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Long-run neutrality of money and inflation in Spanish economy, 1830-1998

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Abstract

In this article, we test a classical model of inflation with rational expectations for the case of Spain during the period 1830–1998. The principal testable implication is that money growth and inflation are cointegrated ruling out speculative bubbles. First, to detect episodes of potential explosive behaviour in the Spanish inflation rate, we use the recursive unit root tests for explosiveness recently proposed by Phillips, Wu, and Yu (2011), and Phillips, Shi, and Yu (2015a,b). Second, we consider the possibility that a linear cointegrated regression model with multiple structural changes would provide a good empirical description of the classical model of inflation for Spain over this long period. Our methodology is based on the instability tests recently proposed in Kejriwal and Perron (2008, 2010) as well as the cointegration tests developed in Arai and Kurozumi (2007) and Kejriwal (2008).

Keywords: Classical model of inflation; Money demand; Money growth; Inflation; Explosiveness; Cointegration; Multiple structural breaks

JEL classification: C22, E31, E41

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1 Introduction

One of the central propositions of monetary theory is monetary models of inflation with forward-looking or rational expectations. Such models impose structural restrictions that are easily evaluated with cointegration models. The solution for the inflation rate resembles the general form of the present value models, as proposed by Campbell and Shiller (1987, 1988a, 1988b). The principal testable implication is that money growth and inflation are cointegrated ruling out speculative bubbles.[1] Specifically, we test for long-run money neutrality which implies that there is an equilibrium relationship between the inflation rate and money growth with a known cointegrating vector $(1,-1)'$.

However, a lack of controls for structural breaks in the series may be reflected in the parameters of the estimated models that can induce misleading results when used for inference or forecasting. In general, structural breaks are a problem for the analysis of economic series, since they are usually affected by either exogenous shocks or changes in policy regimes. As a consequence, the assumption of stability in the long-run relationship between the inflation rate and the money growth would seem too restrictive, such that not allowing for structural breaks would be an important potential shortcoming in the past research that uses cointegration techniques. In our case, the long-run relationship between the inflation rate and money growth has probably changed due to alterations in monetary and fiscal policy, as well as reforms in the financial market. Thus, the information content of the linear classical model of inflation is subject to change over time, and all the empirical modelling studies that have not taken the possible changes and instabilities into account have likely failed to explain the variations in the relationship between the inflation rate and money growth. A visual examination of these variables (see Figures 1 and 2) may allow us to think that the presence of some nonrecurrent shocks with large magnitudes might have affected the evolution of these variables, something that needs to be taken into account when assessing the stochastic properties of time series if meaningful conclusions are to be drawn (see, Perron, 2006).

In this article, we test a classical model of inflation with rational expectations for the case of Spain during the period 1830– 1998 The purpose of this paper is to advance the evidence on the empirical validity of this model in several ways.

First, to detect episodes of potential explosive behaviour in the Spanish inflation rate, we use the recursive unit root tests for explosiveness recently proposed by Phillips, Wu, and Yu (2011), and Phillips, Shi, and Yu (2015a,b). Second, it is well known that misspecifications due to a lack of consideration of structural breaks can bias analyses that are performed using the standard Dickey-Fuller (DF) test statistics for a unit root. Consequently, analyses of the order of integration have to consider the presence of structural breaks. To do this, we first used the GLS-based unit root test statistics proposed in Kim and Perron (2009) and extended in Carrion-i-Silvestre et al. (2009) that allows a

[1] The presence of bubbles has a number of implications. For more details, see Diba and Grossman (1988a, 1988b).

break at an unknown time under both the null and alternative hypotheses. The commonly used tests for unit roots with a structural change in the case of an unknown break date assume that if a break occurs, it does so only under the alternative hypothesis of stationarity. The methodology developed by Kim and Perron (2009) and Carrion-i-Silvestre et al. (2009) solves many of the problems with the standard tests for unit roots with a structural change in the case of an unknown break date.

Finally, in order to control for structural breaks, we make use of recent developments in cointegrated regression models with multiple structural changes. Specifically, we use the approach proposed by Kejriwal and Perron (2008, 2010) to test for multiple structural changes in cointegrated regression models. These authors develop a sequential procedure that not only enables the detection of parameter instability in cointegrated regression models but also allows for consistency in the number of breaks present. Furthermore, we test the cointegrating relationship when multiple regime shifts are identified endogenously. In particular, the nature of the long-run relationship between the inflation rate and money growth is analyzed using the residual based test for the null hypothesis of cointegration with multiple breaks proposed in Arai and Kurozumi (2007) and Kejriwal (2008).

The rest of the paper is organized as follows. A brief description of the underlying theoretical framework is provided in section 2, the methodology and empirical results are presented in sections 3 and 4, respectively, and the main conclusions are summarized in section 5.

2 A classical model of inflation with rational expectations

We use a classical model of inflation with rational expectations, as suggested by Feliz and Welch (1987). The model starts with a version of the Cagan (1956) money demand specification:

$$m_t - p_t = y_t - \alpha i_t + u_t \tag{1}$$

where m_t is the logarithm of the money stock at time t, p_t is the logarithm of the price level at time t, y_t is the logarithm of real output at time t, i_t is the nominal interest rate at time t, and u_t is a zero mean random error term describing a random walk of the form:

$$u_t = u_{t-1} + \psi_t \tag{2}$$

where ψ_t is white noise.

The model assumes a Fisher relationship for the nominal interest rate as follows:

$$i_t = r_t + E[\pi_{t+1} \mid \Phi_{t-k+1}] \tag{3}$$

where $r_t$is the real interest rate, $\pi_{t+1} = p_{t+1} - p_t$ is the logarithmic inflation rate, and Φ_{t-k+1} is the information set at time $t - k + 1$.[2]

The model assumes that real output and the real interest rates follow a random walk with and without drift, respectively:

$$y_t - y_{t-1} = \tilde{y} + \upsilon_{1t} \tag{4}$$

$$r_t - r_{t-1} = \upsilon_{2t} \tag{5}$$

where $\upsilon_{1t}$and υ_{2t} are white noise.

Taking the first differences for equation (1) and combining that with equations (2)-(5), we obtain:

$$\mu_t - \pi_t = \tilde{y} - \alpha \left(E[\pi_{t+1} \mid \Phi_{t-k+1}] - E[\pi_t \mid \Phi_{t-k}] \right) + \varphi_t \tag{6}$$

where μ_t is the logarithmic growth of money and $\varphi_t = \psi_t + \upsilon_{1t} - \alpha \upsilon_{2t}$ is white noise.

Rearranging equation (6), taking expectations on conditional on Φ_{t-k+1}, and solving forward n periods into the future, we obtain the solution to the inflation rate:

$$\begin{aligned} \pi_t \quad &= \quad \mu_t - \tilde{y} + \frac{\alpha}{1+\alpha} \sum_{i=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^i E[\mu_{t+i+1} \mid \Phi_{t-k+1}] \\ -E[\mu_{t+i} \quad &| \quad \Phi_{t-k}]) + \lim_{n \to \infty} \left(\frac{\alpha}{1+\alpha} \right)^n E[\pi_{t+n} \mid \Phi_{t-k+1}] - \varphi_t \end{aligned} \tag{7}$$

For a stable evolution of inflation expectations (and thus the inflation rate) the model imposes the following transversality condition (the "no bubble" condition):

$$\lim_{n \to \infty} \left(\frac{\alpha}{1+\alpha} \right)^n E[\pi_{t+n} \mid \Phi_{t-k+1}] = 0 \tag{8}$$

If equation (8) is satisfied, the no bubbles solution to the inflation rate is:

$$\begin{aligned} \pi_t \quad &= \quad \mu_t - \tilde{y} + \frac{\alpha}{1+\alpha} \sum_{i=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^i E[\mu_{t+i+1} \mid \Phi_{t-k+1}] \\ -E[\mu_{t+1} \quad &| \quad \Phi_{t-k}]) - \varphi_t \end{aligned} \tag{9}$$

Rearranging equation (9) we can obtain a long-run relationship between the inflation rate and money growth:

[2] The model supposes rational expectations, i.e, that individuals use all information available to them to form expectations about inflation rates.

$$
\begin{aligned}
\pi_t - \mu_t \quad &= \quad -\tilde{y} + \frac{\alpha}{1+\alpha}\sum_{i=0}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^i E[\mu_{t+i+1} \mid \Phi_{t-k+1}] \\
-E[\mu_{t+i} \quad &| \quad \Phi_{t-k}]) - \varphi_t \qquad (10)
\end{aligned}
$$

Supposing both the inflation rate and money growth are stationary after first differencing [or I(1)], and with the growth of real output held constant, the left-hand side of (10) is the equilibrium relationship between the inflation rate and money growth with a known cointegrating vector $(1, -1)'$, while the righ-hand side represents the residuals. Further, if the inflation rate and money growth are cointegrated, no bubbles exist.

