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# Math Active Learning Lab: Math 103 College Algebra Notebook

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√MALL

MATH ACTIVE LEARNING LAB

# Math 103 College Algebra Notebook

University of North Dakota

Revised August 2020

# Welcome to the MALL

Welcome to UND's Math Active Learning Lab (MALL)! The MALL is a research-based approach designed to support student engagement with math. The premise of the MALL is that the best way to learn math is by doing math, not by watching someone else do math. This means that most of your time in this course will be spent doing math with the MALL instructors and tutors available to support your learning. The philosophy of the MALL is well described by H. A. Simon's quote

> "Learning results from what the student does and thinks and ONLY from what the student does and thinks. The teacher can advance learning only by influencing what the student does to learn."

For many of you, this is your first college math course. Quite possibly, this course and our expectations may be different from your high school mathematics experiences. We cannot stress strongly enough your role in ensuring your success in this class. More than anything else, your choices will determine your success in this course. Our data shows that students who are successful do the following.

- Attend class (focus group) regularly.
- Work in ALEKS and this Notebook at least 3 days each week.
- Study for written and ALEKS exams.
- Seek help when you need it.

We will be asking you to use the ALEKS resources and to work in your notebooks before coming to class. In your weekly focus group, your instructor will support your learning by facilitating small-group assignments and providing mini-lectures on the more challenging topics.

Instead of sitting in a lecture class for hours each week AND then being expected to do practice problems outside of class, part of your "class time" is spent doing homework in ALEKS. This provides instant feedback and links you to resources as needed. Using ALEKS allows us to individualize the student learning path. Students can move quickly through topics they are familiar with and take the time they need to learn more challenging topics. To help you get the most out of ALEKS, we have created this notebook. If ALEKS and the notebook are still leaving you confused about a topic, we expect you to ask an instructor or tutor for help.

MALL Staff

# Contents

# How to use ALEKS

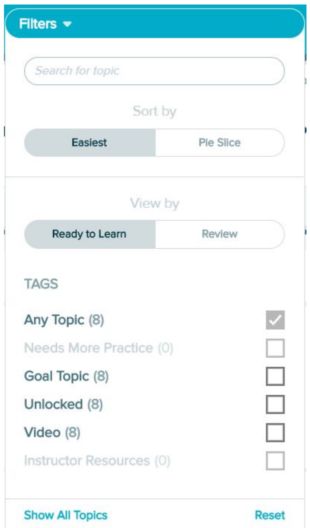
## Working in ALEKS with the Notebook

- Every ALEKS topic is in the Notebook.
- Not every topic in the Notebook will be in YOUR Learning Carousel.
    - If you have already mastered a topic, you will not see the topic in your Learning Carousel.
    - You do NOT need to complete the Notebook for a topic you have already mastered.
- How to work through ALEKS topics
    1. ALEKS presents you with a topic.
    2. Use the table of contents to find the topic in the Notebook.
    3. You will find one of the following icons to help direct your learning.
        - The notes come from the indicated video. You may be asked to select a different video than the first video to pop up.
        - The notes come from the e-book.
            * You may need to scrolll down to find the appropriate topic.
            * Notebook entries are made to look EXACTLY like the e-book material
        - Open the dictionary to show definitions of terms.
        - Directs you to resources your instructor has added.
        - Learning Page The notes come directly from the Learning Page, which is the first page presented to you for each topic.

## The Learning Carousel

- To bring down the Learning Carousel, click the on the upper left side of the ALEKS Learning page.
- indicates a goal topic for the current module
- indicates a locked topic. Click the icon to see what topics must be worked to unlock it.
- No icon means it is a prerequisite topic. Use the Index to find the topic in your Notebook.

- When the Learning Carousel is pulled down, you can
  - Click the Filters for options to filter topics.
  - The Filter menu is shown below.

