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Math Active Learning Lab: Math 107 Precalculus Notebook

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Math 107 Precalculus Notebook

University of North Dakota

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Welcome to the MALL

Welcome to UND's Math Active Learning Lab (MALL)! The MALL is a research-based approach designed to support student engagement with math. The premise of the MALL is that the best way to learn math is by doing math, not by watching someone else do math. This means that most of your time in this course will be spent doing math with the MALL instructors and tutors available to support your learning. The philosophy of the MALL is well described by H. A. Simon's quote

> "Learning results from what the student does and thinks and ONLY from what the student does and thinks. The teacher can advance learning only by influencing what the student does to learn."

For many of you, this is your first college math course. Quite possibly, this course and our expectations may be different from your high school mathematics experiences. We cannot stress strongly enough your role in ensuring your success in this class. More than anything else, your choices will determine your success in this course. Our data shows that students who are successful do the following.

- Attend class (focus group) regularly.
- Work in ALEKS and this Notebook at least 3 days each week.
- Study for written and ALEKS exams.
- Seek help when you need it.

We will be asking you to use the ALEKS resources and to work in your notebooks before coming to class. In your weekly focus group, your instructor will support your learning by facilitating small-group assignments and providing mini-lectures on the more challenging topics.

Instead of sitting in a lecture class for hours each week AND then being expected to do practice problems outside of class, part of your "class time" is spent doing homework in ALEKS. This provides instant feedback and links you to resources as needed. Using ALEKS allows us to individualize the student learning path. Students can move quickly through topics they are familiar with and take the time they need to learn more challenging topics. To help you get the most out of ALEKS, we have created this notebook. If ALEKS and the notebook are still leaving you confused about a topic, we expect you to ask an instructor or tutor for help.

MALL Staff

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How to use ALEKS

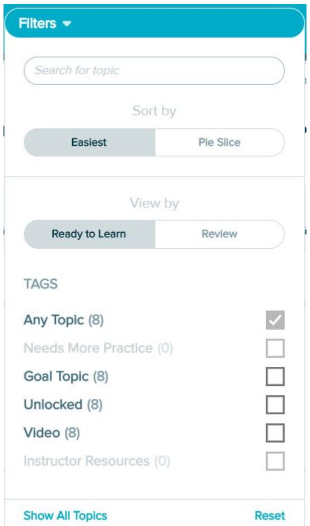
Working in ALEKS with the Notebook

- Every ALEKS topic is in the Notebook.
- Not every topic in the Notebook will be in YOUR Learning Carousel.
  - If you have already mastered a topic, you will not see the topic in your Learning Carousel.
  - You do NOT need to complete the Notebook for a topic you have already mastered.
- How to work through ALEKS topics
  1. ALEKS presents you with a topic.
  2. Use the table of contents to find the topic in the Notebook.
  3. You will find one of the following icons to help direct your learning.
     - Indicates you should watch a video. You may be asked to select a different video than the first video to pop up.
     - You should open the e-book.
       - You may need to scrolll down to find the appropriate topic.
       - Notebook entries are made to look EXACTLY like the e-book material
     - Open the dictionary to show definitions of terms.
     - Directs you to resources your instructor has added.
     - If there is no icon, the material should come directly from the Learning Page, which is the first page presented to you with a new topic.

The Learning Carousel

- To bring down the Learning Carousel, click the on the upper left side of the ALEKS Learning page.
- indicates a goal topic for the current module
- indicates a locked topic. Click the icon to see what topics must be worked to unlock it.
- No icon means it is a prerequisite topic. Use the Index to find the topic in your Notebook.

- When the Learning Carousel is pulled down, you can
  - Click the Filters for options to filter topics.
  - The Filter menu is shown below.



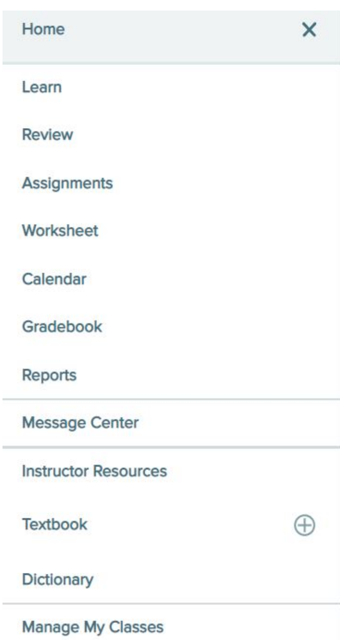
Search for topic You can type in the name of a topic to find it.

TAGS Click in the boxes to show only the topics that are

- goal topics,
- unlocked,
- have videos.

Hamburger Menu

- The Hamburger Menu is in the upper left of your ALEKS screen.
- The options in the Hamburger Menu are shown below.



Home Takes you back to the home screen.

Learn Opens the next topic ALEKS has ready for you to learn.

Review Opens up topics you have learned or mastered for you to review.

Calendar Opens a calendar view of deadlines for weekly modules and exams.

Gradebook Shows your grades for ALEKS modules and exams. The complete and official gradebook is in Blackboard.

Reports Opens a menu of reports that provide additional information about your progress in ALEKS. We encourage you to take a look at these pages.

Technical Support

ALEKS Technical Support is available at `https://www.aleks.com/support/contact_support` or by phone at (800) 258-2374. Call Technical support if you need help with

- accessing your account.
- locating a video.
- questions diplaying correctly.
- other technical issues not related to math content.

Math ______ ________________ ______ ______

Instructor: ____________________ **Email:** ____________________________

Phone: ________________ **Office:** ________________

Focus Group: ____________________

Required Course Materials: ALEKS 18-week access and the ____________________ *Course Notebook*

All email correspondence will go to your official UND email address.

Course prerequisites and content: Topics covered will include: Equations and inequalities; polynomial rational, exponential, logarithmic and trigonometric functions; inverse trigonometric functions; and algebraic and trigonometric methods commonly needed in calculus. Prerequisite: MATH 93 or an appropriate score in the Placement Testing Program.

The Math Active Learning Lab (MALL): Research shows that ____________________ ______________________________, not by listening to someone talk about or present the subject. The primary reason many students do not succeed in traditional math courses is that they do not do the problems or spend enough time engaged with the material.

The MALL is a research-based approach designed to support student engagement with math. Most of your time in this course will be spent doing math, and your instructor will support your learning by facilitating in-class assignments and providing mini-lectures on the more challenging topics. Instructors and tutors are available during the required MALL time to provide just-in-time support.

In a traditional math class, all students are expected to learn at the same pace. In the MALL, the ALEKS learning system allows you to work at you own pace, skip topics you have already mastered, and provides feedback as you are working.

COVID-19: All members of the University community have a role in creating and maintaining a COVID-19 resilient campus. There are several expectations that all community members, including students, are asked to follow for the safety of all:

- maintain physical ________________ of at least 6 feet while in UND facilities,
- wear ______ coverings during interactions with others and in the classroom,
- wash their hands often and use hand sanitizer,
- properly clean spaces that they utilize, and
- if experiencing any symptoms, ________________ and call their health care provider.

- Students electing not to comply with any of the COVID related requirements will not be permitted in the ________________, and may be subject to disciplinary action.

All members of the University community are expected to model positive ______________ both on- and off-campus. Information regarding the pandemic and UND's efforts to create a COVID resilient campus is available on the COVID-19 blog (`http://blogs.und.edu/coronavirus/`). Please subscribe to stay up to date on COVID related information.

Students who test positive for COVID-19 or are identified as a close contact are expected to ____________________ self-isolate/quarantine. If you have tested positive for COVID-19 or have been placed in quarantine due to being identified as a close contact or travel we strongly recommend that you report the information to the Office of Student Rights and Responsibilities at 701.777.2664 or online at `https://veoci.com/veoci/p/w/ss2x4cq9238u`. Doing so will ensure students have the support they need to continue with their academic goals and to protect others.

Due to the evolving circumstances of the COVID-19 pandemic, all information in this syllabus may need to be _____________ to meet the needs of remote instruction. Every effort will be made to operate in a manner consistent with the expectations outlined in this document.

Course Components

Focus Group

- Assignments given during the Focus Group meetings will be completed in small groups.
  - On-time attendance is _____________ to earn full-credit on the assignment.
  - Unless required for the Focus Group activity, cell-phone or computer use will result in a zero for the day.
- Students who do not attend the _______________ meeting, or contact the instructor the first week, will be **DROPPED FROM THE COURSE**.
- Students who do not ______________ their Initial Knowledge Check within two full days of their first class meeting will be **DROPPED FROM THE COURSE**.
- Once a week you will meet in class, the other day you will work in ALEKS in the MALL or remotely.
- Focus Group Absences
  - If due to a serious emergency, absences will usually be excused. Documentation _____________________________.
  - University sanctioned absences must be documented _____________________________ prior to the absence.
  - Travel plans ________________________________ cause for an excused absence.
  - All focus group assignments have a ____ to account for any unexcused absences.
  - Absences will be addressed on a case-by-case basis.

ALEKS

- Weekly module to be completed by _________ at 11:59 pm.
- Can work anywhere you have internet access.
- Deadlines _______________ be extended because of home computer or home internet issues.

MALL Time

- Spend at least 2.5 hours in the MALL working in ALEKS from __________ to __________
- MALL time must be completed in O'Kelly 33 (face-to-face) or virutally through Zoom.
- MALL time is class time, you should be working only on _____________.
- Credit for MALL time is based ________ on front desk check-in/out.
- Check-in with your UND ID when entering **and** check-out when exiting the MALL.
  - Failure to check-in/out results in __ minutes recorded.
  - Check-in/out with another student's ID is academic dishonesty.
- Minutes ________________________________ from one week to another.
- Focus Group time ______________________________ toward your MALL time.
- Food is NOT allowed in the MALL.
- The MALL is the place to get your math questions answered!
- MALL staff are there _____________________.

Notebook

- Graded ____________ in Focus Group.
- ________________ for MALL time and Focus Group.

Topic Goal Extra Credit

- Complete 10 topics in ALEKS by __________ at 11:59 pm.
- Earn a Focus Group bonus point.

Exams

- There will be __ exams.
- Each exam will have 125 pts
  - ALEKS exam: 100 pts
    - Must be completed in the MALL exam area
    - Must be completed by 9:00 pm the ___________________ the written exam.
    - UND ID is required to take your ALEKS exam.
    - All scratch work must be submitted to ____________________ as a PDF within 30 min of test completion.
    - You may not leave your table during an exam without permission.
    - Cell phones must be placed face __________ on the table.
  - Written exam: 25 pts
    - will be given during the Focus Group meeting.

Exam 1: ____________________ Exam 2: ____________________ Exam 3: ____________________

Final Exam

- The final exam will be a comprehensive ALEKS exam.
- All scratch work must be submitted to Blackboard within 30 min of test completion.
- The final ALEKS exam must be completed by Wednesday, December 16 at 7:30 pm.

Grading

- Your course grade will be a weighted average of the following:

Exams	____%
Final Exam	____%
MALL Time	10%
Focus Group Activities	10%
Module Completion	15%

- Grading Scale: A = 90% & above, B = 80–89%, C = 70–79%, D = 60–69%.

Try Score

- Your Try Score reflects your effort in this course.
- The Try Score is composed of:
  - focus group participation,
  - notebook completion,
  - MALL time and
  - module completion.
- This is _______ included in your course grade, but will be shared with your academic advisor.

Finishing the Course Early

- Given the individualized nature of this course it is possible to complete the course _________.
- Each time an exam is given, __________________ students have the option to take the final in place of the scheduled exam.
- To qualify to take the final early
  - the week before the written exam, arrange with the MALL office to take a proctored Knowledge Check
  - ____________ at least 90% of the in the course on this proctored ALEKS Knowledge Check

Academic Honesty

- All students in attendance at the University of North Dakota are expected to be honorable and to observe standards of conduct appropriate to a community of scholars.
- Academic misconduct includes
  - all acts of dishonesty in any academically related matter.
  - any knowing or intentional help or attempt to help, or conspiracy to help, another student.
  - use of _________, books, calculators, _______________ or any electronic devices on exams.
- A student who attempts to obtain credit for work that is not their own (whether that be on a homework assignment, exam, etc.) will receive ________________ for that item of work, and at the professor's discretion, may also receive a failing grade in the course.
- For more information read the Code of Student Life at `https://und.policystat.com/policy/6747183/latest/`.

Accommodations

- Disability
  - Contact me to request disability accommodations, discuss medical information, or plan for an emergency evacuation.
  - To get confidential guidance and support for disability accommodation requests, students are expected to register with DSS at `http://und.edu/disability-services/`, 190 McCannel Hall, or 701.777.3425.
- COVID-19
  - Due to COVID-19 students may need to request course adjustments, flexibility in delivery of content, and increased absenteeism.
  - Students with concerns regarding physically attending class during COVID-19 are encouraged to do the following:
    * Talk with your __________________ to determine appropriate accommodations, as soon as possible
    * Students with a known disability should contact Disability Student Services (DSS).

Starfish

- Important information is available to you through Starfish, which is an online system used to help students be successful.
- When an instructor observes student behaviors or concerns that may impede academic success, the instructor may raise a flag that notifies the student of the concern and/or refer the student to their academic advisor or UND resource.
- Please pay attention to these emails and take the recommended actions. They are sent to help you be successful!
- Starfish also allows you to
  - schedule appointments with various offices and individuals across campus.
  - request help on a variety of topics
  - search and locate information on offices and services at UND
- You can log into Starfish by clicking on Logins on the UND homepage and then selecting Starfish. A link to Starfish is also available in Blackboard once you have signed in.

Notice of Nondiscrimination

- It is the policy of the University of North Dakota that no person shall be discriminated against because of race, religion, age, color, gender, disability, national origin, creed, sexual orientation, gender identity, genetic information, marital status, veteran's status, or political belief or affiliation and the equal opportunity and access to facilities shall be available to all.
- Concerns regarding Title IX, Title VI, Title VII, ADA, and Section 504 may be addressed to:
  - Donna Smith, Director of Equal Employment Opportunity/Affirmative Action and Title IX Coordinator, 401 Twamley Hall, 701.777.4171
  - UND.affirmativeactionoffice@UND.edu
  - Office for Civil Rights, U.S. Dept. of Education, 500 West Madison, Suite 1475, Chicago, IL 60611

Resolution of Problems

Should a problem occur, you should speak to your instructor first. If the problem is not resolved, meet with Dr. Michele Iiams, MALL Director. If the problem continues to be unresolved, go to Dr. Gerri Dunnigan, Mathematics Department Chair, and next to the college Dean. Should the problem persist, you have the right to go to the Provost next, and then to the President.

How to Seek Help When in Distress

- We know that while college is a wonderful time for most students, some students may struggle.
- You may experience students in distress on campus, in your classroom, in your home, and within residence halls.
- Distressed students may initially seek assistance from faculty, staff members, their parents, and other students.
- In addition to the support we can provide to each other, there are also professional support services available to students through the Dean of Students and University Counseling Center.
    - Both staffs are available to consult with you about getting help or providing a friend with the help that he or she may need.
    - For more additional information, please visit the UND Cares program Webpage at `https://und.edu/student-life/student-rights-responsibilities/`.

Time Management

Good time management, good study skills and good organization will help you be successful in this course (and all of your classes). Answer the following questions.

1. To motivate yourself to complete a course, it is helpful to have clear reasons for taking the course. List your goals for taking this course.

2. Taking 12-15 credit-hours is the equivalent of a full-time job. Often students try to work too many hours while taking classes.

   NOTE: Students need to work to pay tuition, rent, buy food, etc., but working too many extra hours for things that are not needed can really impact their success. There is a balance between working to earn money now and having to spend more money later to retake courses.

   (a) Write down the number of of credit-hours you are taking this term and the number of hours you work per week.

   - Number of credit-hours ________________

   - Number of hours worked per week ________________

   (b) The table gives the recommended limit to the number of hours you should work for the number of credit-hours you are taking.

   - How do your numbers from part (a) compare to those in the table?

Number of Credit-Hours	Maximum Number of Hours of Work per Week
3	40
6	30
9	20
12	10
15	0

(c) Keep in mind that other responsibilities in your life, such as your family, might also make it necessary to limit your hours at work even more. What other responsibilities do you have?

(d) It is often suggested that you devote 2 hours of study and homework time outside of class for each credit-hour you take. For example:

12	credit-hours		15	credit-hours
24	study hours		30	study hours
36	total hours		45	total hours

- Based on the number of credit-hours you are taking, how many study hours should you plan for?

  __________ credit hours X 2 = __________ study hours

- What is the total number of hours (class time plus study time) that you should devote to school?

  __________ credit hours + __________ study hours = __________ total hours

- Your MALL course is a 3-credit course. This means you might need to spend up to 9 hours each week in class, working in ALEKS, or studying.
- At least 2 of these hours should be completed in the MALL.

On the next page, write down the times each day (for the next week) that you

- have scheduled classes,
- are scheduled to work
- other non-negotiable commitments (family, organization meetings, etc.)
- times that you plan to work in the MALL
- times that you plan to study outside of the MALL

Time	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
8:00 - 8:30							
8:30 - 9:00							
9:00 - 9:30							
9:30 - 10:00							
10:00 - 10:30							
10:30 - 11:00							
11:00 - 11:30							
11:30 - 12:00							
12:00 - 12:30							
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7:30 - 8:00							
8:00 - 8:30							
8:30 - 9:00							
9:00 - 9:30							
9:30 - 10:00							
10:00 - 10:30							
10:30 - 11:00							

Achieving Your Potential

Read each sentence, then check the box that best describes your behaviors and attitudes as they pertain to this course and to your life in general. Don't overthink the questions. Just ask yourself how you compare to most people. Be honest. There are no right or wrong answers.

	Not at all like me	**Not much like me**	**Somewhat like me**	**Mostly like me**	**Very much like me**
I arrive on time and attend every class meeting.	1	2	3	4	5
I complete assignments on time.	1	2	3	4	5
I have difficulty staying alert and on-task during class.	5	4	3	2	1
I tend to rush through assignments just to get them finished.	5	4	3	2	1
I am willing to learn from my mistakes and use them as an opportunity for growth.	1	2	3	4	5
I have a positive attitude and am ready to learn new ideas and concepts.	1	2	3	4	5
I haven't really set any goals for this course.	5	4	3	2	1
I organize my class materials, handouts, and assignments so that I can easily find information when I need it.	1	2	3	4	5
I have difficulty finding enough time in the day to complete my assignments and study.	5	4	3	2	1
I seek help from my instructor, tutors, classmates, or other resources when I am having difficulty understanding a new topic.	1	2	3	4	5
I'm not really sure why I am going to college.	5	4	3	2	1
I know that I can learn difficult concepts if I work hard and do my best.	1	2	3	4	5

Test Analysis

Have you ever thought of your graded test as a learning experience? There is a lot you can learn about yourself, your study habits, and your test-taking skills by examining your graded test after you get it back.

- Did you do as well as you thought you could?
- Or is there room for improvement?

You may think, ”the test was too hard” or “the teacher didn’t give us enough time”, but, chances are, your instructor has been giving a similar test under similar conditions to many students before you. So let’s see what **YOU** can do to earn a higher score on your next test.

Look at your graded test and analyze if each point loss was due to your having been **unprepared** for that problem, a **concept error**, or a **careless error** .

- Being **underprepared** for a problem means you didn’t know how to do the problem because you hadn’t done the homework that would have prepared you for it. Often an error made is considered an underprepared error if you look at the problem and have no idea where to begin.
- A **concept error** is one where you really didn’t understand the concept behind the problem. No matter how much time was available for a problem like this, you wouldn’t have been able to do it correctly because you have no conceptual understanding of the problem. *This is not a procedural error: you can apply a procedure and still not understand the concept.* Students demonstrate conceptual understanding in mathematics when they provide evidence that they can recognize, label, and generate examples of concepts; use and interrelate models, diagrams, manipulatives, and varied representations of concepts; identify and apply principles; know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; recognize, interpret, and apply the signs, symbols, and terms used to represent concepts. Conceptual understanding reflects a student’s ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.
- A **careless error** is one where you understood the problem and knew how to solve it, but you made a mistake that could have been avoided. Maybe you copied the problem or your handwriting incorrectly, made a relatively minor mistake in calculation, or some similar error.

1. In the chart below, put the **number of points** you missed on each problem under the correct heading. Then find the total in each column.

Problem	**unprepared**	**concept error**	**careless error**
	Total points	**Total points**	**Total points**

2. In which column did you have the most missed points? What does that tell you about yourself?

3. What can you learn from this exercise?

Being Unprepared

Consider the points you lost because you were **unprepared**. Why did you take a test without being fully prepared? Often, activities and responsibilities in life interfere with good intentions about being diligent in attending class, completing the notebook, completing MALL time, and completing the module. It may be time to:

- **re-examine your weekly schedule** and make sure you are devoting a sufficient amount of time to this class. Lay out a time management grid of your schedule making sure to schedule your MALL time and math study time throughout the week.
- **re-commit yourself to succeeding in this class.** Think about your college and career goals and remind yourself of how this course helps you get one step closer to achieving them.

4. List two steps you will take to remedy being unprepared.

Concept Errors
Now consider the **concept errors** point loss. A high total in this column tells you that you didn't understand the concepts very well. You may understand a math concept for the hour you're working on problems, but forget it by the next day; possibly because you didn't do enough homework.

- **Take Knowledge Checks when they appear.** Knowledge Checks (KCs) are the way ALEKS helps you identify topics you are not retaining. Take each KC as if it were a QUIZ (no notebooks, calculators, friends, other websites, etc.) AND to the BEST OF YOUR ABILITY. Topics that you need to revisit will appear again in later modules as they are needed.
- **Get the help you need immediately!** Math concepts build on each other. Each new idea is based on many previous concepts. Make sure you get the help you need immediately, as soon as you find yourself beginning to feel lost, so that the confusion doesn't compound itself - otherwise it can become like a snowball, getting bigger and bigger as it roles through the snow.

If your total loss due to concept errors is fairly large, find out where you can get the help you need. A high concept error total is cause for concern and must be addressed immediately for you to succeed.

5. Which of the following can help you when you are struggling with math?

   (a) your instructor
   (b) MALL tutors
   (c) Reworking and asking questions about previous Focus Group assignments
   (d) Completing your Notebook pages
   (e) All of the above

Careless Errors
Next look at **careless error** point loss. Careless errors are often caused by hurrying during a test or by lack of concentration due to test-anxiety or over-confidence. Here are some strategies that have worked for other students:

- **Do the easiest problems first.** When you first start a test, look it over and note which problems will be easiest for you. Do all those problems first to ensure you don't leave an easy problem blank just because it is at the end of the test. Finishing problems you find easy will help build your confidence! Then go through the rest of the test from beginning to end.
- **Work carefully and neatly.** As you do each problem, try to focus on one step at a time.
- **Review each problem to look for careless errors** when you finish the test. Find and correct common careless errors like arithmetic mistakes and sign errors before you turn in your test.
- Whenever possible, **check your answer.**

A lot of points can be gained by slowing down and being careful.

6. What are things you will do next time to prevent careless errors?

7. Now take half of your careless errors point total and add it back to your test total. What could your test grade have been? Would it have changed the letter grade?

Module 1

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Weekly Checklist

- ☐ Complete MALL time.
- ☐ Work in ALEKS and Notebook at least 4 days a week.
- ☐ Complete the weekly Module and Notebook pages by the due date.
- ☐ Attend Focus Group.
- ☐ Actively participate in Focus Group.
- ☐ Earn extra credit: Complete 10 topics by ______________________.

Translating a sentence into a multi-step equation

Learning Page Click on the first light bulb to complete the following.

More When translating phrases into algebraic expressions, it's helpful to look for the following key words.

Key Words	Operation
sum, increased by, more than, added to	
difference, decreased by, less than, subtracted from	
product, times, twice	
quotient, divided by	

EXAMPLE:
3 is the same as 5 less than the quotient of 16 and a number m.

- "is the same as 3" means $3 =$.
- the "quotient of 16 and a number m" is written $\frac{16}{m}$
- "5 less than" is $\frac{16}{m} - 5$

The equation is $\frac{16}{m} - 5 = 3$

YOU TRY IT:

1. 7 more than the quotient of a number d and 6 is 9.

Converting between radical form and exponent form

Learning Page We can convert between radical and exponential notations by using the following fact.

$$x^{m/n} = \_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_$$

When is this fact true? In order for the equations to be true, ________ must be well defined as a ________ number. It is ________ well defined when n is ________. When n is ________, it is well defined only when ____ is ____________________.

YOU TRY IT:

2. Convert $x^{7/2}$ to radical notation.

3. Convert $\sqrt[3]{x^4}$ to an expression with rational exponents.

Set-builder and interval notation

Learning Page The set $\{x \mid$ ________________ $\}$ is the set of all x such that x is ____________________

________________.

This set is an ________________. It is written using ____________________________.

We can specify an interval using ____________________________, a ________________, or

____________________________ as shown below.

Set Builder Notation	Graph	Interval Notation
$\{x \mid a \leq x \leq b\}$		
$\{x \mid a < x < b\}$		
		$(a, b]$
$\{x \mid a \leq x < b\}$		
		$[a, \infty)$
		(a, ∞)
$\{x \mid x \leq a\}$		
		$(-\infty, a)$

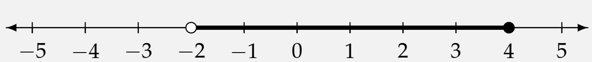
A solid dot shows an endpoint that ____________________.

In interval notation, this is shown using ____________.

A hollow dot shows an endpoint that ____________________.

In interval notation, this is shown using ____________.

EXAMPLE:
Given the set $\{x \mid -2 < x \leq 4\}$, graph the set and write the interval notation.



$(-2, 4]$

YOU TRY IT:

4. Given the set $\{x \mid x \geq -3\}$, graph the set and write the interval notation.

Union and intersection of intervals

Open the Instructor Added Resource which will direct you to a video to complete the following.

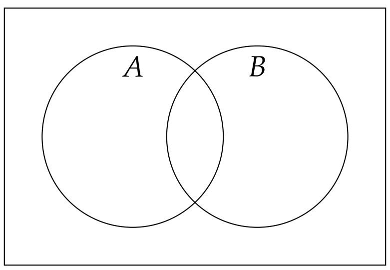
Union and Intersection of Sets

The **union** of sets A and B, denoted __________,

is the set of elements that belong to set A ______

to set B _____________________________.

Shade in $A \cup B$.

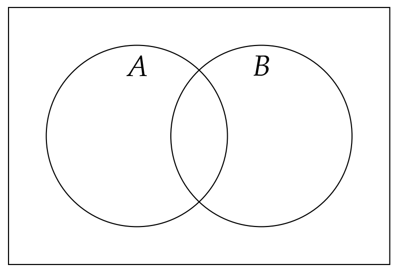


The **intersection** of sets A and B,

denoted ____________, is the set of elements

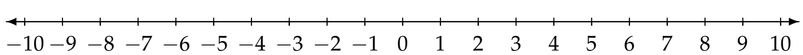
______________________________________.

Shade in $A \cap B$.

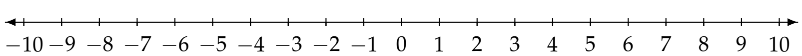


Given the sets $A = \{x \mid x > -6\}$, $B = \{x \mid \ \leq 3\}$, and $C = \{x \mid x \geq 7\}$ find the following. Write your answer in interval notation.

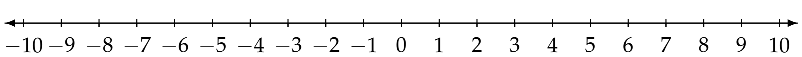
Set A:



Set B:



Set C:



a. $A \cup B =$

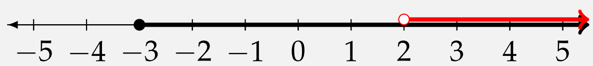
b. $A \cap B =$

c. $B \cap C =$

d. $B \cup C =$

EXAMPLE:
Given $A = \{x \mid x > 2\}$ and $B = \{x \mid x \geq -3\}$.
Find the following.

a) $A \cup B$
We begin by sketching both graphs.



We want values in either of the two intervals.

$A \cup B = [-3, \infty)$

b) $A \cap B = (2, \infty)$
We want the overlap of the two intervals.

YOU TRY IT:
Given $D = \{x \mid x \leq 2\}$ and $E = \{x \mid x > 5\}$. Find the following.

5. $D \cap E$

6. $D \cup E$

Rewriting an algebraic expression without a negative exponent

Learning Page For any ________________ number a and any ________________ ________________ n, we have the following.

Rule 1: $a^{-n} =$ ________

Move ________ to the ____________ and make the ____________________________.

Rule 2: $\dfrac{1}{a^{-n}} =$ ________

Move ________ to the ____________ and make the ____________________________.

YOU TRY IT: Write the following expressions with positive exponents.

7. $4x^{-5}$

8. $\dfrac{3}{x^{-7}}$

Power, product, and quotient rules with negative exponents

Open the e-book to complete the following.

Definition of b^0 and b^{-n}

If b is a nonzero real number and n is a positive integer, then

$b^0 =$ ________	Examples:
$b^{-n} =$ ________ $=$ ________	Examples:

Complete the table for **Properties of Exponents**. Let a and b be real numbers and m and n be integers.

Property	Example	Expanded Form

YOU TRY IT:

9. Simplify $(x^{-2}y^3)\left(\dfrac{3x^3y}{z^{-1}}\right)^{-3}$. Write your answer using only positive exponents.

Special products of radical expressions: Conjugates and squaring

Watch the video *Multiplying Conjugate Radical Expressions and Squaring Two-term Radical Expressions* to complete the following.

Perform the indicated operations.

a.

b.

$(a-b)(a+b) =$ ____________________

$(a-b)^2 =$ ______________________________

Rational exponents: Unit fraction exponents and bases involving signs

Watch the video *Definition of "a" to the* $1/n$ *Power* to complete the following.

Definition of $a^{1/n}$

Let $n > 1$ be an integer. Then, $a^{1/n} =$ __________ provided that $\sqrt[n]{a}$ is a _______________ number.

Verbal Interpretation	**Algebraic Example**
$a^{1/n}$ equals the _______________ of a, provided that the n^{th}-root of a is a _______________ number.	

a.

b.

c.

YOU TRY IT: Simplify the following.

10. $16^{1/4} =$

11. $8^{1/3} =$

Rational exponents: Non-unit fraction exponent with a whole number base

Watch the video *Definition of "a" to the m/n Power* to complete the following.

Definition of $a^{m/n}$

Let m and n be positive integers such that m/n is in lowest terms and $n > 1$. Then if $\sqrt[n]{a}$ is a

________________ number,

$a^{m/n} =$ ________________ $=$ ________________ OR $a^{m/n} =$ ________________ $=$ ________________

Simplify if possible.

a.

b.

c.

d.

e.

f.

YOU TRY IT: Simplify the following.

12. $8^{2/3} =$

13. $81^{3/4} =$

Rational exponents: Negative exponents and fractional bases

If you have not already watched the video *Definition of "a" to the m/n Power* from the previous topic **Rational exponents: Non-unit fraction exponent with a whole number base**, do so now. You may watch the video again for a review.

YOU TRY IT: Simplify. Write your answers without exponents.

14. $\left(\frac{1}{16}\right)^{-3/2}$

15. $27^{-2/3}$

Squaring a binomial: Univariate

Watch the video *Squaring a Binomial* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

The square of a binomial:

Multiply.

YOU TRY IT:

16. Rewrite $(7 - 3y)^2$ without parentheses and simplify.

Complex fraction made of sums involving rational expressions: **Problem type 3**

Watch the video *Simplifying Complex Fractions (Methods I and II)* to complete the following.

Simplify using Method I.

Simplify Using Method II.

Complex fraction made of sums involving rational expressions: Problem type 4

Open the e-book to complete the following.

Simplifying a Complex Fraction: Multiply by the LCD (Method II)

Step 1

Step 2

Step 3

EXAMPLE:

Simplify $\dfrac{2+\frac{2}{x+2}}{1-\frac{3}{x-1}}$.

To start, we will multiply by $\frac{(x+2)(x-1)}{(x+2)(x-1)}$ to eliminate the fraction on the top and the fraction on the bottom.

$$\begin{aligned}
\frac{2+\frac{2}{x+2}}{1-\frac{3}{x-1}} &= \frac{2+\frac{2}{x+2}}{1-\frac{3}{x-1}} \cdot \frac{(x+2)(x-1)}{(x+2)(x-1)} \\
&= \frac{2(x+2)(x-1)+\frac{2(x+2)(x-1)}{x+2}}{1(x+2)(x-1)-\frac{3(x+2)(x-1)}{x-1}} \\
&= \frac{2(x^2+x-2)+2x-2}{x^2+x-2-3x-6} \\
&= \frac{2x^2+2x-4+2x-2}{x^2-2x-8} \\
&= \frac{2x^2+4x-6}{x^2-2x-8}
\end{aligned}$$

YOU TRY IT:

17. Simplify $\dfrac{3-\frac{1}{x+4}}{1+\frac{2}{x-4}}$.

Complex fraction made of sums involving rational expressions: **Problem type 5**

Open the Instructor Added Resource which will direct you to a video to complete the following.

Simplify.

YOU TRY IT: Simplify.

18. $\dfrac{1 - \frac{15}{x} + \frac{56}{x^2}}{1 - \frac{11}{x} + \frac{24}{x^2}}$

Factoring a product of a quadratic trinomial and a monomial

Open the Instructor Added Resource which will direct you to a video to complete the following.

Factor.

Rationalizing a denominator using conjugates: Variable in denominator

Learning Page To rationalize the denominator is to write an ________________________________

To do this, we'll multiply the ________________ and ________________ by the

________________________.

The radical expressions $a + b\sqrt{m}$ and ________________ are **conjugates** of each other.

EXAMPLE: Rationalize the denominator and simplify.

$$\frac{4}{3-2\sqrt{x}} = \frac{4}{3-2\sqrt{x}} \cdot \frac{3+2\sqrt{x}}{3+2\sqrt{x}}$$

$$= \frac{12+8\sqrt{x}}{9-4x}$$

YOU TRY IT: Rationalize the denominator and simplify.

19. $\dfrac{-3}{4+5\sqrt{y}}$

Complex fraction with negative exponents: Problem type 1

Open the e-book to complete EXAMPLE 7: **Simplifying a Complex Fraction (Method II)**.

Solution:

________________________ $= \dfrac{\frac{1}{d^2} - \frac{1}{c^2}}{\frac{1}{d} - \frac{1}{c}}$

First write the expression with

________________________.

$$= \frac{c^2d^2 \cdot \left(\frac{1}{d^2} - \frac{1}{c^2}\right)}{c^2d^2 \cdot \left(\frac{1}{d} - \frac{1}{c}\right)}$$

Step 1: Multiply the numerator and denominator by the ________ of all four individual fractions: ________

$$= \frac{\frac{c^2d^2}{1} \cdot \frac{1}{d^2} - \frac{c^2d^2}{1} \cdot \frac{1}{c^2}}{\frac{c^2d^2}{1} \cdot \frac{1}{d} - \frac{c^2d^2}{1}\frac{1}{c}}$$

Step 2: Apply the ________________ property.

$=$ ________________________

$$= \frac{(c-d)(c+d)}{cd(c-d)}$$

Step 3: Simplify by ________________ and ________________ common factors.

$=$ ________________________

YOU TRY IT:

20. Simplify $\dfrac{u^2 - w^{-1}}{w^2 + u^{-1}}$. Write your answer using only positive exponents.

Complex fraction with negative exponents: Problem type 2

Review **EXAMPLE 7** from the previous topic: Complex fraction with negative exponents: Problem type 1 before working this topic.

EXAMPLE:
Simplify $\dfrac{3x^{-1} - y^{-2}}{2x^{-1} + 4y^{-1}}$. Write your answer using only positive exponents.

$$\begin{aligned}
\frac{3x^{-1} - y^{-2}}{2x^{-1} + 4y^{-1}} &= \frac{\frac{3}{x} - \frac{1}{y^2}}{\frac{2}{x} + \frac{4}{y}} \\
&= \frac{\frac{3}{x} - \frac{1}{y^2}}{\frac{2}{x} + \frac{4}{y}} \cdot \frac{xy^2}{xy^2} \\
&= \frac{\frac{3xy^2}{x} - \frac{xy^2}{y^2}}{\frac{2xy^2}{x} + \frac{4xy^2}{y}} \\
&= \frac{3y^2 - x}{2y^2 + 4xy}
\end{aligned}$$

YOU TRY IT:

21. Simplify $\dfrac{-3xy^{-1} + 2y^{-1}}{4x^{-2} + 5y}$. Write your answer using only positive exponents.

Finding the x and y intercepts of a line given the equation: Advanced

Learning Page To find the x-intercept of a line, ______________________________.

If the x-intercept is a, this means the point __________ lies on the line.

To find the y-intercept of a line, ______________________________.

If the y-intercept is b, this means the point __________ lies on the line.

EXAMPLE:
Find the x and y-intercept of $4x + 3y = -8$.

a) Find the x-intercept. Let $y = 0$.

$$\begin{aligned} 4x + 3(0) &= -8 \\ 4x &= -8 \\ x &= -2 \end{aligned}$$

$(-2, 0)$ is the x-intercept.

b) Find the y-intercept. Let $x = 0$.

$$\begin{aligned} 4(0) + 3y &= -8 \\ 3y &= -8 \\ y &= -\tfrac{8}{3} \end{aligned}$$

$(0, -\frac{8}{3})$ is the y-intercept.

YOU TRY IT:

22. Find the x and y-intercept of $7x - 5y = 3$.

Writing the equation of the line through two given points

Watch the video *Writing an Equation of the Line Passing Through Two Given Points* and complete the following.

Write an equation of the line that passes through the points ________________ and ________________. Write the answer in slope-intercept form.

YOU TRY IT:

23. Write the equation of the line through $(2, -4)$ and $(-1, 3)$.

Writing an equation in slope-intercept form given the slope and a point

Watch the video *Using Slope-Intercept Form to Write an Equation of a Line* and complete the following.

1. Use the slope-intercept form to write an equation of the line that passes through ______________ with slope $m =$ __________.

2. Write the equation using function notation where $y = f(x)$.

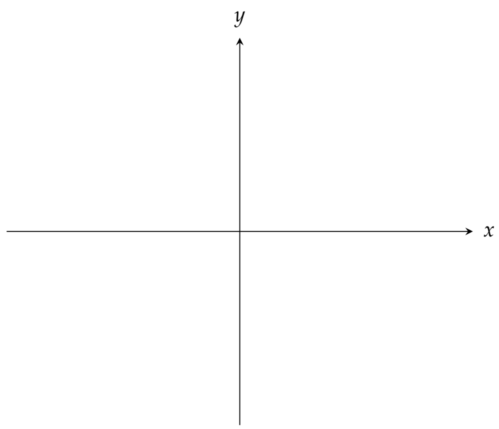
YOU TRY IT:

24. Write the equation of the line with slope $m = \frac{3}{4}$ that passes through $(2, -3)$.

Graping a line given its slope and y-intercept

Watch the video *Using the Slope and y-intercept to Graph a line* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

On the axes, plot and label the point $(0, b)$ and graph the line.



$m =$ ______________ $\implies m =$ ______________

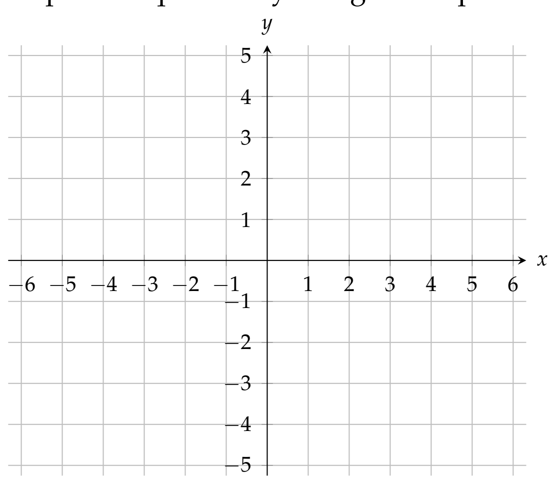
$y - b =$ ________________

Slope-intercept form: ____________________________

a. Write the equation in slope-intercept from and determine the slope and y-intercept.

$m =$ ______ $y -$ intercept:___________

b. Graph the equation by using the slope and y-intercept.



Graphing a line given its equation in standard form

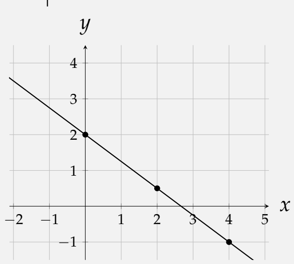
Learning Page First, solve the equation for ______. Then, choose some ______ values and evaluate.

EXAMPLE:
Sketch the graph of $3x + 4y = 8$.
Solve for y.

$$4y = 8 - 3x$$
$$y = 2 - \frac{3}{4}x$$

Find points that lie on the graph.

x	y
0	2
2	$\frac{1}{2}$
4	-1



YOU TRY IT:

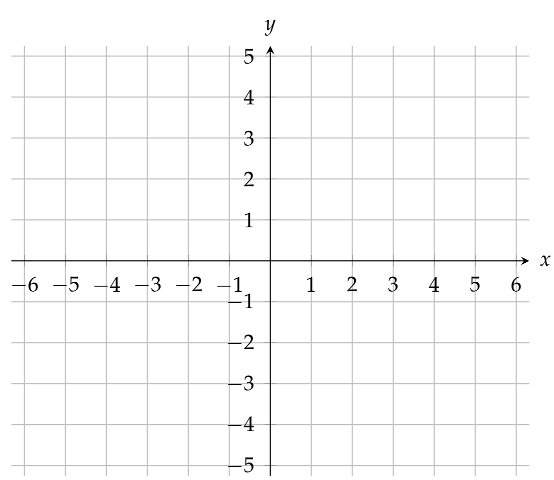
25. Sketch the graph of $2x - 3y = 6$.

Graphing a line by first finding its x and y-intercepts

Open the Instructor Added Resource which will direct you to a video to complete the following.

Sketch the graph of ____________________ by first finding the x and y-intercepts.

x-intercept:



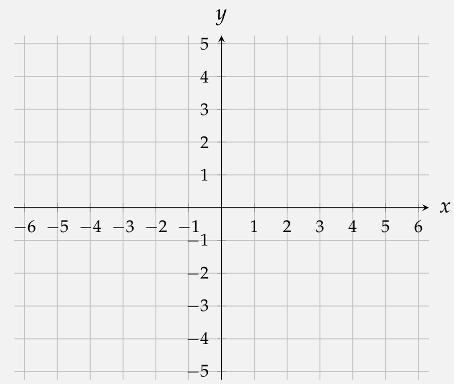
y-intercept:

YOU TRY IT: Consider the line $3x - 4y = 12$.

26. Find the x-intercept.

27. Find the y-intercept.

28. Sketch the graph.



Finding intercepts of a nonlinear function given its graph

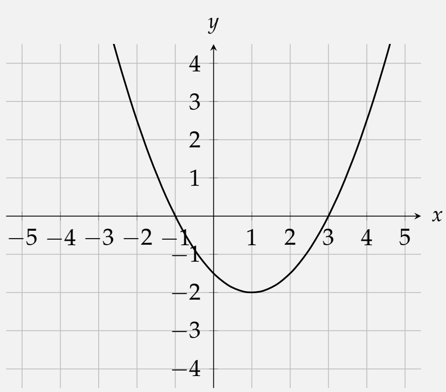
Learning Page The x-intercepts are where the graph intersects the ____________________.

The x-intercepts are also ____________________ of the function.

The y-intercept is where the graph intersects the ____________________.

The graph of a function can have at ______________ __________ y-intercept.

YOU TRY IT: Give all x and y intercepts of the graph below.

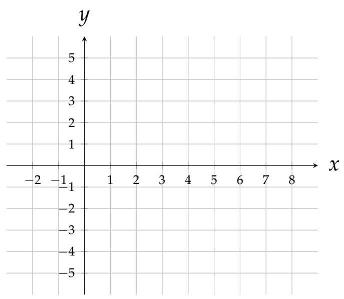


29. x-intercept(s):

30. y-intercept(s):

Finding the x and y intercepts of the graph of a nonlinear equation

Watch the video *Identifying x- and y-intercepts* to complete the following.



x-intercept(s):

y-intercept(s):

Determining x- and y-intercepts from an Equation

Given an equation in x and y,

- Find the ________________ by substituting ______ for ______ in the equation and solving for ______.
- Find the ________________ by substituting ______ for ______ in the equation and solving for ______.

Determine the x- and y-intercepts of the graph of the equation. ______________________

x-intercept(s):

y-intercept(s):

EXAMPLE:
Find the x and y-intercepts of $16x^2 + 25y^2 = 400$.

- Find the x-intercepts.

$$\begin{aligned} 16x^2 + 25 \cdot 0^2 &= 400 \\ 16x^2 &= 400 \\ x^2 &= 25 \\ x &= 5, -5 \end{aligned}$$

The x-intercepts are $(5, 0)$ and $(-5, 0)$.

- Find the y-intercepts.

$$\begin{aligned} 16 \cdot 0^2 + 25y^2 &= 400 \\ 25y^2 &= 400 \\ y^2 &= 16 \\ y &= 4, -4 \end{aligned}$$

The y-intercepts are $(0, 4)$ and $(0, -4)$.

YOU TRY IT:

31. Find the x and y-intercepts of $7x^2 + 5y^2 = 35$.

Writing the equations of vertical and horizontal lines through a given point

Open the e-book to complete the following.

Linear Equations and Slopes of Lines

$Ax + By = C$		$y = k$	$x = k$
Slanted line		Horizontal line	Vertical line
$\nearrow$	$\searrow$	$\leftrightarrow$	$\updownarrow$
______ slope	______ slope	______ slope	______ slope

YOU TRY IT:

32. Write the equation of the vertical line through $(-4, 3)$

33. Write the equation of the horizontal line through $(7, -12)$

Identifying parallel and perpendicular lines from equations

Learning Page Here are some facts about parallel and perpendicular lines.

Parallel Lines:

- Two ______ lines are parallel if and only if they have the ______.
- All ______ lines are parallel to ______.

  Vertical lines are parallel only to other ______.

Perpendicular Lines:

- Two nonvertical lines are perpendicular if and only if the ______ is ______.
- All vertical lines are perpendicular to all ______ lines and vice versa.

  Vertical lines are ______ to horizontal lines and vice versa.

EXAMPLE:
Determine if the lines below are parallel, perpendicular, or neither.

$$5y = 2x + 3$$
$$-5y = 3x + 2$$

We first write the lines in slope-intercept form.

$$y = \frac{2}{5}x + \frac{3}{5}$$
$$y = -\frac{3}{5}x + \frac{2}{5}$$

The slope of the first line is $\frac{2}{5}$ and the slope of the second line is $-\frac{3}{5}$. They are not equal so the lines are NOT parallel. $\frac{2}{5} \cdot -\frac{3}{5} \neq -1$ so the lines are NOT perpendicular.

YOU TRY IT:

34. Determine if the lines below are parallel, perpendicular, or neither.

$$6y = 2x + 3$$
$$-2y = 6x + 2$$

Using i to rewrite square roots of negative numbers

Open the e-book to complete the following.

The Imaginary Number i

- $i =$__________ and $i^2 =$__________
- If b is a positive real number, then $\sqrt{-b} =$__________.

EXAMPLE: Simplify the following.

a) $\sqrt{-36}$

$\sqrt{-36} = i\sqrt{36} = 6i$

b) $\sqrt{-28}$

$\sqrt{-28} = i\sqrt{28} = i\sqrt{2^2 \cdot 7} = 2i\sqrt{7}$

YOU TRY IT: Simplify the following.

35. $\sqrt{-49}$

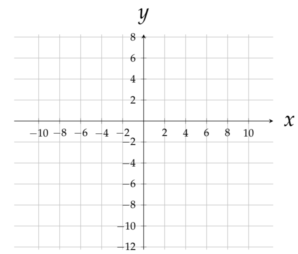
36. $\sqrt{-48}$

Writing equations of lines parallel and perpendicular to a given line through a point

Watch the video *Writing an Equation of a Line Parallel to Another Line* and complete the following.

