

## **ESTIMATION ON THE WEAR AND DURABILITY OF THE HEAD OF THE MILLS USING DIFFERENT MODELS**

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### **ABSTRACT**

The mathematical model presented in this paper is one of the possible mathematical models to use to estimate the wear and lifespan of the head of the milling cutter, which has an appreciable age and initially modeled a relatively similar phenomenon of aggression or erosion of an entity by a high.

The mathematical model of the prey-predator population system, well known in ecology and expressed in a system of differential equations always of the order of AJ Lotka and V. Volterra is successfully applied in other fields: economics, services, prediction of technological evolution, adoption cryptocurrencies, hygiene, communications (security and optimization), medicine, passenger transport forecast, plasma physics.

In this wide context of applications, there are also applications used in the paper, of the prey-predator model, Lotka-Volterra, to the interaction phenomenon between the dental milling cutter and the milled material. The application starts from the assimilation of the dental cutter with the predator that extracts material from the body of the milled material, assimilated with the prey. The milling action, the action of the predator, does not remain without negative results for him, the milling material also losing particles. Both parts of this phenomenon cannot grow as a mass, an experimental aspect that turns into an important hypothesis.

### **INTRODUCTION**

The material of this paper is the phenomenon of wear of dental cutters, being taken from [2]. Some experimental data taken from the literature and the experiences of the authors will be used to calibrate the model.

The working method is mathematical modeling using a classical model, through the partial similarity of the process modeled by each of them.

Among the variants that [\*\*] proposes for the notion of wear, we note those related to the subject of this test: damage, degradation of an object or progressive modification of (some physical characteristics) of a technical part during the operation of the system in which is part.

According [5], "wear is the damaging, gradual removal or deformation of material at solid surfaces".

Among the words that fall into the multitude of words that define the word wear, there are also mechanical erosion and physical erosion.

### **MATERIAL AND METHOD**

For certain similar aspects, from the mathematical models of some natural phenomena, we selected the biological phenomenon of coexistence (interaction) of predatory prey, put in the mathematical form by Lotka and Volterra, [1; 6; 8; 11].

The simplest Lotka - Volterra model is the one that contains two species, the prey and the predator. The predatory species consumes the

prey species. The main hypotheses of this model are the following:

LVH1) The prey population has unlimited food resources;

LVH2) In the absence of predators, the population of prey,  $x$  increases in proportion to its number:

$$\frac{dx}{dt} = \alpha x, \alpha > 0. \quad (1)$$

LVH3) In the absence of security, the population of predators  $y$  decreases in proportion to its number:

$$\frac{dy}{dt} = -\gamma y, \gamma > 0. \quad (2)$$

LVH4) When both populations exist, a decline in the prey population and an increase in the predatory population will occur, both proportional to the individual interaction of the two species:

$$\begin{aligned} -\beta xy, \beta > 0 & \text{ for prey and} \\ \delta xy, \delta > 0 & \text{ for predatory.} \end{aligned}$$

With these hypotheses, the system of two nonlinear, first-order differential equations is born that models the predator-prey interaction, Lotka-Volterra:

$$\begin{aligned} \frac{dx}{dt} &= \alpha x - \beta xy, \frac{dy}{dt} \\ &= -\gamma y \\ &+ \delta xy, \alpha, \beta, \gamma, \delta > 0 \end{aligned} \quad (4)$$

To use this model to describe the interaction between the dental cutter and the milled material, we will modify some of the above hypotheses. First, we will assimilate the mass of the milling material with the measure of the predator population and we will denote it with  $y$ , and the mass of the milled material with the measure of the prey population denoting its mass with  $x$ .

We will accept the following hypothesis:

H5) The mass of the milling cutter as well as the mass of the milled material cannot increase. As a result we will assume:

$$\alpha = \delta = 0 \quad (5)$$

Under these conditions, the system of differential equations that models the interaction between the milling material and the milled material, becomes:

$$\frac{dx}{dt} = -\beta xy, \frac{dy}{dt} = -\gamma y, \beta, \gamma > 0 \quad (6)$$

Se consideră condițiile inițiale:

$$x(t_0) = x_0, y(t_0) = y_0. \quad (7)$$

$t_0$  being the initial time, and  $x_0$  and  $y_0$  the initial masses of the milling material and the milling cutter.

## RESULTS AND DISCUSSIONS

The solution of the system of differential equations (6) with the initial conditions (7) is:

$$\begin{aligned} x(t) &= x_0 e^{\frac{\beta y_0}{\gamma} [e^{-\gamma(t-t_0)} - 1]}, y(t) = \\ &y_0 e^{-\gamma(t-t_0)}. \end{aligned} \quad (8)$$

Losses of milled and milling material result in accordance with the formulas:

$$\begin{aligned} \delta x(t) &= \\ x_0 \left\{ 1 - e^{\frac{\beta y_0}{\gamma} [e^{-\gamma(t-t_0)} - 1]} \right\}, \delta y(t) &= \\ y_0 \left\{ 1 - e^{-\gamma(t-t_0)} \right\}. \end{aligned} \quad (9)$$

The model (8) of the milling of the dental material, contains two parameters:  $\beta$  and  $\gamma$ . It follows from (6) that the physical dimension of these parameters is  $[\beta] = M^{-1}T^{-1}$ ,  $[\gamma] = T^{-1}$ .

