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Support Vector Machine for Regression of Ultimate Strength of Trusses: A Comparative Study

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Abstract. Thanks to the rapid development of computer science, direct analyses have been increasingly used in the design of structures in lieu of member-based design methods using the effective length factor. In a direct analysis, the ultimate strength of a whole structure can be sufficiently estimated, so that the need for member capacity checks is eliminated. However, in complicated structural design problems where many structural analyses are required, the use of direct analyses requires an excessive computation cost. In such cases, Machine Learning (ML) algorithms are used to build metamodels that can predict the structural responses without performing costly structural analysis. In this paper, the support vector machine (SVM) algorithm is employed for the first time to develop a metamodel for predicting the ultimate strength of trusses using direct analysis. Several kernel functions for the SVM model, including linear, sigmoid, polynomial, radial basis function (RBF), are considered. A planar 39-bar nonlinear inelastic steel truss is taken to study the performance of the kernel functions. The results confirm the applicability of the SVM-based metamodel for predicting the ultimate strength of trusses. In particular, the RBF appears to be the best kernel among others. This investigation also provides a deeper understanding of the effect of the parameters on the efficiency of the kernel functions.

Keywords: Direct Analysis, Truss, Machine Learning, SVM, Nonlinear Inelastic Analysis.

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1. Introduction

Allowable stress design (ASD) and load and resistance factored design (LRFD) have been widely used in the analysis and design of structures. Their major advantage is that they allow for an acceptable design of a structure with negligible computing resources through a two-stage design method. In the first stage, the forces of structural members are calculated by using a linear analysis. In the second stage, the members' safety is checked by using provided equations in design codes in which inelastic and nonlinear behaviors of the structure are already considered. However, this indirect design approach is one of the principal disadvantages of ASD and LRFD since the interaction between structural members and the whole structure is not considered and then the structural stability is not checked. In contrast, direct analysis can directly predict structural nonlinear and inelastic behaviors such as $P-\delta$, $P-\Delta$, buckling, etc., and estimate the loadcarrying capacity of the structure. And then the member check is not required. With such advantages, direct analysis has been of the researchers' interest [1-12]. These works also showed that using direct analysis can design a lighter structure compared to using conventional methods. Notwithstanding, direct analysis spends much greater computation efforts compared to ASD and LRFD, especially for complicated structural design problems such as optimization and reliability analysis which require lots of structural analyses. This issue is the major reason preventing the wide application of direct analysis in structural design in practice.

To reduce computation efforts in such complicated structural design problems, using metamodel is considered as a powerful alternative since this approach can efficiently reduce the number of required time-consuming advanced analyses. A metamodel can be simply understood as an approximate function describing the relation between the input data and the output data. This means that the structural responses may be forecasted by using metamodels without performing a time-consuming direct analysis. With the rapid development of computer science, metamodels are easily developed for any problems of structural designs by using machine learning methods (ML) such as deep learning (DL), decision tree (DT), random forest (RF), gradient tree boosting (GTB), and support vector machine (SVM), etc. Up to now, several works on the application of ML in structural design have been published in the literature [13-21]. However, the application of ML to estimate the behaviors of the structure in lieu of direct analysis is still quite limited with few publications. For example, Truong et al. [22] used DL to estimate the ultimate load carrying of steel trusses. In Truong et al. [23], GTB was adopted in the safety evaluation of steel truss structures.

A well-known ML method is the support vector machine (SVM), which was proposed by Vapnik in 1995 [24]. Up to now, SVM is found as a supervised metamodel for solving regression and classification problems. SVM has been widely applied in many fields such as pattern recognition, damage identification, structural reliability analysis, structural health monitoring, etc [25-36]. The results of these works showed that SVM is robust for nonlinear and high-dimensional problems, and requires a small number of samples. So far, no study regarding the application of SVM in truss structure design using direct analysis has been published in the literature. As a consequence, the understanding of using SVM to predict the truss responses has several gaps. Furthermore, when using SVM, the kernel function plays an important role in the performance of the SVM model since it supports mapping the data to a greater dimensional space to get a better interpretation of the SVM model. Notwithstanding, several types of kernel functions can be used in SVM, for example, linear, polynomial, sigmoid, radial basis function, etc. Therefore, it is worth exploring the applicability of SVM as well as the efficiency of different kernel function types in predicting the behavior of nonlinear, inelastic truss structures.

In this work, the application of SVM for the regression of ultimate strength of steel truss structures using direct analysis is studied for the first time. Moreover, we will present the comparison of the influence of different kernel functions on the performance of the SVM model used to predict the ultimate strength. For simplicity, a 39-bar planar steel truss is taken as the case study for the investigation. The ultimate strength of the truss is represented by the ultimate load factor (ULF) which is the ratio of the structural load-carrying capacity and applied loading.