In the empirical section, we test the classical model of inflation with rational expectations in the context of cointegration theory, using a linear model such as:

$$\pi_t = c_1 + c_2 t + \gamma\mu_t + \varepsilon_t \qquad (11)$$

3 Methodology

3.1 A model of recurrent explosive behaviour and a recursive unit root test for explosiveness

Phillips, Wu and Yu (2011, PWY henceforth) and Phillips, Shi, and Yu (2015a,b, PSY henceforth) developed a new recursive econometric methodology for real-time bubble detection that proved to have a good power against mildly explosive alternatives. The focus of the testing algorithm is whether a particular observation comes from an explosive process (H_A) or a normal martingale behaviour (H_0). The testing algorithm is based on a right-tailed unit root test proposed by Phillips, Shi and Yu (2014).

On the one hand, the martingale null is specified as,

$$H_0 : y_t = kT^{-\eta} + y_{t-1} + \varepsilon_t \qquad (12)$$

with constant k and $\eta > 1/2$, and where y_t is the data series of interest (in our case the inflation rate) at period t, ε_t is the error term, and T is the total sample size.

The hypothesis that the parameter $\delta = 1$ implies that y_t is integrated of order one, i.e., $y_t \sim I(1)$.

On the other hand, the alternative is a mildly explosive process, namely,

$$H_A : y_t = \delta_T y + \varepsilon_t \qquad (13)$$

where $\delta_T = (1 + cT^{-\alpha})$ with $c > 0$ and $\alpha \in (0, 1)$. In this case, if $\delta_T > 1$, it implies the explosive behaviour in y_t over sub-period $t \in [T_1, T_2]$.

The methodology developed in PWY and PSY can be applied to test the unit root hypothesis in the standard model of a sustainable y_t described in (12) against an alternative of multiple subperiods of explosive behaviour $[T_1^{(i)}, T_2^{(i)}], i = 1, 2, ...k, k \geq 1]$, for which the y_t dynamics are described in (13). The sustainable dynamics imply that y_t is a process integrated of order one that is interrupted by recurrent episodes of explosive dynamics. This methodology allows to detect episodes of explosive behaviour.

The testing procedure is developed from a regression model of the form,

$$\Delta y_t = \beta_0 + \beta_1 y_{t-1} + \sum_{i=1}^{K} \lambda_i \Delta y_{t-i} + \varepsilon_t \tag{14}$$

where β_0, β_1, and λ_i are model coefficients, K is the lag order, and ε_t is the error term. The key parameter of interest is β_1. We have $\beta_1 = 0$ under the null and $\beta_1 > 0$ under alternative. The model is estimated by ordinary least squares (OLS) and the t-statistics associated with the estimated β_1 are referred to as the ADF statistic.

First, PWY proposed a sup ADF ($SADF$) statistic to test for the presence of explosive behaviour in a full sample. In particular, the test relies on repeated estimation of the ADF model on a forward expanding sample sequence, and the test is obtained as the sup value of the corresponding ADF statistic sequence. In this case, the window size (fraction) r_w expands from r_0 to 1, where r_0 is the smallest sample window width fraction (which initializes computation of the test statistic) and 1 is the largest window fraction (the total sample size) in the recursion. The starting point r_1 of the sample sequence is fixed at 0, so the endpoint of each sample (r_2) equals r_w and changes from r_0 to 1. The ADF statistic for a sample that runs from 0 to r_2 is denoted by $ADF_0^{r_2}$.

The $SADF$ test is then a sup statistic based on the forward recursive regression and is simply defined as,[3]

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2} \tag{15}$$

Second, PSY developed a double-recursive algorithms that enable bubble detection and consistent estimation of the origination (and termination) dates of bubble expansions while allowing for the presence of multiple structural breaks within the sample period. They show that when the sample includes multiple episodes of exuberance and collapse, the PWY procedures may suffer from reduced power and can be inconsistent, thereby failing to reveal the existence of bubbles. This weakness is a particular drawback in analysing long time series or rapidly changing data in which more than one episode of explosive behaviour is suspected.

To overcome this weakness and deal with multiple breaks caused by exuberance and collapse, PSY proposed the backward sup ADF ($BSADF$) statistic defined as the sup value of the ADF statistics sequence over the interval $[0, r_2 - r_0]$. That is,

[3] This notation highlights the dependence of the $SADF$ on the initialization parameter r_0.

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2} \tag{16}$$

where the endpoint of each subsample is fixed at $T_2 = [r_2 T]$ with $r_2 \in [r_0, 1]$, and at the start point of each subsample, $T_1 = [r_1 T]$ varies from 1 to $T_2 - T_0 + 1(r_1 \in [0, r_2 - r_0])$. The corresponding ADF statistics sequence is $\left\{ADF_{r_1}^{r_2}\right\}_{r_1 \in [0, r_2 - r_0]}$.

PSY also proposed a generalized version of the $\sup ADF$ ($SADF$) test of PWY, based on the sup value of the $BSADF$. That is,

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1]} BSADF_{r_2}(r_0) \tag{17}$$

The statistic (17) is used to test the null of a unit root against the alternative of recurrent explosive behaviour, as the statistic (15).

3.2 A linear cointegrated regression model with multiple structural changes

Issues related to structural change have received a considerable amount of attention in the statistics and econometrics literature. Bai and Perron (1998) and Perron (2006, 2008) provide a comprehensive treatment of the problem of testing for multiple structural changes in linear regression models. Accounting for parameter shifts is crucial in cointegration analysis since such analyses normally involves long spans of data that are more likely to be affected by structural breaks. In particular, Kejriwal and Perron (2008, 2010) provide a comprehensive treatment of the problems of testing for multiple structural changes in cointegrated systems.

More specifically, Kejriwal and Perron (2008, 2010) consider a linear model with m multiple structural changes (i.e., $m + 1$ regimes) such as:

$$y_t = c_j + z'_{ft}\delta_f + z'_{bt}\delta_{bj} + x'_{ft}\beta_f + x'_{bt}\beta_{bj} + u_t \qquad (t = T_{j-1} + 1, ..., T_j) \tag{18}$$

for $j = 1, ..., m + 1$, where $T_0 = 0$, $T_{m+1} = T$ and T is the sample size. In this model, y_t is a scalar dependent I(1) variable, $x_{ft}(p_f \times 1)$ and $x_{bt}(p_b \times 1)$ are vectors of I(0) variables while $z_{ft}(q_f \times 1)$ and $z_{bt}(q_b \times 1)$ are vectors of I(1) variables.[4] The break points $(T_1, ..., T_m)$ are treated as unknowns.