Write an equation of the line passing through __________ and parallel to the line ______________.

Pause the video and try graphing the given line and the parallel line yourself.

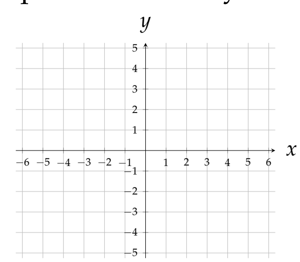


Play the video and check your answers.

Watch the video *Writing an Equation of a Line Perpendicular to Another Line* and complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Write an equation of the line passing through ________ and perpendicular to the line ____________.

Pause the video and try graphing the given line and the perpendicular line yourself.



Play the video and check your answers.

YOU TRY IT: Consider the line $4x + 3y = -6$. Find the equation of a line that is:

37. perpendicular to $4x + 3y = -6$ and contains $(4, -2)$.

38. parallel to $4x + 3y = -6$ and contains $(4, -2)$.

Writing and evaluating a function that models a real-world situation: Advanced

Watch the video *Writing Linear Cost, Revenue, and Profit Functions* and complete the following.

A lawn service company charges __________ for each lawn maintenance call. The fixed monthly cost of __________ includes telephone service and depreciation of equipment. The variable costs include labor, gasoline, and taxes. These amount to __________ per lawn.

a. Write a linear cost function representing the monthly cost $C(x)$ for x maintenance calls.

$$\left(\qquad\right) = \left(\qquad\right) + \left(\qquad\right)$$

b. Write a linear revenue function representing the monthly revenue $R(x)$ for x maintenance calls.

c. Write a linear profit function representing the monthly profit $P(x)$ for x maintenance calls.

d. Determine the number of calls needed per month for the company to make money.

e. If 42 calls are made for a given month, how much money will the lawn service earn or lose?

Interpreting the parameters of a linear function that models a real-world situation: Advanced

Open the Instructor Added Resource which will direct you to a video to complete the following.

Jose is driving to Chicago. Let y represent his distance from Chicago (in miles). Let x represent the time he has been driving (in hours). Suppose that x and y are related by the equation ________________.

a. How far was Jose from Chicago when he began his drive?

b. What is the change in Jose's distance from Chicago for each hour he drives?

This is given by the __________ of the __________.

The slope of ________________ is ________.

This means that for ____________________ that Jose drives, his ______________ to Chicago will ______________ by _______ miles.

YOU TRY IT:
Let y represent the total cost of producing a toy. Let x represent the number of toys produced. Suppose that x and y are related by the equation $1100 + 15x = y$.

39. What is the change in the total cost for each toy made?

40. What is the cost to get started before any toys are made?

Notes from Focus Group:

Notes from Focus Group:

Module 2

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Weekly Checklist

- ☐ Complete MALL time.
- ☐ Work in ALEKS and Notebook at least 4 days a week.
- ☐ Complete the weekly Module and Notebook pages by the due date.
- ☐ Attend Focus Group.
- ☐ Actively participate in Focus Group.
- ☐ Earn extra credit: Complete 10 topics by ____________________.

Solving a quadratic equation needing simplification

Learning Page We first ____________________ the equation with __________ on one _______________.

EXAMPLE:
Solve: $2x^2 - x - 3 = (x+1)^2$.

We expand the right side of the equation.

$$2x^2 - x - 3 = x^2 + 2x + 1$$

Next, rewrite the equation with 0 on one side.

$$x^2 - 3x - 4 = 0$$

Factor.

$$(x-4)(x+1) = 0$$

$$x = 4, -1$$

YOU TRY IT:

41. Solve: $2x^2 + x = (x-2)^2 - 10$

Finding the roots of a quadratic equation with leading coefficient greater than 1

Watch the video *Introduction to Quadratic Equations and the Zero Product Property* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Definition of a Quadratic Equation

Let a, b, and c represent real numbers where $a \neq 0$. A **quadratic equation** in the variable x is an equation of the form

____

Zero Product Property

If $mn = 0$, then __________ or __________.

Solve by applying the zero product property.

EXAMPLE: Solve $5x^2 + 9x - 2 = 0$.

We begin by factoring.

$$\begin{aligned} 5x^2 + 9x - 2 &= 0 \\ (5x-1)(x+2) &= 0 \\ x &= \frac{1}{5}, -2 \end{aligned}$$

YOU TRY IT:

42. Solve $6x^2 + 5x - 4 = 0$.

Finding the roots of a quadratic equation of the form $ax^2 + bx = 0$

Open the e-book to complete the following.

Zero Product Property

If ________________________, then __________________ or __________________.

EXAMPLE:
Solve the equation $2x^2 + 8x = 0$.

$$\begin{aligned} 2x^2 + 8x &= 0 \\ 2x(x+4) &= 0 \end{aligned}$$

$$\begin{aligned} 2x &= 0 \quad \text{or} \quad x + 4 = 0 \\ x &= 0 \quad \text{or} \quad x = -4 \end{aligned}$$

The solution is $x = 0, -4$.

YOU TRY IT:

43. Solve the equation $4x^2 - 20x = 0$.

Writing a quadratic equation given the roots and the leading coefficient

Learning Page We use the ______________________________, which states that if k is a root of the polynomial $P(x) = 0$, then ________________________ is a factor of the polynomial $P(x)$.

EXAMPLE: Write the quadratic equation whose roots are -2 and 3, and whose leading coefficient is 7.
-2 is a root so $x + 2$ is a factor and 3 is a root so $x - 3$ is a factor.

$$\begin{aligned} 7(x+2)(x-3) &= 0 \\ 7(x^2 - 3x + 2x - 6) &= 0 \\ 7(x^2 - x - 6) &= 0 \\ 7x^2 - 7x - 42 &= 0 \end{aligned}$$

YOU TRY IT:

44. Write the quadratic equation whose roots are 5 and -2, and whose leading coefficient is 3.

Introduction to solving an absolute value equation

Watch the video *Introduction to Absolute Value Equations* to complete the following.

Solve the equation.

$|u| = 3$ ⟶

$|u| = 0$ ⟶

$|u| = -3$ ⟶

$|x+1| =$ __________

Let k represent a real number.

1. If __________, $|u| = k$ is equivalent to _______________ or _______________ .
2. If __________, $|u| = k$ is equivalent to _______________.
3. If __________, $|u| = k$ ____________________.

Learning Page The **absolute value** of a number is its ____________________ from _____ on the number line.

YOU TRY IT: Solve for x.

45. $|x| = 8$

46. $|x| = -3$

Solving an absolute value equation: Problem type 2

Watch the video *Solving Absolute Value Equations* to complete the following.

Solve the equations.

a. ____________________ b. ____________________

c. ____________________

Let k represent a real number.

1. If __________, $|u| = k$ is equivalent to _______________ or _______________ .

2. If __________, $|u| = k$ is equivalent to _______________.

3. If __________, $|u| = k$ ____________________.

YOU TRY IT: Solve for x.

47. $|3x + 2| = 8$

48. $|4x - 1| = -7$

Solving an absolute value equation: Problem type 4

Watch the video *Solving an Absolute Value Equation* to complete the following.

Solve the equation.

EXAMPLE:
Solve the following equations.

a) $2|x+5|-10=0$
First isolate $|x+5|$.

$$\begin{aligned} 2|x+5|-10 &= 0 \\ 2|x+5| &= 10 \\ |x+5| &= 5 \end{aligned}$$

Write the equivalent statements without absolute value.

$$\begin{aligned} x+5 &= 5 & \text{or} \quad x+5 &= -5 \\ x &= 0 & \text{or} \quad x &= -10 \end{aligned}$$

So $x=0,-10$

b) $6+4|x+3|=2$

$$\begin{aligned} 4|x+3| &= -4 \\ |x+3| &= -1 \end{aligned}$$

No solution.

YOU TRY IT:
Solve the following equations.

49. $-7|x-5|+4=9$

50. $-3|x-7|+5=-1$

Solving a quadratic equation using the square root property: Exact answers, advanced

Watch the video *Introduction to the Square Root Property* to complete the following.

Square Root Property

If $x^2 = k$, then ____________________.

The solution set is ____________________ or more concisely ____________________.

Solve by applying the square root property.

a.

b.

c.

EXAMPLE: Solve for x.

$$2(x+1)^2 = 16$$

Solve for the squared term.

$$(x+1)^2 = 8$$

Apply the square root property.

$$x+1 = \pm\sqrt{8}$$

$$x = -1 \pm 2\sqrt{2}$$

YOU TRY IT:

51. Solve: $\frac{1}{2}(x-2)^2 - 5 = 0$

Completing the square

Watch the video *Introduction to Completing the Square* and complete the following.

Determine the value of n that makes the polynomial a perfect square trinomial. Then factor as the square of a binomial.

a.

b.

c.

Solving a quadratic equation by completing the square: Exact answers

Watch the video *Solving a Quadratic Equation With Leading Coefficient 1 by Completing the Square* and complete the following.

Solve by completing the square and applying the square root property.

EXAMPLE:
Solve $x^2 - 12x + 33 = 0$ by completing the square.

$$x^2 - 12x + 33 = 0$$
$$x^2 - 12x = -33$$
$$\text{Add } \left(\frac{12}{2}\right)^2 \text{ to each side}$$
$$x^2 - 12x + 36 = -33 + 36$$
Factor the left side.
$$(x - 6)^2 = 3$$
$$x - 6 = \pm\sqrt{3}$$
Apply the square root property.
$$x = 6 \pm \sqrt{3}$$

YOU TRY IT:

52. Solve $x^2 + 2x + 5 = 0$ by completing the square.

Applying the quadratic formula: Exact answers

Watch the video *Introduction to the Quadratic Formula* to complete the following.

1. Factor and apply the zero product rule.

   This method works if the ________________ expression is ________________.

2. Complete the square and apply the square root property.

   This method works in ____________________.

3. Apply the quadratic formula.

   This method works in ____________________.

   State the quadratic formula. ____________________
   Solve.

EXAMPLE: Solve $2x^2 + 6x - 3 = 0$ using the quadratic formula.

$$2x^2 + 6x - 3 = 0$$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{-6 \pm \sqrt{36 + 24}}{4}$$

$$x = \frac{-6 \pm \sqrt{60}}{4}$$

$$x = \frac{-6 \pm 2\sqrt{15}}{4}$$

$$x = \frac{-3 \pm \sqrt{15}}{2}$$

YOU TRY IT:

53. Solve $-4x^2 - 12x + 5 = 0$ by using the quadratic formula.

Solving a quadratic equation with complex roots

Learning Page According to the quadratic formula, the solutions to the quadratic equation ____________________

are as follows.

$$x = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

EXAMPLE:
Solve $5x^2 - 4x + 1 = 0$ using the quadratic formula.

$$5x^2 - 4x + 1 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{4 \pm \sqrt{-4}}{10}$$

$$x = \frac{4 \pm 2i}{10}$$

$$x = \frac{2}{5} \pm \frac{1}{5}i$$

YOU TRY IT:

54. Solve $3x^2 + 2x + 1 = 0$ by using the quadratic formula.

Restriction on a variable in a denominator: Quadratic

Learning Page Division by __________ is not defined. So the expression is ____________________ when its ________________________ is __________.

EXAMPLE:
Find all excluded values for $\frac{y+2}{y^2-9}$.

We must exclude values when the denominator is 0. That is when $y^2 - 9 = 0$.

$$y^2 - 9 = 0$$

$$y^2 = 9$$

$$y = 3, -3$$

$\frac{y+2}{y^2-9}$ is undefined when $y = 3$ or $y = -3$.

YOU TRY IT:

55. Find all excluded values of $\frac{u+7}{u^2-4u+4}$.

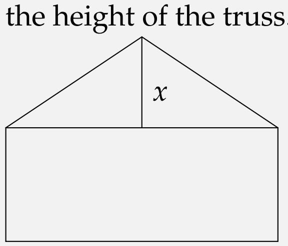
Solving a word problem using a quadratic equation with rational roots

Watch the video *Using a Quadratic Equation in an Application Involving Area* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Sketch the graph from the video and show all work.

YOU TRY IT:

56. The front face of a shed is in the shape shown below. The length of the rectangular region is 3 times the height of the truss. The height of the rectangle is 2 ft more than the height of the truss. If the total area of the front face of the shed is 336 ft^2, determine the length and width of the rectangular region. Let x be the height of the truss.



Solving a word problem using a quadratic equation with irrational roots

Open the Instructor Added Resource which will direct you to a video to complete the following.

The population P of a culture of bacteria is given by ______________________, where t is the time in hours since the culture was started. Determine the time(s) at which the population was ___________. Round to the nearest hour.

EXAMPLE:
If football is kicked straight up with an initial velocity of 128 ft/sec from a height of 5 ft, then its height, h, above the earth is a given by $h = -16t^2 + 128t + 5$. When will the football hit the ground?

The football hits the ground when the height is 0, so we set $h = 0$ and solve for t.

$-16t^2 + 128t + 5 = 0$

Multiply each by -1.

$16t^2 - 128t - 5 = 0$

Use the quadratic formula.

$$x = \frac{128 \pm \sqrt{128^2 - 4(16)(-5)}}{2(16)}$$

$$x = \frac{128 + \sqrt{16704}}{32}, \cancel{\frac{128 - \sqrt{16704}}{32}}$$

There will only be one solution because cannot have a negative time.

$$x = \frac{128 + \sqrt{16704}}{32} \approx 8.04 \text{ sec}$$

YOU TRY IT:

57. If football is kicked straight up with an initial velocity of 128 ft/sec from a height of 5 ft, then its height, h, above the earth is a given by $h = -16t^2 + 128t + 5$. When will the football be at 37 feet?

Solving a rational equation that simplifies to linear: Unlike binomial denominators

Watch the video *Solving a Rational Equation* and complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Solve.

EXAMPLE: Solve the equation.

$$\frac{x}{x-3} = \frac{3}{x-3} - \frac{3}{4}$$

We first note that x cannot be 3.

Multiply both sides of equation by the LCD.

$$4(x-3)\cdot\frac{x}{x-3} = 4(x-3)\cdot\frac{3}{x-3} - 4(x-3)\cdot\frac{3}{4}$$

Simplify.

$$4\cancel{(x-3)}\cdot\frac{x}{\cancel{x-3}} = 4\cancel{(x-3)}\cdot\frac{3}{\cancel{x-3}} - \cancel{4}(x-3)\cdot\frac{3}{\cancel{4}}$$

$$\begin{aligned}4x &= 12 - 3(x-3)\\ 4x &= 12 - 3x + 9\\ 7x &= 21\\ x &= 3\end{aligned}$$

$x = 3$ is a restricted value so there is no solution.

YOU TRY IT: Solve the equation.

58. $\dfrac{3}{4t+4} + 1 = \dfrac{2t-5}{t+1}$

Solving an absolute value inequality: Problem type 3

Watch the video *Introduction to Absolute Value Inequalities* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Solve the equation or inequality.

$|u| =$ ______

$|u|$______

$|u|$______

Solve the inequality.

a.

b.

For a real number $k > 0$,

1. $|u| < k$ is equivalent to ____________________.

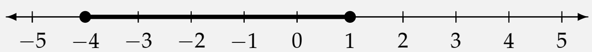
2. $|u| > k$ is equivalent to _______________ or _______________.

EXAMPLE: Graph the solution to the inequality on the number line.

$$|2x+3| \leq 5$$

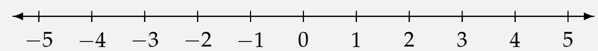
Use the properties from above to rewrite.

$$-5 \leq 2x+3 \leq 5$$
$$-8 \leq 2x \leq 2$$
$$-4 \leq x \leq 1$$



YOU TRY IT: Graph the solution to the inequality on the number line.

59. $|3y-6| > 6$



Solving an absolute value inequality: Problem type 5

Watch the video *Solving an Absolute Value Inequality (Less Than)* to complete the following.

Solve the inequality and write the solution set in interval notation.

For a real number $k > 0$,

1. $|u| < k$ is equivalent to ________________________.

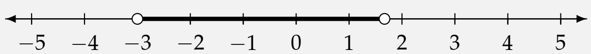
2. $|u| > k$ is equivalent to __________________________________.

EXAMPLE:
Solve the following inequalities.

a) $|3x+2| < 7$
Write the equivalent statements without absolute value.

$$-7 < 3x+2 < 7$$
$$-9 < 3x < 5$$
$$-3 < x < \tfrac{5}{3}$$

The solution in interval notation is $\left(-3, \frac{5}{3}\right)$ and graphically is



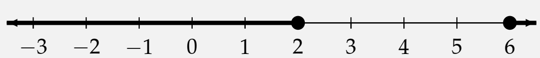
b) $-2|4-x| \leq -4$
First isolate $|4-x|$.

$$|4-x| \geq 2$$

Write the equivalent statements without absolute value.

$$\begin{array}{ccc} 4-x \geq 2 & \text{or} & 4-x \leq -2 \\ -x \geq -2 & \text{or} & -x \leq -6 \\ x \leq 2 & \text{or} & x \geq 6 \end{array}$$

The answer in interval notation is $(-\infty, 2] \cup [6, \infty)$ and graphically is



YOU TRY IT:
Solve the following inequalities.

60. $3 < |2x-1|$

61. $|5-4x| \leq 1$

Solving for a variable in terms of other variables in a rational equation: **Problem type 2**

Learning Page Carefully read the example on the Learning Page.

EXAMPLE: Solve for P.

$$A = P + Prt$$

Factor out P on right.

$$A = P(1+rt)$$

Divide both sides by $1+rt$.

$$\frac{A}{1+rt} = P$$

YOU TRY IT: Solve for d.

62. $S = \frac{n}{2}(a+d)$

Solving an equation using the odd-root property: Problem type 2

Open the Instructor Added Resource which will direct you to a video to complete the following.

Solve for x.

YOU TRY IT: Solve for x.

63. $\frac{1}{2}(x+5)^3 - 64 = 0$

Solving a rational equation that simplifies to quadratic: Binomial denominators, constant numerators

Open the Instructor Added Resource which will direct you to a video to complete the following.

Solve for x. $x \neq$ __________

YOU TRY IT: Solve for x.

64. $\dfrac{1}{x} + \dfrac{1}{x-1} = \dfrac{3}{2}$

Solving a rational equation that simplifies to quadratic: Binomial denominators and numerators

Watch the video *Solving a Rational Equation that Reduces to a Quadratic* and complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Solve the equation.

EXAMPLE: Solve for x.

$$\frac{x-3}{x-1} = \frac{x-2}{x-4} - 1$$

$x = 1$ and $x = 4$ are excluded from the solution.

Multiply both sides by the LCD.

$$(x-1)(x-4)\frac{x-3}{x-1} = \left(\frac{x-2}{x-4} - 1\right)(x-1)(x-4)$$

Simplify.

$$\cancel{(x-1)}(x-4)\frac{x-3}{\cancel{x-1}} = \frac{(x-2)(x-1)\cancel{(x-4)}}{\cancel{x-4}} - 1(x-1)(x-4)$$

$$(x-3)(x-4) = (x-2)(x-1) - (x^2 - 5x + 4)$$

$$x^2 - 7x + 12 = x^2 - 3x + 2 - x^2 + 5x - 4$$

$$x^2 - 9x + 14 = 0$$

$$(x-7)(x-2) = 0$$

$$x = 2, 7$$

YOU TRY IT: Solve for x.

65. $\dfrac{3x+1}{x+5} = \dfrac{x-1}{x+1} + 2$

Solving a rational equation that simplifies to quadratic: Proportional form, advanced

Open the Instructor Added Resource which will direct you to a video to complete the following.

Solve for x.

EXAMPLE: Solve for x.

$$\frac{18}{x^2-8x+12} = \frac{-2x}{x-2}$$

Factor the denominator.

$$\frac{18}{(x-2)(x-6)} = \frac{-2x}{x-2}$$

$x=2$ and $x=6$ are excluded from the solution.

Multiply both sides by the LCD.

$$(x-2)(x-6)\frac{18}{(x-2)(x-6)} = \frac{-2x}{x-2}(x-2)(x-6)$$

Simplify.

$$\cancel{(x-2)(x-6)}\frac{18}{\cancel{(x-2)(x-6)}} = \frac{-2x}{\cancel{x-2}}\cancel{(x-2)}(x-6)$$

$$18 = -2x(x-6)$$

$$18 = -2x^2+12x$$

$$2x^2-12x+18=0$$

$$2(x^2-6x+9)=0$$

$$2(x-3)^2=0$$

$$x=3$$

YOU TRY IT: Solve for y.

66. $\dfrac{2y}{y-6} = \dfrac{12}{y^2-7y+6}$

Solving a radical equation that simplifies to a linear equation: One radical, advanced

Open the e-book to complete the following.

Solving a Radical Equation

Step 1

Step 2

Step 3

Step 4

In solving radical equations, ______________ potentially arise when both sides of the equation are raised to an even power. Therefore, an equation with only ______________ roots will not have extraneous solutions. However, it is still recommended that all potential solutions ______________.

EXAMPLE: Solve for y.

$$\sqrt{y+8}+2=4$$

Isolate the radical.

$$\sqrt{y+8}=2$$

Square both sides.

$$(\sqrt{y+8})^2=(2)^2$$

Simplify.

$$y+8=4$$

$$y=-4$$

Check the solution.

$$\sqrt{-4+8}+2\stackrel{?}{=}4$$

$$\sqrt{4}+2\stackrel{?}{=}4$$

$$4=4$$

$y=-4$ is a solution.

YOU TRY IT: Solve for x.

67. $\sqrt{2x+29}+3=1$

Solving a radical equation that simplifies to a quadratic equation: One radical, advanced

Watch the video *Solving a Radical Equation in which Squaring a Binomial is Required* to complete the following.

Solve the equation.

EXAMPLE: Solve for y.

$$\begin{aligned}\sqrt{y+18}+2 &= y\\ \sqrt{y+18} &= y-2\\ (\sqrt{y+18})^2 &= (y-2)^2\\ y+18 &= y^2-4y+4\\ 0 &= y^2-5y-14\\ 0 &= (y-7)(y+2)\\ y &= -2, 7\end{aligned}$$

Check the solutions.

$$\begin{aligned}\sqrt{-2+18}+2 &\stackrel{?}{=} -2\\ \sqrt{16}+2 &\stackrel{?}{=} -2\\ 4+2 &\stackrel{?}{=} -2\\ 6 &\neq -2\end{aligned} \qquad \begin{aligned}\sqrt{7+18}+2 &\stackrel{?}{=} 7\\ \sqrt{25}+2 &\stackrel{?}{=} 7\\ 5+2 &\stackrel{?}{=} 7\\ 7 &= 7\end{aligned}$$

$y = 7$ is a solution.

YOU TRY IT: Solve for x.

68. $\sqrt{2x+29}+3 = x$

Word problem involving radical equations: Advanced

EXAMPLE:
The distance d (in miles) that an observer can see on a clear day is approximated by $d = \frac{49}{40}\sqrt{h}$, where h is the height of the observer in feet. If Rita can see 24.5 mi, how far above ground is her eye level?

$d = 24.5$ which can also be written as $d = \frac{49}{2}$. We substitute this into the given equation and solve for h.

$$\frac{49}{2} = \frac{49}{40}\sqrt{h}$$

Multiply both sides by $\dfrac{40}{49}$

$$\frac{40}{49}\cdot\frac{49}{2} = \frac{40}{49}\cdot\frac{49}{40}\sqrt{h}$$

$$20 = \sqrt{h}$$

Square both sides.

$$400 \text{ feet} = h$$

YOU TRY IT:

69. If an object is dropped from a height of h meters, the velocity v (in m/sec) at impact is given by $v = \sqrt{19.6h}$. Determine the impact velocity for an object dropped from a height of 10 m.

Solving an equation with exponent $\frac{1}{a}$: Problem type 1

Watch the video *Solving an Equation with Rational Exponents* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Solve the equation.

EXAMPLE: Solve for x.

$$\sqrt[3]{2x-5} = -3$$

Cube both sides.

$$(\sqrt[3]{2x-5})^3 = (-3)^3$$

Simplify.

$$2x - 5 = -27$$
$$2x = -22$$
$$x = -11$$

Check the solution.

$$\sqrt[3]{2(-11)-5} \stackrel{?}{=} -3$$
$$\sqrt[3]{-27} \stackrel{?}{=} -3$$
$$-3 = -3$$

YOU TRY IT: Solve for x.

70. $\sqrt[5]{4x+8} = 2$

Finding the average rate of change of a function

Watch the video *Determining Average Rate of Change* to complete the following.

Determine the average rate of change of the function on the given interval.

$f(x) =$ ______________________ $m =$ ____________________________

a. ________________

b. ________________

c. ________________

EXAMPLE:

Find the average rate of change of $f(x) = x^2 + x - 4$ from $x = 1$ to $x = 3$.

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(3) - f(1)}{3 - 1}$$
$$= \frac{(3^2 + 3 - 4) - (1^2 + 1 - 4)}{2}$$
$$= \frac{8 - (-2)}{2} = 5$$

YOU TRY IT:

71. Find the average rate of change of $f(x) = 3 - 2x - x^2$ from $x = -1$ to $x = 2$.

Word problem involving average rate of change

Learning Page The average rate of change is the ________________ of the line passing through

______________________ and ______________________.

EXAMPLE: Travis is cooking a beef roast. The table below gives the temperature $R(t)$ of the roast in degrees Celsius, at a few times t in minutes after he removed it from the oven. Find the average rate of change for the temperature from 10 to 50 minutes.

Time t	Temperature $R(t)$
0	226.6
10	205.6
30	157.6
50	119.6
70	61.6

The average rate of change over $[x_1, x_2]$ is given by the formula below. In this problem $x_2 = 50$ and $x_1 = 10$. We find the values of the function from the table above.

$$\begin{aligned}\frac{f(x_2) - f(x_1)}{x_2 - x_1} &= \frac{f(50) - f(10)}{50 - 10} \\ &= \frac{119.6 - 205.6}{40} \\ &= \tfrac{-86}{40} = -2.15^{\circ}C \text{ per minute}\end{aligned}$$

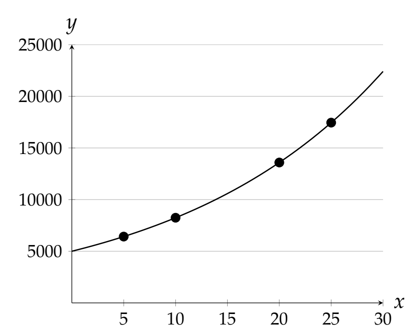
Finding the average rate of change of a function given its graph

Watch the video *Determining Average Rate of Change 1* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

The function given by $y = f(x)$ shows the value of __________ invested at __________ interested compounded continuously after x years.

a. Find the average amount earned per year between the __________ year and __________ year.

$m =$ ____________________

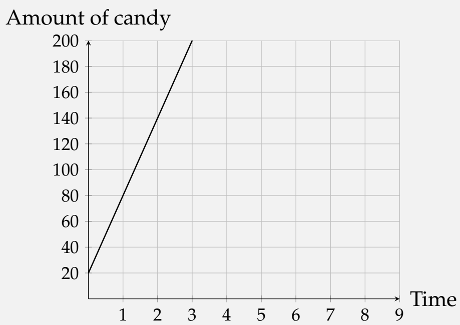


b. Find the average amount earned per year between the __________ year and __________ year.

Finding the initial amount and rate of change given a graph of a linear function

Learning Page Carefully read the example on the Learning Page.

YOU TRY IT:
At a candy factory, a machine is putting candy into a container. The graph shows the amount of candy, in pounds, in the container versus time in minutes.



72. What is the amount of candy in the container at 0 minutes?

73. Describe how the time and amount of candy are related.

Notes from Focus Group:

Notes from Focus Group:

Module 3

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Weekly Checklist

- ☐ Complete MALL time.
- ☐ Work in ALEKS and Notebook at least 4 days a week.
- ☐ Complete the weekly Module and Notebook pages by the due date.
- ☐ Attend Focus Group.
- ☐ Actively participate in Focus Group.
- ☐ Earn extra credit: Complete 10 topics by ____________________.

Evaluating a function: Absolute value, rational, radical

EXAMPLE:
Given $f(x) = \dfrac{x+3}{x-2}$ and
$g(x) = |1-4x|$, find the following.

a) $f(-2)$

$$f(-2) = \frac{-2+3}{-2-2} = \frac{1}{-4} = -\frac{1}{4}$$

b) $g(6)$

$$g(6) = |1-4(6)| = |1-24| = |-23| = 23$$

YOU TRY IT:
Given $f(x) = \dfrac{x-4}{2x+6}$ and
$g(x) = |1-4x|$, find the following.

74. $g(-4)$

75. $f(-3)$

Evaluating a piecewise-defined function

Watch the video *Interpreting a Piecewise-Defined Function* to complete the following.

Evaluate the function for the given values of x.

$$g(x) = \begin{cases} \underline{\hspace{6cm}} & \text{for } x \le -2 \\ \underline{\hspace{6cm}} & \text{for } -2 < x < 3 \\ \underline{\hspace{6cm}} & \text{for } x \ge 3 \end{cases}$$

Pause the video and try these yourself.

a. $g(-3) =$

b. $g(3) =$

c. $g(-2) =$

d. $g(0) =$

e. $g(4) =$

Play the video and check your answers.

Evaluating a cube root function

See a list of cubes Complete the chart below of perfect cubes.

x	x^3	x	x^3
1		6	
2		7	
3		8	
4		9	
5		10	

EXAMPLE:
Given $f(x) = \sqrt[3]{4x+7}$ find $f(-2)$.

$$\begin{aligned} f(-2) &= \sqrt[3]{4(-2)+7} \\ &= \sqrt[3]{-1} = -1 \end{aligned}$$

YOU TRY IT:

76. Given $f(x) = \sqrt[3]{4x+7}$ find $f(5)$.

Variable expressions as inputs of functions: Problem type 2

Watch the video *Evaluating a Function* to complete the following.

Given ____________________. evaluate ____________________.

EXAMPLE:
Given $g(x) = \sqrt{1-4x}$, find $g(x+h)$.

$$\begin{aligned} g(x+h) &= \sqrt{1-4(x+h)} \\ &= \sqrt{1-4x-4h} \end{aligned}$$

YOU TRY IT:

77. Given $f(x) = 3x^2 - 4x + 7$, find $f(x+h)$.

Variable expressions as inputs of functions: Problem type 3

EXAMPLE:
Given $f(x) = 3x^2 - 4x + 7$, find $f(x-2)$.

We substitute $x-2$ into the expression for x.

$$f(x-2) = 3(x-2)^2 - 4(x-2) + 7$$

FOIL and distribute.

$$= 3(x^2 - 4x + 4) - 4x + 8 + 7$$

Distribute and simplify.

$$= 3x^2 - 12x + 12 - 4x + 15$$

$$= 3x^2 - 16x + 27$$

YOU TRY IT:

78. Given $g(x) = \sqrt{1-4x}$, find $g(x^2-4)$.

Domain of a rational function: Excluded values

Learning Page The fraction cannot have a ______________________ of _____.

YOU TRY IT: Find all values of x that are NOT in the domain of g.

79. $g(x) = \dfrac{x+4}{x^2-9}$

Domain of a rational function: Interval notation

Learning Page The domain of any rational function is the set of x for which the ____________________________.

There are ____________________________ on the domain of a rational function.

EXAMPLE:
Find the domain of $f(x) = \dfrac{x-5}{x^2+x-12}$.

We must determine where the denominator is zero. $x^2+x-12 = (x+4)(x-3) = 0$. So $x = 3, -4$. These are the values we want to exclude from the domain.

Domain: $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$

YOU TRY IT:

80. Find the domain of $f(x) = \dfrac{x-2}{x^2-2x-15}$.

Domain of a square root function: Advanced

Open the e-book to complete the following.

Guidelines to Find Domain of a Function

To determine the implied domain of a function defined by $y = f(x)$,

- Exclude values of x that make the ____________________________.
- Exclude values of x that make the ______________________________________.

EXAMPLE:
Find the domain of $g(x) = \sqrt{5x-8}$.

We must determine where $5x-8$ is greater than or equal to zero.

$$\begin{aligned} 5x - 8 &\geq 0 \\ 5x &\geq 8 \\ x &\geq \frac{8}{5} \end{aligned}$$

So the domain is $[\frac{8}{5}, \infty)$

YOU TRY IT:

81. Find the domain of $h(x) = \sqrt{4-7x}$.

Finding the domain of a fractional function involving radicals

Watch the video *Determining Domain and Range of a Function from its Equation* to complete the following.

Write the domain of the function in interval notation.

a.

b.

c.

EXAMPLE: Find the domain of the function.

$$f(x) = \frac{\sqrt{3-x}}{x-1}$$

We must consider two parts.

- We may not have a zero in the denominator, so

$$\begin{aligned} x - 1 &\neq 0 \\ x &\neq 1 \end{aligned}$$

- We also must have 0 or a positive value under the square root.

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \end{aligned}$$

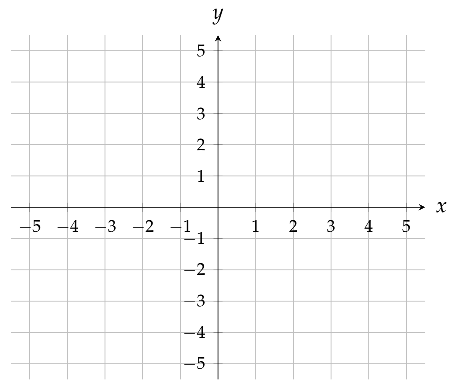
The domain is the intersection of these two sets. In interval notation: $(-\infty, 1) \cup (1, 3)$

YOU TRY IT: Find the domain of the function.

82. $g(x) = \dfrac{4-2x}{\sqrt{9-7x}}$

Domain and range from the graph of a piecewise function

Watch the video *Interpreting Function Values from the Graph* to complete the following.



a. Determine $f(-2)$

b. Determine $f(3)$

c. Find x for which $f(x) = -1$.

d. Find x for which $f(x) = -4$.

e. Determine the x-intercept(s).

f. Determine the y-intercept.

g. Determine the domain.

h. Determine the range.

Domain and range of a linear function that models a real-world situation

Learning Page

- **Description of values for the domain:**

  The domain of a function is the ______________________________.

- **Description of values for the range:**

  The range of a function is the ______________________________.

  To find the range, let's look at the ____________________ for some values of the ____________________.

EXAMPLE:
The Perfect Pickle delivers pickles to its customers. Let C be the total cost to transport the pickles, in dollars. Let P be the amount of pickles transported in pounds. The company can transport up to 30 pounds of pickles. Suppose that C=130P+1500 gives C as a function of P. Describe the domain and range in words and determine the domain and range.

Domain: The domain will be the amount of pickles transported in pounds.
The domain is $[0, 30]$.

- The amount of pickles cannot be negative so the domain must be greater than or equal to 0.
- The company cannot transport more than 30 pounds of pickles so the domain must be less than or equal to 30.
- The amount of pickles could be any amount between 0 and 30.

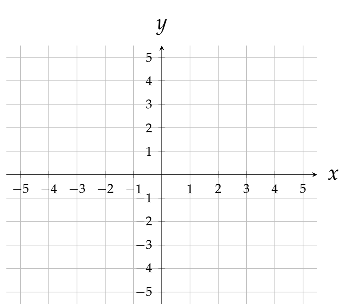
Range: The range will be the cost to transport the pickles in dollars.
The range is $[1500, 5400]$.

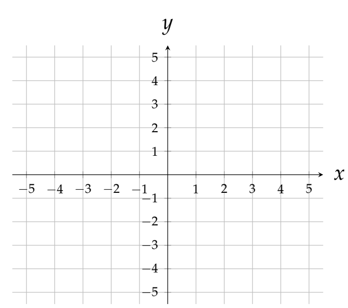
- What would the cost be if 0 pounds of pickles were transported? $C = 1500$.
- What would the cost be if 30 pound of pickles were transported? $C = 130(30) + 1500 = 5400$
- The cost to transport any other amount of pickles will be in between \$1500 and \$5400.

Domain and range from the graph of a continuous function

Watch the video *Determining Domain and Range of a Function from its Graph* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Determine the domain and range.





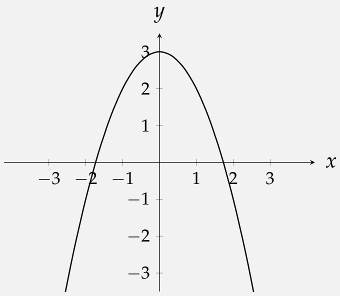
Domain:

Range:

Domain:

Range:

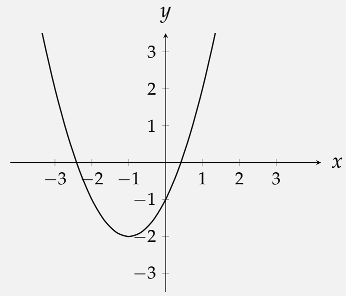
EXAMPLE:
Find the domain and range of the function from the graph.



Domain: $(-\infty, \infty)$
Range: $(-\infty, 3]$

YOU TRY IT:

83. Find the domain and range of the function from the graph.



Identifying functions from relations

Watch the video *Determining Whether a Relation Defines y as a Function of x* to complete the following.

Definition of a Function

Given a relation in x and y, we say that y **is a function of** x if for each value of ________________

____________________, there is ____ in the range.

Determine whether the relation defines y as a function of x.

a. $\{(5,2),(4,-3),(3,1),(5,4)\}$

b. $\{(3,1),(4,2),(-1,2)\}$

YOU TRY IT:
For each relation, determine whether or not it is a function.

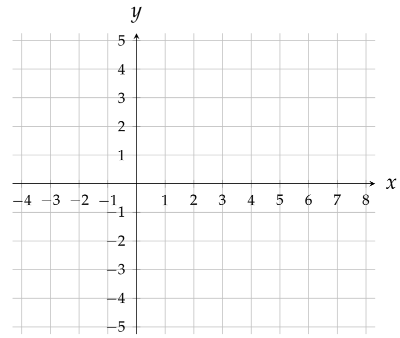
84. $\{(2,3),(-5,1),(0,3),(5,-4)\}$.

85. $\{(1,-2),(-7,3),(1,5),(0,8)\}$.

Vertical line test

Watch the video *Introduction to the Vertical Line Test* to complete the following.

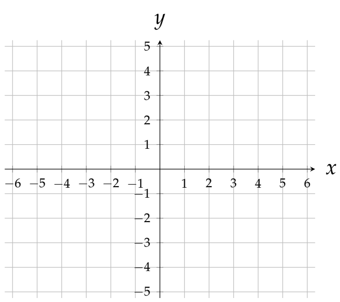
Write the points and plot them on the axes.

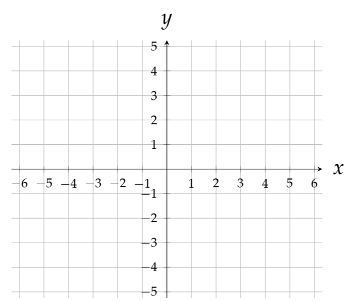


Using the Vertical Line Test

Consider a relation defined by a set of points (x, y) graphed on a rectangular coordinate system. The graph defines y as a function of x if __________ vertical line intersects the graph in ______________________________.

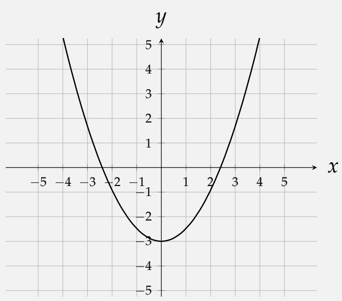
Sketch the graphs from the video below and state if the graph defines y as a function of x.



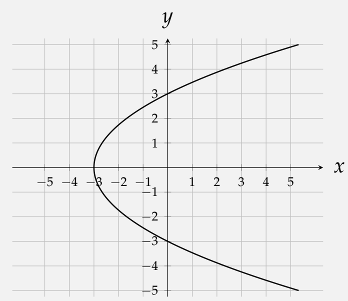


YOU TRY IT: For each relation, determine whether or not it is a function.

86.



87.



Determining whether an equation defines a function: Basic

Watch the video *Determining if a Relation Defines y as a Function of x* to complete the following.

a.

b.

c.

Finding inputs and outputs of a two-step function that models a real-world situation: Function notation

EXAMPLE:
A crew can lay 5 miles of track each day. They need to lay 175 miles of track. The length, L, in miles, that is left to lay after d days is given by the function $L(d) = 175 - 5d$.

a. How many miles of track does the crew have left to lay after 12 days?

We want to substitute 12 in for d to find $L(12)$.

$$\begin{aligned} L(12) &= 175 - 5(12) \\ &= 175 - 60 \\ &= 115 \text{ miles} \end{aligned}$$

b. How many days will it take the crew to lay all of the track?

We want to know when $L(d) = 0$.

$$\begin{aligned} 175 - 5d &= 0 \\ -5d &= -175 \\ d &= 35 \text{ days} \end{aligned}$$

YOU TRY IT:
Steve wants to save \$700 to buy a computer. He saves \$18 each week. The amount A, in dollars he still needs after w weeks is given by the function $A(w) = 700 - 18w$.

88. How much money does Steve still need after 5 weeks?

89. If Steve still needs \$394, how many weeks has he been saving?

Finding inputs and outputs of a function from its graph

Learning Page Each point on the graph of a function f can be written as an ____________________

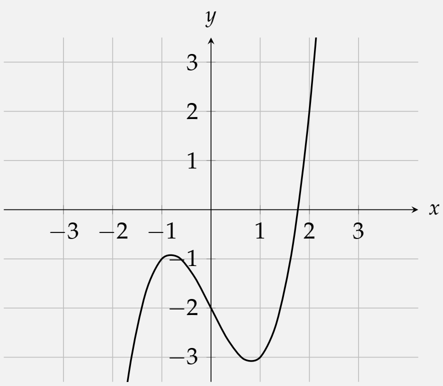
______________________________.

For each point (x, y) on the _______________, the x coordinate gives an _______________ of the function.

The y coordinate gives the corresponding _______________. That is ____________________

The video *Interpreting Function Values from the Graph* may also be helpful. You may find space to take notes under the topic **Domain and range from the graph of a piecewise function**.

EXAMPLE:
Use the graph to find the following.



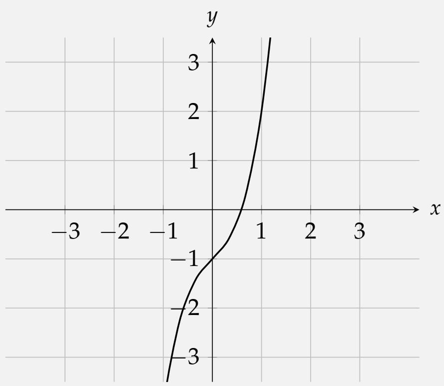
a) $f(2)$

We see the point $(2, 2)$ on the graph, so $f(2) = 2$.

b) One value of x for which $f(x) = -2$

From the graph we see that $f(0) = -2$ so $x = 0$. There are also two other values of x where $f(x) = -2$.

YOU TRY IT:
Use the graph to find the following.



90. $g(1)$

91. One value of x for which $g(x) = -1$

Graphing a parabola of the form $y = ax^2 + c$

Open the e-book to complete the following.

Vertical Shrinking and Stretching of Graphs

Consider a function defined by $y = f(x)$. Let _____ represent a __________ real number.

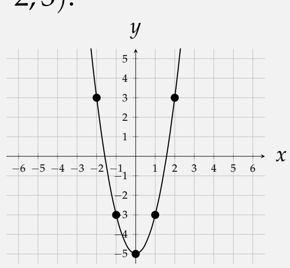
- If __________, then the graph of __________ is the graph of $y = f(x)$ _______________ _______________ by a factor of a.

- If _______________, then the graph of __________ is the graph of $y = f(x)$ ______________________________ by a factor of a.

Note: for any point __________ on the graph of $y = f(x)$, the point __________ is on the graph of $y = af(x)$.

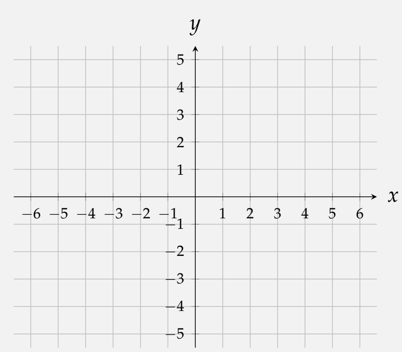
EXAMPLE: Sketch the graph of $y = 2x^2 - 5$.

- We first plot the vertex at $(0, -5)$.
- Next we plot 2 points on either side of the vertex.
  *All parabolas have symmetry so we can use this when finding points.
  - If $x = 1$, then $y = 2(1)^2 - 5 = -3$. Plot $(1, -3)$.
  - If $x = -1$, then $y = 2(-1)^2 - 5 = -3$. Plot $(-1, -3)$.
    We could also have used symmetry. Because the points x values are the same distance from the x value of the vertex, they must have the same y-coordinate.
  - If $x = 2$, then $y = 2(2^2) - 5 = 3$. Plot $(2, 3)$ and using symmetry plot $(-2, 3)$.



YOU TRY IT:

92. Sketch the graph of $y = -\frac{1}{2}x^2 + 3$.



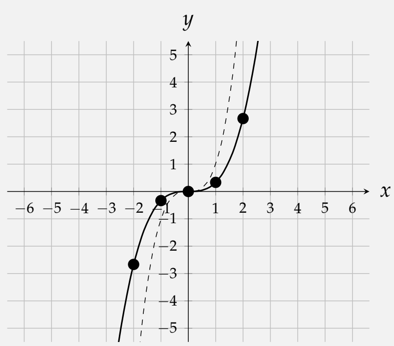
Graphing a cubic function of the form $y = ax^3$

EXAMPLE:
Sketch the graph of $y = \frac{1}{3}x^3$.

We will complete the chart below to obtain the points to graph.

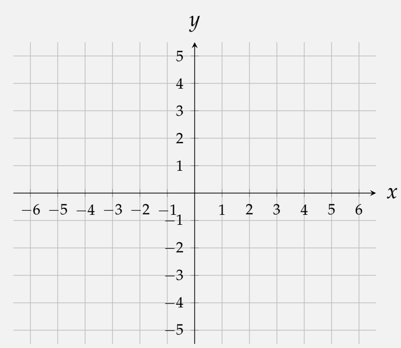
x	$y = \frac{1}{3}x^3$	(x, y)
-2	$y = \frac{1}{3}(-8) = -\frac{8}{3}$	$(-2, -\frac{8}{3})$
-1	$y = \frac{1}{3}(-1) = -\frac{1}{3}$	$(-1, -\frac{1}{3})$
0	$y = \frac{1}{3}(0) = 0$	$(0, 0)$
1	$y = \frac{1}{3}(1) = \frac{1}{3}$	$(1, \frac{1}{3})$
2	$y = \frac{1}{3}(8) = \frac{8}{3}$	$(2, \frac{8}{3})$

The graph of $y = x^3$ is drawn below as a dashed line so you can see how the value of a changes the graph.



YOU TRY IT:

93. Sketch the graph of $y = -\frac{3}{2}x^3$.



Graphing a square root function: Problem type 2

Open the e-book to complete the following.

Vertical Translations of Graphs

Consider a function defined by $y = f(x)$. Let k represent a positive real number.

- The graph of ______________ is the graph of $y = f(x)$ shifted ________________________.
- The graph of ______________ is the graph of $y = f(x)$ shifted ________________________.

Continued on the next page

Horizontal Translations of Graphs

Consider a function defined by $y = f(x)$. Let h represent a positive real number.

- The graph of ______________ is the graph of $y = f(x)$ shifted ________________________.
- The graph of ______________ is the graph of $y = f(x)$ shifted ________________________.

EXAMPLE:
Sketch the graph of $f(x) = \sqrt{x-2} - 3$.

