from the supplier side, making it responsible for repair cost, it is ultimately the manufacturer whose name is damaged when such defects are revealed. In this context, we need to revisit the cost vs. quality equations. Observations of customer behavior towards brand name and reputation suggest that, contrary to the currently dominant model in production where manufacturers are expected to control only Tier 1 supplier and make it responsible for all higher tiers, manufacturers should also have a better hold on the entire supply chain. Said differently, while the current system considers all parts in Tier 1 as equally important, it underestimates the importance of the impact of each piece on the final product. Another flaw of the current system is that, by making common the pieces in several different products, such as different care models of the same manufacturer to reduce the cost, only the supplier of the most common parts will be considered essential and thus get the most attention during quality control. To address the aforementioned concerns, in the present study, we created a parts/supplier ranking algorithm and implemented it into our supply chain system. Upon ranking all suppliers and parts, we calculated the minimum number of the elements, from Tier 1 to Tier 4, that have to be checked in our supply chain. In doing so, we prioritized keeping the cost as low as possible with most inferior possible defects.

PREFACE

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1: INTRODUCTION

“Oh, the things you can find, if you don’t stay behind!” - Dr. Seuss

Over the last decade, due to the growing use of social media where customers and stakeholders share their experiences, on the one hand, and the increase of global competition, on the other hand, companies’ quality, reputation, and brand prestige have become essential, particularly in the car manufacturing industry. This has driven corporations to focus increasingly more on the quality of their products. However, as argued by Goicoechea and Fenollera (2012), the “factor of who assesses the quality, and what is the evaluation based on, is always decisive. However, the bottom line is still determined by customers’ or stakeholders’ requirements” (pp. 619-631).

Consumers’ capacity to share their experience with products on social media in today’s digital world has been viewed as a double-edged sword by companies. On the one hand, via electronic world-of-mouth (e-WOM), brand visibility of a company that produces excellent products can spread globally very quickly, and such products can shift to the “must buy” list. For instance, this was the case of Beats Electronics, a headphone company. In 2013, just only five years after the company was established, positive reviews from consumers brought Beats Electronics a profit of approximately $1.5 billion (Karp, 2014). However, reputation of a well established company can be ruined just as easily. For instance, in 2014, the Takata corporation, manufacturer of auto airbags, suffered a loss of $245 million due to problems with its vehicle airbags (Soble, 2015). Furthermore, while social media can serve as a global news system where the e-WOM spreads extremely quickly, product users frequently lack a deep understanding or knowledge of the products they are reviewing. In the case of the Takata airbags supplied to Toyota, consumers saw only the final result and published reviews claiming that Toyota cars were dangerous because of the airbag system failure.

What the examples provided above illustrate is that no matter which tier supplier is at fault in the process of getting the product from Tier 1 to Tier 4, if a fault is found in the quality of the final product, it is the company that is always to blame. Accordingly, in analyzing the risk associated with a company's reputation, Deloitte's (Papakonstantinidis, 2019) annual survey stated that "reputation damage is the number-one risk concern for business executives around the world (p. 37). This survey also found that about 87% of 300 interviewed senior figures rated reputation risk as "more important or much more important than other strategic risks their companies are facing" (Ibid., p. 3).

A great concern for the companies is the risk that their reputation is damaged as a result of actions of Tier 2 or Tier 3 suppliers. While increasing quality control of Tier 1 suppliers, manufacturers frequently neglect a strict selection for Tier 2 and Tier 3 partners, which can lead to accidents. This calls for a thorough revision of corresponding control regulations, such as introducing the economic standard of quality and the method of finding the lack of selection which should not be left at the chance.

Along with the importance of a company's reputation, another salient concept focused on in the present study is cost and quality control. Quality control is a pivotal part of the manufacturing process, and there has been extensive research on it in the past (e.g., Kim, Lee, Moon, Park, & Hwang, 2011). Yet, the importance of quality has considerably increased specifically in the last two decades. Steve Jobs, the creator of Apple, started a new era of quality by producing and selling high-quality products to middle-class people. In contrast to the 1970s when creating a great product presupposed mass production of inexpensive products, Apple's priorities involved creating of highest-quality models at minimum cost. Said differently, while the main goal of the manufacturing model of the 1970s was to minimize the price and adjust the quality to meet production requirements, in the present-day world of major technological advancements, the dominant model, originally introduced by Apple, prioritizes quality over price. Jobs' vision for Apple was always to charge a premium price for a premium quality product. Apple's least expensive products are usually priced in the mid-range, but they ensure a high-quality user experience through their features (Nielson, 2014). Ultimately, Steve Jobs' model was so influential and well programmed that it expanded to different industries, including also the automobile industry.

In the last decade, the car manufacturing industry has undergone a series of important changes. Companies like Toyota and Nissan have created higher-level luxury sub-brands targeting a certain group of customers. For instance, in 1989, Nissan and Toyota created luxury sub-brands Infiniti and Toyota, respectively. In subsequent years, Nissan and Toyota have experienced a higher demand for their luxury vehicles as compared to their regular vehicles. For instance, the sales of luxury cars like the Maserati increased 70-fold from 1998 to 2014, while the price remained the same. Companies like Maserati, Bugatti, Lamborghini, and Porsche, which are known for building high-performance sports vehicles at a premium price for the wealthier customers, are now starting to move their products into a new group of customers—namely, middle-class families. Accordingly, in 2019, Lamborghini launched their SUV model (Urus), and Bugatti is not much behind with their new SUV model. Likewise, Maserati already launched their Levante SUV in 2017, and Porsche has already been on the market with its 4-door sedan (Panamera) and SUV (Cayenne) for a while. The practice of introducing a line of high-quality premium-priced vehicles are becoming common within the car industry, and even newer companies like Kia Motors have launched their own luxury car lines.

Similar trends are also observed in other industries, such as the electronics industry. One relevant example here is the Beats by Dre headphone company which, in order to attract new customers, changed the purpose of a headphone from an electronic utility to a fashion item. In this respect, Campanella (1990) argued that some managers think that investment in quality programs would always have positive impact on profit, and that ignoring quality is expensive (see also Teli, Majali, & Bhushi, 2013).

In the context of the growing ability of customers to impact company's reputation through social media and the higher risks associated with overlooking quality, there is an urgent need to revise our "cost vs. quality" model and adjust our priorities. A review of the literature undertaken in the present study has shown that Castillo-Villar, Smith, and Simonton's (2012) work on designing a supply chain with consideration for the cost of quality is the only study that considers the cost of quality as an external and global performance measure for the entire supply chain. Accordingly, in the present study, we use Castillo-Villar et al. (2012) model as a foundation of our research and expand the cost of quality beyond the generic supply chain (supplier to manufacturer to retailer) to also include Tier 1, 2, 3 and 4 suppliers, as well as one manufacturer and one retailer.

Furthermore, while keeping the cost low is essential in mass production, due to the changing customer tastes and the growing demand for higher quality, we need to create a method that would enable the manufacturer to have more control over quality at different stages of the supply chain. Despite higher costs and expenses to car manufacturing, this approach would ensure that a brand's prestige will not be damaged due to the fault of a supplier in a higher tier. Since controlling all parts in all four tiers of the supply chain is not a feasible task, in the present study, we propose a new method to rank the most important parts in the supply chain and find the lowest number of parts that manufacturing companies must check in higher tiers of the supply chain to ensure a high standard of production.

CHAPTER 2: LITERATURE REVIEW AND ANALYSIS OF FINDINGS

The cost of quality (COQ) and supply chain introduced in the present study can be divided in the following three components: (1) definition of and different perspectives on quality; (2) the concept of the COQ; and (3) a new approach to the COQ and its consideration as an external factor to manufacturing. In addition, we also used the Google PageRank algorithm and Greedy algorithm to model, optimize, and verify our theory.

This chapter introduces and discusses all relevant concepts. We also analyze the effect of each of these concepts on the evolution of supply chain, the current stage of quality, and reliability of supply chain. Other efforts for the development of the COQ are also discussed.

2.1. Definition of Quality

Quality was an essential concept for our forefathers, and it remains a topic of great interest for scientists and people in business today. Quality is a company's main instrument to gain customer satisfaction and loyalty; it is also one of the most frequently used terms among managers and executives in contemporary organizations (Reeves & Bednar, 1994). Since quality is rather a product of continuous process of improvement (Watson, 2005), it is a cardinal priority for most organizations (Gryna, 2001).

Quality is a multidimensional concept that defies a straightforward definition. Customers view quality according to different criteria based on the product's place in the production marketing chain (Evans & Lindsay, 1999). In the extensive scholarship on quality, it has been variably defined as "fitness for use" (Juran, 1962), "high value" (Feigenbaum, 1956), "loss avoidance" (Taguchi), and "predictable degree of uniformity" (Deming, 1991; see also Reeves & Bednar, 1994; Gryna, 2001). Another definition posits that quality is achieved by putting systems and procedures together into operation and ensuring an effective and efficient functioning of those systems (Sallis, 2002).

According to Tuchman (1981), quality is an "investment of the best skill and effort possible to produce the most admirable results possible; is achieving or reaching for the highest standard rather than being satisfied with the sloppy and fraudulent" (p. 243). Conversely, Crosby (1979) argued that quality cannot be good or bad, high or low, but is simply a requirement to which management must conform; this proposal is otherwise known as "conformance to requirements" (p. 478). In the Deming management method, one must know how to ensure quality of the manufactured products, as the quality is the ultimate aim to ensure customer satisfaction and loyalty (Anderson, Rungtusanatham, & Schroeder, 1994). This means that whatever product or service a company offers, "it must be fit for the purpose the customer intended to use it for" (De Feo, 2015, p. 19). Similarly, Evans (2008) argued that quality influenced by customers' needs and wants is essential to high-performing companies, as it leads to organization receiving substantial patronage and positive word-to-mouth advertising, which often translates into new customers. Furthermore, Eppler (2006) argued that, in addition to ensuring control over organizational processes and ensuring customer satisfaction, an organization must continuously manage for quality, making it a major task with the focus not only on the present, but also on future improvements (Saleh & Marais, 2006). Indeed, previous studies conducted in the 1980s demonstrated that quality improvements lead to higher returns on investments, increased market share, and increased profits (Evans, 2008). In fact, companies can significantly benefit from making continuous improvements to quality, as such improvements help companies to stay in business and generate new jobs (Deming, 1991). Consequently, high quality helps organizations maximize their productivity, increase the demand for their products and services, and minimize costs.

2.2. Theoretical Perspectives on Quality

As a result of consumer desire for higher-quality products and services, attention towards quality grew between business experts. Overall, theoretical perspectives on quality can be broadly categorized into the following two dominant schools of thought:

(1) the one formed during the first half of the 20th century

(2) the other formed in the early 1950s.

The first school of thought, represented by scholars such as Eugene Grant, Walter Shewhart, Ellis Ott, and Edwards Deming, investigated the statistical methods used to deliver top-quality products by applying testing and statistical process control (see Watson, 2005 for a review). The second group of quality experts included Armand Feigenbaum, Edwards Deming, Joseph Juran, and Peter Drucker. Seeking to improve manufacturing performance and business philosophy, these scholars prioritized management-based systems (Watson, 2005).

Feigenbaum (1956) was the first to integrate the concept of quality into a company's full operations by creating a quality system to provide technical and managerial strategies that guarantee customer satisfaction and an economical cost of quality (see also De Feo, 2015, for an in-depth discussion). Feigenbaum's (1956) approach to quality is universally recognized. Called the "quality guru," Feigenbaum (1956) invented a Total Quality Management (TQM) system for bringing together development and improvement efforts in different groups within an organization to enable the functioning marketing, engineering, production, and other services at the most economical levels, resulting in complete customer satisfaction. Praising Feigenbaum's work, Watson (2005) argued that Total Quality Management is not merely a quality method.

According to Watson (2005), the ultimate goal of Total Quality Management is to leverage management methods combined with economic theory and organizational principles so that to achieve a sound business improvement doctrine that would eventually enable financial leadership. The way to outline that quality, from the customer's perspective, comes from the integration of multiple cross-functional workflows throughout a firm.

Along with Feigenbaum’s (1956) work towards outlining a novel approach to quality that would incorporate economics, industrial engineering, and management science, other scholars, including Edwards Deming, Joseph Juran, Philip Crosby, Kaoru Ishikawa, investigated the concept of quality and developed different aspects of quality management and quality control (see Sallis, 2002, for a review).

Different analysts have made thorough investigations of the association as a system. For instance, Feigenbaum (1956) argued that all directions of scholarly research on quality presently appear to agree that associations are frameworks; and they are clearly open frameworks

Srivastava (2008), who was among the first scholars to evaluate the cost of quality (COQ) in an inventory network, measured the COQ in fiscal terms at chosen outsider contract fabricating destinations of a pharmaceutical organization in India. Furthermore, Campanella (1990) also published work on coordinating COQ. According to Campanella (1990), too few directors imagine that interest in quality projects will dependably have a positive effect on the organization, and that overlooking quality is exceptionally costly, while other executives believe that it is uneconomical to work with zero irregularities in the system. Furthermore, serious issues emerge when the executives in an organization from various territories work with clashing points of view on quality. For the most part, once the cost of quality is covered, the processes are used to discover various ventures. These ventures are not simple undertakings, and it remains unclear what moves should be made, and what effects they might have on the quality cost show (Carr, 1992).

Since the customary cost show speaks to the speculated state of value costs and connections, it can just serve to evaluate the circulation of value cost classes regarding absolute quality costs, deals, and benefits. According to Juran (1951), organizations need to consider the production network upstream at all levels, as an initial move towards tending to COQ-related issues over their supply chains. Associations receive various business change techniques to enhance business execution (Feigenbaum, 1956).

Many producers and different market and production analysts emphasized the existence of wide-ranging variations and different sorts of challenges related to production and supply in particular networks. Most previous studies noted that, most of the time, there is a lack either in frameworks or mini parts and components in production networks while seeing the write-ups, whereas the originality of the response or outcome is overlooked. This leads to low COQ and hence the networks where they are to be sent, and their ratio starts coming up with lags (Spens & Bask, 2002). While doing the audit with COQ reference, the administration fails to comprehend the supply as to how to keep the COQ keeping in mind the Supply Chain Management (SCM). As a result, firms build a mutual collaboration network to profit both individually and collectively (Juran, 1951).

Previous research on the quality was not limited to theory testing and information investigation. Specifically, strategies like recreation, artificial neural networks, and fuzzy rationale were also used for streamlining and better guidance and leadership in SCM. For instance, Koh and Tan (2006) used different soft kinds of rationales to keep the supply chain management uplifted and to provide better standards for this purpose. Koh and Tan (2006) observed how to take ahead the products from different producers and providers taking into account the criteria of quality with reference to the cost within a much more reasonable time.

Furthermore, in audit reports, it appeared that, by viewing production networks with an eye towards administration, analysts consider them as fundamental for different linkages and networks amongst producers and consumers (Srivastava, 2008).

To fully understand the significance of the supply chain, we should consider the association as an open framework and keep in view the COQ. Carr (1992) characterized the current association hypothesis as a particular applied and diagnostic construct, with dependence in the light of exact research information and orchestrating, incorporating nature. Juran (1951) developed the idea further, arguing that an association is a framework made out of subsystems and portrayed from its ecological supra framework by identifiable limits. Therefore, it is possible to clarify the interrelationships among subsystems and between the association and its condition, as well as to characterize examples of connections or arrangements of factors. It underlines the multivariate idea of associations and endeavors to clarify how associations work under fluctuating conditions and in particular conditions (Ibid.). Different creators helped the authoritative hypotheses in their particular meanings of coordination. For instance, Gupta (2007) argued that coordination is a science that incorporates all exercises necessary to move products from the first wellsprings of crude materials to definitive customers of the completed items. The creator concurs that it is an all-encompassing science. It does not consider the individual parts of a framework in disconnection; however, it does review the manner by which the parts are associated and proposes better associations. It additionally bolsters the possibility of seeing and characterizing the inventory network as taking control of all products inside the store network, regardless of how unbalanced it may be, to deal with or oversee (Gupta, 2007). According to

Feigenbaum (1956), each action affects whatever is left of the chain; therefore, everything in the entire inventory network condition must be considered.

In addition, characterized coordination has been defined as the procedure whereby the right item is provided to the client at the perfect place, at the correct time, in the proper condition, and for the accurate cost. Accordingly, production network management—the new paradigm of business process renewal—has been at the cutting edge of hierarchical reasoning and research over the previous decade. The benefits from these rebuilding endeavors have mostly been picked up and are relevant from the COQ to the supply chain. Feigenbaum (1956) also described how associations need to re-modify their concentration and move towards coordinated supply chain administration. Following Campanella (1990), the general level of outsourcing remains low. There are more vital purposes behind outsourcing than bringing down expenses. Most organizations have no unmistakable execution measurements for seller administration.

The respondents see data innovation as a noteworthy empowering influence on proper store network administration. A gap between the vital prerequisites of data innovation arrangements and the capacity to meet them exists among the respondents (Carr, 1992). The worldwide investigation firms are occupied with production network rehearsals. This investigation uncovered four levels of store network movement. The initial two levels, which comprise the lion's share of organizations, are inside focused. Schneiderman (1986) spotted the two more elevated amounts, home of the genuine business pioneers, grasp a distinctly the outer core interest. The inner introduction of levels one and two can yield critical funds in zones, for example, stock, process durations, buying, coordination, transportation, and warehousing keeping in view the cost of quality (COQ) (Schneiderman, 1986).

The effect on different parts of the association if the store network finally acclimates to a more considerable amount was depicted by Juran (1951), thirty years before significant enhancements modified the inventory network administration. Specifically, Juran (1951) created a COQ that increased in showcasing productivity and decreased in promoting costs, speaking to a noteworthy outskirt for cost economies. According to Banasik (2009), this is where there is space for generous change, particularly in the execution of the physical conveyance elements of showcasing that constitute an outstanding piece of aggregate showcasing costs. Srivastava (2008) argued that provider-retailer joint effort is a significant qualification between connections and collective effort. Just when specific and restrictive data are traded between supply chain collaborators, coordination attempts happen.

Furthermore, Gryna (2001) shared a similar basic standard—that the re-designing of the inventory network would vastly affect associations. Gryna (2001) argued that, during the 1980s, associations started to look at the practicality of creating vital organizations and associations under the coordination of specialist organizations. As organizations were defined with focused weights, contracting spending plans, transportation deregulation, and a need to progress client benefit levels, they have acquired some segment of their coordination exercises from outsiders.

Feigenbaum (1956) additionally characterized the store network as inter-organization procedures and connections or as how matches of organizations, or significantly bigger gatherings of organizations, arrange their individual exercises to improve things for everyone. The prevalent production network methodology and execution are basic empowering agents for fruitful development. However, the cost-diminishing message of the COQ, rehashed by senior officials over numerous years, has brought about coordination chiefs who are specialists at cutting costs and scaling down.

The development basics require another state of mind. In particular, the present inventory network directors must see how to adjust their activities to not only help but encourage growth. At the point when rebuilding turns into a reality, associations need to think about specific vital components to stay focused. Campanella (1990) described how associations should understand that changing the present supply chain process is not a simple assignment when considering the COQ. Rather, the progressions should be considered in conjunction with the build-up methodology of progress administration. Following Campanella (1990), there are numerous observational examinations that measure the connection between store network magnificence or more normal development, and a remarkable primary concern comes about.

However, organizations have prevailed without a well-overseen supply chain system keeping in view the COQ. In this context, Kethley, Waller, Festervand's (2002) inventory network played an essential role in accomplishing critical objectives as set by the administration. Appropriate plan and combination of coordination into the crucial general arrangement of an association were deemed to be fundamental to the success of any vital arranging (Banasik, 2009).

Furthermore, Ittner (1996) characterized vital arranging as a procedure of distinguishing long-term objectives of the association and expansive advances essential to accomplish those objectives in the long term, along these lines joining the worries and future desires for real partners. Over the previous decade, other store network models were also created (e.g., Ittner, 1996). The reestablished enthusiasm in relevance to the COQ for coordinated inventory network administration ensures that research proceeds. The perfect display has not yet been produced, but some historic work has been done. While a number of distinctive models will be named, only four will be talked about in great detail here (Gryna, 2001).

As noted by Sandoval-Chavez and Beruvides (1997), models must be produced to help associations in their scan for production network advancement. According to Stalk, Evans, and Shulman (1992), when cost weights drive numerous organizations to outsource an ever-increasing number of exercises, capacities-based competitors are incorporating vertically to guarantee that they (rather than a provider or merchant) control the execution of fundamental business forms. Furthermore, Kume (1985) cautioned that, before embarking on updating an essential process, a director should initially ask whether the central issue is quality, cost, or speed of the procedure or, instead, the primary failure of the method to help the COQ system.

In addition, Schiffauerova and Thomson (2006) described huge-scale techniques that are primarily used at the origin of the production network. Overall, there has been extensive research on these store networks. The most particular work began in 1954 when Lesser (1954) presented a multi-commodity coordination configuration to enhance annualized completed item spills out of plants, to the conveyance focuses, to the last clients. Later on, Godfrey and Pasewark (1988) gave an audit for COQ of the development of conveyance procedures. The authors built up and applied a structure for fabricating technique investigation, where they portrayed a progression of stochastic sub-models, which considers annualized item spill outs of crude material sellers employing transitional plants and appropriation echelons to the last customers.

According to Albright and Roth (1992), as model creators struggle so that the audit demonstrates great potential for these models as essential determinants in the longer run, these models are not without their inadequacies. Their exceptional nature causes these issues to be of a considerable scale. Furthermore, Dale and Plunkett (1995) presented models that are frequently hard to explain. The models for COQ are to a great extent deterministic as well as static. The models that consider stochastic components are exceptionally prohibitive. Gupta (2007) concurred that there does not yet appear to be an exhaustive model that is illustrative of the genuine nature of material streams in the store network.

Furthermore, Dale and Plunkett (1995) discussed Effective Cover Reaction (ECR) as one of the most innovative systems for networking and forming a chain for supplies by keeping diverse aspects and examining them in different enterprises. Along similar lines, Bulgak, Alzaman, and Ramudhin (2008) measured the market with relevance to the merchants and their connection to external markets and initiated practically various assumptions they had earlier in order to find out how different agreements can be made for a better supply in the chain at better costs, i.e., the COQ. In addition, Bulgak et al. (2007) also took into account the hierarchal setups. Likewise, Lambert and Pohlen (2001) provided a patterned framework in order to create a better supply chain that can help to improve execution to meet the investor's expectations. Simga-Mugan and Erel's (2000) pattern helped with managing and tackling the lack of connection between the client and the administrator or the owner in almost every supply chain. Overall, it has generally been observed that both the client and provider achieve better production, and their supply is much faster, followed by better fulfillment of the demand.

Furthering this point, Freiesleben (2004) introduced the principal result of an examination venture to examine the impact of various angles on both quality and supply chain administration and the connection between these territories, and subsequently their effect on organizations' execution (see also Plunkett & Dale,1988).

A short introduction to the COQ models on which the present study is based is provided in Sections 2.3 and 2.4 The idea behind the cost of quality was effectively connected in assembling organizations and administrations of organizations (Porter & Rayner, 1992). The COQ model used in the present study is based on the prevention–appraisal–failure (PAF) model proposed by Feigenbaum (1956) and on Juran's (1951) model.

Following Porter and Rayner (1992), the fundamental presumptions of the PAF model are that interest in the examination will decrease costs, facilitate investment in aversion exercises, and diminish expenses. Li (2003) described how the PAF arrangement enables professionals to recognize quality-related costs and to express every classification as far as rates of the aggregate costs. These costs constitute the broadly used conventional prevention–appraisal–failure (PAF) model proposed by Feigenbaum (1956).

Furthermore, Juran (1962) discussed how “gold in the mine” *is characterized as the* “aggregate of avoidable costs of value” (p. 172). According to Juran (1962), costs arising from imperfections are a gold mine where lucrative burrowing should be possible. Known as one of the most influential names in the quality movement, Juran dedicated his life to researching methods of quality improvement and quality leadership in organizations (see Beam, 1997, for an extensive discussion). According to Juran (1962), the primary purpose of an organization is to stay in business to ensure stability of the community and the creation of products and services needed by customers need/want, which provides a satisfactory environment for the company’s associates (Hackman & Wageman, 1995). As many corporations have been severely impacted by quality problems in the form of product deficiencies and failures—which create customer dissatisfaction and demonstrate the high cost of poor quality — it is critical for industrial managers to “breakthrough” into higher levels of performance (Prasad, 1965). Juran (1962) saw the prevention of defects as an essential facet of quality; however, since this was not enough to convince consumers that the product is valuable, he further argued that quality consists of product characteristics that meet customer needs and offer product satisfaction (see Beam, 1997). Fully captivated by the concept of quality, Juran (1962, p. 314) classified the following eight uses of the term “quality” in the industry:

1) *Marketplace quality – the extent to which a specific product meets the requirements of a specific consumer.*
2) *Quality of design – the extent to which a class of products satisfies potential customers.*
3) *Quality of conformance – the extent to which a specific product meets specific design requirements or specifications.*
4) *Consumer preference – the degree to which customers, based on comparative tests, prefer a specific product over its competitors of comparable quality.*
5) *Quality characteristic–feature that distinguishes a given product (in terms of appearance, performance, length of life, dependability, reliability, durability, maintainability, tastes, odor, etc.) from comparable products.*
6) *Quality –A general expression of perceived excellence, devoid of sufficient specificity for further classification.*
7) *Quality –a function in the industry related to the attaining superior performance of a product.*
8) *Quality –a specifically appointed department within a company.*

According to Juran (1979), consumers see quality as two dimensions: features (i.e., quality of design) and absence of deficiencies (i.e., "quality of conformance") (see De Feo, 2015). The first dimension is crucial for the sales income, because customers have different demands and desired levels of quality, meaning that the designer of a company must create products with various sets of features to meet all customers' needs. The second dimension refers to the quality of conformance, which keeps the products free from any possible errors or failures. As De Feo (2015) formulated it, maximizing the quality of compliance leads to lower costs, fewer complaints, and, therefore, less unhappy customers.

On Juran's (1962) view, quality means looking at everything as a repetitive process. The author's approach was scientific, yet very straightforward: look at the process, re-engineer it, combine, strengthen or/and eliminate to make it much more useful. Throughout his career, Juran (1962) extensively wrote about the importance of improving processes while keeping quality costs low; however, only a handful of companies use official quality costing methods. According to Monk (1988), measuring quality costs should be prioritized to ensure that quality specialists and upper management can communicate with each other (see also Porter & Rayner, 1992).

2.3. Introduction to the Cost of Quality (CoQ)

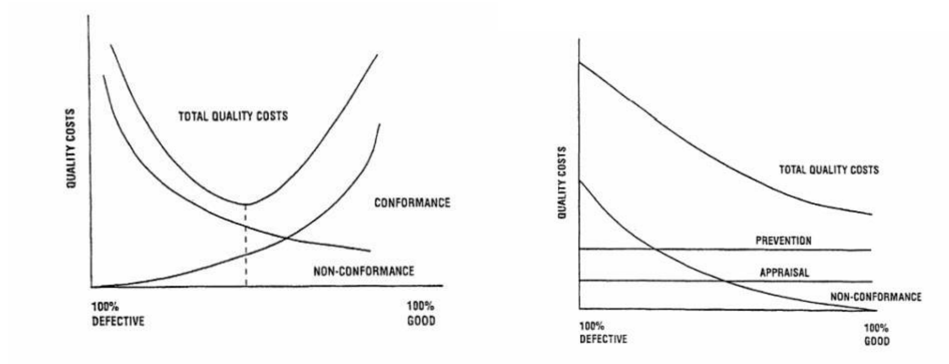
A look at the history of quality—including inspections, specifications, and metrology—suggests that quality has a dramatic impact on both sales' revenue and costs (Gryna, 2001). An interesting fact related to quality is that no business is against quality—In fact, businesses are all for quality, and such an attitude is supposed to result in better quality products and services. However, many companies are naïve, unprepared, and often not serious when it comes to quality, and this behavior usually results in severe damage and quality costs. According to Porter and Rayner (1992), a company's competitiveness can be adversely affected by the costs of correcting failures, redoing things, or apologizing to customers. Throughout the years of past research, quality practitioners have used different terms to describe the costs accrued in a company, including "quality costs," "the cost of poor quality," or "the cost of quality." To avoid any misunderstandings, the general term used throughout this thesis will be the Cost of Quality (CoQ). Srivastava (2008) defines the Cost of Quality as "*the sum of the costs incurred within a company in preventing poor quality, the costs incurred to ensure and evaluate that the quantity requirements are being met, and any other costs resulted from poor quality*" (p. 12). Furthermore, according to Castillo-Villar et al.'s (2012) definition, the CoQ is "a powerful measurement system that translates the implications of poor quality, activities of a quality program and quality improvement efforts into a monetary language for manager" (p. 3).

2.3.1. Classification of the Cost of Quality

In numerous previous studies, the Cost of Quality was argued to be an essential element needed to estimate the amount of money an organization should allocate to ensure the quality of its products. Approximately 30% of a company's total costs are quality costs, which makes quality a significant driver that organizations need to account for if they are to sustain a competitive advantage (Srivastava, 2008). Juran (1951) became, in fact, the pioneer of the quality movement, when he introduced the traditional model for calculating the Cost of Quality (see Fig. 1), long before the Japanese Quality Revolution (see De Feo, 2015 for a discussion). Later on, quality experts including Feigenbaum (1956), Masser (1957), and Crosby (1995) also expressed their concerns with quality in organizations by examining the Cost of Quality models and measuring quality on the process level (Burgess, 1996).

The approach to the COQ calls typically for some kind of categorization. Throughout his career, Juran (1951, p. 741) identified the following four broad categories of quality costs:

- *Internal failure costs* (scrap, rework, failure analysis, etc.), associated with defects found before the transfer of the product to the customer.
- *External failure costs* (warranty charges, complaint adjustment, returned material, etc.), associated with defects found after the product is shipped to the customer.
- *Appraisal costs* (incoming, in-process, and final inspection and testing, product quality audits, maintaining the accuracy of testing equipment, etc.), which determine the degree of conformance to quality requirements.
- *Prevention costs* (quality planning, new product review, supplier evaluation, training, etc.), incurred in keeping failure and appraisal costs to a minimum.



(a) The traditional cost of the quality model (b) The continuous improvement model

Figure 1. Models of quality costs (Ittner, 1996)

Another categorization of quality cost suggested by Ittner (1996) separates Juran's (1951) classification of the expenses into *conformance costs* and *nonconformance costs.* According to this classification, the former type of costs includes the costs of achieving conformance to specifications and has two elements: appraisal and prevention. Furthermore, nonconformance costs arise from the failure to conform to specifications; they too have two elements: internal failure and external failure. Ittner (1996) argued that, contrary to the traditional quality cost theory according to which companies manufacturing defective products can minimize their nonconformance costs by implementing less pricy prevention and appraisal measures, companies should ensure that conformance costs are continuously increased to obtain continuous cuts in nonconformance costs.

Overall, the many models for managing quality costs that have been developed can be broadly categorized into the following four groups: (1) PAF (Prevention, Appraisal, Failure) model; (2) Process-Cost model; (3) Cost-Benefit model; and (4) the Loss Function model (Srivastava, 2008).

Developed by Feigenbaum in 1956 and Masser in 1957, the P-A-F model is one of the oldest COQ models that found great popularity and were extensively used in both manufacturing and in-service industry (Hwang & Aspinwall, 1996; see Figure 2). In their review, Plunkett and Dale (1988) concluded that the "Feigenbaum's" classification is "almost universally" accepted. Since, at the time, companies struggled to maintain CoQ low, PAF's mission was to encourage companies to prevent poor quality, rather than to detect it afterwards. Feigenbaum' (1956) method of categorizing quality costs consisted of the following three types of costs: prevention, appraisal and failure costs (see also Burgess, 1996). The P-A-F method is shown in Fig .2. Porter and Rayner (1992) defined each of Feigenbaum's (1956) categories of costs as follows: (1) prevention costs are associated with the cost of any action taken to research, prevent, or limit the risk of non-conformity; (2) appraisal costs are related to examining the performance of requirements, and (3) failure costs are non-conformance costs that can be internal (scrap rework, re-inspection, redesign) and external (warranty costs, service calls) (p. 353). According to Hwang and Aspinwall (1996), the Prevention-Appraisal- Failure model can be split into a macro and micro model, where the former deals with the external customers and supplier relationship of a company, while the latter is oriented towards the internal customer and supplier within a department. While macro and micro PAF models are similar, they both have benefits and drawbacks depending on their application and environment. The macro weakness of the PAF model includes delayed identification of quality issues, while the micro PAF model creates additional quality costs for the company by being partitioned into departments and divisions before activation (Srivastava, 2008). The advantages offered by the PAF classification include universal acceptance, recognition of different kinds of costs, and having certain criteria to decide whether or not costs are quality-related (Castillo-Villar, Smith, & Simonton, 2012).

Another classification of COQ was proposed by Dahlgaard, Kristensen, and Kanji (1992) who argued that the PAF model is neither sufficient nor adequate to measure the quality costs of a company and, therefore, needs some modification. Specifically, Dahlgaard et al. (1992) proposed a new categorization, where, on one hand, the costs are divided into visible and invisible, while, on the other hand, costs are divided into external and internal (see Table 1). The visible costs are the result of scrap and warranty costs, while the invisible costs are due to internal inefficiencies and loss of goodwill (Srivastava, 2008). Following Deming's ideology about quality costs, Dahlgaard et al. (1992) argued that companies must account for hidden costs as well as for explicit costs and improve measurements to attain an accurate estimation of companies' total quality costs.