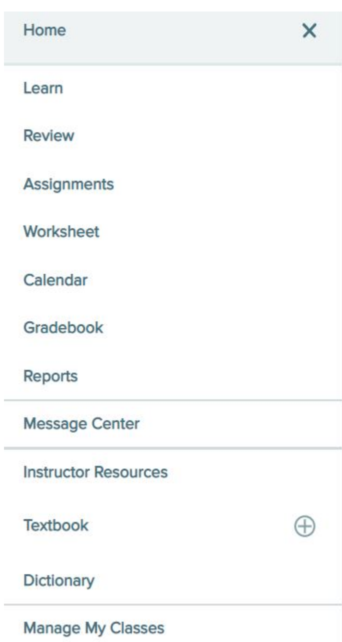



**Search for topic** You can type in the name of a topic to find it.

**TAGS** Click in the boxes to show only the topics that are
- goal topics,
- unlocked,
- have videos.

## Hamburger Menu

- The Hamburger Menu  is in the upper left of your ALEKS screen.
- The options in the Hamburger Menu are shown below.




**Home** Takes you back to the home screen.

**Learn** Opens the next topic ALEKS has ready for you to learn.

**Review** Opens topics you have learned or mastered for you to review.

**Calendar** Opens a calendar view of deadlines for weekly modules and exams.

**Gradebook** Shows your grades for ALEKS modules and exams. The complete and official gradebook is in Blackboard.

**Reports** Opens a menu of reports that provide additional information about your progress in ALEKS. We encourage you to take a look at these pages.

## Technical Support

ALEKS Technical Support is available at `https://www.aleks.com/support/contact_support` or by phone at (800) 258-2374. Call Technical support if you need help with

- accessing your account.
- locating a video.
- questions diplaying correctly.
- other technical issues not related to math content.

# Math ______ __________________ _______ _______

**Instructor:** ____________________ **Email:** ____________________________

**Phone:** ________________ **Office:** ________________

**Focus Group:** ____________________

**Required Course Materials:** ALEKS 18-week access and the ____________________ *Course Notebook*

All email correspondence will go to your official UND email address.

**Course prerequisites and content:** Topics covered will include: equations and inequalities; graphs of equations and functions; linear, quadratic, polynomial, and rational functions; exponential and logarithmic functions; systems of equations; applications and graphs. Prerequisite: Appropriate score in the Placement Testing Program or MATH 93.

**The Math Active Learning Lab (MALL):** Research shows that ____________________ ______________________________, not by listening to someone talk about or present the subject. The primary reason many students do not succeed in traditional math courses is that they do not do the problems or spend enough time engaged with the material.

The MALL is a research-based approach designed to support student engagement with math. Most of your time in this course will be spent doing math, and your instructor will support your learning by facilitating in-class assignments and providing mini-lectures on the more challenging topics. Instructors and tutors are available during the required MALL time to provide just-in-time support.

In a traditional math class, all students are expected to learn at the same pace. In the MALL, the ALEKS learning system allows you to work at you own pace, skip topics you have already mastered, and provides feedback as you are working.

**COVID-19:** All members of the University community have a role in creating and maintaining a COVID-19 resilient campus. There are several expectations that all community members, including students, are asked to follow for the safety of all:

- maintain physical ________________ of at least 6 feet while in UND facilities,
- wear ______ coverings during interactions with others and in the classroom,
- wash their hands often and use hand sanitizer,
- properly clean spaces that they utilize, and
- if experiencing any symptoms, ________________ and call their health care provider.

- Students electing not to comply with any of the COVID related requirements will not be permitted in the ________________, and may be subject to disciplinary action.

All members of the University community are expected to model positive _______________ both on- and off-campus. Information regarding the pandemic and UND's efforts to create a COVID resilient campus is available on the COVID-19 blog (`http://blogs.und.edu/coronavirus/`). Please subscribe to stay up to date on COVID related information.

Students who test positive for COVID-19 or are identified as a close contact are expected to ______________________ self-isolate/quarantine. If you have tested positive for COVID-19 or have been placed in quarantine due to being identified as a close contact or travel we strongly recommend that you report the information to the Office of Student Rights and Responsibilities at 701.777.2664 or online at `https://veoci.com/veoci/p/w/ss2x4cq9238u`. Doing so will ensure students have the support they need to continue with their academic goals and to protect others.