The general model in (18) is a partial structural change model in which the coefficients on only a subset of the regressors are subject to change. In our case, we suppose that $p_f = p_b = q_f = 0$ so that the estimated model is a pure structural change model with all coefficients on the I(1) regressors and constant (slope and the intercept in (11)) allowed to change across regimes:

$$y_t = c_j + z'_{bt}\delta_{bj} + u_t \qquad (t = T_{j-1} + 1, ..., T_j) \tag{19}$$

[4] The subscript b stands for "break" and the subscript f stands for "fixed" (across regimes).

Generally, the assumption of strict exogeneity is too restrictive and therefore the test statistics for testing multiple breaks are not robust to the problem of endogenous regressors. To address the possibility of endogenous I(1) regressors, Kejriwal and Perron (2008, 2010) propose using the so-called dynamic OLS regression (DOLS), in which leads and lags of the first-differences of the I(1) variables are added as regressors, as suggested by Saikkonen (1991) and Stock and Watson (1993):

$$y_t = c_i + z'_{bt}\delta_{bj} + \sum_{j=-l_T}^{l_T} \Delta z'_{bt-j}\Pi_{bj} + u_t^*, \qquad \text{if } T_{i-1} < t \leq T_i \tag{20}$$

for $i = 1, ..., k+1$, where k is the number of breaks, $T_0 = 0$ and $T_{k+1} = T$.

3.3 Structural Break Tests

In this paper we test the parameter instability in cointegrated regressions using the tests proposed in Kejriwal and Perron (2008, 2010). They present issues related to structural changes in cointegrated models that allow for both I(1) and I(0) regressors as well as multiple breaks. They also propose a sequential procedure that permits consistent estimation of the number of breaks, as in Bai and Perron (1998).

Kejriwal and Perron (2010) consider three types of test statistics for testing multiple breaks. First, they propose a sup $Wald$ test of the null hypothesis of no structural break ($m = 0$) versus the alternative hypothesis that there are a fixed (arbitrary) number of breaks ($m = k$):

$$\sup F_T^*(k) = \sup_{\lambda \in \Lambda\varepsilon} \frac{SSR_0 - SSR_k}{\hat{\sigma}^2} \tag{21}$$

where SSR_0 denotes the sum of squared residuals under the null hypothesis of no breaks, SSR_k denotes the sum of squared residuals under the alternative hypothesis of k breaks, $\lambda = \{\lambda_1, ..., \lambda_m\}$ is the vector of breaks fractions defined by $\lambda_i = T_i/T$ for $i = 1, ..., m, T_i$, and T_i are the break dates, and where $\hat{\sigma}^2$ is:

$$\hat{\sigma}^2 = T^{-1}\sum_{t=1}^{T} \tilde{u}_t^2 + 2T^{-1}\sum_{j=1}^{T-1} \varpi(j/\hat{h}) \sum_{t=j+1}^{T} \tilde{u}_t\tilde{u}_{t-j} \tag{22}$$

and $\tilde{u}_t (t = 1, ..., T)$ are the residuals from the model estimated under the null hypothesis of no structural change. Additionally, for some arbitrarily small positive numbers ϵ, $\Lambda_\epsilon = \{\lambda : \mid \lambda_{i+1} - \lambda_i \mid \geq \epsilon, \lambda_1 \geq \epsilon, \lambda_k \leq 1 - \epsilon\}$.

Second, they consider a test of the null hypothesis of no structural break ($m = 0$) versus the alternative hypothesis that there is an unknown number of breaks, given some upper bound $M (1 \leq m \leq M)$:

$$UD\max F_T^*(M) = \max_{1 \leq k \leq m} F_T^*(k) \tag{23}$$

In addition to the tests above, Kejriwal and Perron (2010) consider a sequential test of the null hypothesis of k breaks versus the alternative hypothesis of $k+1$ breaks:

$$SEQ_T(k+1|k) = \max_{1\leq j\leq k+1} \sup_{\tau\in\Lambda_{j,\varepsilon}} T\left\{SSR_T(\hat{T}_1,...,\hat{T}_k\right\} \quad (24)$$

$$-\left\{SSR_T(\hat{T}_1,...\hat{T}_{j-1},\tau,\hat{T}_j,...,\hat{T}_k\right\}/SSR_{k+1} \quad (25)$$

where $\Lambda_{j,\varepsilon} = \left\{\tau : \hat{T}_{j-1} + (\hat{T}_j - \hat{T}_{j-1})\varepsilon \leq \tau \leq \hat{T}_j - (\hat{T}_j - \hat{T}_{j-1})\varepsilon\right\}$. The model with k breaks is obtained by a globally minimizing of the sum of squared residuals, as in Bai and Perron (1998).

3.4 Cointegration tests with structural changes

Kejriwal and Perron (2008, 2010) show that their test can reject the null of no breaks in a purely spurious regression. If anything, their tests have power against spurious regressions. In this sense, tests for breaks in a the long-run relationship are used in conjuction with tests for the presence or absence of cointegration allowing for structural changes in the coefficients.

In this paper, we use the residual-based test for the null of cointegration with an unknown single break against the alternative of no cointegration proposed in Arai and Kurozumi (2007). These authors developed a LM test based on partial sums of residuals in which the break point is obtained by minimizing the sum of squared residuals. They considered three models: i) Model 1, a level shift; ii) Model 2, a level shift with a trend; and iii) Model 3, a regime shift.

The LM test statistic (for one break), $\tilde{V}_1(\hat{\lambda})$, is given by:

$$\tilde{V}_1(\hat{\lambda}) = (T^{-2}\sum_{t=1}^{T} S_t(\hat{\lambda})^2)/\hat{\Omega}_{11} \quad (26)$$

where $\hat{\Omega}_{11}$ is a consistent estimate of the long-run variance of u_t^* in (20), and the dates of the break $\hat{\lambda} = (\hat{T}_1/T,...,\hat{T}_k/T)$ and $(\hat{T}_1,...\hat{T}_k)$ are obtained using the dynamic algorithm proposed in Bai and Perron (2003).

The Arai and Kurozumi (2007) test may be quite restrictive since only a single structural break is considered under the null hypothesis. Hence, the test may tend to reject the null of cointegration when the true data generating process exhibits cointegration with multiple breaks. To avoid this problem, Kejriwal (2008) extends the Arai and Kurozumi (2007) test by incorporating multiple breaks under the null hypothesis of cointegration. The Kejriwal (2008) test for the null of cointegration with multiple structural changes (i.e, with k breaks) is denoted as $\tilde{V}_k(\hat{\lambda})$.

4 Empirical results

In this section we re-examine the issue of a classical model of inflation with rational expectations by using instability tests to account for potential breaks in the long-run relationship between the inflation rate and the money growth as well as by using the cointegration tests with multiple breaks. First, in order to detect episodes of potential explosive behaviour in the Spanish inflation rate, we use the recursive unit root tests for explosiveness recently proposed by PWY and PSY. Second, we use unit root tests to verify that the inflation rate and the money growth are individually integrated of order one. Third, we test the stability of the inflation rate and the money growth relationship (and select the number of breaks) using the test proposed in Kejriwal and Perron (2008, 2010). Next, we verify that the variables are cointegrated with tests for the presence/absence of cointegration allowing for a single or multiple structural changes in the coefficients, as proposed by Arai and Kurozumi (2007) and Kejriwal (2008), respectively. Finally, we estimate the model incorporating the breaks in order to study whether the inflation rate and the money growth relationship (the slope parameter γ) have altered over time.

In our empirical analysis, we use data in the Spanish economy from the period 1830-1998, with 169 annual observations. The time span covered is the longest possible: it begins in 1830, when the first banknotes were put into circulation, and ends in 1998, the year before the peseta was replaced by the euro. The data and sources are: a) consumer index prices from Maluquer de Motes (2013), Table A1.7; b) money supply (M2) from Martín-Aceña (2018), Tables I-6, II-5, IV-4 and V-4.[5] The evolution of the inflation rate, π_t, and money growth, μ_t, appears in Figures 1, 2 and 3, showing a close co-movement between the two series. However, the plots also suggest that the association between π_t and μ_t may have altered over time.