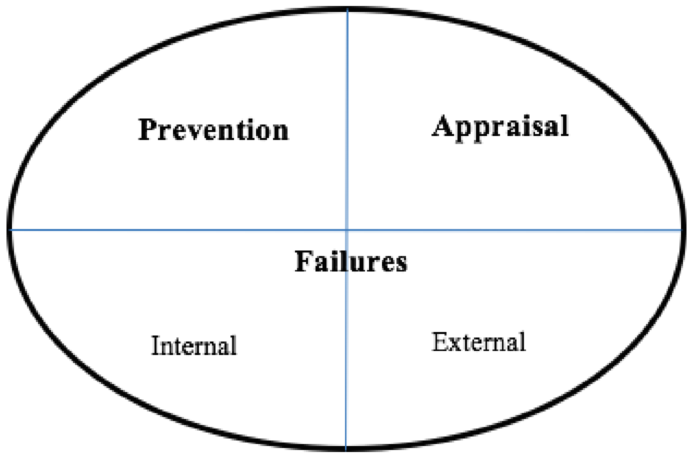


Figure 2: Feigenbaum’s PAF model for COQ [54]

Table 1. Dahlgaard et al.'s (1992) categorization of COQ

	Internal Costs	External Costs	Total
Visible costs	1a. Scrap 1b. Prevention/appraisal	• 2. Warranty costs • (complaints)	1+2
Invisible costs	3a. Costs due to internal inefficiencies 3b. Prevention/appraisal costs	• 4. Loss of goodwill (loss of future sales)	3+4
Total	1+3	2+4	1+2+3+4

Crosby (1996) argued that zero defects are the ultimate goal of company performance and that Feigenbaum's (1956) cost of poor-quality indicator should be used the business measurement standard to evaluate the extent of a product's nonconformance with customer requirements.

2.3.2. New Approach to the Cost of Quality

The cost of quality is a central concept in supply chain management and optimization. It includes prevention, appraisal, internal failure, and external failure costs. Except for external failure, none of these costs are related to actual quality of production provided to the final customers. External failure costs include costs from two streams—namely, the cost of taking actions on warranty claims and the cost of not satisfying customer needs.

An alternative approach proposed in the present study to calculate this cost is as follows:

the value of quality is total loss estimation caused by a unit of defected production. Introducing this value is equal to transforming multi-objective optimization problem of improving quality and cost to a single-objective problem using the weighted sum method.

Therefore, in the novel approach proposed in this thesis, each manufacturer in the supply chain is assumed to have input quality (i.e., probability that at least one part used to make a unit of production is defected) and cost and output quality (i.e., probability that production is defected).

A fast algorithm simulates the cost and quality impact of each node from the final manufacturer to all direct and indirect suppliers implemented. This algorithm is based on the backward value and costs propagation. Initially, the value of quality is known for the final node, and the derivatives of variables within nodes are then calculated numerically. This method does not require recalculating the whole network for each derivative.

2.4. PageRank

This section reviews the literature on Google page ranking. Various secondary sources are examined to form a theoretical structure that contributes to enhancing the understanding about Google PageRank. Some of the subdomains of Google page ranking covered in this chapter include the algorithm on which page ranking is based, the importance of page ranking, influence of Google page ranking, as well as its relevance to search engine optimization (SEO). In what follows, we first introduce the basic concepts and ideas related to Google page ranking, which is followed by an in-depth analysis.

2.4.1. Google PageRank

The concept of Google PageRank (PR) correlates with the PR calculations, and the idea has been introduced by Google. The PageRank technology was developed to evaluate and rate the webpages in terms of significance and quality (Lin, Ding, Hu, & Wang, 2015). According to Zambuk, Gital, Boukary, Jauro, and Chiroma (2019), online users are generally trying to improve the page rankings by using SEO; it is where Google Page Ranking comes in practice. The entire process of Google PageRank is based on numbers that include the rank at which a website will be displayed. To Google as a search engine, the number-based ranking of webpages also indicates the importance of those webpages (Zambuk et al., 2019). The webpages available online are assigned specific ranks based on the number series from 0 to 10. The higher a webpage is ranked on the number series, the higher its position of display on the ranking list of websites would be (Zambuk et al., 2019).

Many previous studies found that Google page ranking is closely related to search and SEO strategy, and that the ranking score demonstrates the effectiveness of SEO strategy. The technology considers page ranks as votes, and some votes are usually more prominent and significant than others (Pant, Ramirez, & Reeves, 2019). Murrugarra, Miller, and Mueller (2016) argued that Google employs the system of page ranks to calculate link votes and measures the importance of webpages based on the ranks allotted to each page. However, when it comes to the rank of the pages during a search, different factors are viewed along with the calculated link votes to realize that a webpage is ranked well in the search. Based on the unique structure for ranking webpages, Google places page rank at the heart of its technologies (Murrugarra et al., 2016). To improve the functionality of Google, many engineers are constantly trying to improve the system of search, and one of the key elements being focused with regard to search tools is its page ranking.

2.4.2. The PageRank Algorithm

According to Zeitlyn and Hook (2019), PageRank was initially developed based on an extremely intuitive and powerful idea. Specifically, PageRank was developed using an algorithm which helps to confirm that some pages are more appealing and could be ranked by users higher than other pages (Zeitlyn & Hook, 2019). By the system of PageRank algorithm calculations, the pages that are visited more frequently by users are mechanically categorized as high rankers. The algorithm functions by running Markov chain through which user behavior of web surfing is expressed and modeled. The modeling of user behavior by the PageRank algorithm is supported by direct web graph along with assessing random jump in the algorithm (Suzuki & Ishii, 2018). With the technological advancements, Google infused some new and effective features to offer a more efficient webpage ranking. Although the new features of web ranking by Google are highly efficient and productive, scientifically speaking, PageRank is one of the most elementary features that Google uses for web search and page ranking (Song, 2018). Although the PageRank algorithm introduced by Google is widely regarded among different tools for web searching, the algorithm always requires constant updates to ensure an effective page ranking (Song, 2018).

2.4.3. Importance of PageRank

Page ranking is associated with the importance of a webpage. The system functions by calculating the links of other websites that combine on one webpage. Mohan and Kurmi (2017) argued that this represents combining the links and recommending or voting for the host webpage. With an increase of the number of links attached to a webpage, the ranking of that also increases (Mohan & Kurmi, 2017). Overall, multiple factors are involved in the improvement of ranking of a webpage. Two critical factors here are the importance of a webpage and its relevance with the subject of the search. This can be understood through an example of a webpage designed to conduct online banking. Imagine some other bank finds the webpage you have developed and considers it to be an excellent explanation of the online banking process. The other bank also aims to present your webpage as an informative tool to its customers and present a link of your webpage on their website. The moment your webpage is linked with other websites, the page ranking of your webpage will immensely improve (Mohan & Kurmi, 2017). According to Gupta and Singh (2016), page ranking is highly important, because the technology is among key factors used by Google to promote relevant and critical searches. Google uses page ranking to evaluate those search results that can be presented at the top of list charts of Google search. Gupta and Singh (2016) further argued that the significance of page ranking is also evident from the fact that Google considers PageRank as its trademark for authentic web search. Furthermore, other search engines also employ different page ranking algorithms to present a rank-based search of quality links. Therefore, page ranking is important, because it uses elements like quality, relevance and importance to present search results for a specific search item (Gupta & Singh, 2016).

2.4.4. Revolution in Search

The introduction of the PageRank technology by Google revolutionized the way in which information is searched online. Page ranking emerged as a new way of ranking where ranks were assigned to pages by incorporating the measures that calculated the links attached to a webpage (Goel, Kumar, Kumar, & Chopra, 2019). The idea of measuring links to evaluate the importance of webpages can be easily applied to academic research. When a study is being conducted by academic researchers, the papers cited to develop the main document are collected by viewing the times a paper has been cited by other papers. The relevance and importance of a paper is thus evaluated by the number of times it has been cited by others. Said simply, the more articles cite a paper, the more important this paper would be considered by academic researchers. This example successfully explains the whole idea of search results rankings presented by Google where most relevant searches appear at the top of the list (Goel et al., 2019).

According to Chipman (2019), before Google joined the search engine market with its PageRank technology, users experiences many difficulties in searching for information online. Search engines like Yahoo and Infoseek did not have technologies as efficient as the one used by Google. Hence, Google's PageRank revolutionized the domain of searching through the Internet by making online search easier, quicker and more efficient. This occurred because the PageRank technology was efficient and fast in calculating the relevance of links that matched the information being searched (Chipman, 2019).

2.4.5. Increasing PageRank of a Webpage

According to Chae and Seong (2017), the page rank assigned to a website by Google is the indicator of importance of that web page. Said differently, the higher the rank of a website, more important it is considered by Google. Therefore, it is essential to explore the factors that can make a website to be seen as important by Google. The factors that Google uses to evaluate the importance of a website are incoming or inbound links and the type of incoming links. Therefore, to have a higher page rank or to increase page rank of a web site, it must focus on getting other websites linked to it. By building a large number of links with other websites, the rank of a webpage can improve significantly. However, two aspects that needs to be considered while building links for a website are relevance with other websites and continuing with the process of building links in a slow manner to avoid being flagged for spamming. Also, it is essential to ensure that only appropriate types of links are chosen to improve page ranking. That is, a website should have links to webpages with higher page rankings (Chae & Seong, 2017).

2.4.6. Summary

The page ranking technology of Google, which relies on calculations and algorithms for webpages that are mostly linked by other relevant webpages has effectively improved the way information is searched today and reshaped the digital searches using the Internet. The page rank system introduced by Google has revolutionized the digital search engine industry. The page ranking relies on calculations and algorithms for webpages that are mostly linked by other relevant webpages. The system introduced by Google has seen immense success and reshaped the digital searches using the Internet.

2.5 Greedy Algorithm and Optimization

The section lays down the theoretical foundation of our research based on greedy algorithm and its use for optimization. By reviewing relevant literature, we discuss a variety of aspects and explain the concepts of greedy algorithm and optimization. Specific emphasis is placed on the on role and effectiveness of greedy algorithms in solving optimization problems. The section begins with an outline of the basic and fundamental concepts of greedy algorithm followed by standard greedy algorithms, explanation of optimization problems, and a discussion of the role of greedy algorithms in solving complex problems. Several different types of standard greedy algorithms are also briefly discussed outlined.

2.5.1. Greedy Algorithm

Greedy algorithm can be defined as an ideal algorithmic model used to solve optimization problems. The greedy algorithmic process is purely intuitive and offers a simple approach to obtain optimized solutions for complex problems (Chiang & Mu, 2016). As argued by Moran and Bouchaud (2018), the process of greedy algorithm is based on making optimal choices at different stages of the model, with the aim of finding an optimized solution for the entire problem. The structure of greedy algorithm involves taking the complete data set relevant to the problem and providing rules for each element at each stage of the algorithmic process. The rules defined for every element support the process in finding an optimal solution to solve the problem (Moran & Bouchaud, 2018). Greedy algorithm has the following two properties based on which a problem can be solved. The first property is called "greedy choice property," which ensures that a global optimized solution is obtained by making optimal choices at each stage of the process.

Furthermore, the second property is 'optimal substructure', which suggests that the problems with optimal substructure must have a solution that can be treated as an optimal solution for subproblems (Moran & Bouchaud, 2018).

Therefore, greedy algorithm is also suitable to solve problems for which there is always an optimal solution at any given step of the process (Geissmann, Leucci, Liu, Penna, & Proietti, 2019). As the process completes, a solution is discovered for the entire problem, and the algorithm provides an optimal way to solve the entire problem. Accordingly, it is essential to ascertain that the problem confirms with the two basic properties of an ideal issue, which can be solved using greedy algorithm. Next, compulsory elements or components required within the solution are determined, and an iterative process is established to examine sub-problems. The most widely known advantages of using greedy algorithms include the ease of formulation and implementation and reduced time to reach a solution (Geissmann et al., 2019).

2.5.2. Standard Greedy Algorithm

The standard greedy algorithms are paradigms that proceed through a step-to-step process by taking a single input into account at each step of the process (Schwartz, Singh, & Yazdanbod,2019). Several standard greedy algorithms have been extensively investigated in previous research.

As greedy algorithmic solutions are based on the idea of establishing a solution piece by piece, the focus of the model is always over the next piece or activity that can offer a quick relief to the problem (Liu, Zhao, & Ren, 2016). The problem pertaining to the category of activity selection problem can be explained by using an example. For instance, a person is provided with ‘n’ activities along with the beginning and the finishing times. The problem is to combine the total number of activities which a person can perform while working on only one activity at a time during the entire process. Activities can later be sorted according to the finishing time of each activity so that to realize the next activity as the least time-consuming activity (Liu et al., 2016).

2.5.3. Job Sequencing Problem

The job sequencing problem is another standard greedy algorithm where a collection of jobs is built so that every job is bound by a deadline and profit. When the job is finished before its approaching deadline, a profit can be received by the job performer (Yang, Ban, & Xing, 2019). The problem also considers that every job takes only a single unit of time; consequently, the lowest possible deadline for any job from the collection is 1. By generating subsets for the assigned jobs while simultaneously keeping a track over the feasibility of the jobs, it is possible to maximize the total profit from the job (Paiva & Carvalho, 2017).

2.5.4. Water Connection Problem

A third exemplary type of standard greedy algorithm is where the problem is related to improper connectivity of water pipes within a colony. As argued by Maidamisa and Eckson (2017), this problem suggests that all houses in the colony have a single pipe going into every single house and coming out from the other end. Accordingly, every house of the colony holds one incoming pipe and one outgoing pipe for water attainment. The way in which the tanks and taps must be installed requires that houses with only an outgoing pipe will receive a tank, while the houses with only an incoming pipe will get a tap installed. Let us suppose that the number of pipes and number of houses are denoted by two integers 'n' and 'p', respectively. Then, an efficient solution needs to be explored for the network of connecting pipes from one house to another within the colony (Maidamisa & Eckson, 2017).

2.5.5. Optimization Problems

The optimization problem can be defined as the problem of finding the best possible solution amongst different feasible solutions. One of the simplest explanations of an optimization problem is related to the findings of shortest distance/path between two points, or to the identification of a path with the lowest weight (Khalil, Dai, Zhang, Dilkina, & Song, 2017). An optimization problem has several general characteristics. The first characteristic is associated with instances for which a possible input is explored. The second characteristic relates to solving the instances while each instance comprises of a giant set of solutions. Finally, the third characteristic concerns the ease of measuring value or cost of a specific solution (Khalil et al., 2017).

For the solution of every optimization problem, there is an essential step that must be considered. This step focuses on the choice of the optimization model that must align with the optimization problem and its expected solutions. Considering that every algorithm is designed to solve a different type of optimization problem, an optimization model should be critically selected (Bertsimas, Sim, & Zhang, 2017). The types of optimization problems include discrete optimization/continuous optimization, constrained optimization/unconstrained optimization, multi-objective optimizations, and deterministic optimization/stochastic optimization. However, not all optimization problems can be solved using greedy algorithm techniques (Bertsimas et al., 2017).

The method of greedy algorithm is simple and efficient to solve optimization problems. The method follows a simple rule of taking everything that looks best to develop a solution of the entire problem. A variety of optimization problems can be solved using greedy algorithms (Chen, Song, Zhang, & Wang, 2016). In general, every optimization problem can have the following two categories of solutions: (1) feasible solutions and (2) optimal solutions. The difference between these two categories of solutions pertains to the difference between the possible solution and the best possible solution (Chen et al., 2016). In what follows, we discuss several optimization problems.

2.5.6. Huffman Coding

The Huffman coding is an efficient technique of data compression without the loss of any information during the process (Siahaan, 2017). In one of the branches of computing, the information is encoded in the shape of bits denoted as 1 and 0. All computers use multiple strings of bits to encode information that helps to direct a computer via specific instructions that should be performed. Different types of data—such as videos, audio clips, video games, pictures, or movies— can be encoded in the form of bit strings. Everything that receives a code holds a certain message, and every message consists of symbols (Siahaan, 2017). The symbols that repeatedly appear through the process of coding are encoded in the name of shorter bit strings.

2.5.7. Dijkstra's Algorithm

Dijkstra's algorithm is the algorithm that involves exploration of the shortest path in-between two nodes over a weighted graph. This algorithm follows an approach of creating trees that represent the shortest paths from one source, or vertex, to any other given point on the graph (Adzhar, Salleh, Yusof, & Ahmad, 2019). One of the key conditions for using Dijkstra's algorithm is related to its application, which can only be done to a weighted graph. Another important requirement is that the graph must have a non-negative weight over all edges of the surface. The algorithm functions by maintaining two sets where the first set consists of vertices and the tree of shortest paths, while second set contains vertices that are not planted over the shortest path tree (Adzhar et al., 2019).

2.5.8. Summary

The greedy algorithm paradigm is applied to solve problems by making suitable and appropriate choice at every stage of the process. When there is continuous and consistent focus on finding the best possible solution, the process successfully ends with finding an optimal solution. In this section, we reviewed several types of optimization problems. Based on this review, we can conclude that greedy algorithm plays a significant role in finding an optimal solution for a variety of optimization problems.

2.6 Reliability of Supply Chain

This section focuses on an essential function of business called supply chain management. We focus on the aspect of reliability and the role that reliability plays in improvement of business operations. To this end, we focus on different theoretical aspects related to the function of supply chain management and its reliability. The sources reviewed for this part of our study are secondary. Some of the critical components discussed in this section are basic concepts of supply chain management, importance of reliability in supply chain, and the models that can help to measure reliability of a supply chain for a business.

2.6.1. Supply Chain Management

According to Bozarth and Handfield (2016), supply chain management can be defined as the management of activities to support a smooth supply for a business. One of the key areas of business that receives substantial contribution from efficient route of supplies is operations and production. The basic concept of supply chain management involves the following two core ideas. The first idea is that there are multiple organizations involved in making a product reach the end user. Furthermore, the second idea suggests that most of the organization focus only on the internal areas of supply chain that lies within the four walls of the organization. In this respect, Chen (2019) argued that it is critical to understand that efficiency of a supply chain directly depends on the attention paid to the entire supply chain activities (rather than on the focus on the receiving ends of the supply chain). Said differently, supply chain management presupposes a conscious effort to effectively and efficiently provide supplies to a firm.

Hugos (2018) identified the following most prominent activities of the chain of supplies: product procurement, sourcing, warehousing, production, development, and logistics. Each major activity has further supporting activities to support the function in forming smooth chains of supplies for the business and its continuity. Different organizations are integrated together for form chains of supplies for the business. The integration of the organizations involved to build supply chains is based on two different types of flows: physical flows and information flows (Hugos, 2018). In what follows, these flows are discussed in further detail.

2.6.2. *Physical Flows*

According to Taschner and Charifzadeh (2020), the physical flow of supply chain refers to movement of tangible goods that are further used in storing or production of the final product. Physical flow of supply change is a visible element of the entire supply chain process and activities.

2.6.3. Information Flows

The information flow within a supply chain is related to the exchange of data between partners handling different activities of the entire supply chain process (Taschner & Charifzadeh, 2020). The information flow is an intangible component of the whole supply chain process and is associated with the long-term goals of managing routine flow of materials.

2.6.4. Business Continuity with Supply Chain

Following Azadegan, Mellat Parast, Lucianetti, Nishant, & Blackhurst (2019, risk management is one of the crucial aspects of sustaining smooth operations of business in the present-day dynamic business world. While, on the one hand, the global business environment has become more interconnected, on the other hand, greater risks have become involved in the process of maintaining a balance between flow of demand and supply. In this global context, businesses require proper risk management frameworks to assist business operations towards a better continuity. Risk management frameworks are built by combining different types of systems associated with business continuity. Therefore, the development and execution of business continuity plans are directly concerned with the components involved in risk management frameworks. Risk management structures are also developed by considering threats and uncertainties from the external environment. The systems of business continuity are developed as a result of risk evaluation (Azadegan et al., 2019).

As argued by Brindley (2017), key elements involved in business continuity systems include policies, procedures, plans and priorities. The aspects of such procedures involve activities such as work-force management, financial management, sales management, customer service, and supply chain management. An efficient and effective flow of supplies for a business helps the business in manufacturing products that match the demand. When a company is capable of providing products in alignment with the demand, the efforts contributes towards generating financial support, which further works in continuity of the entire business process. On the other hand, whenever supply chains of a business go through period of disruptions, it becomes challenging to manage the continuity of the business (Azadegan, Syed, Blome, & Tajeddini, 2020). In the event that supplies, or raw material are not received in the required periods, the company becomes incapable of manufacturing products according to the demand in the market. In this context, an effective supply chain management is necessary to promote continuity of the business.

2.6.5. Optimization of Supply Chain Efficiency

According to Bai and Li (2016), the general notion of optimization presupposes attaining the maximum level of performance or profit with minimum inputs. The optimization of supply chain refers to performing supply chain activities at a lower cost that can yield a higher profit margin. Such type of an optimized process of supply chain helps an organization to gain a competitive advantage that is difficult to be imitated by the rivals in the industry (Abeysekara, Wang, & Kuruppuarachchi, 2019). A number of factors--including technology, capabilities of supply chain partners and other resources of the firm—affect the degree to which a supply chain process is optimized for the business.

Furthermore, Brunaud, Laínez-Aguirre, Pinto, & Grossmann (2019) also suggested that technological resources should be efficiently used within the process of supply chain to improve the performance of different networks of a supply chain. Relevant modern-day technology resources that are widely used in supply chain processes today include artificial intelligence (AI), Blockchain, and Internet-of-Things (IoT). The appropriate use to modern technology and resources contributes to the development of a high performing chain of supplies which promotes the responsiveness of a business. The more responsive a business becomes, the better it can attend to the changing needs of customer and provide superior quality of customer experience (Brunaud et al., 2019).

2.6.6. Reliability of Supply Chain Process

According to Amelkin and Vohra (2020), reliability is a crucial issue within the entire process of supply chain management of a firm. The reliability of supply chain is predetermined by extent to which the chain of supplies of a business is consistent in performance. Therefore, consistency plays a vital role in reliability of a supply chain. Globally, supply chain managers and professionals share the following three core priorities for supply networks of a business: (1) responsiveness towards demand; (2) reduced inventory levels; and (3) reliability of the supply chain. The ideas of reliability in a supply chain are interrelated with the optimization of supply lines through data and visibility (Spiliotopoulou, Donohue, & Gürbüz, 2016). Managing reliability is the most prominent issue for today's supply chain processes and requires an operative supply chain system, high productivity, and low cost.

Therefore, as argued by Jia, Cui, and Xing (2018), highly reliable supply chains are the fundamental attribute of successful management of supply chain in dynamic business contexts. Since the competition is cutthroat in global markets characterized by frequent demand changes, in order to save cost and time, businesses need supply chains that are highly reliable. The modern supply chain managers are aware of the important role of reliability and optimization in making a business survive through external pressures. Accordingly, significant attention has been paid to supply chain models which help infuse reliability into the supply chain activities of a business. Several relevant models focus on reducing cost of suppliers to maintain reliability; alternatively, other models stress improving the production capacity while managing the same input. When forming a network of suppliers, businesses should be aware about a number of suppliers and the benefits offered by each of them (Jia et al., 2018).

2.6.7. SCOR Model to Measure Reliability

As described by Akkucuk (2016), the SCOR model has emerged as a management tool to improve the decisions associated with the chain of supplies. Every decision regarding the supply chain activities of a business is concerned not only with business suppliers but also with the customers. The mode of SCOR can explain the required processes to attend to the demand of the customers. The term ‘SCOR’ refers to supply chain operation reference, and the model is highly useful in terms of identification of the basis through which the process of supply chain can be improved (Akkucuk, 2016).

Delipinar and Kocaoglu (2016) argued that the SCOR model can successfully measure the reliability of supply chains of a business. The use of the SCOR model for a business is associated with multiple benefits. As the model functions by going through multiple stages of the entire supply chain process, it becomes easier for an organization to analyze different stages of a supply chain. The model guides the business about the level at which a supply chain is advanced and efficient. Moreover, the model is highly proficient in identifying the issues within a supply chain process of the business (Delipinar & Kocaoglu, 2016).

In this section, we have developed a theoretical framework regarding the role of reliability within supply chain of a business. The basic concept of supply chain concerns the movement and transfer of materials to essential business units. In essence, such transfer is in a flow of supply, be it physical or in the form of information. The efficiency of a supply chain process is demonstrated by the extent of reliability of the process. However, the reliability of the supply chain largely depends on the level of optimization within the process activities of a supply chain.

2.7 Summary

This chapter has reviewed the current theoretical perspectives on the CoQ and relevant methodologies used to investigate supply chains. Despite the richness of different ways of finding, addressing, and eliminating a problem from the defect sources, all methods reviewed in this chapter are typically used when a problem has already occurred. That is, all reviewed methodologies have one common characteristics of being reactive. In the car manufacturing industry, after a car leaves the production line, it is used by the customer, who then provides feedback on the vehicle to the manufactures, and only after that the manufacturer can identify and solve a problem if it occurs. In this competitive market, companies cannot make mistakes without having to pay for them. Moreover, at present, the cost of making a mistake is nowadays higher than ever before.

CHAPTER 3: MODELING AND OPTIMIZATION

In the present study, we divided the entire process into two main sections. In the first section, a simple formulation was created with basic four tiers of the supply chain. In the second section, we used three tools—namely, Excel, MATLAB and AnyLogic 8—to test the proof of concept, identify, establish the ground rules and assumptions for the present study, densify the best tools for our study, and see if we need to build our toolbox. This section presents an overview of methodology to create the basic supply chain model, introduce a ranking system for each part/supplier within the entire supply chain, and create different basic test scenarios (numerical examples). The complete process flow used for this study is given in Figure 3.

3.1. Creating the Basic Supply Chain Model

Supply chain can be represented as weighted directed graph without cycles.[1] Vertex (node) is a manufacturer, and edge is a supply line. Edge with weight k shows that manufacturer *y* needs *k* parts from his supplier *x* (or *k* parts number *x*) to produce one-part number *y*. If more than one edge has node *y* as their head, manufacturer y needs parts from multiple suppliers. Manufacturer *y* chooses random parts from supply regarding type and quantity. If one of the parts used by manufacturers y is bad, the produced part will be defected.

Node 1 produces the final production (that can also be named part). The entire supply chain may include manufacturers controlled by the same corporation or by different structures. If there is no direct control over some suppliers, they can be recommended to use more inspections.

The proposed supply chain model (see Figure 4) consists of four tier suppliers, one manufacturer, and one retailer.

[1] While it is possible to calculate cycled graphs, this would require using more complicated models. Cycled graph would mean that production of manufacturer X needs one of his suppliers.

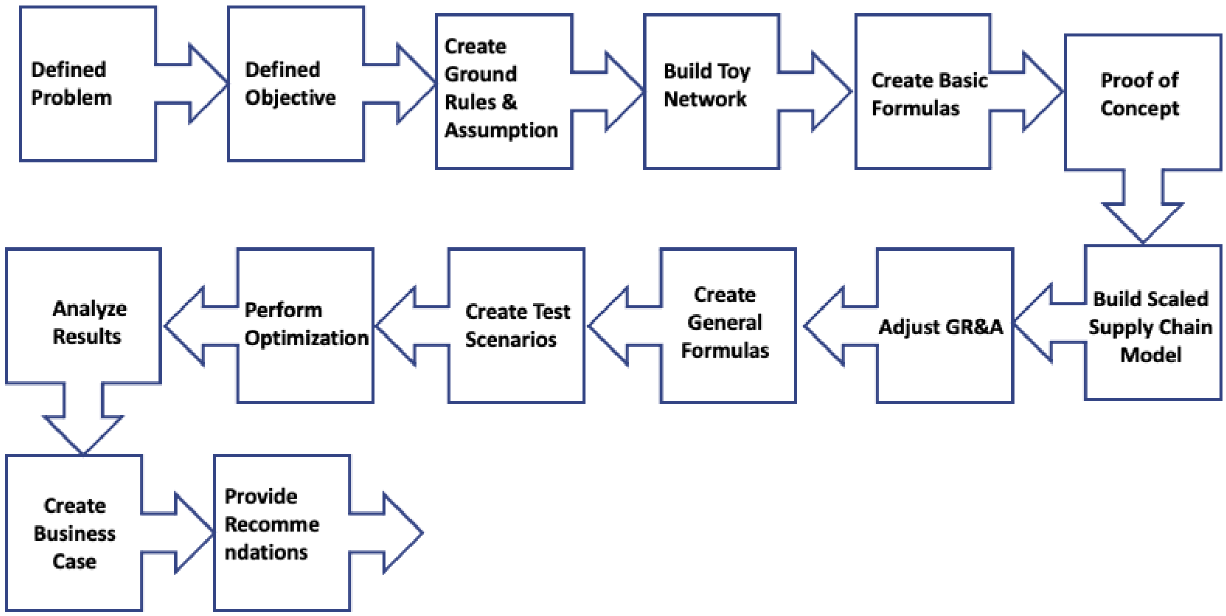


Figure 3: Process Flowchart

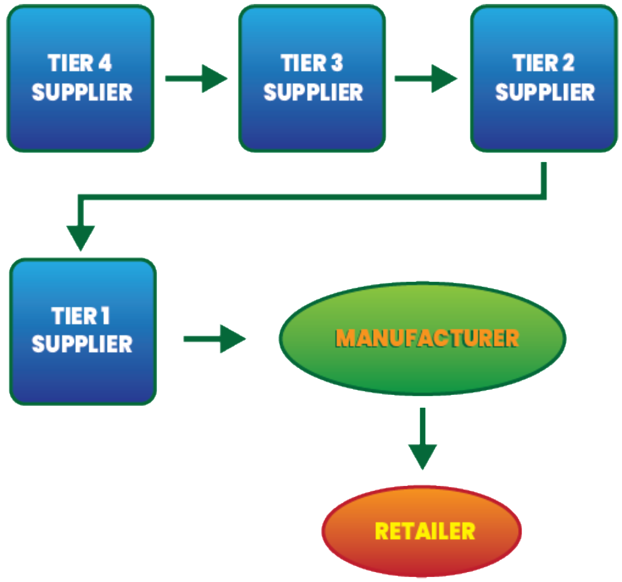


Figure 4: Supply Chain Model

The flow of the supply chain model leads from Tier 4 to retailer and based on the ground rules and assumptions, each tier supplier follows the product/part flow (see Figure 3).

In what follows, we will explain the flow chart in further detail—specifically, with regard to how one undetected bad part can travel from higher tier (tier 4) to the manufacturer and end up in the hands of the end user or final customer. As we can see in Figure 4, each supplier can produce four different types of products, of which only one type can be classified as "good," while the other three kinds are categorized as "bad" in part type classification.

Good parts are the parts that have good raw materials and have gone through a high-quality manufacturing processing and assembly process. Theoretically, the only parts that should exit the supplier and reach the next destination should be parts that fall in this class. However, as we know, this is not always the case. In reality, three classes of parts classified as bad also end up reaching the next destination. These parts are of the following three types: (1) good raw materials with bad manufacturing; (2) bad raw materials with bad manufacturing; and (3) bad raw material with good manufacturing.

During the inspection, due to operator- or process-related errors, some of these bad parts are identified as good and find their way to the next destination, which, in our case, will be next tiers in supply chain. Owing to different reasons, such as training of the personnel, poka-yoke process, or different layers of building inspection throughout the entire process, different manufacturers or suppliers can have different inspection rates of error; of note, however, error rate is always never zero. Importantly, when parts move from one supplier to the next one, the chances of inspection in a higher tier are even lower and cannot get detected later on in the supply chain. This becomes even more explicit nowadays due to the advancements of the parts.