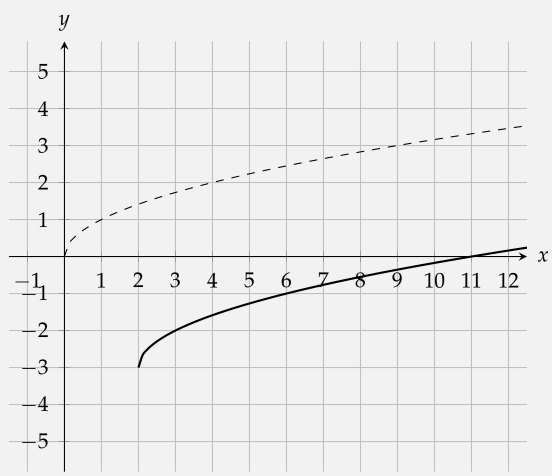
This is the graph of $f(x) = \sqrt{x}$ shifted 2 units right and 3 units down so the **leftmost point** will be $(2, -3)$.

The graph will have x values greater than or equal to 2.

We will find 3 other points on the graph.

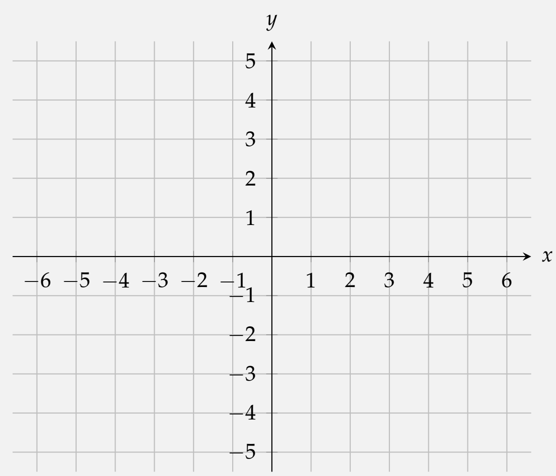
x	$f(x) = \sqrt{x-2} - 3$	(x, y)
3	$f(3) = \sqrt{3-2} - 3 = -2$	$(3, -2)$
6	$f(6) = \sqrt{6-2} - 3 = -1$	$(6, -1)$
11	$f(11) = \sqrt{11-2} - 3 = 0$	$(11, 0)$

The graph of $y = \sqrt{x}$ is drawn below as a dashed line so you can see how the graph has shifted.



YOU TRY IT:

94. Sketch the graph of $g(x) = \sqrt{x+1} + 2$.



Transforming the graph of a function by reflecting over an axis

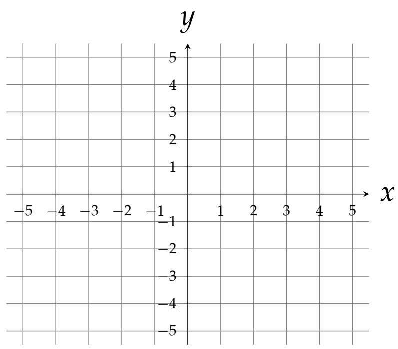
Watch the video *Investigating Reflections Across the x and y-Axes* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

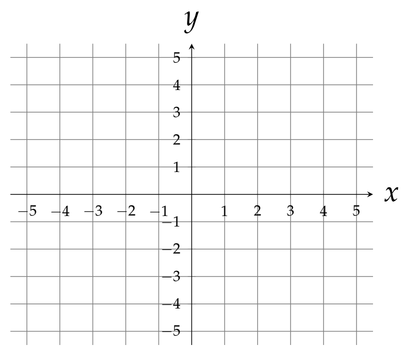
Reflections Across the x and y-Axes

Consider a function defined by $y = f(x)$.

- The graph of ______________ is the graph of $y = f(x)$ reflected across the ______________.
- The graph of ______________ is the graph of $y = f(x)$ reflected across the ______________.

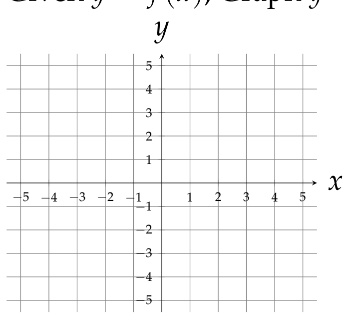
Sketch the blue graph from the video using a dashed line and the red graph using a solid line.





Sketch the graph of $y = f(x)$ using a dashed line and the transformed graph using a solid line.

Given $y = f(x)$, Graph $y = f(-x)$.



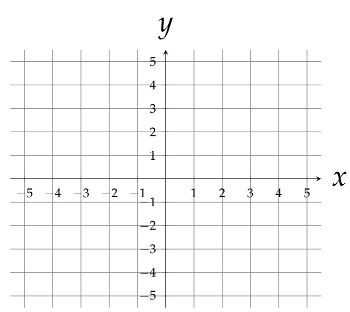
Points on $y = f(x)$:

________, ________, ________, ________

Points on $y = f(-x)$:

________, ________, ________, ________

Given $y = f(x)$, Graph $y = -f(x)$.



Points on $y = f(x)$:

________, ________, ________, ________

Points on $y = -f(x)$:

________, ________, ________, ________

Transforming the graph of a function by shrinking or stretching

Watch the video *Investigating Horizontal Shrinking and Stretching* to complete the following.

Horizontal Shrinking and Stretching of Graphs

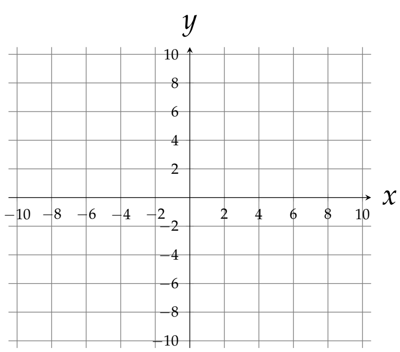
Consider a function defined by $y = f(x)$. Let ______ represent a __________ real number.

- If __________, then the graph of __________ is the graph of $y = f(x)$ ______________ ______________ by a factor of a.
- If ______________, then the graph of __________ is the graph of $y = f(x)$ ______________________________ by a factor of a.

Note: for any point __________ on the graph of $y = f(x)$, the point __________ is on the graph of $y = f(ax)$.

Sketch the graph of $y = f(x)$ using a dashed line and the transformed graph using a solid line.

Given $y = f(x)$, Graph $y = f(3x)$.



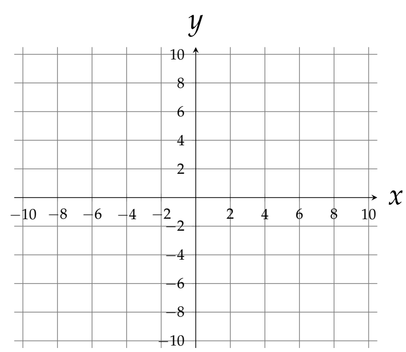
Points on $y = f(x)$:

__________, __________, __________

Points on $y = f(3x)$:

__________, __________, __________

Given $y = f(x)$, Graph $y = f(\frac{1}{3}x)$.



Points on $y = f(x)$:

__________, __________, __________

Points on $y = f(\frac{1}{3}x)$:

__________, __________, __________

Transforming the graph of a function using more than one transformation

Open the e-book to complete the following.

Steps for Graphing Multiple Transformations of Functions

To graph a function requiring multiple transformations, use the following order.

1.

2.

3.

4.

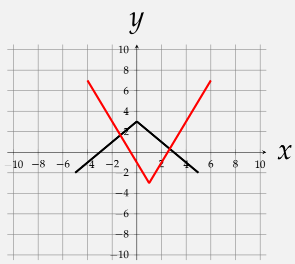
EXAMPLE:
The graph of $y = f(x)$ is shown. Draw the graph of $y = -2f(x-1)+3$.

This is the graph of $y = f(x)$ that is

- stretched vertically by a factor of 2
- reflected across the x-axis
- shifted right 1 and up 3.

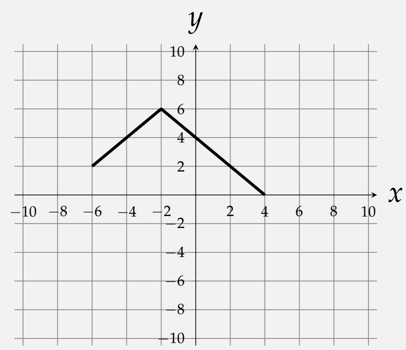
Consider the following points:

Original	Stretch	Reflect	Shift
$(-5,-2)$	$(-5,-4)$	$(-5,4)$	$(-4,7)$
$(0,3)$	$(0,6)$	$(0,-6)$	$(1,-3)$
$(5,-2)$	$(5,-4)$	$(5,4)$	$(6,7)$



YOU TRY IT:

95. The graph of $y = g(x)$ is shown. Draw the graph of $y = \frac{1}{2}g(x+2)-3$.



Transforming the graph of a quadratic, cubic, square root, or absolute value function

Possible transformations on a graph are reflecting about an axis, shifting, stretching, and shrinking. The chart below summarizes all the possible transformations of parent functions.

Transformations of functions

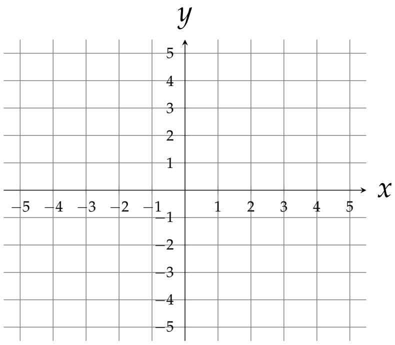
Consider a function defined by $y = f(x)$. If h, k, and a represent positive real numbers, then the graphs of the following functions are related to $y = f(x)$ as follows.

Transformation	**Effect on the Graph of f**	**Changes to Points on f**
Vertical Translations of Graphs		
$y = f(x) + k$	Shift ________ units	Replace (x, y) by ______
$y = f(x) - k$	Shift ________ units	Replace (x, y) by ______
Horizontal translations		
$y = f(x - h)$	Shift ________ units	Replace (x, y) by ______
$y = f(x + h)$	Shift ________ units	Replace (x, y) by ______
Vertical stretch/shrink	Vertical ______ if $a > 1$	
$y = af(x)$	Vertical ______ if $0 < a < 1$	Replace (x, y) by ______
	Graph is stretched/shrunk vertically by a factor of ___	
Horizontal stretch/shrink	Horizontal ______ if $a > 1$	
$y = f(ax)$	Horizontal _____ if $0 < a < 1$	Replace (x, y) by ______
	Graph is shrunk/stretched horizontally by a factor of ___	
Reflection		
$y = -f(x)$	Reflection across the _____	Replace (x, y) by ______
$y = f(-x)$	Reflection across the _____	Replace (x, y) by ______

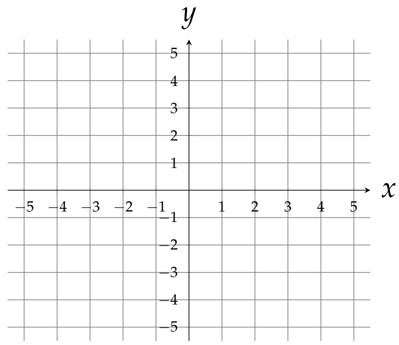
Matching parent graphs with their equations

Basic functions and Their Graphs

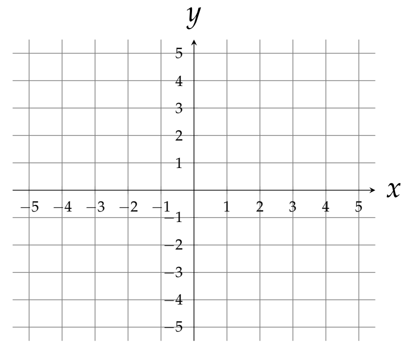
1. Linear functions
$f(x) = mx + b$



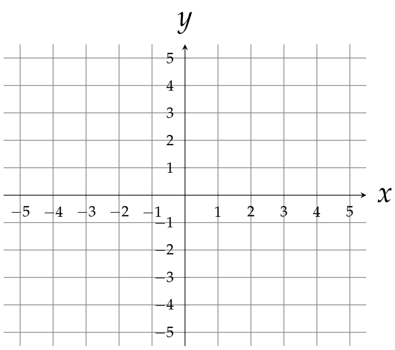
Constant functions
$f(x) = b$



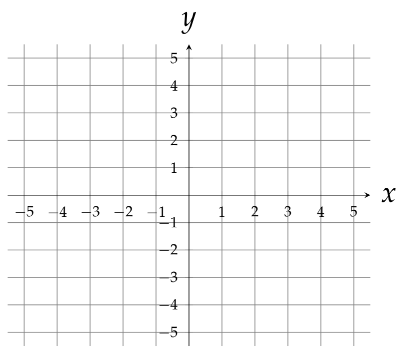
2. Identity function
$f(x) = x$



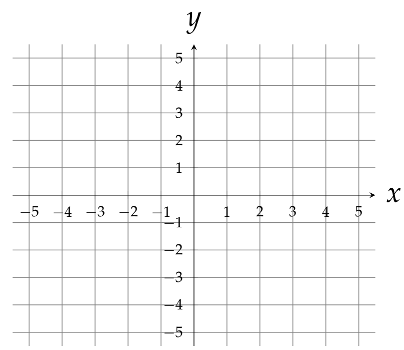
3. Quadratic function
$f(x) = x^2$



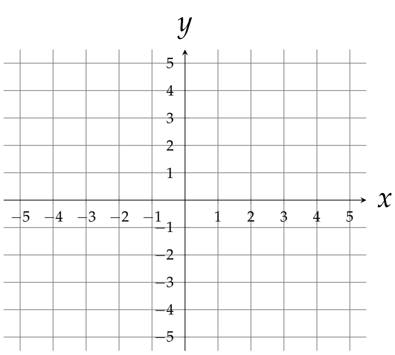
4. Cube function
$f(x) = x^3$



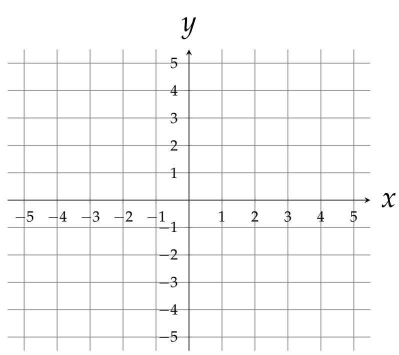
5. Square root function
$f(x) = \sqrt{x}$



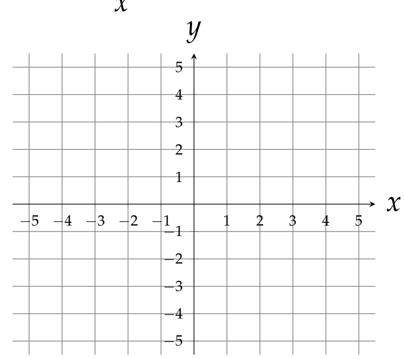
6. Cube root function
$f(x) = \sqrt[3]{x}$



7. Absolute value function
$f(x) = |x|$



8. Reciprocal function
$f(x) = \dfrac{1}{x}$



How the leading coefficient affects the graph of a parabola

Learning Page An equation of the form ____________ $(a \neq 0)$ describes a ________________ whose vertex is at the __________.

The value of the ____________________ tells us how the parabola looks.

(a) A ____________ leading coefficient (________) gives a parabola that opens ____________.

A ____________ leading coefficient (________) gives a parabola that opens ____________.

(b) A ____________ parabola has a leading coefficient a ____________________.

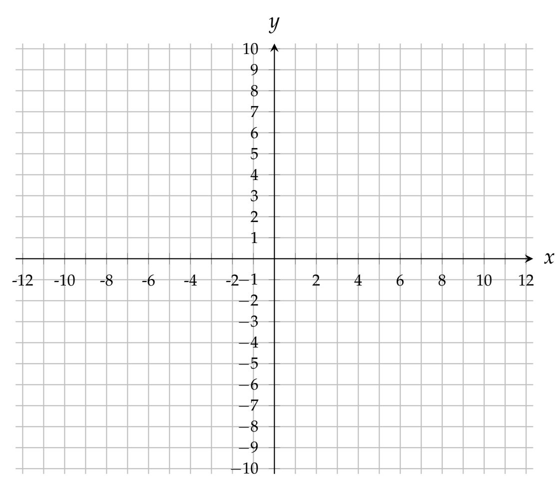
A ____________ parabola has a leading coefficient a ____________________.

Translating the graph of an absolute value function: Two steps

Open the Instructor Added Resource which will direct you to a video to complete the following.

Sketch the graph of ________________.

Translations:



x-intercept(s):

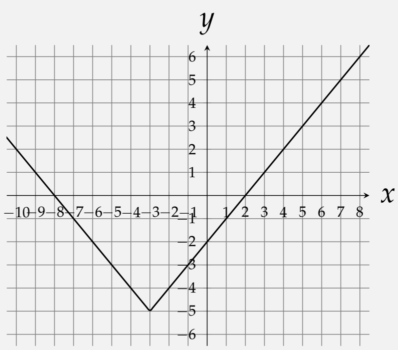
y-intercept:

EXAMPLE: Sketch the graph of $g(x) = |x+3| - 5$.

- This is the graph of $g(x) = |x|$ shifted left 3 and down 5.
- We also find the x and y intercepts to obtain the graph.
  - Let $x = 0$ to find the y-intercept: $y = |0+3| - 5 = 3 - 5 = -2$.
  - Let $y = 0$ to find the x-intercept(s):

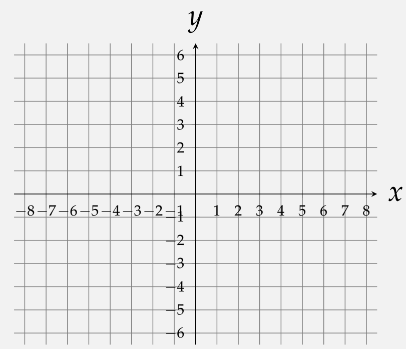
$$|x+3| - 5 = 0$$
$$|x+3| = 5$$

$$x + 3 = 5 \quad \text{or} \quad x + 3 = -5$$
$$x = 2 \quad \text{or} \quad x = -8$$



YOU TRY IT:

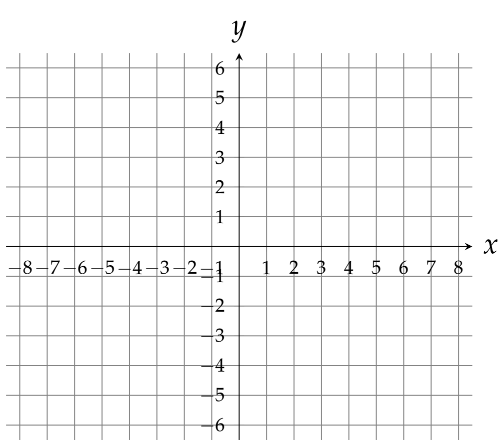
96. Sketch the graph of $f(x) = |x+2| - 3$.



Writing an equation for a function after a vertical and horizontal translation

Open the Instructor Added Resource which will direct you to a video to complete the following.

Using translations of the base graph $y = |x|$, write the equation of the graph shown below.

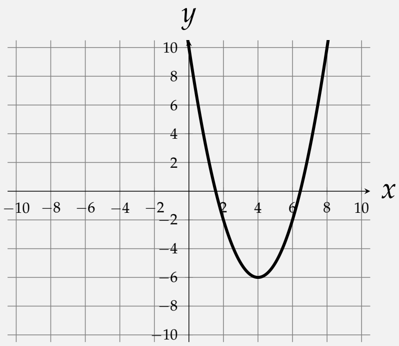


The base graph has been moved _____ units to the _______________ and _____ units _______________.

The equation of the graph is ________________________.

YOU TRY IT: Write the equation of the graph given below.

97.



Notes from Focus Group:

Notes from Focus Group:

Module 4-Review Module

To help you review for your upcoming exam, this module contains all of the topics from the modules since the last exam. Topics that you have already mastered will not appear in your carousel.

- ☐ Complete this module before you take the ALEKS exam.
- ☐ Each exam has two parts.
  - The ALEKS exam (100 pts)
    - The ALEKS exam must be taken in the MALL.
    - The ALEKS exam is a Comprehensive Knowledge Check.
      - Your score is the number of topics you have mastered out of the number of topics you should have mastered by this point.
    - If you lose topics on your ALEKS exam, your Review Module completion grade will not change.
    - Your scratch work for the ALEKS exam must be numbered and turned in through Blackboard.
  - The Written exam (25 pts)
    - Take your written exam in class the day of your focus group.
    - To study for the written exam:
      - Rework your old Focus Group assignments.
      - Rework any topics in ALEKS you may have lost on the ALEKS exam.

	Score
ALEKS Exam	
Written Exam	

Module 5

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Weekly Checklist

- ☐ Complete MALL time.
- ☐ Work in ALEKS and Notebook at least 4 days a week.
- ☐ Complete the weekly Module and Notebook pages by the due date.
- ☐ Attend Focus Group.
- ☐ Actively participate in Focus Group.
- ☐ Earn extra credit: Complete 10 topics by ____________________.

Identifying the center and radius to graph a circle given its equation in standard form

Watch the video *Identifying the Center and Radius of a Circle from Standard Form* to complete the following.

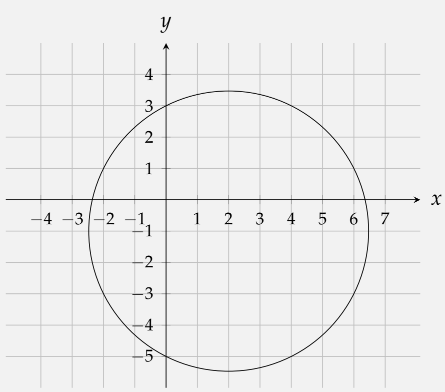
An equation of a circle is given. Identify the center and radius.

Equation	**Standard form**	**Center**	**Radius**
$(x-h)^2+(y-k)^2=r^2$	$(x-h)^2+(y-k)^2=r^2$	(h,k)	r

EXAMPLE:
Find the center and radius of the circle and sketch the graph.

$$(x-2)^2+(y+1)^2=20$$

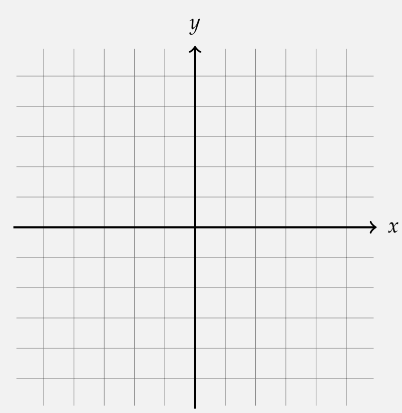
Center: $(2,-1)$ and Radius: $\sqrt{20}=2\sqrt{5}$



YOU TRY IT:

98. Find the center and radius of the circle and sketch the graph.

$$(x+1)^2+(y+3)^2=4$$



Identifying the center and radius of a circle given in general form: Basic

Watch the video *Given an Equation of a Circle in General Form Write the Standard Form* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

$(x-8)^2 + (y+3)^2 = 28$ ________________ form

$x^2 + y^2 - 16x + 6y + 45 = 0$ ________________ form

Write the equation in standard form $(x-h)^2 + (y-k)^2 = r^2$. Then identify the center and radius.

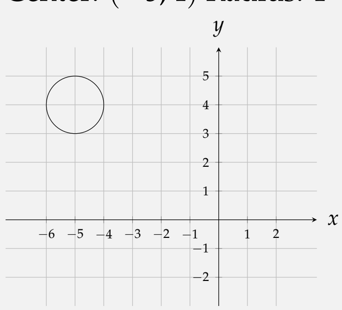
EXAMPLE:
Find the center and radius of the circle and sketch the graph.

$$2x^2 + 20x + 2y^2 - 16y + 80 = 0$$

Before completing the square, we must divide by 2 so the coefficient of x and y is a 1.

$$\begin{aligned} 2x^2 + 20x + 2y^2 - 16y + 80 &= 0 \\ x^2 + 10x + y^2 - 8y &= -40 \\ x^2 + 10x + 25 + y^2 - 8y + 16 &= -40 + 25 + 16 \\ (x+5)^2 + (y-4)^2 &= 1 \end{aligned}$$

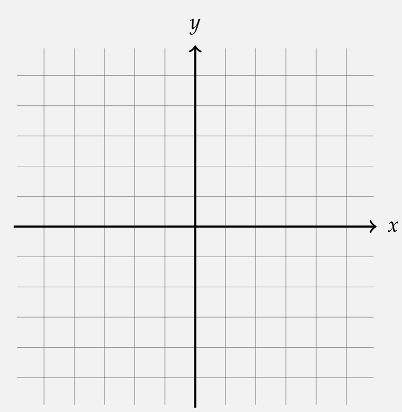
Center: $(-5, 4)$ Radius: 1



YOU TRY IT:

99. Find the center and radius of the circle and sketch the graph.

$$3x^2 - 18x + 3y^2 - 24y = 0$$



Writing an equation of a circle given its center and a point on the circle

Learning Page The standard form of an equation of a circle with center (h, k) and radius r is

____

We need to find ______, which is the __________________ from the ____________ to a point on the circle.

So we can find r using the ______________________________.

EXAMPLE: Write the equation of the circle with center $(5, -1)$ and passing through $(1, 3)$.

- We begin by finding the radius. This is the distance between the two given points.

$$r = \sqrt{(5-1)^2 + (-1-3)^2}$$
$$r = \sqrt{16+16}$$
$$r = \sqrt{32} = 4\sqrt{2}$$

- We have the center and radius so we write the equation.

$$(x-5)^2 + (y+1)^2 = 32$$

YOU TRY IT:

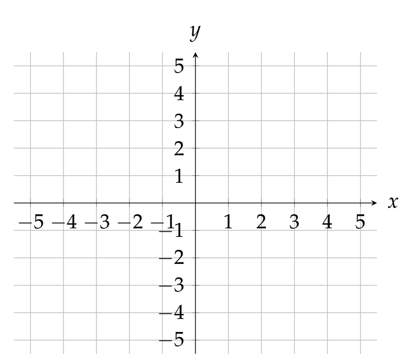
100. Write the equation of the circle with center $(-3, 5)$ and passing through $(4, 5)$.

Finding where a function is increasing, decreasing, or constant given the graph: Interval notation

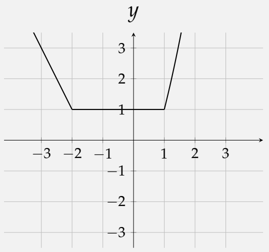
Watch the video *Determining the Intervals over Which a Function Increases Decreases or is Constant with Open Intervals* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Sketch the graph of $f(x)$. Use interval notation to write the intervals over which f is

1. increasing.
2. decreasing.
3. constant.



EXAMPLE: Determine where the function below is increasing, decreasing, or constant.

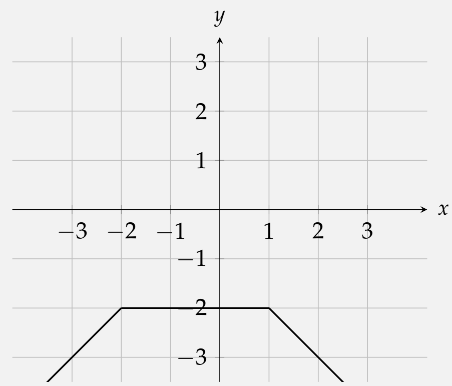


The function is

- Increasing on $(1, \infty)$
- Decreasing on $(-\infty, -2)$
- Constant on $(-2, 1)$

YOU TRY IT: Determine where the function below is increasing, decreasing, or constant.

101.



Finding where a function is increasing, decreasing, or constant given the graph

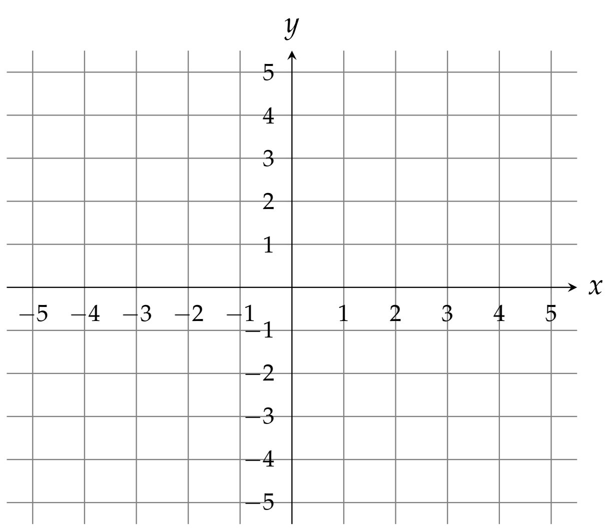
Learning Page If the graph is **horizontal**, the function is ______________________.

If the graph is **falling**, the function is ______________________.

If the graph is **rising**, the function is ______________________.

Finding values and intervals where the graph of a function is zero, positive, or negative

Open the Instructor Added Resource which will direct you to a video to complete the following.



a. Is $f(-2)$ negative?

b. For which value(s) of x is $f(x) < 0$?

c. For which value(s) of x is $f(x) = 0$?

d. For which value(s) of x is $f(x) > 0$

Finding local maxima and minima of a function given the graph

Watch the video *Introduction to Relative Maxima and Minima* to complete the following.

Relative Minimum and Relative Maximum Values

- $f(a)$ is a **relative maximum** of f if there exists an open interval containing a such that

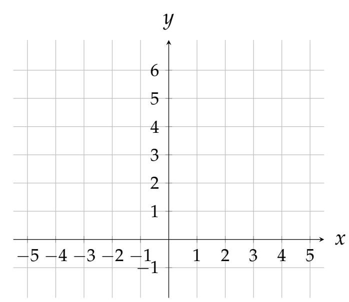
  ____________________ for all x in the interval.

- $f(a)$ is a **relative minimum** of f if there exists an open interval containing a such that

  ____________________ for all x in the interval.

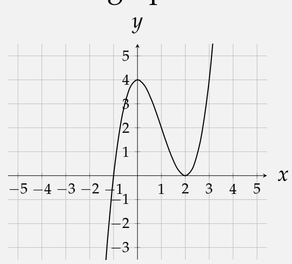
Note: An __________ interval is an interval in which the endpoints are ________________________.

a. Determine the relative maxima.



b. Determine the relative minima.

EXAMPLE:
Use the graph of the function f below to find:



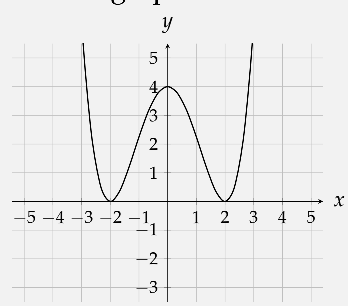
a) Local maximum and minimum values of f

- Local maximum value: 4
- Local minimum value: 0

b) Values at which f has a local maximum and minimum

- Local maximum at $x = 0$
- Local minimum at at $x = 2$

YOU TRY IT:
Use the graph of the function f below to find:



102. All local maximum and minimum values of f

103. All values at which f has a local maximum and minimum

Finding the absolute maximum and minimum of a function given the graph

Learning Page We will use the following information about absolute maximums and minimums, vertical asymptotes, and "holes".

Suppose the domain of a function f is an interval.

- Absolute maximums and minimums:

  The absolute ____________ of f is the ____________ ________________ of any point on the graph of f.

  The absolute ____________ of f is the ____________ ________________ of any point on the graph of f.

- Vertical asymptotes:

  Suppose the graph of f has a vertical asymptote, ________.

  As the x-coordinatesof the graph of f approach a, the y-coordinates approach ________ or ________.

  If the y-coordinates approach ________, then the function will __________ have an absolute ____________.

  If the y-coordinates approach ________, then the function will __________ have an absolute ____________.

- "Holes":

  A "hole" in the graph of f is show as a __________ __________.

  A "hole" is a point that is __________ on the graph of f.

  If a "hole" in the graph of f has a ____________ y-coordinate than any point on the graph of f, then the function does __________ have an absolute ____________.

  If a "hole" in the graph of f has a ____________ y-coordinate than any point on the graph of f, then the function does __________ have an absolute ____________.

Even and odd functions: Problem type 1

Watch the video *Introduction to Even and Odd Functions* to complete the following.

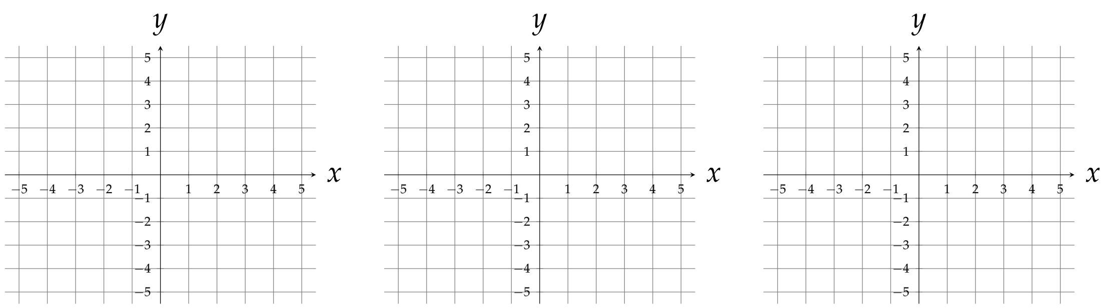
Even and Odd Functions

- A function f is an **even function** if ____________________ for all x in the domain of f.

  The graph of an even function is symmetric with respect to the ____________________.

- A function f is an **odd function** if ____________________ for all x in the domain of f.

  The graph of an odd function is symmetric with respect to the ____________________.



EXAMPLE: Determine if the following are even, odd or neither.

a) $f(x) = 4x^3 - x + \frac{1}{x}$

$$f(-x) = 4(-x)^3 - (-x) + \frac{1}{-x}$$
$$= -4x^3 + x - \frac{1}{x}$$
$$-f(x) = -1(4x^3 - x + \tfrac{1}{x}) = -4x^3 + x - \frac{1}{x}$$

$f(-x) = -f(x)$ so f is odd.

b) $f(x) = 2x^5 + 3x^3 - 5$

$$f(-x) = 2(-x)^5 + 3(-x)^3 - 5$$
$$= -2x^5 - 3x^3 - 5$$
$$-f(x) = -(2x^5 + 3x^3 - 5)$$
$$= -2x^5 - 3x^3 + 5$$

$f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$ so f is neither even or odd.

c) $f(x) = 3x^4 + 5x^2 - 7$

$$f(-x) = 3(-x)^4 + 5(-x)^2 - 7$$
$$= 3x^4 + 5x^2 - 7$$

$f(-x) = f(x)$ so f is even.

YOU TRY IT: Determine if the following are even, odd or neither.

104. $g(x) = \dfrac{2}{x-3}$

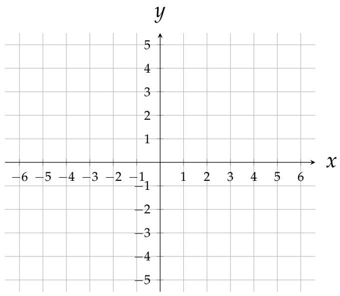
105. $g(x) = 3|x| + 2$

106. $g(x) = x^3 - x$

Determining if graphs have symmetry with respect to the x-axis, y-axis, or origin

Watch the video *Introduction to Symmetry* to complete the following.

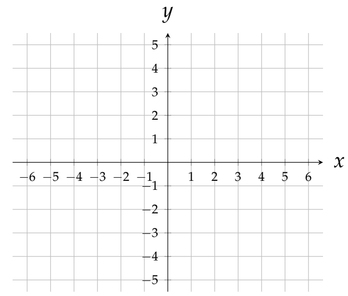
Symmetry with respect to the x-axis



Every point (x, y) has a

mirror image ________________.

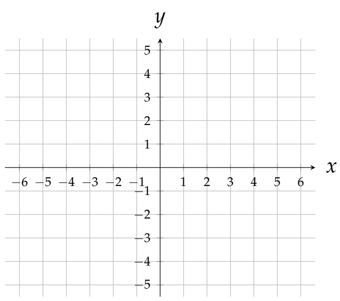
Symmetry with respect to the y-axis



Every point (x, y) has a

mirror image ________________.

Symmetry with respect to the origin

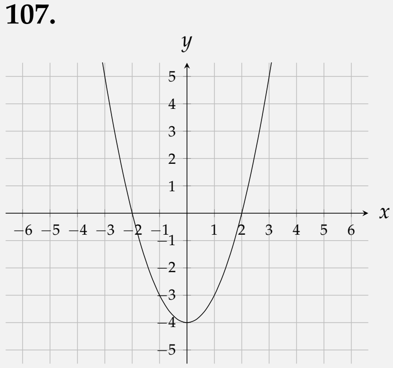


Every point (x, y) has a

mirror image ________________.

YOU TRY IT: Determine what kind of symmetry (if any) applies to the graph.

107.



Testing an equation for symmetry about the axes and origin

Watch the video *Testing for Symmetry* to complete the following.

Tests for Symmetry

Consider an equation in the variables x and y.

- The graph of an equation is **symmetric with respect to the** ____________ if substituting ____________________ in the equation results in an equivalent equation.
- The graph of an equation is **symmetric with respect to the** ____________ if substituting ____________________ in the equation results in an equivalent equation.
- The graph of an equation is **symmetric with respect to the** ____________ if substituting ____________________ and ____________________ in the equation results in an equivalent equation.

Determine whether the graph of the equation is symmetric with respect to the x-axis, y-axis, origin, or none of these.

a.

b.

EXAMPLE:
Determine whether the graph of the equation is symmetric with respect to the x-axis, the y-axis, or the origin.

$$x^2y^2 + xy = 4$$

- Replace y with $-y$.

$$x^2(-y)^2 + x(-y) = 4$$
$$x^2y^2 - xy = 4$$

This is not equivalent to $x^2y^2 + xy = 4$ so it is not symmetric to the x-axis.

- Replace x with $-x$.

$$(-x)^2y^2 + (-x)(y) = 4$$
$$x^2y^2 - xy = 4$$

This is not equivalent to $x^2y^2 + xy = 4$ so it is not symmetric to the y-axis.

- Replace x with $-x$ and y with $-y$.

$$(-x)^2(-y)^2 + (-x)(-y) = 4$$
$$x^2y^2 + xy = 4$$

This is equivalent to $x^2y^2 + xy = 4$ so it is symmetric to the origin.

YOU TRY IT:

108. Determine whether the graph of the equation is symmetric with respect to the x-axis, the y-axis, or the origin.

$$5x^2 + 8y^2 = 14$$

Introduction to the composition of two functions

Watch the video *Composing Functions* to complete the following.

Composition of Functions

The **composition of** f **and** g, denoted $f \circ g$ is defined by $(f \circ g)(x) =$ ________________. The domain of $f \circ g$ is the set of real numbers x in the domain of g such that

__________________________.

Evaluate the given functions for

$f(x) =$ ________________ $g(x) =$ ________________ $h(x) =$ ________________

a.

b.

Composition of two functions: Basic

Learning Page Given functions f and g, we can define a new function, $f \circ g$, as follows.

$$(f \circ g)(x) = \_\_\_\_\_\_\_\_\_\_\_\_$$

This function is called the **composition** of ________________.

Note that a value of $f \circ g$ is obtained as follows.

First evaluate the function ________________ to obtain ________.

Then evaluate the function ________________ to obtain ____________.

The domain of $f \circ g$ is the set of all values x for which ________ is in the ____________________.

EXAMPLE:
Given $f(x) = x^2 - 3x$ and $g(x) = \sqrt{x-1}$, find $(f \circ g)(5)$.

$$\begin{aligned}(f \circ g)(5) &= f(g(5)) \\ &= f(\sqrt{5-1}) \\ &= f(2) \\ &= 2^2 - 3(2) \\ &= -2\end{aligned}$$

YOU TRY IT:
Given $f(x) = 3x^2 + 2x + 3$ and $g(x) = 1 - \frac{1}{x}$, find $(f \circ g)(1)$.

109. $(f \circ g)(1)$

Expressing a function as a composition of two functions

Watch the video *Decomposing a Function* to complete the following.

Find two functions f and g such that $h(x) = (f \circ g)(x)$.

Composition of a function with itself

Open the Instructor Added Resource which will direct you to a video to complete the following.

Given ______________________, find and simplify $(f \circ f)(x)$.

EXAMPLE:
Given $f(x) = x^2 + 2$ and $g(x) = \frac{1}{x-4}$, find the following.

a) $(f \circ f)(x)$

$$f(f(x)) = f(x^2+2)$$
$$(x^2+2)^2 + 2$$
$$= (x^4 + 2x^2 + 2x^2 + 4) + 2$$
$$= x^4 + 4x^2 + 6$$

b) $(g \circ g)(x)$

$$g(g(x)) = g\left(\tfrac{1}{x-4}\right)$$
$$= \frac{1}{\frac{1}{x-4} - 4}$$
$$= \frac{1}{\frac{1}{x-4} - 4} \cdot \frac{x-4}{x-4}$$
$$= \frac{x-4}{1-4(x-4)}$$
$$= \frac{x-4}{1-4x+16}$$
$$= \frac{x-4}{17-4x}$$

YOU TRY IT:
Given $f(x) = \frac{3}{x}$ and $g(x) = x^2 - 5$, find the following functions and their domains.

110. $(f \circ f)(x)$

111. $(g \circ g)(x)$

Composition of two functions: Advanced

Watch the video *Composing Functions and Determining Domain 1* to complete the following.

For the given functions, evaluate $(q \circ m)(x)$ and write the domain in interval notation.

EXAMPLE:
Given $f(x) = \frac{x}{x+2}$ and $g(x) = \frac{1}{x-4}$, find the following functions and their domains.

a) $(f \circ g)(x)$

$$\begin{aligned} f(g(x)) &= f\left(\tfrac{1}{x-4}\right) \\ &= \frac{\frac{1}{x-4}}{\frac{1}{x-4}+2} \\ &= \frac{\frac{1}{x-4}}{\frac{1}{x-4}+2} \cdot \frac{x-4}{x-4} \\ &= \frac{1}{1+2(x-4)} \\ &= \frac{1}{2x-7} \end{aligned}$$

We must exclude 4 from the domain and we must also exclude values of x where $\frac{1}{x-4} + 2 = 0$. We solve this equation for x.

$$\begin{aligned} \tfrac{1}{x-4} + 2 &= 0 \\ 1 + 2(x-4) &= 0(x-4) \\ 1 + 2x - 8 &= 0 \\ 2x &= 7 \\ x &= \tfrac{7}{2} \end{aligned}$$

The domain of $f \circ g$ is $(-\infty, \frac{7}{2}) \cup (\frac{7}{2}, 4) \cup (4, \infty)$.

YOU TRY IT:
Given $f(x) = \frac{3}{x}$ and $g(x) = \frac{x-1}{x-4}$, find the following functions and their domains.

112. $(g \circ f)(x)$

Word problem involving composition of two functions

Open the e-book and read **EXAMPLE 10** to complete the following.

At a popular website the cost to download individual songs is ______________ per song. In addition, a first time visitor to the website has a one-time coupon for ______________ off.

a. Write a function to represent the cost $C(x)$ (in \$) for a first-time visitor to purchase x songs.

$C(x) =$ ______________

The cost function is a ____________________________________.

b. The sales tax for online purcahses depends on the location of the business and customer. If the sales tax rate on a purchase is _________, write a function to represent the total cost $T(a)$ for a first-time visitor who buys a dollars in songs.

$T(a) =$ ______________ $=$ _________

The total cost is the ____________________________________.

c. Find $(T \circ C)(x)$ and interpret the meaning in context.

$(T \circ C)(x) = T(C(x)) =$ _________ $=$ __________________ $=$ __________________

$(T \circ C)(x)$ represents the ____________________________________ for a first-time visitor to the website.

d. Evaluate $(T \circ C)(10)$ and interpret the meaning in context.

$(T \circ C)(10) =$ __________________ $=$ _________

The ______________ for a first-time visitor to ____________________________.

Quotient of two functions: Basic

Watch the video *Evaluating Functions for a Given Value of x* to complete the following.

Evaluate the functions for the given values of x.

$f(x) =$ ________________ $g(x) =$ ________________ $h(x) =$ ________________

a.

b.

Sum, difference, and product of two functions

Watch the video *Introduction to Operations on Functions* to complete the following.

Sum, Difference, Product, and Quotient of Functions

Given the functions f and g, the functions $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$ are defined by:

$$(f+g)(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$(f-g)(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$(f \cdot g)(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\left(\frac{f}{g}\right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

The domains of the functions $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$ are all real numbers in the

________________________ of the individual functions f and g.

For $\frac{f}{g}$ we further restrict the domain to ____________________________.

Find $(f+g)(x)$.

EXAMPLE:
Given $f(x) = x^2 - 3x$ and $g(x) = \sqrt{4x-1}$, find the function and its domain.

$$\begin{aligned}(f \cdot g)(x) &= f(x) \cdot g(x) \\ &= (x^2 - 3x)\sqrt{4x-1}\end{aligned}$$

The domain of f is $(-\infty, \infty)$ and the domain of g is $[\frac{1}{4}, \infty)$ so the domain of $f \cdot g$ is the intersection of the two domains. Interval notation: $[\frac{1}{4}, \infty)$.

YOU TRY IT:
Given $f(x) = 3x^2 + 2x$ and $g(x) = 1 - \frac{1}{x}$, find the function and its domain.

113. $(g \cdot f)(x)$

Combining functions to write a new function that models a real-world situation

EXAMPLE:
A website designer creates videos on how to create websites. He sells the video packages for \$40 each. His one-time initial cost to produce a package is \$5000. The cost to ship each video is \$2.80.

a. Write a function that represents the cost $C(x)$ to produce and ship x video packages.

   $C(x) = 2.8x + 5000$

b. Write a function that represents the revenue $R(x)$ for selling x video packages.

   $R(x) = 40x$

c. Evaluate $(R - C)(x)$ and interpret its meaning in the context of this problem.

   $(R - C)(x) = 40x - (2.8x + 5000) = 37.2x - 5000$
   This represents the profit for selling x video packages.

YOU TRY IT:
An artist makes jewelry from polished stones. The rent for her studio and utilities comes to \$640 per month. It also costs her \$3.50 for supplies to make one necklace. She sells the necklaces for \$25 each.

114. Write a function $C(x)$ that represents the cost to produce x necklaces during a one month period.

115. Write a function $R(x)$ that represents the revenue for selling x necklaces.

116. Evaluate $(R - C)(x)$ and interpret its meaning in the context of this problem.

Combining functions: Advanced

Watch the video *Combining Functions and Finding Domain* to complete the following.

Given ________________ and ________________, evaluate the given function and write the domain in interval notation.

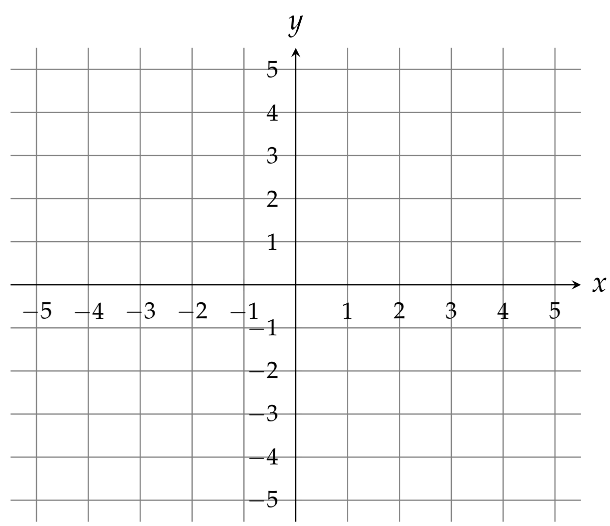
a.

b.

Graphing a piecewise-defined function: Problem type 1

Open the Instructor Added Resource which will direct you to a video to complete the following.