Due to the evolving circumstances of the COVID-19 pandemic, all information in this syllabus may need to be ______________ to meet the needs of remote instruction. Every effort will be made to operate in a manner consistent with the expectations outlined in this document.

## Course Components

**Focus Group**

- Assignments given during the Focus Group meetings will be completed in small groups.
  - On-time attendance is _______________ to earn full-credit on the assignment.
  - Unless required for the Focus Group activity, cell-phone or computer use will result in a zero for the day.
- Students who do not attend the _________________ meeting, or contact the instructor the first week, will be **DROPPED FROM THE COURSE**.
- Students who do not ________________ their Initial Knowledge Check within two full days of their first class meeting will be **DROPPED FROM THE COURSE**.
- Once a week you will meet in class, the other day you will work in ALEKS in the MALL or remotely.
- Focus Group Absences
  - If due to a serious emergency, absences will usually be excused. Documentation ________________________________.
  - University sanctioned absences must be documented _______________________________ prior to the absence.
  - Travel plans ____________________________________ cause for an excused absence.
  - All focus group assignments have a _______________________________________________ to account for any unexcused absences.
  - Absences will be addressed on a case-by-case basis.

**ALEKS**

- Weekly module to be completed by __________ at 11:59 pm.
- Can work anywhere you have internet access.
- Deadlines _________________ be extended because of home computer or home internet issues.

**MALL Time**

- Spend at least 2.5 hours in the MALL working in ALEKS from ________ to ________
- MALL time must be completed in O'Kelly 33 (face-to-face) or virutally through Zoom.
- MALL time is class time, you should be working only on __________.
- Credit for MALL time is based ______ on front desk check-in/out.
- Check-in with your UND ID when entering **and** check-out when exiting the MALL.
  - Failure to check-in/out results in __ minutes recorded.
  - Check-in/out with another student's ID is academic dishonesty.
- Minutes __________________________ from one week to another.
- Focus Group time _________________________ toward your MALL time.
- Food is NOT allowed in the MALL.
- The MALL is the place to get your math questions answered!
- MALL staff are there ________________.

**Notebook**

- Graded __________ in Focus Group.
- _____________ for MALL time and Focus Group.

**Topic Goal Extra Credit**

- Complete 10 topics in ALEKS by ________ at 11:59 pm.
- Earn a Focus Group bonus point.

**Exams**

- There will be __ exams.
- Each exam will have 125 pts
  - ALEKS exam: 100 pts
    - Must be completed in the MALL exam area
    - Must be completed by 9:00 pm the _______________ the written exam.
    - UND ID is required to take your ALEKS exam.
    - All scratch work must be submitted to ________________ as a PDF within 30 min of test completion.
    - You may not leave your table during an exam without permission.
    - Cell phones must be placed face ________ on the table.
  - Written exam: 25 pts
    - will be given during the Focus Group meeting.

Exam 1: ________________ Exam 2: ________________ Exam 3: ________________

**Final Exam**

- The final exam will be a comprehensive ALEKS exam.
- All scratch work must be submitted to Blackboard within 30 min of test completion.
- The final ALEKS exam must be completed by Wednesday, December 16 at 7:30 pm.

**Grading**

- Your course grade will be a weighted average of the following:

| | |
|---|---|
| Exams | ____% |
| Final Exam | ____% |
| MALL Time | 10% |
| Focus Group Activities | 10% |
| Module Completion | 15% |

- Grading Scale: A = 90% & above, B = 80–89%, C = 70–79%, D = 60–69%.

**Try Score**

- Your Try Score reflects your effort in this course.
- The Try Score is composed of:
  - focus group participation,
  - notebook completion,
  - MALL time and
  - module completion.
- This is _______ included in your course grade, but will be shared with your academic advisor.