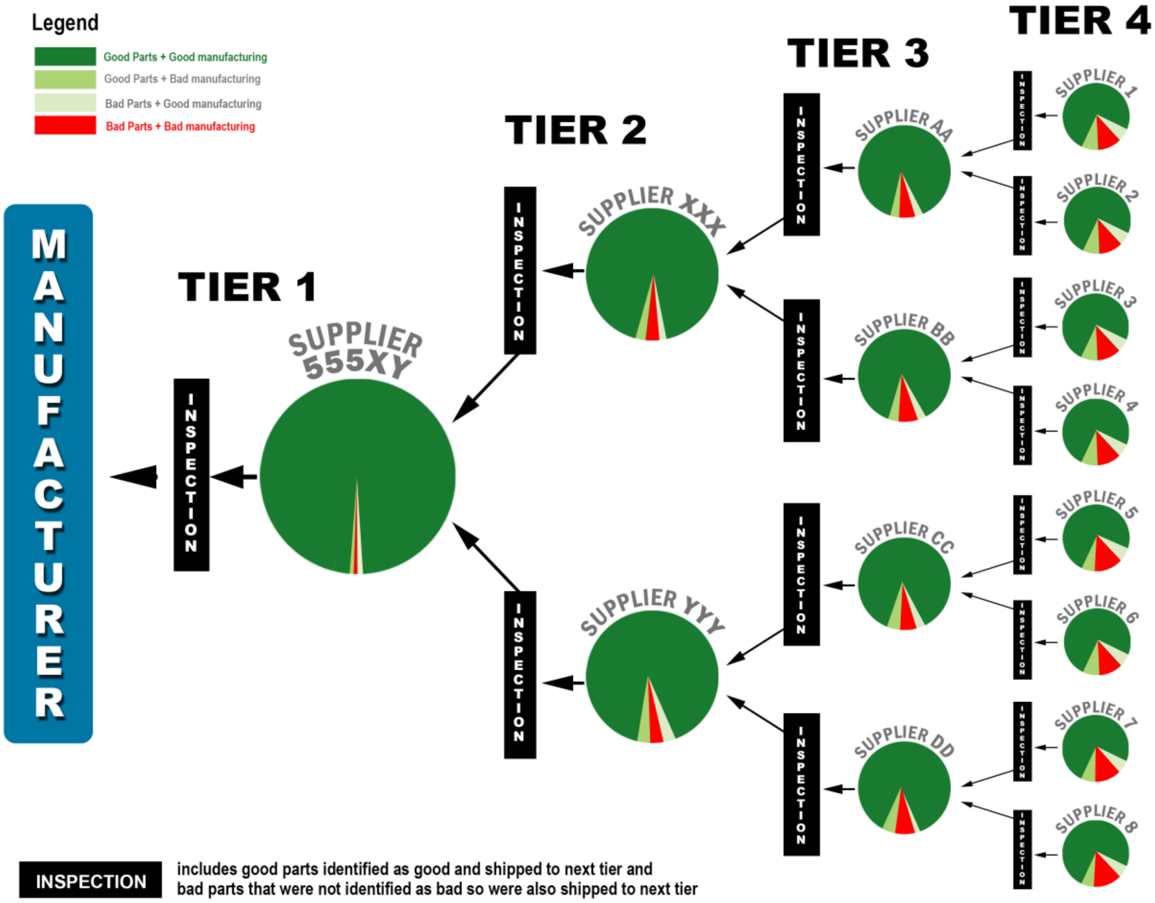


Figure 5: Part flow from Higher tier in supply chain to manufacturer

In final manufacturing, more items come as Line Replacement Units (LRU). Relevant examples of this include the whole dash assembly in car manufacturing that comes as one piece to the production line or landing gear in plane manufacturing.

Of note, with the advancement of technology and globalization of manufacturing and supply chain, the use of LRU is becoming a preferred method. A positive aspect of this development is that, today, manufacturers can lower their costs, as they no longer need to hire workers with specialized training, and they can let supplier hire, train, and handle qualified employees. This way of production also allows for more complex parts to be assembled by a more specialized supplier.

However, a limitation of this style of production is that it increases the rate of defects in higher tiers of the supply chain. For example, in today’s car manufacturing, entire dash comes as one assembled piece and, to the final manufacturer, it is like a black box where any defects one tier higher cannot be checked.

3.1.1. Assumptions

In this section, we outline assumptions and ground rules for simple/toy network. In subsequent sections, these assumptions are adjusted based on the results of the initial trial and proof of concepts. The assumptions are as follows:

- Fixed costs and economies of scale are not considered in the model. Optimization has an impact on the quantity of parts that should be produced (due to a reduction in the number of scrapped or replaced parts or increasing demand on the final production). Fixed cost impacts the breakeven point, which, in turn, influences the company’s survival; however, companies in a high tier have many buyers, and non-linear effect will not be significant.

Therefore, costs will depend on the number of items produced, inspected, and reworked in a linear way with consideration of scrap fraction.

- Demand on the product does not depend on supply chain optimization. Supply chain nodes do not have deficit or surplus of parts which impact costs.
- If multiple inspections are done on the node, cost and probabilities of type I and II errors of each inspection on the node are the same. The part that successfully passes all inspections is sent out; if at least one of inspections is not passed, the part goes to rework.
- Defect levels are neglected. Any part can be either defected or good. If the defected part is used, then production will also be defected.
- This model includes n tier of suppliers, one manufacturer, and one retailer.
- For simplicity of calculation, the whole system is divided into several subsystems and, at the end, we combine all the subsystems, meaning that each supplier considers manufacturer for the last tier.
- Each tier in our system does 100% inspection before shipping parts to the next customer; inspection error rate during each process is taken into account.
- Single sourcing is assumed during the entire process.
- Constant demand is assumed.
- Two types of error are considered after 100% inspection: error type I, in which good parts classified as bad, and error type II, in which bad products are identified as good.
- All parts categorized as defected are returned to the manufacturer.
- For simplicity of calculation, we assume that all defected parts returned to the manufacturer go to scrap. Therefore, the returned parts will not enter the rework cycle in manufacturing.

- All tiers of suppliers sell the parts as good with warranty, even those that have gone through rework, so there is no "as is" part with discount for all top four tiers.
- "As is" parts are sold only by manufacturer to retailer, and there is a discount factor on pricing (the reason behind this assumption is that, at this stage, the cost of scrapping the goods is too high).

Figures 6 through 10 show the parts flow in detail, starting from tier 4 and finishing with the retailer. Each supplier produces 4 different types of products (GR&A), performs 100% self-inspection (GR&A), examines two different types of error (GR&A), and evaluates the rework process.

As observed in the figures, in each tier, bad parts can be sent to next tier at discounted rates and the receiving manufacturer in lower tier is the deciding authority on whether to use or reject those parts. Based on this assumption, discounted parts were not included in the calculation. This assumption helped simplify the calculation and focus was put on the total number of failures transferring from one tier to the next due to the rate of error. This assumption was correct in all tiers except final manufacturer to retailer, reason being that scraping the final product is too expensive for manufacturer unless it is safety related.

Open or refurbished items sold at discounted prices to customer can be used as an example. Normally, open or refurbished products come with a shorter warranty time (i.e., three months) and, as mentioned above, the reason is that the final product is too complex and too expensive to scrap.

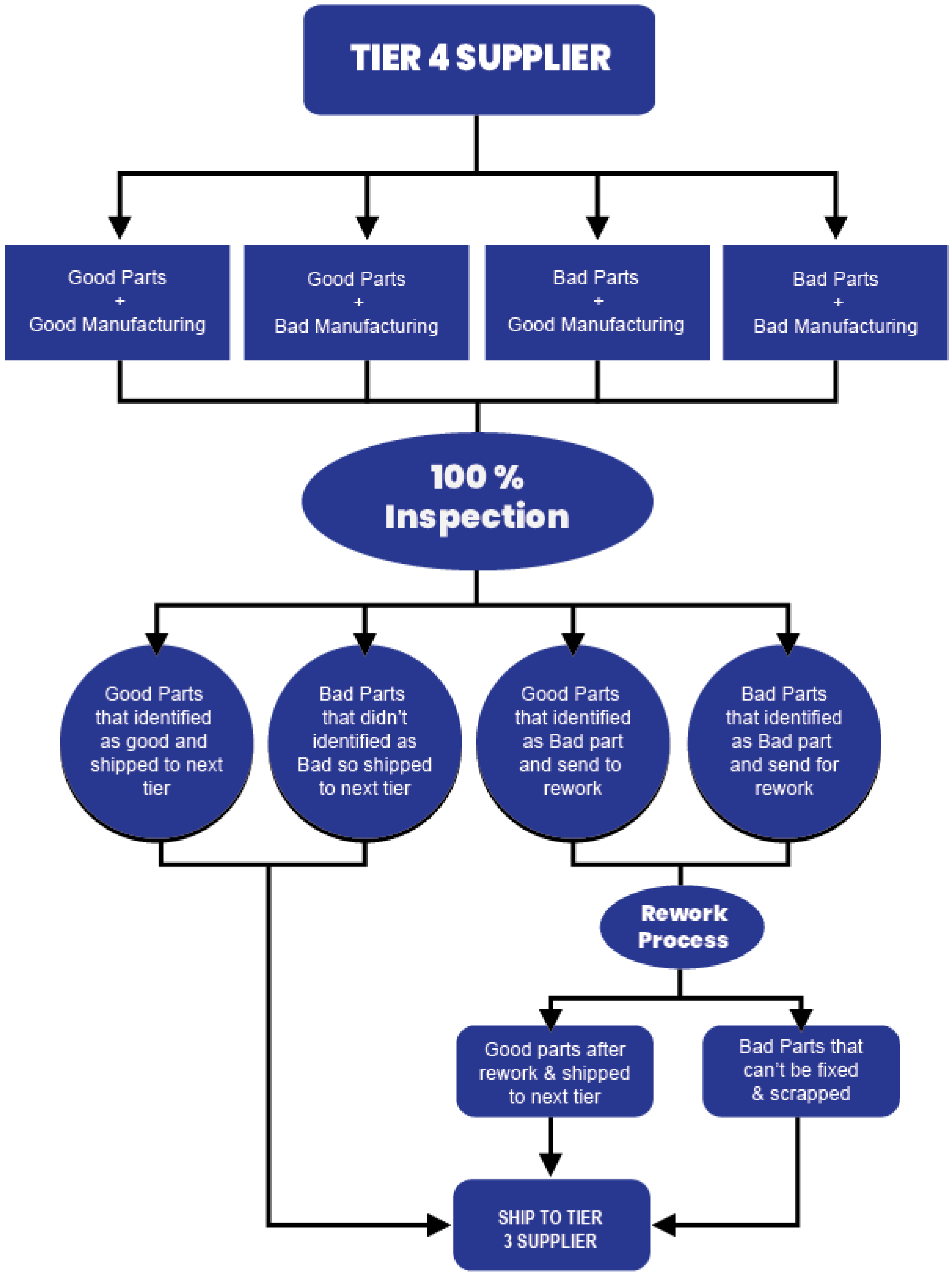


Figure 6: Tier 4 to Tier 3-part flow

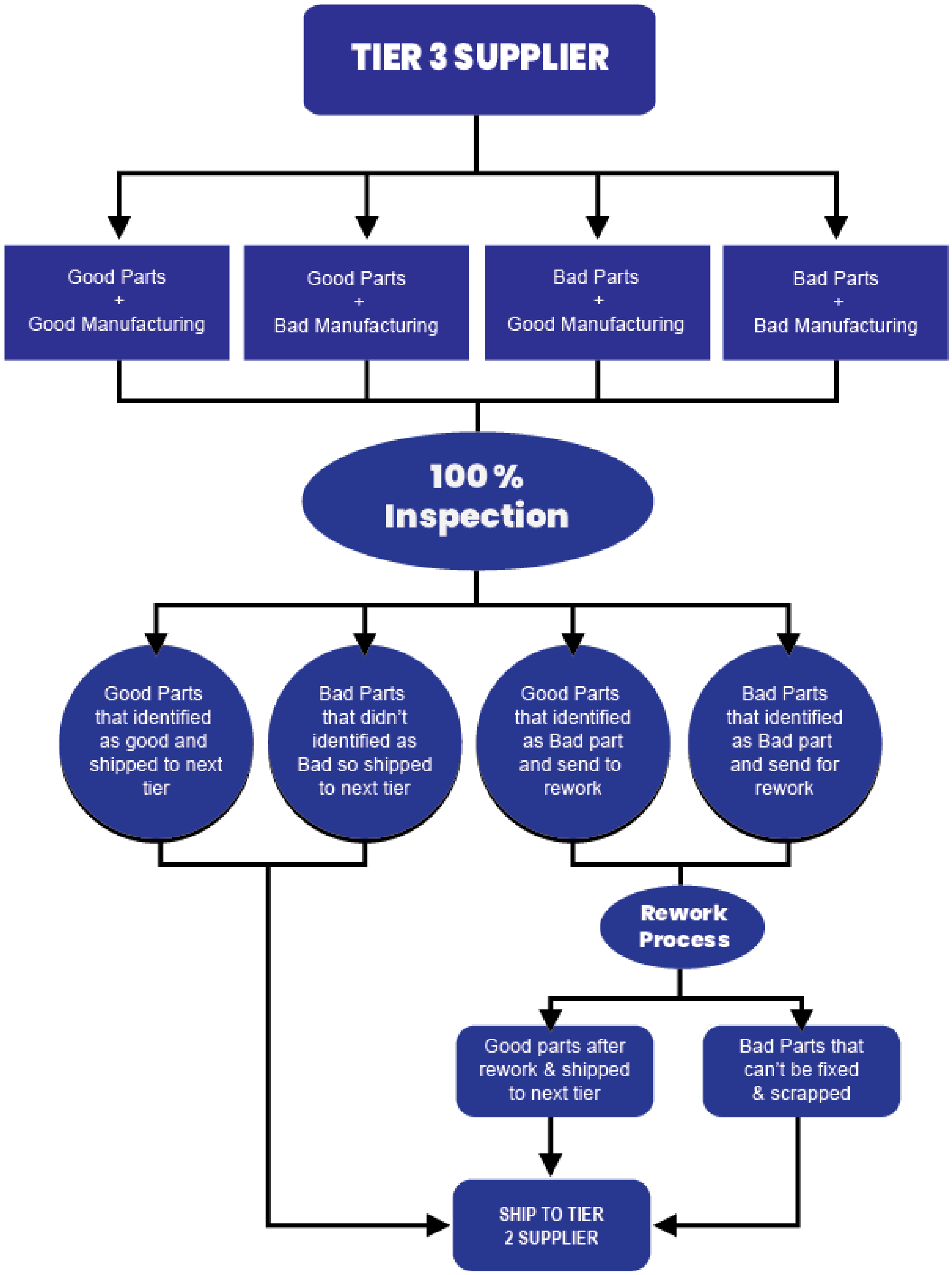


Figure 7: Tier 3 to Tier 2-part flow

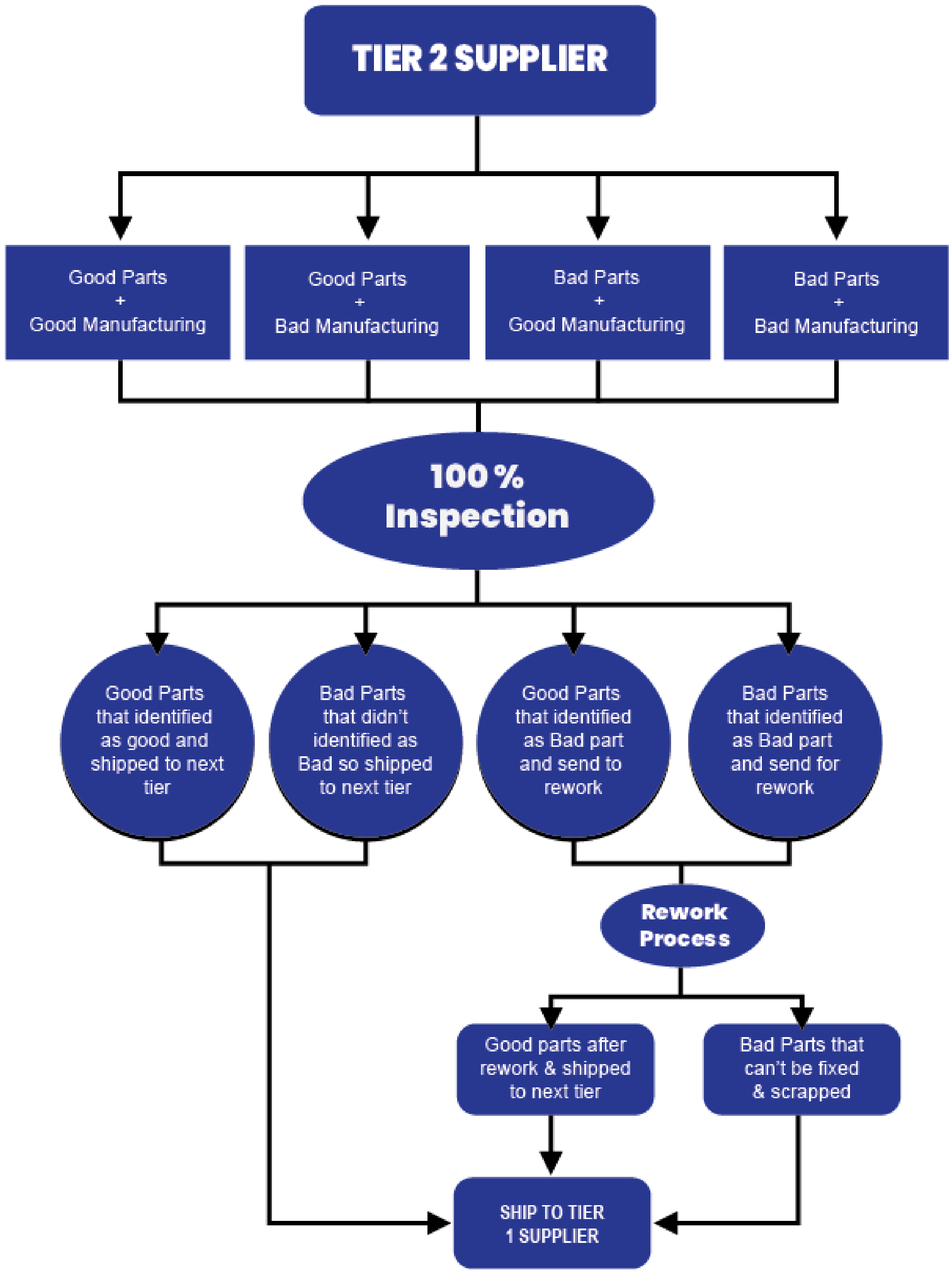


Figure 8: Tier 2 to Tier 1-part flow

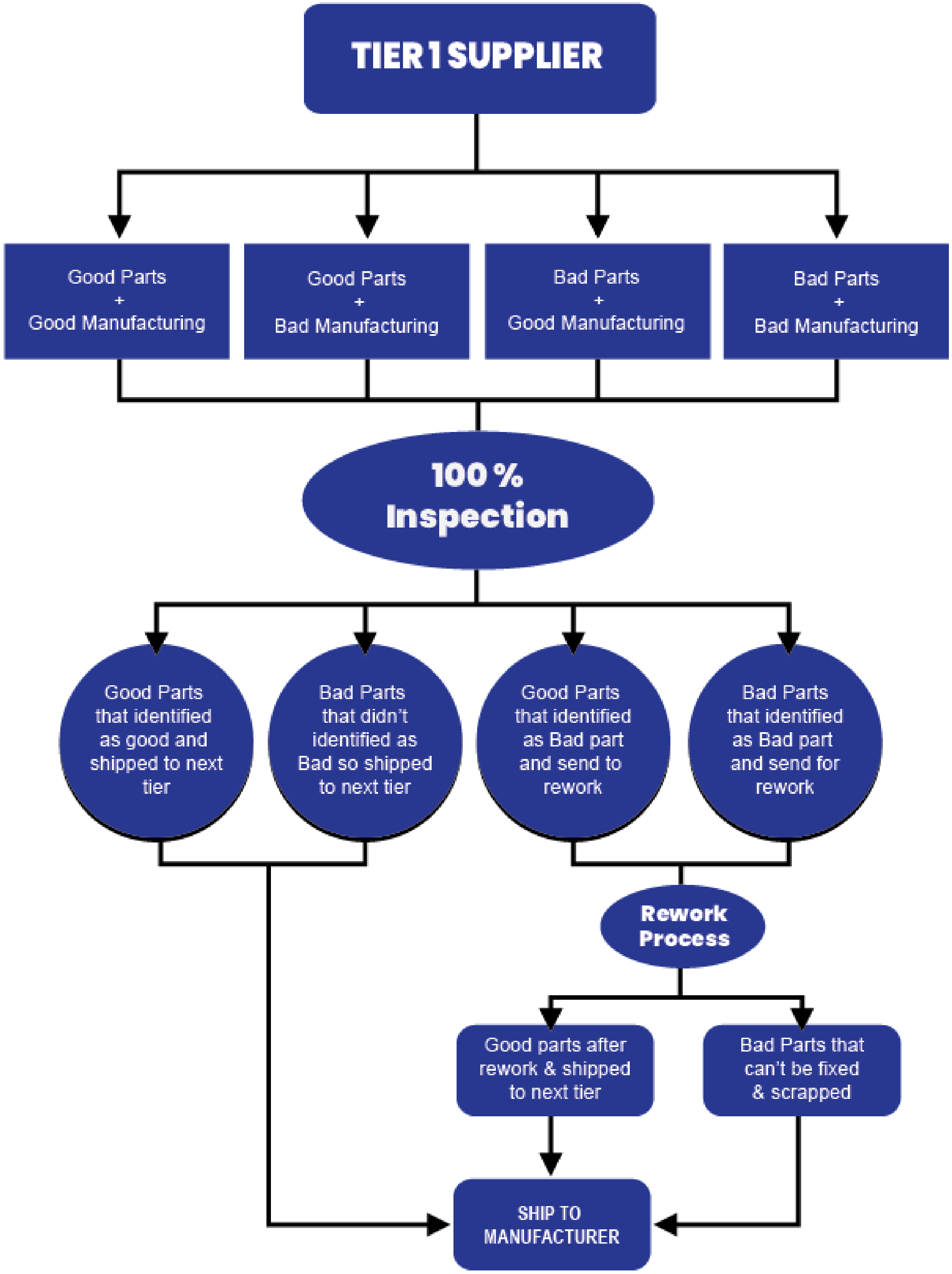


Figure 9: Tier 1 to manufacturer part flow

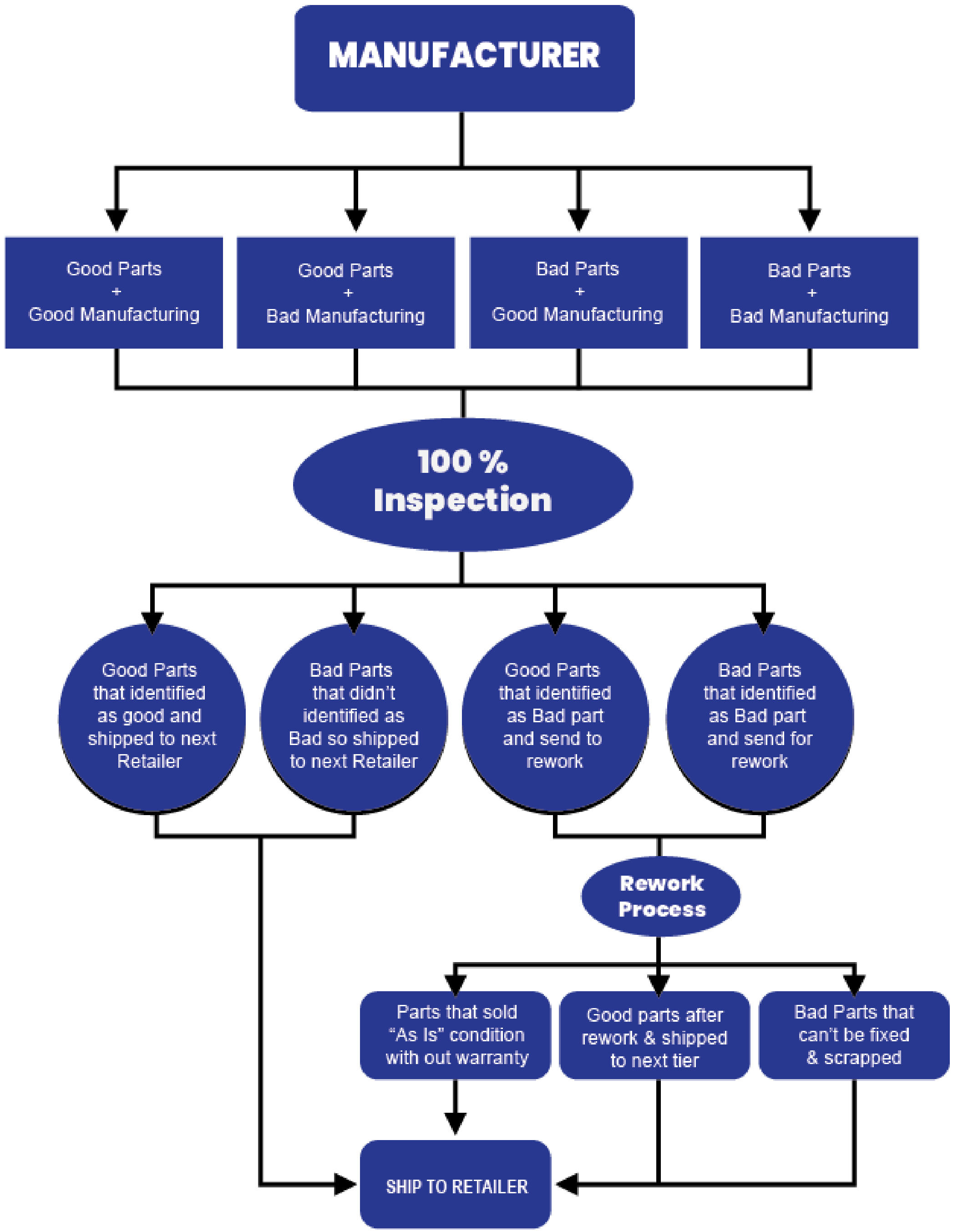


Figure 10: Manufacturer to Retailer part flow

In what follows, we introduce the parameters, variables, and basic formulas used to build our simple/toy supply chain network.

3.1.2. Parameters

In this section, we introduce the variables used during the method verification, proof of concept, and subsequent optimization.

I Set of suppliers

J Set of manufacturing plants

K Set of retailers

W Number of parts delivered from one supplier to the next supplier

Ysi Fraction defective at selected supplier i

Yij Inspection error rate at manufacturing plant J

Yrk Fraction defective at selected retailer K

Dem Customer demand (each supplier is customer for supplier in a higher tier)

FCP Fixed cost for prevention activities

VCP Variable cost for prevention activities (i.e., the cost incurred to keep failure and appraisal costs at a minimum)

VCP1: Cost of time spent for preparation, meeting attendance, presentation, etc.

VCP2: Cost of time spent on recurring statistical process control application, including measurement, chart preparation, data analyses, CP and Cpk calculation, etc.

VCP3: Cost of time spent on conducting, analyzing, reporting, etc. of machine and process capability research

VCP4: Cost of time spent on writing, distributing, controlling quality policies and procedures

VCP5: Cost of time spent on quality system audits, including scheduling, preparation, audit, follow-up, report presentation, and presentation to management

VCP6: Cost of time for capability evaluation of new subcontractors and revaluation of current subcontractors

VCP7: Cost of time spent by personnel for quality related training provided by internal or external sources

VCP8: Cost of quality related training provided by outside sources, including fees, travel & business expenses, etc.

VCP9: Cost of time spent on maintenance of equipment and machinery performed on a planned schedule.

VCP10: Cost of time spent on non-problem related visits to customers

FCI: Fix Cost of inspection at each manufacturing plant before shipping parts out

VCI: Variable cost of inspection at each manufacturing plant before shipping parts out

ICF: Total internal cost of failure

COF: Cost due to a failure of purchased component from supplier to meet quality requirements

DCM: Direct cost of manufacturing for each item that goes to rework

RC: Rework cost per item

Pg: Price of goods sold as "No defect, 100% good with warranty"

Pdef: Price of goods sold as "as is" with no warranty (only in manufacturer)

Cinsp: Cost of 100% inspection, incurred to determine the degree of conformance to quality requirements *n:* number of suppliers, and $n = 1 \rightarrow 5$

Wn: Number of parts delivered from supplier in one tier to the next supplier in a lower tier. The following formulas were created for calculation:

$$\mathrm{Wn} = (\mathrm{GpBmn} + \mathrm{BpGmn} + \mathrm{BpBmn} + \mathrm{GpGmn}) - \mathrm{SCRAPn} \quad \textbf{(1)}$$

Good parts from a supplier:

$$Gpn = (1 - Ysin) \times Wn \quad \textbf{(2)}$$

Bad parts from a supplier:

$$Bpn = Wn - Gpn \quad \textbf{(3)}$$

Rework rate at each manufacturing plant:

$$\varnothing n = \frac{(\ GpSrn\ + BpSrn\ )}{Wn} \quad \textbf{(4)}$$

Good parts from a supplier and good manufacturing:

$$GpGmn = Gpn \times (1 - Ysin) \quad \textbf{(5)}$$

Good parts from a supplier and bad manufacturing:

$$GpBmn = Gpn \times (Yijn) \quad \textbf{(6)}$$

Bad parts from a supplier with good manufacturing:

$$BpGmn = Bpn \times (1 - Ysi) \quad \textbf{(7)}$$

Bad parts from a supplier with bad manufacturing:

$$GpBmn = Gpn \times (Yijn) \quad \textbf{(8)}$$

Good parts send out after 100% inspection:

$$GpSon = GpBmn \times (1 - Ysin) \quad \textbf{(9)}$$

Bad part unidentified during 100% inspection, so send out:

$$\mathrm{BpSon}\ = (\ \mathrm{GpBmn}\ +\ \mathrm{BpBmn}\ \ ) \times\ (\ \mathrm{Yijn}\ ) + (\ \mathrm{BpGmn}\ \ \times\ \mathrm{Ysin}\ ) \quad \textbf{(10)}$$

Bad parts identified as defected during 100% inspection and sent for rework:

$$BpSrn = (GpBmn + BpGmn + BpBmn) - (BpSon) \quad \textbf{(11)}$$

Good parts identified as bad and sent for rework:

$$GpSrn = (Wn - GpSon - BpSon - BpSrn) \quad \textbf{(12)}$$

Good parts after rework (it costs more for the company due to adding rework cost):

$$GpArn = (GpSrn + BpSrn) \times (1 - Yirn) \quad \textbf{(13)}$$

Defective product that cannot economically be repaired:

$$SCRAPn = (GpBmn + BpGmn + BpBmn + GpGmn) - (GpArn + GpSon + BpSon) \quad \textbf{(14)}$$

Cost-related calculation:

ICF: Total internal cost of failure (cost due to the bad manufacturing process)

$$ICFn = FCPn + (DCMn + Crn) \times \varnothing n \times (1 - Yijn) \times BpSrn + (DCMn + Crn) \times \varnothing n \times (1 - Yijn) \times GpSrn + BpBmn \times (DCMn + Crn) \times \varnothing n \times (1 - Yijn) \quad \textbf{(15)}$$

where

FCI: Fix cost of inspection at each manufacturing plant before shipping parts out to customer

DCM: Direct cost of manufacturing for each item going for rework

Cr: Cost of rework per item

Cp: Prevention cost that is equal to all activities related to the preventing of poor quality:

$$Cpn = FCPn + VCPn \times (1 - Ysin) \times Wn \times (1 - Yijn) \quad \textbf{(16)}$$

where

VCI: Variable cost of inspection at each manufacturing plant before shipping parts out to customer.

Cinspn: Appraisal cost or cost of 100% inspection is a cost that incurred to determine the degree of conformance to quality requirements:

$$\mathrm{Cinspn} = \mathrm{FCIn} + \mathrm{VCIn} \times (1 - \mathrm{Ysin}) \times \mathrm{Wn} \quad \textbf{(17)}$$

This equation calculates the probability of a faulty product being sent out from a higher tier in the supply chain (i.e., Tier 4 or Tier 3) or defected auto parts being used in the final product due to inspection error rate in each supplier and manufacturing setting.

$$P(An) = [(BpGmn + BpBmn + GpBmn) \times Yijn]/[\ (GpGmn + GpBmn + BpGmn + BpBmn) - (SCRAPn)] \quad \textbf{(18)}$$

where P(A) is the probability of defected parts not being detected in each supplier and sent to the next tier, while n represents the tier of a supplier in the supply chain.

The multiplication of probability theorem to see the possible percentage of the defected final product due to fault in auto parts provided from Tier 1 to Tier 4 or higher:

$$P\ (P(Total)) = P(((An))) + \{P(((An))) \times P\ (An - 1 \mid An)\} + \{P(((An))) \times P\ (An - 1 \mid An) \times P\ (An - 2 \mid An - 1\ Ç\ An)\} + \cdots \quad \textbf{(19)}$$

3.1.3. Basic Supply Chain Model and Proof of Concept

The following example shows a small supply chain network with four tiers of the supplier, with a single supplier in each tier using a single product. This small toy network (see Figure 11) is the simplest possible supply chain network for manufacturing, extended from Tier 4 to the retailer.

The toy network in Figure 11 consists of four tiers of suppliers and one manufacturer. As can be seen in Figure 11, each horizontal group of numbered stars represents one tier in the supply chain (Stars 1-5 represent Tier 4 supplier; Stars 6-9 represent Tier 3; Stars 10-15 represent Tier 2 supplier, and Stars 16-29 represent Tier 1 supplier). All suppliers in Tier 1 connect with Star 30, i.e., the manufacturer.

As the toy network outlines, some Suppliers provide auto parts to more than one Supplier. For instance, Supplier 10 from Tier 2 provides auto parts to Supplier 7 in Tier 1, while using only one Supplier to gather the auto parts needed for its production. Therefore, for Supplier 10, we have one in-link and seven out-links. In the car manufacturing industry, similarly to companies that produce electronics and provide services and parts used as stereo or as board computers in an auto vehicle, several companies provide parts to different Suppliers in all tiers.

Basically, Figure 11 demonstrates that some suppliers provide parts for the next tier along with providing parts for the tier in the same number (i.e., Supplier 3). According to our calculation, not all suppliers in the same tier have the same value, and the rank of each supplier changes during the calculation based on the weight of the in-link supplier.

The results of toy network calculation (Figure 12) suggest that the manufacturer has the most value as ranking, as the parts go through inspection before they are shipped to retailer.

However, the important components are suppliers in Tier 2 who have more value to control than suppliers in Tier 1. The defect in Suppliers 10 and 15 from Tier 2 has more impact on the failure of the final product as compared to that in Suppliers 16,17, 21, 22, 26, 28, and 29. Importantly, as Supplier 7 in Tier 3 has a higher value than those in Tier 2 and most of Tier 1, any defects in the raw material unidentified during inspection will cause a high number of recalls. Table 2 reports the calculated PrtRnk for the toy network.