4.1 Explosive dynamics in the inflation rate

The methodology developed in PWY and PSY was originally proposed to test for recurrent explosive behaviour for the U.S.stock market. In this paper, we use the methodology developed in PWY and PSY to examine whether the Spanish inflation rate series exhibits bubble behaviour at any point time over the sample period.

For our empirical application, the lag order K is selected by using the Bayesian information criterion (BIC) with a maximum lag order of 5, as suggested by Campbell and Perron (1991). We set the smallest windows size according to the rule $r_0 = 0.01 + 1.8/\sqrt{T}$ recommended by PSY, giving the minimum length of a sub-sample as 13 years. The origination (termination) of an explosive episode is defined as the first chronological observation for which test statistic exceed (falls below) its corresponding critical value.

Table 1 reports the $SADF$ and $GSADF$ tests for the null hypothesis of a

[5] The data of money supply for the period 1936-1040 has been interpolated.

unit root against the alternative of an explosive root in the Spanish inflation rate. The various critical values for each of the two test are also reported. We conduct a Monte Carlo simulation with 2,000 replications to generate the $SADF$ and $GSADF$ statistics sequences and the corresponding critical values at the 10, 5 and 1 per cent levels. As seen in Table 1, we can not reject the unit root null hypothesis in favour of the explosive alternative at the 1 % significance level for the $SADF$ and $GSADF$ tests. Neither test exceeds its respective 10%, 5% and 1% right-tail critical values, giving any evidence that Spanish inflation rate had not explosive subperiods. Consequently, we conclude from both summary tests that there is no evidence of bubbles.

Next, we conduct a real-time bubble monitoring exercise for the Spanish inflation rate using the PSY strategy. The PSY procedure also has the capability to identify downturns and adjustments in the inflation rate.

To locate the origin and conclusion of the explosive behaviour and the adjustments episodes, Figure 4 plots the profile of the $GSADF$ statistic for the Spanish inflation rate series. We compare the $GSADF$ statistic with the 99% $GSADF$ critical value for each observation of interest. The initial start-up sample for the recursive regression covers the period 1831-1854 (14% of the full sample). Figure 4 identifies episodes of explosive inflation rate behaviour, and it permits us to date-stamp their origination and termination, as well as the potential adjustments. Next, we also conduct a real-time bubble monitoring exercise for the Spanish inflation rates using the PWY strategy. Figure 5 plots the $SADF$ test statistics against the corresponding 99% critical value sequence.

According to Figures 4 and 5, there is no speculative bubble behaviour in Spanish inflation rate series over the period 1831-1998.

4.2 Stationarity of the time series

The first step in our analysis is to examine the time series properties of the series by testing for a unit root over the full sample. Trend breaks appear to be prevalent in macroeconomic time series, and unit root tests therefore need to make allowances for these breaks if they are to avoid the serious effects that unmodelled trend breaks have on power.[6] In a seminal paper, Perron (1989) shows that failure to account for trend breaks present in the data results in unit root tests with zero power, even asymtotically. Consequently, when testing for a unit root, allowing for this kind of deterministic structural change has to become a matter of regular practice. To avoid this pitfall, we run tests to assess whether structural breaks are present or not in π_t and μ_t series.

We have used the GLS-based unit root tests with multiple structural breaks under both the null and the alternative hypotheses proposed in Carrion-i-Silvestre et al. (2009). The commonly used tests for unit root with a structural change in the case of an unknown break date (Zivot and Andrews (1992), Perron (1997), Vogelsang and Perron (1998), Perron and Vogelsang (1992a, 1992b)), assume that if a break occurs, it does so only under the alternative hypothesis of sta-

[6] See, *inter alia*, Stock and Watson (1996, 1999, 2005) and Perron and Zhu (2005).

tionarity. The methodology developed by Carrión-i-Silvestre et al. (2009) solves many of the topical problems in standard unit root tests with a structural change in the case of an unknown break date.[7] Carrion-i-Silvestre et al. (2009) consider the modified unit root tests (M-class tests) analysed by Stock (1999), Perron and Ng (1996) and Ng and Perron (2001), and the P_T^{GLS}, MP_T^{GLS}, MZ_α^{GLS}, MSB^{GLS} and MZ_t^{GLS} tests.

The results of applying the Carrion-i-Silvestre-Kim-Perron tests to Model 0 are shown in Table 2, allowing for up to one or two breaks, respectively. As Table 2 shows, the null hypothesis of a unit root with one or two structural breaks that affects the level (intercept) of the times series cannot be rejected by any of the tests at the 1% level of significance. [8] Consequently, we can conclude that the π_t and μ_t variables could are $I(1)$ with one single or two different structural breaks.

4.3 Long-run relationship

Once the order of integration of the series has been analysed, we estimate the long-run or cointegration relationship between π_t and μ_t.

If there is cointegration in the demeaned specification given in (11), such cointegration would occur when $c_2 = 0$, which corresponds to deterministic cointegration and implies that the same cointegrating vector eliminates both the deterministic and stochastic trends. However, if the linear stationary combinations of $I(1)$ variables have nonzero linear trends (which occurs when $\Phi \neq 0$), as given in (11), this would correspond to a stochastic cointegration.[9] In both cases, the parameter γ is the estimated long-run cointegrating coefficient between π_t and μ_t.

First, we estimate and test the coefficients of the cointegration equation by means of the dynamic ordinary least squares (DOLS) method of Saikkonen (1991) and Stock and Watson (1993) and following the methodology proposed by Shin (1994). This estimation method provides a robust correction to the possible presence of endogeneity in the explanatory variables, as well as serial correlation in the error terms of the OLS estimation. Additionally, to overcome the problem of the low power in classical cointegration tests in the presence of persistent roots in the residuals from the cointegration regression, Shin (1994) suggests a new test in which the null hypothesis is that of cointegration. Therefore, in the first place, we estimate a long-run dynamic equation that includes the leads and lags of all the explanatory variables, i.e., the so-called DOLS regression:

$$\pi_t = c + \Phi t + \gamma \mu_t + \sum_{j=-q}^{q} \gamma_j \Delta \mu_{t-j} + \upsilon_t \tag{27}$$

[7] See Carrión-i-Silvestre et al. (2009) for more details.

[8] The critical values were obtained from simulations using 1,000 steps to approximate the Wiener process and 10,000 replications.

[9] See Ogaki and Park (1997) and Campbell and Perron (1991) for an extensive study of deterministic and stochastic cointegration.

The coefficient from the DOLS regression and the results of the Shin test are reported in Table 3. The null of deterministic cointegration between π_t and μ_t is not rejected at the 1% level, with an estimated value for γ of 0.71. However, this estimate is significantly different from one at the 1% level according to a Wald test on the null hypothesis $\hat{\gamma} = 1$, following a χ_1^2 distribution and denoted as W_{DOLS} in Table 3. Thus, the cointegration vector is not (1, -1) as predicted by the theory. Overall, the results of the estimated value for γ using DOLS method imply that a 10 percentage-point increase in money growth is associated with 7.1 percentage-point higher inflation rate in the full sample. This suggests the presence of a partial effect in the long-run in the sense that the inflation rate was not adjusted to be fully compensated for higher money growth.

Accounting for parameter shifts is crucial in cointegration analysis since this type of analysis normally involves long spans of data, which are more likely to be affected by structural breaks. In particular, our data covers one hundred and sixty-eight years of the history of the series, and during that period of time, the long-run relationship between the inflation rate and money growth has probably changed due to alterations in monetary and fiscal policy, as well as reforms in the financial market. Thus, the information content of the linear classical model of inflation with rational expectations is subject to change over time, and all the empirical modelling studies that have not taken the possible changes and instabilities into account have likely failed to explain the variations in the relationship between the inflation rate and money growth. Therefore, as we argued before, it is very important to allow for structural breaks in our cointegration relationship.