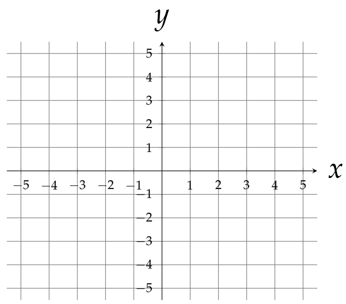
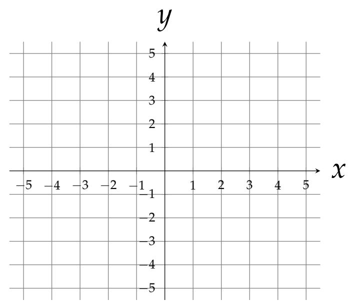
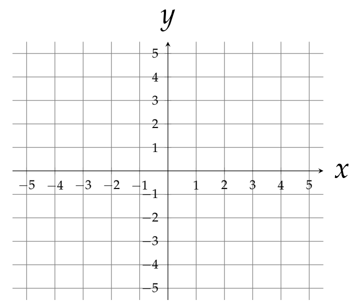
$$g(x) = \begin{cases} \_\_\_\_\_\_ & \text{if } -2 \le x < -1 \\ \_\_\_\_\_\_ & \text{if } -1 \le x < 0 \\ \_\_\_\_\_\_ & \text{if } 0 \le x < 1 \\ \_\_\_\_\_\_ & \text{if } 1 \le x < 2 \end{cases}$$

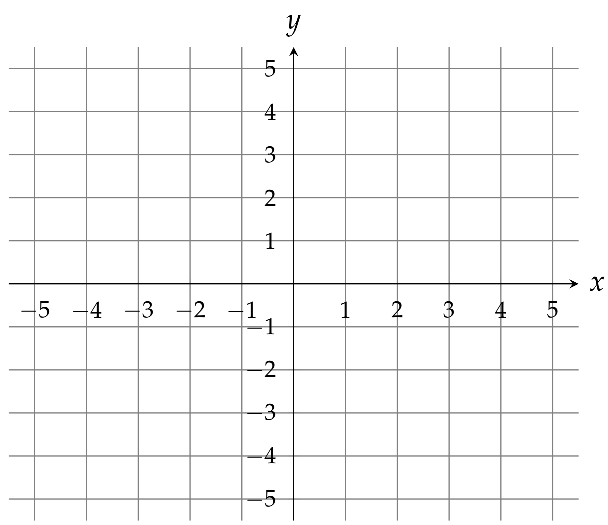


Graphing a piecewise-defined function: Problem type 2

Watch the video *Graphing a Piecewise-Defined Function* to complete the following.

Graph. $m(x) = \begin{cases} \underline{\hspace{2cm}} & \text{if } -4 < x < -1 \\ \underline{\hspace{2cm}} & \text{if } -1 \leq x < 3 \\ \underline{\hspace{2cm}} & \text{if } x \geq 3 \end{cases}$



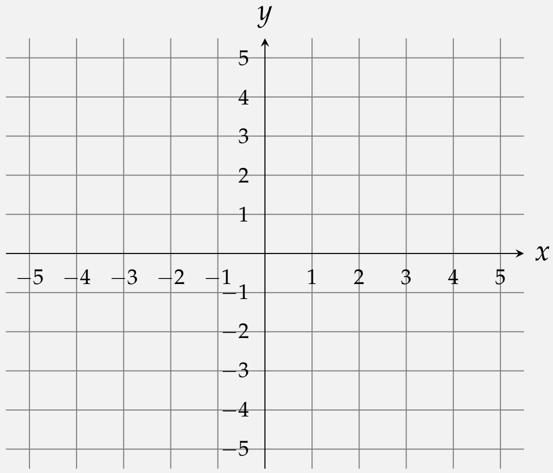
The graph of $f(x)$ is **continuous** if there are no "holes" or "jumps" in the graph. In other words, you can draw the graph without lifting your pencil.

Graphing a piecewise-defined function: Problem type 3

If you did not complete the video *Graphing a Piecewise-Defined Function* under the topic **Graphing a piecewise-defined function: Problem type 2**, click the video link now and complete the work.

YOU TRY IT:

117. Sketch the graph of $f(x) = \begin{cases} -2 & \text{if } x < -3 \\ x+1 & \text{if } -3 \leq x \leq 2 \\ 4 & \text{if } x > 2 \end{cases}$



Finding a difference quotient for a linear or quadratic function

Watch the video *Finding a Difference Quotient for a Nonlinear Function* to complete the following. *NOTE:* This may not be the first video that pops up. Select the appropriate video in the video box.

Given ____________________, find the difference quotient.

EXAMPLE:
Find the difference quotient for $f(x) = 3x^2 - 4x + 5$.
First, find $f(x+h)$.

$$\begin{aligned} f(x+h) &= 3(x+h)^2 - 4(x+h) + 5 \\ &= 3(x^2 + 2xh + h^2) - 4x - 4h + 5 \\ &= 3x^2 + 6xh + 3h^2 - 4x - 4h + 5 \end{aligned}$$

Now find $\dfrac{f(x+h) - f(x)}{h}$.

$$\begin{aligned} &\frac{f(x+h) - f(x)}{h} \\ &= \frac{3x^2 + 6xh + 3h^2 - 4x - 4h + 5 - (3x^2 - 4x + 5)}{h} \\ &= \frac{3x^2 + 6xh + 3h^2 - 4x - 4h + 5 - 3x^2 + 4x - 5}{h} \\ &= \frac{6xh + 3h^2 - 4h}{h} \\ &= \frac{h(6x + 3h - 4)}{h} \\ &= 6x + 3h - 4 \end{aligned}$$

YOU TRY IT:

118. Find the difference quotient for $f(x) = -4x^2 + 5x - 3$.

Finding a difference quotient for a rational function

Open the Instructor Added Resource which will direct you to a video to complete the following.

YOU TRY IT: Find the difference quotient for

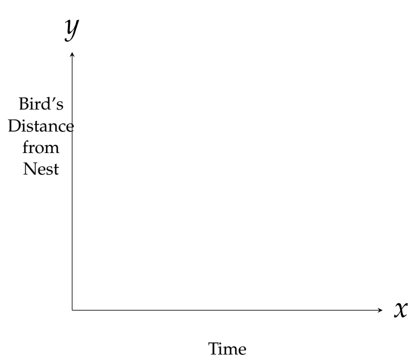
119. $f(x) = \dfrac{5}{x-3}$.

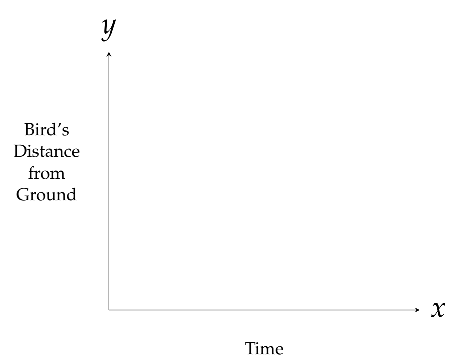
Choosing a graph to fit a narrative: Basic

Open the Instructor Added Resource which will direct you to a video to complete the following.

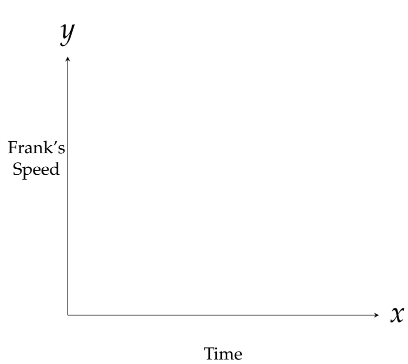
Sketch the graph that best describes the scenario below.

(a) A ______________ flies ______________ from its nest to go hunting.





(b) Frank drives at a ______________ speed for a while.

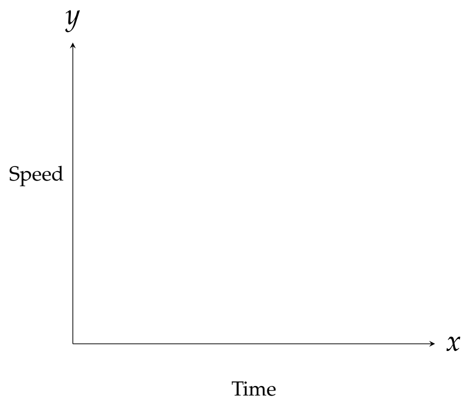


Choosing a graph to fit a narrative: Advanced

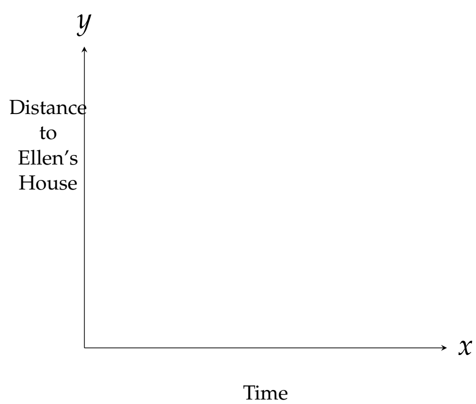
Open the Instructor Added Resource which will direct you to a video to complete the following.

Sketch the graph that best describes the scenario below.

(a) Hector begins his jogging workout by running ____________ for about a minute. Once he hits a comfortable pace, he runs at that pace for ________ minutes. Then he gradually __________ to a stop over the next few minutes.



(b) Tina is delivering a pizza to Ellen's house. She drives at a ________ speed toward the house until she hits a traffic jam and has to ________ for several minutes. After, she starts up again and drives at a ________ speed than before.



Notes from Focus Group:

Notes from Focus Group:

Module 6

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Weekly Checklist

- ☐ Complete MALL time.
- ☐ Work in ALEKS and Notebook at least 4 days a week.
- ☐ Complete the weekly Module and Notebook pages by the due date.
- ☐ Attend Focus Group.
- ☐ Actively participate in Focus Group.
- ☐ Earn extra credit: Complete 10 topics by ______________________.

Graphing a parabola of the form $y = a(x-h)^2 + k$

Watch the video *Graphing a Parabola Given an Equation in Vertex Form* to complete the following.

Given $h(x) =$ ____________________

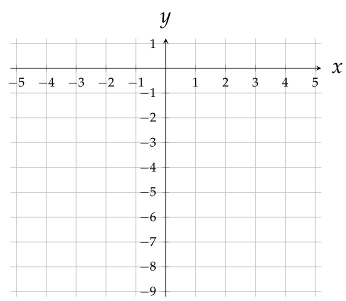
a. Determine whether the graph of the parabola opens upward or downward.

b. Identify the vertex.

c. Determine the x-intercepts.

d. Determine the y-intercept.

e. Sketch the function.



f. Determine the axis of symmetry.

g. Determine the minimum or maximum value of the function.

h. Domain:

Range:

Finding the maximum or minimum of a quadratic function

Watch the video *Applying the Vertex Formula and Graphing a Parabola* to complete the following.

Given $g(x) =$ ____________________.

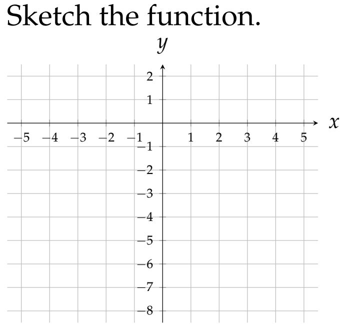
a. Determine whether the graph of the parabola opens upward or downward.

b. Identify the vertex.

c. Determine the x-intercept(s).

d. Determine the y-intercept.

e. Sketch the function.



f. Determine the axis of symmetry.

g. Determine the minimum or maximum value of the function.

h. Domain: Range:

Rewriting a quadratic function to find its vertex and sketch its graph

Watch the video *Graphing a Parabola Given its Equation in Standard Form* to complete the following.

Given $d(x) =$ ____________________

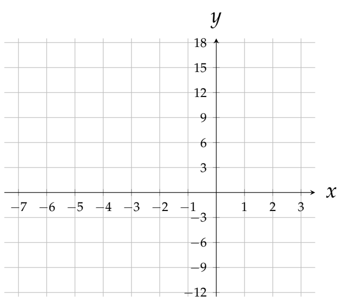
a. Determine whether the graph of the parabola opens upward or downward.

b. Identify the vertex.

c. x-intercept(s):

d. y-intercept:

e. Sketch the function.



f. Determine the axis of symmetry.

g. Determine the minimum or maximum value of the function.

h. Domain:

Range:

Finding the vertex, intercepts, and axis of symmetry from the graph of a parabola

Open the e-book to complete the following.

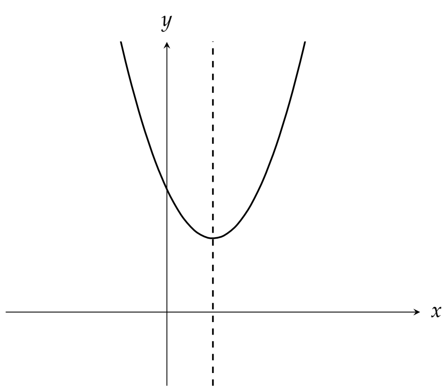
Quadratic Function

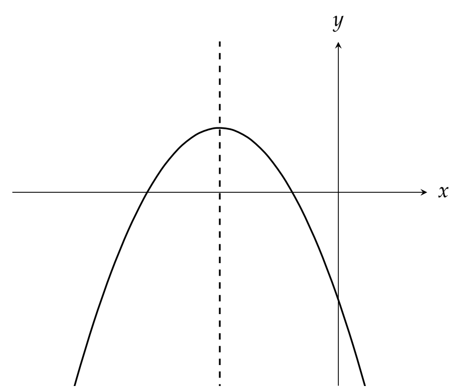
A function defined by ________________ ($a \neq 0$) is called a **quadratic function**. By completing the square $f(x)$ can be expressing in vertex form as $f(x) = a(x-h)^2 + k$.

- The graph of f is a ____________ with vertex ________.
- If ________, the parabola opens ____________, and the ____________ is the ____________ point. The ____________ value of f is ____.
- If ________, the parabola opens ____________, and the ____________ is the ____________ point. The ____________ value of f is ____.
- The ____________________ is ________. This is the ____________ line that passes through the ____________.

On the graphs below, label

- the axis of symmetry with $x = h$
- the vertex with (h, k)
- the value of a with $a > 0$ or $a < 0$





Finding the x-intercept(s) and the vertex of a parabola

Learning Page

Finding the x-intercept(s)

An x-intercept is the ____________________ of a point where the graph ________________________.

A parabola can have ____________, ________, or ________ x-intercepts.

At each point where a graph ____________ the x-axis, the y-coordinate is ________.

To find any x-intercepts of the parabola, we let ____________ and solve the resulting ________________ equation.

Finding the vertex:

The vertex lies on the ____________________.

Word problem involving the maximum or minimum of a quadratic function

Watch the video *Interpreting the Vertex of a Parabola in an Application* to complete the following.

A fireworks mortar is launched straight upward from a pool deck platform 3 m off the ground at an initial velocity of 42 m/sec. The height of the mortar can be modeled by ________________, where $h(t)$ is the height in meters and t is the time in seconds after launch.

a. Determine the time at which the mortar is at its maximum height. Round to 2 decimal places.

b. What is the maximum height? Round to the nearest meter.

Word problem involving optimizing area by using a quadratic function

Watch the video *Applying a Quadratic Function in Geometry* to complete the following.

Suppose that a family wants to fence in an area of their yard for a garden. One side is already fenced from the neighbor's property.

Draw the picture to illustrate this example.

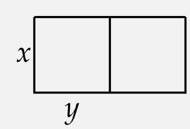
a. If the family has enough money to buy ________ ft of fencing, what dimensions would produce the maximum area for the garden?

Constraint equation: ________________ = ________________

Area equation: ________________

b. What is the maximum area?

YOU TRY IT: Two pens are to be built adjacent to one another from 120 ft of fencing.



120. What dimensions should be used to maximize the area of an individual coop?

121. What is the maximum area of an individual coop?

Writing the equation of a quadratic function given its graph

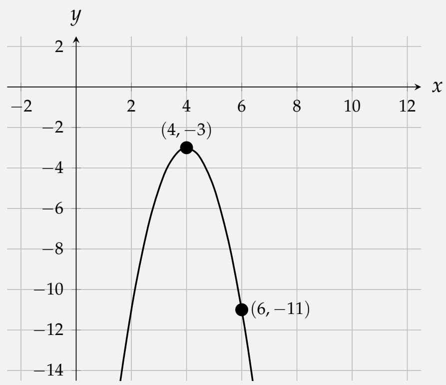
Learning Page The graph of a quadratic function is a ____________________.

Any quadratic function f whose graph has vertex ________________ can be written in the following form.

$$f(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_, \text{ where } a \neq 0$$

Carefully read the example on the Learning Page and the example below.

EXAMPLE:
Find the equation of the quadratic function f whose graph is shown below.



A parabola with vertex (h, k) has the form: $y = a(x-h)^2 + k$

The graph has vertex $(4, -3)$ so we have $y = a(x-4)^2 - 3$.

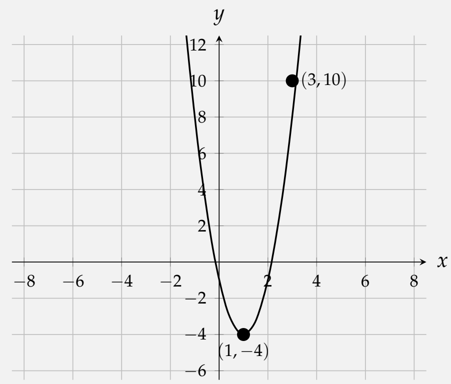
We need to find a. We use the other given point: $(6, -11)$, which gives us an x and a y value to substitute and solve then for a.

$$\begin{aligned} y &= a(x-4)^2 - 3 \\ -11 &= a(6-4)^2 - 3 \\ -8 &= a(2)^2 \\ -2 &= a \end{aligned}$$

Equation of parabola: $y = -2(x-4)^2 - 3$

YOU TRY IT:
Find the equation of the quadratic function f whose graph is shown below.

122.



Identifying polynomial functions

Watch the video *Introduction to Polynomial Functions* to complete the following.

Definition of a Polynomial Function

Let n be a whole number and $a_n, a_{n-1}, a_{n-2}, \ldots, a_1, a_0$ be ______________________, where $a_n \neq 0$. Then a function defined by

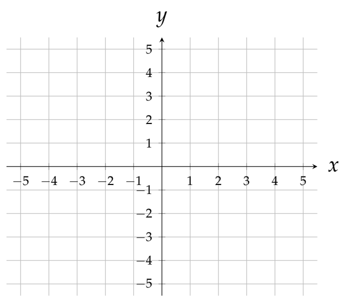
$$f(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

is called a **polynomial function of degree** ______.

Polynomial Function

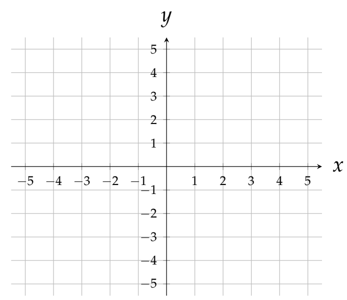
Not a Polynomial Function

$f(x) =$ ______



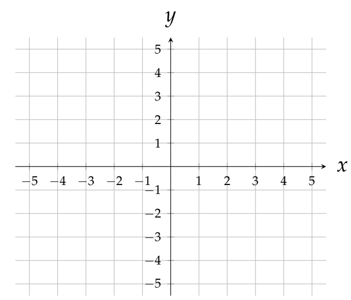
degree = ______

$f(x) =$ ___________



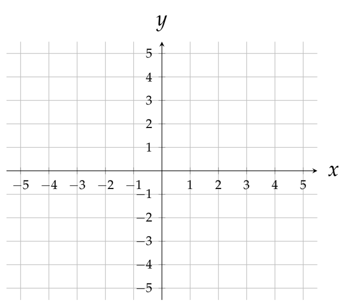
degree = ______

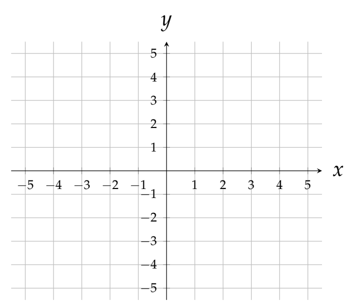
$f(x) =$ _________________

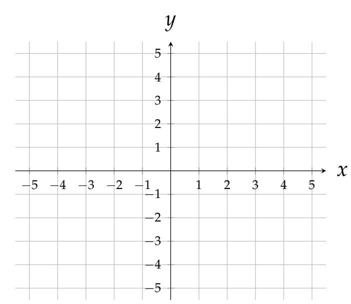


degree = ______

Graph three functions that are NOT polynomials.







EXAMPLE:
Identify which of the following are polynomials.

a) $A(x) = 3x^5 - 2x^3 + 5x^{-4}$
This is not a polynomial because the exponent on the term $5x^{-4} = \frac{5}{x^4}$ is not a whole number.

b) $B(x) = x^3 + \sqrt{5}x^2 - 3x + \sqrt{7}$
This is a polynomial. All coefficients are real numbers and all exponents are whole numbers.

c) $C(x) = \dfrac{3-x}{7}$
This is a polynomial, it can be rewritten as $C(x) = \frac{3}{7} - \frac{1}{7}x$. All coefficients are real numbers and all exponents are whole numbers.

d) $D(x) = \dfrac{4-x^2}{x-1}$
This is not a polynomial. It is a ratio of polynomials so is a rational function.

YOU TRY IT:
Identify which of the following are polynomials.

123. $a(x) = 3x^5 - 2\sqrt{x} + 5x^2$

124. $b(x) = \dfrac{5x^4 - 2x^2 + x}{3}$

125. $c(x) = -6$

126. $d(x) = 2x(x+4)(x-7)(x+1)^4$

Finding zeros of a polynomial function written in factored form

Learning Page The ____________________ of f are the real numbers x for which ____________.

So we set ____________________ and ________________.

For a product to __________, at least one of the __________ must __________ 0.

YOU TRY IT:

127. Find the zeros of $f(x) = 3x^2(x^2-9)(x+4)$

Finding zeros and their multiplicities given a polynomial function written in factored form

Watch the video *Determining Zeros and Multiplicities* to complete the following.

Determine the zeros of the function and state their multiplicities.

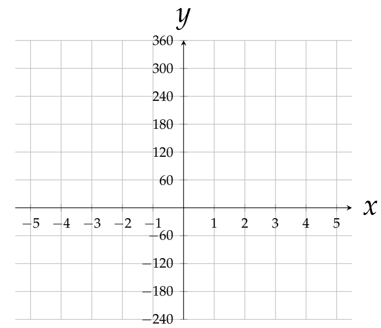
$f(x) =$ ____________________

Zero: __________ Multiplicity: __________

Zero: __________ Multiplicity: __________

Zero: __________ Multiplicity: __________

Zero: __________ Multiplicity: __________



EXAMPLE:
Consider the polynomial

$$p(x) = -4x(x-3)^2(x+7)^3(x-1).$$

List each zero and its multiplicity.

Zeros of multiplicity one: 0, 1
Zero of multiplicity two: 3
Zero of multiplicity three: -7

YOU TRY IT:

128. Consider the polynomial
$$q(x) = 5x^2(x-1)^4(x+5)^2(x+6).$$
List each zero and its multiplicity.

Finding x and y intercepts given a polynomial function

Learning Page A y-**intercept** is the ____________________ of a point where the graph ____________________ the ____________________.

A function's graph has ____________ y-intercept.

To find it, we find the ______________________________.

Continued on the next page

Watch the video *Identifying Zeros and Multiplicities* to complete the following.

Given a polynomial function defined by $y = f(x)$:

The values of x in the ______________ of f for which ______________ are called the ______________ of the function. These are also called the ______________________ of the equation ______________.

Determine the zeros of the function and state their multiplicities.

EXAMPLE:
Find all intercepts of $p(x) = 3x^3 + x^2 - 2x$.

a) y-intercept
$p(0) = 3(0)^3 + 0^2 - 2(0) = 0$
$(0,0)$ is the y-intercept.

b) x-intercept

$$\begin{aligned} 3x^3 + x^2 - 2x &= 0 \\ x(3x^2 + x - 2) &= 0 \\ x(3x-2)(x+1) &= 0 \\ x &= 0, \frac{2}{3}, -1 \end{aligned}$$

$(0,0)$, $\left(\frac{2}{3}, 0\right)$, and $(-1,0)$ are x-intercepts.

YOU TRY IT:

129. Find all intercepts of $q(x) = 2x^4 - 2x^3 - 24x^2$.

Determining the end behavior of the graph of a polynomial function

Open the e-book to complete the following.

Notation for Infinite Behavior of $y = f(x)$

$x \to \infty$	is read as ________________ This means that x becomes infinitely large in the ____________ direction
$x \to -\infty$	is read as ________________ This means that x becomes infinitely large in the ____________ direction
$f(x) \to \infty$	is read as ________________ This means that the y value becomes infinitely large in the ____________ direction
$f(x) \to -\infty$	is read as ________________ This means that the y value becomes infinitely large in the ____________ direction

The Leading Term Test

Consider a polynomial function given by

$$f(x) = \underline{\hspace{6cm}}$$

As $x \to \infty$ or as $x \to -\infty$, f eventually becomes forever increasing or forever decreasing and will follow the general behavior of ____________.

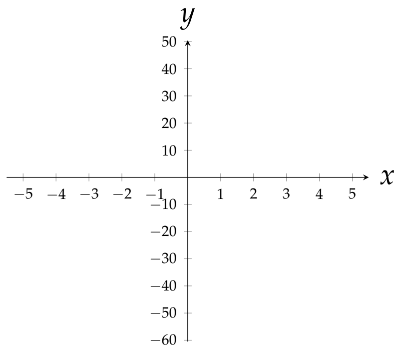
Compete the chart below, then sketch a graph in each box that represents the correct end behavior.

n is even				n is odd			
a_n positive		a_n negative		a_n positive		a_n negative	
As $x \to -\infty$, $f(x) \to$ ___	As $x \to \infty$, $f(x) \to$ ___	As $x \to -\infty$, $f(x) \to$ ___	As $x \to \infty$, $f(x) \to$ ___	As $x \to -\infty$, $f(x) \to$ ___	As $x \to \infty$, $f(x) \to$ ___	As $x \to -\infty$, $f(x) \to$ ___	As $x \to \infty$, $f(x) \to$ ___

Determining end behavior and intercepts to graph a polynomial function

Watch the video *Graphing a Polynomial Function* to complete the following.

Graph $k(x) =$ ____________________



leading term:

y-intercept:

Zeros:

Matching graphs with polynomial functions

Open the e-book to complete the following.

Touch Points and Cross Points

Let f be a polynomial function and let c be a real zero of f. The point ________ is an x-intercept of the graph of f. Furthermore,

- If c is a zero of ________ multiplicity, the graph ____________ the x-axis at c.

  The point $(c, 0)$ is called a ________________.

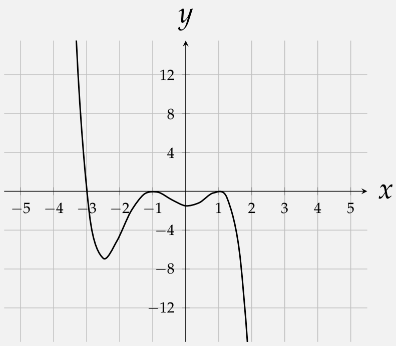
- If c is a zero of ________ multiplicity, the graph ____________ the x-axis at c.

  The point $(c, 0)$ is called a ________________.

EXAMPLE:
Sketch the graph of

$$f(x) = -\tfrac{1}{2}(x-1)^2(x+3)(x+1)^2.$$

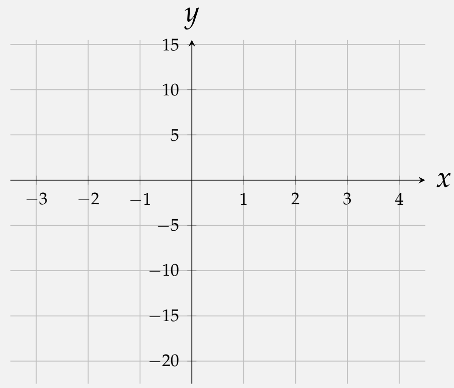
- The touch points of f are $(1,0)$ and $(-1,0)$.
- The cross point of f is $(-3,0)$.
- The y-intercept of f is $\left(0,-\frac{3}{2}\right)$.
- The degree of f is 5 and a_n is negative so as $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to \infty$.



YOU TRY IT:

130. Sketch the graph of

$$g(x) = x^2(x+1)^2(x-3)(x-2).$$



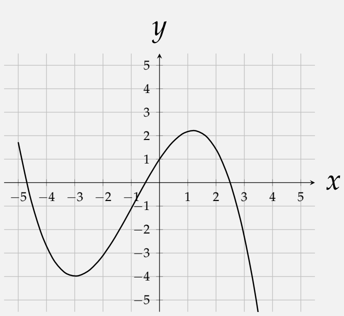
Inferring properties of a polynomial function from its graph

Watch the video *Turning Points of a Graph of a Polynomial Function* to complete the following.

Number of Turning Points of a Polynomial Function

Let f represent a polynomial function of ____________________. Then the graph of f has at most __________ turning points.

YOU TRY IT: Below is the graph of a polynomial function f with real coefficients. Use the graph to answer the following questions.



131. At what x-values does f have local minima?

132. What is the sign of the leading coefficient of f?

133. What is the lowest possibility for the degree of f?

Finding a polynomial of a given degree with given zeros: Real zeros

Learning Page The **Factor Theorem** tells us the following.

A number c is a ________________ of a polynomial $f(x)$ if and only if ________________ is a ________________ of __________.

We also get that, if c is a zero of __________________________, then ________________ is a ________________.

YOU TRY IT:

134. Find a polynomial $p(x)$ of degree 5 that has zeros -2, 0, 1 (multiplicity 2), 7.

Polynomial long division: Problem type 3

Watch the video *Long Division of Polynomials with a Nonlinear Divisor* to complete the following.

EXAMPLE:
Use polynomial long division to evaluate:
$(x^4 + 3x^3 + x - 5) \div (x^2 - 3)$

$$
\begin{array}{r|l}
 & \quad x^2 + 3x + 3 \\
\hline
x^2 - 3 & x^4 + 3x^3 \qquad\qquad + x - 5 \\
 & -x^4 \qquad + 3x^2 \\
\hline
 & \quad 3x^3 + 3x^2 + x \\
 & -3x^3 \qquad + 9x \\
\hline
 & \qquad 3x^2 + 10x - 5 \\
 & \quad -3x^2 \qquad + 9 \\
\hline
 & \qquad\qquad 10x + 4
\end{array}
$$

So $(x^4 + 3x^3 + x - 5) \div (x^2 - 3)$

$$= x^2 + 3x + 3 + \frac{10x + 4}{x^2 - 3}$$

YOU TRY IT:
Use polynomial long division to evaluate:

135. $(2x^5 + x^4 - x^3 - x - 1) \div (x^2 - 2x + 1)$

Dividing a polynomial by a monomial: Univariate

Learning Page Carefully read the example on the Learning Page

YOU TRY IT: Divide.

136. $\dfrac{3x^4 - 6x^3 + 9x}{3x^2}$

Synthetic division

Watch the video *Introduction to Synthetic Division* to complete the following.

EXAMPLE:
Use synthetic division to evaluate:
$(x^4 - 14x^2 + 5x - 9) \div (x + 4)$

$$\begin{array}{r|rrrrr} -4 & 1 & 0 & -14 & 5 & -9 \\ & & -4 & 16 & -8 & 12 \\ \hline & 1 & -4 & 2 & -3 & 3 \end{array}$$

So $(x^4 - 14x^2 + 5x - 9) \div (x + 4)$

$$= x^3 - 4x^2 + 2x - 3 + \frac{3}{x+4}$$

YOU TRY IT:
Use synthetic division to evaluate:

137. $(2x^4 - x^3 - 3x - 1) \div (x - 2)$

The Factor Theorem

Watch the video *Introduction to the Factor Theorem* to complete the following.

Factor Theorem

Let $f(x)$ be a polynomial.

1. If $f(c) = 0$, then ________________ is a ________________ of $f(x)$.

2. If ________________ is a factor of $f(x)$, then ________________.

Use the Factor Theorem to determine if the given binomial is a factor of $f(x)$.

$$f(x) = x^4 + 11x^3 + 41x^2 + 61x + 30$$

a. ________________

b. ________________

EXAMPLE:
Use the Factor Theorem to determine whether $x + 1$ is a factor of $p(x) = -3x^3 + 4x^2 - 2x - 6$.

$$\begin{aligned} p(-1) &= -3(-1)^3 + 4(-1)^2 - 2(-1) - 6 \\ &= 3 + 4 + 2 - 6 \\ &= 3 \end{aligned}$$

$p(-1) \neq 0$ so $x + 1$ is not a factor of $p(x)$.

YOU TRY IT:

138. Use the Factor theorem to determine whether $x + 4$ is a factor of $q(x) = x^3 - 13x + 12$.

Solving a system of linear equations using elimination with multiplication and addition

Watch the video *Solving a System of Equations Using the Addition Method* to complete the following. *NOTE:* This may not be the first video that pops up. Select the appropriate video in the video box.

Solve the system by using the addition method.

EXAMPLE: Solve the system of equations using elimination.

$$2x - 3y = -2$$
$$3x - 2y = 12$$

Multiply the first equation by -3 and the second equation by 2.

$$-3(2x - 3y) = -2(-3)$$
$$2(3x - 2y) = 12(2)$$

Simplify the equations. Note that we have a $6x$ in one equation and a $-6x$ in the other.

$$-6x + 9y = 6$$
$$6x - 4y = 24$$

Add the two equations together and solve for y.

$$-6x + 9y = 6$$
$$6x - 4y = 24$$

$$5y = 30$$
$$y = 6$$

Use one of the equations to solve for x.

$$2x - 3(6) = -2$$
$$2x - 18 = -2$$
$$2x = 16$$
$$x = 8$$

The solution is the ordered pair $(8, 6)$.

YOU TRY IT: Solve the system of equations using elimination.

139.

$$-2x + 5y = 14$$
$$7x + 6y = -2$$

Solving a word problem using a system of linear equations of the form $Ax + By = C$

Learning Page Carefully read through the example on the Learning Page.

YOU TRY IT:

140. John and Alycia bought school supplies. John spent \$10.65 on 4 notebooks and 5 pens. Alycia spent \$7.50 on 3 notebooks and 3 pens. What is the cost of 1 notebook and what is the cost of 1 pen?

Solving a system of linear and quadratic equations

Watch the video *Solving a Nonlinear System of Equations by using the Substitution Method* to complete the following.

Solve the system by using the substitution method.

EXAMPLE: Solve the system.

$$2x^2 - y = 8$$
$$7x + y = -4$$

We can use elimination and eliminate y.

$$\begin{aligned} 2x^2 - y &= 8 \\ 7x + y &= -4 \\ \hline 2x^2 + 7x &= 4 \\ 2x^2 + 7x - 4 &= 0 \\ (2x-1)(x+4) &= 0 \\ x &= \frac{1}{2}, -4 \end{aligned}$$

We must find the corresponding y-values. We can use either of the original equations.

- If $x = \frac{1}{2}$, we use the second equation and find
$$y = -7\left(\frac{1}{2}\right) - 4 = -\frac{15}{2}$$
- If $x = -4$, again use the second equation and
$$y = -7(-4) - 4 = 24$$

The solutions are $\left(\frac{1}{2}, -\frac{15}{2}\right)$ and $(-4, 24)$.

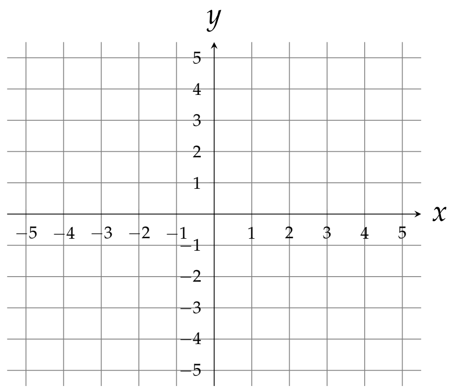
YOU TRY IT:

141. Solve the system.

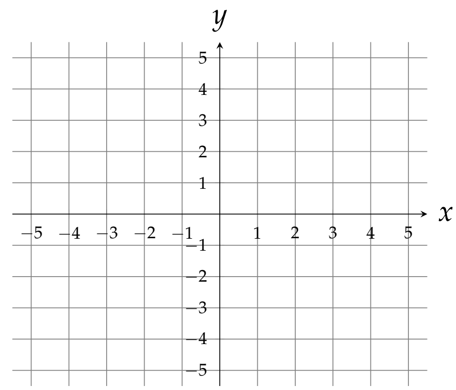
$$y = x^2 + 1$$
$$y - x = 2$$

Graphically solving a system of linear and quadratic equations

Watch the video *Introduction to Nonlinear Systems of Equations* to complete the following. *NOTE:* This may not be the first video that pops up. Select the appropriate video in the video box.



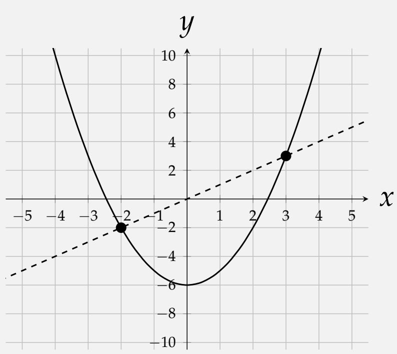
$$y = x^2 - 2$$
$$2x - y = 2$$



$$4x^2 = 4 - y^2$$
$$16y^2 = 144 + 9x^2$$

EXAMPLE:
Graph the system of equations to find its solution.

$$y = x$$
$$y = x^2 - 6$$



From the graph we can see that the solutions are $(-2, -2)$ and $(3, 3)$.

Using a given zero to write a polynomial as a product of linear factors: Real zeros

Watch the video *Factoring a Polynomial Given a Zero of the Polynomial* to complete the following.

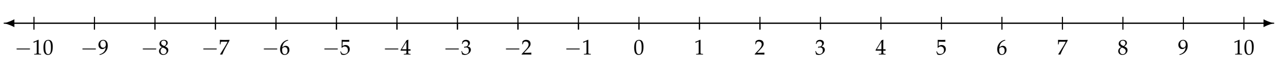
a. Factor $f(x) =$ ________________ given that $\frac{1}{4}$ is a zero.

b. Solve ________________

Solving a quadratic inequality written in factored form

Open the Instructor Added Resource which will direct you to a video to complete the following.

Graph the solution to the inequality ________________.



$x =$ ________ $x =$ ________ $x =$ ________

Solving a quadratic inequality

Watch the video *Solving Quadratic Inequalities* to complete the following.

Solve the inequality.

EXAMPLE: Graph the solution to the inequality.

$$x^2 - x < 12$$

We rewrite the inequality, then factor.

$$x^2 - x - 12 < 0$$
$$(x-4)(x+3) < 0$$

- We want the values of x that make $(x-4)(x+3)$ less than zero (negative).
- $(x-4)(x+3)$ is equal to zero when $x = 4$ or $x = -3$.

$$\begin{array}{ccccc} & 0 & & 0 & \\ \text{———————} & | & \text{———————} & | & \text{———————} \\ & -3 & & 4 & \end{array}$$

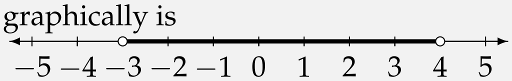
We will test a point in each interval on the number line above.

- For $x = -4$, we have $(-)(-) = +$
- For $x = 0$, we have $(-)(+) = -$
- For $x = 5$, we have $(+)(+) = +$

Note that we do not need the VALUE, just whether it will be positive or negative.

$$\begin{array}{ccccc} + + + + + + + + + & 0 & - - - - - - - - & 0 & + + + + + + + + + \\ \text{———————} & | & \text{———————} & | & \text{———————} \\ & -3 & & 4 & \end{array}$$

The solution in interval notation is $(-3, 4)$. And graphically is

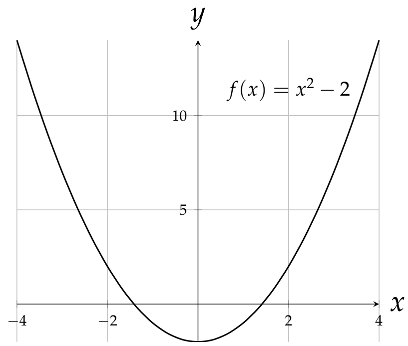


YOU TRY IT: Graph the solution to the inequality.

142.

$$2x^2 - 9x \geq 5.$$

An alternative method to the one shown before is to graph the parabola and determine the answer from the graph. Solve $x^2 - 2 \geq 0$



We can find the x-intercepts of the graph $(\sqrt{2}, 0)$ and $(-\sqrt{2},, 0)$. We want the x values where the graph lies on or above the x-axis.

The solution is $(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$.

Notes from Focus Group:

Notes from Focus Group:

Module 7

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Weekly Checklist

- ☐ Complete MALL time.
- ☐ Work in ALEKS and Notebook at least 4 days a week.
- ☐ Complete the weekly Module and Notebook pages by the due date.
- ☐ Attend Focus Group.
- ☐ Actively participate in Focus Group.
- ☐ Earn extra credit: Complete 10 topics by ____________________.

Finding the intercepts, asymptotes, domain, and range from the graph of a rational function

Watch the video *Introduction to Rational Functions* to complete the following.

a. As $x \to -\infty$, $f(x) \to$ __________

b. As $x \to 4^-$, $f(x) \to$ __________

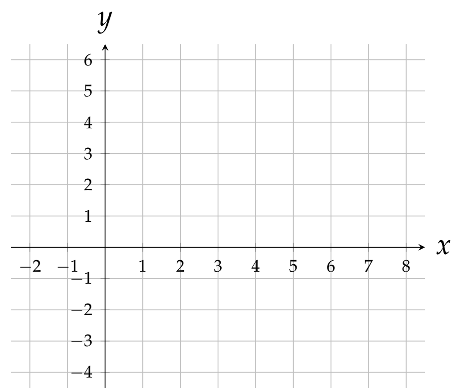
c. As $x \to 4^+$, $f(x) \to$ __________

d. As $x \to \infty$, $f(x) \to$ __________

e. Increasing:

f. Decreasing:

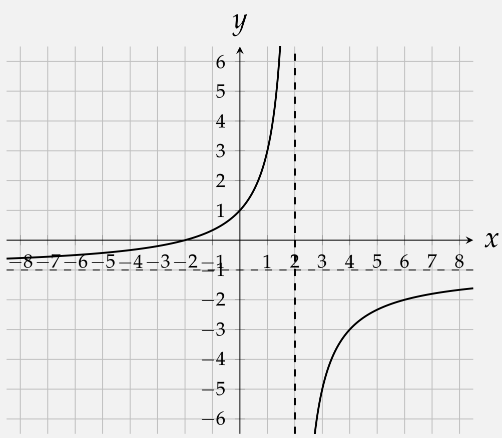
g. Domain:



h. Range:

In mathematics, $\to$ means the word ________________.

YOU TRY IT: Use the graph to answer the following questions about $f(x)$.



143. Find the domain of $f(x)$.

144. Find the range of $f(x)$.

145. Find all asymptotes of $f(x)$.

Finding the asymptotes of a rational function: Constant over linear

Learning Page

Vertical asymptote(s):

A rational function in ____________________ form has vertical asymptotes at the ____________________

of the ______________________________.

Horizontal asymptote(s):

A rational function can have ____________________ horizontal asymptote.

To find the horizontal asymptotes (if any), we compare the ____________________ of the numerator

with the ______________________ of the denominator.

- If _______________, the horizontal asymptote is _______________.
- If _______________, the horizontal asymptote is given by ______________________________.
- If _______________, there is __________ horizontal asymptote.

YOU TRY IT:

146. Find all vertical and horizontal asymptotes of the function $f(x) = \dfrac{7}{3x-2}$.

Finding the asymptotes of a rational function: Linear over linear

Open the e-book to complete the following.

Definition of a Vertical Asymptote

The line ______________ is a **vertical asymptote** of the graph of a function f if $f(x)$ approaches

______________ or ________________________ as x approaches __________ from either side.

Identifying Vertical Asymptotes of a Rational Function

Consider a rational function f defined by ___________________, where $p(x)$ and $q(x)$ have

____________________________ other than 1.

If c is a ____________________________, then ______________ is a ______________ asymptote of the graph of f.

Definition of a Horizontal Asymptote

The line ______________ is a **horizontal asymptote** of the graph of a function f if

____________________________ infinity or negative infinity.

Identifying Horizontal Asymptotes of a Rational Function

Let f be a rational function defined by

$$f(x) = \frac{a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + b_{m-2}x^{m-2} + ... + b_1 x + b_0}$$

The definition of $f(x)$ indicates that _____ is the ______________ of the ______________ and _____ is the ______________ of the ______________.

1.

2.

3.

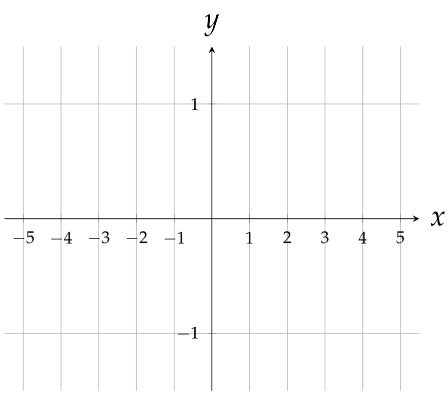
YOU TRY IT:

147. Find all vertical and horizontal asymptotes of the function $f(x) = \dfrac{-5x+4}{3x-2}$.

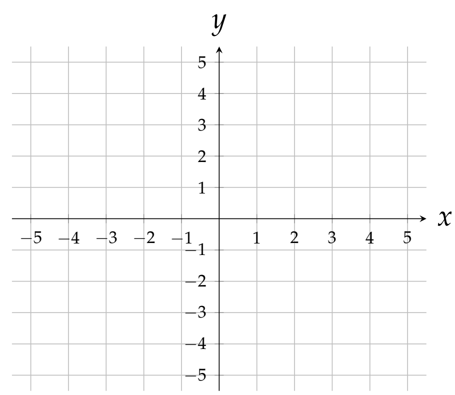
Finding horizontal and vertical asymptotes of a rational function: Quadratic numerator or denominator

Watch the video *Identifying Vertical Asymptotes Algebraically* to complete the following.

a. $m(x) =$ ____________________



b. $f(t) =$ ____________________



Watch the video *Identifying Horizontal Asymptotes Algebraically* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

a. $h(x) =$ ____________________

b. $m(x) =$ ____________________

c. $t(x) =$ ____________________

EXAMPLE:
Find all asymptotes of
$$f(x) = \frac{x+1}{(x-2)(x+3)}.$$

- The numerator and denominator share no common factors other than 1.
    - To find the vertical asymptotes we consider the zeros of the denominator which are 2 and -3.
- Vertical asymptotes:
    - $x = 2$
    - $x = -3$
- Horizontal asymptote:
    - We look at the degree of the top compared to the degree of the bottom.
    - As x gets large, y will get close to zero so the horizontal asymptote is $y = 0$.

YOU TRY IT:

148. Find all asymptotes of
$$f(x) = \frac{x^2}{x^2-9}.$$

Matching graphs with rational functions: Two vertical asymptotes

Watch the video *Graphing a Rational Function* to complete the following.

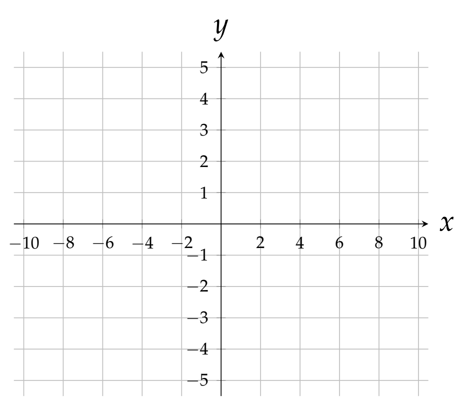
Graph $r(x) =$ ______________________ $=$ ______________________

y-intercept:

x-intercept(s):

Vertical asymptote(s):

Horizontal or slant asymptote:



Graphing a rational function: Constant over linear

Open the e-book to complete the following.

Graphing a Rational Function

Consider a rational function f defined by $f(x) = \dfrac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials with no common factors.

1. Determine the ____________________ by evaluating __________.

2. Determine the ____________________ by finding the real solutions of _______________.

   The value of $f(x)$ equals zero when ____________________.

3. Identify any _________________________ and graph them as dashed lines.

4. Determine whether the function has a ______________________________ or a slant asymptote (or neither), and graph the asymptote as a dashed line.