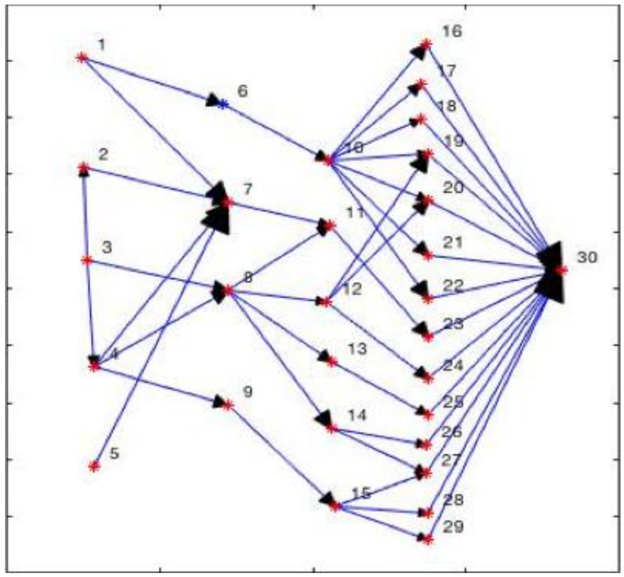


Figure 11: A Simple Toy Network

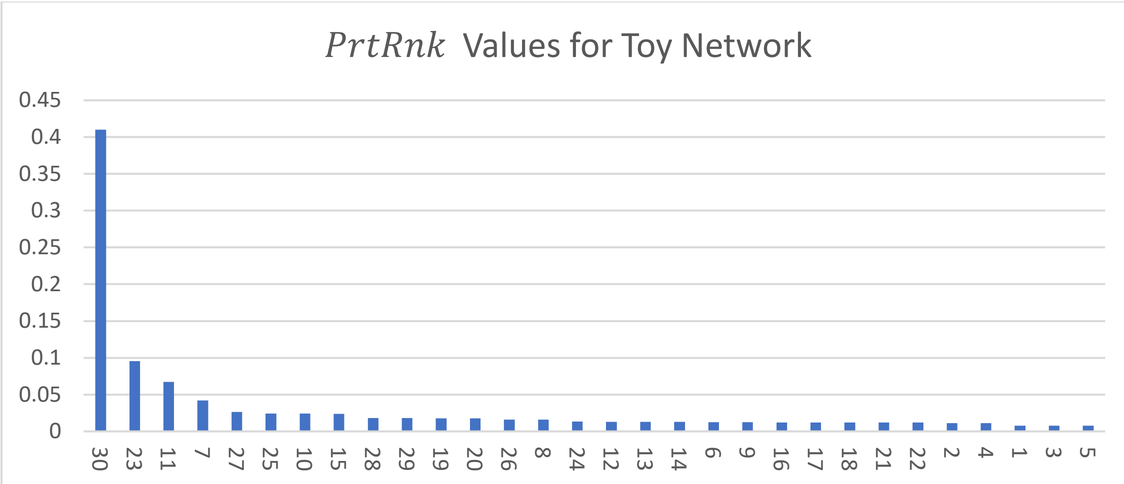


Figure 12.Toy network PrtRnk

Table 2. PrtRnk results for the toy network

	Supplier No.
0.409860683	30
0.09558889	23
0.067397034	11
0.041835689	7
0.026441588	27
0.024402316	25
0.024210732	10
0.023927035	15
0.018077031	28
0.018077031	29
0.017761224	19
0.017761224	20
0.016037759	26
0.015796771	8
0.013249574	24
0.012824705	12
0.012824705	13
0.012824705	14
0.012677835	6
0.01246035	9
0.012184853	16
0.012184853	17
0.012184853	18
0.012184853	21
0.012184853	22
0.011009624	2
0.011009624	4
0.007673202	1
0.007673202	3
0.007673202	5

As mentioned earlier, the purpose of this test was to look into the new proposed system that calculates the weight of each supplier in the entire supply chain. The outcomes of proposed ranking system showed that not all suppliers have the same value with regard to quality control, because, regardless of which supplier is at fault, when a defected final product is recalled, the manufacturer is always to blame. Therefore, in our system, it is the manufacturer's responsibility to take ownership and responsibility in higher tiers of the supply chain and keep a check on hot and high-ranked suppliers.

3.1.4. Testing Push vs. Pull Parts Concept

Upon obtaining the proof of concept on creating weight system for its further use for inspection purposes, we employed Prologic 8 to verify the following:

- whether creating new toolbox is necessary, and whether commercially available tools can be used.
- creating ground rules and assumptions on Pull VS Push part from one supplier to the next one.

The general parameters used in the model were as follows:

- Damping factor, defined in the PowerPoint document to calculate prtRank.
- Inspection scenario, defined as the inspection policy to be used in the model (see values 0 to 3 explained below).
- Fraction inspected, defined as the fraction of the suppliers that will be inspected by the manufacturer, if the inspection scenario defines the manufacturer as an inspector.
- Rework cost, defined as the cost of reworking one par.
- Inspection cost, defined as the cost to inspect one part.
- Fraction inspected Tier 1, defined as the fraction of suppliers interested in inspecting the previous tier, if inspection scenario is 1 or 2.

- Fraction inspected Tier 2, defined as the fraction of suppliers interested in inspecting the supplier that is two tiers up if the inspection scenario is 2 (these suppliers are not necessarily interested in inspecting one tier up).
- Fraction self-inspected, defined as the fraction of suppliers interested in inspecting themselves.
- Quality standard, defined as the fraction of good quality parts sold with warranty as compared to the total number of parts, according to the quality guidelines. The following four inspection scenarios were considered for external inspections:

0: No external inspection

1: External inspection from the first tier below

2: External inspection from the first tier below, and from the second tier below

3: External inspection from the manufacturer, if the prtRank is sufficiently high.

The parameters for each supplier were as follows:

- ID: the unique identifier for the supplier.
- Tier: the tier at which the supplier is placed (0 to 4, where 0 is the manufacturer).
- numParts: the number of parts that arrive from the tier above to Tier 4 supplier. This only counts for Tier 4, since Tier 3 will receive parts from Tier 4, for instance. Tier 4 also receives parts. If Tiers 3, 2, 1 or 0 have a value, it will be ignored.
- Fraction defective supplier, this reflects the fraction of bad parts received by Tier 4 suppliers. If Tiers 3, 2, 1 or 0 have a value, it will be ignored.
- Fraction defective manufacturing, which represents the fraction of parts that will end up being bad after the manufacturing process.

- Inspection rate error, this represents the fraction of inspections that will fail. If an inspection fails, the following will occur: if the part is good, the supplier will think that the part is bad; if the part is bad, the supplier will think that the part is good.
- Fraction of fail rework, which represents the fraction of parts that will be scrapped after rework (without any inspection)
- Fraction to sell "as is", which represents the fraction of parts that the manufacturer will consider having doubtful quality after the rework process and that will not be sold with any warranty. This parameter will be ignored for all tiers except for Tier 0.
- Network, which represents the id values of the suppliers in lower tiers that are connected to the supplier. The format is that all id values are separated by a comma.
- Inspection policy, which is an implicit parameter indicating whether or not the supplier will inspect itself. This is supposed to be used in the optimization.

The simulation model is generated based on the flowchart (see Figure 13) and represents how a supplier works in any tier.

As parts arrive, the manufacturing starts for that part. A part has the following three important variables:

- **isGood** (defines whether or not the part has good manufacturing; this variable is true by default).
- **perceptionGood** (defines whether or not the supplier thinks that the part has good manufacturing; this variable is true by default).
- **hasWarranty** (this variable is true by default and will be false only for a part, if the manufacturer reworks that part and is unsure about the quality).

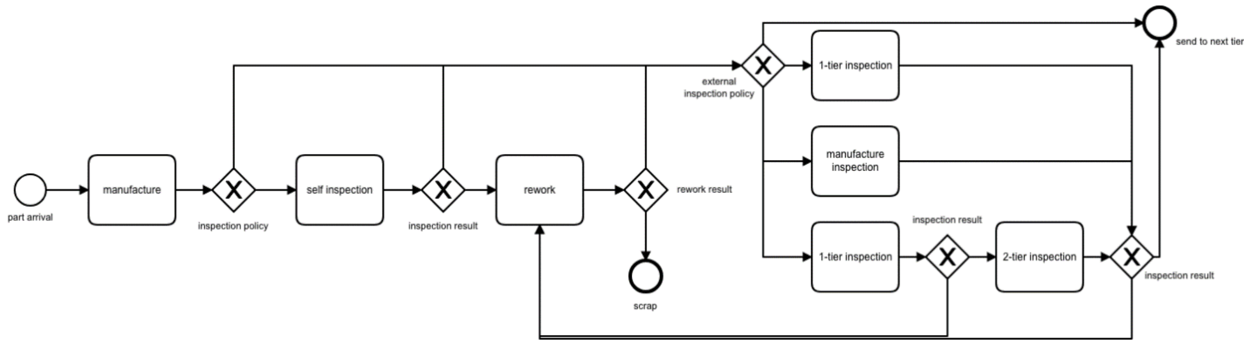


Figure 13. Prologic8 Model

The number of parts that will arrive if the tier is Tier 4 is indicated by the **numParts** variable**.** Each part will have the path randomly defined based on the defined network from the parameter, **Network.**

During the manufacturing process, the variable isGood will be true with the probability of (1- **Fraction defective manufacturing).** The inspection policy will define if there is self-inspection, and this will be indicated by the parameter, **Inspection policy.** During self-inspection, the part is inspected, and the perceptionGood variable will be equal to the isGood variable with the probability of (1- **Inspection rate error);** otherwise, it will be equal to the opposite of isGood. If perceptionGood is false, the part will go to rework; otherwise, it will go to the external inspection policy. During the rework process, the part will be fixed (even if it is already good) and scrapped with the probability of **Fraction of fail rework**. This is independent of the state of the part. Therefore, a good part could end up being scrapped with a low probability. However, any part that is not scrapped is considered good with 100% certainty; yet, if the supplier is the manufacturer (Tier 0), then there is a (**Fraction to sell as is)** chance for it to be sold without warranty; after rework, the external policy is checked.

If the **inspection scenario** is 0, the part is sent to the next tier. If **inspection scenario** is 1 or 2, the part will be sent to inspection by the supplier in tier (**Tier 1).** If the inspection results in the part value of **perceptionGood** being false, the part will be reworked again; otherwise, if **inspection scenario** is 1, the part will be sent to the next tier; if **inspection scenario** is 2, the part will be sent to be examined by the supplier in tier **(Tier 2).** Again, if the inspection result, in the part value of **perceptionGood,** is false, the part will be reworked again or, alternatively, sent to the next tier. If **inspection scenario** is 3, the part will be sent to the manufacturer inspection. Again, if the inspection results in the part value of **perceptionGood** are false, the part will be reworked again; otherwise, the part will be sent to the next tier.

3.1.5. Retailer

The retailer might buy allegedly good parts with warranty (i.e., the parts where perceptionGood = true coming from the manufacturer) or sold “as is” without warranty (these will be a fraction from the parts reworked by the manufacturer, where this fraction is “**Fraction to sell ‘as is’”).** From the parts sold with warranty, the retailer will return all parts which are coded as Good = false.

3.1.6. Part Rank

According to the literature (Page, 1998), there are three ways to calculate part ranks:

$$PR(A) = \sum_{i=0}^{n} \frac{PR(i)}{L(i)} \tag{20}$$

$$PR(A) = (1 - d) + d \sum_{i=0}^{n} \frac{PR(i)}{L(i)} \tag{21}$$

$$PR(A) = \frac{(1 - d)}{N} + d \sum_{i=0}^{n} \frac{PR(i)}{L(i)} \tag{22}$$

where

PR(A) is the part rank of Supplier A.

PR(i) is the part rank of Supplier i.

L(i) is the number of outgoing connections from Supplier i.

N is the number of ingoing connections to Supplier A. d is the damping factor.

N is the total number of suppliers.

Eq. (20) is the only one that is an actual probability; however, it only works if all suppliers have ingoing connections. Otherwise, it converges down to zero as the part rank for all suppliers.

Eq. (21) was created in order to overcome this problem where nodes without ingoing connections have to be taken out. However, this formula does not lead to a probability, and the page rank can be considerably above 1.

Eq. (22) was used as well, even though it was difficult to see why it would be interesting to use it instead. However, it was implemented in the model and can be added as a parameter, if needed.

In order to generate a part rank that is actually a probability, part ranks have to be normalized, so the final equation used in the model was as follows (see Eq. (23)):

$$PR(A) = \frac{(1-d) + d\sum_{i=0}^{n}\frac{PR(i)}{L(i)}}{\sum_{i=0}^{N} PR(i)} \quad \textbf{(23)}$$

For the purpose of this simulation, it made no difference whether or not the PR was normalized.

3.1.7. Optimization

The optimization problem has to take into consideration costs and defects, as well as what is natural to do in any industry, i.e., to minimize the costs while maintaining an acceptable level of service. Therefore, the optimization model was defined as follows (see Eq. (24)):

$$\begin{cases} min(RC + IC) \\ subject\ to: \frac{GQ}{N} > quality\ standard \end{cases} \quad \textbf{(24)}$$

where

RC = sum of all rework costs

IC = sum of all inspection costs

GQ = total number of sold good quality parts that were not returned for warranty

N = total number of parts

Quality standard is a parameter that could be, for example, 99%.

3.2. Creating Model

In this section, we adjust basic equations and parameters used to build the toy network, as well as to calculate and test proof of concept, for further use in real-world applications and real data.

3.2.1. Model Inputs

This section contains input parameters of model. These parameters should be provided in order to make calculations and optimization.

$M_{i,Yp}$ – Probability of defective manufacturing

$M_{i, Yir1}$ – Type I error of inspection probability (good part did not pass inspection)

$M_{i, Yir2}$ – Type II error of inspection probability (missed bad part)

$M_{i,Pr}$ – Probability of successful rework (for part reworked)

$M_{i,Cr}$ – Cost of rework

$M_{i,Dr}$ – Discount of rework (loses due to using reworked part)

$M_{i,Cinsp}$ – Cost of inspection

$M_{i,Cm}$ – Cost of manufacturing

$M_{i,Inum}$ – Number of inspections (including self-inspection by supplier)

$M_{i,Tn}$ – Tier number

Input parts represented as vector

$M_{i,IT,k}$ – Type of product needed to produce 1 unit of production $M_{i,IQ,k}$ – Quantity of product needs to produce 1 unit of production where k is an index of the part in list of input of manufacturer i (not in the global list of parts in model).

Introduce supply chain matrix that can be calculated using $M_{i,IT,k}$, $M_{i,IQ,}$.

$G_{i,j}$ – Number of parts that have to be supplied from node i to node j in order to produce one part at node j. Zero value means that manufacturer j does not need parts number, i.

n – Number of nodes

Q_{val} – Value of quality (total loss from every defected part)

3.2.2. *Intermediate and Output Values*

Calculating probabilities related to successful producing and reworking part

Probability that at least one raw part is bad:

$$M_{i,Ys} = 1 - \prod_k \left(1 - M_{(M_{i,IT,k}),P\_iB}{}^{M_{i,IQ,k}}\right) \qquad \textbf{(25)}$$

Probability that product is good:

$$M_{i,P\_Gp} = (1 - M_{i,YS})\,(1 - M_{i,Yp}) \qquad \textbf{(26)}$$

Probability that product is bad:

$$M_{i,P\_Bp} = 1 - (1 - M_{i,YS})(1 - M_{i,Yp}) \qquad \textbf{(27)}$$

Probability that bad part is sent out:

$$M_{i,P\_BpSo} = M_{i,P\_Bp} M_{i,Yir2}{}^{M_{i,Inum}} \qquad \textbf{(28)}$$

Probability that good part is sent out after inspection:

$$M_{i,P\_GpSo} = M_{i,P\_Gp}(1 - M_{i,Yir1})^{M_{i,Inum}} \quad \textbf{(29)}$$

Probability that bad part is sent to rework:

$$M_{i,P\_BpSr} = M_{i,P\_Bp}\left(1 - M_{i,Yir2}{}^{M_{i,Inum}}\right) \quad \textbf{(30)}$$

Probability that good part is sent to rework:

$$M_{i,P\_GpSr} = M_{i,P\_Gp}\left(1 - (1 - M_{i,Yir1})^{M_{i,Inum}}\right) \quad \textbf{(31)}$$

Probability that good part is sent out after rework:

$$M_{i,P\_GpAr} = (M_{i,P\_BpSr} + M_{i,P\_GpSr})M_{i,Pr} \quad \textbf{(32)}$$

Probability that good part is sent out:

$$M_{i,P\_GpT} = M_{i,P\_GpSo} + M_{i,P\_GpAr} \quad \textbf{(33)}$$

Probability that part goes to scrap:

$$M_{i,P\_SCRAP} = (M_{i,P\_GpSr} + M_{i,P\_BpSr})(1 - M_{i,Pr}) \quad \textbf{(34)}$$

Probability that the sent part is good:

$$M_{i,P\_iB} = 1 - \frac{M_{i,P\_BpSo}}{M_{i,P\_BpSo} + M_{i,P\_GpT}} \quad \textbf{(35)}$$

3.2.3. Calculating Costs

In this section, we discuss cost calculation for optimization process. This calculation is critical, as it was used to determine the final cost of the product. We calculated the cost in relation to the number of defects in each supplier, as well as in final manufacturing. The result of these formulas was one of the decisive factors to determine whether or not an inspection was necessary in a specific supplier.

Cost of raw parts necessary to produce one-part number *i*:

$$M_{i,RAW} = \sum_k M_{(M_{i,IT,k}),Cost} M_{i,IQ,k} \quad \textbf{(36)}$$

Cost of producing one part of type number *i:*

$$M_{i,Cost} = \frac{\left(M_{i,Cm} + M_{i,Cmsp} M_{i,Inum} + M_{i,Cr}\left(M_{i,P\_BpSr} + M_{i,P\_GpSr}\right) + M_{i,P\_GpAr} M_{i,Dr} + M_{i,RAW}\right)}{1 - M_{i,P\_SCRAP}} \quad \textbf{(37)}$$

3.2.4. Structure of Cost and Quality Expressions Analysis

Every manufacturer accepts raw materials with a quality level and produces products of varying levels of quality. All variables that impact the quality can be assumed to be intermediate. Input of manufacturer:

- $M_{i,RAW}$ - Cost of raw materials
- $M_{i,YS}$ - Quality of raw materials (probability that at least one input part is set necessary to produce 1 unit of production is bad)

Output of manufacturer:

- $M_{i,COST}$ - Cost of production
- M_{i,P_iB} - Probability that produced part is bad

3.3 Simulation of Quality and Cost Propagation

If quality M_{i,P_iB} changes at one node, then it causes a change in $M_{j,Ys}$ for direct buyers. Therefore, it changes M_{j,P_iB}. As a result, it will, directly or indirectly, change quality and cost at all manufacturers who use part number *i*.

The main aim of simulation of quality and cost propagation was to evaluate the impact of quality and cost of every manufacturer on the final production cost and quality. Overall, optimization in both production cost and quality is a multi-objective optimization problem. To this end, linear scalarization method can be used. In order to use it, the losses incurred by sending defected production should be evaluated. The objective function can be written as shown in Eq. (38).

$$F_{obj} = M_{1,COST} + Q_{val} M_{1,P\_iB} \quad \textbf{(38)}$$

Cost and quality propagation within nodes and between nodes should be modeled.

Idea of the method:

Aim:

Estimate how changing cost and quality of each type of production impacts objective function. Weighted sum of cost and quality of final production is considered as objective function in this algorithm.

Steps:

0. Decision maker decides what i the value of quality Q_{val} is (loss estimation caused by sending out one unit of final production).

1. Effects of the final production cost $M_{1,COST}$ and quality M_{1,P_iB} are easy to calculate using direct differentiation

$$\frac{\partial F_{obj}}{\partial M_{1,COST}} = 1$$

$$\frac{\partial F_{obj}}{\partial M_{1,P\_iB}} = Q_{val}$$

2. Calculate how changing input cost and quality of node impacts its output cost and quality (input cost does not impact output quality):

$$\frac{\partial M_{i,P\_iB}}{\partial M_{i,Ys}}, \frac{\partial M_{i,COST}}{\partial M_{i,RAW}}, \frac{\partial M_{i,COST}}{\partial M_{i,Ys}}$$

This shows how input costs and quality affect output cost and quality. These values are calculated numerically (of note, most calculations described in Section 1 should be performed twice adding delta to each input value of each node $M_{i,RAW}$, $M_{i,Ys}$).

3. Backward cost and quality effect propagation. It is known how quality and cost of node *i* production influence final quality and costs. Node *j* is supplier of *i*. Node's *j* output quality/cost impacts node's *i* quality/cost. Therefore, node *j* impacts the final node's quality/cost, as node *i* impacts the final node's quality/cost, and *j* impacts *i*.

The algorithm in Step 3 calculates quality and cost of node impact on the final node. The algorithm can be used for the node when it does not have buyers, as those are not processed yet (the final node processed in Step 1).

The results of applying this algorithm show the effect of changing cost and quality of each node on the final result (assuming nothing else has changed). Those results can be used as the basis for inspection optimization.

3.3.1. Direct and Indirect Effects

In this section, we use total and partial derivatives. While total derivative means that both direct and indirect effects are taken into account, partial derivative means that only direct effects are considered. For sake of clarity, only direct effects should be described.

Let us assume that quality and cost of parts directly affected by the total cost of raw materials are needed on this node and their quality (see Eq. (39)-(40)).

$$M_{i,P\_iB} = M_{i,P\_iB}\left(M_{i,RAW}, M_{i,Ys}\right) \quad \textbf{(39)}$$

$$M_{i,COST} = M_{i,COST}\left(M_{i,RAW}, M_{i,Ys}\right) \quad \textbf{(40)}$$

Direct effect of cost and quality of parts from supplier to buyer can be expressed as shown in Eq. (41)-(42).

$$M_{j,RAW} = \sum_{i=1}^{n} G_{i,j} M_{i,COST} \quad \textbf{(41)}$$

$$M_{j,YS} = 1 - \prod_{i=1}^{n} \left(1 - M_{i,P\_iB}\right)^{G_{i,j}} \quad \textbf{(42)}$$

3.3.2. Propagating Cost and Quality Within Node

This section focuses on the propagation of quality and cost from node input to node output. Quality does not depend on cost of input quality.

$$\frac{\partial M_{i,P\_iB}}{\partial M_{i,RAW}} = 0$$

Output quality depends on input quality, and output cost depends on quality of raw materials and raw materials cost.

$$\frac{\partial M_{i,P\_iB}}{\partial M_{i,Ys}}, \frac{\partial M_{i,COST}}{\partial M_{i,RAW}}, \frac{\partial M_{i,COST}}{\partial M_{i,Ys}}$$

The values of derivatives depend only on parameters of node *i*. For every node, the following variables are calculated numerically (see below).

The method of calculation is as follows: changing values of $M_{i,Ys}$, $M_{i,RAW}$ for small values and evaluating corresponding M_{i,P_iB}, $M_{i,COST}$

$$\frac{\partial M_{j,YS}}{\partial M_{i,P\_iB}} = \frac{G_{i,j}}{\left(1 - M_{i,P\_iB}\right)} \prod_{i=1}^{n} \left(1 - M_{i,P\_iB}\right)^{G_{i,j}} = \frac{\left(1 - M_{j,YS}\right) G_{i,j}}{\left(1 - M_{i,P\_iB}\right)}$$

$$\frac{\partial M_{j,RAW}}{\partial M_{i,COST}} = G_{i,j}$$

$$\frac{\partial M_{j,RAW}}{\partial M_{i,P\_iB}} = 0$$

$$\frac{\partial M_{j,YS}}{\partial M_{i,COST}} = 0$$

Objective function depends on quality and cost of final production, however indirect dependence on higher tier costs and quality evaluated assuming $M_{1,COST}$ and M_{1,P_iB} as composite functions.

$$F_{obj} = M_{1,COST} + Q_{val} M_{1,P\_iB}$$

3.3.3. Propagation of Parameters Between Nodes

This section discusses the propagation of quality and cost between nodes.

Following group of derivatives denotes quality and cost propagation from supplier output to producer input

$$\frac{\partial M_{j,P\_iB}}{\partial M_{i,P\_iB}} = \frac{\partial M_{j,P\_iB}}{\partial M_{j,RAW}} \frac{\partial M_{j,RAW}}{\partial M_{i,P\_iB}} + \frac{\partial M_{j,P\_iB}}{\partial M_{j,YS}} \frac{\partial M_{j,YS}}{\partial M_{i,P\_iB}} =$$

$$= \left[\frac{\left(1 - M_{j,YS}\right) G_{i,j}}{\left(1 - M_{i,P\_iB}\right)} \right] \frac{\partial M_{j,P\_iB}}{\partial M_{j,YS}}$$

$$\frac{\partial M_{j,P\_iB}}{\partial M_{i,COST}} = 0$$

$$\frac{\partial M_{j,COST}}{\partial M_{i,P\_iB}} = \frac{\partial M_{j,COST}}{\partial M_{j,RAW}} \frac{\partial M_{j,RAW}}{\partial M_{i,P\_iB}} + \frac{\partial M_{j,COST}}{\partial M_{j,YS}} \frac{\partial M_{j,YS}}{\partial M_{i,P\_iB}} =$$

$$= \left[\frac{(1 - M_{j,YS})G_{i,j}}{(1 - M_{i,P\_iB})} \right] \frac{\partial M_{j,COST}}{\partial M_{j,YS}}$$

$$\frac{\partial M_{j,COST}}{\partial M_{i,COST}} = \frac{\partial M_{j,COST}}{\partial M_{j,RAW}} \frac{\partial M_{j,RAW}}{\partial M_{i,COST}} + \frac{\partial M_{j,COST}}{\partial M_{j,YS}} \frac{\partial M_{j,YS}}{\partial M_{i,COST}} =$$

$$= \frac{\partial M_{j,COST}}{\partial M_{j,RAW}} G_{i,j}$$

Following group of derivatives denotes quality and cost propagation supplier input to producer input

$$\frac{\partial M_{j,YS}}{\partial M_{i,YS}} = \frac{\partial M_{j,YS}}{\partial M_{i,P\_iB}[I_{i,j}]} \frac{\partial M_{i,P\_iB}[I_{i,j}]}{\partial M_{i,YS}} + \frac{\partial M_{j,YS}}{\partial M_{i,COST}[I_{i,j}]} \frac{\partial M_{i,COST}[I_{i,j}]}{\partial M_{i,YS}}$$

$$\frac{\partial M_{j,RAW}}{\partial M_{i,RAW}} = \frac{\partial M_{j,RAW}}{\partial M_{i,P\_iB}[I_{i,j}]} \frac{\partial M_{i,P\_iB}[I_{i,j}]}{\partial M_{i,RAW}} + \frac{\partial M_{j,RAW}}{\partial M_{i,COST}[I_{i,j}]} \frac{\partial M_{i,COST}[I_{i,j}]}{\partial M_{i,RAW}}$$

$$\frac{\partial M_{j,RAW}}{\partial M_{i,YS}} = \frac{\partial M_{j,RAW}}{\partial M_{i,P\_iB}[I_{i,j}]} \frac{\partial M_{i,P\_iB}[I_{i,j}]}{\partial M_{i,YS}} + \frac{\partial M_{j,RAW}}{\partial M_{i,COST}[I_{i,j}]} \frac{\partial M_{i,COST}[I_{i,j}]}{\partial M_{i,YS}}$$

$$= \frac{\partial M_{j,COST}}{\partial M_{j,YS}} \frac{\partial M_{j,YS}}{\partial M_{i,P\_iB}} = \frac{\partial M_{j,COST}}{\partial M_{j,YS}} \left[\frac{(1 - M_{j,YS})G_{i,j}}{(1 - M_{i,P\_iB})} \right]$$

$$\frac{\partial M_{j,YS}}{\partial M_{i,YS}} = \frac{(1 - M_{j,YS})G_{i,j}}{(1 - M_{i,P\_iB}[I_{i,j}])} \frac{\partial M_{i,P\_iB}[I_{i,j}]}{\partial M_{i,YS}} + 0 \frac{\partial M_{i,COST}[I_{i,j}]}{\partial M_{i,YS}}$$

$$\frac{\partial M_{j,YS}}{\partial M_{i,YS}} = \frac{(1 - M_{j,YS})G_{i,j}}{(1 - M_{i,P\_iB}[I_{i,j}])} \frac{\partial M_{i,P\_iB}[I_{i,j}]}{\partial M_{i,YS}}$$

$$\frac{\partial M_{j,RAW}}{\partial M_{i,RAW}} = 0 \frac{\partial M_{i,P\_iB}[I_{i,j}]}{\partial M_{i,RAW}} + G_{i,j} \frac{\partial M_{i,COST}[I_{i,j}]}{\partial M_{i,RAW}}$$

$$\frac{\partial M_{j,RAW}}{\partial M_{i,RAW}} = G_{i,j} \frac{\partial M_{i,COST}[I_{i,j}]}{\partial M_{i,RAW}}$$

$$\frac{\partial M_{j,RAW}}{\partial M_{i,YS}} = \frac{\partial M_{j,RAW}}{\partial M_{i,P\_iB}[I_{i,j}]} \frac{\partial M_{i,P\_iB}[I_{i,j}]}{\partial M_{i,YS}} + \frac{\partial M_{j,RAW}}{\partial M_{i,COST}[I_{i,j}]} \frac{\partial M_{i,COST}[I_{i,j}]}{\partial M_{i,YS}}$$

$$\frac{\partial M_{j,RAW}}{\partial M_{i,YS}} = 0 \frac{\partial M_{i,P\_iB}[I_{i,j}]}{\partial M_{i,YS}} + G_{i,j} \frac{\partial M_{i,COST}[I_{i,j}]}{\partial M_{i,YS}}$$

$$\frac{\partial M_{j,RAW}}{\partial M_{i,YS}} = G_{i,j} \frac{\partial M_{i,COST}[I_{i,j}]}{\partial M_{i,YS}}$$

Summary of quality and cost effects is presented in Table 3.

Table 3. Summary of quality and cost effects

	Quality →	Cost →
→ Quality	$\frac{\partial M_{j,P\_iB}}{\partial M_{i,P\_iB}} = \left[\frac{(1-M_{j,YS})G_{i,j}}{(1-M_{i,P\_iB})}\right]\frac{\partial M_{j,P\_iB}}{\partial M_{j,YS}}$	$\frac{\partial M_{j,P\_iB}}{\partial M_{i,COST}} = 0$
→ Cost	$\frac{\partial M_{j,COST}}{\partial M_{i,P\_iB}} = \left[\frac{(1-M_{j,YS})G_{i,j}}{(1-M_{i,P\_iB})}\right]\frac{\partial M_{j,COST}}{\partial M_{j,YS}}$	$\frac{\partial M_{j,COST}}{\partial M_{i,COST}} = \frac{\partial M_{j,COST}}{\partial M_{j,RAW}} G_{i,j}$

3.3.4. Backward Propagation of Quality and Cost Effects from Final Manufacturer

The objective function in Eq. (38) includes production costs on the entire supply chain and estimation of losses due to defected parts.

$$F_{obj} = M_{1,COST} + Q_{val} M_{1,P\_iB}$$

Therefore, the effect of final producer's cost and quality on objective function can be written as shown in below Equations.

$$\frac{\partial F_{obj}}{\partial M_{1,COST}} = 1$$

$$\frac{\partial F_{obj}}{\partial M_{1,P\_iB}} = Q_{val}$$

The final aim of propagation analysis to calculate how much each node quality and cost impact the final quality and cost.

