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What are graphical models?

Sara Geneletti

Department of Epidemiology and Public Health, Imperial College

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Outline

1. Introduction
2. What is a DAG?
3. What can it do?
4. What does it mean?
5. Heuristic tool
6. Formal tool
7. Causality
8. Useful info

Introduction

Uses

- Physics
- Genetics
- Psychology - Path analysis, Structural equation models
- Statistics
- Causal inference

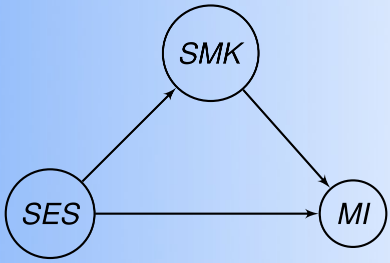
Types

- Directed
- Directed Acyclic
- Unidirected
- Chain graphs

What is a DAG?

DAGs are *directed acyclic graphs*

- All arrows have direction
- No cycles $A \rightarrow B \rightarrow A$
- Arrows are *not* causal unless extra assumptions made - time ordering, intervention



What does it do?

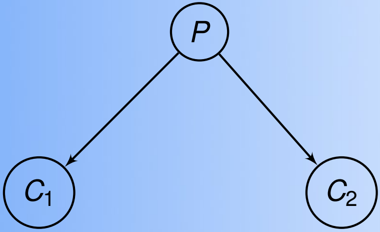
DAGs are used to encode conditional independence statements

- In words if we know about C, knowing about A gives us no extra clues about B (and vice-versa)
- Formally, we write $A \perp\!\!\!\perp C | B$ [1]
- which means $p(A, C|B) = p(A|B)p(C|B)$
- Although DAGs have arrows, they DO NOT automatically mean causal relationships
- rather an arrow means dependence/association and lack of an arrow means independence/no association

Simple example - inheritance

1. Two children are siblings
2. If you know the DNA of one, you know something about the DNA of the other
3. they are associated

Simple example - inheritance



1. Two children are siblings
2. If you know the DNA of one, you know something about the DNA of the other
3. they are associated
4. If you know their parents' DNA however
5. knowing about one child tells you nothing new about the other
6. they are independent GIVEN the parents

Qualitative approach

- DAGs can be constructed to make sense of a particular set of relationships
- Make it easier for - qualitative and quantitative researchers to understand one another
- Pictorial representation can highlight uncertainty and bias

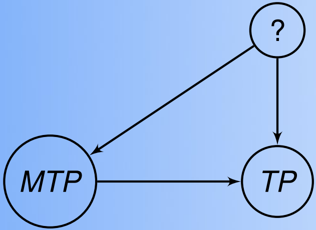
Caveats

- a DAG that expresses assumptions about relationships (i.e. pre-data analysis) does not necessarily correspond to reality
- Putative associations/causal relations need to be tested against data where possible and assessed carefully

Constructing a DAG

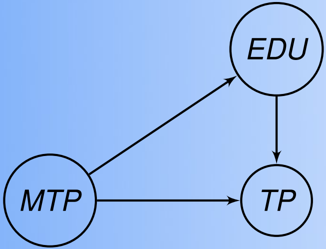
- A teenager whose mother had children as a teenager is more likely to have children herself

Constructing a DAG



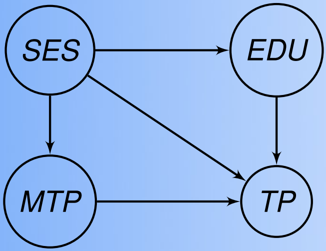
- A teenager whose mother had children as a teenager is more likely to have children herself
- BUT there are factors that influence both these events

Constructing a DAG



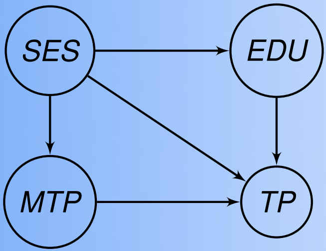
- A teenager whose mother had children as a teenager is more likely to have children herself
- BUT there are factors that influence both these events
- Education (full-time vs school leaver) is one of these
- But surely that is influenced in its own way by?? Anyone?

Constructing a DAG



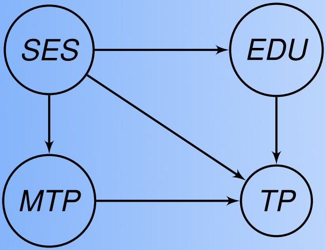
- A teenager whose mother had children as a teenager is more likely to have children herself
- BUT there are factors that influence both these events
- Education (full-time vs school leaver) is one of these
- But surely that is influenced in its own way by?? Anyone?
- SES

Constructing a DAG



- This is a simple example - could add
  - Ethnicity
  - Low-self esteem
  - Substance abuse
  - history of violence
- Some of these could be unobserved or reported with bias
- e.g. low-self esteem or substance abuse