5. Determine where the function crosses the _______________ or slant asymptote (if applicable).

6. If a test for ____________________ is easy to apply, use ____________________ to plot additional points. Recall:

   - f is an even function (symmetric to the _______________) if ____________________.
   - f is an odd function (symmetric to the _______________) if ____________________.

7. Plot at least one point on the intervals defined by the x-intercepts, vertical asymptotes, and any points where the function crosses a horizontal or slant asymptote.

8. Sketch the function based on the information found in steps 1-7.

Graphing a rational function: Linear over linear

Open the e-book to read EXAMPLE 7: **Graphing a Rational Function** to complete the following steps.

Graph $f(x) =$ ______________________

Solution:

1. **Determine the ________________.**

   $$f(0) = \frac{(0)+3}{(0)-2} = \text{\_\_\_\_\_\_\_\_}$$

   The y-intercept is ____________.

2. **Determine the ____________.**

   $\frac{x+3}{x-2} = 0$ when ________________ or $x = -3$

   The x-intercept is ____________.

3. **Identify the ____________ asymptotes.**

   The polynomial ________ has a zero at $x = 2$,

   and the numerator $x + 3$ is ____________ for $x = 2$.

   The graph has one vertical asymptote, _____.

   The vertical asymptotes occur at the values of x for which the denominator is ________ and the numerator is ____________.

4. **Determine whether f has a horizontal or slant asymptote.**

   The degree of the ____________ is ____________ to the degree of the ____________. Therefore, the graph has a horizontal asymptote given by the ratio of leading coefficients of the numerator and denominator.

   ____________ is the horizontal asymptote.

5. **Determine where f ____________ the horizontal asymptote (if at all).**

   Solve the equation ____________.
   $\frac{x+3}{x-2} = 1 \implies x+3 = x-2 \implies 3 = -2$ (contradiction)

   The graph of f ______________________ cross its horizontal asymptote.

6. **Test for symmetry.**
   $f(-x) = \frac{-x+3}{-x-2}$ does not equal $f(x)$ or $-f(x)$. The function is ____________ even nor odd and is not symmetric with respect to the y-axis or origin.

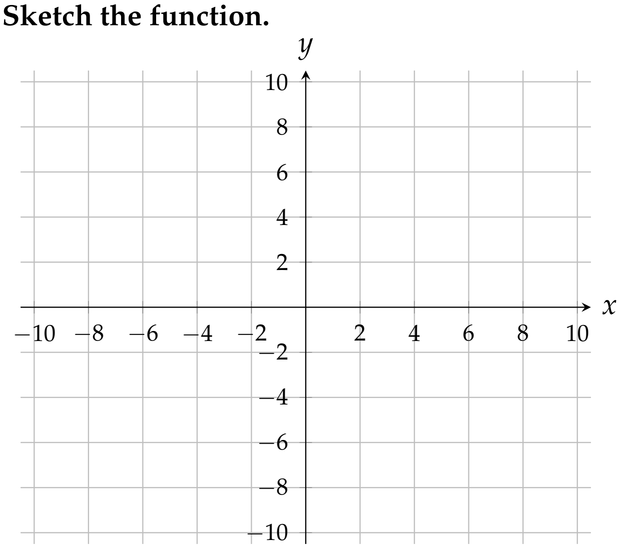
7. **Determine the behavior of f on each interval.**
   Determine the sign of the function on the intervals shown defined by the x-intercept at $x = -3$ and the

   vertical asymptote ____________.

Continued on the next page.

Interval	**Test Point**	**Comments**
$(-\infty, -3)$		•Since $f(x)$ is ______________ on this interval, $f(x)$ must approach the horizontal asymptote $y = 1$ from below as _________.
$(-3, 2)$		•Since $f(x)$ is ____________ on this interval, the graph crosses the x-axis at the intercept $(-3, 0)$ and continues downward (through the y-intercept). As x approaches the vertical asymptote $x = 2$ from the ____________, ______________.
	$(3, 6)$	•Since $f(x)$ is ____________ on this interval, as x approaches the vertical asymptote $x = 2$ from the ____________, $f(x) \to$ _________. •Since $f(x)$ is positive on this interval, $f(x)$ must approach the horizontal asymptote from ____________ as $x \to \infty$.

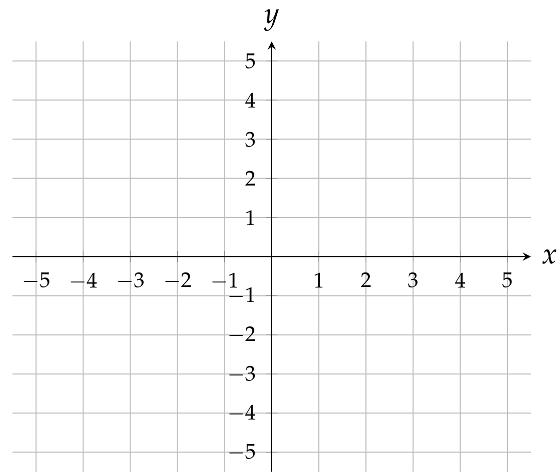
8. **Sketch the function.**



Graphing a rational function with holes

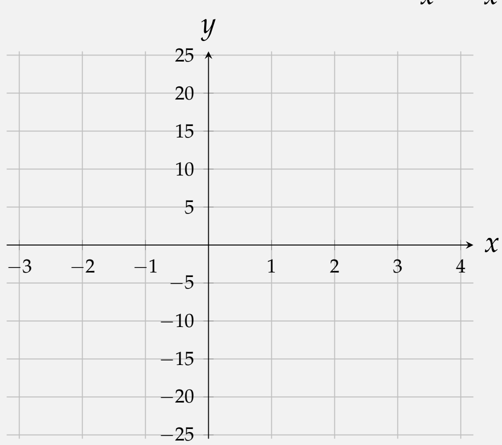
Open the Instructor Added Resource which will direct you to a video to complete the following.

Sketch the graph of $f(x) =$ ____________________.



YOU TRY IT:

149. Sketch the graph of $g(x) = \dfrac{3x^2}{x^3 - x^2}$.



Writing the equation of a rational function given its graph

Learning Page We must determine the equation of a rational function $f(x) = \dfrac{p(x)}{q(x)}$ given its graph. To do this, we will use the following properties.

Property 1:

If __________ is a vertical asymptote of the graph, then __________ is a factor of the ____________________ $q(x)$.

Property 2:

If __________ is an x-intercept of the graph, then _______________ is a factor of the ____________________ $p(x)$.

If there is __________ x-intercept and the rational function is in _______________ form, then _______________

has _________________________.

Property 3:

If __________, $a \neq 0$, is a ______________________________ of the graph, then the degree of the

numerator $p(x)$ _______________ the degree of the denominator $q(x)$, and a equals the ____________________

____.

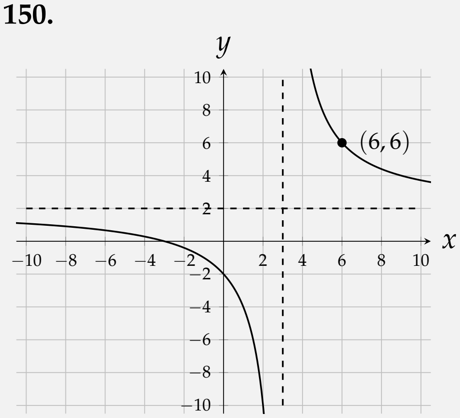
Property 4:

If __________ is a horizontal asymptote of the graph, then the ____

____.

YOU TRY IT: Write the equation of the graph given below.

150.



Horizontal line test

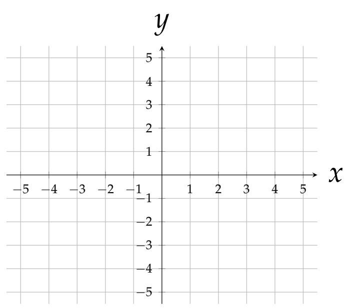
Watch the video *Applying the Horizontal Line Test* to complete the following.

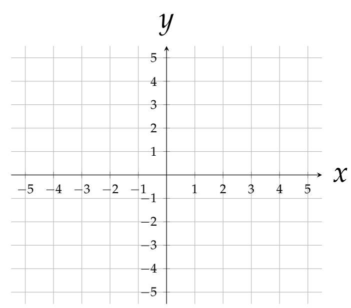
Horizontal Line Test

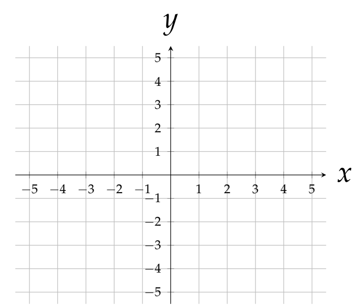
A function defined by $y = f(x)$ is ________________ if ____________________

intersects the graph in ________________________.

Determine if the relation defines y as a one-to-one function of x.







Graphing the inverse of a function given its graph

Learning Page To get the graph of ________, we take the ________________________ of ____ and

________________ them.

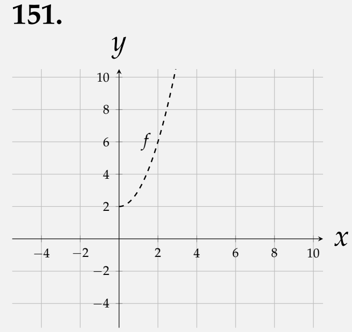
That is, we ____________ the ________________________ of each ________________________.

We see that the graph of f^{-1} is the ________________ of the ____________________ over the

line ____________.

YOU TRY IT: The graph of $f(x)$ is given. Sketch the graph of $f^{-1}(x)$ on the same axes.

151.



Determining whether two functions are inverses of each other

Watch the video *Determining Whether Two Functions are Inverses* to complete the following.

Determine whether the two functions are inverses.

$f(x) =$ ________________ and $g(x) =$ ________________

Let f be a ______________________ function. Then g is the inverse of f if the following conditions are both true.

1.

2.

YOU TRY IT:

152. Determine if $f(x) = 3x + 7$ and $g(x) = \frac{x-3}{7}$ are inverses.

Finding, evaluating, and interpreting an inverse function for a given linear relationship

EXAMPLE: Steve is walking and his distance D in miles from Fargo after x hours of walking is given by $D(x) = 11.6 - 4x$.

a. Describe in words what $D^{-1}(x)$ means.

   With a function and its inverse we are "switching" the domain and range.
   The input for $D^{-1}(x)$ will be a distance and the output will be a time.

   $D^{-1}(x)$ represents the amount of time in hours that Steve has walked when he is x miles from Fargo.

b. Find $D^{-1}(x)$.

$$\begin{aligned} y &= 11.6 - 4x \\ x &= 11.6 - 4y \\ x - 11.6 &= -4y \\ \tfrac{x-11.6}{-4} &= y \\ D^{-1}(x) &= \tfrac{11.6-x}{4} \end{aligned}$$

Inverse functions: Linear, discrete

Learning Page For a given ____________________ function f, there is a related function, ________,

which is the ________________________.

The function f maps ________________ if and only if f^{-1} maps ________________.

So, the ____________________ are the ____________________ and vice versa.

More precisely,

________________ if and only if ________________.

There is a general method to find the inverse of a function that is defined by an equation.

Step 1:

Step 2:

Step 3:

Step 4: .

The **composition** of a function with its ________________ always gives an ________________ equal to

the ________________. **Continued on the next page.**

Watch the video *Introduction to Inverse Functions* to complete the following.

f= __________,__________,__________,__________

Domain:

Range:

f^{-1}=__________,__________,__________,__________

Domain:

Range:

EXAMPLE:
Given $f = \{(1,3),(2,4),(5,7)\}$, find the following

a) f^{-1}
The inverse function f^{-1} reverses the ordered pairs of f.
$f^{-1} = \{(3,1),(4,2),(7,5)\}$.

b) $f^{-1}(7)$
From part a) we see $f^{-1}(7) = 5$.

c) $(f^{-1} \circ f)(1)$

$$(f^{-1} \circ f)(1) = f^{-1}(f(1)) = f^{-1}(3) = 1$$

YOU TRY IT:
Given $g = \{(3,0),(2,5),(4,6),(7,9)\}$, find the following.

153. g^{-1}

154. $g^{-1}(5)$

155. $(g^{-1} \circ g)(7)$

EXAMPLE:
Given $g(x) = 3x - 7$, find the following

a) $g^{-1}(x)$

$$\begin{aligned} g(x) &= 3x - 7 \\ y &= 3x - 7 \\ x &= 3y - 7 \\ x + 7 &= 3y \\ \frac{x+7}{3} &= y \\ g^{-1}(x) &= \frac{x+7}{3} \end{aligned}$$

b) $(g \circ g^{-1})(4)$
From the definition of inverse function we know $(g \circ g^{-1})(x) = x$ for all x in the domain. So $(g \circ g^{-1})(4) = 4$.

YOU TRY IT:
Given $f(x) = \frac{1}{7}x + 5$.

156. Find $f^{-1}(x)$

157. Find $(f \circ f^{-1})(-3)$.

Inverse functions: Cubic, cube root

Open the Instructor Added Resource which will direct you to a video to complete the following.

YOU TRY IT:

158. Find the inverse of $f(x) = (x+4)^3$.

Inverse functions: Rational

Watch the video *Finding the Inverse of a Rational Function* to complete the following.

Write an equation for the inverse function for the one-to-one function.

$t(x) =$ ____________________

EXAMPLE: Find the inverse of $f(x) = \dfrac{x-3}{x+2}$.

$$y = \frac{x-3}{x+2}$$

Switch x and y.

$$x = \frac{y-3}{y+2}$$

Multiply both sides by $y+2$

$$x(y+2) = \frac{y-3}{\cancel{y+2}} \cdot \cancel{y+2}$$

Distribute the x.

$$xy + 2x = y - 3$$

Bring all terms with y to one side and all other terms to other side.

$$xy - y = -2x - 3$$

Factor out y.

$$y(x-1) = -2x - 3$$

Divide by $x - 1$

$$y = \frac{-2x-3}{x-1}$$

$$f^{-1}(x) = \frac{-2x-3}{x-1}$$

YOU TRY IT:

159. Find the inverse of $g(x) = \dfrac{2x-1}{x-4}$.

Inverse functions: Quadratic, square root

Watch the video *Finding the Inverse of a Function with a Restricted Domain* to complete the following.

a. Graph $f(x) =$________________; ____________.

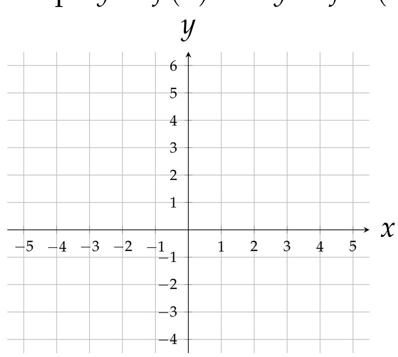
b. From the graph of f, is f a one-to-one function?

c. Write the domain of f in interval notation.

d. Write the range of f in interval notation.

e. Write an equation for $f^{-1}(x)$.

f. Graph $y = f(x)$ and $y = f^{-1}(x)$ on the same coordinate system.



g. Write the domain of $f^{-1}(x)$ in interval notation.

h. Write the range of $f^{-1}(x)$ in interval notation.

EXAMPLE:

a) Find the inverse of $f(x) = \sqrt{x-4}+3$.

$$\begin{aligned} f(x) &= \sqrt{x-4}+3 \\ y &= \sqrt{x-4}+3 \\ x &= \sqrt{y-4}+3 \\ x-3 &= \sqrt{y-4} \\ (x-3)^2 &= y-4 \\ x^2-6x+9+4 &= y \\ f^{-1}(x) &= x^2-6x+13 \quad \text{for } \ x \geq 3 \end{aligned}$$

We need the extra condition $x \geq 3$ because otherwise $f^{-1}(x)$ is NOT one-to-one.

b) Find the inverse of $g(x) = x^2+2x-4$ where $x \geq -1$.

$$\begin{aligned} g(x) &= x^2+2x-4 \\ y &= x^2+2x-4 \\ x &= y^2+2y-4 \\ x &= y^2+2y+1-1-4 \\ x &= (y+1)^2-5 \\ x+5 &= (y+1)^2 \\ \sqrt{x+5} &= y+1 \\ \sqrt{x+5}-1 &= f^{-1}(x) \end{aligned}$$

YOU TRY IT:

160. Find the inverse of $f(x) = \sqrt{3x-1}+2$.

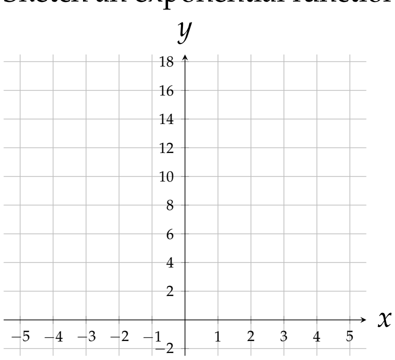
161. Find the inverse of $g(x) = x^2-6x-4$ where $x \geq 3$.

Graphing an exponential function and its asymptote: $f(x) = b^{-x}$ or $f(x) = -b^{ax}$

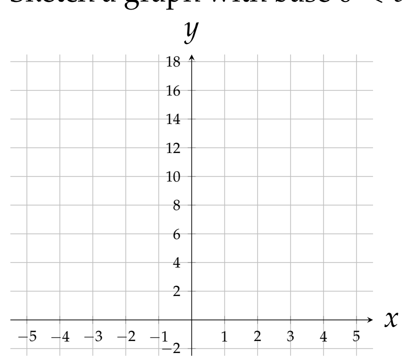
Learning Page **Background:**

For a ______________ $b \neq 1$, a function of the form ______________ is an exponential function with base b.

Sketch an exponential function with base $b > 1$.



Sketch a graph with base $0 < b < 1$.



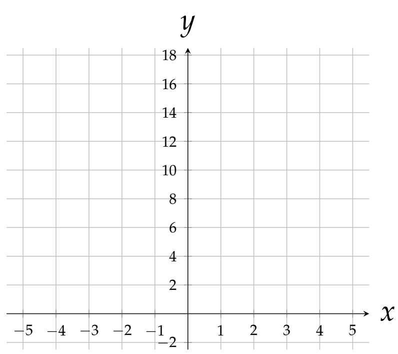
Graphing an exponential function and its asymptote: $f(x) = a(e)^{x-b} + c$

Watch the video *Graphing Exponential Functions* to complete the following. *NOTE:* This video may not pop up immediately. Select it from the list of videos in the video box.

Graph the functions.

a. $g(x) =$ __________

x	$g(x)$
0	
1	
2	
3	
-1	
-2	
-3	



b. $k(x) =$ __________

x	$k(x)$
0	
1	
2	
3	
-1	
-2	
-3	

Table for an exponential function

Learning Page The table gives ____________ x and their corresponding ____________ $h(x)$.

We use the rule $\left(\frac{a}{b}\right)^{-n} =$ ____________

YOU TRY IT: Complete the tables below.

162.

x	$g(x) = 5^x$
0	
1	
2	
3	
-1	
-2	
-3	

163.

x	$f(x) = (\frac{1}{3})^x$
0	
1	
2	
3	
-1	
-2	
-3	

The graph, domain, and range of an exponential function

Open the e-book to complete the following.

Graphs of $f(x) = b^x$

The graph of an exponential function defined by $f(x) = b^x$ where $b > 0$ and $b \neq 1$ has the following properties.

1. If $b > 1$, f is an ________________ exponential function.

   If $0 < b < 1$, f is a ________________ exponential function.

2. The domain is ____.

3. The range is ________________.

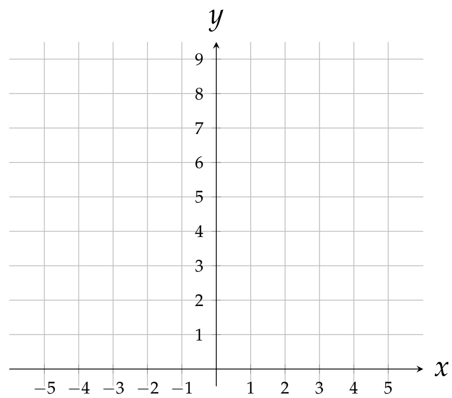
4. The line ________ is a ____________________.

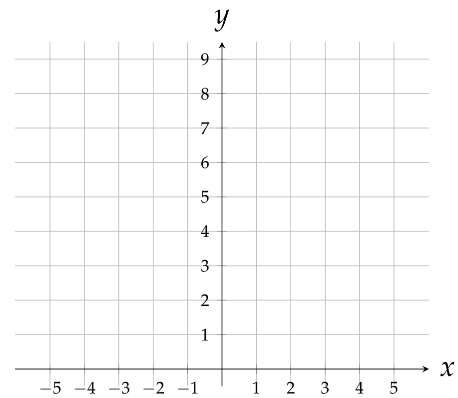
5. The function passes through the point ________ (this is the y-intercept) because $f(0) = b^0 = 1$.

Translating the graph of an exponential function

Watch the video *Graphing an Exponential Function Using Transformations* to complete the following.

Graph $g(x) =$ ______________

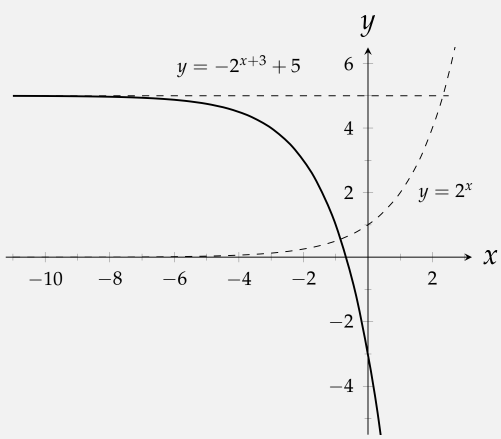




EXAMPLE:
Sketch the graph of $y = -2^{x+3} + 5$.

This is the graph of $y = 2^x$ transformed by

- Shifting down 5 units
- Shifting left 3 units
- Reflecting across the x-axis



YOU TRY IT.

164. Sketch the graph of $y = 3^{x-2} - 4$

Transforming the graph of a natural exponential function

Learning Page Some ways to transform the graph of a function.

1.

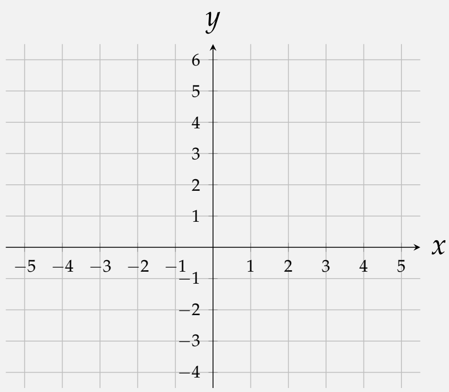
2.

3.

In what order is it a good idea to perform the transformations? ____________________

YOU TRY IT: Sketch the graph of $y = e^{x-1} - 3$

165.



Evaluating an exponential function that models a real-world situation

EXAMPLE:
The dollar value $c(t)$ of a car that is t years old is given by $c(t) = 19,900(0.86)^t$. Find the value initial value of the car and the value of the car after 11 years.

- Initial value
  The initial value will be the value of the car at 0 years so we compute $c(0)$.
  $c(0) = 19,900(0.86)^0 = \$19,900$
- Value after 11 years
  We are computing $c(11)$.
  $c(11) = 19,900(0.86)^{11} \approx \$3787$

YOU TRY IT:
A radioactive substance has a half-life of 14 hours. The amount $a(t)$ in grams of a sample remaining after t hours is given by

$$a(t) = 2800\left(\frac{1}{2}\right)^{\frac{t}{14}}$$

166. Find the initial amount in the sample.

167. Find the amount remaining after 30 hours.

Evaluating an exponential function with base e that models a real-world situation

Watch the video *Applying an Exponential Function to Newton's Law of Cooling* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

The temperature $T(t)$ of an object set to cool is modeled by

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

where T_a is the __________________ of the surrounding __________

T_0 is the ______________ temperature of the object

t is the ______________ since the hot object was set to cool

k is a ______________ related to the physical __________________ of the object

A cake comes out of the oven at __________ and is placed on a cooling rack in a __________ kitchen.

After checking the temperature several minutes later, it is determined that the cooling rate k is __________.

a. Write a function that models the temperature $T(t)$ (in °F) of the cake t minutes after being removed from the oven.

b. What is the temperature of the cake 10 min after coming out of the oven? Round to the nearest degree.

c. It is recommended that the cake should not be frosted until it has cooled to under 100°F. If Jessica waits 1 hr to frost the cake, will the cake be cool enough to frost?

EXAMPLE:
A bacteria population size increases according to $P(t) = 1700e^{0.18t}$ where t is measured in hours. Find the initial number in the population and the number after 7 hours.

- Initial number
  We want the number of bacteria after 0 hours so we compute $P(0)$.
  $P(0) = 1700e^{0.18(0)} = 1700$
- Number after 7 hours $P(7) = 1700e^{0.18(7)} \approx 5993$

YOU TRY IT:
The velocity $v(t)$ in m/s of an object falling near Earth's surface is given by $v(t) = 49(1 - e^{-0.22t})$ where t is measured in seconds.

168. Find the velocity of the object after 4 seconds.

Notes from Focus Group:

Notes from Focus Group:

Module 8-Review Module

To help you review for your upcoming exam, this module contains all of the topics from the modules since the last exam. Topics that you have already mastered will not appear in your carousel.

☐ Complete this module before you take the ALEKS exam.

☐ Each exam has two parts.

- The ALEKS exam (100 pts)
    - The ALEKS exam must be taken in the MALL.
    - The ALEKS exam is a Comprehensive Knowledge Check.
        - Your score is the number of topics you have mastered out of the number of topics you should have mastered by this point.
    - If you lose topics on your ALEKS exam, your Review Module completion grade will not change.
    - Your scratch work for the ALEKS exam must be numbered and turned in through Blackboard.
- The Written exam (25 pts)
    - Take your written exam in class the day of your focus group.
    - To study for the written exam:
        - Rework your old Focus Group assignments.
        - Rework any topics in ALEKS you may have lost on the ALEKS exam.

	Score
ALEKS Exam	
Written Exam	

Module 9

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Weekly Checklist

☐ Complete MALL time.

☐ Work in ALEKS and Notebook at least 4 days a week.

☐ Complete the weekly Module and Notebook pages by the due date.

☐ Attend Focus Group.

☐ Actively participate in Focus Group.

☐ Earn extra credit: Complete 10 topics by ______________________.

Solving an exponential equation by finding common bases: Linear exponents

Watch the video *Solving an Exponential Equation by Using the Equivalence Property* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Equivalence Property of Exponential Expressions

Let b, x, and y be real numbers with $b > 0$ and $b \neq 1$. Then,

$$b^x = b^y \text{ implies that } \_\_\_\_\_\_\_\_\_\_\_\_$$

Solve.

Check:

Learning Page For any positive number A such that $A \neq 1$, we have the following.

________________ if and only if ____________

We can write each side of our equation with the ________________ and then apply this property.

EXAMPLE: Solve for x.

$$32^{x-4} = 64$$

Rewrite each side with base 2.

$$(2^5)^{x-4} = 2^6$$

Simplify exponent on left.

$$2^{5x-20} = 2^6$$

Use property from above.

$$5x - 20 = 6$$

$$5x = 26$$

$$x = \frac{26}{5}$$

YOU TRY IT: Solve for x.

169. $4^{x+2} = \dfrac{1}{2^x}$

Solving an exponential equation by finding common bases: Linear and quadratic exponents

Open the Instructor Added Resource which will direct you to a video to complete the following.

YOU TRY IT: Solve for x.

170. $16^{7x-7} = 4^{x^2+4x+7}$

Converting between logarithmic and exponential equations

Watch the video *Converting from Logarithmic Form to Exponential Form* to complete the following.

Write each equation in exponential form.

a.

b.

______ is the same as ______

c.

Watch the video *Converting from Exponential Form to Logarithmic Form* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Write each equation in logarithmic form.

a.

b.

______ is the same as ______

c.

EXAMPLE:

a) Write $\log_5 x = y$ as an exponential equation.

$$\log_5 x = y$$
$$5^y = x$$

b) Write $c^6 = 3$ as a logarithmic equation.

$$c^6 = 3$$
$$\log_c 3 = 6$$

YOU TRY IT:

171. Write $\log_4 5 = x$ as an exponential equation.

172. Write $7^y = 9$ as a logarithmic equation.

Converting between natural logarithmic and exponential equations

Learning Page For any numbers a, b, and c, with a and c positive ($a \neq 1$), we have the following equivalence.

$\log_a c = b$ if and only if ______________

The first is a ______________ equation, and the second is an ______________ equation.

However, when the base is __________, we do __________ write ______________.

Instead, we write __________, which is read as ____________________

e is a special ______________ number. Its value is $e =$______________

So, when the base of the logarithm is e, we write the relationship as follows.

____________________ if and only if ____________________

EXAMPLE:

a) Write $\ln 8 = x$ as an exponential equation.

$$\ln 8 = x$$
$$e^x = 8$$

b) Write $e^y = 2$ as a logarithmic equation.

$$e^y = 2$$
$$\ln 2 = y$$

YOU TRY IT:

173. Write $\ln x = 5$ as an exponential equation.

174. Write $e^r = t$ as a logarithmic equation.

Evaluating logarithmic expressions

Watch the video *Evaluating Common and Natural Logarithms* to complete the following.

Simplify the expressions.

a.

b.

YOU TRY IT: Simplify the expressions.

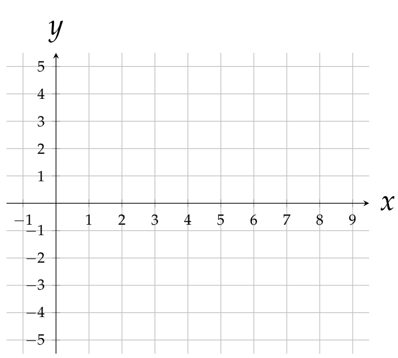
175. $\log_5 \frac{1}{125}$

176. $\ln e^5$

Graphing a logarithmic function: Basic

Watch the video *Graphing a Logarithmic Function* to complete the following.

Graph ____________________.

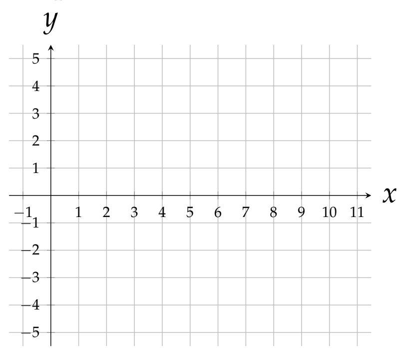


	y
	0
	2
	-2

Translating the graph of a logarithmic function

Watch the video *Using Transformations to Graph a Logarithmic Function* to complete the following.

Graph. ____________________________

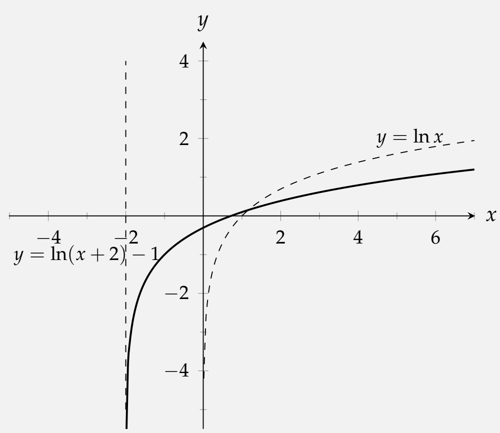


EXAMPLE:

Sketch the graph of $y = \ln(x + 2) - 1$.

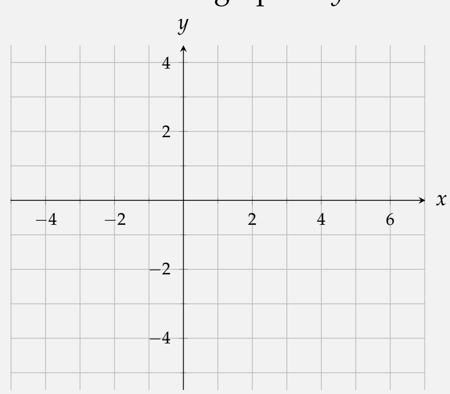
This is the graph of $y = \ln x$ shifted

- left 2 units
- down 1 unit



YOU TRY IT:

177. Sketch the graph of $y = -2\ln(x + 3) + 1$.

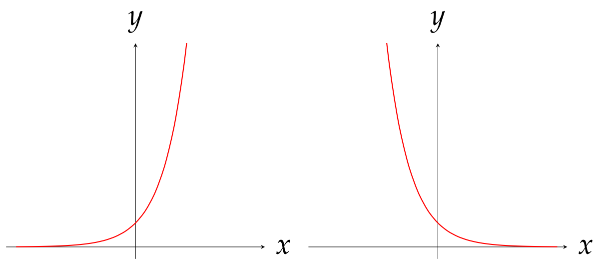


The graph, domain, and range of a logarithmic function

Open the e-book to complete the following.

Graphs of Exponential and Logarithmic Functions

Exponential Functions



$b > 1$ $\quad$ $0 < b < 1$

Domain: ____________

Range: ____________

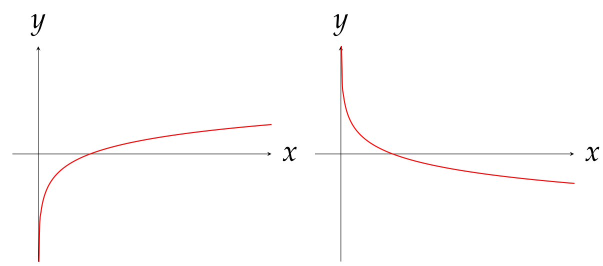
Horizontal asymptote: ____________

Passes through: ____________

If $b > 1$, the function is ____________.

If $0 < b < 1$, the function is ____________.

Logarithmic Functions



$b > 1$ $\quad$ $0 < b < 1$

Domain: ____________

Range: ____________

Vertical asymptote: ____________

Passes through: ____________

If $b > 1$, the function is ____________.

If $0 < b < 1$, the function is ____________.

Solving an equation of the form $\log_b a = c$

Learning Page For any numbers a, b, and c, with a and c positive ($a \neq 1$), we have the following relationship.

________________________ if and only if ________________

EXAMPLE: Solve.

$$\log_2 x = -3$$

Use the relationship above.

$$2^{-3} = x$$

$$\frac{1}{8} = x$$

YOU TRY IT: Solve.

178. $\log_x 2 = \frac{1}{3}$

Basic properties of logarithms

Open the e-book to complete the following.

Product Property of Logarithms

Let b, x, and y be positive real numbers where $b \neq 1$. Then

$$\log_b(xy) = \underline{\qquad\qquad\qquad\qquad}$$

The logarithm of a product equals the ____.

Quotient Property of Logarithms

Let b, x, and y be positive real numbers where $b \neq 1$. Then

$$\log_b\left(\frac{x}{y}\right) = \underline{\qquad\qquad\qquad\qquad}$$

The logarithm of a quotient equals the _______________ of the logarithm of the _______________ and the _______________ of the _______________.

Power Property of Logarithms

Let b, x, and y be positive real numbers where $b \neq 1$. Let p be any real number. Then

$$\log_b x^p = \underline{\qquad\qquad\qquad\qquad}.$$

Properties of Logarithms

Let b, x, and y be positive real numbers where $b \neq 1$, and let p be any real number. Then the following properties of logarithms are true.

1. $\log_b 1 = \underline{\qquad}$
2. $\log_b b = \underline{\qquad}$
3. $\log_b b^p = \underline{\qquad}$
4. $b^{\log_b x} = \underline{\qquad}$

Domain of a logarithmic function: Advanced

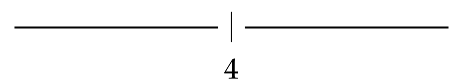
Watch the video *Identifying the Domain of a Logarithmic Function* to complete the following.

The domain of ______________ is restricted to ______________

Write the domain in interval notation.

a. $f(x) =$ ______________

b. $r(x) =$ ______________



YOU TRY IT: Find the domain of the function. Write your answer in interval notation.

179. $g(x) = \log(x+7)$

Expanding a logarithmic expression: Problem type 1

Watch the video *Applying the Product Property of Logarithms* to complete the following.

Product Property of Logarithms

Let b, x, and y be positive real numbers where __________. Then,

$$\_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Example:

Write the logarithm as a sum and simplify if possible.

EXAMPLE: Expand $\log\left(\frac{x^3y^5}{z}\right)$.

Use the Quotient Property

$$\log\left(\frac{x^3y^5}{z}\right) = \log(x^3y^5) - \log z$$

Use the Product Property

$$= \log x^3 + \log y^5 - \log z$$

Use the Power Property

$$= 3\log x + 5\log y - \log z$$

YOU TRY IT:

180. Expand $\ln\left(\frac{xz^2}{y^5}\right)$.

Expanding a logarithmic expression: Problem type 2

Watch the video *Writing a Logarithmic Expression in Expanded Form* to complete the following.

Write the expression as the sum or difference of logarithms.

EXAMPLE: Expand $\log\left(\dfrac{x^3y^5}{\sqrt{z}}\right)$.

$$\begin{aligned}\log\left(\frac{x^3y^5}{\sqrt{z}}\right) &= \log(x^3y^5) - \log\sqrt{z}\\ &= \log x^3 + \log y^5 - \log\sqrt{z}\\ &= 3\log x + 5\log y - \frac{1}{2}\log z\end{aligned}$$

YOU TRY IT:

181. Expand $\ln\left(\dfrac{3\sqrt{x}}{y^5z^4}\right)$.

Writing an expression as a single logarithm

Watch the video *Writing the Sum or Difference of Logarithms as a Single Logarithm 2* to complete the following.

Write the logarithmic expression as a single logarithm with coefficient 1, and simplify if possible.

EXAMPLE: Write $\frac{1}{2}\ln y - \frac{1}{3}\ln x + \ln 2$ as a single log.

$$\begin{aligned}\frac{1}{2}\ln y - \frac{1}{3}\ln x + \ln 2 &= \ln y^{1/2} - \ln x^{1/3} + \ln 2 \\ &= \ln\sqrt{y} - \ln\sqrt[3]{x} + \ln 2 \\ &= \ln\left(\frac{\sqrt{y}}{\sqrt[3]{x}}\right) + \ln 2 \\ &= \ln\left(\frac{2\sqrt{y}}{\sqrt[3]{x}}\right)\end{aligned}$$

YOU TRY IT:

182. Write $\log(x-1) + \log 3 - 3\log x$ as a single log.

Using properties of logarithms to evaluate expressions

Learning Page Let a, b, and c be any real numbers, with a and c positive, and $a \neq 1$.

We have the following definition of **logarithm**.

________________ if and only if ________________

From this definition, we get the following fact.

____

However, when the base is __________, we do __________ write ____________.

Instead we write __________, which is read as ____________________.

________________ if and only if ________________

From this definition, we get the following fact.

____

We also have the following properties of logarithms.

Logarithm of a product:	$\log_a M + \log_a N =$ ____________
Logarithm of a quotient:	$\log_a M - \log_a N =$ ____________
Logarithm of a power:	$p\log_a M =$ ____________

YOU TRY IT: Use the properties of logarithms to evaluate the expression.

183. $6\ln e^4 - \ln e^3$

Solving a multi-step equation involving a single logarithm: Problem type 1

Open the e-book to read EXAMPLE 8: **Solving a Logarithmic Equation** to complete the following steps.

Solve. ____________________

Solution:

$4\log_3(2t-7)=8$

$\log_3(2t-7)=2$ Isolate the ____________ by ____________ both sides by 4.

The equation is in the form ________________ where ________________

________________ Write the equation in ____________ form.

$2t-7=9$

$t=8$

Check: $4\log_3(2t-7)=8$

$4\log_3[2(8)-7]\stackrel{?}{=}8$

$4\log_3 9\stackrel{?}{=}8$

$4\cdot 2\stackrel{?}{=}8\checkmark$

YOU TRY IT: Solve.

184. $5\log_6(7x+1)=10$

Solving a multi-step equation involving a single logarithm: Problem type 2

Watch the video *Solving a Logarithmic Equation by Writing Exponential Form* to complete the following.

Solving Logarithmic Equations by Using Exponential Form

Step 1 Given a logarithmic equation, ______________ the ________________ on ________________ of the ______________.

Step 2 Use the __________________________ to write the equation in the form ______________ where k is a constant.

Step 3 Write the equation in ________________________.

Step 4 __________ the equation from ______________.

Step 5 __________ the potential solution(s) in the ________________________.

Solve.

Check:

EXAMPLE: Solve.

$$\log_3(x-1) - \log_3 4 = 2$$

Use Quotient Property of Logs.

$$\log_3 \frac{x-1}{4} = 2$$

Use Def of Log.

$$\frac{x-1}{4} = 3^2$$

$$x - 1 = 36$$

$$x = 37$$

YOU TRY IT: Solve.

185. $-7 + \log_4(x+3) = -5$

Solving a multi-step equation involving natural logarithms

Open the Instructor Added Resource which will direct you to a video to complete the following.

Solve for x.

YOU TRY IT: Solve for x.

186. $\ln(x+2) = 4$

Solving an equation involving logarithms on both sides: Problem type 1

Watch the video *Solving a Logarithmic Equation 2* to complete the following.

Solve ______________________

YOU TRY IT: Solve the equation.

187. $\log_3 x + \log_3(x+6) = 3$

Solving an equation involving logarithms on both sides: Problem type 2

Watch the video *Solving a Logarithmic Equation by Using the Equivalence Property* to complete the following.

Equivalence Property of Logarithmic Expressions

Let b, x, and y be positive real numbers with $b \neq 1$. Then,

__________________ implies that ______________

Solve __________________.

EXAMPLE:
Solve.

$$\begin{aligned}
\log_5(x+18)+\log_5(x-6) &= 2\log_5 x\\
\log_5((x+18)(x-6)) &= \log_5 x^2\\
(x+18)(x-6) &= x^2\\
x^2+12x-108 &= x^2\\
12x-108 &= 0\\
12x &= 108\\
x &= 9
\end{aligned}$$

YOU TRY IT:
Solve.

188. $\log_2 x+\log_2(x-4)=\log_2(x+24)$

Solving an exponential equation by using logarithms: Exact answers in logarithmic form

Watch the video *Solving an Exponential Equation by Using Logarithms 3* to complete the following.

Solve ____________________________

EXAMPLE: Solve.

$$\begin{aligned} 4^{x+2} &= 7^x \\ \ln 4^{x+2} &= \ln 7^x \\ (x+2)\ln 4 &= x\ln 7 \\ x\ln 4 + 2\ln 4 &= x\ln 7 \\ x\ln 4 - x\ln 7 &= -2\ln 4 \\ x(\ln 4 - \ln 7) &= -2\ln 4 \\ x &= \frac{2\ln 4}{\ln 4 - \ln 7} \end{aligned}$$

YOU TRY IT: Solve.

189. $e^{x-2} = 9$

Finding the time given an exponential function with base e that models a real-world situation

Watch the video *Solving Two Equations for a Specified Variable* to complete the following. *NOTE:* This video may not pop up immediately. Select it from the list of videos in the video box.

Solve for k.

Solve for D.

Finding the initial amount and rate of change given an exponential function

Learning Page A function in the following form models ______________________.

______________ (where $a > 0, b > 0,$ and $b \neq 1$)

Here, y is an ___________ and t is the ______________________. Note the following.

- The constant ________ is the __________________, that is, the value of __________________.

- The constant ________ tells whether the functions models ______________________.

    - If ___________, then the function models ___________.

    - If ___________, then the function models ___________.

- From the value of ________, we can also ge the ___________ of growth or decay.

    - If ___________, then b equals ___________, where r is the __________________.

      That is ________ is the __________________ (expressed as a decimal) for each ______________________.

    - If ___________, then b equals ___________, where r is the __________________.

      That is ________ is the __________________ (expressed as a decimal) for each ______________________.

Notes from Focus Group:

Notes from Focus Group:

Module 10

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Weekly Checklist

- ☐ Complete MALL time.
- ☐ Work in ALEKS and Notebook at least 4 days a week.
- ☐ Complete the weekly Module and Notebook pages by the due date.
- ☐ Attend Focus Group.
- ☐ Actively participate in Focus Group.
- ☐ Earn extra credit: Complete 10 topics by ____________________.

Converting between degree and radian measure: Problem type 1

Watch the video *Converting between Degrees to Radians and Vice Versa* to complete the following.

Converting Between Degree and Radian Measure

- To convert from degrees to ____________, multiply the degree measure by __________.
- To convert from radians to ____________, multiply the radian measure by __________.

Number of radians in one revolution: $\frac{2\pi r}{r} =$ __________ radians $=$ __________

1. Convert __________ to radians.
2. Convert __________ to degrees.

EXAMPLE:

Convert $\frac{5\pi}{3}$ to degrees.

$\frac{5\pi}{3} \cdot \frac{180^\circ}{\pi} = 300^\circ$

YOU TRY IT:

190. Convert 165° to radians.

Sketching an angle in standard position

Learning Page An angle θ in standard position has its ______________ side on the

____________________________.

If θ is positive, we rotate the terminal side in the ____________________________ direction.

If θ is negative, we rotate the terminal side in the ________________________ direction.

An angle that makes a ______________ revolution of the circle sweeps _________ radians.

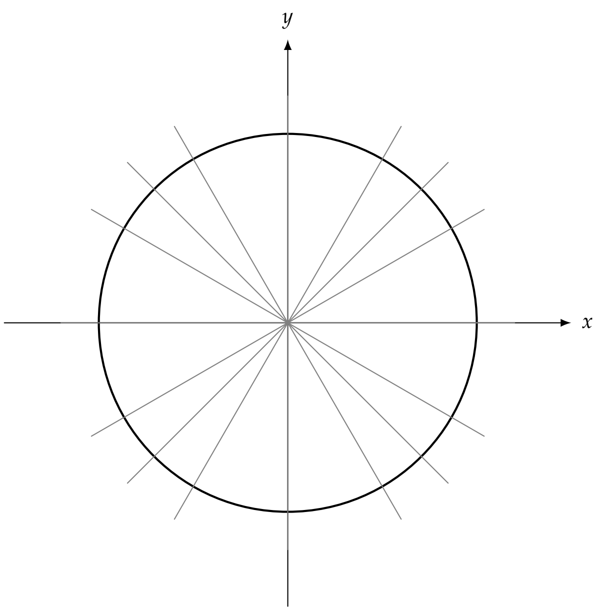
Using this fact, we can find the radian measures of some "quarter turn" angles.

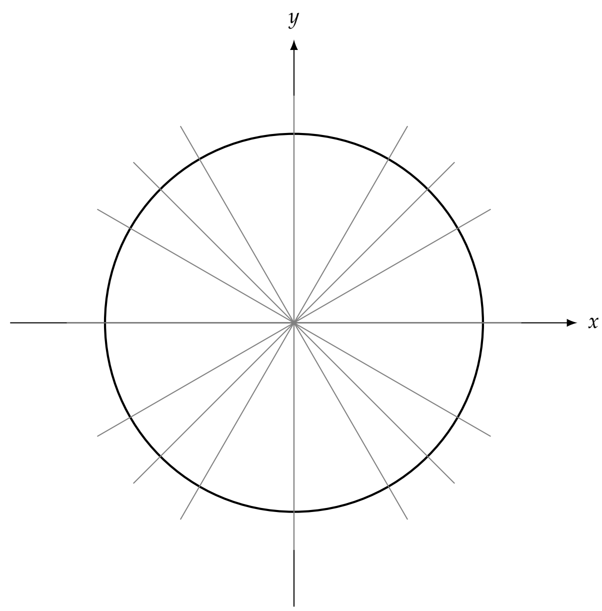
$\frac{1}{4}$ revolution =_____ radians

$\frac{1}{2}$ revolution =_____ radians

$\frac{3}{4}$ revolution =_____ radians

On the circles below label the angles using radians on the unit circle. On the left circle use positive angles and the right circle use negative angles.