Let us assume that values $\frac{\partial F_{obj}}{\partial M_{i,COST}}$, $\frac{\partial F_{obj}}{\partial M_{i,P\_iB}}$ were already calculated for manufacturer *i*. Then, the derivatives can be written as shown in below

$$\frac{dF_{obj}}{dM_{i,COST}} = \sum_{j=1}^{n} \frac{\partial F_{obj}}{\partial M_{j,P\_iB}} \frac{\partial M_{j,P\_iB}}{\partial M_{i,COST}} + \frac{\partial F_{obj}}{\partial M_{j,COST}} \frac{\partial M_{j,COST}}{\partial M_{i,COST}}$$

$$\frac{dF_{obj}}{dM_{i,P\_iB}} = \sum_{j=1}^{n} \frac{\partial F_{obj}}{\partial M_{j,P\_iB}} \frac{\partial M_{j,P\_iB}}{\partial M_{i,P\_iB}} + \frac{\partial F_{obj}}{\partial M_{j,COST}} \frac{\partial M_{j,COST}}{\partial M_{i,P\_iB}}$$

The impact of cost and quality can be computed using Eq. (43) and (44), respectively.

$$\frac{\partial F_{obj}}{\partial M_{i,COST}} = G_{i,j} \sum_{j=1}^{n} \frac{\partial M_{j,COST}}{\partial M_{j,RAW}} \frac{\partial F_{obj}}{\partial M_{j,COST}} \quad \textbf{(43)}$$

$$\frac{\partial F_{obj}}{\partial M_{i,P\_iB}} = \left[\frac{(1 - M_{j,YS})G_{i,j}}{(1 - M_{i,P\_iB})} \right] \sum_{j=1}^{n} \frac{\partial M_{j,P\_iB}}{\partial M_{j,YS}} \frac{\partial F_{obj}}{\partial M_{j,P\_iB}} + \frac{\partial M_{j,COST}}{\partial M_{j,YS}} \frac{\partial F_{obj}}{\partial M_{j,COST}} \quad \textbf{(44)}$$

Above equations and results can also be written using the cost of raw parts necessary to produce one part (M*RAW*) and the probability of at least one raw part being bad (*Ys*).

$$F_{obj} = M_{1,COST} + Q_{val} M_{1,P\_iB}$$

$$\frac{\partial F_{obj}}{\partial M_{1,YS}} = \frac{\partial F_{obj}}{\partial M_{1,COST}} \frac{\partial M_{1,COST}}{\partial M_{1,YS}} + \frac{\partial F_{obj}}{\partial M_{1,P\_iB}} \frac{\partial M_{1,P\_iB}}{\partial M_{1,YS}}$$

$$\frac{\partial F_{obj}}{\partial M_{1,RAW}} = \frac{\partial F_{obj}}{\partial M_{1,COST}} \frac{\partial M_{1,COST}}{\partial M_{1,RAW}} + \frac{\partial F_{obj}}{\partial M_{1,P\_iB}} \frac{\partial M_{1,P\_iB}}{\partial M_{1,RAW}}$$

$$\frac{\partial F_{obj}}{\partial M_{1,YS}} = \frac{\partial M_{1,COST}}{\partial M_{1,YS}} + Q_{val} \frac{\partial M_{1,P\_iB}}{\partial M_{1,YS}}$$

$$\frac{\partial F_{obj}}{\partial M_{1,RAW}} = \frac{\partial M_{1,COST}}{\partial M_{1,RAW}} + \frac{\partial F_{obj}}{\partial M_{1,P\_iB}} 0 = \frac{\partial M_{1,COST}}{\partial M_{1,RAW}}$$

$$\frac{dF_{obj}}{dM_{i,RAW}} = \sum_{j=1}^{n} \frac{\partial F_{obj}}{\partial M_{j,RAW}} \frac{\partial M_{j,RAW}}{\partial M_{i,RAW}} + \frac{\partial F_{obj}}{\partial M_{j,COST}} \frac{\partial M_{j,COST}}{\partial M_{i,RAW}}$$

$$\frac{dF_{obj}}{dM_{i,YS}} = \sum_{j=1}^{n} \frac{\partial F_{obj}}{\partial M_{j,RAW}} \frac{\partial M_{j,RAW}}{\partial M_{i,YS}} + \frac{\partial F_{obj}}{\partial M_{j,COST}} \frac{\partial M_{j,COST}}{\partial M_{i,YS}}$$

$$\frac{dF_{obj}}{dM_{i,RAW}} = \sum_{j=1}^{n} \frac{\partial F_{obj}}{\partial M_{j,RAW}} \frac{\partial M_{j,RAW}}{\partial M_{i,RAW}} + \frac{\partial F_{obj}}{\partial M_{j,YS}} 0 = \sum_{j=1}^{n} \frac{\partial F_{obj}}{\partial M_{j,RAW}} \frac{\partial M_{j,RAW}}{\partial M_{i,RAW}}$$

$$\frac{dF_{obj}}{dM_{i,YS}} = \sum_{j=1}^{n} \frac{\partial F_{obj}}{\partial M_{j,RAW}} \frac{\partial M_{j,RAW}}{\partial M_{i,YS}} + \frac{\partial F_{obj}}{\partial M_{j,YS}} \frac{\partial M_{j,YS}}{\partial M_{i,YS}}$$

3.3.5. Analysis of Quality and Cost Propagation Model

The model of cost and probability of defect propagation is based on Eq. (38), Eq. (39), and Eq. (43)-(44).

The main variables in this model are $\frac{\partial F_{obj}}{\partial M_{i,COST}}$ and $\frac{\partial F_{obj}}{\partial M_{i,P\_iB}}$ that represent how cost and quality of part sent out by every manufacturer affect the cost of good part of the final production. Eq. (43)-(44) are recursive and should be used for modeling manufacturer *i* before all its suppliers. This can be done when the graph does not have cycles.

Computation complexity of this algorithm is comparable to calculating basic values for all manufacturers and can be expressed as shown below: O(nt+s)

where

n – number of manufacturers

t – number of tiers

s – number of edges

The main limitation of model is its linearity. If parameters are significantly changed, the basic calculation algorithm has to be redone. On other hand, the number of such changes is significantly less than n.

3.3.6. Verification and Optimization

This section discusses adding or removing inspections to minimize cost and improve quality. Optimization variables are as follows:

$M_{i,Inum}$ – number of inspections on each node Objective function is shown in below

$$F_{obj} = M_{1,COST} + Q_{val} M_{1,P\_iB}$$

And constraints are shown as:

$$L_{i,\min} \le M_{i,Inum} \le L_{i,\max} \ ,$$

$$M_{i,Inum} \text{ is integer}$$

where $L_{i,\min}, L_{i,\max}$ - are minimum and maximum possible number of inspections on node.

3.3.7. Linear Estimation of Inspections Benefit

We then calculated the ratio of cost and quality on each node that would take place if manufacturer *i* adds or removes inspection, while all other manufacturers do not change number of inspections (see Eq. (44)-(45)).

$$M_{i,I+} = \frac{M_{i,COST}\left[M_{i,Inum} = M_{i,Inum} + 1\right] - M_{i,COST}\left[M_{i,Inum} = M_{i,Inum}\right]}{M_{i,P\_iB}\left[M_{i,Inum} = M_{i,Inum} + 1\right] - M_{i,P\_iB}\left[M_{i,Inum} = M_{i,Inum}\right]} \quad \textbf{(44)}$$

$$M_{i,I-} = \frac{\Delta M_{i,COST}\left[\max(M_{i,Inum} - 1, 0)\right] - M_{i,COST}\left[M_{i,Inum} = M_{i,Inum}\right]}{\varepsilon + \Delta M_{i,P\_iB}\left[\max(M_{i,Inum} - 1, 0)\right] - M_{i,COST}\left[M_{i,Inum} = M_{i,Inum}\right]} \quad \textbf{(45)}$$

where square brackets denote a change in the number of inspections, ε - is an infinitely small constant.

$M_{i,I+}$ and $M_{i,I-}$ show first approximation of inspection effect.

We then compared the contribution of quality impact to cost impact to evaluate how much this node may pay to improve quality (see Eq. (46)).

$$Q_{i,Cost} = \frac{\partial F_{obj}}{\partial M_{1,P\_iB}} / \frac{\partial F_{obj}}{\partial M_{i,COST}} \quad \textbf{(46)}$$

Coefficient of inspection effectiveness was computed as follows (see Eq. (47)):

$$E_i = \begin{cases} \frac{Q_{i,Cost}}{M_{i,I+}}, (M_{i,Inum} \le L_{i,\max}) \wedge \left(\frac{Q_{i,Cost}}{M_{i,I+}} > 1 \right) - \\ 0, \; otherwise \end{cases} \quad \textbf{(47)}$$

$$- \begin{cases} \frac{Q_{i,Cost}}{M_{i,I-}}, (L_{i,\min} \le M_{i,Inum}) \wedge \left(\frac{Q_{i,Cost}}{M_{i,I-}} > 1 \right) \\ 0, \; otherwise \end{cases}$$

While positive values of E_i show that inspection should be added, negative values demonstrate that inspection should be removed. The values of E_i represent the first approximation. Changing values of inspections on some nodes will change the effectiveness of inspections on other nodes.

3.3.8. Greedy Algorithm of Optimization

Greedy algorithm used for inspection optimization.

- $L_{in}=n$
- Calculate E_i using Eq. (47) and number K of non-zero elements in E_i
- If min (L_{in}, K) = 0 stop iteration algorithm
- Choose min (L_{in}, K) nodes with the maximum absolute values from Ei.
- Change number of inspections for nodes chosen.
- If objective function value becomes higher than L_{in}, then rollback inspections change and set L_{in}= floor(K/2), else L_{in}=K.
- Go to step 2.

The main idea of the algorithm outlined above is finding vector of values $M_{i,Inum}$, providing values of objective function that cannot be improved by changing the number of inspections on a single node. In some cases, this solution may be improved by cancelling inspections on some nodes and adding them on other ones; however, solution obtained can be considered as the near optimal solution. In the limiting case $M_{1,P\_iB} \approx 1$, removing inspection cannot worsen quality; however, it saves costs. This effect can take place when initial network parameters result in a large value of M_{1,P_iB} or when Q_{val} is underestimated.

3.4 DefectRank Approach

As discussed in Section 2.4, PageRank is a powerful tool to estimate activity on webpage. The main idea of this Google tool it is simulating user behavior. Every page has the probability that a user starts surfing from, the matrix of moving to other page probabilities, and the probability to end surfing. Based on this, the PageRank algorithm calculates the vector of probabilities of a random user at a random moment.

Defect propagation can use the PageRank analogy to find the suppliers who contribute the most to total defect probability (see Table 4). The idea behind this methodology is to consider the supply chain as one directed weighted graph and treat each supplier the same way Google treats each webpage in their search engine optimization algorithm. The implementation of PageRank algorithm in supply chain is very different. One main difference is the damping factor (d), which is a constant (0.85) that shows the probability of a user randomly clicking on the link to a webpage without following any backlink. In supply chain the random move from one supplier to the next does not exist, so this number is kept as close as possible to zero (0.001).

Table 4. PageRank and DefectRank analogy

PageRank calculation	DefectRank calculation
User in: User starts at a random page	Final node Quality of final node production has a significant value
Propagation User goes from page source to one of links (considering weight) It transfers value from the link source to the destination	Propagation Value of defect propagates backward by the supply lines direction. Quality of node production has value because is supplies parts to node that has value.
User out: Probability that user goes out	Inspection reduce probability of fail
Multiple links do not have multiple effects; they can only redistribute probability of transfer	Multiplication of link effect A→ B, if B needs several parts of A, then defects of part provided by A will cause more defects on node B
Adding node to network redistributes PageRank	If node S added to A (S→A) – it will not change value of defect propagated by line A→B
Page rating meaning Probability (weight) that user is on this page	Defect rank refers to the extent to which defects on this node impacts the final node quality

3.4.1. DefectRank calculation

The assumptions underlying DefectRank calculation were as follows:

- Costs not taken into consideration.
- No type I errors taken into consideration.
- Defected part and bad manufacturing probability is sufficiently small to use a linear model. Probability of having at least two defected raw details or at least one defected detail and bad manufacturing can be neglected.

Defect weight shows how much change of defect on node *i* impacts defect on the final node. It was calculated using below equations.

$$W_1 = 1$$

$$W_i = \sum_{j=1}^{n} M_{j,Yir2} G_{i,j} \quad \textbf{(46)}$$

DefectRank shows how much the final node quality could be improved if the node defects are eliminated Eq. (47).

$$D_i = W_i M_{i,P\_iB} \quad \textbf{(47)}$$

The DefectRank algorithm is similar to 3.2.

$$E_i = \begin{cases} \dfrac{D_i Q_{val}}{M_{i,Cinsp}}, (M_{i,Inum} \leq L_{i,\max}) \wedge \left(\dfrac{D_i Q_{val}}{M_{i,Cinsp}} > 1 \right) \\ 0,\ otherwise \end{cases} -$$

$$- \begin{cases} \dfrac{M_{i,Yir2} M_{i,Cinsp}}{D_i Q_{val}}, (L_{i,\min} \leq M_{i,Inum}) \wedge \left(\dfrac{M_{i,Yir2} M_{i,Cinsp}}{D_i Q_{val}} > 1 \right) \\ 0,\ otherwise \end{cases}$$

Term $\frac{D_i Q_{val}}{M_{i,Cinsp}}$ represents the ratio between estimated value of quality improvement in nominator and the cost of inspection needed to achieve this quality improvement. If this value is exceeding 1 and the limit allows adding more inspections, then adding inspection is suggested.

Term $\frac{M_{i,Yir2} M_{i,Cinsp}}{D_i Q_{val}} = \left(\frac{D_i Q_{val}}{M_{i,Cinsp}} \right)^{-1} M_{i,Yir2}$ represents the effect of removing inspection.

The ratio of the cost of inspection and value of quality lost. Coefficient $M_{i,Yir2}$ shows that removing inspection more significantly impacts quality than adding one.

In what follows, we provide the specifications and assumptions used to create the supply chain network.

- In this network, we consider a single supplier that produces only one part for the manufacturer.
- Inspection error rate (Yir) is constant between all suppliers and manufacturer and measures 0.009, meaning that, for every 1000 bad auto parts (due to bad part, bad manufacturing, or both), nine are not detected during inspection and will be shipped to the next tier supplier.

- We considered that Tier 4 supplier receives 100,000 raw materials and, after the assembly stage, those parts are shipped to the next tier.
- For the purpose of the present study, we decided that the auto parts detected by manufacturer as bad due to a defect from the supplier will not be shipped back to the supplier but sent out to scrap.
- Demand is constant throughout the entire network and, as we start with 100,000 parts from Tier 4, the number shipped to Tier 3 will be 100,000 minus parts scrapped in Tier 4 facility.

3.5 Scenarios and Summary of Parameters Description

For this project, 18 different scenarios were built to cover all possible conditions for inspection of all tiers within the supply chain. The 18 scenarios were divided into three main categories:

1- Non-optimization cases used for baseline and comparison

2- Cases that included Type 1 and Type 2 errors

3- Cases that eliminated Type 1 error and focused exclusively on Type 2 error

Table 6 summarizes all scenarios used for this study, and outlines all parameters used for input, basic output, advanced output and inspection effect analysis in the supply chain.

It is important to note that more scenarios can be considered for this test based on the number of tiers, the number of suppliers in each tier and ground rules and assumptions, however, the base line scenario should be kept the same (no optimization). In case scenarios that included Type I and Type II errors, the focus was to test the compatibility of the model to calculate more complex inputs.

Table 5 Case descriptions

Case	Description
1	No optimization
2	DefectRank analysis with threshold = 0.001
3	Multiple inspections are allowed only on final node
4	Multiple inspections are allowed only on Final, T1
5	Multiple inspections are allowed only on Final, T1, T2
6	Multiple inspections are allowed on all tiers
7	Double inspections are allowed only on T1
8	Double inspections are allowed only on T1, T2
9	Double inspections are allowed on all tiers
10	No optimization: Yir1=0
11	DefectRank analysis with threshold = 0.001: Yir1=0
12	Multiple inspections are allowed only on final node: Yir1=0
13	Multiple inspections are allowed only on Final, T1: Yir1=0
14	Multiple inspections are allowed only on Final, T1, T2: Yir1=0
15	Multiple inspections are allowed on all tiers: Yir1=0
16	Double inspections are allowed only on T1: Yir1=0
17	Double inspections are allowed only on T1, T2: Yir1=0
18	Double inspections are allowed on all tiers: Yir1=0

Table 6. Parameters Name and description

Name	Meaning	Comment
Input data		
Yp	Probability of defective manufacturing	High quality selective inspection can provide this data. More accurate results can be obtained by warranty claims statistics
Yir1	Type I error of inspection (good part failed to pass)	Addition selective inspection should be used to obtain this value
Yir2	Type II error of inspection (missed bad part)	Addition selective inspection should be used to obtain this value
Pr	Probability of successful rework	Rework model is simplified: successful rework provides good part with no probability of defect. Failed rework means part goes to scrap. If rework is turned off, this value should be equal to 0 as well as **Cr**
Cr	Cost of rework	Average costs of rework (no fixed and variable costs considered). If rework is turned off, this value should be equal to 0 as well as **Pr**
Dr	Discount of rework	Reworked parts sold cheaper, however model neglects probability of defect after rework
Cinsp	Cost of inspection	Cost of inspection (no fixed/variable costs assumed). All production of node undergoes integer number of inspections. Each inspection has same independent error type I and II rates. Part passed all inspections is sent out. Part failed to pass at least one inspection sent to rework.
Cm	Cost of manufacturing	Manufacturing costs per part (no scale effect considered)
Inum	Number of inspections	Number of inspections. Inspections reduce probability of defected part sent out. However, it increases probability of error type I (if it was not set to zero) and costs **It left for compatibility**
Tn	Tier number	Final manufacturer is tier 0. Their direct suppliers – tier 1. Next level supplier – tier 2.

Name	Meaning	Comment
inputType	Type of raw parts needed	Those 3 variables are arrays. Length of arrays represents number of different parts needed to produce one part of this type. Input inspections have same effect on quality and cost as normal inspections but don't affect parts supplier sends to other nodes
inputQty	Quantity of raw parts from this supplier needed	
inputInsp	Number of input inspections	
Basic output data		
raw_cost	*Input Cost*: (Cost of raw materials)	Total cost of raw materials. In accordance to model, raw materials can be bought only from nodes present in network. This value is calculated using **Cost** of all direct suppliers. **This is main input parameter of node**
Ys	*Input Quality*: (probability that raw materials are bad)	Probability that at least one part taken to produce part of production is bad. This value is calculated using **P_iB** of all direct suppliers **This is main input parameter of node**
P_Gp	probability that product is good	Probability that product is good (if manufacturing was good and all raw parts was good)
P_Bp	probability that product is bad	Probability that product is bad (if manufacturing was bad or at least one pars was bad)
P_BpSo_I	probability that bad part sent out	Probability that part is bad, and it sent out
P_GpSo_I	probability that good part sent out after inspection	Probability that part is good, and it sent out
P_BpSr_I	probability that bad sent to rework	Probability that part is bad, and it sent to rework
P_GpSr_I	probability that good part sent to rework	Probability that part is bad, and it sent to rework
P_GpAr_I	probability that good part sent out after rework	Probability that part is good, and it sent out after rework
P_GpT_I	probability that good part sent out	Probability that part is good, and it sent out after rework

Name	Meaning	Comment
P_SCRAP_I	Probability that part goes to SCRAP	Probability that part goes to scrap. It impacts part cost, because all expenses of scrapped parts redistributed to parts sent out
P_iB_I	*Output Quality*: (Probability that if part was sent it is bad)	Probability that if part sent out it is bad. **This is main output parameter of node**
Cost_I	*Output Cost*: (of part)	Probability that if part sent out it is bad. **This is main output parameter of node**
index "_I"	Value depends on number of input inspections	This variable is array. Since buyers may choose different number of input inspections of this node production, values for all possible number of input inspections are stored. Number of input inspections equal to index (starting from 1) reduced by one.

Name	Meaning	Comment
Advanced output data		
Derivatives within node		
dCost_I_draw_cost	Effect of raw cost on this part cost	Internal derivatives represent how changes of input parameters impacts output node parameters. Significant change of parameters inside or outside this node will impact those values. Those derivatives depend on number of input inspections buyer choose.
dP_iB_I _dYs	Effect of raw quality on this part quality	
dCost_I _dYs	Effect of raw quality on this part cost	
Inspection effect analysis		
AddinspQ	Summary expenses of additional inspection per bad part not sent out	Analyze adding/removing inspection on this node. This value represents cost of prevention sending out one detected unit of final production. In first approximation: if AddinspQ is acceptable cost of preventing sending out defected part, inspection should be added. If **RemInspQ** is too high cost, inspection can be removed. Adding/removing inspection impacts effect of inspections on other nodes too.
RemInspQ	Summary saving of cancelled inspection per bad part sent out	
Backward cost and quality impact propagation		
Derivative dA/dB represents effect of changing variable A by small change of variable B, while other variables (except ones depending on B) are unchanged. Objective function is weighted sum of quality and cost on final node.		
QtyInFinal	Quantity of this parts in final production	Quantity of this type production, that have to be manufactured to produce one unit of final production (neglecting scrap fraction on all nodes)
dF_obj_draw_cost	Effect of this part input cost on objective function	Derivative of objective function with respect to input cost (cost of raw materials) of this part.
dF_obj_dYs	Effect of this part input quality on objective function	Derivative of objective function with respect to input quality of this node (probability that at least one raw part taken to produce one part of production is bad).
dF_obj_dP_iB_I	Effect of this part output quality on objective function	Derivative of objective function with respect to output quality with each possible number of input inspections

Name	Meaning	Comment
dF_obj_dCost_I	Effect of this part output cost on objective function	Derivative of objective function with respect to output cost with each possible number of input inspections
dF_obj_dSn_P_iB	Effect of output quality of each supplier on objective function	Derivative of objective function with respect to output quality of each supplier of this node.
dF_obj_dSn_Cost	Effect of output cost of each supplier on objective function	Derivative of objective function with respect to output cost of each supplier of this node. It also equals number of parts of this type needed to produce one unit of final production (with respect to scrap rate)
TotalQualityImpact	Effect of reducing rate of defect to 0 on objective function	Effect of reducing defect rate to 0 (for each number of inspections) without consideration to possible expenses of quality improve
QValue	Value of quality (estimated loss due to selling defected production	Loss estimation of selling one unit of defected production. It includes external failure costs (defected production refurbishing, replacement and reputation loss). This value scalarize multi-objective optimization problem to single-objective problem. It should be provided by decision maker. Currently it defined in MATLAB code
Fobj	Objective function: Cost of final production including value of quality	Objective function. It is linear combination of **Cost**+**P_iB***QValue** of final production

CHAPTER 4: VERIFICATION, VALIDATION AND RESULTS

In this chapter, we report the results obtained from two sets of trials—first, the simple toy network which was used for proof of concept and, second, general model including the entire supply chain from Tier 1 to Tier n and the unlimited number of suppliers and parts in each tier.

4.1 Results of Simulation with a Simple Network

In this section, we present the result of simulation with a simple network that contains eight suppliers in four tiers and one manufacturer (see Figure 14). This simple toy network was designed to verify: 1- the concept of part rank idea and if each supplier has different weight considering the entire supply chain as one; 2- test the “pull” parts from manufacturer versus “push” parts from supplier. In the first case, the manufacturer can choose to purchase more parts from a supplier with less inspection rate of error and number of defects, however, in the second case, all suppliers can send any parts they produce at their facility.

As can be seen in Figure 15, the manufacturer has the highest values of rank (weight), so it is the most important part of the process to inspect before sending parts to customer. In our model, all eight suppliers in Tier 4 have identical weight values, and the reason behind this is that they do not have any in-links, which means they are the starting point within our system. These suppliers only have out-links, and they only send parts and do not receive any parts from a higher tier. As can be seen in Figure 15, there are suppliers in Tier 3 (e.g., Supplier 6) with a higher value in the final product than that of Supplier 22 in Tier 2.

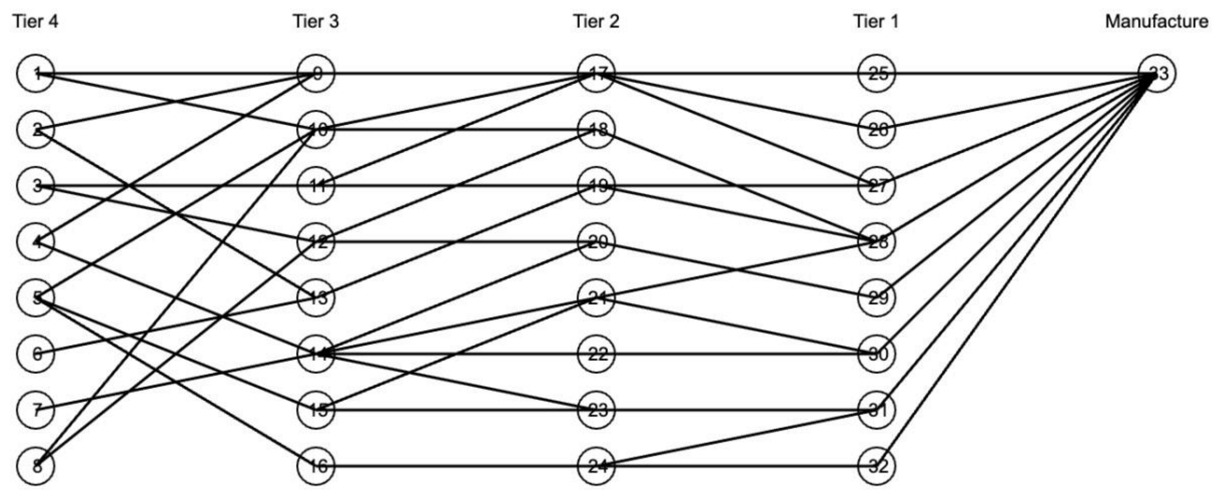


Figure 14. Toy Network

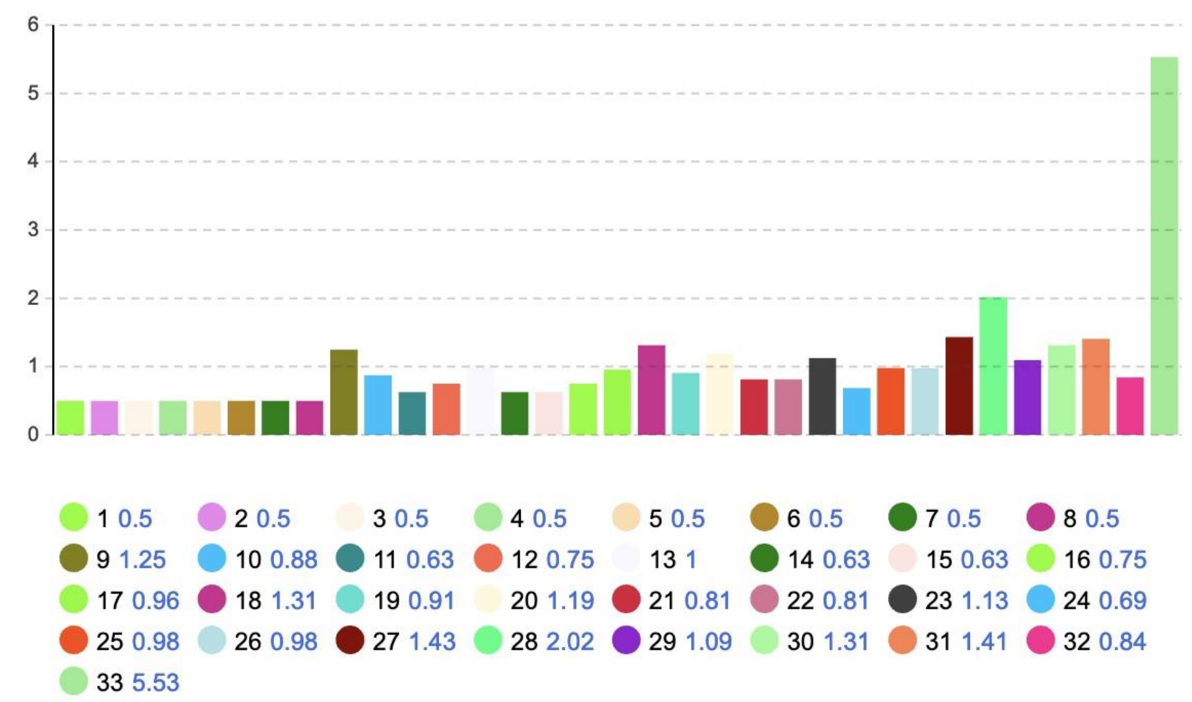
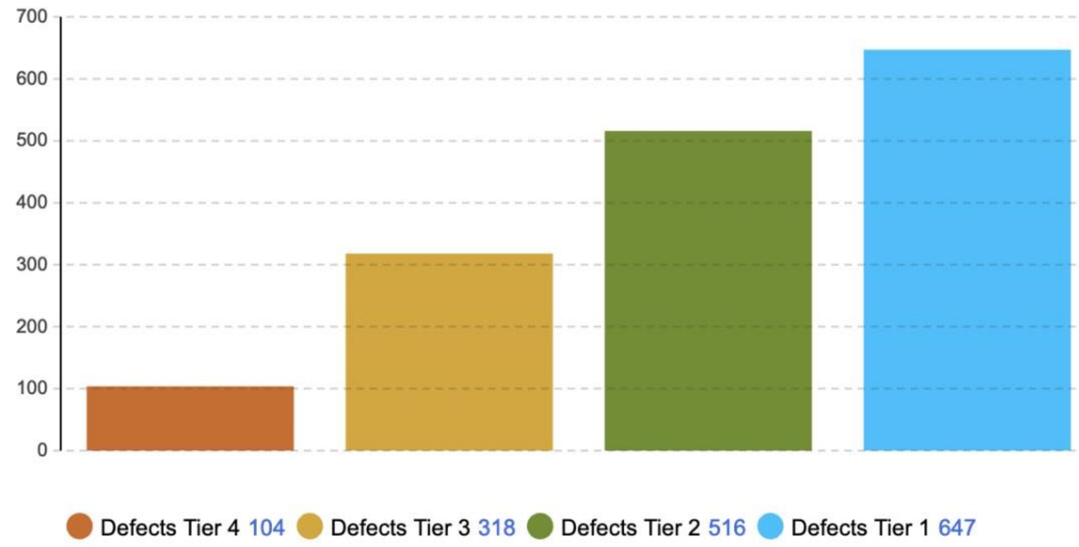


Figure 15. Calculated PrtRnk

After creating the toy network and calculating the weight (rank) of each supplier within our toy network, we created three different scenarios to verify our claim. In Scenario 1, we did not use any type of optimization and did not consider inspection in the entire supply chain by the manufacturer (see Figure 16). We modeled a standard industry system according to which each supplier is responsible only for their own quality and they randomly inspect 50% of their own product. Figure 15 shows the result of this scenario. As seen in Figure 16, the cost is low because the number of inspection checks is kept at the minimum; however, the number of defects increases from higher to lower tiers. This result is consistent with our expectations, as discussed previously, for every type of good product, there are three types of bad product (see Figure 5).

In Scenario 2, we did consider another level on inspection. Therefore, in this case, each supplier did internal inspection (50% of the product), underwent external inspection, and got inspected by a higher tier supplier (see Figure 17). As can be seen in Figure 17, the number of defects did not considerably change in Tier 4 but did decrease by over 50%. In this case, the total cost also doubled due to the addition of extra layers of external inspection.

In Scenario 3, we expanded our work from self-inspection and only one tier of higher external inspection to 50% self-inspection and two tiers of higher external inspection. As a result, the number of defects and quality significantly decreased and, in two tiers, zero defects were observed (see Figure 18). The interesting finding in this trial was that the number of defects in Tier 1 did not change, as they only got inspected by the manufacturer and did not have an extra level of quality inspection. With regard to the cost, congruently with our expectation, it increased by another 50%, as we added one level extra as compared to Scenario 2. Although we verified the effect of adding extra level of inspection, the tools we used in this trial allowed for only a limited number of suppliers and parts in each tier.



Rework Costs:$17,200.00
Inspection Costs:$195,350.00
Total Costs:$212,550.00

Figure 16. Scenario 1: 50% of suppliers inspect themselves

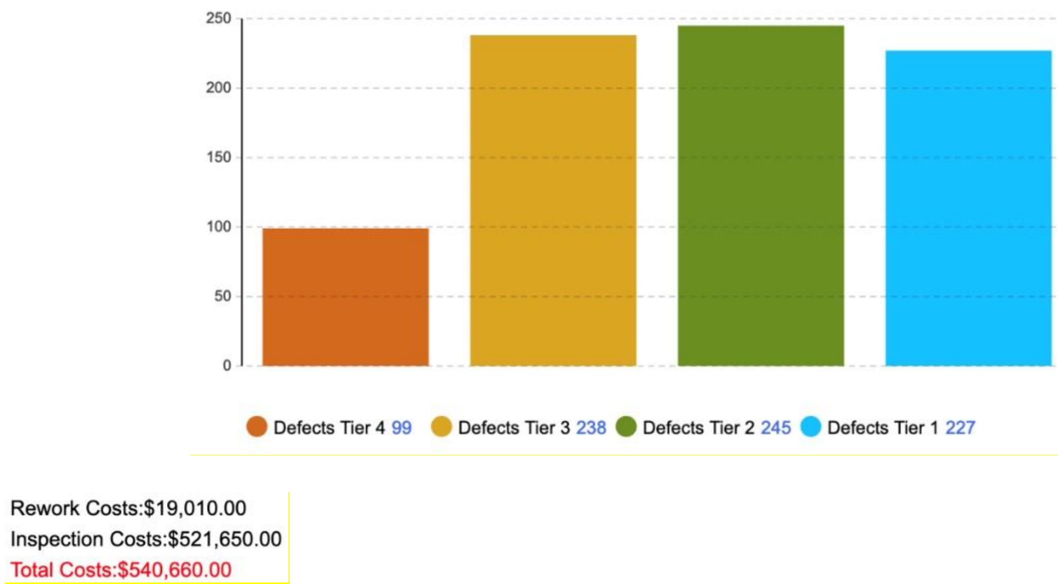


Figure 17. Scenario 2: 50% of suppliers inspect themselves and get inspected by a lower tier

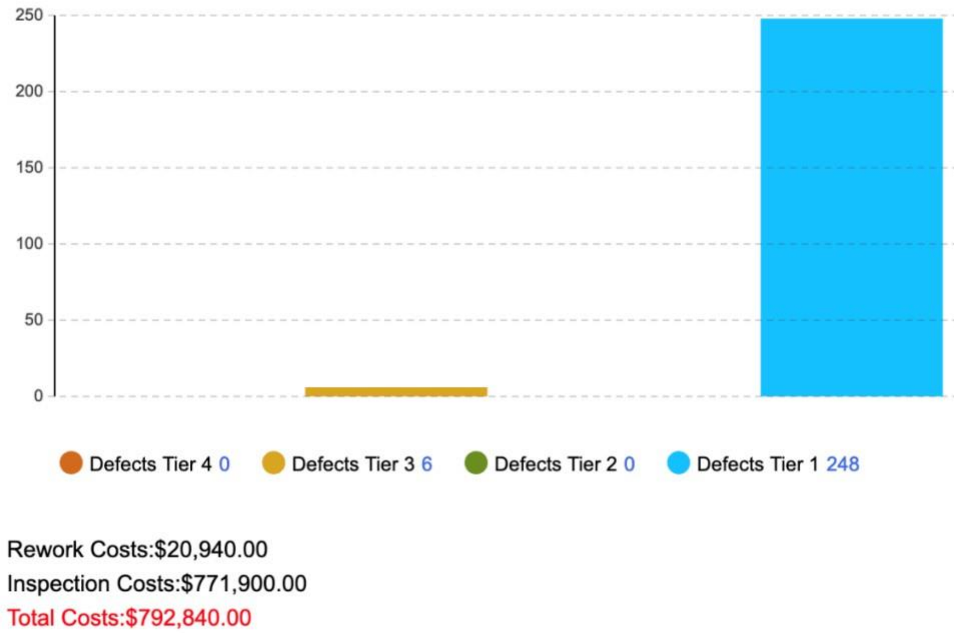


Figure 18. Scenario 3: 50% of suppliers inspect themselves and get inspected by two lower tiers

4.2 Results of Simulation with the General Model

Cost_distrib.png shows costs distribution in objective function (see Figure 19). **Av_Insp.png** in Figure 20 shows average number of inspections by cases and tiers. A more detailed inspection distribution is shown in files **Insp_distrib_C<N>.png** where **<N>** represents the number of cases. Color scale for distribution number is the same for all cases (see Figure 21). Distribution of costs by tier is shown in **Cost_by_tier_C<N>.png,** where **<N>** is the number of cases (see Figure 22).