Incorporating data

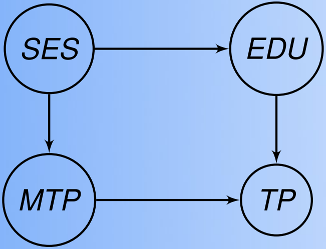


$$
\begin{aligned}
Pr(TP|MTP) &= \\
\sum_{SES,EDU} & Pr(TP|MTP, SES, EDU) Pr(EDU|SES) Pr(MTP|SES) Pr(SES)
\end{aligned}
$$

can use frequencies from contingency tables to estimate $Pr(TP|MTP)$ and Odds Ratio

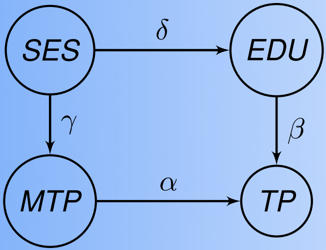
- The graph tells us how to factorise the distribution of variables into smaller simple parts
- Helps to estimate using a modular approach - see later

Incorporating data



- We can do a path analysis [2] by assuming linear relationships between the variables
- For example, if we think that the influence of SES on TP is mediated only by MTP and EDU
- i.e. $TP \perp\!\!\!\perp SES | (MTP, EDU)$ then

Incorporating data



- We can do a path analysis [2] by assuming linear relationships between the variables
- For example, if we think that the influence of SES on TP is mediated only by MTP and EDU
- i.e. $TP \perp\!\!\!\perp SES | (MTP, EDU)$ then
- $TP = \alpha MTP + \beta EDU$
- $MTP = \gamma SES$ and $EDU = \delta SES$

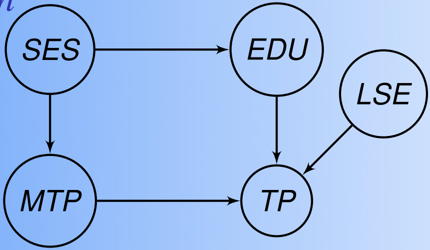
Does the DAG correspond to reality?

True?

- So you have a DAG that represents your belief about the relationships
- Does it fit with observed data?

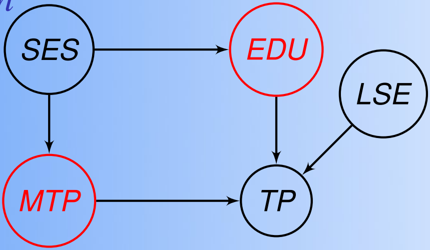
1. What conditional independences does DAG encode?
2. Moralisation criteria (see next slide)
3. Use e.g. χ^2 or Mantel-Haenszel test (or Bayesian network software) to determine if true in data
4. Regressions - if adding a variable to reg makes no difference to the outcome - maybe there is no dependence (not 100%).

Moralisation



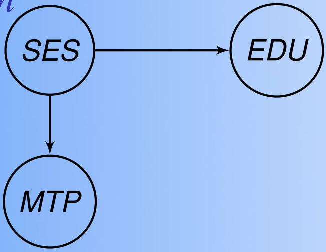
- Say you care about relationship between EDU and MTP

Moralisation



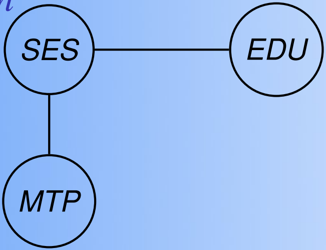
- Say you care about relationship between EDU and MTP
- Exclude all variables that are not ancestors of EDU and MTP -only SES here

Moralisation



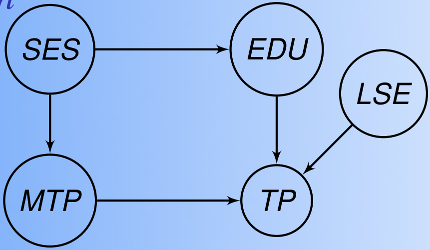
- Say you care about relationship between EDU and MTP
- Exclude all variables that are not ancestors of EDU and MTP -only SES here
- Join (marry - hence moralise) parents of common children (none here)
- remove direction from arrows

Moralisation



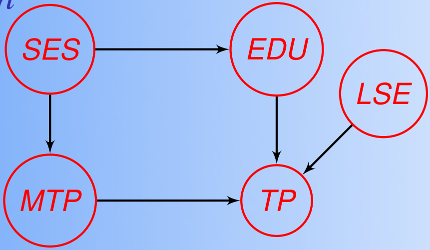
- Say you care about relationship between EDU and MTP
- Exclude all variables that are not ancestors of EDU and MTP -only SES here
- Join (marry - hence moralise) parents of common children (none here)
- remove direction from arrows
- all paths from EDU and MTP go through SES - $MTP \perp\!\!\!\perp EDU \mid SES$
- i.e. mother being a teen mum is only associated to daughter's education via SES - makes sense?