The mathematical model (8) of the cutter-dental interaction does not

contain an important process control parameter, namely the cutter speed,  $n$  [rot / min], related to the angular velocity  $\omega$  and the rotation frequency  $\nu$ , through the relations:

$$\omega = \frac{\pi n}{30}, \nu = \frac{\omega}{2\pi}. \quad (10)$$

One of the ideas that can be used to introduce this parameter in the model (8) is related to the physical size of the parameters involved in the process. In this way, another parameter that does not appear in (9) will be introduced, namely the force of the milling cutter on the dental material,  $F$ . From a dimensional point of view, the following formulas for the parameters of the model (8) can be accepted:

$$\beta = \beta_0 \nu \cdot \frac{HV_y}{HV_x} \cdot \frac{g}{F}, \gamma = \gamma_0 \nu \cdot \frac{HV_x}{HV_y} \cdot \frac{F}{y_0 g}, \quad (11)$$

in which:

$\beta_0$ ,  $\gamma_0$ , si  $\gamma_0$ , are dimensionless constants,

$HV_x$ ,  $HV_y$  are the Vickers hardnesses of the milled material, respectively of the material from which the milling head is built.

For the calculation of the two dimensionless parameters  $\beta_0$ , and  $\gamma_0$ , a we have the option of using experimental data, if they exist.

Obviously, considering the model hypotheses, only positive values will be accepted for the two dimensionless parameters.

Using experimental data from [10], values are obtained for the temporal variation of the mass of the dental cutter and of the milled material, which give the following values for the two dimensionless parameters:

$$\gamma = 0.000000522027 \text{ s}^{-1}, \beta = 0.000007025375 \text{ kg}^{-1} \text{ s}^{-1}. \quad (12)$$

We used the values for the material parameters and the established working regime:

$$\begin{aligned} HV_x &= 180 \text{ (575 MP)}, HV_y &= 280 \text{ (900 MPa)}, F &= 3.5 \text{ N}, \nu &= 200 \text{ s}^{-1}, g &= 9.81 \text{ ms}^{-2} \end{aligned} \quad (13)$$

The initial mass of the dental cutter being  $y_0 = 4.701 \text{ g}$ , and that of the processed material,  $x_0$ , being 6,180 g, and the final values of the mass of the processed material varied between 5,482 and 6,102 g. In fig. 1 and 2 are given the temporal variations of the functions that shape the evolution of the masses of the two interacting bodies: the dental milling cutter and the material that is subjected to milling. The same figures show the experimental data that were used to estimate the dimensionless model parameters.

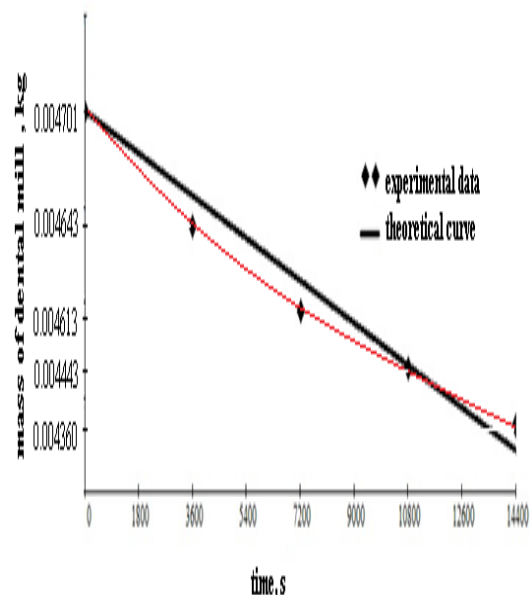


Fig. 1 The variation in time of the mass of material lost by the head of the dental cutter, due to wear.

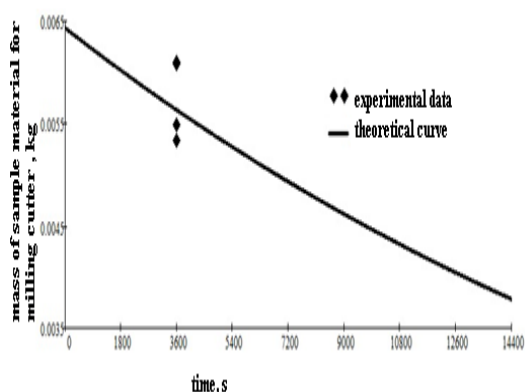


Fig. 2 Time variation of the milled material sample.