The bar plot in Figure 22 shows costs distribution cost categories of distribution. Scrap includes costs of raw materials, manufacturing, inspections, reworks for parts sent to scrap. Those costs are excluded from the corresponding categories. Raw material costs are not shown on the plot, as only cost added on each tier is shown. Inspection distribution for all cases is shown in **Insp_distrib_All.png**. Bar stacks represent distribution of inspection numbers. Stacks in the groups represent tiers. Figure 22 contains six tiers (T0-T5). Groups of stacks represent cases; the legend shows the number of inspections. Tier 0 contains only one node; however, its size was scaled. Cost distribution for all cases so shown in Cost_by_tier_All.png (see Figure 23).

This plot shows cost distribution by tiers in each case. Costs including quality value are shown in COQ_by_Tier.png (see Figure 24). Figure 25 shows cost and quality transfer through supply chain.

Value of input quality is the value of all parts needed on this tier and is negative. The bar of this value overlaps by bars representing positive values.

Value of output quality is quality value of all parts sent out from this tier. The first bar in group for each case shows the value of quality of the final production. The difference between **Value of output quality** and **Value of input quality** is the contribution of the tier to the total value of quality. If **Value of input quality** is higher than **Value of output quality**, then the bar **Value of output quality** is below the X axis. File weighted_P_iB.png contains logarithmic data for weighted quality (see Figure 26). The weight of each node is based on its value of quality. Log-scale is used for a better illustration.

Figure 19 outlines how the model calculated the cost distribution in each tier based on different scenarios. For instance, in scenario number seven (7) a double inspection was assumed in the first two tiers only. This means that the final manufacturer inspects all tier 1 and tier 2 suppliers, in addition to the 100 % self-inspection perform by each supplier within the entire supply chain. Suppliers in tier 1 perform inspection in all tier 2 suppliers. When double inspection is allowed in the first two tiers, the cost of rework and the cost of inspection decreases dramatically, which is caused by performing additional inspection by manufacturer in tier 1 and tier 2. This results in a lower number of defects passing through inspection and moving to lower tiers. By using a higher quality material, the number of defects decreases, resulting in less rework and scrap cost.

Although the cost of scrap and rework is lower in tier 1 for case scenario number 7, the cost of inspection went up. The increase of cost was expected because the manufacturer is performing additional levels of inspection at a higher tier. No changes are observed in tier 3 and higher because those tiers are not considered in this case for additional inspection. The same calculation was performed in all 18 scenarios and results were used to perform the final analysis and select the best scenario.

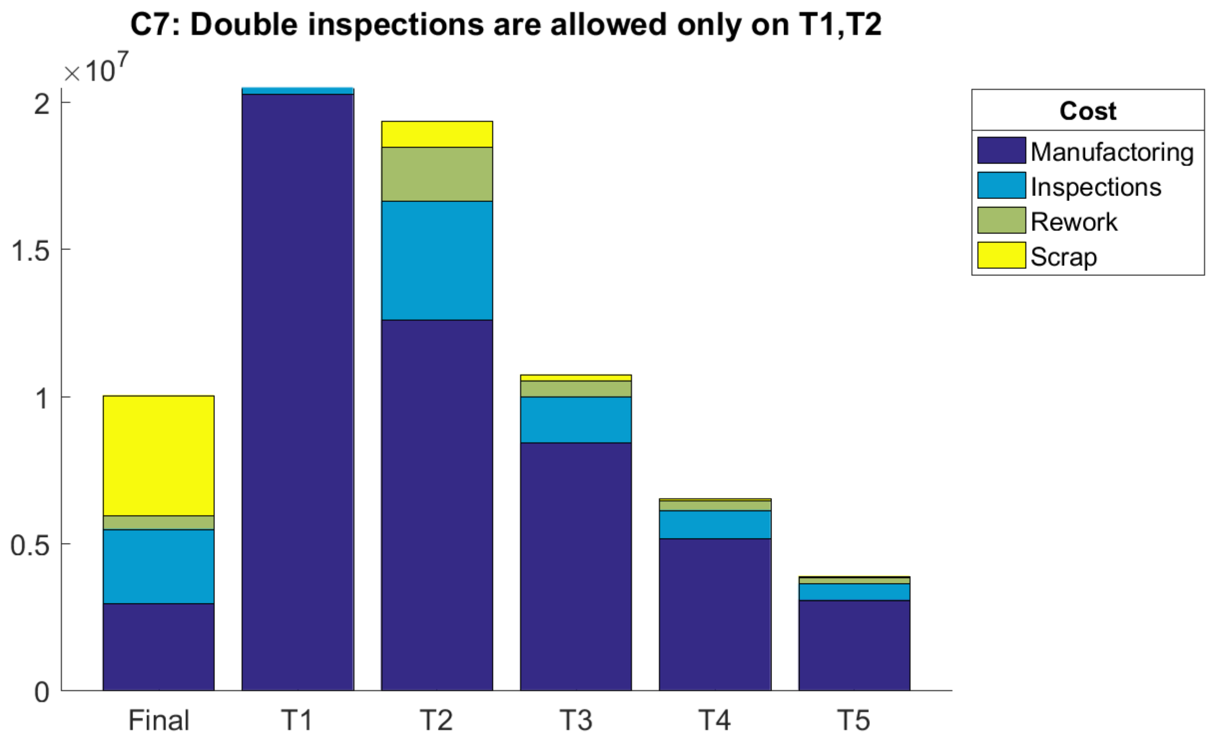


Figure 19. Cost Distribution by Tiers in Case 7

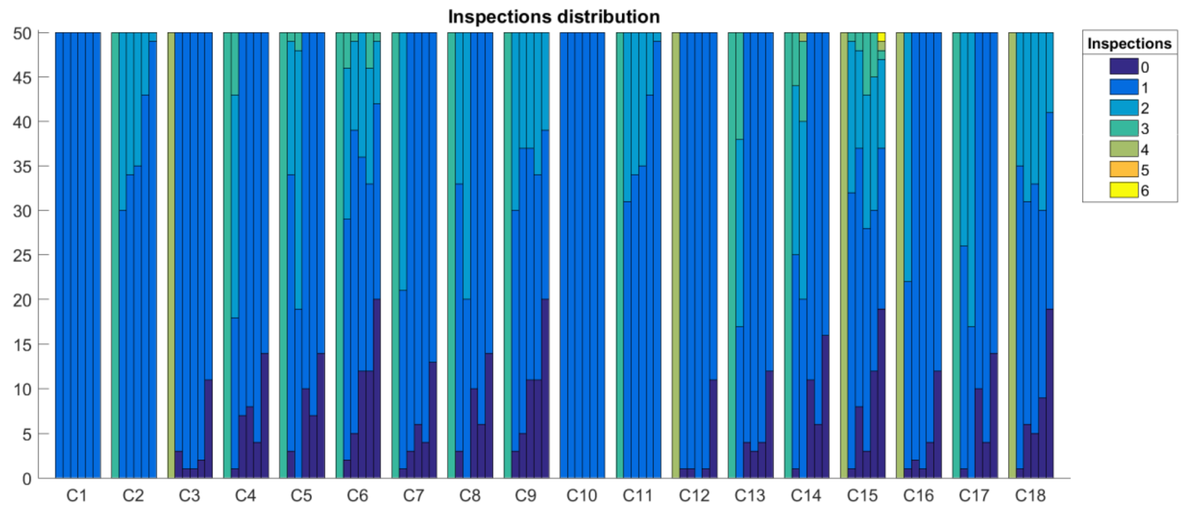


Figure 20. Inspection Distribution by Tiers in All Cases

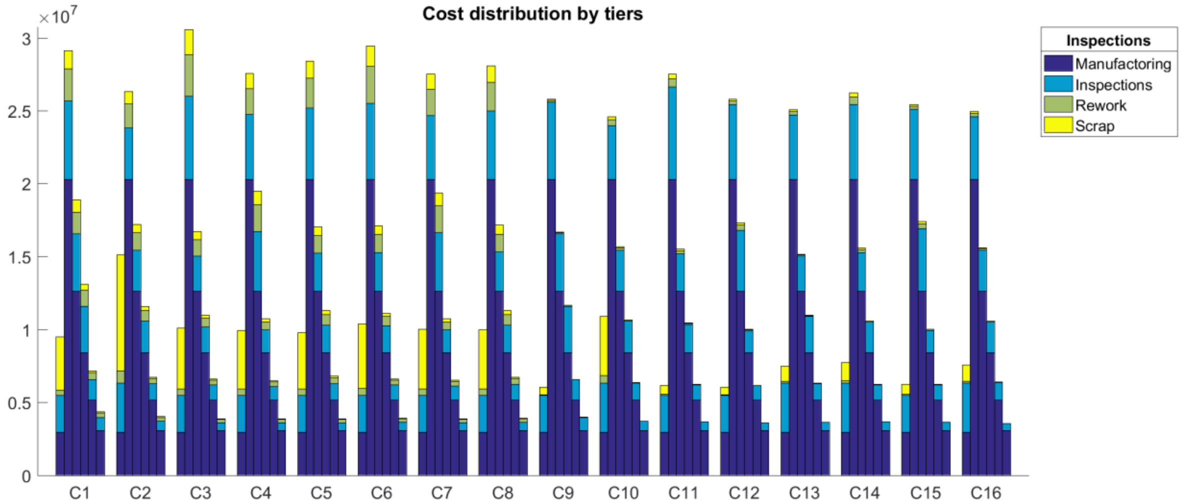


Figure 21. Cost distribution by tiers in all cases

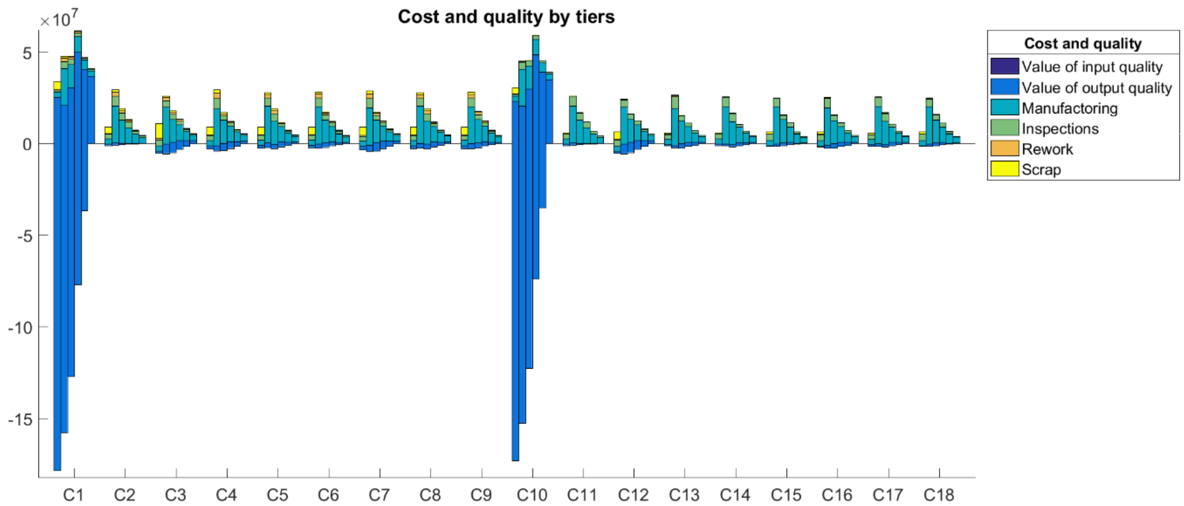


Figure 22. Cost distribution by tiers in all cases

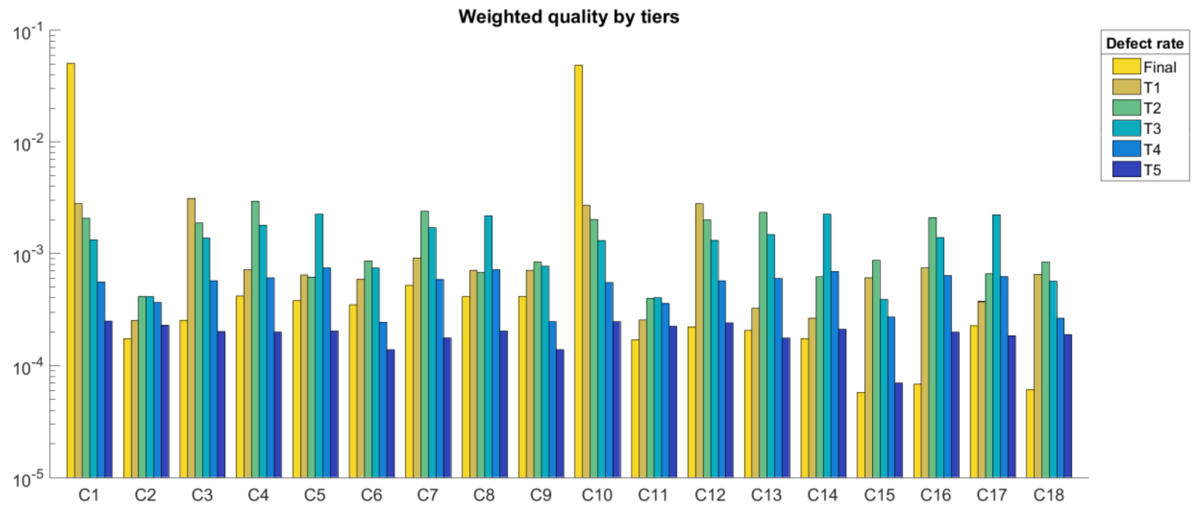


Figure 23. Weighted quality by tiers in all cases

4.3 Results

A summary of the results for all cases is provided in Table 6. The value of quality is $1.0424 \cdot 10^9$ per defected unit (i.e., preventing sending out 1 of 1000 defected units is better than decreasing the cost price by 10^6, which amounts to 10% of the product cost price).

Table 6 shows two case sets.

In both case sets, the best value of objective function is obtained in Case 4. This case includes the most flexible method of optimization among all cases simulated. Case 1 shows basic scenario with one inspection per node. Adding inspection on the final manufacturer node (Case 2) significantly improves quality at a small cost of inspection. If error type I is enabled, cost would increase more significantly, as inspection increases the scrap rate. Adding inspection to every node of T1 further improves quality, but it is not cost-effective.

Using optimization techniques allows for choosing nodes of T1 where additional inspection will be added. Selective choice of T1 inspections in Case 4 based on convex optimization algorithm reduces the value of objective function by 0.58%. It also reduces the cost of the final product due to a reduction in the scrap rate and, at the same time, improves quality. Due to a less accurate accounting of non-linear effects, the simplified DefectRank method provides objective function decline of 0.45%.

DefectRank requires a small number of parameters, while it can provide good results of optimization that are slightly worse than those afforded by using the method based on cost and quality impact. If inspections have false-positive errors, negative effect of redundant inspections will worsen the solution. Since DefectRank neglects error type I, if a model includes it, the result of defect would be worse than when only inspection on T0 is added.

Table 7. Summary of Inspection Optimization Problem Cases

	No error type I			
CA SE	Description	Cost of final product, 10^6	Rate of defect, 10-6	Objective function, 106
1	One inspection on T0-T5	10.424	1313	11.925
2	Two inspections on T0, one inspection on T1-T5	10.458	108	10.581
3	Two inspections on T0-T1, one inspection on T2-T5	10.656	20	10.678
4	Convex optimization on T0 and T1	10.451	55	10.514
5	DefectRank optimization on T0 and T1	10.457 **e I enabled**	59	10.524
	Error type			
CASE SE	Description	Cost of Final product, 10^6	Rate of defect, 10-6	Objective function, 106
1	One inspection on T0-T5	11.438	1394	13.033
2	Two inspections on T0, one inspection on T1-T5	11.669	117	11.802
3	Two inspections on T0-T1, one inspection on T2-T5	12.158	21	12.182
4	Convex optimization on T0 and T1	11.673	92	11.779
5	DefectRank optimization on T0 and T1	11.808	60	11.876

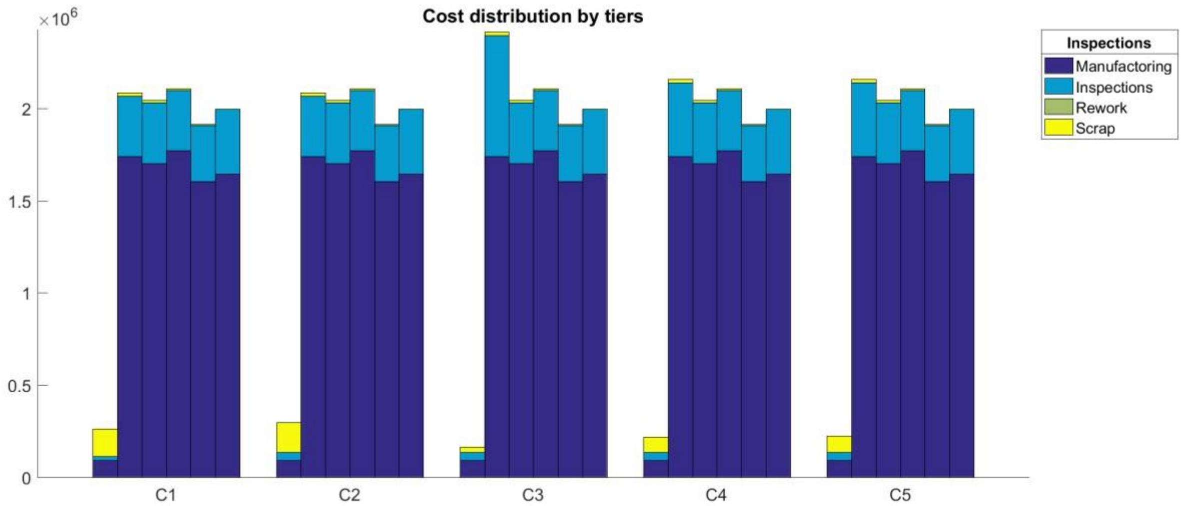


Figure 24. Cost distribution by tiers at Yir1=0

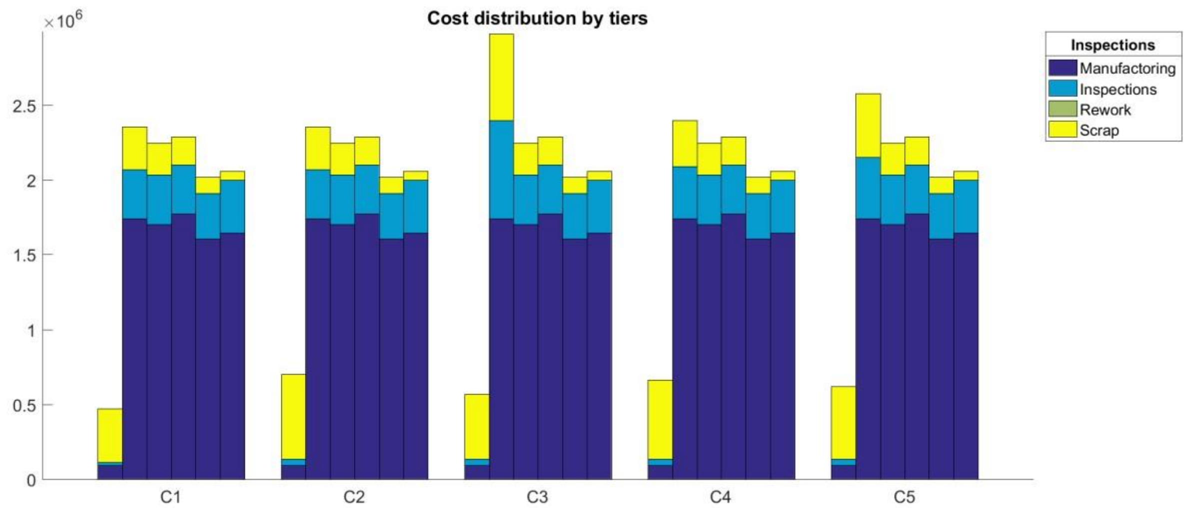


Figure 25. Cost distribution by tiers at Yir1>0

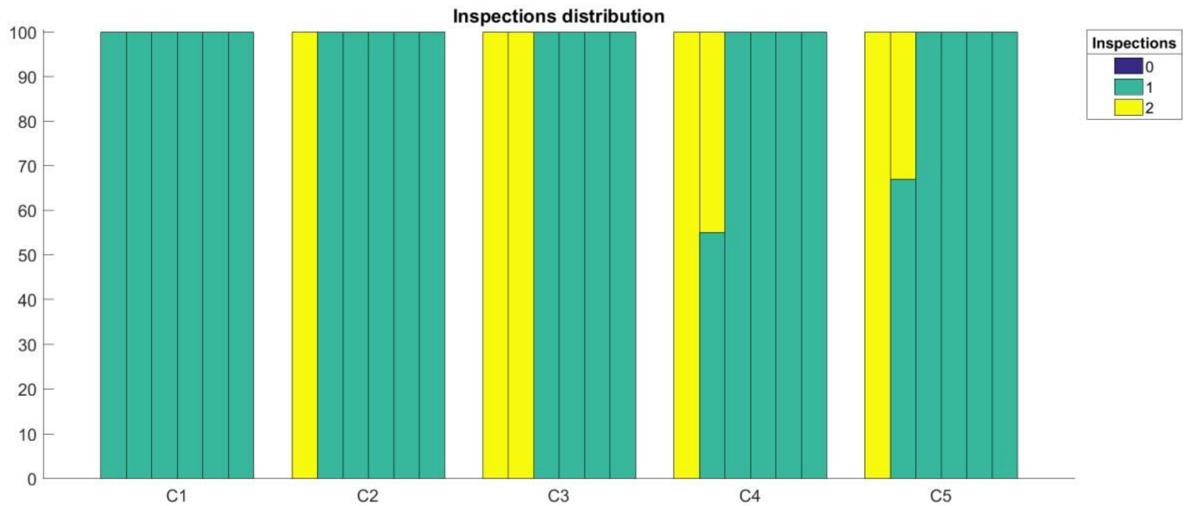


Figure 26. Inspection distribution by tiers at Yir1=0

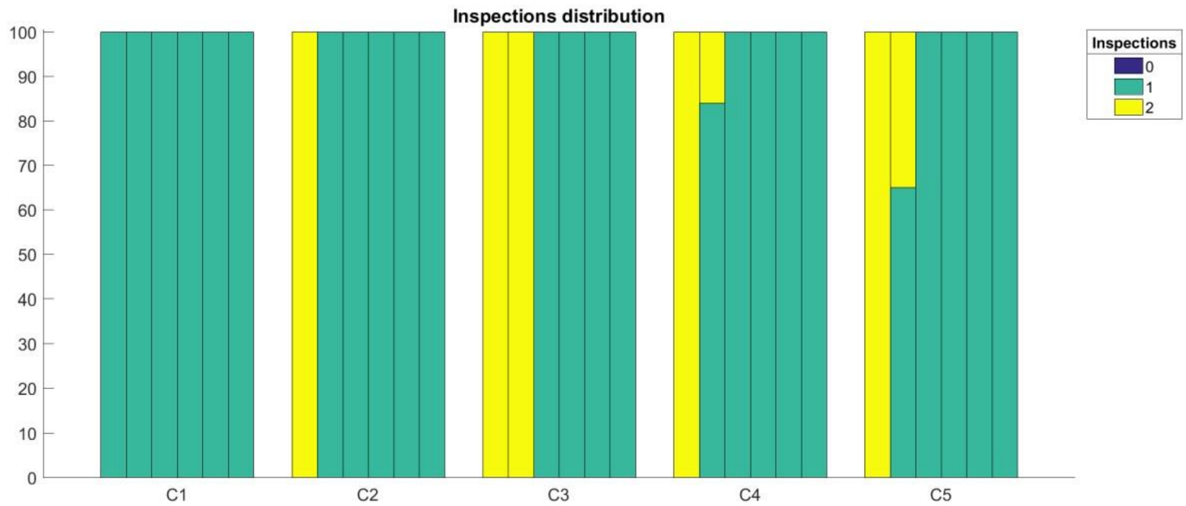


Figure 27.Inspection distribution by tiers in at Yir1>0

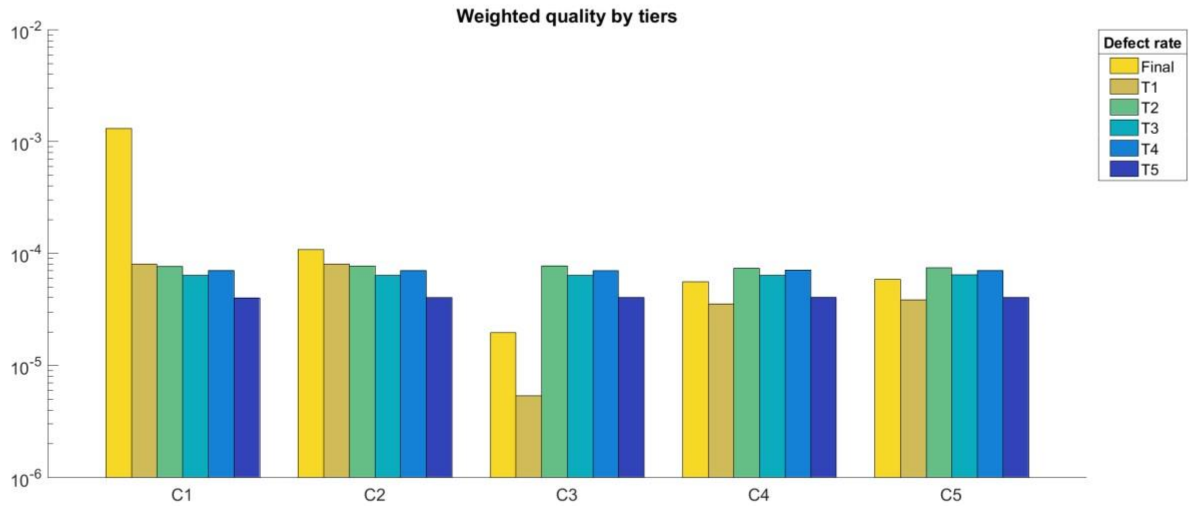


Figure 28. Weighted quality by tiers at Yir1=0

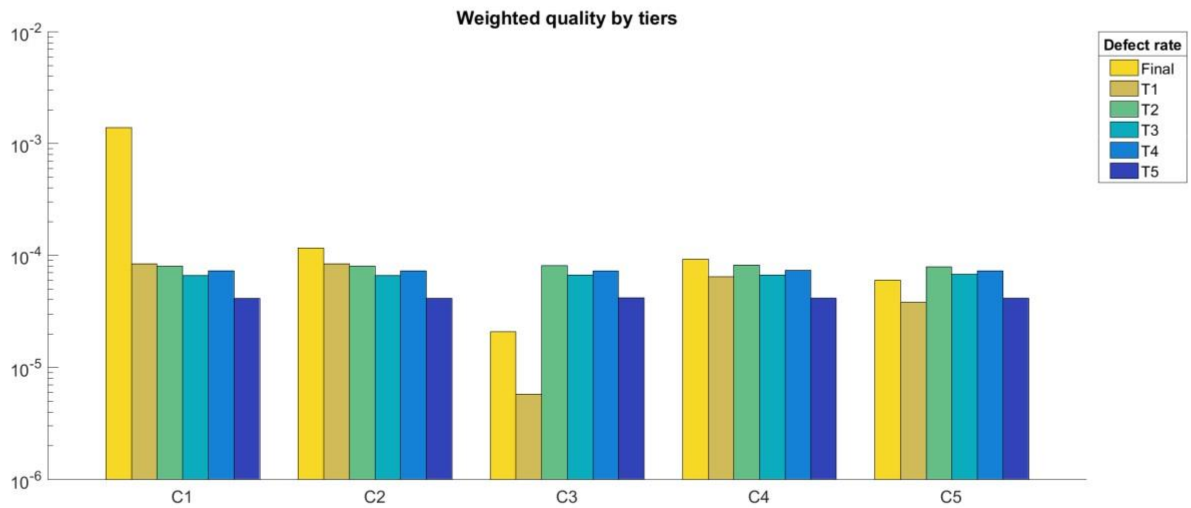


Figure 29.Weighted quality by tiers at Yir1>0

Case set 1 (Yir1=0) results are shown in Figures 26, 28, and 30. Case set 2 (Yir1>0) results are shown in Figures 27, 29, and 31.

The main component of expenses is manufacturing costs. Inspections cost is significant too. If Yir1>0, scrap expenses are higher due to the high rate of error type I, which stacks from double inspections. Further detail is provided in Figures 26-27. False positive errors can be considered as an indirect cost of inspection. Since convex optimization algorithm considers those expenses, it reduces the number of nodes with double inspections, and the defect rate increases. Zero rate of false positive errors increases efficiency of inspections; accordingly, Figures 28-29 show different number of inspections in Case 4. DefectRank neglects this factor and shows the same rate of inspections.

In Cases 1-3 and 5, considering false positive errors increases the rate of defect of the final production even if all other parameters, including the number of inspections, remain the same.

Convex optimization provides near best solution within the mathematic model of cost and quality calculation. The DefectRank method works well if error type I can be neglected. However, if false-positive error rate of inspections is high, the results of DefectRank are less optimal.

Therefore, the DefectRank method can be used in the case of limited data access.

4.4 Summary

Taken together, the results reported in this chapter demonstrate that, at the current stage, the PartRank/DefectRank algorithm can be effectively used to determine the importance of each part or supplier so that, if it is financially feasible, to add another layer of inspection in the entire supply chain, regardless of the number of tiers.

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

5.1. Ranking of Supplier

The results of testing the ranking system proposed in the present study demonstrate that, with regard to quality control, not all suppliers have the same value. This is so because, regardless of which supplier is at fault when a defected final product is recalled, the manufacturer is always to blame. Therefore, our system suggests that it is the manufacturer's responsibility to adopt more ownership and responsibility in the higher tier of a supply chain and to check on high-ranked suppliers. Furthermore, our results also emphasize that manufactures do not have to wait until customers discover defects in the manufactured parts but can instead take action to prevent such occurrences. Said differently, different sections of the industry should be proactive, rather than reactive. The algorithm proposed in the present study suggests genuinely proactive action, thereby ensuring reliability of the supply chain. At present, the many types of software and tracking systems that are available for commercial use are all based on helping users to take quickest action in the event of a failure. It should also be mentioned that, today, many industries, such as car manufacturing or aerospace industry, seek to make common the parts they produce for their different products using bigger Line Replacement Units (LRU). However, this LRU-based approach has two important consequences. First, an undetected failure in a higher tier will result in more failures in different products. Second, it becomes more difficult to detect failures in final products because an LRU comes as one big unit, such as the landing gear of an airplane.

5.2. Cost vs. Quality Importance in the Manufacturing Setting

An analysis of the current numbers of recalls in the car manufacturing industry suggests that the quality control system requires a thorough revision and adaptation, as both the market and consumer behavior undergo many and varied changes.

The results reported in this study let us draw the following two conclusions:

1) Inspection error rate in each manufacturing facility prevents the detection of all bad auto parts causing some defected items to be shipped and used in the final product.
2) If each manufacturer inspects two tiers above the supplier vs. inspecting only one tier, the number of defects will dramatically decrease, while the cost will double.

Based on these findings, we argue that a revision in the quality control of companies is necessary to make it economically feasible. Priority is needed during inspection, as the inspection is particularly important when it is performed on a specific supplier, as in the case of a failure of the final product where the most critical auto parts will cause the most damage to the manufacturer.