The paper's rest is organized as follows. Section 2 presents the data collection method. Section 3 briefly introduces the SVM algorithm and proposes a framework to estimate the truss ULF using direct analysis. In section 4, the numerical results and discussions of the case study are presented. Finally, Section 5 draws conclusions and directions for further studies.

2. Data Collection

Without losing the generality, a 39-bar planar steel truss with the geometry presented in Fig. 1 is considered to illustrate the study presented in this paper. All bars are assumed to have the circle cross-section shape with the cross-sectional area in the range [645.16, 11290.3] (mm²). The yield strength of steel is 344.5 (Mpa) and the elastic modulus is 200 (Gpa). The horizontal and gravity loads are assumed at all nodes and equal to 150 and 200 (kN), respectively. In light of this, the SVM developed model has 39 input variables which are the cross-sectional areas of the bars, and 1 output variable that is the ULF of the truss. To develop the SVM model in this study, a data set including 5,000 samples is used. As shown later in the next sections, this number of samples is sufficient to build the SVM model for the prediction of the ULF of the truss.



Fig. 1. 39-bar planar truss.



Fig. 2. Histogram of ultimate load-factor data.

The structural ULF is predicted by using the direct analysis method proposed by Thai and Kim [37] in this work because this method is highly accurate and robust to estimate the nonlinear inelastic behaviors of truss structures. Furthermore, yielding, buckling, reloading, and unloading failure mechanisms are considered by using the nonlinear constitutive model proposed by Blandford [38]. Both compressive and tensile areas, elastic and inelastic regions are also considered. Nonlinear equations are solved by using the generalized displacement control method [39] which can adjust automatically the step size and self-adapt to the loading direction's change. The detail of this direct analysis method can be found in [37, 40-43].



Fig. 3. A hyperplane for a two-dimensional training set.

Table 1.	Formula and	l important	parameters	of SVM	model	using co	mmon	kernels	5
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Kernel Function	Formular	Parameter		
Linear	$K(x_n, x_i) = (x_n, x_i)$	C and γ		
Radial basis function (RBF)	$K(x_n, x_i) = \exp\left(-\gamma \left\ x_n - x_i\right\ ^2 + C\right)$	C and γ		
Sigmoid	$K(x_n, x_i) = \tanh(\gamma(x_n, x_i) + r)$	C , γ , and r		
Polynomial	$K(x_n, x_i) = \tanh\left(\gamma(x_n, x_i) + r\right)^d$	C , γ , r , and d		

A database consisting of 5,000 samples is generated as follows. The input data which are 39 cross-sectional areas of the bars is randomly created in the range [645.16,11290.3] (mm²). And then, the above direct analysis is used to estimate the truss ULF. The histogram of 5,000 obtained ULF is presented in Fig. 2. As can be seen in this figure, the ULF of the truss is in the range [0.5;3.5] with an average value is about 2.1.

3. Proposed SVM-based Framework to Estimate Truss ULF

3.1. SVM Algorithm

SVM was developed by Vapnik [24]. It is a supervised learning algorithm for solving regression and classification problems. For a classification problem, the basis of SVM is to find a hyperplane so that all data points $x_i \in \mathbb{R}^n$ in the same tag are on the same side of the plane and the distance between the nearest point of each tag to the hyperplane is the largest. In other words, the best hyperplane is found by maximizing the width of the margin. This basic idea is presented in Fig. 3 for a two-dimensional training set. For such a regression problem in this study, the output (i.e., ULF) is a real number. Therefore, it becomes very difficult due to the infinite possibilities of hyperplanes. The framework to estimate the truss ULF by SVM is presented in the following.

Assuming the model used in SVM as:

$$f(x_i) = w^T h(x_i) + b \tag{1}$$

where h() is the basic function, w is the weight vector, and b is the bias value. b and w are determined by minimizing the below-regularized error function:

$$C\frac{1}{N}\sum_{n=1}^{N}L_{\varepsilon}\left(f\left(x_{n}\right)-y_{n}\right)+\frac{1}{2}\|w\|^{2}$$
(2)

where C is a constant parameter, E_{ε} is an insensitive error function proposed by Vapnik [24] as

$$L_{\varepsilon}(f(x_n) - y_n) = \begin{cases} 0, & \text{if } |f(x_n) - y_n| < \varepsilon \\ |f(x_n) - y_n| - \varepsilon, & \text{otherwise} \end{cases}$$
(3)

in which ε is a pre-defined tolerance. To simplify Eq. (2), the slack variables ξ_n and ξ_n^* are used as the deviation of training data outside the ε -zone. Eq. (2) is rewritten as

$$C\sum_{n=1}^{N} \left(\xi_{n} + \xi_{n}^{*}\right) + \frac{1}{2} \left\|w\right\|^{2}$$
(4)

subject to:

 $\xi_n \ge 0, \xi_n^* \ge 0, y_i - f(y_i) \le \varepsilon + \xi_i^*, f(y_i) - y_i \le \varepsilon + \xi_i^-$ (5) This optimization problem can be solved by transforming

it into a dual problem:

$$y(x) = \sum_{n=1}^{N} (\alpha_n - \alpha_n^*) K(x, x_n) + b$$
(6)

subject to: $0 \le \alpha_n \le C$, $0 \le \alpha_n^* \le C$ (7)

where α_n and α_n^* are the Lagrange multipliers, $K(x, x_n)$ is a kernel function. The kernel function is used to map the input data to a high dimensional space where it is easier to use linear separation. Obviously, the accuracy of the SVM model depends on C, ε and the kernel parameters. In this study, we consider four common kernels as presented in Table 1.

3.2. SVM-based Procedure to Predict Truss ULF

The SVM-based procedure for predicting the ULF of steel trusses is proposed as follows:

Table 2. Comparison of different kernel ty
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Step 1: Develop dataset

The data (obtained as in Section 2) is divided into two groups of training $(\mathbf{X}_{train}, \mathbf{Y}_{train})$ and testing $(\mathbf{X}_{test}, \mathbf{Y}_{test})$. Data will then be scaled down to the range [0,1] as follows:

$$x_i^{scale} = \frac{x_i}{x_i^{\max}} \tag{8}$$

$$y_i^{scale} = \frac{y_i}{y^{\max}} \tag{9}$$

where x_i^{max} and y^{max} are the maximum values of the input x_i and the output y_i data, respectively.

Step 2: Define a training model

To build an SVM model, four main components need to be defined, including: (1) the loss function, (2) the type of the kernel, (3) the parameters for the kernel, and (4) the value of epsilon (ε). For predicting the ULF of the truss, the common loss function "least squares" is used.

Step 3: Train the SVM model defined in Step 2

The training data is used for training the SVM model, while the test data is used for evaluating the performance of the model.

Kernel	Optimal parameters					MSE	D 2	Adjusted D?	
function	С	γ	r	d	ε	WISE	N2	Aujusteu K-	
Linear	1.0	1.0	n/a	n/a	0.01	0.00213	0.8564	0.8505	
RBF	1.0	0.1	n/a	n/a	0.01	0.00123	0.9177	0.9143	
Sigmoid	10	0.001	1.0	n/a	0.01	0.00211	0.8582	0.8524	
Polynomial	100	1.0	1.0	3.0	0.01	0.00142	0.9042	0.9003	



Fig. 4. Effect of ε on the SVM model using RBF kernel.

The grid-search and k-fold methods (with k=5) are used to find the optimal parameters for the SVM model with different kernel functions. Grid-search is a robust method for hyperparameter tuning of an SVM model by all the possible combinations evaluating of hyperparameters. The k-fold is a cross-validation method where the data is divided into k equal-sized groups, and then k SVM models are trained by using each group as the test data and the remaining groups as the training data. The average performance of k models is considered as the final performance of the SVM model. For developing the SVM model, the programming language Python and the open-source software library Tensorflow are used.

4.1. The Effects of Kernel Functions

In this section, the effects of four kernel functions such as linear, RBF, sigmoid and polynomial on the SVM model are studied. Corresponding to each kernel function, the optimal values for C, γ , r, d, ε and are calculated by using the grid-search method. The value lists of their parameters are [0.1, 1.0, 10, 100, 1000], [0.001, 0.01, 0.1, 1.0], [0.01, 0.1, 1.0, 10, 100], [1, 2, 3, 5, 10], and [0.0001,0.001,0.01,0.1].



Fig. 5. Effect of C on the SVM model using RBF kernel.