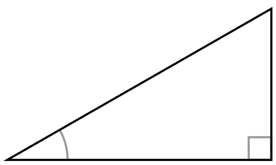




Sine, cosine, and tangent ratios: Numbers for side lengths

Watch the video *Define Trigonometric Functions of Acute Angles* to complete the following.

Label the triangle as shown in the video.



sine:

cosine:

tangent:

cosecant:

secant:

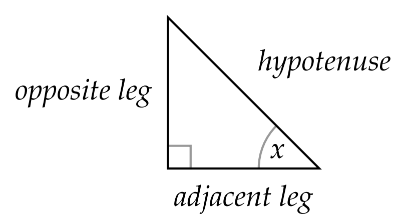
cotangent:

Sine, cosine, and tangent ratios: Variables for side lengths

Learning Page A **trigonometric ratio** is a ratio of ____________________.

For an acute angle with measure x, these ratios are defined as follows.

$\sin x =$



$\cos x =$

$\tan x =$

Coterminal angles

Learning Page Two angles are **coterminal** if they have the ______________________________.

For a given angle θ, ____________ or ____________ a __________________

gives an angle ____________ with ________.

A ____________ revolution is ____________, which is 360°.

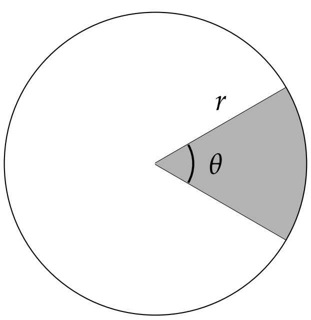
So, to get an angle coterminal with θ, we __________________ a multiple of ______ radians

or 360 degrees.

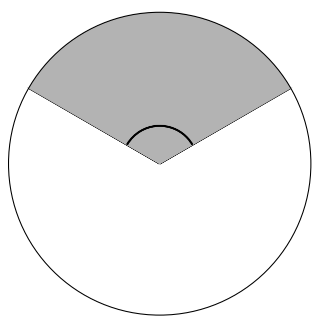
Area of a sector of a circle

Watch the video *Determining the Area of a Sector* to complete the following.

Find the area of the sector of a circle with radius r and central angle θ.



Find the exact area of the sector of the circle shown. Then approximate to the nearest tenth of a centimeter. **Label the circle as shown in video.**



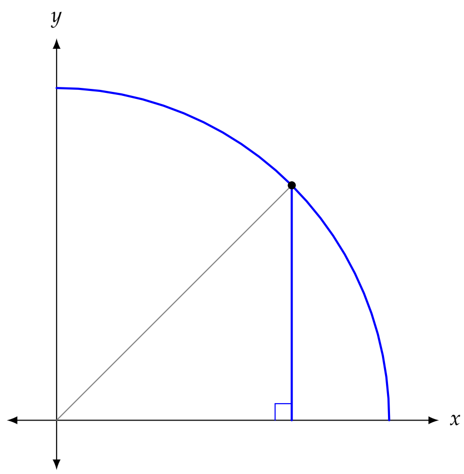
Finding coordinates on the unit circle for special angles

Open the Instructor Added Resource which will direct you to a video to complete the following.

$\cos 0 =$ ________ $\cos \frac{\pi}{2} =$ ________ $\cos \pi =$ ________ $\cos \frac{3\pi}{2} =$ ________

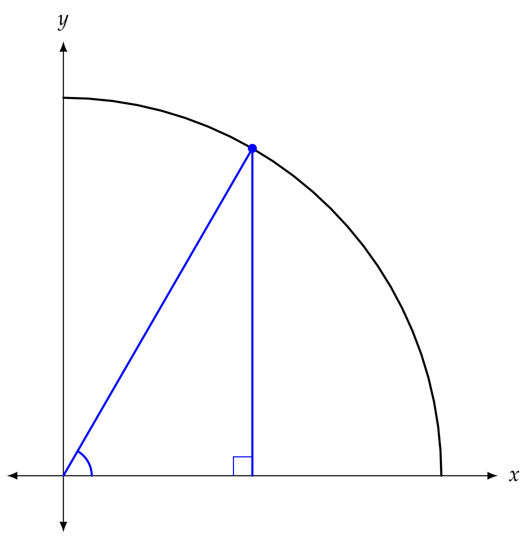
$\sin 0 =$ ________ $\sin \frac{\pi}{2} =$ ________ $\sin \pi =$ ________ $\sin \frac{3\pi}{2} =$ ________

On the triangle below, label the angle and side lengths of the triangle as they are labeled in the video.



Show the work to find $\sin \frac{\pi}{4}$ and $\cos \frac{\pi}{4}$.

On the triangle below, label the angle and side lengths of the triangle as they are labeled in the video.



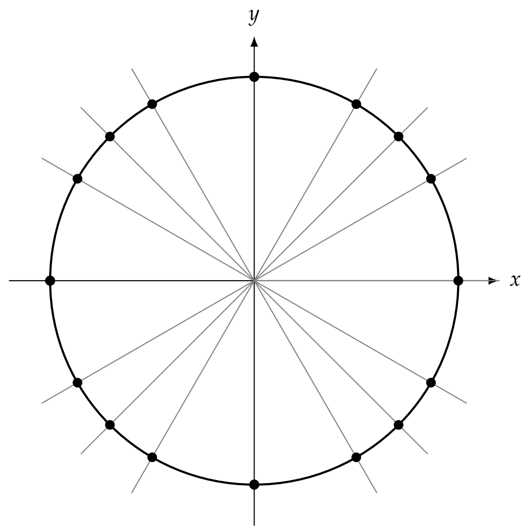
Show the work to find $\sin \frac{\pi}{3}$.

$\cos \frac{\pi}{3} =$ __________

From the triangle above we have $\cos \frac{\pi}{6} =$ __________ and $\sin \frac{\pi}{6} =$ __________.

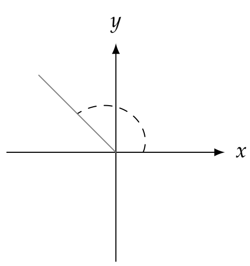
Continued on the next page.

On the circle given below, label the coordinates of each intersection point with the unit circle.



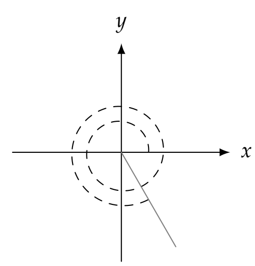
Reference angles: Problem type 1

Watch the video *Finding Reference Angles* to complete the following.



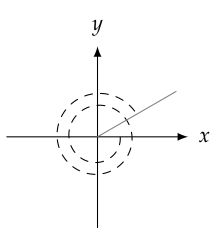
$\theta =$ ____________

$\theta' =$ ____________



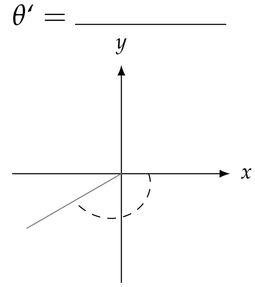
$\theta =$ ____________

$\theta' =$ ____________



$\theta =$ ____________

$\theta' =$ ____________

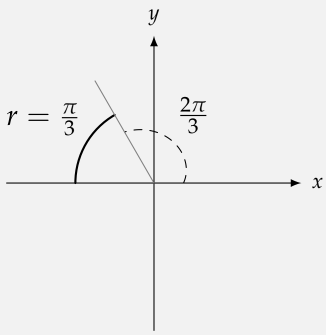


$\theta =$ ____________

$\theta' =$ ____________

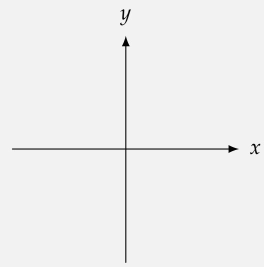
EXAMPLE:
Find the reference angle, r, for the given angle.

a) $\theta = \dfrac{2\pi}{3}$



YOU TRY IT:
Find the reference angle, r, for the given angle.

191. $\theta = \dfrac{7\pi}{6}$



Reference angles: Problem type 2

Open the e-book to complete the following.

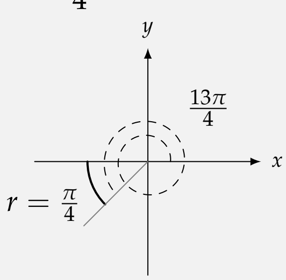
Definition of a Reference Angle

Let θ be an angle in standard position. The **reference angle** for θ is the ____________________

formed by the ________________________ and the ____________________.

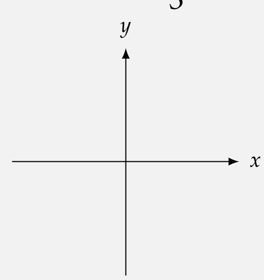
EXAMPLE:
Find the reference angle, r, for the given angle.

a) $\theta = \dfrac{13\pi}{4}$



YOU TRY IT:
Find the reference angle, r, for the given angle.

192. $\theta = -\dfrac{8\pi}{3}$



Trigonometric functions and special angles: Problem type 1

Watch the video *Using the Unit Circle to Evaluate the Trigonometric Functions of Special Angles* to complete the following.

Evaluate the functions.

a.

b.

Trigonometric functions and special angles: Problem type 2

Learning Page Complete the following fundamental identities.

$\sec\theta =$

$\csc\theta =$

$\tan\theta =$

$\cot\theta =$

Trigonometric functions and special angles: Problem type 3

Complete the chart below.

θ	$\cos\theta$	$\sin\theta$
0		
$\frac{\pi}{6}$		
$\frac{\pi}{4}$		
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		
π		
$\frac{3\pi}{2}$		

Finding trigonometric ratios from a point on the unit circle

Watch the video *Evaluating Trigonometric Functions Using the Unit Circle* to complete the following.

Suppose that the real number t corresponds to the point ________ on the unit circle. Evaluate the six trigonometric functions of t.

$\sin t =$

$\sec t =$

$\cos t =$

$\cot t =$

$\csc t =$

$\tan t =$

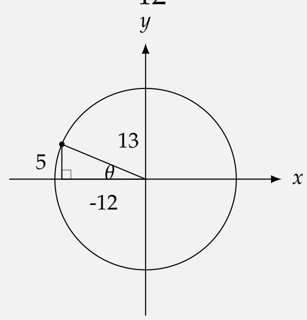
EXAMPLE:

Suppose θ is an angle whose terminal side intersects the unit circle at $\left(-\frac{12}{13}, \frac{5}{13}\right)$. Find the exact value of the following.

a) $\sin\theta = \frac{5}{13}$
The y coordinate of the intersection point gives us $\sin\theta$.

b) $\cos\theta = -\frac{12}{13}$
The x coordinate of the intersection point gives us $\cos\theta$.

c) $\tan\theta = -\frac{5}{12}$



d) $\sec\theta = -\frac{13}{12}$

e) $\csc\theta = \frac{13}{5}$

f) $\cot\theta = -\frac{12}{5}$

YOU TRY IT:

Suppose θ is an angle whose terminal side intersects the unit circle at $\left(\frac{40}{41}, -\frac{9}{41}\right)$. Find the exact value of the following.

193. $\sin\theta =$

194. $\cos\theta =$

195. $\tan\theta =$

196. $\sec\theta =$

197. $\csc\theta =$

198. $\cot\theta =$

Evaluating expressions involving sine and cosine

Learning Page We will have to know the sine and cosine of some common trigonometric angles.

We should also keep in mind some facts about angles in ____________ position.

- A rotation ________________ gives a ____________ angle.

  A rotation ________________ gives a ____________ angle.

- When finding sine and cosine values of a ____________ angle, we can use the

  ____________ ________________ angle.

EXAMPLE:
Given $\theta = \frac{3\pi}{4}$, evaluate each expression.

$$\begin{aligned}\sin(-\theta) &= \sin\left(-\frac{3\pi}{4}\right)\\ &= \sin\left(\frac{5\pi}{4}\right)\\ &= -\frac{\sqrt{2}}{2}\end{aligned}$$

$$\begin{aligned}\sin^2\theta &= \sin^2\left(\frac{3\pi}{4}\right)\\ &= \left(\sin\frac{3\pi}{4}\right)^2\\ &= \left(\frac{\sqrt{2}}{2}\right)^2\\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\sin(2\theta) &= \sin\left(2\cdot\frac{3\pi}{4}\right)\\ &= \sin\left(\frac{3\pi}{2}\right)\\ &= -1\end{aligned}$$

YOU TRY IT:
Given $\theta = \frac{7\pi}{6}$, evaluate each expression.

199. $\sin(-\theta) =$

200. $\sin^2\theta =$

201. $\sin(2\theta) =$

Even and odd properties of trigonometric functions

Learning Page A function $f(x)$ is **odd** if ________________ for all x in its domain.

A function $f(x)$ is **even** if ________________ for all x in its domain.

If ________________ of these properties hold, then f is ________________ odd nor even.

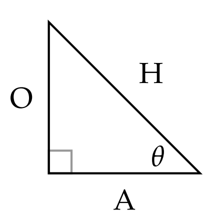
Open the e-book to complete the following.

Even and Odd Properties of Trigonometric Functions

Function	Evaluate at t and $-t$	Property
Sine	$\sin t = y$ and $\sin(-t) =$ ______	$\sin(-t) =$ __________ __________ function
Cosine	$\cos t = x$ and $\cos(-t) =$ ______	$\cos(-t) =$ __________ __________ function
Cosecant	$\csc t = \frac{1}{y}$ and $\csc(-t) =$ ______	$\csc(-t) =$ __________ __________ function
Secant	$\sec t = \frac{1}{x}$ and $\sec(-t) =$ ______	$\sec(-t) =$ __________ __________ function
Tangent	$\tan t = \frac{y}{x}$ and $\tan(-t) =$ ______	$\tan(-t) =$ __________ __________ function
Cotangent	$\cot t = \frac{x}{y}$ and $\cot(-t) =$ ______	$\cot(-t) =$ __________ __________ function

Finding trigonometric ratios given a right triangle

Learning Page State the **Pythagorean Theorem** for the triangle given below.



Click on trigonometric ratios to define the following in terms of the sides of the right triangle given.

$\sin\theta =$

$\tan\theta =$

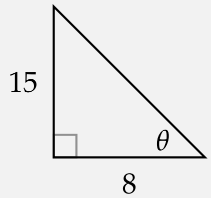
$\csc\theta =$

$\cos\theta =$

$\sec\theta =$

$\cot\theta =$

EXAMPLE:
Find $\cos\theta$ from the triangle given.



Find the length of H, the hypotenuse.

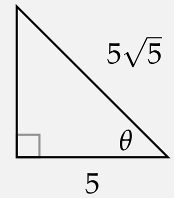
$$\begin{aligned} 8^2 + 15^2 &= H^2 \\ 289 &= H^2 \\ 17 &= H \end{aligned}$$

$\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

$\cos\theta = \dfrac{8}{17}$.

YOU TRY IT:

202. Find $\sin\theta$ from the triangle given.



Determining the location of a terminal point given the signs of trigonometric values

Learning Page Let θ be an angle with terminal point (x, y) on the unit circle.

The signs of x and y depend on the quadrant in which the terminal point lies.
Complete the following chart determining the sign in each quadrant using + for positive and - for negative.

	Quadrant I	Quadrant II	Quadrant III	Quadrant IV
x	$+$	$-$		
y	$+$	$+$		
$\sin\theta$				
$\cos\theta$				
$\tan\theta$				
$\sec\theta$				
$\csc\theta$				
$\cot\theta$				

Finding values of trigonometric functions given information about an angle: Problem type 1

Watch the video *Evaluating Trigonometric Functions of Any Angle* to complete the following.

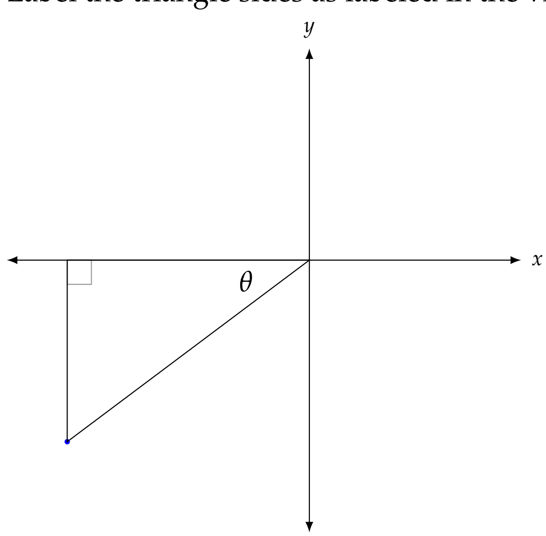
The point ______________ is on the terminal side of an angle θ drawn in standard position. Find the values of the six trigonometric functions of θ.

Finding values of trigonometric functions given information about an angle: Problem type 2

Open the Instructor Added Resource which will direct you to a video to complete the following.

Let θ be an angle in quadrant III such that ________________________. Find the exact value of the following.

Label the triangle sides as labeled in the video.



a) $\cos\theta =$

b) $\tan\theta =$

c) $\sec\theta =$

d) $\csc\theta =$

e) $\cot\theta =$

Finding values of trigonometric functions given information about an angle: Problem type 3

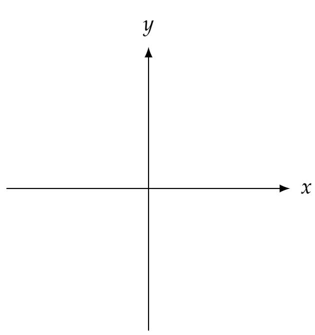
Watch the video *Evaluating a Trigonometric Function from Given Information by Using Fundamental Identities* to complete the following.

Given $\sin\theta =$ ____________ in Quadrant III, use the fundamental trigonometric identities to find $\cos\theta$.

Finding values of trigonometric functions given information about an angle: Problem type 4

Watch the video *Using Reference Angles to Evaluate a Trigonometric Function from Given Information* to complete the following.

Given $\tan\theta =$ ____________ and ____________ < 0, find the values of $\sin\theta$ and $\cos\theta$.



YOU TRY IT: Given $\cos\theta = -\frac{8}{17}$ and θ is in Quadrant II, find the exact value of the following.

203. $\sin\theta =$

204. $\tan\theta =$

205. $\sec\theta =$

206. $\csc\theta =$

207. $\cot\theta =$

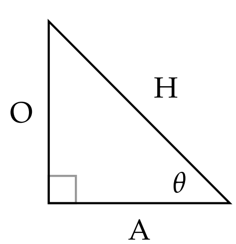
Using trigonometry to find a length in a word problem with one right triangle

Watch the video *Apply Trigonometric Functions to Find an Unknown Distance* to complete the following.

A ______ foot boat ramp makes a ______ angle with the water. What is the height of the ramp above the water at the ramp's highest point?

Using a trigonometric ratio to find a side length in a right triangle

Learning Page Define the following in terms of the sides of the right triangle given.



$\sin\theta =$

$\cos\theta =$

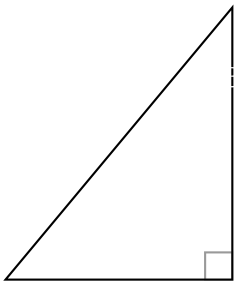
$\tan\theta =$

Using the Pythagorean Theorem to find a trigonometric ratio

Watch the video *Evaluating Trigonometric Functions Given Two Sides of a Right Triangle* to complete the following.

Given a right triangle with legs of length ____ cm and ____ cm, find the values of the six trigonometric functions of the smallest angle.

Label the graph as shown in the video.



Find the value of c.

$\sin\theta =$

$\cos\theta =$

$\tan\theta =$

$\csc\theta =$

$\sec\theta =$

$\cot\theta =$

Notes from Focus Group:

Notes from Focus Group:

Module 11

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Weekly Checklist

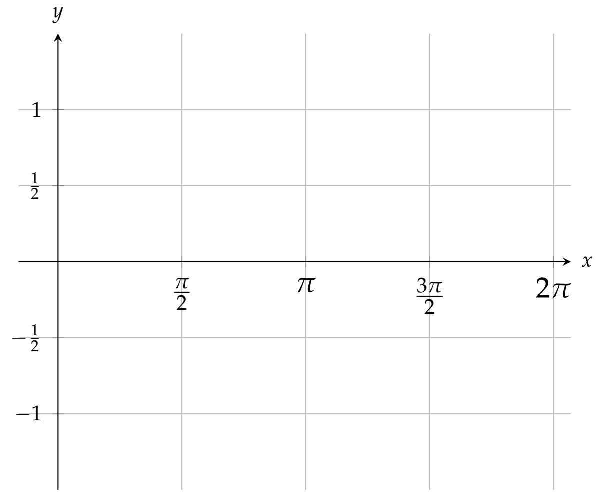
- ☐ Complete MALL time.
- ☐ Work in ALEKS and Notebook at least 4 days a week.
- ☐ Complete the weekly Module and Notebook pages by the due date.
- ☐ Attend Focus Group.
- ☐ Actively participate in Focus Group.
- ☐ Earn extra credit: Complete 10 topics by ____________________.

Sketching the graph of $y = a \sin x$ or $y = a \cos x$

Watch the video *Graphing* $y = A\sin(x)$ to complete the following.

Graph the function and identify the key points on one full period.

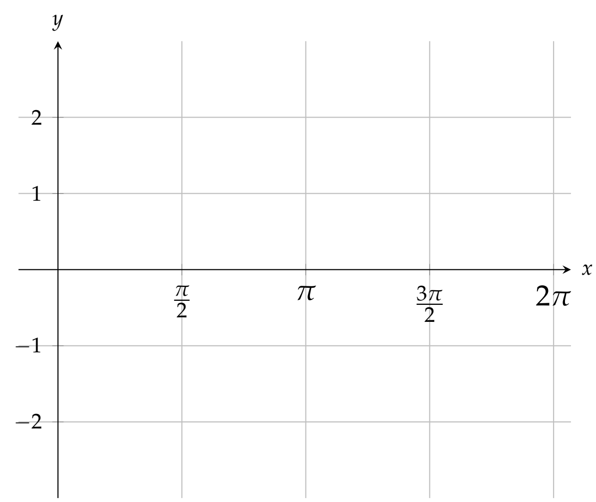
____



Watch the video *Graphing* $y = A\cos(x)$ to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Graph the function and identify the key points on one full period.

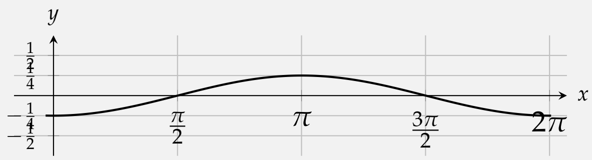
____



EXAMPLE: Sketch the graph of $y = -\frac{1}{4}\cos x$.

We will consider how the y-coordinates of the five key points change from the graph of $y = \cos x$.

x	$y = \cos x$	$y = -\frac{1}{4}\cos x$
0	1	$-\frac{1}{4}$
$\frac{\pi}{2}$	0	0
π	-1	$\frac{1}{4}$
$\frac{3\pi}{2}$	0	0
2π	1	$-\frac{1}{4}$



The amplitude of $y = -\frac{1}{4}\cos x$ is $\frac{1}{4}$.

YOU TRY IT:

208. Sketch the graph of $y = 5\sin x$.

Sketching the graph of $y = \sin bx$ or $y = \cos bx$

Open the Instructor Added Resource which will direct you to a video to complete the following.

Sketch the graph of ____________________.

Amplitude and Period of the Sine and Cosine Functions

For $y = A\sin Bx$ and $y = A\cos Bx$ and $b > 0$, the amplitude and period are

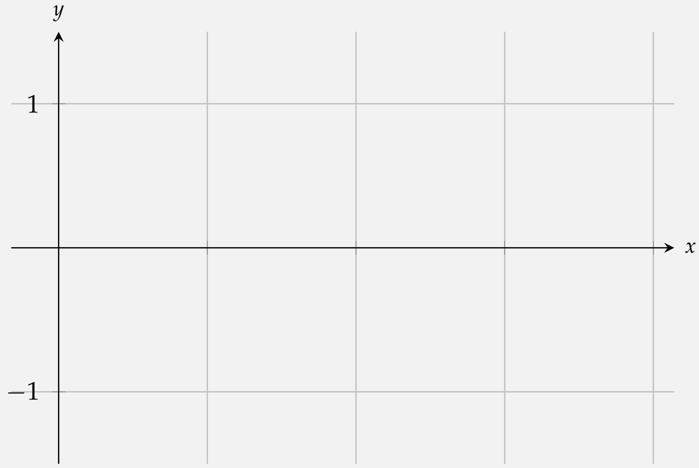
Amplitude $=$ __________ and Period $=$ __________

Period $=$ __________ $=$ __________

Clearly label all values on the x-axis.

YOU TRY IT:

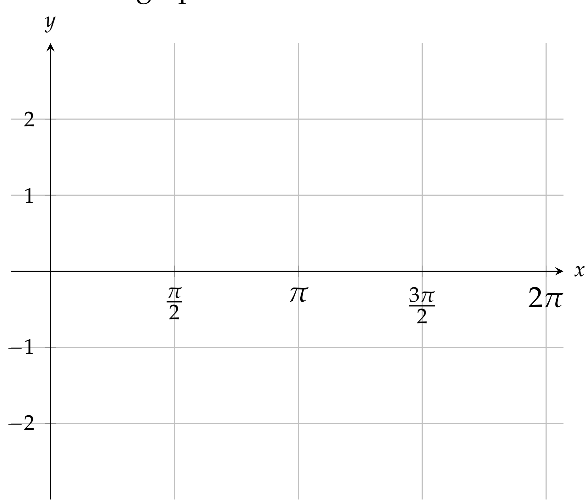
209. Sketch the graph of $y = \sin 2x$. Clearly label all values on the x-axis.



Sketching the graph of $y = \sin x + d$ or $y = \cos x + d$

Open the Instructor Added Resource which will direct you to a video to complete the following.

Sketch the graph of _________________________.



Sketching the graph of $y = \sin(x + c)$ or $y = \cos(x + c)$

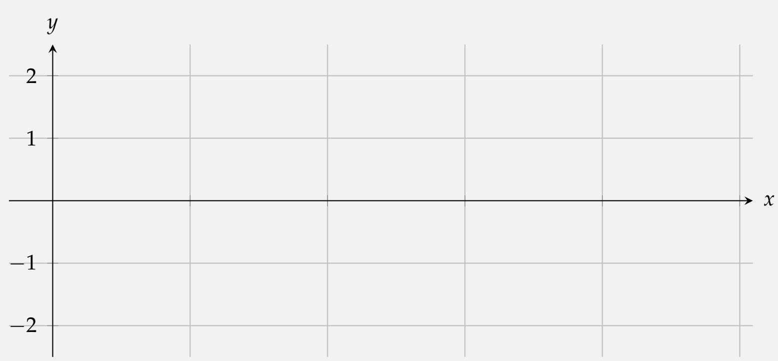
Open the e-book to complete the following.

Characteristics of the Graphs of $y = \sin x$ and $y = \cos x$

- The domain is ________________.
- The range is ________________.
- The period is ________________.
- The graph of $y = \sin x$ is symmetric with respect to the ________________.
  - $y = \sin x$ is an ________________ function.
- The graph of $y = \cos x$ is symmetric with respect to the ________________.
  - $y = \cos x$ is an ________________ function.
- The graphs of $y = \sin x$ and $y = \cos x$ differ by a ___________________________.

YOU TRY IT:

210. Sketch the graph of $y = \sin(x - \frac{\pi}{4})$.



Sketching the graph of $y = a\sin(x + c)$ or $y = a\cos(x + c)$

Open the e-book to complete the following.

Properties of the General Sine and Cosine Functions

Consider the graphs of $y = A\sin(Bx - C) + D$ and $y = A\cos(Bx - C) + D$ with $B > 0$

1. The ________________ is ________.

2. The period is ________.

3. The phase shift is ________.

4. The vertical shift is ________.

5. One full cycle is given on the interval ____________________.

6. The domain is ________________.

7. The range is ________________________.

Sketching the graph of $y = a\sin bx$ or $y = a\cos bx$

Watch the video *Graphing* $y = A\sin Bx$ *and* $y = \cos Bx$ to complete the following.

Identify the amplitude and period and graph the function. Clearly label all values on the x-axis.

____

$A =$ ________

$P =$ ________

Sketching the graph of $y = a\sin(bx) + d$ or $y = a\cos(bx) + d$

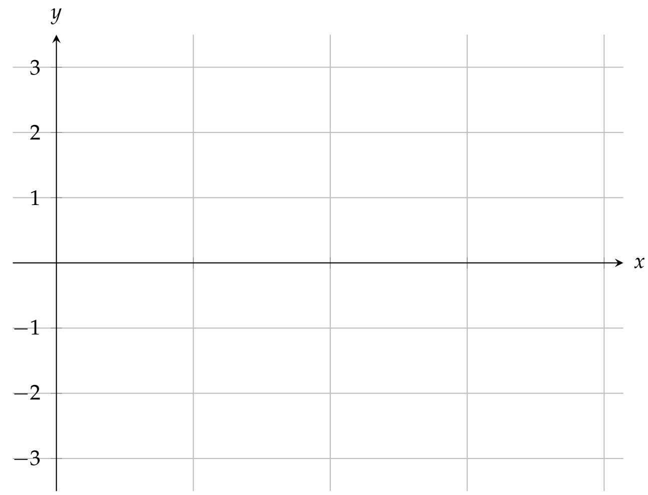
Open the Instructor Added Resource which will direct you to a video to complete the following.

Sketch the graph of ______________________. Clearly label all values on the x-axis.

Period = __________ = __________.

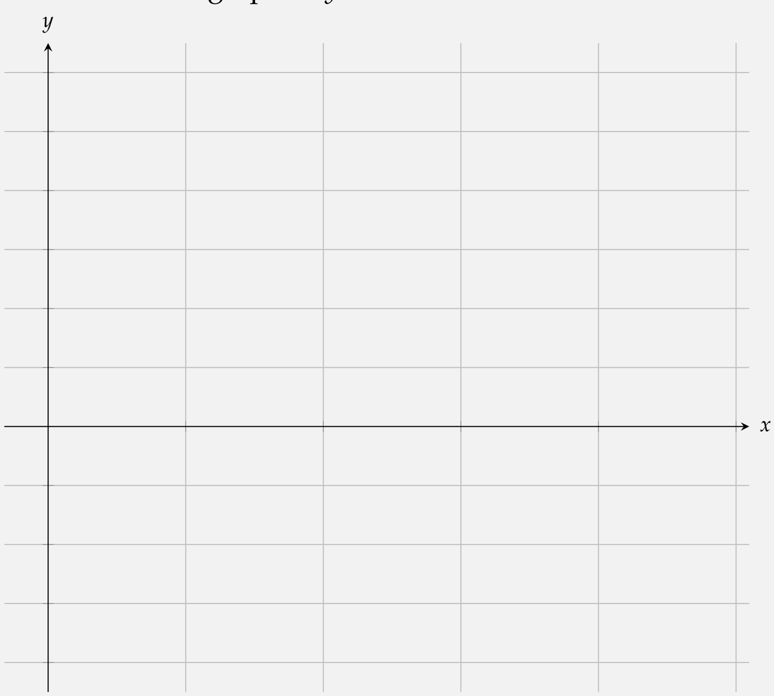
Amplitude = _____

Shift: ____________________



YOU TRY IT:

211. Sketch the graph of $y = -4\cos 3x + 2$.



Sketching the graph of $y = a\sin(bx + c)$ or $y = a\cos(bx + c)$

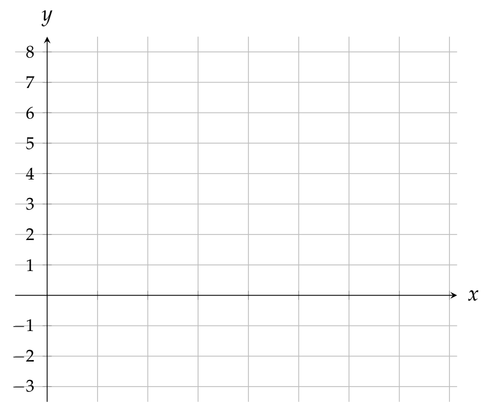
Watch the video *Graph a function of the Form* $y = A\sin(Bx - C) + D$ to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Graph the function $h(x) =$ ______________________. Clearly label all values on the x-axis.

Period: __________

Phase shift: __________

Vertical shift: __________



Amplitude and period of sine and cosine functions

Learning Page The sine and cosine functions are ____________________.

This means that their graphs ______________________________.

For instance, ____________________ has a period of __________

So, ______________________________ for all x.

The amplitude of a sine or cosine function is ____

____.

For example, consider $y = \sin x$ again.

It has a maximum _______________ and a minimum _______________.

The distance between these is _____, so the amplitude is _____.

More generally, we can consider ____________________ and ____________________, with $b > 0$.

For each, the amplitude is __________. The period is __________.

YOU TRY IT: For the function $y = -3\cos 5x$:

212. Period:

213. Amplitude:

Amplitude, period, and phase shift of sine and cosine functions

Open the Instructor Added Resource which will direct you to a video to complete the following.

Find the period, phase shift, and amplitude of ____________________________.

Period: Amplitude: Phase shift:

YOU TRY IT: Find the period, phase shift, and amplitude of $y = -3\sin(4x - \frac{\pi}{2})$.

214. Amplitude:

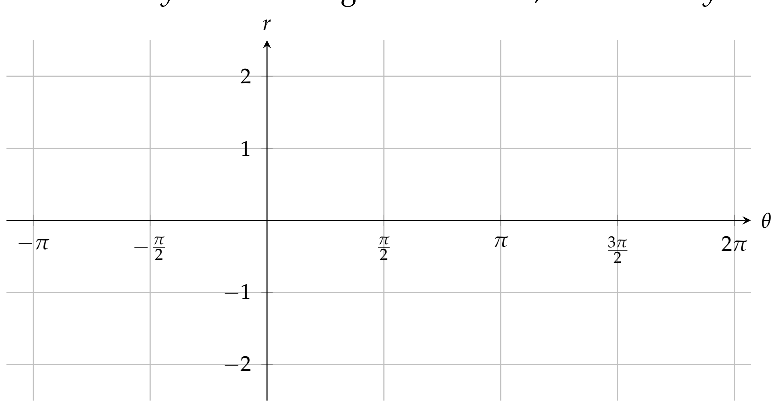
Period:

Phase shift:

Sketching the graph of a secant or cosecant function: Problem type 1

Watch the video *Graph the Parent Secant Function* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

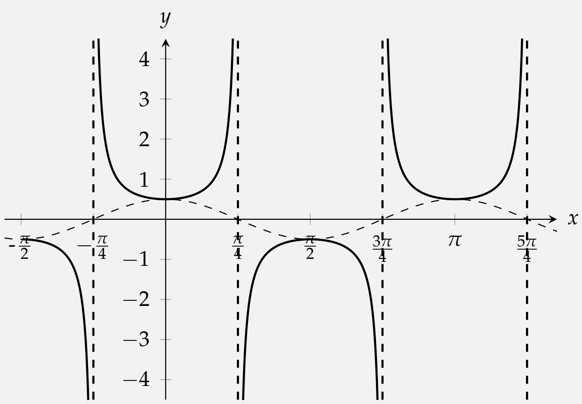
First sketch $y = \cos x$ using a dashed line, then sketch $y = \sec x$ using a solid line.



EXAMPLE:

Sketch the graph of $y = \frac{1}{2}\sec 2x$.

- Reminder that $y = \frac{1}{2}\sec 2x = \frac{1}{2} \cdot \frac{1}{\cos 2x}$.
- We use the graph of $y = \frac{1}{2}\cos 2x$ as the guide function.
  - The amplitude is $\frac{1}{2}$.
  - The period is $\frac{2\pi}{2} = \pi$.
- The graph will have vertical asymptotes when $\cos 2x = 0$.
  - This happens when $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \ldots$.



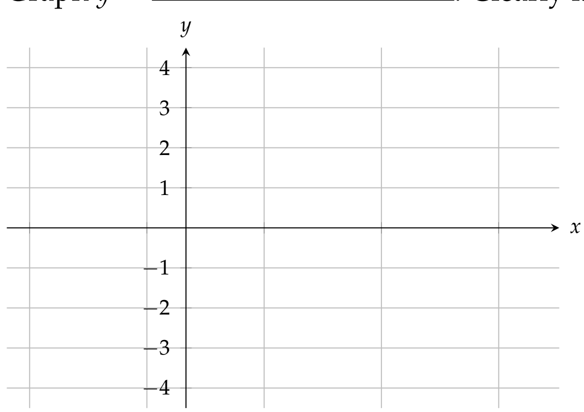
YOU TRY IT:

215. Sketch the graph of $y = -4\sec 3x$.

Sketching the graph of a tangent or cotangent function: Problem type 1

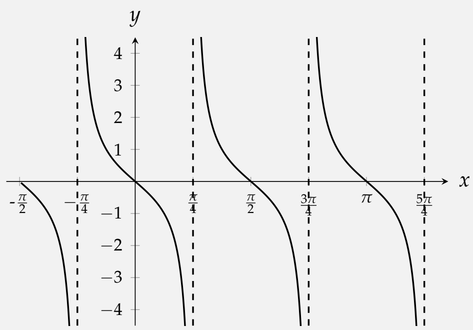
Watch the video *Graph the Cotangent Function with Transformations* to complete the following.

Graph $y =$ ________________. Clearly label all values on the x-axis.



EXAMPLE: Sketch the graph of $y = -\tan 2x$.

- Reminder that $y = \tan 2x = \frac{-\sin 2x}{\cos 2x}$.
    - The graph will have vertical asymptotes where $\cos 2x = 0$.
      This happens when $x = -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \ldots$
    - The graph will be 0 where $-\sin 2x = 0$.
      This happens when $x = -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \ldots$
- The period is $\frac{\pi}{2}$.
- The graph is a reflection over the x-axis of $y = \tan x$.



YOU TRY IT:

216. Sketch the graph of $y = \tan(x - \frac{\pi}{6})$.

Matching graphs and equations for secant, cosecant, tangent, and cotangent functions

Open the e-book to complete the following.

Graphs of the Cosecant and Secant Functions

	$y = \csc x$	$y = \sec x$
Domain		
Range		
Amplitude		
Period		
Vertical Asymptotes		
Symmetry		

Continued on the next page

Graphs of the Tangent and Cotangent Functions

	$y = \tan x$	$y = \cot x$
Domain		
Range		
Amplitude		
Period		
Vertical Asymptotes		
Symmetry		

Writing the equation of a sine or cosine function given its graph: Problem type 1

Learning Page Look at the general graphs for the equations $y = a \sin bx$ and $y = a \cos bx$, where $b > 0$.

For each, the amplitude and period are as follows.

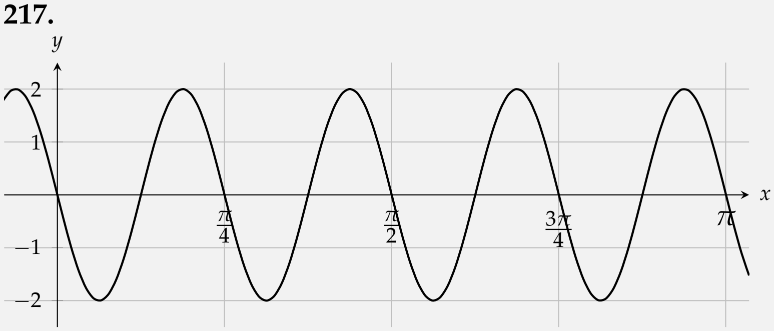
amplitude = ________ period = ________

We can find b by first finding the ________________.

The period is the ________________ of an interval in which the graph ____________________ exactly ____________ cycle.

YOU TRY IT: Write an equation to describe the graph below.

217.



Word problem involving a sine or cosine function: Problem type 1

Learning Page For $a \neq 0$, equations of the form ________________ and ________________ can be used to model simple ________________ motion.

Here, ____ is the ________________ from the ______________ position and ____ is the ____________.

Displacement is the ____,

and it can be ________________ or ________________.

For example, the displacement is negative when the current position is ________________________.

We use the ____________ equation if ________________________.

We use the ____________ equation if ____.

The ____________ is the time it takes for one complete cycle of the motion. It is given by ________.

The ________________ is the maximum displacement from 0 (____________). It is given by ________.

To find the sign of a, we need to consider the ____________________ of the graph.

Word problem involving a sine or cosine function: Problem type 2

Learning Page Many periodic phenomena can be described by sine or cosine ______________ of time ____.

These functions might take the following forms (for ________).

$$f(t) = \underline{\hspace{4cm}} \quad \text{or} \quad f(t) = \underline{\hspace{4cm}}$$

The **amplitude** of these functions is ________, and the period is ________.

The ________________ value of these functions is ____________, and the ________________ is ____________.

The frequency of these functions is the ________________ of ________________ per unit of ____________.

The period is the time it takes for ____________ cycle. So, the ____________________ is the ____________________ of the ________________.

$$\text{frequency} = \frac{1}{\underline{\hspace{2cm}}} = \underline{\hspace{2cm}}$$

Values of inverse trigonometric functions

Watch the video *Evaluating the Inverse Sine Function* to complete the following.

The Inverse Sine Function

The **inverse sine function**, denoted ______ or ______ is the ______ of the restricted sine function $y = \sin x$ for ______. Therefore,

$$y = \sin^{-1} x \Leftrightarrow \underline{\hspace{4cm}}$$

$$\underline{\hspace{4cm}} \Leftrightarrow \sin y = x$$

a.

b.

c.

Watch the video *Evaluating the Inverse Cosine Function* to complete the following.*NOTE:* This may not be the first video to pop up. Select it from the list in the video box.

$$y = \cos^{-1} x \Leftrightarrow \underline{\hspace{4cm}} \qquad -1 \leq x \leq 1 \qquad \underline{\hspace{1cm}} \leq y \leq \underline{\hspace{1cm}}$$

$$y = \underline{\hspace{4cm}} \Leftrightarrow \cos y = x \qquad -1 \leq x \leq 1 \qquad \underline{\hspace{1cm}} \leq y \leq \underline{\hspace{1cm}}$$

Evaluate the function.

a.

b.

EXAMPLE: Find the exact value of the following.

a) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

- We want the angle whose sine is $-\frac{\sqrt{3}}{2}$.
- It must lie in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

b) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

- We want the angle whose cosine is $\frac{\sqrt{2}}{2}$.
- It must lie in $[0, \pi]$.

$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

c) $\tan^{-1}(-1)$

- We want the angle whose tangent is -1.
- It must lie in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$\tan^{-1}(-1) = -\frac{\pi}{4}$

YOU TRY IT: Find the exact value of the following

218. $\cos^{-1}\left(-\frac{1}{2}\right)$

219. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

220. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Composition of a trigonometric function with its inverse trigonometric function: Problem type 1

Watch the video *Composing Trigonometric Functions and Inverse Trigonometric Functions* to complete the following.

Find the exact values.

a.

b.

Composition of a trigonometric function with its inverse trigonometric function: Problem type 2

More

More about the composition of a trigonometric function with its inverse

For each of cosine, sine and tangent, we will consider the valid ________________

(domain) of the ________________ function, and the valid ________________ (range) of

the ________________ function.

Let's look at the composition of cosine and inverse cosine.

Function	Inputs	Outputs
$y = \cos\theta$	θ in ________________	y in ________________
$\theta = \cos^{-1} y$	y in ________________	θ in ________________

- $\cos(\cos^{-1} y) =$ _____

  Note that the ________________ function is ________________________________.

  The input _____ must be in the interval ________________.

  If y is in ________________, then ____________.

  But if y is ____________ in ________________, then $\cos^{-1} y$ is ____________________

  and so is $\cos(\cos^{-1} y)$.

- $\cos^{-1}(\cos\theta) =$ _____

  Note that the ________________ function is ________________.

  The input _____ can be ____________________________.

  The ________________ function is ________________________________.

  The output _____ must be in the interval ________________.

  So, if θ is in ________________, then ____________

  But if θ is ____________ in ________________, then we must find the ____________ β in

  ________________ such that $\cos\beta =$ ____________.

Continued on the next page

More about the composition of a trigonometric function with its inverse

Now, we'll look at the composition of sine and inverse sine.

Function	Inputs	Outputs
$y = \sin\theta$	θ in ____________________	y in ____________________
$\theta = \sin^{-1} y$	y in ____________________	θ in ____________________

- $\sin(\sin^{-1} y) =$ _____

  Note that the ____________________ function is ____.

  The input _____ must be in the interval ____________________.

  If y is in ____________________, then _______________.

  But if y is _______________ in ____________________, then $\sin^{-1} y$ is _________________________

  and so is $\sin(\sin^{-1} y)$.

- $\sin^{-1}(\sin\theta) =$ _____

  Note that the ____________________ function is _______________.

  The input _____ can be ___________________________________.

  The ____________________ function is ____.

  The output _____ must be in the interval ____________________.

  So, if θ is in ____________________, then _______________

  But if θ is _______________ in ____________________, then we must find the _______________ β in

  ____________________ such that $\sin\beta =$ _______________.

Finally, let's look at the composition of tangent and inverse tangent.

Function	Inputs	Outputs
$y = \tan\theta$	θ in ____________________ but __________ equal to __________ where k is an __________ integer	y in ____________________
$\theta = \tan^{-1} y$	y in ____________________	θ in ____________________

EXAMPLE:
Find the exact value of the following.

a) $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$

- The answer must lie in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
- $\frac{2\pi}{3}$ is not in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$$\begin{aligned} \sin^{-1}\left(\sin \frac{2\pi}{3}\right) &= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{\pi}{3} \end{aligned}$$

b) $\cos\left(\cos^{-1} \frac{1}{4}\right)$

- $\frac{1}{4}$ is in $[-1, 1]$ so we use the inverse property.

$\cos\left(\cos^{-1} \frac{1}{4}\right) = \frac{1}{4}$

c) $\tan(\tan^{-1}(-1))$

- We may have any real number here to apply the inverse property.

$\tan(\tan^{-1}(-1)) = -1$

YOU TRY IT:
Find the exact value of the following

221. $\cos^{-1}\left(\cos \frac{3\pi}{4}\right)$

222. $\tan(\tan^{-1} 4)$

223. $\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$

Composition of a trigonometric function with the inverse of another trigonometric function: Problem type 1

Watch the video *Using a Right Triangle to Compose Trigonometric Functions and Inverse Trigonometric Functions* to complete the following.

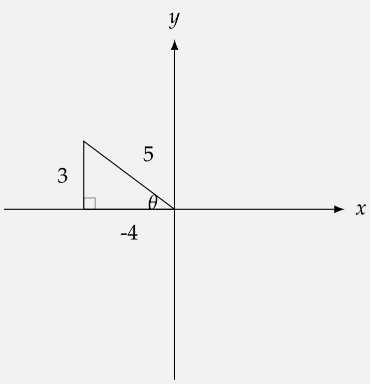
Find the exact value.

Composition of a trigonometric function with the inverse of another trigonometric function: Problem type 2

If you have not watched the video *Using a Right Triangle to Compose Trigonometric Functions and Inverse Trigonometric Functions*, do so now and complete the work in the box in the previous topic.

EXAMPLE:
Find the exact value of $\tan\left(\cos^{-1}\left(-\frac{4}{5}\right)\right)$.

- Let $\theta = \cos^{-1}\left(-\frac{4}{5}\right)$.
- Then $\cos\theta = -\frac{4}{5}$.
- We know that θ must be in quadrant II.



- From the triangle we want to find $\tan\theta$.

$\tan\left(\cos^{-1}\left(-\frac{4}{5}\right)\right) = -\frac{3}{4}$

YOU TRY IT:

224. Find the exact value of $\cos\left(\sin^{-1}\frac{12}{13}\right)$

Composition of a trigonometric function with the inverse of another trigonometric function: Problem type 3

Open the Instructor Added Resource which will direct you to a video to complete the following.

Find the exact value of ____________________________.

YOU TRY IT:

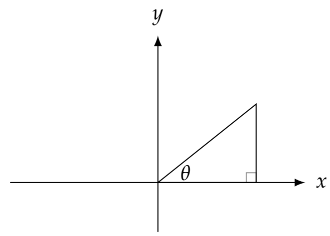
225. Find the exact value of $\tan\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$

Composition of trigonometric functions with variable expressions as inputs: Problem type 1

Watch the video *Writing a Trigonometric Expression as an Algebraic Expression* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Write the given expression as an algebraic expression.
It is not necessary to rationalize the denominator.