We now consider the tests for structural changes that are proposed in Kejriwal and Perron (2008, 2010). Since we have used a 20% trimming, the maximum numbers of breaks we may have under the alternative hypothesis is 3. Given the span of the data, it seems unreasonable to expect the occurrence of two or more breaks. Moreover, the intercept and the slope in equation (27) are permitted to change. Table 4 presents the results of the stability tests as well as the number of breaks selected by the sequential procedure (SP) and the BIC and LWZ proposed by Bai and Perron (2003). The four test statistic results do suggest instability at 5% level of significance. Further, the SP and LWZ results do no suggest any instability, although the BIC selects two breaks, which provides evidence against the stability of the long-run relationship. Overall, the results of the Kejriwal-Perron tests suggest a cointegrated model with two breaks estimated at 1915 and 1947 and three regimes, 1831-1914, 1915-1946 and 1947-1998.[10]

As can be seen in Figure 2, the first break, dated in 1915, coincides with the start of the First War World, a period when the acceleration of money growth was exceptionally high, especially in 1917 (21.9%) and 1918 (37.9%). During the years of this war, the Spanish balance of payments benefited from

[10] The two breaks estimated at 1915 and 1947 are similar to those reported in Riera i Brunera and Blasco-Martel (1996) in the context of estimating a money demand function for Spain over the period 1883-1998.

neutrality, accounting for more than two-thirds of the expansion in liquidity.[11] The second break, dates in 1947 (see Figure 2), coincides with the start of monetary financing of public deficits (seigniorage). Since 1947 until 1959, the monetary base expanded at a much higher rate than in the previous years, with the exception of the war periods. Furthermore, most of the monetary expansion was due to the financing of the public deficits, so the policy regime can be qualified as "fiscal-policy dominant".[12]

Since the above reported stability tests also reject the null coefficient of stability when the regression is a spurious, we still need to confirm the presence of cointegration among the variables. With that end in mind, we use the residual based test of the null of cointegration against the alternative of cointegration with unknown multiple breaks proposed in Kejriwal (2008), $\tilde{V}_k(\hat{\lambda})$.

Arai and Kurozumi (2007) show that the limit distribution of the test statistic, $\tilde{V}_k(\hat{\lambda})$, depends only on the timing of the estimated break fraction $\hat{\lambda}$ and the number of I(1) regressors m.[13] Since we are interested in the stability of the inflation rate-money growth coefficient, γ, we only consider model 3, which permits a slope shift as well as a level shift. Table 5 shows the results of the Arai-Kurozumi-Kejriwal cointegration tests allowing for two breaks. As before, the level of trimming used is 15%. As a result, we find that test $\tilde{V}_2(\hat{\lambda})$ cannot reject the null of cointegration with two structural breaks at 1% level of significance. Therefore, we conclude that π_t and μ_t are cointegrated with two structural changes estimated at 1915 and 1947.

To compare the coefficients obtained from the break models with those reported from models without any structural break, we estimate the cointegration equation (27) with a two-breaks model. The results with the subsamples are presented in the last three columns of Table 3. The null of the deterministic cointegration between π_t and μ_t is not rejected at the 5% level of significance in the three regimes. In the first regime, ocurring in 1831-1914, the estimated cointegrating coefficient is positive and significant, indicating that a 10 percentage-point increase in money growth is associated with 2.4 percentage-point higher of inflation rate. This value is less than half of the estimate in the full sample (7.1 percentage point). This suggests the presence of a partial effect in the long-run in the sense that the inflation rate was not adjusted to be fully compensated for higher money growth.

For the second regime, dated between 1915-1946, the coefficient is positive and significant, indicating that a 10 percentage-point increase in money growth is associated with almost 10 percentage-point higher of inflation rate. Moreover, this estimate is not be significantly different from one at the 1% level, according to a Wald test on the null hypothesis $\hat{\gamma} = 1$. In this case the cointegration vector is (1, -1), as predicted by the theory. This implies the presence of a full

[11] For more details, see Martín-Aceña (2018).

[12] For more details, see Escario, Gadea and Sabaté (2012), and Bajo-Rubio, Díaz-Roldán and Esteve (2014).

[13] In our case, the critical values for the test are then simulated for the corresponding break fractions using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of *i.i.d.* $N(0, 1)$ random variables.

effect in the long-run in the sense that the inflation rate was adjusted to be fully compensated for higher money growth.

In the last episode detected (1947-1998), the coefficient is positive and significant, indicating that a 10 percentage-point increase in money growth is associated with 7.9 percentage-point higher of inflation rate. This indicates the presence of a partial effect in the long-run in the sense that the inflation was not been adjusted to be fully compensated for higher money growth.

Overall, the results suggest that ignoring structural changes in the long-run cointegration relationships may understate the extend of the correlation between the inflation rate, π_t, and money growth, μ_t, since the response of the present value of inflation to a change in money growth changes over time. Our results for the full sample support the existence of a partial effect in the long-run in the sense that the inflation rate was not adjusted to be fully compensated for higher money growth. Only in the second regime (1915–1946) was the inflation rate adjusted to be fully compensated for higher money growth.

5 Conclusions

In this article, we test a classical model of inflation with rational expectations for the case of Spain during the period 1830–1998. The principal testable implication is that money growth and inflation are cointegrated, ruling out speculative bubbles. Specifically, we test for long-run money neutrality, which implies that there is a equilibrium relationship between the inflation rate and money growth with a known cointegrating vector $(1, -1)'$.

First, to detect episodes of potential explosive behaviour in the Spanish inflation rate, we use the recursive unit root tests for explosiveness recently proposed by Phillips, Wu, and Yu (2011), and Phillips, Shi, and Yu (2015a,b). Second, we consider the possibility that a linear cointegrated regression model with multiple structural changes provides a good empirical description of the classical model of inflation for Spain over this long period. Our methodology is based on the instability tests recently proposed in Kejriwal and Perron (2008, 2010) as well as the cointegration tests developed in Arai and Kurozumi (2007) and Kejriwal (2008).

Second, we find that there is not speculative bubble behaviour in the Spanish inflation rate series. Third, the results obtained in our study are consistent with the existence of linear cointegration between the inflation rates and money growth series, with a vector (1, -0.71). Thus, the cointegration vector is not (1, -1), as predicted by the theory. Finally, the results suggest a cointegration model with two breaks estimated at 1915 and 1947 and three regimes, 1831-1914, 1915-1946 and 1947-1998.

Overall, the results suggest that ignoring structural changes in the long-run cointegration relationships may understate the extend of correlation between the inflation rate and money growth, since the response of the present value of inflation to a change in money growth changes over time. Our results for the full sample support the existence of a partial effect in the long-run in the sense that the inflation rate was not adjusted to be fully compensated for higher money growth.

Only in the second regime (1915–1946) was the inflation rate adjusted to be fully compensated for higher money growth. Furthermore, most of the monetary expansion was due to the financing of the public deficits, so the policy regime can be qualified as "fiscal-policy dominant".

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Table 1
Tests for explosive behaviour in the Spanish inflation rate, π_t, from 1831 to 1998

Unit root tests	Estimated Value	Finite Critical Value		
		1%	5%	10%
$SADF$	-6.011	1.887	1.300	1.033
$GSADF$	0.282	2.740	2.085	1.789

Notes:
*, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively.

Table 2

M unit root tests with multiple structural breaks from Carrion-i-Silvestre et al. (2009) [a,b,c]

Variable	Model	P_T^{GLS}	MZ_t^{GLS}	MZ_α^{GLS}	MP_T^{GLS}	MSB^{GLS}
π_t	0 ($\hat{T}_1$)	68.636	-0.362	-0.662	63.384	0.547
μ_t	0 ($\hat{T}_1$)	69.121	-0.302	-0.554	63.995	0.546
π_t	0 ($\hat{T}_1$,$\hat{T}_2$)	81.295	-0.365	-0.610	75.324	0.598
μ_t	0 ($\hat{T}_1$,$\hat{T}_2$)	46.931	-0.378	-0.907	41.543	0.416

Notes:

[a] A ** denotes significance at the 5% level.