In this context, it is critical that manufacturers understand consumers' needs and adjust their priorities accordingly. Looking at the successful products that, in the last years, have brought genuine change to customers' lives and great wealth to those products' manufacturers, it can easily be seen that customers are no longer interested in purchasing products to survive—rather, what comes to the forefront these days is enjoying life. Accordingly, most successful companies of the last decade—such as Facebook, Instagram, Tweeter, Amazon, as well as numerous luxury brand cars—all produce items that offer enjoyment, rather than have utilitarian value. This trend was also noted by Steve Jobs and, in 2019, Apple Inc. became the most valuable company in the world, overcoming its rival ARAMCO Saudi Oil company. Accordingly, better prospects and opportunities open for those companies that understand this trend and adjust their strategies, accordingly, using some proactive system to increase their quality even doing so would raise the final product cost.

5.3. Future work

Results reported in the present study offer many venues for further research in this area. Since the proposed algorithm is the first algorithm to target parts and suppliers within the entire supply chain, irrespective of tier of the supplier before it fails, it offers a wide array of opportunities for future research, ranging from data collection to algorithm itself.

For instance, future studies could combine the actual failure data in a ranking system to adjust weight of each supplier. Such combination of input data can help to dynamically calculate new weight for each individual part and adjust the number of inspections for different suppliers. The results of further research in this vein could offer suppliers a valuable opportunity to evaluate their true performance and, subsequently, improve quality of their products. Another line of research worth examining in the future concerns the calculation and modeling section that, for instance, can be done using different models of optimizations.

REFERENCES

Abeysekara, N., Wang, H. & Kuruppuarachchi, D. (2019). Effect of supply-chain resilience on firm performance and competitive advantage. *Business Process Management Journal*, *25*(7). Retrieved from https://www.emerald.com/insight/content/doi/10.1108/BPMJ-09-2018-0241/full/html

Adzhar, N., Salleh, S., Yusof, Y., & Ahmad, M.A. (2019). Routing problem in rectangular mesh network using shortest path based Greedy method. *Journal of Physics: Conference Series*, 1358(1), 012079. Retrieved from https://iopscience.iop.org/article/10.1088/17426596/1358/1/012079/pdf

Agirre, E., & Soroa, A. (2009). Personalizing PageRank for word sense disambiguation. *EACL '09 Proceedings of the 12th Conference of the European Chapter of the Association for Computational Linguistics.* Athens. Retrieved from https://www.aclweb.org/anthology/E09-1005.pdf

Akkucuk, U. (2016). SCOR model and the green supply chain. In U. Akkuluk (Ed.), *Handbook of research on waste management techniques for sustainability* (pp. 108-124). Hershey, PA: IGI Global.

Albright, T., & Roth, H. (1992). The measurement of quality costs: An alternative paradigm. *Accounting Horizons*, pp. 15-27. Retrieved from https://maaw.info/ArticleSummaries/ArtSumAlbrightRoth92.htm

Amelkin, V., & Vohra, R. (2020). Strategic formation and reliability of supply chain networks. *Proceedings of the 21st ACM Conference on Economics and Computation* (pp. 77-78). Retrieved from https://dl.acm.org/doi/abs/10.1145/3391403.3399450

Anderson, J., Rungtusanatham, M., & Schroeder, R. (1994). A theory of quality management underlying the Deming management method. *The Academy of Management Review, 19*(3), 472-509.

Arasu A., (2011). PageRank Computation and the Structure of the Web: Experiments and Algorithms. *Computer Science Department, Stanford University*. Retrieved from http://www.ramb.ethz.ch/CDstore/www2002/poster/173.pdf

Avrachenkov, K., Litvak, N., Nemirovsky, D., & Osipova, N. (2007). Monte Carlo methods in PageRank computation: When one iteration is sufficient. *SIAM Journal on Numerical Analysis , 45*(2), 890-904.

Azadegan, A., Mellat Parast, M., Lucianetti, L., Nishant, R., & Blackhurst, J. (2019). Supply chain disruptions and business continuity: An empirical assessment. *Decision Sciences, 51*(1), 38-73.

Azadegan, A., Syed, T.A., Blome, C., & Tajeddini, K. (2020). Supply chain involvement in business continuity management: Effects on reputational and operational damage containment from supply chain disruptions. *Supply Chain Management: An International Journal, 25*(6). Retrieved from https://www.emerald.com/insight/content/doi/10.1108/SCM-08-2019-0304/full/html

Banasik, M. (2009). *A study of the costs of quality in a renewable resource environment*. Lubbock, Texas:Texas Tech University.

Bai, X., & Liu, Y. (2016). Robust optimization of supply chain network design in fuzzy decision system. *Journal of Intelligent Manufacturing, 27*(6), 1131-1149.

Beam, H. (1997). Juran: A lifetime of influence. *The Academy of Management Executive, 11*(4), 96-98.

Berkhin, P. (2005). A survey on PageRank computing. *Internet Math*, *2*(1), 73-120.

Bertsimas, D., Sim, M., & Zhang, M. (2017). A practically efficient approach for solving adaptive distributional robust linear optimization problems. *Management Science.* Retrieved from http://www.optimization-online.org/DB_FILE/2016/03/5353.pdf

Bhamidipati, N. (2009). Comparing Scores Intended for Ranking. *IEEE Transactions on Knowledge and Data Engineering*, *21*(1), 21-34

Bianchini M., Gori, M., & Scarcelli, F. (2005). Inside PageRank. *ACM Transactions on Internet Technology* , *5*(1), 92-128.

Bidoki, A. & Yazdani, N. (2007). DistanceRank: An intelligent ranking algorithm for web pages. *Information Process and Management*. Retrieved from https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.494.2614&rep=rep1&type=pdf

Boldi P., Santini, M., & Vigna, S. (2005). PageRank as a function of the damping factor. *WWW '05 Proceedings of the 14th international conference on World Wide Web.* Chiba.

Bollen, J., van de Sompel, H., & Rodriguez M. (2008). Towards usage-based impact metrics: First Results from the MESUR project. *Joint Conference on Digital Libraries*, Pittsburgh. Retrieved from https://www.researchgate.net/publication/1920727_Towards_Usagebased_Impact_Metrics_-_First_Results_from_the_MESUR_Project

Bozarth, C. C., & Handfield, R. B. (2016). *Introduction to operations and supply chain management*. London: Pearson.

Brindley, C. (ed.) (2017). *Supply chain risk*. London: Taylor & Francis.

Brunaud, B., Laínez-Aguirre, J.M., Pinto, J.M., & Grossmann, I.E. (2019). Inventory policies and safety stock optimization for supply chain planning. *AIChE Journal*, *65*(1), 99-112.

Bulgak, A., Alzaman, A., & Ramudhin, A. (2008). Incorporating the cost of quality in supply chain design. *Quality Maintenance Engineering, 14(1)*, 71-86.

Burgess, T. (1996). Modelling quality-cost dynamics. *International Journal of Quality & Reliability Management*, *13*(3), 8-26.

Campanella, J. (ed.) (1990). *Principles of quality costs: Principles, implementation and use*. Milwaukee: ASQ Quality Committee.

Carr, L. (1992). "Applying cost of quality to a service business. *MIT Sloan Management Review*. Retrieved from https://sloanreview.mit.edu/article/applying-cost-of-quality-to-a-servicebusiness/

Castillo-Villar, K., Smith, N., & Simonton, J. (2012). A model for supply chain design considering the cost of quality. *Applied Mathematical Modelling*, *36*(12), 5920-5935.

Chae, Y.H., & Seong, P.H. (2017). Find root cause by using Google PageRank algorithm.

Chakrabarti, (2002). Mining the web: Discovering knowledge from hypertext data. Morgan-Kaufmann Publishers.

Chakrabarti, S. (2007). Dynamic personalized pagerank in entity-relation graphs. *WWW '07 Proceedings of the 16th international conference on World Wide Web.* Banff, Alberta.

Chen, C. J. (2019). Developing a model for supply chain agility and innovativeness to enhance firms' competitive advantage. *Management Decision*, *57*, 1511-1534.

Chen, K., Song, M. X., Zhang, X., & Wang, S. F. (2016). Wind turbine layout optimization with multiple hub height wind turbines using greedy algorithm. *Renewable Energy*, *96,* 676686.

Chiang, Y. H., & Mu, S. C. (2016). Formal derivation of greedy algorithms from relational specifications: A tutorial. *Journal of Logical and Algebraic Methods in Programming*, *85*(5), 879-905.

Chipman, H. (2019). The math of networks: Random walks and Google PageRank. Retrieved from https://docs.google.com/document/d/1r9V3USScuSqbSW_zW8X2XwC6lpgKKo6y6qond UyFUA/edit

Chris P., & Lee, G. H. (2007). A two-stage algorithm for computing PageRank and multistage generalizations. *Internet Mathematics*, *4*(4), 299-327.

Dahlgaard, J., Kristensen, K., & Kanji, G. (1992). Quality costs and total quality management. *Total Quality Management*, *3(*3), 211-221.

Dai, L., & Freris, N. (2017). Fully distributed PageRank computation with exponential convergence. Preprint arXiv 1705.09927

Dale B., & Plunkett, J. (1995). *Quality costing*. (2nd ed.). London: Chapman and Hall.

De Feo, J. (2015). *Juran's quality management and analysis*. (6th ed.). New York, NY: McGraw Hill Education.

Delipinar, G.E., & Kocaoglu, B. (2016). Using SCOR model to gain competitive advantage: A Literature. *Procedia-Social and Behavioral Sciences*, *229*, 398-406.

Deming, W. (1991). *Quality, productivity and competitive position*. Cincinnati: Quality Enhancement Seminars.

Devi, P., Gupta A., & Dixit, A. (2014). Comparative study of HITS and PageRank link based ranking algorithms. *International Journal of Advanced Research in Computer and Communication Engineering*, *3*(2), 5749-5754.

Erjia Y., & Ying, D. (2011). Discovering author impact: A PageRank perspective. *Information Processing & Management*, 47(1), 125-134.

Evans, J. (2008). *Managing for quality and performance excellence*. (9th ed.). Mason, Ohio:

South-Western.

Evans, J., & Lindsay, W. (1999). *The management and control of quality*. Cincinnati, Ohio: South-Western College.

Feigenbaum, A. (1956). Total quality control. *Harward Business Review 34*(6), 93-101.

Franceschet, M. (2011). PageRank: standing on the shoulders of giants. *Communications of the ACM*, *54*(6), 91-101.

Freiesleben, J. (2004). On the limited value of cost of quality models," *Total Quality Management*, *15*(7), 959–969.

Fujimura, M., Inoue, T., & Sugisaki, M. (2005). The EigenRumor algorithm for ranking blogs. *WWW 2005*, May 10-14, Chiba, Japan. Retrieved from https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.136.7250&rep=rep1&type=pdf

Geissmann, B., Leucci, S., Liu, C. H., Penna P., & Proietti, G. (2019). Dual-dode greedy algorithms can save energy. In *30th International Symposium on Algorithms and Computation (ISAAC 2019)* (Vol. 149, pp. 64-1). Schloss Dagstuhl-Leibnitz-Zentrum für Informatik.

Gleich, D., Berkhin, P., & Zhukov, L. (2005). Fast parallel pagerank: A linear system approach. Retrieved from https://www.researchgate.net/publication/51991696_Fast_Parallel_PageRank_A_Linear_System_Approach

Goel, S., Kumar, R., Kumar, M., & Chopra, V. (2019). An efficient page ranking approach based on vector norms using sNorm (p) algorithm. *Information Processing & Management*, *56*(3), 1053-1066.

Goicoechea, I., & Fenollera, M. (2012). Quality management in the automotive industry. In Katalinib, B. (Ed.), *DAAAM International Scientific* (pp. 619-632). Vienna: DAAAM International.

Godfrey, J., & Pasewark, W. (1988). Controlling quality costs. *Management Accounting, 69,* 4851

Gryna, F. M. (2001). *Quality planning and analysis: From product development through use*. (4th ed.). New York, NY: McGraw-Hill.

Gupta, P. (2007). Reduce appraisal costs. *Quality*, *X*(X), xx-xx.

Gupta, D., & Singh, D. (2016). April. User preference based page ranking algorithm. *2016 International Conference on Computing, Communication and Automation (ICCCA)*, 166-171.

Hackman, J., & Wageman, R. (1995). Total quality management: Empirical, conceptual, and practical issues. *Administrative Science Quarterly*, 40(2), 309-342.

Haveliwala, T. (1999). *Efficient computation of PageRank.* San Francisco: Stanford.

Haveliwala, T. (2003). Topic-sensitive PageRank: A context-sensitive ranking algorithm for Web search. *IEEE Transactions on Knowledge and Data Engineering, 15*(4), 784-796.

Heidemann J., Klier, M., & Probst, F. (2009). Identifying key users in online social networks: A PageRank-based approach. Retrieved from https://www.uibk.ac.at/wipl/team/team/papers/identifyingkeyusersinonlinesocialnetworks.pdf

Hugos, M. H. (2018). *Essentials of supply chain management*. Hoboken, NJ: John Wiley & Sons.

Hwang, G., & Aspinwall, E. (1996). Quality cost models and their application: A review. *Total Quality Management,* 7(3), 267-281.

Ittner, C. D. (1996). Exploratory evidence of the behavior of quality cost. *Operations Research*, *44*(1), 114-130.

Ishii, H., & Tempo, R. (2010). Distributed Randomized Algorithms for the PageRank Computation. *IEEE Transactions on Automatic Control*, *55*(9), 1987-2002

Jia, X., Cui, L., & Xing, L. (2018). New insights into reliability problems for supply chains management based on conventional reliability model. *Maintenance and Reliability*, *20* (3),465-470.

Jiang, H., Ge, Y., Zuo, D., & Han, D. (2008). TimeRank: A method of improving ranking scores by visited time. *2008 International Conference on Machine Learning and Cybernetics*, Kunming, pp. 1654-1657. doi: 10.1109/ICMLC.2008.4620671.

Jing, Y., & Baluja, S. (2008). VisualRank: Applying PageRank to large-scale image search. *IEEE Transactions on Pattern Analysis and Machine Intelligence, 30*(11), 1877-1890.

Jones, M. (2017). Querying the archive: Data mining from apriori to PageRank. In Daston, L. (Ed.), *Science in the Archives: Pasts, Presents, Futures*, pp. 311-314. Retrieved from http://www.columbia.edu/~mj340/jones_2017_querying.pdf

Juran, J. (1962). *Quality control handbook*. New York, NY: McGraw-Hill.

Juran, J. (1951). *Quality control handbook*. New York, NY: McGraw-Hill.

Kamvar, S., Haveliwala, T., & Golub, G. (2004). Adaptive methods for the computation of PageRank. *Linear Algebra and its Applications , 386*, 51-65.

Kamvar, S., Haveliwala, T., Manning, C., & Golub, G. (2003). Exploiting the block structure of the Web for computing PageRank. San Francisco: Stanford. Retrieved from https://nlp.stanford.edu/pubs/blockrank.pdf

Karp, H. (2014, date). Company paying more than $2.5 Billion for Beats Electronics in $3 billion deal. *The Wall Street Journal.* Retrieved from XXX

Kethley, R., Waller, B., & Festervand, T. (2002). Improving customer service in the real estate industry: A property selection model using Taguchi loss functions. *Total Quality Management*, *13*(6), 739-748.

Khalil, E., Dai, H., Zhang, Y., Dilkina, B., & Song, L. (2017). Learning combinatorial optimization algorithms over graphs. *Advances in Neural Information Processing Systems*, *30*, 63486358.

Kim, K., Lee, S.W., Moon. B., Park, C., & Hwang, J. (2011). IPL-P: In-page logging with PCRAM. *Proceedings of the VLDB Endowment*, *4*(12), 1363-1366. Retrieved from http://www.vldb.org/pvldb/vol4/p1363-kim.pdf

Kleinberg, J. (1998). Authoritative sources in a hyperlinked environment. *Journal of ACM*, *46* (5), 604-632.

Kloumann, I. U., Ugander, J. & Kleinberg, J. (2017). Block models and personalized PageRank. *Proceedings of the National Academy of Sciences*, *114*, 33-38.

Koh, S., & Tan, K. (2006). Translating knowledge of supply chain uncertainty into business strategy and actions. *Journal of Manufacturing Technology Management*, *17*(4), 472-485**.**

Krapivin, M., & Marchese, M. (2008). Focused page rank in scientific papers ranking. *Digital Libraries: Universal and Ubiquitous Access to Information*, vol. 5362, pp. 144-153.

Kumar, K., Abhaya, & Mukoko, F. (2013). PageRank algorithm and its variations: A survey report. *IOSR Journal of Computer Engineering*, *14*(1), 38-45.

Kume, H. (1985). Business management and quality cost: The Japanese view. *Quality Progress*, pp. 13-18.

Kurland O., & Lee, L. (2010). PageRank without hyperlinks: Structural reranking using links induced by language models. *ACM Transactions on Information Systems , 28*(4).

Lambert, D., & Pohlen, T. (2001). Supply chain metrics. *The International Journal of Logistics Management*, *12*(1), 1-19.

Lamberti, F., Sanna, A., & Demartini, C. (2009). A relation-based page rank algorithm for semantic web search engines. *IEEE Transaction of KDE*, *21*(1), 123-136.

Langville, A., & Meyer, C. (2004). Deeper inside PageRank. *Internet Mathematics*, *1*(3), 335-380.

Lesser, W. (1954). Cost of quality. *Industrial Quality Control*, *11*(5), 11-14.

Li, M.-H. (2003). Quality loss functions for the measurement of service quality. *International Journal of Advanced Manufacturing Technology, 21,* 29-37.

Lian, X., & Chen, L. (2010). Ranked query processing in uncertain databases. *IEEE Transactions on Knowledge and Data Engineering*, 22(3), 420-236.

Lin, Z., Ding, G., Hu, M., & Wang, J. (2015). Semantics-preserving hashing for cross-view retrieval. In *IEEE Conference on Comouter Vision and Pattern Recognition*, pp. 3864-3872.

Liu, J., Zhao, C.C., & Ren, Z.G. (2016). The application of greedy algorithm in real life. 2016 *International Conference on Manufacturing Construction and Energy Engineering (MCEE*), 107-113.

Ma, N., Guang, J., & Zhao, Y. (2008). Bringing PageRank to the citation analysis. *Information Processing & Management , 44*(2), 800-810 .

Maidamisa, A.A., & Eckson, N. (2017). Robust water distribution system using minimal spanning tree algorithm. *International Journal of Computer Applications, 158*(1). Retrieved from https://www.ijcaonline.org/archives/volume158/number1/maidamisa-2017-ijca912724.pdf

Mohan, K., & Kurmi, J. (2017). A technique to improved PageRank algorithm in perspective to optimized normalization technique. *International Journal of Advanced Research in Computer Science, 8*(3), 702-706.

Moran, J., & Bouchaud, J. P. (2018). Greedy algorithms and Zipf laws. *Journal of Statistical Mechanics: Theory and Experiment*, *2018*(4), 043402.

Murrugarra, D., Miller, J., & Mueller, A.N. (2016). Estimating propensity parameters using Google PageRank and genetic algorithms. *Frontiers in Neuroscience*, *10*. Retrieved from https://www.frontiersin.org/articles/10.3389/fnins.2016.00513/full

Neiman, O., & Solomon, S. (2016). Simple deterministic algorithms for fully dynamic maximal matching. *ACM Transactions on Algorithms (TALG), 12*(1), 7.

Nielson, S. (2014, Feb 6). Apple's premium pricing strategy and product differentiation. *Yahoo Finance*. Retrieved from https://finance.yahoo.com/news/apple-premium-pricingstrategy-product-191247308.html

Page, L. B. (1998). The PageRank citation ranking: Bringing order to the Web . *Proceedings of the 7th International World Wide Web Conference.* Brisbane.

Paiva, G. S., & Carvalho, M.A.M. (2017). Improved heuristic algorithms for the job sequencing and tool switching problem. *Computers & Operations Research*, *88*, 208-219.

Pandurangan G., Raghavan, P., & Upfal, E. (2002). Using PageRank to characterize web structure. *Computing and Combinatorics Lecture Notes in Computer ScienceSpringer, Berlin, Heidelberg,* 2387. Retrieved from http://cs.brown.edu/research/pubs/pdfs/2002/Pandurangan-2002-UPC.pdf

Pant, B., Ramirez, R., & Reeves, L. (2019). Google PageRank explained via power iteration.

Pastor-Satorras, R., & Castellano, C. (2016). *Distinct types of eigenvector localization in networks. Scientific Reports, 6***,** 18847. doi:10.1038/srep18847

Pecina P., Tortal, A., Papavassilio, V., Tamchyna, A., & Genabith, V. (2015). Domain adaptation of statistical machine translation with domain-focused web crawling. *Language Resources and Evaluation, 49*(1), 147-193.

Plunkett, J., & Dale, B. (1988). Quality costs: A critique of some 'economic cost of quality' models. *International Journal of Production Research, 26(11*1713-1726.

Porter, L., & Rayner, P. (1992). Quality costing for total quality management. *International Journal of Production Economics, 27*(1), 69-81.

Prasad, S. (1965). Review of "Managerial breakthough: A new concept of the manager's job" by J. Juran. *Management Science, 12*(4), 89-90.

Rani, P. (2013). A review of web page ranking algorithm. *International Journal of Advanced Research in Computer Engineering & Technology, 2*(3), 1052-1054.

Reeves, C., & Bednar, D. (1994). Defining quality: Alternatives and implications. *The Academy of Management Review, 19*(3), 419-445.

Saleh, J., & Marais, K. (2006). Highlights from early history of reliability engineering. *Reliability and System Engineering, 91*(2), 249-256.

Sallis, E. (2002). Total quality management in education (3rd ed.). London: Kogan Page.

Sandoval-Chavez, D., & Beruvides, M. (1997). A state-of-the-art matrix analysis of the cost related to quality. *Sixth International Conference on Management of Technology* II, pp. 1253-1260.

Sargolzaei, P., & Soleymani, F. (2010). PageRank problem, survey and future research directions. *International Mathematical Forum, 5*(19), 937-956.

Schwartz, R., Singh, M., & Yazdanbod, S. (2019). Online and offline greedy algorithms for routing with switching costs. *arXiv preprint arXiv:1905.02800*.

Schiffauerova, A., & Thomson, V. (2006). A review of research on cost of quality models and best practices. *International Journal of Quality and Reliability Management*, *23*(6), 647-669.

Schneiderman, A. (1986). Optimum quality costs and zero defects: Are they contradictory Concepts? *Quality Progress*, 28-31. Retrieved from https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.531.421&rep=rep1&type=pdf

Sharma, D. & Sharma, A. (2010). Comparative analysis of web page ranking algorithms. *International Journal of Advanced Trends in Computer Science and Engineering, 2*(8), 2670-2676.

Siahaan, A. P. U. (2017). Huffman text compression technique. *SSRG International Journal of Computer Science and Engineering, 3*(8), 103-108.

Simga-Mugan, C., & Erel, E. (2000). Distribution of quality costs: Evidence from an aeronautical firm," *Total Quality Management, 12*(1), 227-234.

Soble, J. (2015, Feb 5). Takata, maker of faulty airbags, expects wider loss. *New York Times.* Retrieved from https://www.nytimes.com/2015/02/06/business/international/takatamaker-of-airbags-expects-wider-loss-as-recall-costs-mount.html

Song, B. (2018). *Virtual web for PageRank computing*. (Unpublished doctoral dissertation). University of Missouri-Columbia.

Spens, K., & Bask, A. (2002). Developing a framework for supply chain management. *The International Journal of Logistic Management,* 13(1), 73-88.

Spiliotopoulou, E., Donohue, K., & Gürbüz, M. Ç. (2016). Information reliability in supply chains: The case of multiple retailers. *Production and Operations Management*, *25*(3), 548-567.

Stalk G., Evans, P., & Shulman, L. (1992). Competing on capabilities: The new rules of corporate strategy. *Harvard Business Review*, pp. 57-69. Retrieved from http://prismfoundation.org/wp-content/uploads/2016/10/HBR-Capabilities.pdf

Srivastava, S. (2008). Towards estimating cost of quality in supply chains. *Total Quality Management and Business Excellence, 19*(3), 193-208.

Suzuki, A., & Ishii, H. (2018). Distributed randomized algorithms for PageRank based on a novel interpretation. *2018 Annual American Control Conference (ACC)*, 472-477.

Taschner, A., & Charifzadeh, M. (2020). Supply chains, supply chain management, and management accounting. In A. Taschner & M. Charifzadeh (Eds.), *Management Accounting in Supply Chains* (pp. 1-14). Wiesbaden: Springer.

Teli, S.N., Majali, V., & Bhushi, U. (2013). Assessment of cost of poor-quality using knowledge-based system. International Journal of Engineering research and Technology, *2*(2). Retrieved from https://www.researchgate.net/publication/317902983_Assessment_of_Cost_of_Poor_Quality_Using_Knowledge_based_System

Tyagi, R., & Dev, D. (2016). Review of ranking algorithms. *International Journal of Innovative Research in Computer and Communication Engineering, 4*(4), 5205-5209.

Watson, G. H. (2005). Feigenbaum's enduring influence. *Quality Progress*, 51-54. Retrieved from http://gregoryhwatson.eu/images/8-QP_Watson_-_November2005_-_Feigenbaum_'s_Enduring_Influence.pdf

Xing, W., & Ghorbani, A. (2004). Weighted PageRank algorithm. *Proceedings of the Second*

Annual Conference on Communication Networks and Services Research (CNSR'04). Retrieved from https://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.454.5022&rep=rep1&type=pdf

Xu, Z., Luo, X., Zhang, S., Wei, X., Mei, L., & Hu, C. (2014). Mining temporal explicit and implicit semantic relations between entities using web search engines. *Future Generation Computer Systems*, *37*, 468-477.

Yang, J., Ban, X., & Xing, C. (2019). Using greedy random adaptive procedure to solve the user selection problem in mobile crowdsourcing. *Sensors*, *19*(14), 3158. Retrieved from https://www.ncbi.nlm.nih.gov/pmc/articles/PMC6679560/

Yates E., & Dixon, L. (2015). PageRank as a method to rank biomedical literature by importance. *Source code for Biology and Medicine*, *10*(16). Retrieved from https://scfbm.biomedcentral.com/articles/10.1186/s13029-015-0046-2

Zambuk, F. U., Gital, A.Y., Boukary, S., Jauro, F., & Chiroma, H. (2019). Evaluation of iterative PageRank algorithm for webpage ranking. *2019 4th International Conference on Electrical, Electronics, Communication, Computer Technologies and Optimization Techniques (ICEEECCOT)*, 365-370.

Zeitlyn, D., & Hook, D. W. (2019). Perception, prestige and PageRank. *PloS One*, *14*(5), e0216783.

Zhang, L. & Suganthan, P. (2016). A survey of randomized algorithms for training neural networks. *Information Sciences, 364*, 146-155.

APPENDICES

APPENDIX 1: A Brief Introduction to PageRank Algorithm

The present study was mainly inspired by topics from PageRank and Search Engine Optimization. This Appendix provides a short introduction to the history and uses of this algorithm in different applications.

A1. Google PageRank

According to Lin, Ding, Hu, and Wang (2015), numerous ranking methods to identify an author's effect and views in a research field are available. Such methods include citations, PageRank, h- index, weighted PageRank, and publications; however, most of these methods depend on the investigated topic. According to Berkhin (2005), the PageRank algorithm is a method used to compute a relative rank of website pages depending on their link structure. Computations are major components used in web search ranking systems, which earning the technique a wide range of applications. In recent years, the PageRank vector has been extensively used to calculate global importance scores; later, each score was recomputed for a new web graph crawl. However, in both instances, there is a need to compute many PageRanks corresponding to a variety of teleportation vectors for many topics and user preferences.

PageRank and associated algorithms prioritize links rather than contents of web pages (Devi, Gupta, & Dixit, 2014). Such algorithms are effective when ranking web pages against a variety of parameters, such as input parameters, result relevancy, outcome significance, and methodology. However, the disadvantages of these algorithms include their time response, accuracy, relevance, and significance of the acquired results (Sharma and Sharma, 2010). Therefore, there is a need for a more efficient web page ranking algorithm that would efficiently sort out the claimed challenges and be compatible with the global standards of this technology.

Another possibility is using graph-based algorithm that depends on the structure web page links. Such technique could be based on backlinks. Therefore, rank is calculated based on the significance of pages, and the results are computed during indexing, rather than during queries (Jones, 2017). Ranks can be calculated via computing hubs followed by authorities' scores of the present pages with the most relevant content listed at the top, followed by the least relevant content. This helps to sort efficiency challenges, since returned pages will have more relevance and significance, and the topic drift is taken into account (Kleinberg, 1998).

As argued by Dai and Freris (2017), the modeling of web graphs majors on capturing the extent of distributions witnessed on the web, thereby demonstrating total reliance on the local features of the web graph, as the disbursement of the PageRank values on the web and distribution do not depend on search engine optimization. According to Dai and Freris (2017), PageRank values on the web work with the power law. Dai and Freris (2017) explained models of the web graph while remaining loyal to previously studied degree distributions. The authors also analyzed the models and compared the analyses from web snapshots and graphs created through simulations of the new model.

Furthermore, according to Spens and Bask (2002), the PageRank algorithm works with probability to estimate the correspondence of clear semantics and the recognized authoritative documents in human perception. In this case, perfectly defined semantics with clear elucidation efficiently respond to qualitative bibliometric search queries. However, priority has to be established with regard to the needs of factors relevant to such modeling, which trades the computational cost to avoid the limitations of local maxima (Xing & Ghorbani, 2004). Considering the technique used in page content, this algorithm results in highly accurate outcomes, as the calculation of page weights is performed considering the outgoing links and the title tag of the specified page during searching, despite its dependence on the popularity of a web page (Erjia & Ying, 2011).

As argued by Yates and Dixon (2015), PageRanks ranks a page by assigning varying weights based on the following three factors: (1) a relative position in a page; (2) the link's tag; and (3) the length of the anchor text. Here, the first factor (i.e., relative position) proves to be less effective, suggesting that the logical locality does not always match the physical locality. Previous research on the effect of collusion as a nepotistic linkage in PageRank's web graph commonly reported PageRank increase. This depends on the reset probability, random walk, and the initial PageRank of the colluding set. As a result of the power-law distribution, highly ranked websites have no benefit from the collusion. Instead of relying on a page-to-page link, adjacency matrices can be constructed from an agent to the object link. Here, the following three vectors are used in getting score calculation of writing: (1) hubs; (2) reputation; and (3) authority. Since input and output links are not included the algorithm, this approach is suitable for blog ranking. According to Bidoki and Yazdani (2007), this technique is also based on reinforcement learning that is based logarithmic distance between pages. Here, the algorithm considers a real user by the number of pages that can be accessed faster with high quality. Therefore, when a new page is inserted between the two pages, huge calculations for the distance vector are necessary (Fujimura, Inoue, & Sugisaki, 2005).

Another important factor used for ranking is visitor time. The use of sequential clicking makes it possible to calculate vectors; it involves applying the random surfing model. It is useful if two pages possess different contents, but the same link structures, and the approach works better when used without a server log (Jiang, Ge, Zuo, & Han, 2008). According to Jiang et al. (2018), a PageRank can be determined based on the analysis of the tag heat in the social annotation web. In this way, very accurate ranking results can be obtained, and any new data sources can be more effectively indexed. Of note, the co-occurrence attribute of a tag, which may its influence its weight, is not considered. Likewise, Xu, Luo, Zhang, Wei, Mei, and Hu (2014) established that web page ranking for any semantic search engine uses the data generated from questions in addition to the used annotated sources. Through an efficient handling of the search page, ranking becomes simpler, as every page is annotated in relation to a given ontology, which is in essence a complex task (Lamberti, Sanna, & Demartini, 2009).

In PageRank, training queries underlie models, and each new query is incorporated according to the merged weighted scores of the model. This, in turn, provides results for the users' question, and those of a similar kind where a restricted number of factors are used to calculate similarity (Lee et al., 2009). Furthermore, Zhang and Suganthan (2016) found that the items to be used in tagging include the following three randomized algorithms: (1) proportional frequency sampling (2) move-to-set; and (3) frequency move-to-set. Accordingly, due to the many amounts of tags dictated by the method, tag popularity increases. However, this method has its restrictions, such as the lack of toleration to any other choice of model, ranking rules, and any other regulations. According to Bhamidipati (2009), based on the score fusion techniques, PageRank is used when two pages have a similar ranking. That is, PageRank does not consider a case when score vector T is created from a specified distribution. Similarly, Lian and Chen (2010) also reported that PageRank can be used as a discourse tool when retrieving moving objects from uncertain databases. The technique uses the probabilistic ranked query and the J-Probabilistic ranked query on join methods (Pandurangan, Raghavan, & Upfal, 2002). Since the technique employs the R-tress, the procedure is very fast. Using the same method, even though it requires only a restricted number of parameters like time and the number of prank candidates, the results are very encouraging.

Furthermore, Chakrabarti (2002) and Krapivin and Marchese (2008) provided a detailed coverage of Web crawling, mining and ranking techniques associated with information recoveries such as classification and clustering. Furthermore, Sarlós et al. (2011) demonstrated that, just as PageRank expresses quality with time over a complete web, personalized PageRank displays a link-based quality around user-selected pages. The current personalized PageRank algorithms can serve online questions with imitations to a constrained choice of pages. Sarlós et al. (2011) used a novel algorithm that precomputes a compressed database and attained total personalization. Therefore, a compact database can serve an online response to random user chosen personalization. This algorithm uses replicated random walks proving that, for a constant error probability, the size of a database is linear in a number of pages of the web. This approximation approach that involves asymptotic worst-case lower bounds that display on some graphs, the exact personalized PageRank values, can only be obtained from a quadratic database.