Fill in the side lengths on the triangle below.

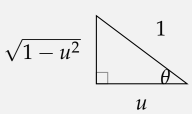


Composition of trigonometric functions with variable expressions as inputs: Problem type 2

If you have not already done so, watch the video *Writing a Trigonometric Expression as an Algebraic Expression* and take notes under the previous topic.

EXAMPLE:
Find the exact value of $\sin\left(\cos^{-1} u\right)$.

- Let $\theta = \cos^{-1}(u)$.
- Then $\cos\theta = \frac{u}{1}$.
- We use the Pythagorean Theorem to find the third side of the triangle.



- From the triangle we want to find $\sin\theta$.

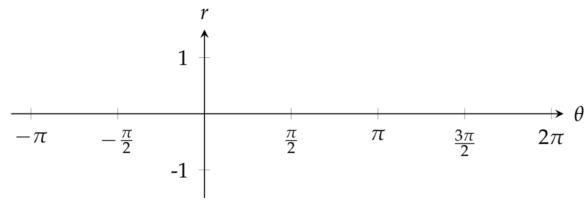
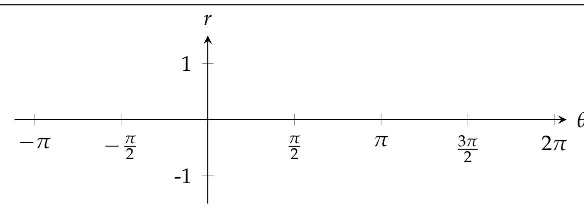
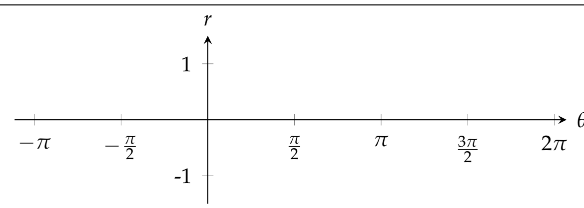
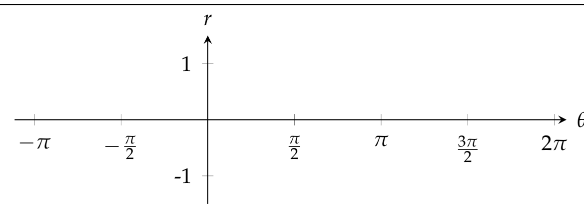
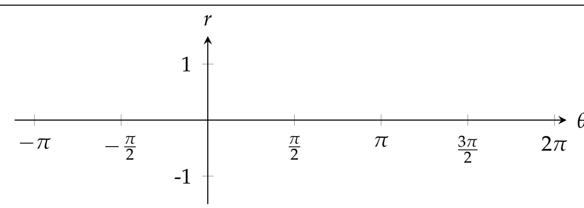
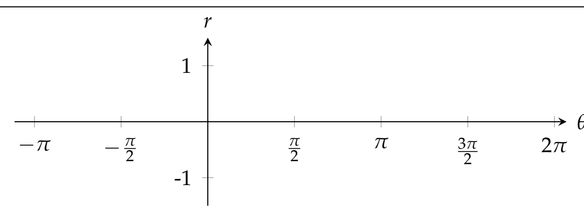
$\sin\left(\cos^{-1} u\right) = \sqrt{1-u^2}$

YOU TRY IT:

226. Find the exact value of $\cos\left(\tan^{-1} \frac{\sqrt{1-u^2}}{u}\right)$

Domains and ranges of trigonometric functions

Learning Page Complete the chart below.

Function	Domain	Range	Graph
$y = \cos x$			
$y = \sin x$			
$y = \sec x$			
$y = \csc x$			
$y = \tan x$			
$y = \cot x$			

Notes from Focus Group:

Notes from Focus Group:

Module 12-Review Module

To help you review for your upcoming exam, this module contains all of the topics from the modules since the last exam. Topics that you have already mastered will not appear in your carousel.

- ☐ Complete this module before you take the ALEKS exam.
- ☐ Each exam has two parts.
  - The ALEKS exam (100 pts)
    - The ALEKS exam must be taken in the MALL.
    - The ALEKS exam is a Comprehensive Knowledge Check.
      - Your score is the number of topics you have mastered out of the number of topics you should have mastered by this point.
    - If you lose topics on your ALEKS exam, your Review Module completion grade will not change.
    - Your scratch work for the ALEKS exam must be numbered and turned in through Blackboard.
  - The Written exam (25 pts)
    - Take your written exam in class the day of your focus group.
    - To study for the written exam:
      - Rework your old Focus Group assignments.
      - Rework any topics in ALEKS you may have lost on the ALEKS exam.

	Score
ALEKS Exam	
Written Exam	

Module 13

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Weekly Checklist

- ☐ Complete MALL time.
- ☐ Work in ALEKS and Notebook at least 4 days a week.
- ☐ Complete the weekly Module and Notebook pages by the due date.
- ☐ Attend Focus Group.
- ☐ Actively participate in Focus Group.
- ☐ Earn extra credit: Complete 10 topics by ______________________.

Simplifying trigonometric expressions

Watch the video *Simplifying a Trigonometric Expression* to complete the following.

Simplify.

Fundamental Identities

Reciprocal Identities	$\csc x =$	$\sec x =$	$\cot x =$
	$\sin x =$	$\cos x =$	$\tan x =$
Quotient Identities	$\tan x =$	$\cot x =$	
Pythagorean Identities			
Even and Odd Identities	$\sin(-x) =$	$\cos(-x) =$	
	$\csc(-x) =$	$\sec(-x) =$	
	$\tan(-x) =$	$\cot(-x) =$	

EXAMPLE:

Simplify $\dfrac{\sec x}{\csc x}$.

$$\begin{aligned} \frac{\sec x}{\csc x} &= \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} && \text{Reciprocal} \\ &= \frac{1}{\cos x} \cdot \frac{\sin x}{1} && \text{Algebra} \\ &= \frac{\sin x}{\cos x} && \text{Algebra} \\ &= \tan x && \text{Quotient} \end{aligned}$$

YOU TRY IT:

227. Simplify $\dfrac{\cot x}{\cos x}$.

Verifying a trigonometric identity

Learning Page To prove a trigonometric identity, we use algebraic transformations and the following **fundamental trigonometric identities**. Try to complete the chart below from memory, then check your work on the Learning Page.

Reciprocal identities:

$\sin u =$ $\cos u =$ $\tan u =$

$\csc u =$ $\sec u =$ $\cot u =$

Quotient Identities:

$\tan u =$ $\cot u =$

Pythagorean identities:

1) 2) 3)

Proving trigonometric identities: Problem type 1

Watch the video *Simplifying a Trigonometric Expression by Combining Fractions* to complete the following. *NOTE:* This may not be the first video to pop up. Select it from the list in the video box.

Simplify.

Proving trigonometric identities: Problem type 2

Watch the video *Introduction to Verifying Trigonometric Identities* and complete the following.

Guidelines for Verifying a Trigonometric Identity

1. Work with __________________ of the equation (usually the ______________ ______________) and keep the other side in mind as your __________________.

2. Look for opportunities to apply the ____________________________.

   - If the expression is a ________________________________, consider the ________________________________.

   - If __________________ are present, look to see if the terms can be grouped in one of the forms of a ____________________________.

   - If an expression involves a __________________, consider using the ____________________________________.

3. Apply basic algebraic techniques such as

   -
   -
   -
   -

4. Consider writing expressions explicitly in terms of _______________________.

Verify the identity.

EXAMPLE:

Verify the identity $\cot x + \tan x = \sec x \csc x$.

$$\begin{aligned}
\cot x + \tan x &= \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} && \text{Quotient} \\
&= \frac{\cos^2 x}{\sin x \cos x} + \frac{\sin^2 x}{\cos x \sin x} && \text{Algebra} \\
&= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x} && \text{Algebra} \\
&= \frac{1}{\sin x \cos x} && \text{Pythagorean} \\
&= \frac{1}{\sin x} \cdot \frac{1}{\cos x} && \text{Algebra} \\
&= \csc x \sec x && \text{Reciprocal}
\end{aligned}$$

YOU TRY IT:

228. Verify $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$.

Proving trigonometric identities: Problem type 3

If necessary, review the Guidelines for Verifying a Trigonometric Identity found under the previous topic **Proving trigonometric identities: Problem type 2**.

Double-angle identities: Problem type 1

Watch the video *Find the Sine, Cosine and Tangent of 2 times Theta* to complete the following.

Use the given information to find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

$\sin\theta =$ __________ for θ in Quadrant IV.

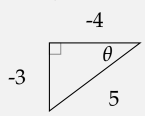
EXAMPLE:
Given $\cos\theta = -\frac{4}{5}$ and θ is in Quadrant III, find the following.

a) $\sin 2\theta$
We will use the Double-Angle identity but we first find $\sin\theta$. We can do this in two ways.

- Use the Pythagorean Identity.

$$\begin{aligned}\sin^2\theta + \cos^2\theta &= 1\\ \sin^2\theta + \left(-\tfrac{4}{5}\right)^2 &= 1\\ \sin^2\theta &= 1 - \tfrac{16}{25}\\ \sin^2\theta &= \tfrac{9}{25}\\ \sin\theta &= -\tfrac{3}{5}\end{aligned}$$

- Make a right triangle as we did previously.



$$\begin{aligned}\sin 2\theta &= 2\sin\theta\cos\theta\\ &= 2\left(-\tfrac{3}{5}\right)\left(-\tfrac{4}{5}\right)\\ &= \tfrac{24}{25}\end{aligned}$$

b) $\cos 2\theta$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta\\ &= \left(-\tfrac{4}{5}\right)^2 - \left(-\tfrac{3}{5}\right)^2\\ &= \tfrac{16}{25} - \tfrac{9}{25}\\ &= \tfrac{7}{25}\end{aligned}$$

YOU TRY IT:
Given $\sin\theta = \frac{1}{3}$ and θ is in Quadrant II, find the following.

229. $\sin 2\theta$

230. $\cos 2\theta$

Double-angle identities: Problem type 2

Open the e-book to complete the following.

Double-Angle Formulas

$\sin 2\theta =$ ____________________

$\tan 2\theta =$ ____________________

$\cos 2\theta =$ ____________________ $=$ ____________________ $=$ ____________________

Sum and difference identities: Problem type 3

Watch the video *Applying the Sum Formula for Cosine Given Values of* $\sin u$ *and* $\cos u$ to complete the following.

Find the exact value of $\cos(\alpha + \beta)$ subject to the given conditions.

$\sin \alpha =$ ________ for α in Quadrant III. $\cos \beta =$ ________ for β in Quadrant II.

Open the e-book to complete the following.

Sum and Difference Formulas

Sine Formulas	$\sin(u+v) =$ $\sin(u-v) =$
Cosine Formulas	$\cos(u+v) =$ $\cos(u-v) =$

EXAMPLE:
Verify the identity

$$\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = \sqrt{2}\sin x.$$

$$\begin{aligned} &\sin(x + \tfrac{\pi}{4}) + \sin(x - \tfrac{\pi}{4}) \\ &= \sin x \cos \tfrac{\pi}{4} + \cos x \sin \tfrac{\pi}{4} + \sin x \cos \tfrac{\pi}{4} - \cos x \sin \tfrac{\pi}{4} \\ &= 2 \sin x \cos \tfrac{\pi}{4} \\ &= 2 \sin x \cdot \tfrac{\sqrt{2}}{2} \\ &= \sqrt{2} \sin x \end{aligned}$$

YOU TRY IT:

231. Verify the identity

$$\frac{\sin(x+y)}{\cos x \sin y} = \tan x \cot y + 1.$$

Proving trigonometric identities using double-angle properties

Watch the video *Verify an Identity Using Double Angle Identities* to complete the following. *NOTE:* This may not be the first video to pop up. Select it from the list in the video box.

Verify the identity.

Half-angle identities: Problem type 2

Watch the video *Finding the Sine and Cosine of a Half-Angle* to complete the following.

Given that $\cos\alpha =$ ________ for ________ $< \alpha <$ ________, find

a.

b.

c.

Half-Angle Formulas

$\sin\frac{\alpha}{2} =$ ____________________

$\cos\frac{\alpha}{2} =$ ____________________

$\tan\frac{\alpha}{2} =$ ____________________ $=$ ____________________ $=$ ____________________

The sign _____ or _____ is determined by the ______________ in which the angle _____ lies.

Finding solutions in an interval for a basic equation involving sine or cosine

Watch the video *Introduction to Trigonometric Equations* to complete the following. You will only complete part a. in the video.

Solve the equation ______________________ over the interval $[0, 2\pi)$.

EXAMPLE: Find all solutions to $3\sqrt{2} + 6\cos x = 0$ on the interval $[0, 2\pi)$.

We begin by solving for $\cos x$.

$$\begin{aligned} 3\sqrt{2} + 6\cos x &= 0 \\ 6\cos x &= -3\sqrt{2} \\ \cos x &= -\tfrac{\sqrt{2}}{2} \end{aligned}$$

We are asking,"What angleshave cosine equal to $-\frac{\sqrt{2}}{2}$?"

$$x = \tfrac{3\pi}{4}, \tfrac{5\pi}{4}$$

YOU TRY IT:
Find all solutions to $4\sin x + 2\sqrt{3} = 0$ on

232. the interval $[0, 2\pi)$.

Solving a basic trigonometric equation involving sine or cosine

Watch the video *Introduction to Trigonometric Equations* to complete the following. You will only complete part b. of the video.

Solve the equation ______________________ over the set of real numbers.

EXAMPLE:
Find all solutions to $3\sqrt{2} + 6\cos x = 0$ on the set of all real numbers.
We want all multiples of the angles we found in part a).

$$x = \frac{3\pi}{4} + 2\pi n \qquad \text{for any integer } n$$
$$x = \frac{5\pi}{4} + 2\pi n$$

YOU TRY IT:
Find all solutions to $4\sin x + 2\sqrt{3} = 0$ on

233. the set of all real numbers.

Finding solutions in an interval for a basic tangent, cotangent, secant, or cosecant equation

Open the e-book to complete **EXAMPLE 1**.

Solve over $[0, 2\pi)$.

$2\tan x =$ ____________________

$3\tan x =$ __________   __________ $\tan x$ to both sides.

_______________ $= \frac{\sqrt{3}}{3}$ __________by 3.

$x =$ __________, __________

YOU TRY IT:

234. Find all solutions to $4\tan\theta + 4 = 0$ on $[0, 2\pi)$.

Finding solutions in an interval for a trigonometric equation with a squared function: Problem type 1

Watch the video *Solving a Quadratic Trigonometric Equation by the Square Root Property* to complete the following.

Solve the equation over the interval $[0, 2\pi)$.

EXAMPLE:
Find all solutions to $\tan^2 x - 1 = 0$ on $[0, 2\pi)$.

$$\begin{aligned} \tan^2 x - 1 &= 0 \\ \tan^2 x &= 1 \\ \tan x &= \pm 1 \\ x &= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \end{aligned}$$

YOU TRY IT:

235. Find all solutions to $2\cos^2 x - 1 = 0$ on $[0, 2\pi)$.

Finding solutions in an interval for a trigonometric equation with a squared function: Problem type 2

Watch the video *Solving a Quadratic Trigonometric Equation by Factoring* to complete the following.

Solve the equation over the interval $[0, 2\pi)$.

Finding solutions in an interval for a trigonometric equation in factored form

Learning Page If a __________________ is ______________, then at least one of the __________________ must be ______________.

EXAMPLE:
Find all solutions to $(\sin x + 3)(\sin x - 1) = 0$ on $[0, 2\pi)$.

We begin by setting each factor equal to zero.

$$(\sin x + 3)(\sin x - 1) = 0$$

$$\begin{array}{ll} \sin x + 3 = 0 & \sin x - 1 = 0 \\ \sin x = -3 & \sin x = 1 \\ \text{No solution} & x = \dfrac{\pi}{2} \end{array}$$

YOU TRY IT:

236. Find all solutions to $(2\cos x - 1)(\cos x + 1) = 0$ on $[0, 2\pi)$.

Finding solutions in an interval for a trigonometric equation using Pythagorean identities: Problem type 1

Watch the video *Solving a Trigonometric Equation Using Pythagorean Identities* to complete the following.

Solve the equation over the interval $[0, 2\pi)$.

Finding solutions in an interval for a trigonometric equation with an angle multiplied by a constant

Open the Instructor Added Resource which will direct you to a video to complete the following.

Find all solutions to ______________________ on $[0, 2\pi]$.

YOU TRY IT:

237. Find all solutions to $2\cos 2x + 1 = 0$ on $[0, 2\pi)$.

Solving a trigonometric equation modeling a real-world situation

Learning Page Carefully read through the example on the Learning Page and take notes below.

Finding solutions in an interval for an equation with sine and cosine using double-angle identities

Open the Instructor Added Resource which will direct you to a video to complete the following.

Find all solutions to ______________________ on $[0, 2\pi]$.

YOU TRY IT:

238. Find all solutions to $\sin 2x = \cos x$ on $[0, 2\pi)$.

Notes from Focus Group:

Notes from Focus Group:

Module 14

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Weekly Checklist

- ☐ Complete MALL time.
- ☐ Work in ALEKS and Notebook at least 4 days a week.
- ☐ Complete the weekly Module and Notebook pages by the due date.
- ☐ Attend Focus Group.
- ☐ Actively participate in Focus Group.
- ☐ Earn extra credit: Complete 10 topics by ____________________.

Using trigonometry to find angles of elevation or depression in a word problem

Open the e-book to complete **EXAMPLE 4: Computing an Angle in a Construction Application**.

Find the pitch of the roof to the nearest tenth of a degree.

Solution:

The pitch angle is the angle that the __________ makes with the _______________. Using either right triangle formed by the 6-ft altitude and half the width of the house we have

$$\tan\theta = \underline{\hspace{3cm}}$$

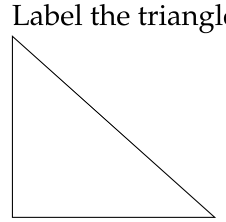
$$\theta = \underline{\hspace{4cm}} \approx \underline{\hspace{3cm}}$$

Solving a right triangle

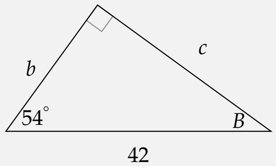
Watch the video *Solving a Right Triangle* to complete the following.

Solve the right triangle.

Label the triangle below.



EXAMPLE:
Solve for the unknowns in the given triangle.



- Find B.
  $B = 180^\circ - 90^\circ - 54^\circ = 36^\circ$
- Find c.

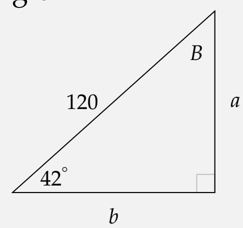
$$\begin{aligned} \tfrac{c}{42} &= \sin 54^\circ \\ c &= 42 \sin 54^\circ \\ c &\approx 34.0 \end{aligned}$$

- Find b.

$$\begin{aligned} \tfrac{b}{42} &= \sin 36^\circ \\ b &= 42 \sin 36^\circ \\ b &\approx 24.7 \end{aligned}$$

YOU TRY IT:

239. Solve for the unknowns in the given triangle.



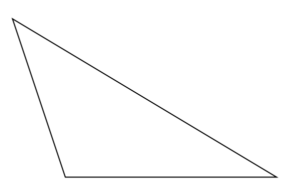
Solving a triangle with the law of sines: Problem type 1

Watch the video *Applying the Law of Sines Given SAA* to complete the following.

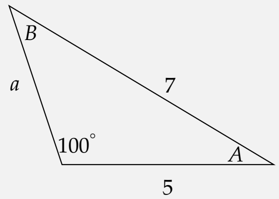
Solve triangle ABC subject to the given conditions.

$A =$ __________, $B =$ __________, $a =$ __________

Label the triangle below.



EXAMPLE:
Consider the triangle drawn below. Find the unknown values.



Use the Law of Sines to find B.

$$\begin{aligned} \frac{\sin 100^{\circ}}{7} &= \frac{\sin B}{5} \\ 5 \sin 100^{\circ} &= 7 \sin B \\ \sin B &= \frac{5 \sin 100^{\circ}}{7} \\ B &= \sin^{-1}\left(\frac{5 \sin 100^{\circ}}{7}\right) \\ B &\approx 44.7^{\circ} \end{aligned}$$

We know the sum of the angles of a triangle is 180°, so

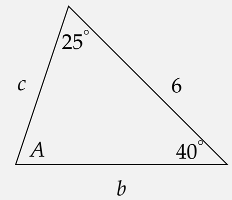
$$\begin{aligned} A &= 180^{\circ} - 100^{\circ} - \sin^{-1}\left(\frac{5 \sin 100^{\circ}}{7}\right) \\ A &\approx 35.3^{\circ} \end{aligned}$$

We can also use the Law of Sines to find the length of a.

$$\begin{aligned} \frac{\sin 100^{\circ}}{7} &= \frac{\sin\left(80^{\circ} - \sin^{-1}\left(\frac{5 \sin 100^{\circ}}{7}\right)\right)}{a} \\ a \sin 100^{\circ} &= 7 \sin\left(80^{\circ} - \sin^{-1}\left(\frac{5 \sin 100^{\circ}}{7}\right)\right) \\ a &= \frac{\sin\left(80^{\circ} - \sin^{-1}\left(\frac{5 \sin 100^{\circ}}{7}\right)\right)}{\sin 100^{\circ}} \\ a &\approx \frac{7 \sin 35.3^{\circ}}{\sin 100^{\circ}} \\ a &\approx 4.1 \end{aligned}$$

YOU TRY IT:

240. Consider the triangle drawn below. Find the unknown values.



Solving a triangle with the law of sines: Problem type 2

Open the Instructor Added Resource which will direct you to a video to complete the following.

For each case, draw the picture of the triangle(s) and state the number of possible solutions.

- $a < h$ # of solutions: ________

- $a = h$ # of solutions: ________

- $a > b$ # of solutions: ________

- $h < a < b$ # of solutions: ________

Continued on the next page

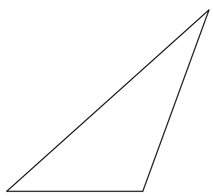
Watch the video *Use the Law of Sines to Solve a Triangle SSA (Two Solutions)* to complete the following. *NOTE:* This may not be the first video to pop up. Select it from the list in the video box.

Solve the triangle ABC using the given information.

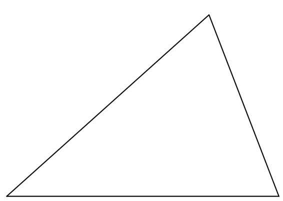
$a =$ ________, $b =$ ________, $A =$ ________

$\sin 45^{\circ} =$ ________________ $\implies h = 18 \sin 45^{\circ} \approx$ ________________

Label the triangle below.



Label the triangle below.



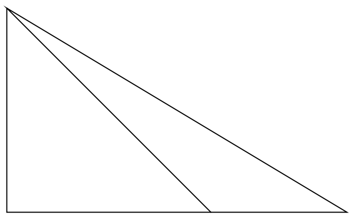
Solving a word problem with the law of sines

Watch the video *Using the Law of Sines in an Application* to complete the following.

From a point along a straight road, the angle of elevation to the top of a hill is __________.

From __________ ft farther down the road, the angle of elevation to the top of the hill is __________. How high is the hill?

Label the triangle below.

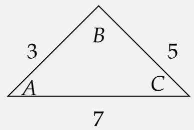


Solving a triangle with the law of cosines

Watch the video *Using the Law of Cosines to Solve a Triangle SAS* to complete the following.

Solve triangle ABC given that $a =$ __________, $c =$ __________, and $B =$ __________.

EXAMPLE:
Consider the triangle drawn below. Find the unknown values.



We begin by finding c using the Law of Cosines.

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C \\ 3^2 &= 5^2 + 7^2 - 2(5)(7)\cos C \\ 9 &= 74 - 70\cos C \\ -65 &= -70\cos C \\ \tfrac{13}{14} &= \cos C \\ \cos^{-1}\left(\tfrac{13}{14}\right) &= C \\ C &\approx 21.8^\circ \end{aligned}$$

We now use the Law of Sines to find either A or B.

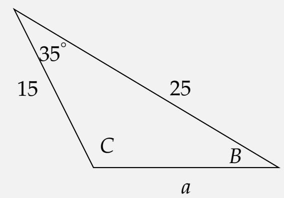
$$\begin{aligned} \frac{\sin B}{7} &= \frac{\sin\left(\cos^{-1}\left(\frac{13}{14}\right)\right)}{3} \\ \sin B &= 7 \cdot \frac{\frac{3\sqrt{3}}{14}}{3} \\ \sin B &= \frac{\sqrt{3}}{2} \\ B &= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \\ B &= 120^\circ \end{aligned}$$

Now we find A.

$$\begin{aligned} A &= 180 - 120 - \cos^{-1}\left(\tfrac{13}{14}\right) \\ A &= 60 - \cos^{-1}\left(\tfrac{13}{14}\right) \\ A &\approx 38.2 \end{aligned}$$

YOU TRY IT:

241. Consider the triangle drawn below. Find the unknown values.



Solving a word problem with the law of cosines

Open the e-book to complete the following.

Law of Cosines

If $\triangle ABC$ has sides of lengths a, b, and c opposite vertices A, B, and C, respectively, then

1.

2.

3.

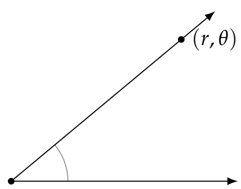
Plotting points in polar coordinates

Open the e-book to complete the following.

Polar Coordinate System

A **polar coordinate system** consists of a fixed point O called the __________ (or origin), and a ray called the ______________, with endpoint at the pole. Each point P in the plane is defined by an ordered pair __________, where r is the ___________________________________ ______________________________.

On the picture below, label the pole, the polar axis, θ, and r.



- If $r > 0$, point P is located ____________________ from the pole in the direction of _____.
- If $r < 0$, point P is located ____________________ from the pole in the direction of __________ (the direction ____________________).
- If $r = 0$, point P is located ____________________.

θ is a directed angle from the ____________________ to line OP.

- $\theta > 0$ is measured _________________________ from the polar axis.
- $\theta < 0$ is measured _________________________ from the polar axis.

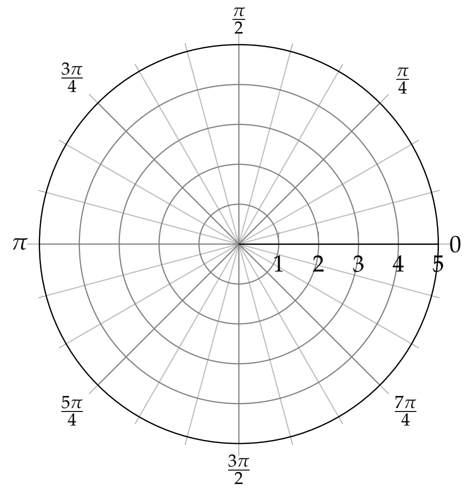
The polar axis in a polar coordinate system is usually aligned with the _________________________ in a rectangular coordinate system.

Continued on the next page

Watch the video *Plotting Points in a Polar Coordinate System* to complete the following.

Plot the points whose polar coordinates are given. Label each point on your graph using the letters given.

a. P____________ b. Q____________ c. R____________ d. S____________

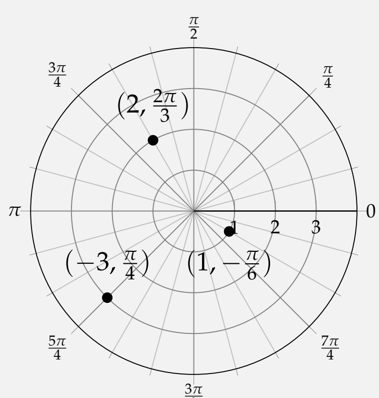


EXAMPLE:
Plot the following points on the given axes.

a) $\left(2, \frac{2\pi}{3}\right)$

b) $\left(-3, \frac{\pi}{4}\right)$

c) $\left(1, -\frac{\pi}{6}\right)$

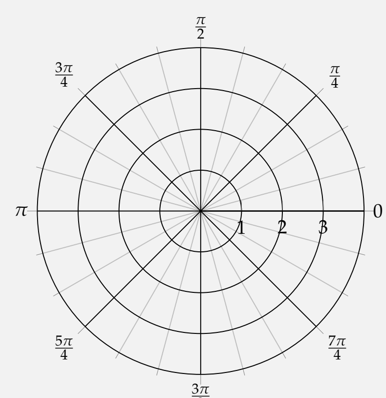


YOU TRY IT:
Plot the following points on the given axes.

242. $\left(3, \frac{5\pi}{4}\right)$

243. $\left(-1, \frac{3\pi}{2}\right)$

244. $\left(2, -\frac{7\pi}{6}\right)$



Multiple representations of polar coordinates

Learning Page In the polar coordinate system, a point with coordinates (r, θ) has infinitely many representations.

- For example, both ____________ and ____________ represent the point with coordinates ____________.

  This is true because adding or subtracting multiples of _______ gives ______________________ angles.

- Also, ______________ and ______________ are other representations of ______________.

EXAMPLE: Find two additional polar representations of the point $\left(7, \frac{2\pi}{3}\right)$.

$$\left(7, \frac{2\pi}{3}\right) = \left(7, \frac{2\pi}{3} - 2\pi\right) = \left(7, -\frac{4\pi}{3}\right)$$

$$\left(7, \frac{2\pi}{3}\right) = \left(-7, \frac{2\pi}{3} + \pi\right) = \left(-7, \frac{5\pi}{3}\right)$$

YOU TRY IT: Find two additional polar representations of the point $\left(-3, \frac{5\pi}{6}\right)$.

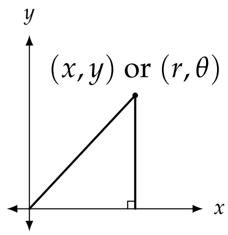
245. $\left(-3, \frac{5\pi}{6}\right) =$

246. $\left(-3, \frac{5\pi}{6}\right) =$

Converting rectangular coordinates to polar coordinates: Special angles

Open the Instructor Added Resource which will direct you to a video to complete the following.

On the graph, label x, y, r, and θ.



State the two equations given in the video.

__________ = __________

__________ = __________

Find polar coordinates of the point __________.

Two possible polar representations: __________ or __________

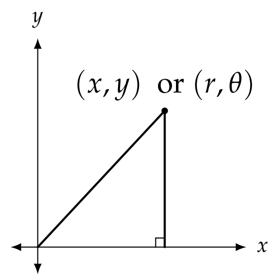
YOU TRY IT: Find polar coordinates of the following rectangular point.

247. $(\sqrt{3}, -\sqrt{3})$

Converting polar coordinates to rectangular coordinates

Learning Page Consider a point P in quadrant I with polar coordinates ____________, where r and θ are ________________. On the graph below, label x, y, r, and θ.

From right triangle trigonometry, we get that the ________________ coordinates (r, θ) and the ____________________ coordinates ____________ are related by two equations.



$$\frac{x}{r} = \underline{\hspace{2cm}} \quad \text{and} \quad \frac{y}{r} = \underline{\hspace{2cm}}$$

Multiplying each side of the equations by ____ gives the following.

$$\underline{\hspace{5cm}} \quad \text{and} \quad \underline{\hspace{5cm}}$$

As it turns out, we can use these formulas to find the rectangular coordinates ____________ for ________ polar coordinates ____________.

Watch the video *Converting From Polar Coordinates to Rectangular Coordinates* to complete the following.

Convert the ordered pair in polar coordinates to rectangular coordinates.

EXAMPLE: Find the rectangular coordinates of the polar point $\left(6, \frac{2\pi}{3}\right)$.

$$\begin{aligned} x &= r\cos\theta \\ &= 6\cos\tfrac{2\pi}{3} \\ &= 6\left(-\tfrac{1}{2}\right) \\ &= -3 \end{aligned} \qquad \begin{aligned} y &= r\sin\theta \\ &= 6\sin\tfrac{2\pi}{3} \\ &= 6\tfrac{\sqrt{3}}{2} \\ &= 3\sqrt{3} \end{aligned}$$

So $\left(6, \frac{2\pi}{3}\right) = (3, 3\sqrt{3})$

YOU TRY IT: Find the rectangular coordinates of the following polar point.

248. $\left(-3, \frac{5\pi}{6}\right)$

Converting an equation written in polar form to one written in rectangular form: Problem type 1

Learning Page To convert an equation in polar form to rectangluar form, we can use the following **polar-rectangular relationships**.

________________ and ________________

________________ and ________________

Watch the video *Converting Equations from Polar to Rectangular Coordinates* to complete the following.

Write an equivalent equation using rectangular coordinates.

a. ____________ b. ____________ c. ____________ d. ____________

EXAMPLE:
Convert the following polar equations to rectangular equations.

a) $\theta = \dfrac{2\pi}{3}$
We know $\tan\theta = \frac{y}{x}$ so we have

$$\tan\frac{2\pi}{3} = \frac{y}{x}$$
$$-\sqrt{3} = \frac{y}{x}$$
$$y = -\sqrt{3}x$$

b) $r = 5\cos\theta$
Multiply both sides by r.

$$r \cdot r = r \cdot 5\cos\theta$$
$$r^2 = 5r\cos\theta$$
$$x^2 + y^2 = 5x$$

YOU TRY IT:
Convert the following polar equations to rectangular equations.

249. $r\sin\theta = -5$

250. $r = -5\sin\theta$

Converting an equation written in polar form to one written in rectangular form: Problem type 2

Try to write the polar-rectangular relationships from memory. Check your work on the Learning Page

____________________ and ____________________

____________________ and ____________________

EXAMPLE: Convert the following polar equations to rectangular equations.

a) $r = \dfrac{2}{3\cos\theta - 5\sin\theta}$

Multiply both sides by $3\cos\theta - 5\sin\theta$.

$$\begin{aligned}(3\cos\theta - 5\sin\theta)r &= \frac{2(3\cos\theta - 5\sin\theta)}{3\cos\theta - 5\sin\theta}\\ 3r\cos\theta - 5r\sin\theta &= 2\\ 3x - 5y &= 2\end{aligned}$$

YOU TRY IT: Convert the following polar equations to rectangular equations.

251. $r = \dfrac{-3}{4\cos\theta + 7\sin\theta}$

Converting an equation written in rectangular form to one written in polar form

Learning Page To convert an equation in rectangular form to polar form, we can use the following polar-rectangular relationships.

____________________ and ____________________

____________________ and ____________________

Watch the video *Convert an Equation of a Circle from Rectangular to Polar Coordinates* to complete the following.

Convert the equation in rectangular coordinates to polar coordinates.

EXAMPLE: Convert the following rectangular equations to polar equations.

a) $y = -\sqrt{3}x$
We know $\tan\theta = \frac{y}{x}$ so we have

$$\begin{aligned} \tfrac{y}{x} &= -\sqrt{3} \\ \tan\theta &= -\sqrt{3} \\ \theta &= \tfrac{2\pi}{3} \end{aligned}$$

b) $x^2 + y^2 - 2x = 0$
We know $x^2 + y^2 = r^2$ and $x = r\cos\theta$.

$$\begin{aligned} r^2 - 2r\cos\theta &= 0 \\ r^2 &= 2r\cos\theta \\ r &= 2\cos\theta \end{aligned}$$

c) $y = 3$
We know $y = r\sin\theta$.

$$\begin{aligned} r\sin\theta &= 3 \\ r &= \tfrac{3}{\sin\theta} \\ r &= 3\csc\theta \end{aligned}$$

YOU TRY IT: Convert the following rectangular equations to polar equations

252. $y = -x$

253. $x^2 + y^2 + 3y = 0$

254. $x = 1$

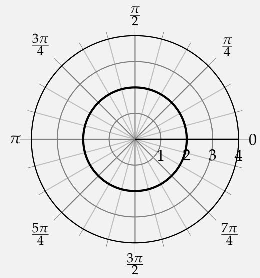
Graphing a polar equation: Basic

Learning Page The graph of a polar equation is the set of all points __________ that satisfy the equation.

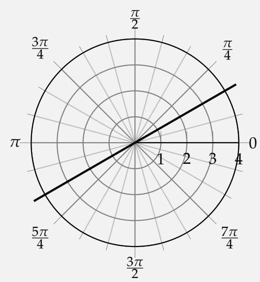
- $r = a$ is the graph of a _______________ centered at the _______________ with radius _______________.
- $\theta = a$ is the graph of a _______________ through the _______________ making an angle _____ with the ____________________ (line has _________________________).
- $r\cos\theta = a$ is the graph of a ____________________ line _____ units from the pole.
  - If __________, the line is to the _______________ of the pole.
  - If __________, the line is to the _______________ of the pole.
- $r\sin\theta = a$ is the graph of a ____________________ line _____ units from the pole.
  - If __________, the line is _______________ the pole.
  - If __________, the line is _______________ the pole.

EXAMPLE:
Sketch the graph of the polar equations.

a) $r = 2$

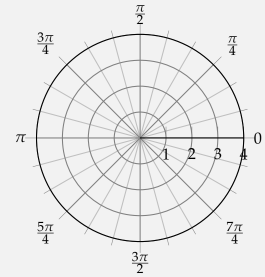


b) $\theta = \frac{\pi}{6}$

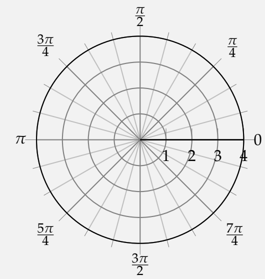


YOU TRY IT:
Sketch the graph of the polar equations.

255. $r = -3$



256. $r = -3\csc\theta$



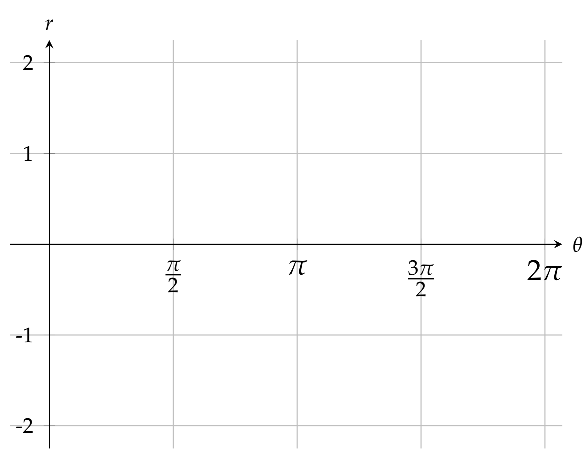
Graphing a polar equation: Circle

Open the Instructor Added Resource which will direct you to a video to complete the following.

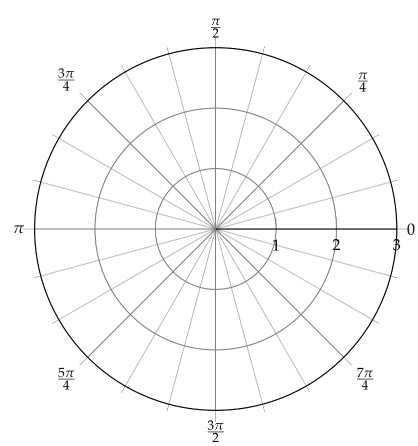
The polar equation of a circle will have the form

________________, __________________________, or __________________________.

Rectangular graph: Clearly label all points.

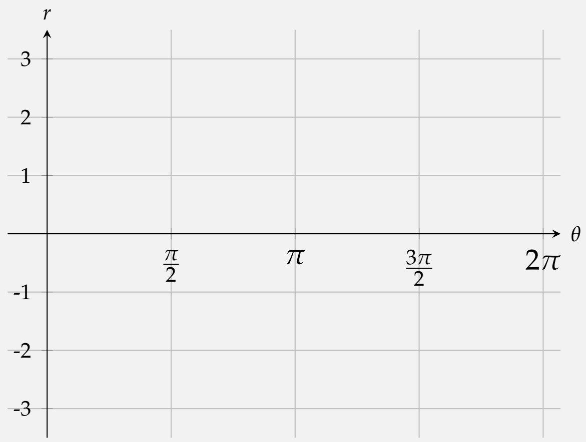


Polar graph: Clearly label all points.

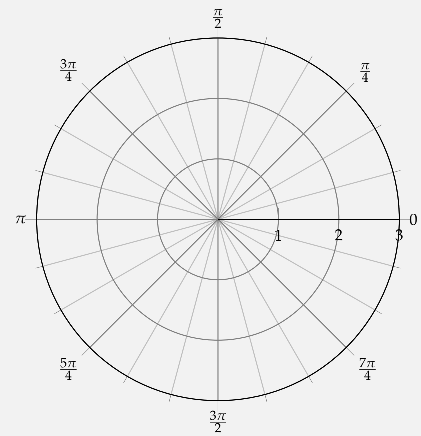


YOU TRY IT: Sketch the graph $r = 3\cos\theta$.

257. Rectangular graph: Clearly label all points.



258. Polar graph: Clearly label all points.

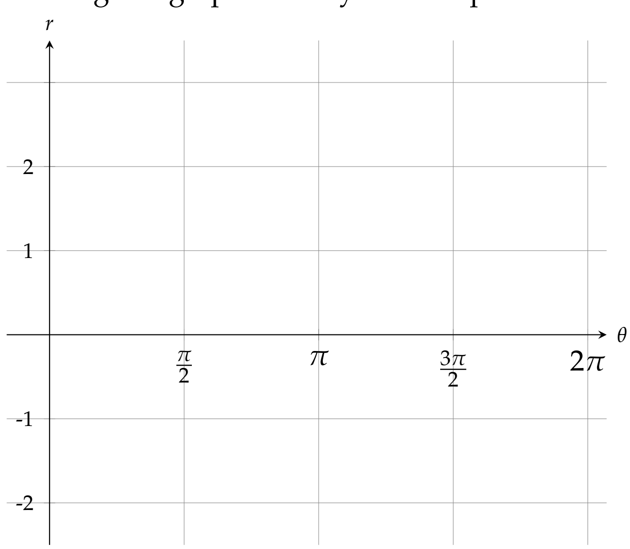


Graphing a polar equation: Limaçon

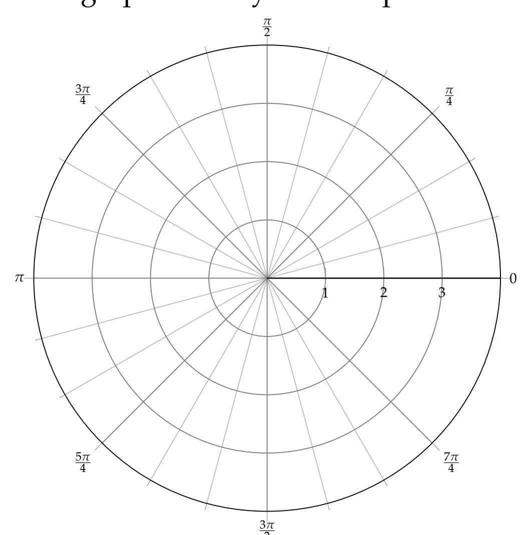
Open the Instructor Added Resource which will direct you to a video to complete the following.

Sketch the graph of $r = 1 - 2\sin\theta$.

Rectangular graph: Clearly label all points.



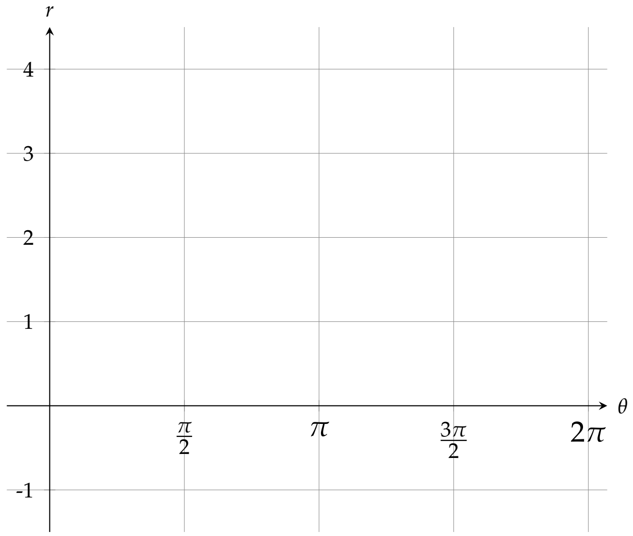
Polar graph: Clearly label all points.



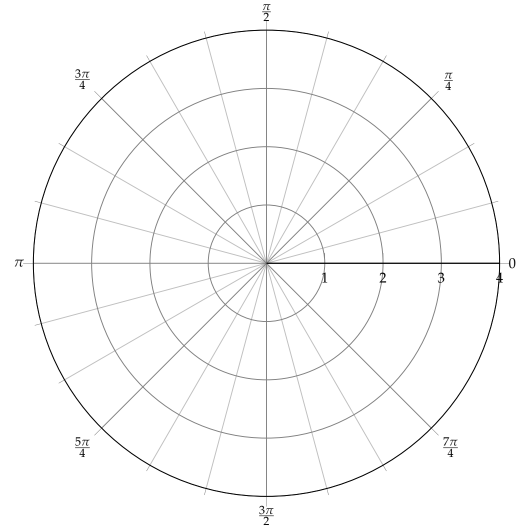
Continued on the next page

Sketch the graph of $r = 2 + 2\sin\theta$.

Rectangular graph: Clearly label all points.

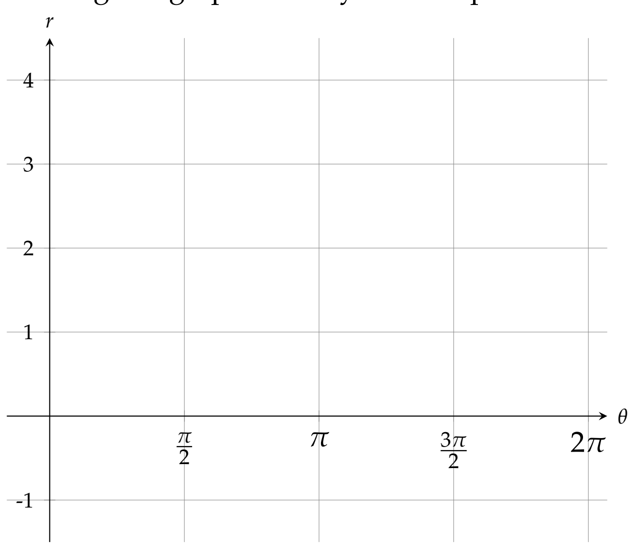


Polar graph: Clearly label all points.

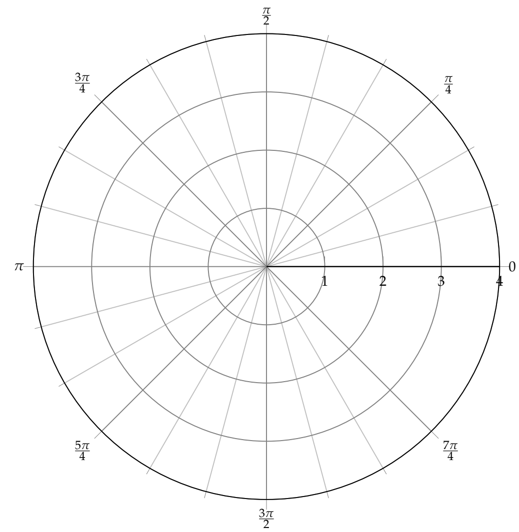


Sketch the graph of $r = 2 + \sin\theta$.

Rectangular graph: Clearly label all points.



Polar graph: Clearly label all points.



A polar equation of a **cardioid** will be of the form $r = a\cos\theta \pm a$ or $r = a\sin\theta \pm a$.