**Finishing the Course Early**

- Given the individualized nature of this course it is possible to complete the course _________.
- Each time an exam is given, __________________ students have the option to take the final in place of the scheduled exam.
- To qualify to take the final early
  - the week before the written exam, arrange with the MALL office to take a proctored Knowledge Check
  - ____________ at least 90% of the in the course on this proctored ALEKS Knowledge Check

**Academic Honesty**

- All students in attendance at the University of North Dakota are expected to be honorable and to observe standards of conduct appropriate to a community of scholars.
- Academic misconduct includes
  - all acts of dishonesty in any academically related matter.
  - any knowing or intentional help or attempt to help, or conspiracy to help, another student.
  - use of _________, books, calculators, _______________ or any electronic devices on exams.
- A student who attempts to obtain credit for work that is not their own (whether that be on a homework assignment, exam, etc.) will receive ________________ for that item of work, and at the professor's discretion, may also receive a failing grade in the course.
- For more information read the Code of Student Life at `https://und.policystat.com/policy/6747183/latest/`.

**Accommodations**

- Disability
  - Contact me to request disability accommodations, discuss medical information, or plan for an emergency evacuation.
  - To get confidential guidance and support for disability accommodation requests, students are expected to register with DSS at `http://und.edu/disability-services/`, 190 McCannel Hall, or 701.777.3425.
- COVID-19
  - Due to COVID-19 students may need to request course adjustments, flexibility in delivery of content, and increased absenteeism.
  - Students with concerns regarding physically attending class during COVID-19 are encouraged to do the following:
    - Talk with your _________________ to determine appropriate accommodations, as soon as possible
    - Students with a known disability should contact Disability Student Services (DSS).

**Starfish**

- Important information is available to you through Starfish, which is an online system used to help students be successful.
- When an instructor observes student behaviors or concerns that may impede academic success, the instructor may raise a flag that notifies the student of the concern and/or refer the student to their academic advisor or UND resource.
- Please pay attention to these emails and take the recommended actions. They are sent to help you be successful!
- Starfish also allows you to
  - schedule appointments with various offices and individuals across campus.
  - request help on a variety of topics
  - search and locate information on offices and services at UND
- You can log into Starfish by clicking on Logins on the UND homepage and then selecting Starfish. A link to Starfish is also available in Blackboard once you have signed in.

**Essential Studies:** This course addresses the Essential Studies *Learning Goal* of *Quantitative Reasoning*. Quantitative reasoning is competency and comfort in working with numerical data, using it to reason and solve quantitative problems from a wide array of authentic contexts and everyday life situations, and to create and clearly communicate sophisticated arguments supported by quantitative evidence, such as by using words, tables, graphs, mathematical equations, etc., as appropriate. You should expect to focus on these intellectual skills as part of this course.

This is an Essential Studies Math, Science, & Technology course. Mathematics is a body of knowledge based on patterns, abstraction and logical reasoning, often involving quantity, structure, space, or change. Mathematics uses formal reasoning to investigate relationships between abstract patterns.

- Many courses in mathematics involve numerical skills and quantitative reasoning.
- ES courses in mathematics should give students some experience in abstract reasoning as well as the use of such reasoning to reach conclusions about the world.

**Notice of Nondiscrimination**

- It is the policy of the University of North Dakota that no person shall be discriminated against because of race, religion, age, color, gender, disability, national origin, creed, sexual orientation, gender identity, genetic information, marital status, veteran's status, or political belief or affiliation and the equal opportunity and access to facilities shall be available to all.
- Concerns regarding Title IX, Title VI, Title VII, ADA, and Section 504 may be addressed to:
  - Donna Smith, Director of Equal Employment Opportunity/Affirmative Action and Title IX Coordinator, 401 Twamley Hall, 701.777.4171
  - UND.affirmativeactionoffice@UND.edu
  - Office for Civil Rights, U.S. Dept. of Education, 500 West Madison, Suite 1475, Chicago, IL 60611

**Resolution of Problems**

Should a problem occur, you should speak to your instructor first. If the problem is not resolved, meet with Dr. Michele Iiams, MALL Director. If the problem continues to be unresolved, go to Dr. Gerri Dunnigan, Mathematics Department Chair, and next to the college Dean. Should the problem persist, you have the right to go to the Provost next, and then to the President.

