Local Determinacy with Non-separable Utility

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Abstract

This paper introduces general formulations for both technology (with input substitution) and non-separable utility (compatible with balanced growth and stationary worked hours) into a benchmark RBC model. It is shown that intertemporal substitution and input substitutability lead to local determinacy and rule out stationary sunspot equilibria when labor demand is downward-sloping, in contrast with recent results obtained under the assumption of separable utility. The main intuition behind this result is shown to work as follows: in contrast with separable preferences, increasing the elasticity of intertemporal substitution in consumption necessarily implies *decreasing* the elasticity of constant-consumption labor supply, when utility is non-separable and concave, which affects unfavorably the occurrence of local indeterminacy.

Keywords: real business cycles, intertemporal substitution, capital-labor substitution, externalities, indeterminacy, sunspots.

Journal of Economic Literature Classification Numbers: C62, D58, D91, E32.

1 Introduction

A major challenge of the literature exploring expectation-driven business cycles is to provide models that pass the empirical tests, e.g. calibration. Bennett and Farmer [5] have argued that non-separable utility is a key element to achieve this objective. However, Hintermaier [6] has recently shown that assuming Cobb-Douglas technology and a downward-sloping labor demand rules out local indeterminacy, in contrast with the examples by Bennett and Farmer [5] which turn out to violate concavity. On the other hand, Pintus [9] has stressed that local indeterminacy may occur with *small* externalities (and downward-sloping labor demand): this requires both risk aversion to be small enough (very close to zero, indeed, for reasonable parameter values) and input substitutability to be large enough (greater than two). However, he assumes that utility is separable in consumption and leisure. Taking all recent results together, one wonders whether or not departing from Cobb-Douglas technology (which is key in Pintus [9]) may overturn the result by Hintermaier [6] and reconcile local indeterminacy with small externalities, in the spirit of the examples with

low risk aversion studied in Bennett and Farmer [5].

In this paper, I address this open question and show that input substitution and non-separable utility *compatible with both balanced growth and stationary worked hours* do not help to improve the plausibility of local indeterminacy and sunspots. More precisely, I demonstrate that the result by Hintermaier [6] generalizes to any technology with constant (private) returns to scale, when utility belongs to the class of utility functions considered in King *et al.* [7] (see also King and Rebelo [8]). Under the maintained assumption of a downward-sloping labor demand, I show, first, that local indeterminacy requires both the elasticity of capital-labor substitution to be less than one (complementary inputs) and the coefficient of risk aversion to be less than the elasticity of input substitution (low risk aversion). Then I demonstrate that local indeterminacy is ruled out when the value of capital share is smaller than some upper bound (for example, less than a half for reasonable parameter values). In other words, complementary inputs do not lead to indeterminacy with negatively sloped labor demand when utility is non-separable, which puts some light on a conjecture stated in footnote 6 of Bennett and Farmer [5, p. 130]. As a corollary, I derive the expression of a minimal level of externalities below which indeterminacy is ruled out and illustrate that, for reasonable parameter values, this critical level of increasing returns is large and well outside the range of recent estimates (for instance, Basu and Fernald [2], based on constant shares of input costs).

The analysis also delivers an intuitive description that may help to understand the effects of non-separable utility on local indeterminacy, in terms of two key parameters stressed by Bennett and Farmer [5] (the elasticity of intertemporal substitution - the inverse of risk aversion - and the slope of the constant-consumption labor supply curve). In that context, the intuition behind the main result may be presented as follows. As already known from the literature (e.g. Benhabib and Farmer [3]), a greater elasticity of labor supply helps to get local indeterminacy for lower externalities. When utility is non-separable (and of the King *et al.* [7] form) and when labor demand has a negative slope, local indeterminacy requires both inputs to be complementary (or, equivalently, a very elastic rate of return on capital investment) and risk aversion to be small enough. Moreover, when capital externalities are small enough, indeterminacy requires the coefficient of (relative) risk

aversion to be less than the elasticity of capital-labor substitution. Therefore, when technology, starting from the Cobb-Douglas configuration, is getting closer to the Leontief case, risk aversion has therefore to decrease to zero. However, imposing that utility is concave in both consumption and labor implies that the lower risk aversion, the *lower* the (constant-consumption) labor supply elasticity to the real wage, which in fact goes to zero when risk aversion goes to zero. In other words, although intuition may suggest that decreasing risk aversion should increase the plausibility of indeterminacy (indeed, this is the case when separable utility is almost linear; see Pintus [9]), this necessarily reduces labor supply elasticity when utility is non-separable and concave, which makes indeterminacy *less* likely. In contrast, separable utility allows to treat labor supply elasticity as a parameter that is *independent* of risk aversion. This also explains why *complementary* inputs are needed to get indeterminacy. As noted above, labor supply is close to inelastic when risk aversion is small so that wage and interest rate have to be elastic enough to move labor hours, i.e. technology has to be close to the Leontief case (see also the discussion in Bennett and Farmer [5, pp. 129-30]).

It turns out that, in our model with input substitution and non-separable utility, there is no net gain in terms of indeterminacy being more plausible. It is worth noting that our formulation of non-separability does not nest the separable case studied by Pintus [9]: equation (3) below shows that separability implies *logarithmic* consumption utility, which is less general than separable utility as assumed in Pintus [9].

The rest of the paper is organized as follows. Section 2 presents the extended Ramsey model, following Bennett and Farmer [5] and Pintus [9], and derives its perfect-foresight competitive equilibria. Section 3 establishes some necessary conditions for indeterminacy to occur and establishes the main result. Finally, Section 4 concludes, while the proofs are gathered in two appendices.