[b] Structural breaks affect the intercept (Model 0: level shift or "crash"). $\hat{T}$ numbers of breaks.

[c] The critical values were obtained from simulations using 1,000 steps to approximate the Wiener process and 10,000 replications.

Table 3
Estimation of long-run relationships: Stock-Watson-Shin cointegration tests[a,b,c,d]

Parameter estimates	Model without structural breaks	Two-breaks model		
	Full sample 1831-1998	First regime 1831-1914	Second regime 1915-1946	Third regime 1947-1998
γ	0.71	0.24	0.99	0.79
	(7.26)	(1.96)	(2.87)	(2.45)
Tests:				
C_μ	0.048	0.033	0.050	0.051
W_{DOLS}	8.48*	33.77*	0.003	0.40

Notes:

[a] t-statistics are in brackets. Standard Errors are adjusted for long-run variance. The long-run variance of the cointegrating regression residual is estimated using the Barlett window which is approximately equal to $INT\left(T^{1/2}\right)$ as proposed in Newey and West (1987).

[b] We choose $q = INT\left(T^{1/3}\right)$ as proposed in Stock and Watson (1993).

[c] C_μ is LM statistics for cointegration using the DOLS residuals from deterministic cointegration, as proposed in Shin (1994). *, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively. The critical values are taken from Shin (1994), table 1, from $m = 1$.

[d] W_{DOLS} is a Wald test on the null hypothesis $\hat{\gamma} = 1$, distributed as a χ^2_1. *, **, and *** denote significance at the 1%, 5%, and 10% levels, respectively.

Table 4
Kejriwal-Perron tests for testing multiple structural breaks in cointegrated regression models: equations (20) and (27)[a,b,c]

	Specifications[a]		
$y_t = \{\pi_t\}$	$z_t = \{1, \mu_t\}$	$x_t = \{\emptyset\}$	$M = 3$
	$q = 2$	$p = 0$	$h = 32$
	Tests[b]		
$\sup F_T(1)$	$\sup F_T(2)$	$\sup F_T(3)$	$UD\max$
9.10**	8.38**	7.13**	9.10**
	Number of Breaks Selected	Breaks	
		$\hat{T}_1$	$\hat{T}_2$
SP	0	—	—
LWZ	0	—	—
BIC	2	1915	1947

Notes:

[a] y_t, z_t, q, p, h, and M denote the dependent variable, the regressors, the number of $I(1)$ variables (and the intercept) allowed to change across regimes, the number of $I(0)$ variables, the minimum number of observations in each segment, and the maximum number of breaks, respectively.

[b] *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

[c] The critical values are taken from Kejriwal and Perron (2010), Table 1.10 (critical values are available on Pierre Perron's Web site), non-trending case with $q_b = 1$.

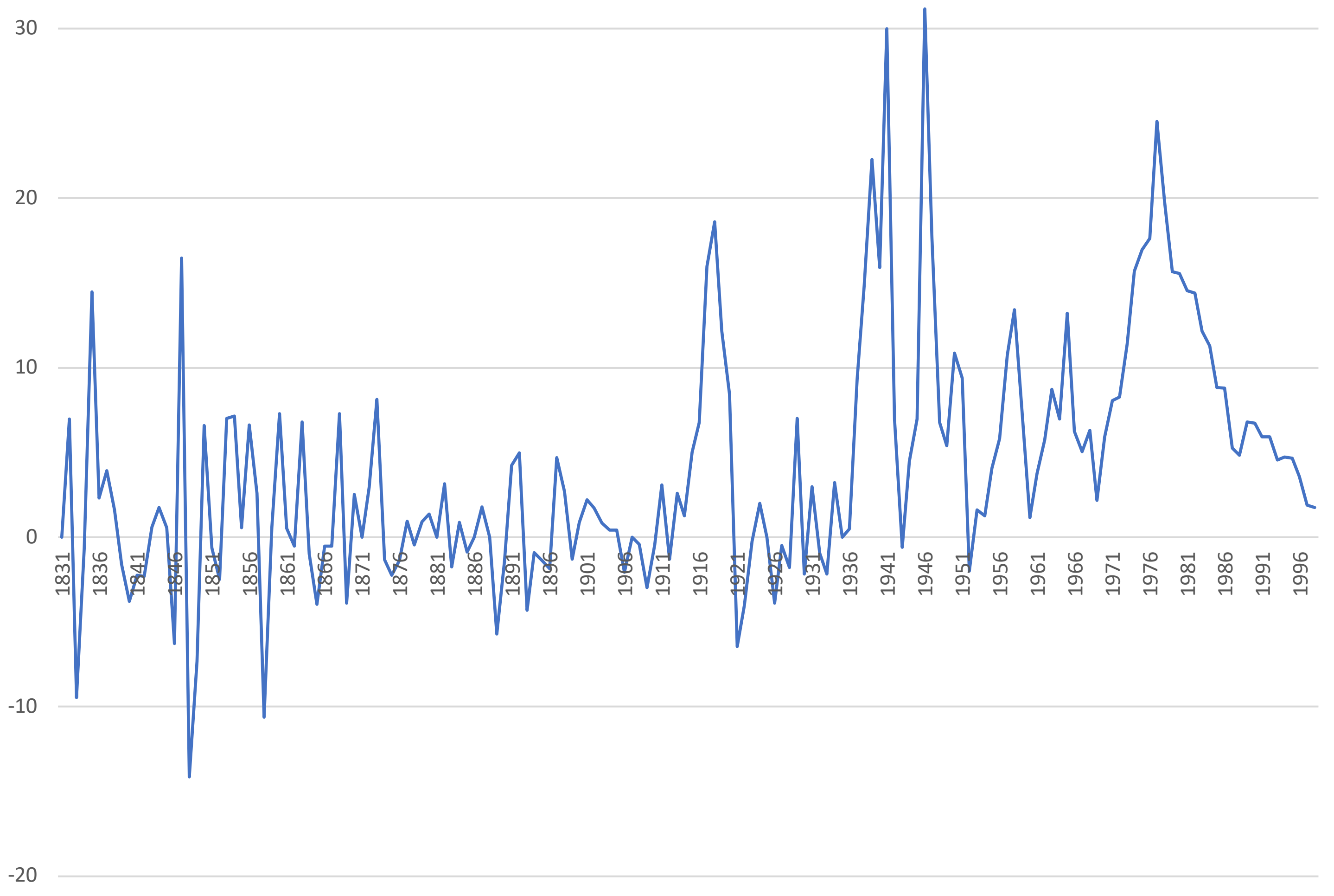
Table 5
Arai-Kurozumi-Kejriwal cointegration tests with multiple structural breaks: equations (20) and (27)[a,b]

Two-breaks model				
Test $\tilde{V}_2(\hat{\lambda})$	$\hat{\lambda}_1$	$\hat{T}_1$	$\hat{\lambda}_2$	$\hat{T}_2$
0.083***	0.51	1915	0.70	1947

Notes:
[a] *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.
[b] Critical values are obtained from simulations111 using 500 steps and 2000 replications. The Wiener processes are approximated by partial sums of *i.i.d.* $N(0,1)$ random variables.