**How to Seek Help When in Distress**

- We know that while college is a wonderful time for most students, some students may struggle.
- You may experience students in distress on campus, in your classroom, in your home, and within residence halls.
- Distressed students may initially seek assistance from faculty, staff members, their parents, and other students.
- In addition to the support we can provide to each other, there are also professional support services available to students through the Dean of Students and University Counseling Center.
  - Both staffs are available to consult with you about getting help or providing a friend with the help that he or she may need.
  - For more additional information, please visit the UND Cares program Webpage at `https://und.edu/student-life/student-rights-responsibilities/`.

# Time Management

Good time management, good study skills and good organization will help you be successful in this course (and all of your classes). Answer the following questions.

1. To motivate yourself to complete a course, it is helpful to have clear reasons for taking the course. List your goals for taking this course.

2. Taking 12-15 credit-hours is the equivalent of a full-time job. Often students try to work too many hours while taking classes.

   **NOTE:** Students need to work to pay tuition, rent, buy food, etc., but working too many extra hours for things that are not needed can really impact their success. There is a balance between working to earn money now and having to spend more money later to retake courses.

   (a) Write down the number of of credit-hours you are taking this term and the number of hours you work per week.

   - Number of credit-hours ________________
   - Number of hours worked per week ________________

   (b) The table gives the recommended limit to the number of hours you should work for the number of credit-hours you are taking.

   - How do your numbers from part (a) compare to those in the table?

| Number of Credit-Hours | Maximum Number of Hours of Work per Week |
|---|---|
| 3 | 40 |
| 6 | 30 |
| 9 | 20 |
| 12 | 10 |
| 15 | 0 |

(c) Keep in mind that other responsibilities in your life, such as your family, might also make it necessary to limit your hours at work even more. What other responsibilities do you have?

(d) It is often suggested that you devote 2 hours of study and homework time outside of class for each credit-hour you take. For example:

| | | | | |
|---|---|---|---|---|
| 12 | credit-hours | | 15 | credit-hours |
| 24 | study hours | | 30 | study hours |
| 36 | total hours | | 45 | total hours |

- Based on the number of credit-hours you are taking, how many study hours should you plan for?

  ________ credit hours X 2 = ________ study hours

- What is the total number of hours (class time plus study time) that you should devote to school?

  ________ credit hours + ________ study hours = ________ total hours

- Your MALL course is a 3-credit course. This means you might need to spend up to 9 hours each week in class, working in ALEKS, or studying.
- At least 2 of these hours should be completed in the MALL.

On the next page, write down the times each day (for the next week) that you

- have scheduled classes,
- are scheduled to work
- other non-negotiable commitments (family, organization meetings, etc.)
- times that you plan to work in the MALL
- times that you plan to study outside of the MALL

| Time | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
|---|---|---|---|---|---|---|---|
| 8:00 - 8:30 | | | | | | | |
| 8:30 - 9:00 | | | | | | | |
| 9:00 - 9:30 | | | | | | | |
| 9:30 - 10:00 | | | | | | | |
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| 10:00 - 10:30 | | | | | | | |
| 10:30 - 11:00 | | | | | | | |

# Test Analysis

Have you ever thought of your graded test as a learning experience? There is a lot you can learn about yourself, your study habits, and your test-taking skills by examining your graded test after you get it back.

- Did you do as well as you thought you could?
- Or is there room for improvement?

You may think, ”the test was too hard” or “the teacher didn’t give us enough time”, but, chances are, your instructor has been giving a similar test under similar conditions to many students before you. So let’s see what **YOU** can do to earn a higher score on your next test.

Look at your graded test and analyze if each point loss was due to your having been **unprepared** for that problem, a **concept error**, or a **careless error** .

- Being **underprepared** for a problem means you didn’t know how to do the problem because you hadn’t done the homework that would have prepared you for it. Often an error made is considered an underprepared error if you look at the problem and have no idea where to begin.
- A **concept error** is one where you really didn’t understand the concept behind the problem. No matter how much time was available for a problem like this, you wouldn’t have been able to do it correctly because you have no conceptual understanding of the problem. *This is not a procedural error: you can apply a procedure and still not understand the concept.* Students demonstrate conceptual understanding in mathematics when they provide evidence that they can recognize, label, and generate examples of concepts; use and interrelate models, diagrams, manipulatives, and varied representations of concepts; identify and apply principles; know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; recognize, interpret, and apply the signs, symbols, and terms used to represent concepts. Conceptual understanding reflects a student’s ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.
- A **careless error** is one where you understood the problem and knew how to solve it, but you made a mistake that could have been avoided. Maybe you copied the problem or your handwriting incorrectly, made a relatively minor mistake in calculation, or some similar error.