It is noticed that I care bodies interact lost weight. The curves have an exponential appearance even if, during the working interval, due to the values of the parameters, their curvature is less visible.

Finally, write the solution (8), introducing the other parameters that describe the phenomenon:

$$y(t; v, F) = y_0 e^{-\gamma_0 v \frac{HVx}{HVy} \frac{F}{y_0 g} (t-t_0)} \quad (14)$$

for the mass lost by the dental cutter, and:

$$x(t; v, F) = x_0 e^{\frac{\beta_0}{\gamma_0} \left( \frac{HVy \cdot g}{HVx \cdot F} \right)^2 \cdot \gamma_0 [y(t) - y_0]} \quad (15)$$

For the milled table, we passed as additional arguments the working frequency of the milling head and pressing force, as the most important control parameters. The functions of mass loss of the milled material and of the milling cutter are defined:

$$\delta x(t) = x_0 - x(t), \delta y(t) = y_0 - y(t), \quad (16)$$

and cutter efficiency ratio:

$$e(t) = \frac{\delta x(t)}{\delta y(t)}, t > 0. \quad (17)$$

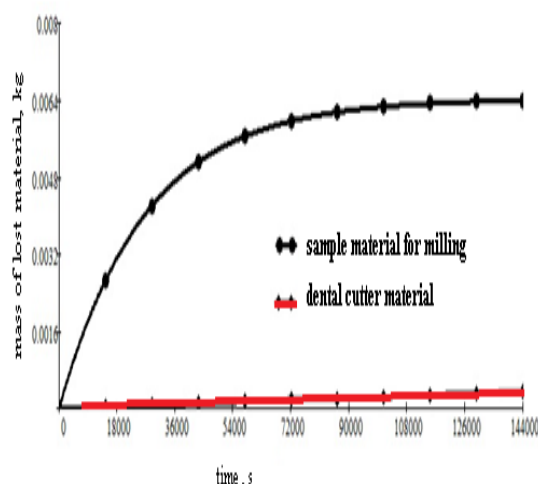


Fig. 3 Variation in time of material mass losses: milling and milling.

Curves with an appearance similar to that in Figure 4, obtained experimentally for wear measures, are found, for example in [3; 4; 7; 9]. In the literature, the search for effective measures of wear continues. In this paper we expressed wear in terms of mass, but in [10], which exposes most of the experimental research, we also expressed wear in geometric terms (changes in lengths, thicknesses or angles).

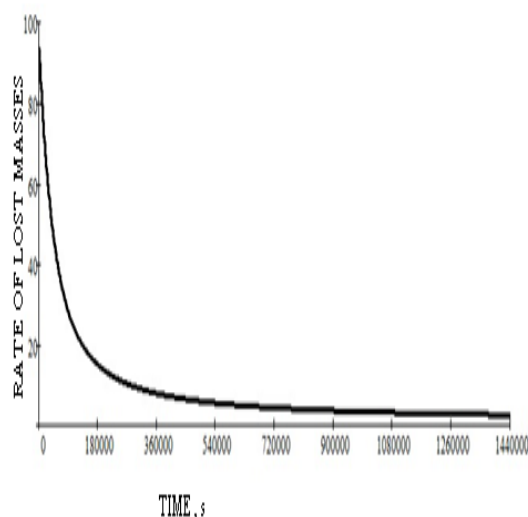


Fig. 4 Time variation of the ratio between the loss of mass of the milling material and the loss of material of the milling cutter.

With the help of the graphical representations of fig. 3 and 4 it can be seen that the material and configuration of the cutter material allows it to be efficient, ie using a small amount of cutter material more material of the milled material is removed. However, this effect is visible for times much longer than four hours of operation, after which the experimenter noticed the inefficiency of the cutters. From this point of view, the direct model extracted for the interaction phenomenon between the milling cutter and the milled material from the prey-predator model is not very successful. Some improvements are expected, but they can lead to complications in solving the system of differential equations.

### CONCLUSIONS

The idea of modeling the cutter-milled material interaction starting from the prey-predator model is, in principle, successful for short simulation times. The behavior from which the asymptotic phenomena are noticeable, however, is large in relation to the time found experimentally, at which the milling cutter becomes inefficient.

The model does not lead to solving the problem completely, without experimental information, and if the experimental information is used, as we did in the case presented in this paper, the model becomes an approximate one.

This model opens a research direction, possibly very profitable, in which the prey-predator model is applied to cutting phenomena, erosion, friction, etc. In order to obtain the best possible results, qualitatively and quantitatively, it is necessary to introduce other functions in the system of predator prey equations, which will

shape the aggression of the predator. The modification of the system of prey-predator equations was frequently made even in biology, resulting in complex nonlinear models that can even model the extinction of some species. It is obvious that in order to obtain better results in modeling the phenomenon of the interaction of the cutter with the milling material, the introduction of a function or at least some coefficients to measure the aggressiveness or efficiency of the teeth of the cutter head, is a necessity.

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