4. Results and Discussion

Three factors are used to compare the efficiency of SVM models, including mean-squares-error (MSE), R², and adjusted R². The formulas of these indicators are as follows:

$$MSE = \frac{\sum_{i=1}^{N} (y_i - y_i')^2}{N}$$
(10)

$$R^{2} = 1 - \frac{\sum_{i=1}^{N} (y_{i} - y_{i}')^{2}}{\sum_{i=1}^{N} (y_{i} - \overline{y})^{2}}$$
(11)

$$\overline{R}^{2} = 1 - \frac{\left(1 - R^{2}\right)\left(N - 1\right)}{N - p - 1}$$
(12)

Where y_i and y_i' are true and estimated values, respectively; \overline{y} is the mean value of y_i ; *N* is the number of samples; and, *p* is the number of independent variables. Table 2 presents the obtained results for the considered truss structure, including the optimal parameters for each type of kernel function, MSE, R^2 , and adjusted R^2 . As can be seen from Table 2, RBF has the best performance since its MSE of 0.00123 is the smallest, and its R^2 and adjusted R^2 are the greatest. The second best one is "Polynomial" while "Linear" is the worst one. In light of this, in the next sections, RBF is chosen as the kernel of the considered SVM model.

4.2. The Effects of Parameters of RBF Kernel

The effects of the parameters of the SVM model using the RBF kernel are investigated. To do it, each parameter is changed while others are kept as the optimal values obtained above. The results are presented in Figs. 4, 5, and 6 corresponding to the effect of ε , *C*, and γ . It should be noted that the computing time presented in these figures is for one running time using the k-fold method with k = 5. It is shown that all parameters ε , *C*, and γ have influences on the performance of the SVM model using the RBF kernel. Figure 4 shows that ε smaller than 0.01 is good enough for the SVM model. However, the computing time is slightly increased when ε is decreased.

Figure 5 indicates that the good range for C is [1,10]. In this range, computing time is increased when C increases. Note that C is a regularization parameter for SVM models. A smaller value of C means that the hyperplanes allow more misclassifications. Figure 6 shows that a good range for γ is [0.01, 0.1]. γ affects the

partitioning in the feather space. If it is too small, the model is too constrained and cannot forecast the complexity of the data and the model can be under-fitting. In contrast, if γ is too great the model can be over-fitting. Furthermore, the computing time is significantly increased when γ increases.



Fig. 6. Effect of γ on the SVM model using RBF kernel.

4.3. The Effects of Parameters of RBF Kernel

The performance of the optimal SVM model is presented in this section. The kernel is RBF with C = 1.0, $\gamma = 0.1$, and $\varepsilon = 0.01$. The data with 5,000 samples is divided into 2 data: training data with 4,000 samples and test data with 1,000 samples. One random training process is performed. The results for training data is as MSE = 0.00044, R² = 0.9699, and $\overline{R}^2 = 0.9696$. The results for test data is MSE = 0.00117, R² = 0.9265, and $\overline{R}^2 = 0.9235$. Obviously, the trained model is very good when the value of MSE is small and both R² and \overline{R}^2 are very high.

Figures 7 and 8 present the relationship between the exact and predicted values for training data and test data. In these figures, the fit functions are presented in the form of y = ax+b, where a is defined as the slope and b is the difference. A good model has a ≈ 1 and b ≈ 0 . In light of this, Figs. 7 and 8 show that the trained model is very accurate for training data since a = 0.9634 and b = 0.0815. In addition, the R² of 0.9703 is very high. The results for test data is also very good with a = 0.939, b= 0.1325 and R² = 0.927. Therefore, it can be concluded that SVM is very robust for estimating the ultimate load-carrying capacity of steel trusses using direct analysis.

5. Conclusion

In this paper, an machine learning (ML) framework based on the support vector machine (SVM) algorithm to estimate the ultimate load factor of truss structures via direct analysis was presented for the first time. Four types of kernel functions, including linear, sigmoid, polynomial, radial basis function (RBF) for the SVM model were considered. The work used a 39-bar planar nonlinear inelastic steel truss to study the applicability and effectiveness of the proposed SVM model. The numerical results showed that the accuracy and stability of the SVM model are dependent on the type of kernel used and its parameter values. For the considered structure, the RBF kernel appeared to be the best type for the SVM model to predict the ULF of the truss. Regarding the SVM model using RBF, all parameters ε , C, and γ affected the performance of the SVM model. Moreover, each parameter had a good range for the performance of the SVM model. Particularly, these good ranges were obtained as: (1) ε not greater than 0.01, (2) C in [1,10], and (3) γ in [0.01, 0.1]. Besides, the computation cost was increased when ε decreased, *C*, and γ increased. The aforementioned results could provide a deeper understanding of the performance of the SVM method for predicting the ultimate strength of steel trusses using direct analysis. Further study should consider more

examples, such as complex steel space trusses and frames, and the application of the proposed SVM-based

framework in practical design problems of steel structures, like optimization or reliability assessment.



Fig. 7. Relationship between exact and predicted values of ULF for training data.



Fig. 8. Relationship between exact and predicted values of ULF for test data.

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