1. In the chart below, put the **number of points** you missed on each problem under the correct heading. Then find the total in each column.

| **Problem** | **unprepared** | **concept error** | **careless error** |
|---|---|---|---|
| | | | |
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| | | | |
| | **Total points** | **Total points** | **Total points** |
| | | | |

2. In which column did you have the most missed points? What does that tell you about yourself?

3. What can you learn from this exercise?

**Being Unprepared**
Consider the points you lost because you were **unprepared**. Why did you take a test without being fully prepared? Often, activities and responsibilities in life interfere with good intentions about being diligent in attending class, completing the notebook, completing MALL time, and completing the module. It may be time to:

- **re-examine your weekly schedule** and make sure you are devoting a sufficient amount of time to this class. Lay out a time management grid of your schedule making sure to schedule your MALL time and math study time throughout the week.
- **re-commit yourself to succeeding in this class.** Think about your college and career goals and remind yourself of how this course helps you get one step closer to achieving them.

4. List two steps you will take to remedy being unprepared.

**Concept Errors**
Now consider the **concept errors** point loss. A high total in this column tells you that you didn't understand the concepts very well. You may understand a math concept for the hour you're working on problems, but forget it by the next day; possibly because you didn't do enough homework.

- **Take Knowledge Checks when they appear.** Knowledge Checks (KCs) are the way ALEKS helps you identify topics you are not retaining. Take each KC as if it were a QUIZ (no notebooks, calculators, friends, other websites, etc.) AND to the BEST OF YOUR ABILITY. Topics that you need to revisit will appear again in later modules as they are needed.
- **Get the help you need immediately!** Math concepts build on each other. Each new idea is based on many previous concepts. Make sure you get the help you need immediately, as soon as you find yourself beginning to feel lost, so that the confusion doesn't compound itself - otherwise it can become like a snowball, getting bigger and bigger as it roles through the snow.

If your total loss due to concept errors is fairly large, find out where you can get the help you need. A high concept error total is cause for concern and must be addressed immediately for you to succeed.

5. Which of the following can help you when you are struggling with math?
   (a) your instructor
   (b) MALL tutors
   (c) Reworking and asking questions about previous Focus Group assignments
   (d) Completing your Notebook pages
   (e) All of the above

**Careless Errors**
Next look at **careless error** point loss. Careless errors are often caused by hurrying during a test or by lack of concentration due to test-anxiety or over-confidence. Here are some strategies that have worked for other students:

- **Do the easiest problems first.** When you first start a test, look it over and note which problems will be easiest for you. Do all those problems first to ensure you don't leave an easy problem blank just because it is at the end of the test. Finishing problems you find easy will help build your confidence! Then go through the rest of the test from beginning to end.
- **Work carefully and neatly.** As you do each problem, try to focus on one step at a time.
- **Review each problem to look for careless errors** when you finish the test. Find and correct common careless errors like arithmetic mistakes and sign errors before you turn in your test.
- Whenever possible, **check your answer.**

A lot of points can be gained by slowing down and being careful.

6. What are things you will do next time to prevent careless errors?

7. Now take half of your careless errors point total and add it back to your test total. What could your test grade have been? Would it have changed the letter grade?

# Module 1

## Contents



## Weekly Checklist

☐ Complete MALL time.

☐ Work in ALEKS and Notebook at least 3 days a week.

☐ Complete the weekly Module and Notebook pages by the due date.

☐ Attend Focus Group.

☐ Actively participate in Focus Group.

☐ Earn extra credit: Complete 10 topics by ____________________.

## Evaluating an expression with a negative exponent: Positive fraction base

Learning Page

For any ____________________ rational number ________ and any _________________________ $n$, we have the following.

$$\left(\frac{a}{b}\right)^{-n} = \underline{\qquad\qquad\qquad} = \underline{\qquad\qquad\qquad}$$

Simplify and write the answer with positive exponents.