A polar equation of a **limacon without a loop** will be of the form

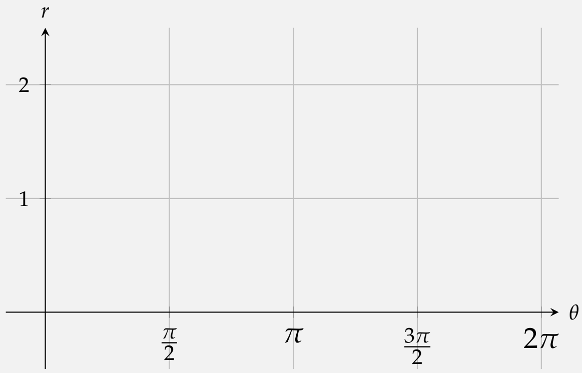
$$r = a\cos\theta \pm b \quad \text{or} \quad r = a\sin\theta \pm b \quad \text{where } a < b.$$

A polar equation of a **limacon with a loop** will be of the form

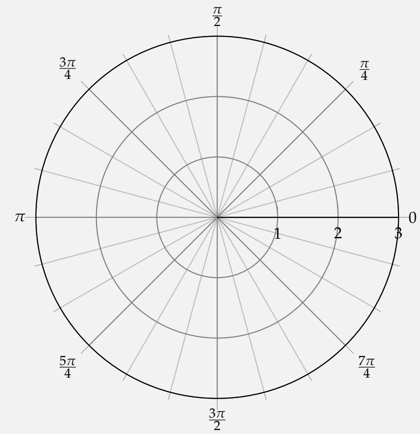
$$r = a\cos\theta \pm b \quad \text{or} \quad r = a\sin\theta \pm b \quad \text{where } a > b.$$

YOU TRY IT: Sketch the graph $r = 1 - \cos\theta$.

259. Rectangular graph: Label all points.

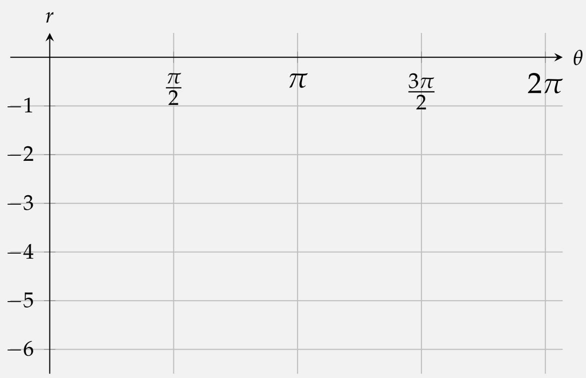


260. Polar graph: Label all points.

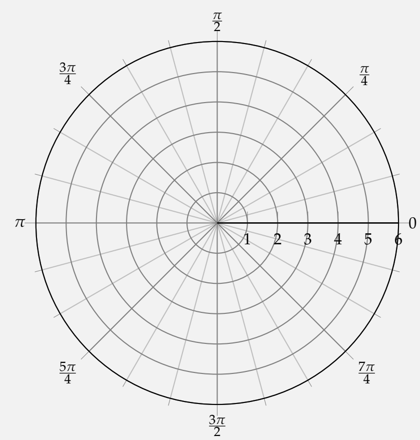


YOU TRY IT: Sketch the graph $r = -4 + 2\sin\theta$.

261. Rectangular graph: Label all points.

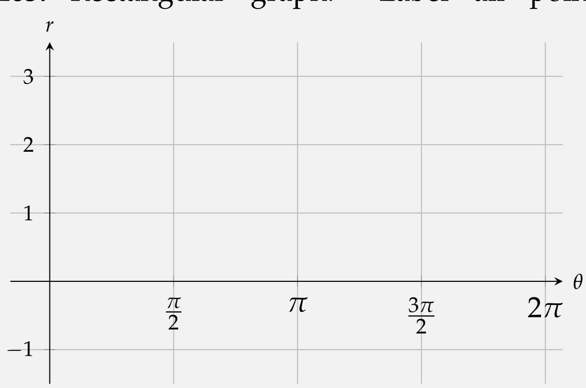


262. Polar graph: Label all points.

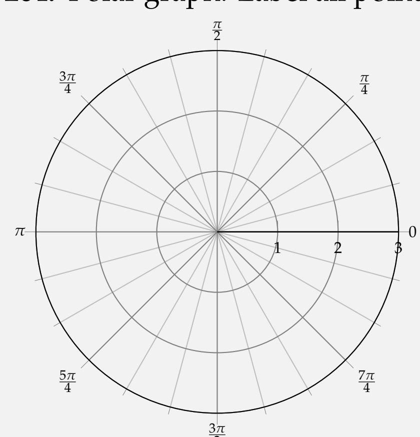


YOU TRY IT: Sketch the graph $r = 1 - 2\cos\theta$.

263. Rectangular graph: Label all points.



264. Polar graph: Label all points.

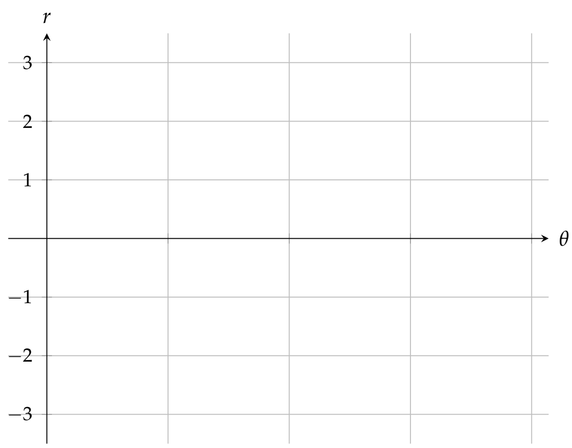


Graphing a polar equation: Rose

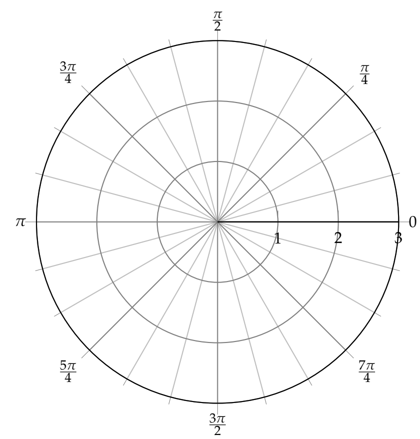
Open the Instructor Added Resource which will direct you to a video to complete the following.

Sketch the graph of ________________________.

Rectangular graph: Clearly label the points.



Polar graph: Clearly label the points.



A polar equation of a **rose** will be of the form

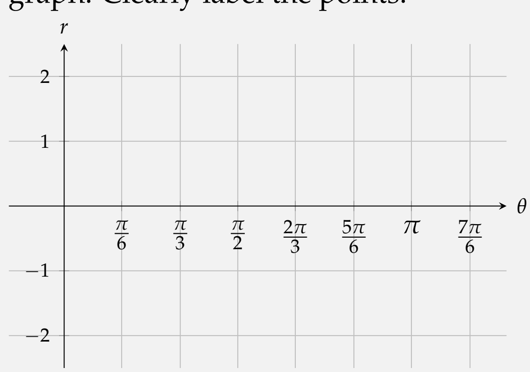
________________________ or ________________________

If n is _______________, the rose will have _______ leaves.

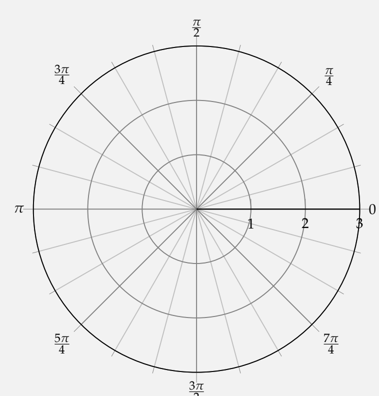
If n is _______________, the rose will have _______ leaves.

YOU TRY IT: Sketch the graph $r = 2\cos 3\theta$.

265. Sketch the polar graph as a rectangular graph. Clearly label the points.



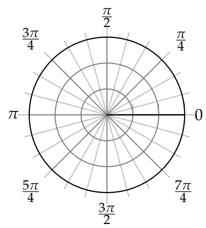
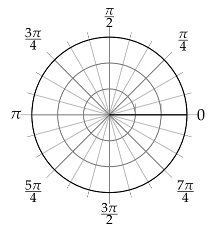
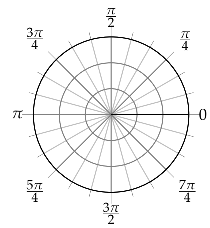
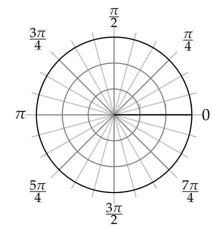
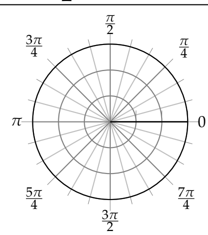
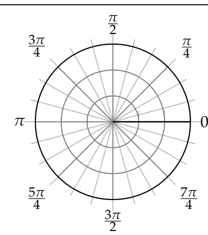
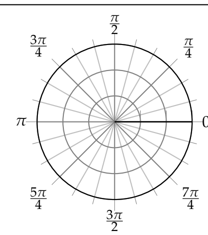
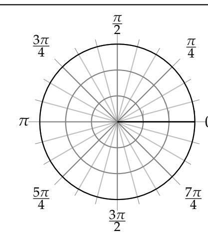
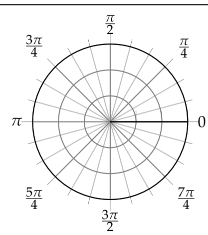
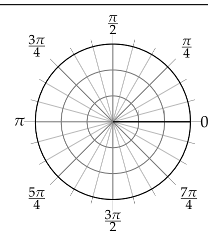
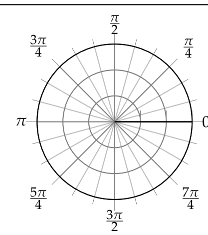
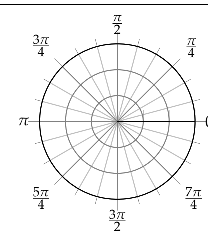
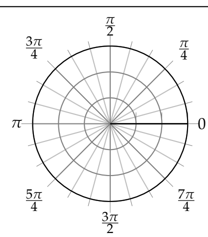
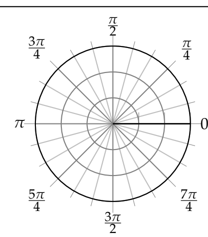
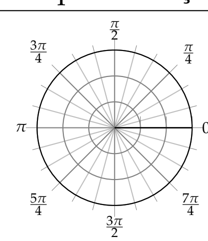
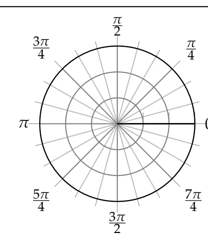
266. Sketch the polar graph. Label your points.



Matching polar equations with their graphs

Open the e-book to complete the following.

Common Polar Equations and Their Graphs

Lines	 $\theta = 0$ x-axis $\theta = \frac{\pi}{2}$ y-axis	 $\theta = a$ Line through origin	 $r = \frac{a}{\sin\theta}$ Horizontal line	 $r = \frac{a}{\cos\theta}$ Vertical line
Circles and Spirals	 $r = a$ Circle	 $r = a\cos\theta$ Circle	 $r = \sin\theta$ Circle	 $r = a\theta$ Spiral
Limaçons $r = a \pm b\cos\theta$ $r = a \pm b\sin\theta$	 $a < b$ limaçon with a loop	 $a = b$ cardioid	 $b < a < 2b$ dimpled limaçon	 $a \geq 2b$ convex limaçon
Roses $r = a\cos n\theta$ $r = a\sin n\theta$	 $r = a\sin 2\theta$ 4-petal rose	 $r = a\sin 4\theta$ 8-petal rose	 $r = a\sin 3\theta$ 3-petal rose	 $r = a\sin 5\theta$ 5-petal rose

Identifying symmetries of graphs given their polar equations

Open the Instructor Added Resource which will direct you to a video to complete the following.

Tests for Symmetry in Polar Coordinates

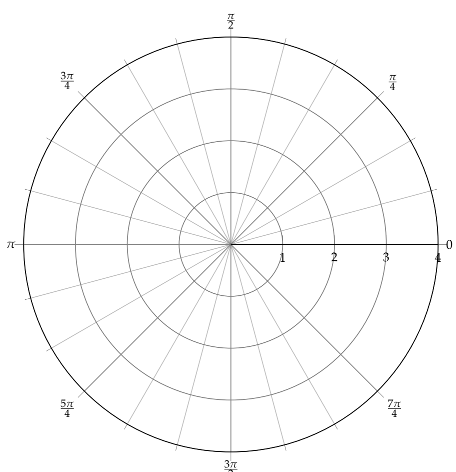
For an equation in polar coordinates, if the indicated substitution produces an equivalent equation, then the graph has the indicated type of symmetry.

1. Symmetry with respect to the **polar axis**: Replace ________________ by ________________
2. Symmetry with repect to the **line** $\boldsymbol{\theta = \frac{\pi}{2}}$: Replace ________________ by ______________________
3. Symmetry with respect to the **pole**: Replace ________________ by ________________

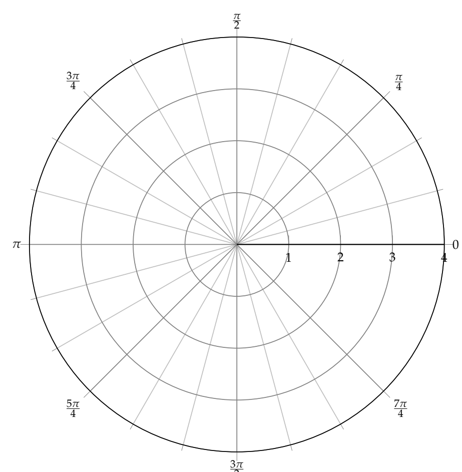
$\cos(-\theta) =$ ________________ $\sin -\theta) =$ ________________ $\sin -\theta) =$ ______________________

Sketch the graphs below from the video that have the appropriate symmetry.

Symmetric to the polar axis (x-axis)



Symmetric to $\theta = \frac{\pi}{2}$ (y-axis)



Notes from Focus Group:

Notes from Focus Group:

Module 15-Final Review

To help you review for your upcoming final exam, this module contains all of the topics from the course. Topics that you have already mastered will not appear in your carousel.

- ALEKS final exam
  - The ALEKS final exam must be taken in the MALL.
  - The ALEKS final exam is a Comprehensive Knowledge Check.
  - The ALEKS final exam must be completed by ______________________.
- To study for the final exams:
  - Complete this ALEKS Final Review Module.
  - Rework the problems on your old exams.
  - Review your old Focus Group assignments.

Solutions

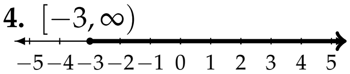
Module 1

1. $7 + \dfrac{d}{6} = 9$

2. $\sqrt{x^7}$

3. $x^{4/3}$

4. $[-3, \infty)$



5. $\emptyset$

6. $(-\infty, 2] \cup (5, \infty)$

7. $\frac{4}{x^5}$

8. $3x^7$

9. $\frac{1}{27x^{11}z^3}$

10. 2

11. 2

12. 4

13. 27

14. 64

15. $\frac{1}{9}$

16. $49 - 42y + 9y^2$

17. $\frac{3x^2-x-44}{x^2+2x-8}$

18. $\frac{x-7}{x-3}$

19. $\frac{-12+15\sqrt{y}}{16-25y}$

20. $\frac{u(wu^2-1)}{w(w^2u+1)}$

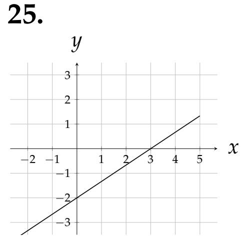
21. $\frac{-3x^3+2x^2}{4y+5x^2y^2}$

22. x-intercept: $\frac{3}{7}$
y-intercept: $-\frac{3}{5}$

23. $y = -\frac{7}{3}x + \frac{2}{3}$

24. $y = \frac{3}{4}x - \frac{9}{2}$

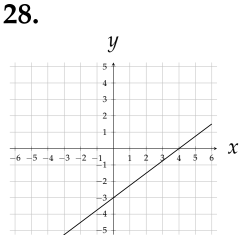
25.



26. 4

27. -3

28.



29. $x = -1, 3$
y-intercept: $-\frac{3}{2}$

30. $y = -\frac{3}{2}$

31. x-intercepts: $(\sqrt{5}, 0), (-\sqrt{5}, 0)$
y-intercepts: $(0, \sqrt{7}), (0, -\sqrt{7})$

32. $x = -4$

33. $y = -12$

34. perpendicular

35. $7i$

36. $4i\sqrt{3}$

37. $y = \frac{3}{4}x - 5$

38. $y = -\frac{4}{3}x + \frac{10}{3}$

39. \$15 per toy produced

40. \$1100

Module 2

41. $x = -3, -2$

42. $x = -\frac{4}{3}, \frac{1}{2}$

43. $x = 0, 5$

44. $3x^2 - 9x - 30 = 0$

45. $-8, 8$

46. No solution

47. $2, -\frac{10}{3}$

48. No solution

49. No solution

50. $x = 5, 9$

51. $x = 2 \pm \sqrt{10}$

52. $x = -1 \pm 2i$

53. $x = \frac{-3 \pm \sqrt{14}}{2}$

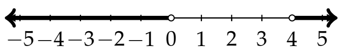
54. $x = -\frac{1}{3} \pm i\frac{\sqrt{2}}{3}$

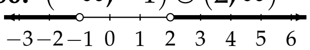
55. $u = 2$

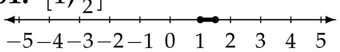
56. length: 24 ft
height: 10 ft

57. $x = 4 \pm \sqrt{14}$ sec
$x \approx .26$ sec, 7.74 sec

58. $t = \frac{27}{4}$

59. 

60. $(-\infty, -1) \cup (2, \infty)$


61. $[1, \frac{3}{2}]$


62. $d = \frac{2S - an}{n}$

63. $x = 4\sqrt[3]{2} - 5$

64. $x = \frac{1}{3}, 2$

65. $x = -\frac{1}{3}$

66. $y = 3, -2$

67. No Solution

68. $x = 10$

69. 14 m/sec

70. $x = 6$

71. -3

72. 20

73. As time increases, the amount of candy in the container increases by 60 pounds per minute.

Module 3

74. 17

75. undefined

76. 3

77. $3x^2 + 6xh + 3h^2 - 4x - 4h + 7$

78. $\sqrt{17 - 4x^2}$

79. $x = 3, -3$

80. $(-\infty, -3) \cup (-3, 5) \cup (5, \infty)$

81. $(-\infty, \frac{4}{7}]$

82. $(-\infty, \frac{9}{7})$

83. Domain: $(-\infty, \infty)$
Range: $[-2, \infty)$

84. Function

85. Not a Function

86. Function

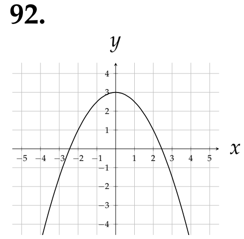
87. Not a Function

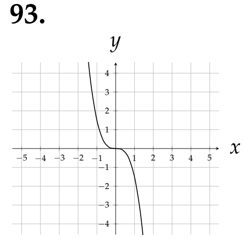
88. \$610

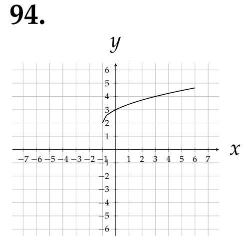
89. 17 weeks

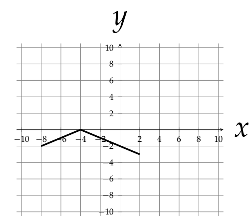
90. 2

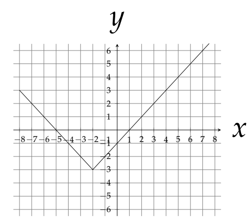
91. 0

92.


93.


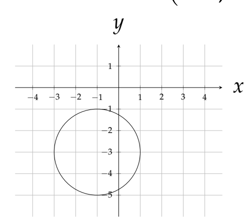
94.


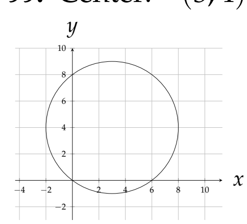
95.


96.


97. $y = (x - 4)^2 - 6$

Module 5

98. Center: $(-1, -3)$, Radius: 2


99. Center: $(3, 4)$, Radius: 5


100. $(x + 3)^2 + (y - 5)^2 = 49$

101. Increasing on $(-\infty, -2)$
Decreasing on $(1, \infty)$
Constant on $(-2, 1)$

102. local min value: 0
local max value: 4

103. max at $x = 0$
min at $x = -2, 2$

104. neither

105. even

106. odd

107. y-axis

108. symmetric to the x-axis, y-axis and the origin

109. 3

110. $(f \circ f)(x) = x$

111. $(g \circ g)(x) = x^4 - 10x^2 + 20$

112. $\frac{3-x}{3-4x}$
D: $(-\infty, 0) \cup (0, \frac{3}{4}) \cup (\frac{3}{4}, \infty)$

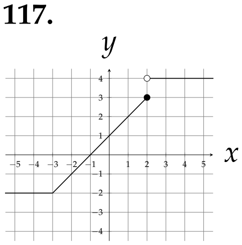
113. $3x^2 - x - 2$
D: $(-\infty, 0) \cup (0, \infty)$

114. $C(x) = 3.5x + 640$

115. $R(x) = 25x$

116. $(R - C)(x) = 21.5x - 640$
Represents the monthly profit for selling x necklaces.

117.



118. $-8x - 4h + 5$

119. $\frac{-5}{(x-3)(x+h-3)}$

Module 6

120. 20 ft by 15 ft

121. 300 ft^2

122. $y = 3(x-1)^2 - 4$

123. Not a polynomial

124. polynomial

125. polynomial

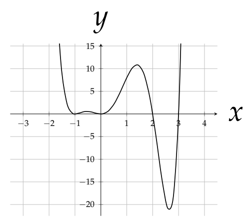
126. polynomial

127. $0, 3, -3, -4$

128. Zero of multiplicity one: -6
Zeros of multiplicity two: 0,-5
Zero of multiplicity four: 1

129. x-intercepts: $(0,0)$, $(-3,0)$, $(4,0)$
y-intercept: $(0,0)$

130.



131. $x = -3$

132. negative

133. 3

134. $p(x) = x(x+2)(x-1)^2(x-7)$

135. $2x^3 + 5x^2 + 7x + 9 + \frac{10x-10}{x^2-2x+1}$

136. $x^2 - 2x + \frac{3}{x}$

137. $2x^3 + 3x^2 + 6x + 9 + \frac{17}{x-2}$

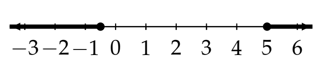
138. $q(-4) = 0$ so $x + 4$ is a factor.

139. $(-2, 2)$

140. A notebook is \$1.85 and a pen is \$0.65.

141. $\left(\frac{1+\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right), \left(\frac{1-\sqrt{5}}{2}, \frac{5-\sqrt{5}}{2}\right)$

142.



Module 7

143. $(-\infty, 2) \cup (2, \infty)$

144. $(-\infty, -1) \cup (-1, \infty)$

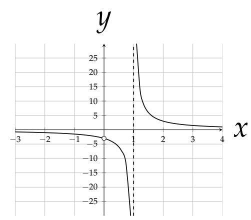
145. Vertical asymptote: $x = 2$
Horizontal asymptote: $y = -1$

146. Vertical: $x = \frac{2}{3}$
Horizontal: $y = 0$

147. Vertical: $x = \frac{2}{3}$
Horizontal: $y = -\frac{5}{3}$

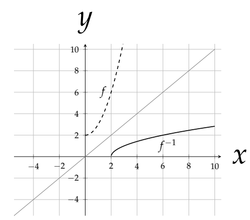
148. $x = 3, x = -3, y = 1$

149.



150. $f(x) = \frac{2(x+3)}{x-3}$

151.



152. $(f \circ g)(x) = \frac{3x+40}{7}$ so f and g are NOT inverses.

153. $g^{-1} = \{(0,3), (5,2), (6,4), (9,7)\}$

154. 2

155. 7

156. $f^{-1}(x) = 7x - 35$

157. -3

158. $f^{-1}(x) = \sqrt[3]{x} - 4$

159. $g^{-1}(x) = \frac{4x-1}{x-2}$

160. $f^{-1}(x) = \frac{1}{3}x^2 - \frac{4}{3}x + \frac{5}{3}$
for $x \geq 2$

161. $g^{-1}(x) = \sqrt{x+13} + 3$

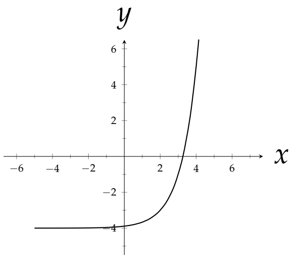
162.

x	$g(x) = 5^x$
0	1
1	5
2	25
3	125
-1	$\frac{1}{5}$
-2	$\frac{1}{25}$
-3	$\frac{1}{125}$

163.

x	$f(x) = (\frac{1}{3})^x$
0	1
1	$\frac{1}{3}$
2	$\frac{1}{9}$
3	$\frac{1}{27}$
-1	3
-2	9
-3	27

164.



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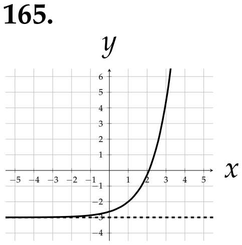


166. 2800 grams

167. 634 grams

168. 29 m/s

Module 9

169. $x = -\frac{4}{3}$

170. $x = 3, 7$

171. $4^x = 5$

172. $\log_7 9 = y$

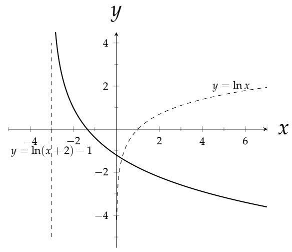
173. $e^5 = x$

174. $\ln t = r$

175. -3

176. 5

177.



178. $x = 8$

179. $(-7, \infty)$

180. $\ln x + 2\ln z - 5\ln y$

181. $\ln 3 + \frac{1}{2}\ln x - 5\ln y - 4\ln z$

182. $\log\left(\frac{3x-3}{x^3}\right)$

183. 21

184. $x = 5$

185. $x = 13$

186. $x = e^4 - 2$

187. $x = 3$

188. $x = 8$

189. $x = \ln 9 + 2$

Module 10

190. $\frac{11\pi}{12}$

191. $r = \frac{\pi}{6}$

192. $r = \frac{\pi}{3}$

193. $-\frac{9}{41}$

194. $\frac{40}{41}$

195. $-\frac{9}{40}$

196. $\frac{41}{40}$

197. $-\frac{41}{9}$

198. $-\frac{40}{9}$

199. $\frac{1}{2}$

200. $\frac{1}{4}$

201. $\frac{\sqrt{3}}{2}$

202. $\frac{2}{\sqrt{5}}$

203. $\frac{15}{17}$

204. $-\frac{15}{8}$

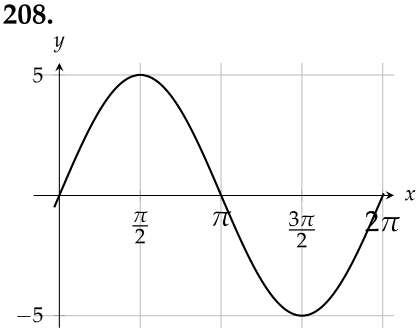
205. $-\frac{17}{8}$

206. $\frac{17}{15}$

207. $-\frac{8}{15}$

Module 11

208.



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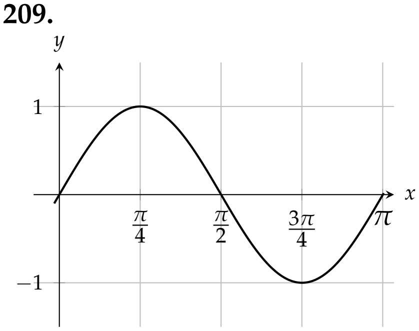

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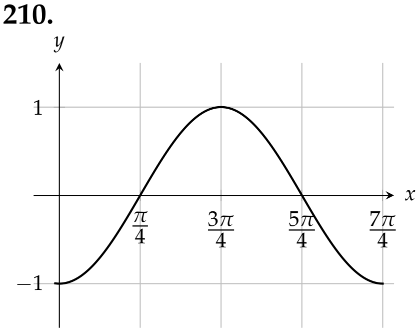

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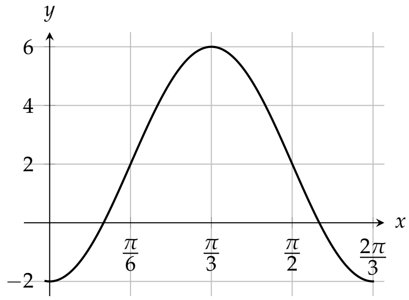
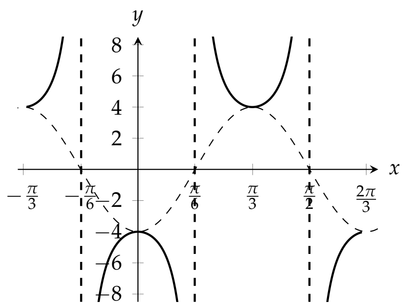


212. $\frac{2\pi}{5}$

213. 3

214. Amp: 3
Period: $\frac{\pi}{2}$
Phase shift: $\frac{\pi}{8}$

215.



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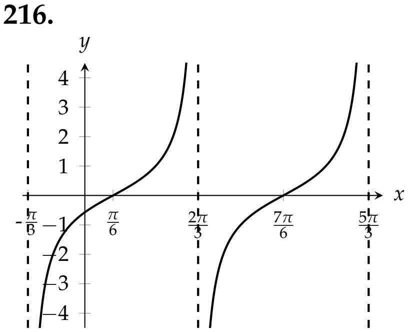


217. $y = -2\sin 8x$

218. $\frac{2\pi}{3}$

219. $\frac{\pi}{6}$

220. $-\frac{\pi}{4}$

221. $\frac{3\pi}{4}$

222. 4

223. $-\frac{\pi}{6}$

224. $\frac{5}{13}$

225. 1

226. u

Module 13

227. $\csc x$

228.

$$\begin{aligned}\sec x + \tan x &= \frac{1}{\cos x} + \frac{\sin x}{\cos x}\\ &= \frac{1+\sin x}{\cos x}\\ &= \frac{1+\sin x}{\cos x}\cdot\frac{1-\sin x}{1-\sin x}\\ &= \frac{1-\sin^2 x}{\cos x(1-\sin x)}\\ &= \frac{\cos^2 x}{\cos x(1-\sin x)}\\ &= \frac{\cos x}{1-\sin x}\end{aligned}$$

229. $-\frac{4\sqrt{2}}{9}$

230. $\frac{7}{9}$

231.

$$\begin{aligned}&\frac{\sin(x+y)}{\cos x\sin y}\\ &= \frac{\sin x\cos y + \cos x\sin y}{\cos x\sin y}\\ &= \frac{\sin x\cos y}{\cos x\sin y} + \frac{\cos x\sin y}{\cos x\sin y}\\ &= \tan x\cot y + 1\end{aligned}$$

232. $x = \frac{4\pi}{3}, \frac{5\pi}{3}$

233. $x = \frac{4\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$

234. $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

235. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

236. $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

237. $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

238. $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

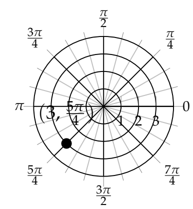
Module 14

239. $B = 48, a \approx 80.3, b \approx 89.2$

240. $A = 115^\circ, b \approx 2.8, c \approx 4.3$

241. $a \approx 15.4$, $B \approx 34.1^\circ$, $C \approx 110.9^\circ$

242.



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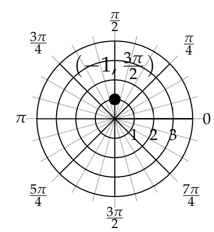

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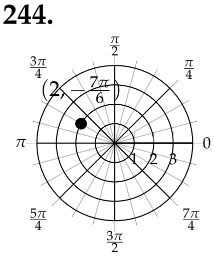


245. $(3, \frac{11\pi}{6})$, answers may vary

246. $(3, -\frac{\pi}{6})$, answers may vary

247. $(\sqrt{6}, \frac{7\pi}{4})$, answers may vary

248. $(\frac{3\sqrt{3}}{2}, -\frac{3}{2})$

249. $y = -5$

250. $x^2 + y^2 = -5y$

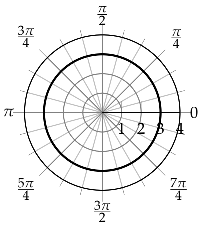
251. $4x + 7y = -3$

252. $\theta = \frac{3\pi}{4}$, answers may vary

253. $r = -3\sin\theta$

254. $r = \sec\theta$

255.



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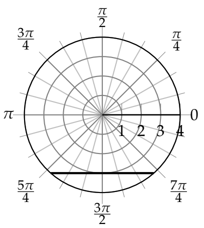

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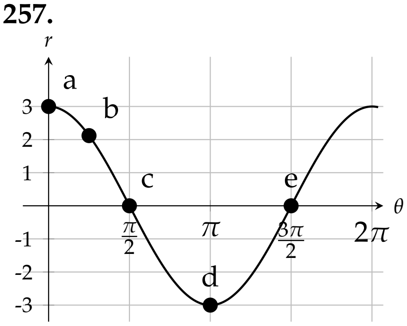

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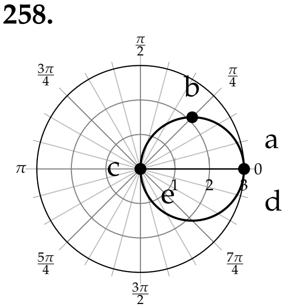

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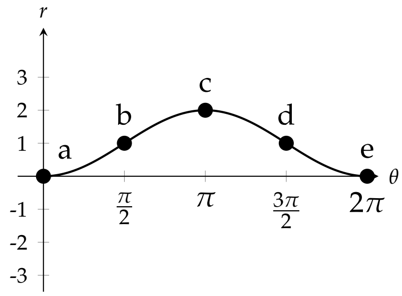

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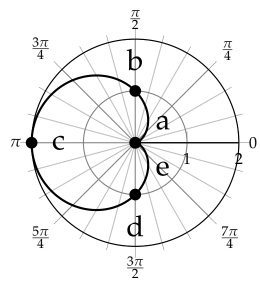

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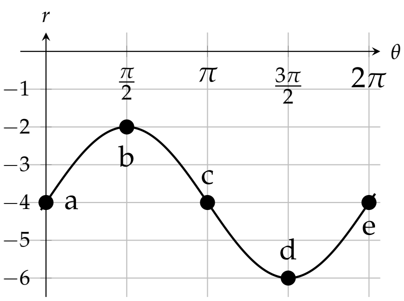

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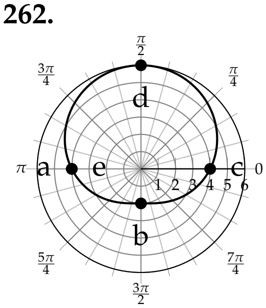

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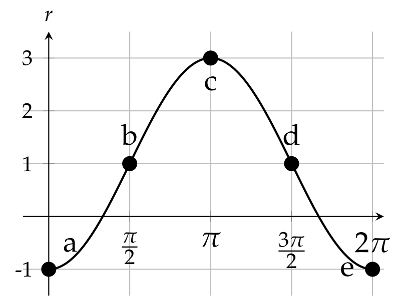

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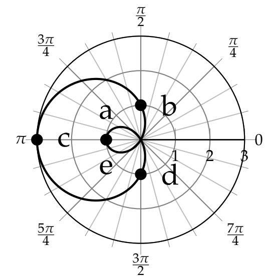

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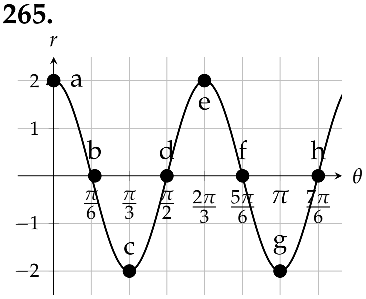

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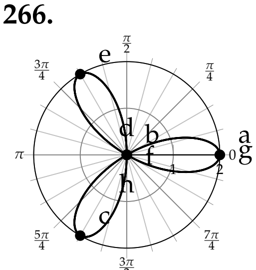

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ARITHMETIC PROPERTIES			
Associative:	addition: $a+(b+c)=(a+b)+c$ multiplication: $a(bc)=(ab)c$	**Identity:**	addition: $0+a=a$ multiplication: $1\cdot a=a$
Commutative:	addition: $a+b=b+a$ multiplication: $ab=ba$	**Inverse:**	addition: $a+(-a)=0$ multiplication: $a\cdot\frac{1}{a}=1,\ a\neq 0$
Distributive:	$a(b+c)=ab+ac$		

FRACTIONS			
Adding:	$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$	**Multiplying:**	$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$
Subtracting:	$\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$	**Dividing:**	$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$

FACTORING	
Difference of Two Squares $a^2-b^2=(a-b)(a+b)$ $a^2+b^2=$Does not factor	**Sum and Difference of Two Cubes** $a^3+b^3=(a+b)(a^2-ab+b^2)$ $a^3-b^3=(a-b)(a^2+ab+b^2)$
Perfect Square Trinomials $a^2-2ab+b^2=(a-b)^2$ $a^2+2ab+b^2=(a+b)^2$	

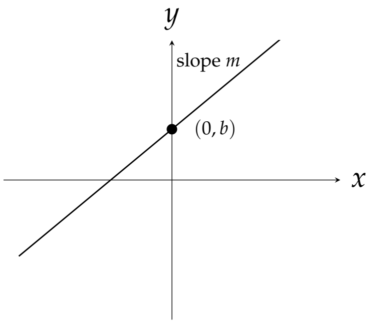
DISTANCE AND MIDPOINT FORMULAS	
Distance between (x_1,y_1) and (x_2,y_2) $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	Midpoint between (x_1,y_1) and (x_2,y_2) $m=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

ABSOLUTE VALUE			
Statement	Equivalent Statement	Statement	Equivalent Statement
$\lvert x\rvert=a$	$x=a$ or $x=-a$	$\lvert x\rvert\leq a$	$-a\leq x\leq a$
$\lvert x\rvert=\lvert y\rvert$	$x=y$ or $x=-y$	$\lvert x\rvert\geq a$	$x\leq -a$ or $x\geq a$

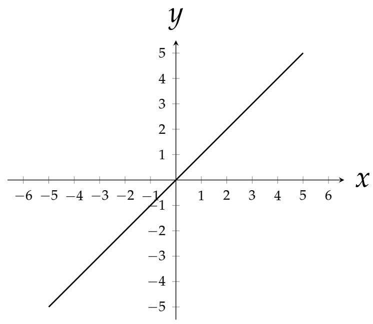
CIRCLE
Standard Form of a Circle with center (h,k) and radius r: $(x-h)^2+(y-k)^2=r^2$

COMMON GRAPHS

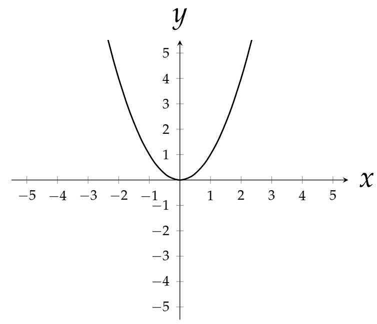
$f(x) = mx + b$



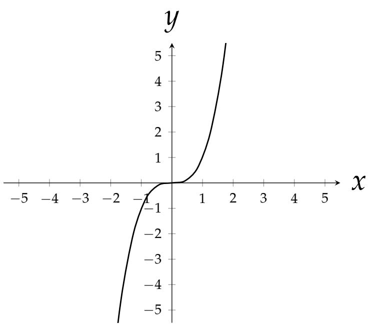
$f(x) = x$



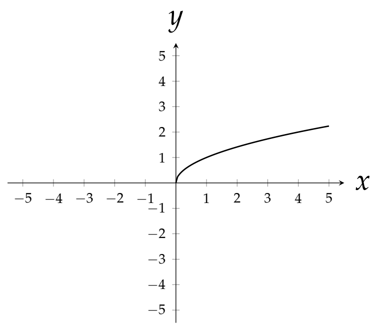
$f(x) = x^2$



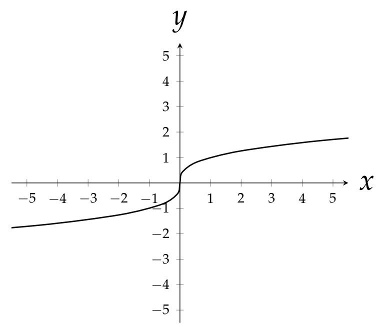
$f(x) = x^3$



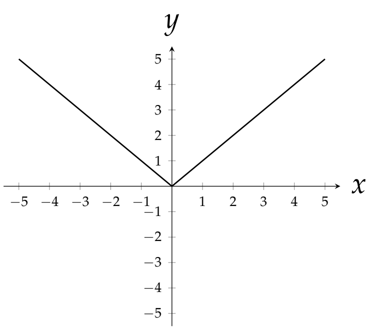
$f(x) = \sqrt{x}$



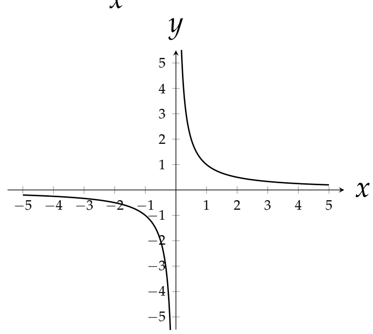
$f(x) = \sqrt[3]{x}$



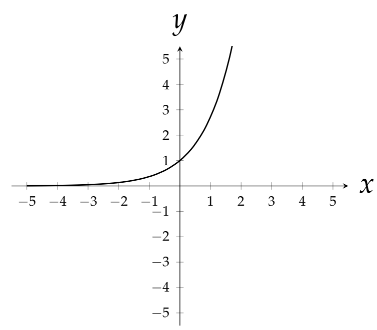
$f(x) = |x|$



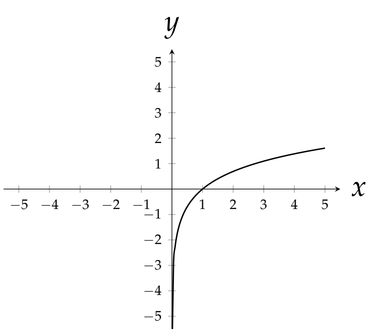
$f(x) = \dfrac{1}{x}$



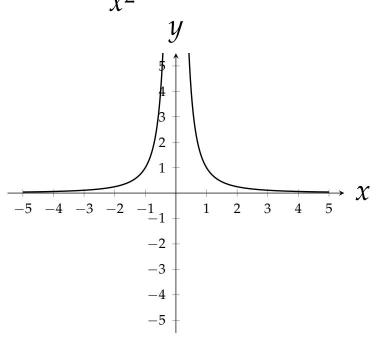
$f(x) = e^x$



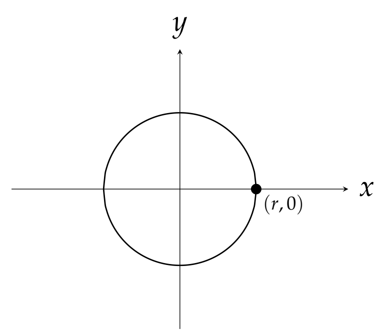
$f(x) = \ln x$

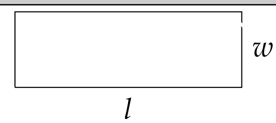
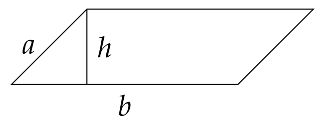
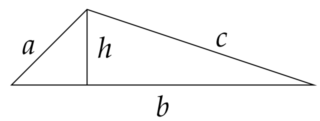
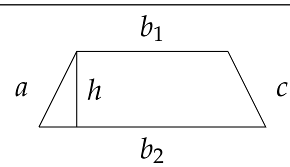
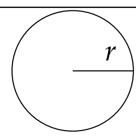
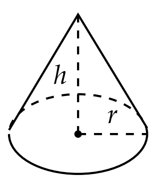
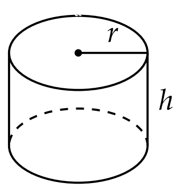
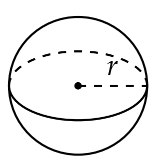
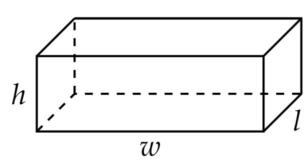


$f(x) = \dfrac{1}{x^2}$



$x^2 + y^2 = r^2$



GEOMETRY			
Rectangle		Perimeter $P = 2l + 2w$	Area $A = lw$
Parallelogram		Perimeter $P = 2a + 2b$	Area $A = bh$
Triangle		Perimeter $P = a + b + c$	Area $A = \frac{1}{2}bh$
Trapezoid		$\mathrm{P} = a + b_1 + b_2 + c$	Area $A = \left(\frac{b_1 + b_2}{2}\right)h$
Circle		Circumference $C = 2\pi r$	Area $A = \pi r^2$
Right Circular Cone		Volume $V = \frac{1}{3}\pi r^2 h$	Surface Area $A = \pi r\sqrt{r^2 + h^2}$
Right Circular Cylinder		Volume $V = \pi r^2 h$	Surface Area $A = 2\pi rh$
Sphere		Volume $V = \frac{4}{3}\pi r^3$	Surface Area $A = 4\pi r^2$
Parallelepiped		Volume $V = lwh$	Surface Area $A = 2(lw + lh + wh)$

PROPERTIES OF EXPONENTS

$a^m \cdot a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^{m-n}$	$(a^n)^m = a^{nm}$	$(ab)^m = a^m b^m$
$a^0 = 1, a \neq 0$	$a^{-n} = \frac{1}{a^n}$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	

DEFINITION OF LOGARITHM

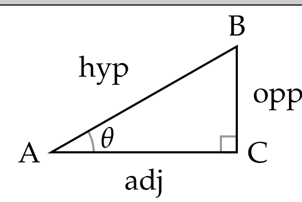
$$log_a x = y \iff a^y = x \qquad \ln x = y \iff e^y = x$$

LAWS OF LOGARITHMS

$\log_a m + \log_a n = \log_a mn$	$\ln m + \ln n = \ln mn$
$\log_a m - \log_a n = \log_a \frac{m}{n}$	$\ln m - \ln n = \ln \frac{m}{n}$
$\log_a m^n = n \log_a m$	$\ln m^n = n \ln m$

RIGHT TRIANGLE TRIGONOMETRY

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\tan\theta = \frac{\text{opp}}{\text{adj}}$
$\csc\theta = \frac{\text{hyp}}{\text{opp}}$	$\sec\theta = \frac{\text{hyp}}{\text{adj}}$	$\cot\theta = \frac{\text{adj}}{\text{opp}}$



QUOTIENT IDENTITIES

$\tan x = \frac{\sin x}{\cos x}$ $\qquad$ $\cot x = \frac{\cos x}{\sin x}$

RECIPROCAL IDENTITIES

$\sec x = \frac{1}{\cos x}$ $\qquad$ $\csc x = \frac{1}{\sin x}$

$\tan x = \frac{1}{\cot x}$ $\qquad$ $\cot x = \frac{1}{\tan x}$

PYTHAGOREAN IDENTITIES

$\sin^2 x + \cos^2 x = 1$ $\qquad$ $\tan^2 x + 1 = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

SUM AND DIFFERENCE IDENTITES

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

DOUBLE-ANGLE FORMULAS

$\sin 2x = 2 \sin x \cos x$ $\qquad$ $\cos 2x = \cos^2 x - \sin^2 x$

HALF-ANGLE FORMULAS

$\sin^2 x = \frac{1-\cos 2x}{2}$ $\qquad$ $\cos^2 x = \frac{1+\cos 2x}{2}$

LAW OF SINES

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

LAW OF COSINES

$a^2 = b^2 + c^2 - 2bc \cos A$ $\qquad$ $c^2 = a^2 + b^2 - 2ab \cos C$

$b^2 = a^2 + c^2 - 2ac \cos B$