Critical values:	10%	5%	1%
$\tilde{V}_2(\hat{\lambda})$	0.075	0.097	0.152

Figure 1. Spain: inflation rate (%), 1851-1998



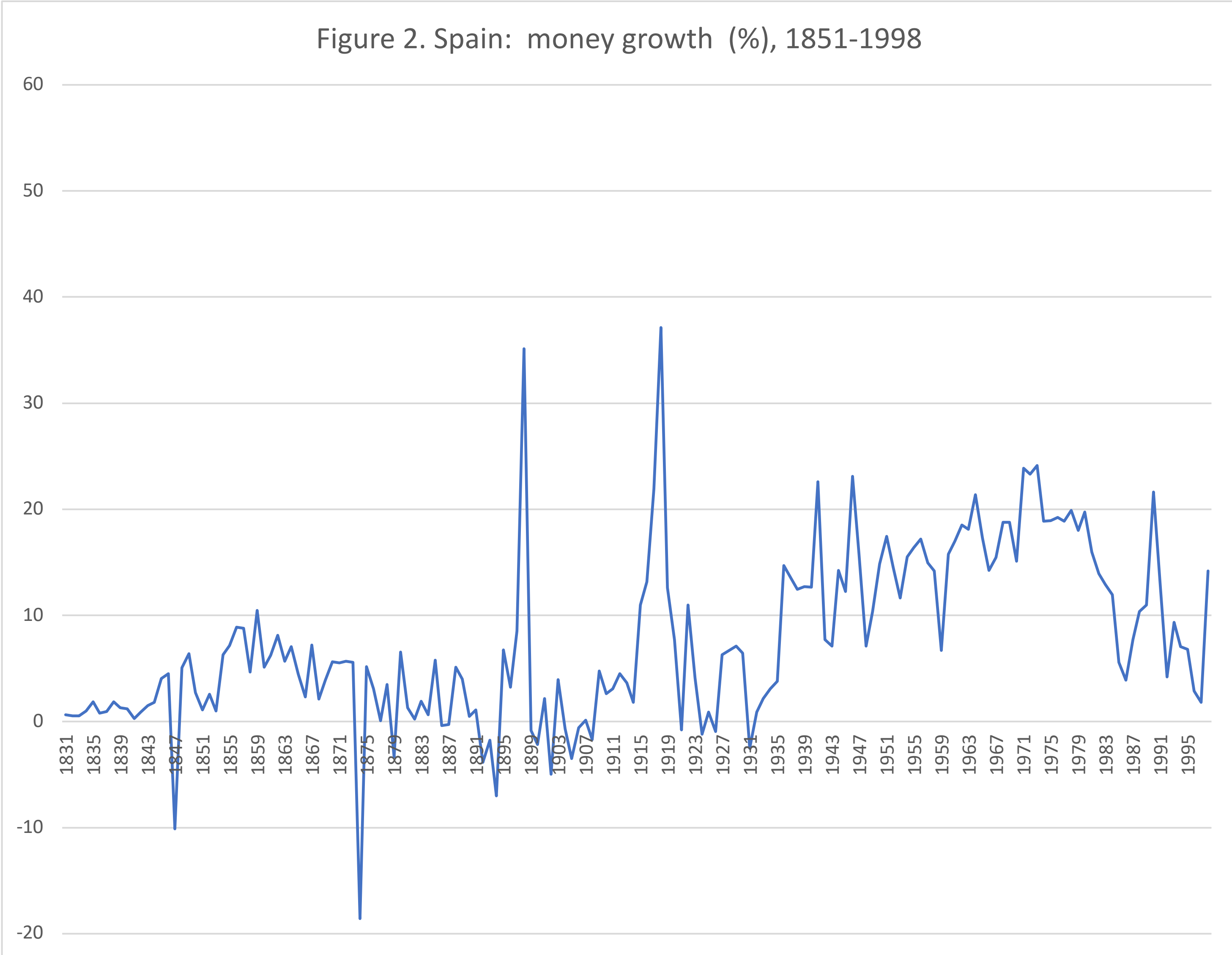


Figure 2. Spain: money growth (%), 1851-1998
60
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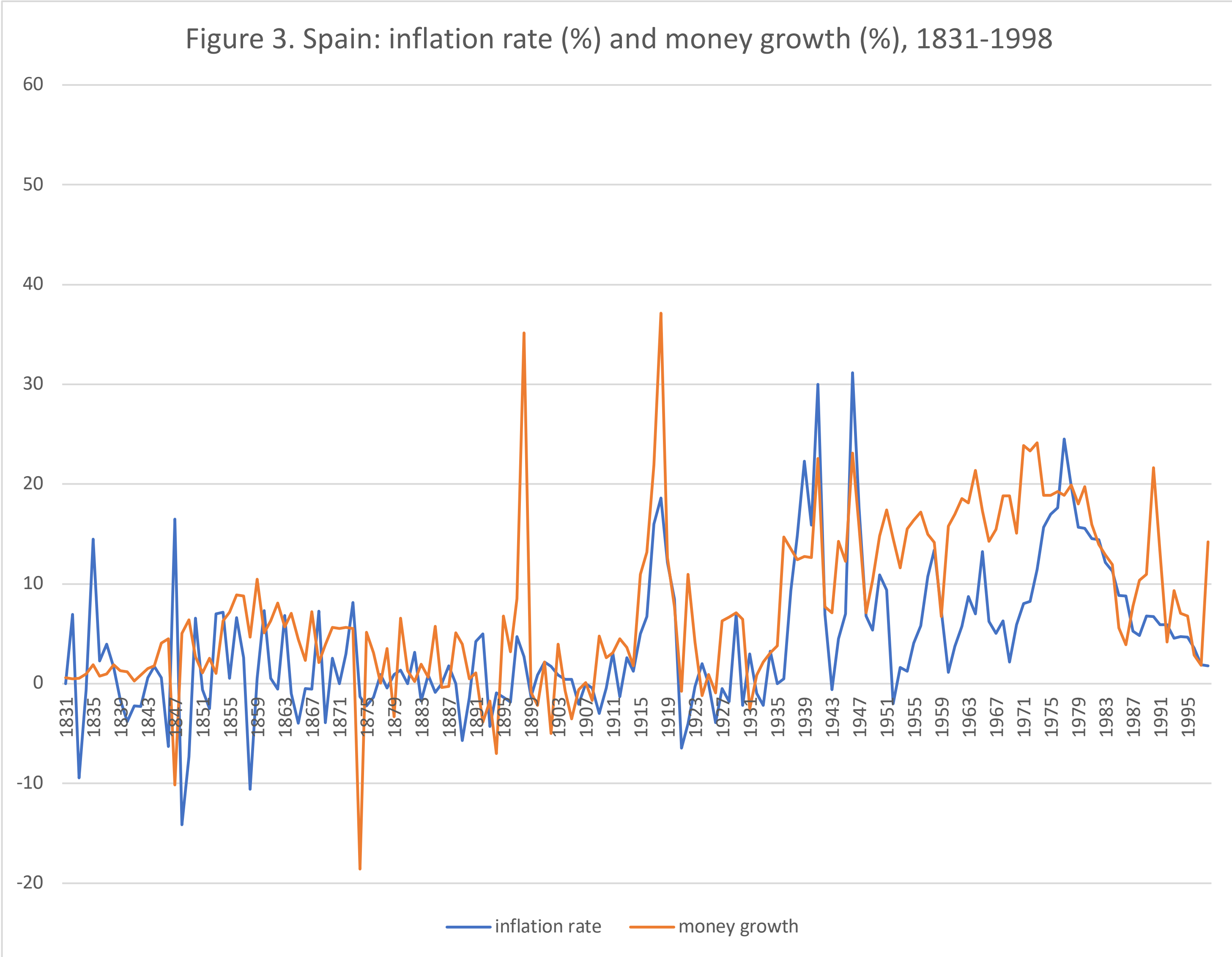

Figure 3. Spain: inflation rate (%) and money growth (%), 1831-1998
60
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40
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20
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1851
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1995
inflation rate
money growth

Figure 4
Date-stamping bubble periods in the Spanish inflation rate:
The GSADF test