**EXAMPLE:**

$$\left(\frac{-2}{3}\right)^{-4} = \left(\frac{3}{-2}\right)^{4} = \frac{3^4}{(-2)^4} = \frac{81}{16}$$

**YOU TRY IT:**

**1.** $\left(\frac{5}{-2}\right)^{-3} =$

## Evaluating an expression with a negative exponent: Negative integer base

Learning Page

For any ____________________ number _____ and any _________________________ $n$, we have the following.

$$a^{-n} = \underline{\qquad\qquad\qquad}$$

Rewrite the following without an exponent.

**EXAMPLE:**

$$(-4)^{-3} = \frac{1}{(-4)^3} = \frac{1}{-64} = -\frac{1}{64}$$

**YOU TRY IT:**

**2.** $(-5)^{-2} =$

## Rewriting an algebraic expression without a negative exponent

Learning Page

For any ______________ number $a$ and any ________________________ $n$, we have the following.

**Rule 1:** $a^{-n} =$ __________

Move __________ to the ______________ and make the ________________________.

**Rule 2:** $\frac{1}{a^{-n}} =$ __________

Move __________ to the ______________ and make the ________________________.

**YOU TRY IT:** Write the following expressions with positive exponents.

**3.** $4x^{-5}$

**4.** $\dfrac{3}{x^{-7}}$

## Power and quotient rules with negative exponents: Problem type 1

Watch the video *Simplifying an Exponential Expression by Using the Power Properties of Exponents* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Simplify.

**YOU TRY IT:**

**5.** Simplify $\left(\frac{3x^3}{x^7}\right)^{-3}$

## Converting between radical form and exponent form

Watch the video *Converting Between Radical Notation and Rational Exponents* to complete the following.

Convert each expression to radical notation. Assume all variables represent positive real numbers.

1.

2.

3.

Convert each expression to an expression with rational exponents. Assume that all variables represent positive real numbers.

4.

5.

**YOU TRY IT:**

**6.** Convert $x^{7/2}$ to radical notation.

**7.** Convert $\sqrt[3]{x^4}$ to an expression with rational exponents.

## Rational exponents: Unit fraction exponents and bases involving signs

Watch the video *Definition of "a" to the* $1/n$ *Power* to complete the following.

**Definition of $a^{1/n}$**

Let $n > 1$ be an integer. Then, $a^{1/n} =$ ________ provided that $\sqrt[n]{a}$ is a ____________ number.

| Verbal Interpretation | Algebraic Example |
|---|---|
| $a^{1/n}$ equals the ____________ of $a$, provided that the $n^{th}$-root of $a$ is a ____________ number. | |

Simplify if possible.

a.

b.

c.

**YOU TRY IT:** Simplify the following.

**8.** $16^{1/4} =$

**9.** $8^{1/3} =$

## Rational exponents: Non-unit fraction exponent with a whole number base

Watch the video *Definition of "a" to the m/n Power* to complete the following.

**Definition of $a^{m/n}$**

Let $m$ and $n$ be positive integers such that $m/n$ is in lowest terms and $n > 1$. Then if $\sqrt[n]{a}$ is a

________________ number,

$a^{m/n} =$ ________________ $=$ ________________ OR $a^{m/n} =$ ________________ $=$ ________________

Simplify if possible.

a.

b.

c.

d.

e.

f.

**YOU TRY IT:** Simplify the following.

**10.** $8^{2/3} =$

**11.** $81^{3/4} =$

## Rational exponents: Negative exponents and fractional bases

If you have not already watched the video *Definition of "a" to the m/n Power* from the previous topic **Rational exponents: Non-unit fraction exponent with a whole number base**, do so now. You may watch the video again for a review.

**YOU TRY IT:** Simplify. Write your answers without exponents.

**12.** $\left(\frac{1}{16}\right)^{-3/2}$

**13.** $27^{-2/3}$

## Simplifying the square root of a whole number greater than 100

Learning Page One of the properties of square roots is the ____________________ property.