A2. The Google Search Engine in PageRank

Gleich, Berkhin, and Zhukov (2005) studies the structure of Google search engine, including the PageRank technique, a Hubs, and HITS, an authority-based ranking technique described by Kleinberg (1999). Along with other popularity-based ranking techniques, these three techniques were used to evade search engine spamming (see also Chakrabarti, 2002). Furthermore, through a survey of web resource discovery for elaboration, Pecina, Tortal, Papavassilio, Tamchyna, and Genabith (2015) addressed the focused web crawling to reach pages related to a specified topic. In another survey report, Kaushar-Kumar, Abhaya, and Mukoko (2013) compared the reports on variant page ranking algorithms through a numeric analysis. Kaushar-Kumar et al. (2013) discussed PageRank and Weighted PageRank both separately and in combination using VOL. Web mining as a concept in the PageRank calculation was explained in detail, and a detailed comparison of the four algorithms was performed done in tabular form.

Furthermore, in a study on PageRank as a method to rank biomedical literature by importance, Elliot and Louise (2015) argued that, in order to overcome article overload, what is essential is a ranking of the significance of literature to an optimal level. Since present ranking techniques are based on unfinished citation counts, this results in “inbound” links that do not consider the importance of citations. PageRank was first developed to rank webpages in search engines. Accordingly, Google can be modified into a bibliometric tool to quantify the significance of weightings within a citation network. Such modification can be attained with the computation of PageRank on commodity cluster hardware, followed by linear correlation with the counts of citations. PageRank is important in the quantification of relative importance, and it has can sort inadequacy problems in citation counts. Therefore, there is a broad consensus among scholars that PageRank is a practicable supplement to current bibliometric ranking methods. For instance, Lian and Chen (2006) applied the Google PageRank algorithm to evaluate the relative significance of all documents within a review family of articles written between 1893 and 2003. The authors found a strong positive correlation between the Google number and that of citations of each publication, as well as identified unique papers, also called germs, there were the outliers generated from linear relations.

As specified by Sargolzaei and Soleymani (2010), PageRank is patented as Google’s trademark in the U.S. Google assigns numeric weighting to each webpage, and this PageRank signifies the site’s importance to Google. PageRank is generated from a theoretical value obtained by probability on a logarithmic scale. PageRank of a specific page depends, in a very straightforward fashion, on the number of inbound links or the PageRank values of the pages giving out the links. Additionally, the vector serves as a feature in ranking, while the generic

PageRank matches uniform teleportation. Non-uniform teleportation also makes sense, since it results in topical PageRank. While the computation of most non-uniform teleportation is complex, in some instances, it can be optimized, which makes PageRank a fascinating calculative phenomenon that has stirred numerous studies across many disciplines. Since simple methods used in the numerical analysis of matrix computations are difficult to implement with an order 100 matrix, previous studies employed most numerical techniques used in computing the PageRank vector; interestingly, despite its low efficiency, it was found to be reliable in terms of performance. As mitigation of slow convergence of the power method, the use of extrapolation, aggregation, as well as disaggregation and lumping, were also proposed as accelerator techniques.

In summary, as a ranking tool that quantifies the significance of each web page based on the link structure of the web, PageRank plays a significant role in the Google search engine (Ishii & Tempo). Ishii and Tempo (2010) reviewed PageRank's problem set-up and proposed a series of distributed randomized schemes for PageRank's computation. Here, the pages can be locally updated values by communicating with those ones connected by links (Lei & Chen, 2015). Therefore, the schemes asymptomatically converge through a mean square manner to the correct PageRank values. Ishii and Tempo (2010) also provided an in-depth discussion of the close relations to the multiple agent consensus challenges.

A3. PageRank Computing

In a review of previous research on PageRank computing, Berkhin (2005) found that the constituents of the algorithm's vector act as the authority weights for web pages, not considering the page contents, and are mostly based on the link structure of that same web. Therefore, PageRank is an algorithm typically used as a web search ranking constituent (Kamvar, Haveliwala, Manning, & Golub, 2003). Thus, underscores the significance of the discussed model and the information structures based on the algorithm's processing capabilities and functions. Since computing PageRanks is a complex activity, there have been efforts to develop building sets of personalized PageRank vectors (Kloumann, Ugander, & Kleinberg, 2017). Along with raking *per se*, the algorithms can be used in other tasks, such as accelerating computing, in optimal arrangement of the computations, and enhancing the algorithm's stability (Avrachenkov, Litvak, Nemirovsky, & Osipova, 2007). Alternative models that cause similar authority indices as those of PageRank are also considered, along with an elucidation of link-based search personalization, in turn, listing the aspects of PageRank infrastructure, from the related measures of convergence to the link preprocessing.

Haveliwala (1999) reviewed several efficient methods of computing PageRank as a ranking technique for hypertext documents. Accordingly, PageRank can be computed for extremely large web's sub-graphs using the machines that have limitations of their main memory. The running time dimensions on a number of memory configurations were found for the computation of PageRank over a 24 million-page Stanford web base collection. Several convergences of PageRank methods were analyzed based on the prompted organization of the involved pages. Haveliwala (1999) reported the convergence results at the ultimate; this was useful to determine the number of repetitions necessary to achieve a helpful PageRank assignment in the absence or presence of search queries. Kamvar et al. (2003) also proposed a novel algorithm for a quick computation of PageRank as a hyperlink dependent estimation technique used to determine the importance of web pages. The original PageRank algorithm uses the power technique to compute successive repeats that congregate to the principal eigenvector of a Markov matrix represented as the web link graph. On the other hand, the new algorithm proposed by Kamvar et al. (2003), called the Quadratic Extrapolation, accelerates the convergence of power technique through detracting off approximations of the non-principal eigenvectors that form a present iterate of the power technique. Quadratic extrapolation takes into account the advantage the first eigenvector's value of a Markov matrix and is used to compute all non-principal eigenvectors using successive repeats of the power technique. Empirically, Kamvar et al. (2003) found that the Quadratic Extrapolation speeds up the computation of PageRank by 25% to 300% on a web graph that contains approximately 80 million nodes and has minimal overhead. This finding is meaningful for the PageRank community, because it is a fast way of determining the dominant eigenvector of a matrix considered to be too large for standard fast techniques.

Furthermore, a study by Rani (2013) concerning the use of PageRank as a link exploration algorithm used for Internet access by the Google search engine yielded other useful information. Since PageRank is a numeric value that embodies the significance of a page on the web, it is instrumental for the task of computing importance via counting of linked page numbers, and thus backlinks (Arasu, 2011). When the backlinks are from an important page, they are considered to have more weight than when they are from a less important one, whereby a link from a page to another is regarded as a vote. By calculating the significance of pages from the acquired votes, Google uses this technique to display important pages as results. It is thus an effective approach to calculate a numeric page value, since it represents the page importance on the web.

A4. Methods of Making A Google PageRank

Initially, in order to improve the ranking of search query results, only one PageRank vector was computed using a web's link structure. This made it possible to evaluate the relative importance of a process that was not dependent on a specific search query (Haveliwala 1999, 2003). However, for the sake of higher accuracy, it was proposed to compute a set of PageRank vectors using a representative set of pre-defined topics. The use of biased PageRank vectors to query-specific significance scores for pages at the time of query produced query-specific importance scores for pages at query time. This approach proved to be more accurate, thus generating rankings as opposed to that of one generic PageRank vector. For ordinary keyword search engines, topic-specific PageRank scores are calculated for the pages that contain a set of predefined keywords. For contextual searches, a user computes topic-sensitive scores with the topic of context in which the query exists.

Agirre and Soroa (2009) proposed a new graph-based technique based on the use of the knowledge in an LKB for unsubstantiated Word sense disambiguation. This algorithm effectively uses the full graph of the LKB, outperforming existing methods in the English dataset with all words. Agirre and Soroa (2009) also described how the algorithm can be used with languages other than English and, with WordNet as the only requirement, still yield impeccable results. Furthermore, the results of the analysis of the algorithm's performance showed the technique is very efficient and can be tuned faster, thereby curbing time wastage. Likewise, Kamvar et al. (2003) argued that, in PageRank, the web link graph has a nested block structure where most hyperlinks link pages in a host to other pages in the same host, as well as to other hosts that do not possess pages in the same domain.

The structure to accelerate the computation of PageRank employs a three-stage algorithm which consists of the following three steps. First, the local PageRanks of Pages are autonomously computed for every host by the link structure of the host. Second, the local PageRanks are weighted for their importance corresponding to the host. Finally, the standard PageRank algorithm, which employs the weighted cumulative of the local PageRanks as the starting vector, can be performed. Empirically, the algorithm accelerates the computation PageRank by a factor of two. A variant of the algorithm that effectively computes many variants and personalized PageRanks, as well as re-computes PageRanks after nodes have been updated, can also be used.

In summary, PageRank is a well-known link algorithm that is used to analyze the rank of web pages and to independently estimate the significances of such pages (Haveliwala, 2003). Furthermore, the questions and user-sensitive extensions of PageRank using a foundation set with biased PageRank vectors to personalize ranking into a one that can be tracked were also proposed. The author thus reviewed various approaches of personalizing PageRank, as well as provided a detailed discussion of tradeoffs of each of the approaches.

A5. Text vs. Image Search Rankings

Jing and Baluja (2008) found that, due to the relative ease of understanding and processing wordings, commercial picture searches mostly rely on the methods that are very much akin to the search of texts. In recent years, numerous studies demonstrated that employing picture-based features can be effectively used to give either substitutive or additional indications for usage in this process. However, it remains unclear whether such techniques can be equally applicable for the analysis of a large number of common web queries, as well as whether the measures to improve search quality would involve additional computational costs. A challenge in picture ranking is that there is a need to recognize authority nodes on incidental visual resemblance graph and propose a visual rank that can analyze any visual link structures in the images.

Images considered as authorities are images that perfectly match the search queries. To better grasp the performance of this kind of approach in reality, a series of large-scale tests can be done. For instance, recovering messages from around many popular commodity queries could result in a significant improvement in user satisfaction and relevance of the findings as compared to previous Google image search results. Keeping a modest computational monetary expense is essential to ensure that such procedures can be put to shape. Likewise, Kamvar, Haveliwala, and Golub (2003) reported that the convergence trends of the pages in the PageRank algorithm do not exhibit uniform distribution. Specifically, most pages converge very fast to their true PageRank, with relatively few pages taking more time to converge. Furthermore, the slow converging pages are normally the ones with a high PageRank. Later, a simple algorithm that aids in accelerating PageRank computation was devised. In this algorithm, called the Adaptive PageRank, the PageRank of the included pages that have been converged is not recomputed at each repetition after the convergence. This algorithm was found to accelerate the computation of page rank by 30%.

Although the rank of a web page largely depends on its content and visitors, it is possible to generate a measure of the page's rank (Bianchini, Gori, & Scarcelli, 2005). However, this depends on the topological outline of the web, since PageRank is an effective method of ascribing a score to web pages based on their connection to other web pages. In this article, PageRank is analyzed in-depth to reveal its essential features with reference to solidity, convolution of the computational scheme, and the essential role of the parameters used in the computation. Furthermore, Bianchini et al. (2005) also presents a circuit examination that allows individuals to better understand the dissemination of page scores, the ways various web communities are interrelated to each other, the roles that pages play with and without links, and the mysteries for the advancement of web pages.

As argued by Ma, Guang, and Zhao (2008), a substitutive method to measure the importance of papers depending on their PageRank needs to be developed. This method could become a useful extension of common integer counting citations and should be followed by large-scale experimentation where PageRank is used for the citation analysis. First, one can compute the PageRank values of the papers and then run distributional characteristics in comparison with the customarily used number of citations, followed by a detailed analysis. Additionally, PageRank is extensively used in various research domains, such as biochemistry and molecular biology, which highlights the usefulness of applying PageRank to the citation analysis. Upon publication of Gleich et al.'s (2005) paper, there have been many attempts to use the PageRank for the query independent organization of web pages. For instance, Matthew et al. (2006) demonstrated that using features that are not dependent on the link structure of the web can significantly outperform PageRank results. Accordingly, a boost in accuracy can be achieved using the information on the frequency of page visits. For instance, an alternative machine learning algorithm to merge the samples and other static variables depending on the anchor wordings and domain features is Rank Net. In this article, the consequential model achieved a static rating pairwise with the accuracy of 67.3% vs. 56.7% for PageRank and 0% for random.

A6. The Markov Chain

Boldi, Santini, and Vigna (2005) defined PageRank as the static of a Markov chain, whereby the chain is generated by upsetting the transition matrix prompted by a web graph, with a damping factor alpha (α) that diffuses evenly. In recent years, the comportment of PageRank in relation to variations of α has been demonstrated to be helpful in link-spam exposure. However, an exploratory validation of the value chosen as α is still to be made. In this respect, Boldi et al. (2005) offered the first mathematical analysis of PageRank as α changed. In particular, the authors demonstrated that, contrary to the common belief, in a real-world graph, the values of α that are closer to one always never give a useful ranking. Boldi et al. (2005) provided a closed-type formula for PageRank byproducts of any order together with a leeway of the power model that estimates them with convergence O for the K^{th} byproduct. Finally, Boldi et al. (2005) demonstrated a deep linkage between repeated computation and methodical conduct by showing that the K^{th} repetition of the power technique gives precisely the PageRank value generated from using a Maclaurin polynomial degree of K. The latter consequences give room for the application of systematic techniques in further research on PageRank.

Furthermore, Langville and Meyer (2004) elaborated on a specific reorganization suited for the PageRank problem that minimizes the computation of the PageRank vectors. This is sized down to one of a solving that is much smaller as a system, followed by the use of forwarding substitution to attain total solution vectors. Upon a comparison of the theoretical rates of convergence of the initial PageRank algorithm with those of a new reorganized PageRank algorithm, Langville and Meyer (2004) showed that the new one will not do further works than the existing one. Ultimately, the results of Langville and Meyer’s(2004) experimental comparison of five datasets demonstrated that can give an acceleration of up to factor six. Based on these findings, the authors concluded that the suggested reorganization could offer potential additional benefits.

Likewise, Chris and Lee (2007) demonstrated a two-staged algorithm for a fast computation of the PageRank vector. This algorithm is based on the following observation. The uniform time distinct Markov chain related to PageRank is lump able and as a result of the lump able subset of nodes are the dangling nodes. Therefore, convergence time is only a fraction of the needed requirement for the standard PageRank as stated by Google. Upon the analysis of 451,237 pages, convergence was attained at 20% of that period. The algorithm was found to replace basic practices that are generally unrectified and used to be ignored until the final computational steps in a process that does not speed up convergence. A comparison showed that the algorithm is generally usable and reaches the targeted acceleration. Overall, there are two variations that incorporate multiple stages of algorithms: while the first variation portrays an ordinary PageRank vector being computed, the second one shows a generalized version of PageRank being computed where web pages are divided into various categories, each of them integrating different personalization vectors. The latter stage stands for the main modeling extension and presents bigger suppleness and a probably more refined model for web traffic.

A7. PageRank and Social Networks

As argued by Heidemann, Klier, & Probst (2009), online social networks have gradually advanced into a worldwide conventional channel that generates a rising socioeconomic effect. However, most of the online social networks have to solve the issue of how to influence their fast-growing markets to attain sustainable returns. To this end, more efficient advertising methods, along with sophisticated consumer loyalty programs that adopt user maintenance, are necessary. Accordingly, key users regarding connectivity and communication play a pivotal role in this technique. However, relevant qualitative methods for key users' identification in online social media networks integration models and research results on their connectivity and communication are currently lacking. Depending on the design science research patterns, a novel PageRank-based approach was proposed. To demonstrate applicability of this approach, Brin et al. (1999) used an openly accessible dataset from Facebook.com and compared the results with other available approaches acting as substitutes.

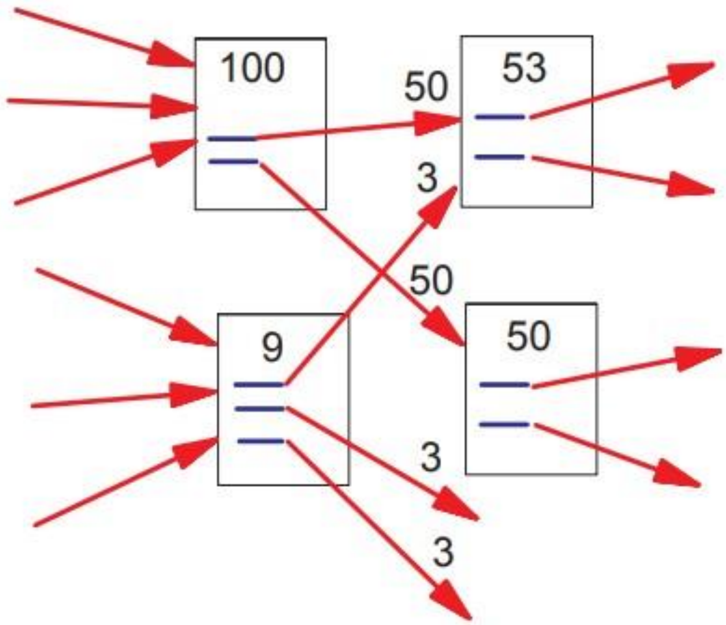


Figure 30:A simple calculation of PageRank (Brin et al., 1999)

A7. Developments

Ying et al. (2009) emphasized that PageRank developed a synergy to data retrieval as a way of improving ranking. In the Page Rank, the ranking of documents is performed based on the graphs' topology and the nodes' weights. Therefore, PageRank has considerably advanced information retrieval, which, in turn, has allowed Google to stay at the top of the search engine market industry. the Page Rank has been extensively used in bibliometrics to evaluate the research impact of the damping factor, thereby facilitating ranking. Accordingly, various damping factors are believed to offer more insight into authors' ranking. Specifically, there is evidence that weighted PageRank algorithms and an author's co-citation network and citation rank strongly correlated with the PageRank values that have varying damping factors. This proves that the h-index, citation link, and PageRank do not correlate with central measures.

Furthermore, Tyagi and Dev (2016) also indicated that the PageRank can be narrowed down to web crawling as a distinct component. Today, whenever any information is needed, most users resort to the web, as it is a fast and reliable way. Back in the days when web crawlers were not available yet, users experienced difficulties in accessing important information. However, with the invention of web crawling, users can get access whatever information they need at a given time. By definition, a web crawler, or topic-specific crawler, is a set of instructions that gathers only relevant data. Accordingly, web crawlers should be robust, high-quality, efficient, scalable, and yield high performance. PageRank was developed to evaluate the significance of web pages though their link structures (David, 2015). It has a mathematical structure that commonly applies to graphs of networks in all domains. Today, the algorithm is widely used in bibliometrics, data network analysis, social purposes, and hyperlink forecast and endorsement. Among other applications, it is also used in road networks systems' analysis and in various disciplines, such as physics, chemistry, neuroscience, and biology.

As argued by Brin et al. (1999), eb pages become inherently significant when they fit a subject matter that fits a given user's interests, attitude, and knowledge. Therefore, PageRank is a method of factual and mechanical rating of web pages while efficiently assessing human interest and attention. In a comparison of PageRank to any idealized random web search engine, Brin et al. (1999) effectively displayed a computation of PageRank and its application in searching for the searcher's navigation. Furthermore, Matthew and Pedro (2005) introduced a model that, based on probability, connects page content and the hyperlink structure in an intelligent random surfer's form. This model fundamentally accommodates fundamentally any question relevance function that is currently in use. The model produces results of a higher quality as compared to those afforded by PageRank and has time and storage that match the need of present-day largescale search engines.

In a review of all issues surrounding PageRank, such as the basic PageRank model, accessible and endorsed elucidation approaches, storing concerns, presence, exclusivity and merging features, and probable modifications to the basic model, Langville and Meyer (2004) proposed substitutes to the customary solution means, sensitivity, and conditioning and the updating problem while speculating on the necessary areas of future research. A local graph apportioning algorithm was reported to always demonstrate a cut close to the selected starting vertex with the running time that depended on the size of the small part of the cut vs. the size of the data in the graph. In the symposium, Langville and Meyer (2004) also demonstrated a partitioning algorithm that uses the variation of PageRank with quantified starting disbursement. The authors derived a collaborative result for the PageRank vector that resembles those for random walks, thereby showing an organization of vertices generated by a PageRank vector that reveals a cut with small conductance. Langville and Meyer (2004) elaborated an advanced algorithm for computing estimate PageRank vectors which enables finding set times that are proportional to any sizes.

Specifically, a cut can have a conductance not exceeding O; then its small part has to be at least 2^b as the volume, in time $O\ (2^b\ log^2\ ^m/_{\varphi^2})$ where m is the number of edges. Upon merging of the small sets generated by the partitioning algorithm, Langville and Meyer (2004) acquired a cut with a conductance \not as well as an estimated optimal balance in time $O(m\ log^4\ m/_{\varphi^2})$.

Based on the findings reported by Arasu (2011), the PageRank computation can be considerably speeded up. In particular, Arasu (2011) proposed an innovative open text word sense disambiguation technique that merges the use of logical inferences with PageRank-like algorithms done on graphs generated from basic language papers. This technique can be used to evaluate accuracy on annotated texts. Moreover, it can also be used to show its constant outclassing of the accuracy of other knowledge-based word sense disambiguation methods. Previously, Chris et al. (2003) classified PageRank and HITS as the most common web page ranking algorithms. Although both algorithms were ranked by in-degree, HITS accentuated common fortification between authority and hub pages, whereas PageRank stressed hyperlink weight regularization and web surfing, where both depended on the indiscriminate walk model. These two concepts were methodically generalized into a merged structure. While the ranking outline has a huge algorithm space, the HITS and PageRank occupy two extreme ends of the space. At present, research on many regularized ranking algorithms that are intermediary between HITS and PageRank and also obtain closed-type solutions is underway.

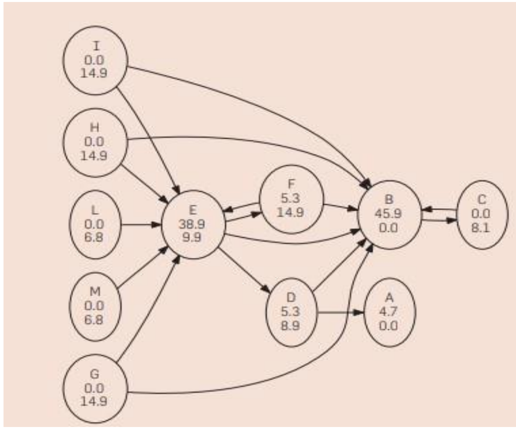


Figure 31: A HITS with a solution in PageRank (Massimo, 2011)

According to Dai and Freris (2017), PageRank has acquired significance in a wide range of applications and domains; after the algorithm proved to be efficient in determining node significance in huge graphs, it became the pioneering idea underlying the Google search engine. In the sector of disbursed computing alone, the algorithm's vectors, as well as other random based qualities, have gained usage in a vast variety of applications, ranging from significant nodes, load equilibrating, search to recognition of connectivity structures. However, thus far, minimal efforts have been directed towards designing reliable, effective and completely disbursed algorithms used in computing PageRank. In part, this gap can be explained by the fact that, due to communication bandwidth limitations and convergence rates, customary matrix vector multiplication approaches iterative methods cannot perfectly adapt to the disbursed setting.

A possible solution can involve quick random walk-based distributed algorithms used in computing PageRank in overall graphs, an indication that strong bunds do exist on round complexities. This begins with a presentation on the algorithm that takes O (log n $/\varepsilon$) rounds with higher potentials on either directed or undirected graphs and n id the network size and ε is the reset probability used in the computation of PageRank and it is a constant. An algorithm that takes an efficient round in undirected graphs has also been discussed. The two discussed algorithms are scalable, since each of the nodes processes and transmits the only minimal number of bits each round, thereby working for the distributed computing model (Neiman & Solomon, 2016). In the case of directed graphs, an algorithm that shows efficiency with running time, but needs a polynomial figure of bits to be processed and be transmitted per node in each round has been discussed. The first completely disbursed algorithm that can be used in computing models with proven efficiency of their running time was proposed.

Focusing on SALSA, Bahmani et al. (2010) analyzed Monte Carlo methods of incremental computation of PageRank, Personalized PageRank, and several other random walk-based techniques. Using large-scale and ever-growing social networks like Twitter, the authors assumed that the graph of friendship is maintained in the distributed shared memory. In the case of a global PageRank, social networks are assumed to possess n nodes, and m argumentatively chosen edges come in a random organization. Therefore, if a reset probability of ε is present, the total work required to maintain precise estimations of the PageRank of each node always exists, which gives the technique a competitive edge over other bounds used for incremental PageRank. For example, if an individual innocently re-computes PageRank when each edge comes, the basic power iteration technique requires full time, and the Monte Carlo requires $O(mn/\varepsilon)$ total time, which makes the two excessively expensive. This means that an individual can efficiently handle deletions. Then, author then major on the computation of a topic, k that has been personalized by the PageRank beginning from a seed node with an assumption that personalized PageRank work with a power law with the exponent α < 1 (Pastor-Satorras & Castellano, 2016). The authors demonstrated that, if they store R > q in n random walks starting from each node for a big constant q, then the expected number of calls reaching the distributed social network database is $O\ ({}^{\kappa}/_{R(1-\alpha)})$.

Therefore, Pastor-Satorras and Castellano (2016) concluded that the algorithm is fast for real-time queries directed to a dynamic social network by using Twitter.

In another relevant study, Chakrabarti (2007) reported on the experiments with Citeseer's ER graph and a big number of other real Citeseer questions in comparison to PageRank propinquity search establishments. Chakrabarti (2007) analyzed the competitors' strategies of success which, when implemented by PageRank, can lead to more advancements. When processing, HubRank works by computing and indexing, which can give sketchy random walk fingerprints belonging to small fractions of carefully selected nodes by considering the statistics of the query log. During querying, small but active subgraphs bordering with the nodes that have indexed fingerprints are recognized. The fingerprints are adaptively entered into different resolutions to generate approximate PageRank vectors, also called PPV, which, when remaining active, are ready for iterative computation.

The extemporized recovery duty is to discover documents that are largely pertinent to an entered query. Inspired by PageRank algorithms, Kurland and Lee (2010) proposed a re-ranking method to allow retrieval that works for settings without hyperlink data. However, Kurland and Lee (2010) reorganized the documents in an originally recovered set through exploiting understood asymmetric associations between the documents. The process takes into consideration the generation links, suggesting that the language model prompted from a single model offers a higher potential to the text of another. Upon the analysis of vast amounts of re-ranking techniques dependent on the central measures in graphs created with generation links, Kurland and Lee (2010) concluded that incorporating centrality into the standard language model-based recovery efficiently increases precision at top ranks, and that the perfect consequential performance is always comparable and superior to the one of a state-of-the-art pseudo feedback- based recovery approach. The advantages of the language-based model method in inducing inter-document links by their comparison to notions of similarities are also discussed at length. In summary, the techniques for inducing centrality are considerably more efficient than the methods based on specific characteristics of the documents.

According to Massimo (2011), as a web page ranking method, PageRank has been fundamental for the development and success of the Google search engine. Google continues to use PageRank to identify the most important pages. The major ideology behind this algorithm is evaluating the importance of a web page. In a review of various techniques of web data retrievals, such as sociometric, bibliometrics, and econometrics, Massimo and colleagues reviewed PageRank as Google's search engine algorithm.

A8. PageRank Calculation

In order to determine the importance and the frequency at which pages may be visited and cited, different PageRank methods or algorithm have been used. In a discussion of the basic elements of PageRank algorithm, Chen et al. (2006) stated that "given a network of N nodes i = 1, 2, …, N, with direct links that represent references from an initial node to a target (cited) node, the Google number G_t for the i^{th} node is defined by the formula

$G_t = (1 – d) \sum G_j/k_j + d/N$" , Furthermore, according to Chen et al. (2016) "is a free parameter

> that controls the performance of the Google PageRank algorithm; the pre-factor algorithm (1-d) in the first term gives the fraction of the random walks that continue to propagate the along the links, … the first describes the propagation of the probability distribution of the random walk in which a walk at the node j propagates to node i with probability $1/k_j$, where k_j is the out-degree of node j while the second term describes the uniform injection of probability into the network in which each node receives a contribution d/N at each step."

As argued by Fall (2003), the relationship between page rank, set of pages, out-degree of the rank, and the dumping factor d can be used to establish an algorithm that defines the criterion for determining PageRank. Accordingly, Fall's (2003) algorithm is as follows: $R(u) = d \sum R(v)/Nv + (1 - d)$ where $v \mathcal{E} B_u$.

Furthermore, Page (1998) provided the following definition of the page ranks algorithm: "let *u* be a web page, F_u be the set of pages that point to *u* and let $N_u = [F_u]$ be the number of links from *u* and let *c* be a factor used for normalization. By defining a simple ranking R which is a simplified version of PageRank as follows; $R(u) = c \sum R(v)/Nv$ where $v \varepsilon B_u$" (p. X).

However, Page (1998) came out with another method where square matrix was used to determine the page rank. As Page (1998) articulated, "let A be a square matrix with rows and columns corresponding to web pages. Let $A_{u,v} = 1/Nu$ if there is an edge from *u* to *v* and $A_{u,v} =$ 0 if not.

Then if we treat R as a vector over web pages, then we have R = *c*AR. So, an eigenvector of A with eigenvalue *c*". According to Page (1998), these values can be used to come up with a graph where we can read values to come to *p* with a matrix. Page's (1998) matrix algorithm was derived from the following formula:

"$R'(u) = c \sum R'(v)/Nv + cE(u)$

Furthermore, according to Franceschet (2010), there is a direct relationship between the number of times a page is visited and the degree of importance of that page. Franceschet (2010) defined the following more formal algorithm to determine a Page Rank:

> "A little more formally, the method can be described as follows. Let us denote by qi the number of distinct outgoing (hyper)links of page i. Let $H = (h_{i,j})$ be a square matrix of size equal to the number n of Web pages such that $h_{i,j} = 1/q_i$ if there exists a link from page i to page j and $h_{i,j} = 0$ otherwise. The value $h_{i,j}$ can be interpreted as the probability that the random surfer moves from page i to page j by clicking on one of the distinct links of page i. The PageRank π_j of page j is recursively defined as $\pi_i = \sum \pi_i h_{i,j}$."

However, according to Massimo Franceschet (2010), this algorithm has two problems that prevent us from getting the final solution—namely, dangling nodes and trapping of the surfer in the pool of Web graph.

Yet, as Franceschet (2010) argues, a power method can be used to help compute the PageRank. He adds that the power method, a simple iteration method to find the dominant eigenpair of a matrix developed by von Mises and Pollaczek-Geiringer. It works as follows on the Google matrix G. Let π (0) = u = 1/n e. Repeatedly compute π(k+1) = π(k)G until ||π(k+1) − π(k)|| < ϙ, where || · || measures the distance between the two successive PageRank vectors and ϙ is the desired precision (Franceschet, 2010).

Furthermore, according to Haveliwala (1999), "The process can also be expressed as the

> following eigenvector calculation, providing useful insight into PageRank. Let M be the square, stochastic matrix corresponding to the directed graph G of the web, assuming all nodes in G have at least one outgoing edge. If there is a link from page j to page i, then let the matrix entry M_{ij} have the value $1/Nj$. Let all other entries have the value 0. One iteration of the previous x point computation corresponds to the matrix-vector multiplication M × Rank. Repeatedly multiplying Rank by M yields the dominant eigenvector Rank_ of the matrix M. Because M corresponds to the stochastic transition matrix over the graph G, PageRank can be viewed as the stationary probability distribution over pages induced by a random walk on the web."

Haveliwala (1999) further argued that, by use of residual vector, the iteration convergence can be determined and M being stochastic has got an eigenvalue of 1, giving the result as rank, since the multiplication of Rank by 1 is still Rank. Furthermore, Haveliwala (1999) added that introducing a new matrix by which probability edges of transitions $(1 - c/N)$ in every node pair in G results in the following formulation: $M' = cM + (1-c) \times {}^{1}/_{N}(N \times N)$, explaining that "the reason as to why this modification results in better quality of PageRank is because decay factor is introduced".

INDEX

VITA

Farshad Rabib was born in, Iran, to Zahra and Moharam. He is the fourth of five children; his siblings are David, Javid, Maryam and Shabnam. Since childhood, Farshad enjoyed science related subjects, partly because of the many hours spent helping his father fix mechanical problems on his semi-truck, and partly because his mother, who did not have the opportunity to gain a higher education, put all her efforts to encourage Farshad and his siblings to study hard and expand their wealth of knowledge. As result, Farshad graduated from high school at the age of sixteen years old and went on to study Industrial Engineering for his undergraduate degree.

Farshad moved to US in 2004 as a political refugee when he was in his twenties. Before moving to US, however, Farshad lived in Turkey, where he worked as a teacher for the United Nations (UNHCR). There, he taught mathematics and other STEM subjects to over 700 students of different age range, from more than six different countries and different ethnicities. After moving to US, Farshad decided to continue his education in engineering. The path to a higher education was not easy, however, Farshad's ambition and determination paid off, and he graduated with a master's degree in Electrical Engineering from the University of Tennessee in December 2014. Farshad joined the doctoral program in Industrial and Systems Engineering at the University of Tennessee in August 2016.

While studying for his PhD, Farshad worked full time positions in various industries, starting with Lead Project Engineer at Nissan Automotive, to Engineer/Scientist for the Navy Warfare Center, to Senior Reliability Engineer at Johns Manville, and currently working as a Lead Systems Engineer at Boeing Defense.