Moralisation



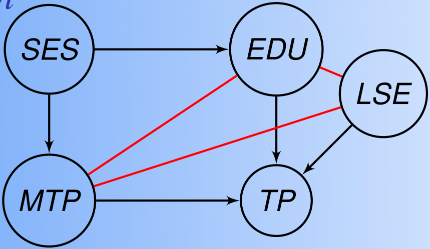
- Say you care about relationship between TP and SES

Moralisation



- Say you care about relationship between TP and SES
- Exclude all variables that are not ancestors of EDU and MTP - all variables are ancestors of TP

Moralisation



- Say you care about relationship between TP and SES
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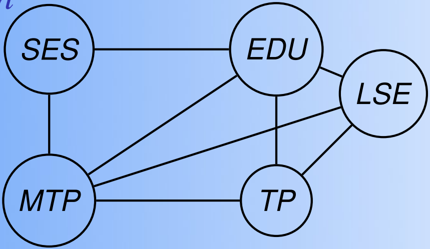
Moralisation



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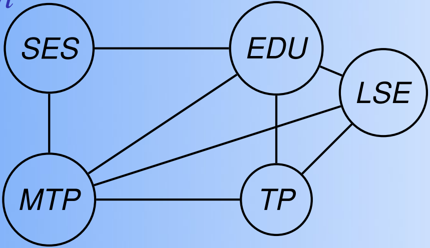

- Say you care about relationship between TP and SES
- Exclude all variables that are not ancestors of EDU and MTP - all variables are ancestors of TP
- Join parents of common children
- remove direction from arrows
- all paths from SES and TP go through EDU and MTP - $TP \perp\!\!\!\perp SES | (MTP, EDU)$
- i.e. being a teen mum is only associated to SES via mother's teen mum status and education - not plausible, need more confounders!

Getting DAGs from data

Data mining

- There are various methods for extracting DAGs from data
- Most ask what the conditional independences are between variables (using e.g. χ^2 tests) and construct a series of DAGs
- There are also loads of computer programmes that take data and turn it into DAGs

Simple example

Political affiliation (PA), abuse as a child (AC) and abusive parent (AP) [3]

Contingency table

Obs		PA			
AC	AP	l	s	r	tot
1	1	**12**	**27**	**58**	
	0	**7**	**28**	**30**	
0	1	**9**	**5**	**9**	
	0	**19**	**15**	**18**	
	tot				

.

Simple example

Political affiliation (PA), abuse as a child (AC) and abusive parent (AP)

Contingency table

Obs		PA			
AC	AP	l	s	r	tot
1	1	**12**	**27**	**58**	97
	0	**7**	**28**	**30**	65
		19	55	88	162
0	1	**9**	**5**	**9**	23
	0	**19**	**15**	**18**	52
	tot	28	20	27	75

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Simple example

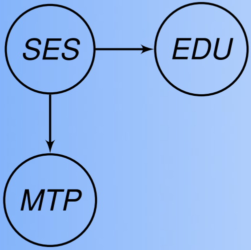
Political affiliation (PA), abuse as a child (AC) and abusive parent (AP)

Contingency table

Obs		PA				Exp		PA			
AC	AP	l	s	r	tot	AC	AP	l	s	r	tot
1	1	**12**	**27**	**58**	97	1	1	**12**	**33**	**53**	97
	0	**7**	**28**	**30**	**65**		0	**8**	**22**	**35**	65
		19	55	88	162			19	55	88	162
0	1	**9**	**5**	**9**	23	0	1	**9**	**6**	**8**	23
	0	**19**	**15**	**18**	52		0	**19**	**14**	**19**	52
	tot	28	20	27	75		tot	28	20	27	75

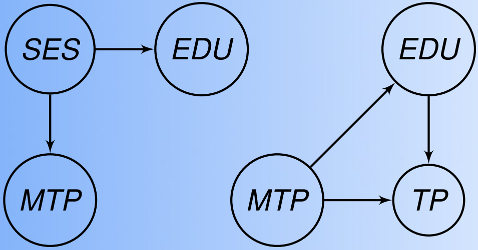
The two tables are very similar and “say” that $PA \perp\!\!\!\perp AP | AC$

DAGs are modular



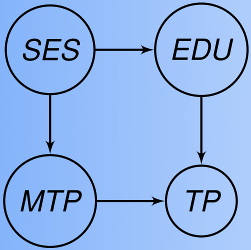
- Data source 1: SES,EDU, MTP

DAGs are modular



- Data source 1: SES,EDU, MTP
- Data source 2: MTP, EDU and TP

DAGs are modular



- Data source 1: SES,EDU, MTP
- Data source 2: MTP, EDU and TP
- Can join two sources to make inference about SES and TP!

Causal inference

Types

- Potential outcomes/Counterfactuals (Rubin [4],Pearl [5])
- Causal Graphs (Pearl[5],Greenland, Robins [6])
- Decision theory (Dawid [7],Geneletti [8],Didelez [9])

General issues

- no causation w/out manipulation
- Means need to be careful about observational data
- typically there are unobserved confounders, reporting bias etc
- Causality is an external assumption

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