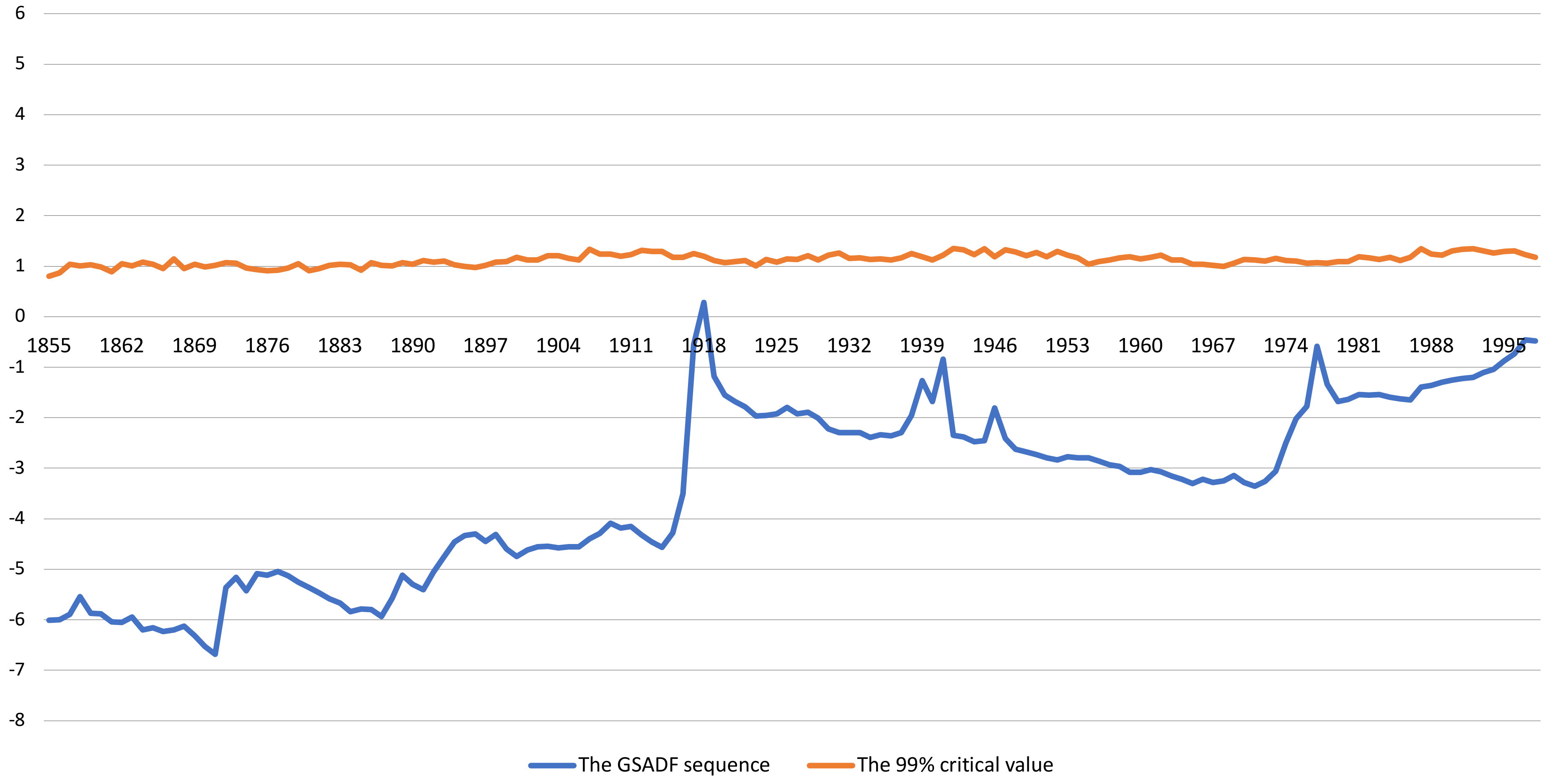
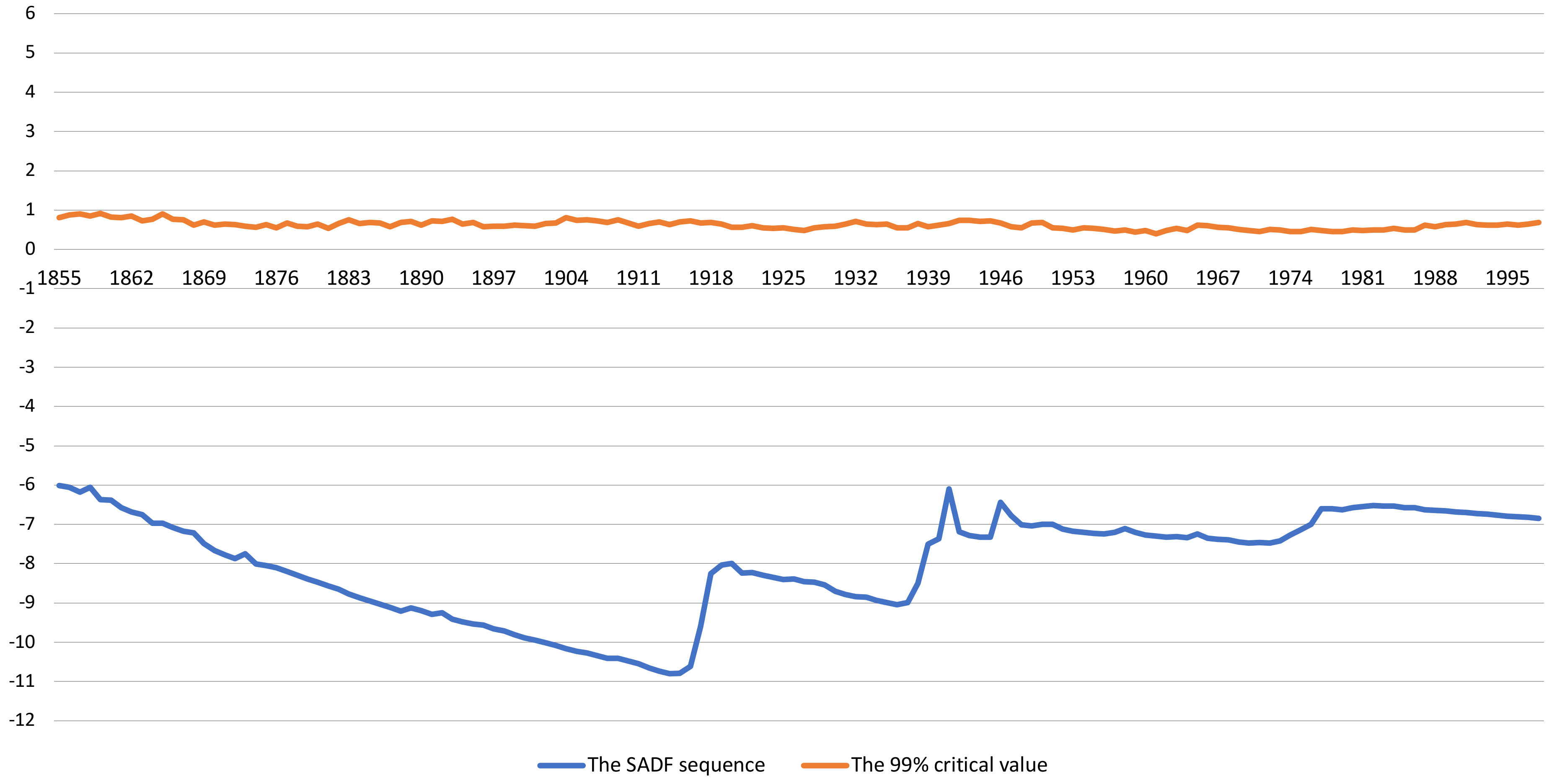


Figure 5
Date-stamping bubble periods in the Spanish inflation rate:
The SADF test



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