$\sqrt{a \times b} =$ ____________________ for any _________________________ numbers $a$ and $b$.

We want to find the greatest factors that are perfect squares.

More In the space below, write twelve perfect squares.

**EXAMPLE:** Simplify $\sqrt{252}$

$$\begin{aligned} \sqrt{252} &= \sqrt{4} \cdot \sqrt{63} \\ &= \sqrt{4} \cdot \sqrt{9} \cdot \sqrt{7} \\ &= 2 \cdot 3 \cdot \sqrt{7} \\ &= 6\sqrt{7} \end{aligned}$$

**YOU TRY IT:** Simplify.

**14.** $\sqrt{294}$

## Factoring a product of a quadratic trinomial and a monomial

Watch the video *Factoring Out the Greatest Common Factor* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Factor out the greatest common factor.

1.

2.

## Restriction on a variable in a denominator: Linear

Learning Page

Division by _____ is _______________ defined, so the expression is ____________________ when its _________________________ is _____.

We must find all values for which the expression is ____________________.

So we set the _________________________ equal to _____ and solve.

Find all excluded values for the expression.

**EXAMPLE:** $\frac{2x+8}{5x+15}$

$$\begin{aligned} 5x+15 &\neq 0 \\ 5x &\neq -15 \\ x &\neq -3 \end{aligned}$$

**YOU TRY IT:**

**15.** $\frac{4x+1}{3-9x}$

## Estimating a square root

Learning Page Complete the following table of square roots.

| | |
|---|---|
| | |
| | |
| | |
| | |
| | |

## Solving a linear equation with several occurrences of the variable: Variables on both sides and two distributions

Watch the video *Solving a Linear Equation in One Variable* to complete the following.

If you have not already done so, complete the definition box, **Solving a Linear Equation in One Variable**, on the next page under the topic ”Solving a linear equation with several occurrences of the variable: Variables on both sides and fractional coefficients”.

Solve.

**EXAMPLE:**
Solve the equation $3(x+4)-13=2(3x+4)+6$.

$$3(x+4)-13=2(3x+4)+6$$
Distribute the 3 and 2
$$3x+12-13=6x+8+6$$
Combine like terms on each side
$$3x-1=6x+14$$
Move $x$'s to one side, constant to other
$$-3x=15$$
Divide by $-3$
$$x=-5$$

**YOU TRY IT:**

**16.** Solve the equation $5(7+3x)=4(x-1)$

## Solving a linear equation with several occurrences of the variable: Variables on both sides and fractional coefficients

Open the e-book to complete the following.

**Solving a Linear Equation in One Variable**

**Step 1:** ______________ both sides of the equation.

- Use the ______________ property to clear ___________________.
- Combine _________________________.
- Consider clearing ______________ or ______________ by ______________ both sides of the equation by the _____________________________ of all terms.

**Step 2:** Use the addition property of equality to collect the ________________________ on ___________________ of the equation and the ___________________ terms on the other side.

**Step 3:** Use the multiplication property of equality to make the ___________________ of the variable term equal to _____.

**Step 4:** ______________ the potential solution in the ___________________ equation.

**Step 5:** Write the ___________________ set.

**YOU TRY IT:**

**17.** Solve. $\frac{2}{3}y - \frac{5}{6} - 3 = \frac{1}{2}y - 5$

## Solving a linear equation with several occurrences of the variable: Fractional forms with binomial numerators

Watch the video *Solving a Linear Equation in One Variable Containing Fractions* to complete the following.

Solve. ________________________

**EXAMPLE:**
Solve the equation.

$$\frac{x+1}{2} = \frac{x-4}{6}$$

Multiply both sides of the equation by the LCD.

$$6 \cdot \frac{x+1}{2} = 6 \cdot \frac{x-4}{6}$$

Simplify.

$$3(x+1) = x-4$$

Distribute the 3.

$$3x+3 = x-4$$

Combine like terms.

$$2x = -7$$

$$x = -\frac{7}{2}$$

**YOU TRY IT:**
Solve the following equations.

**18.** $\dfrac{x-2}{5} - \dfrac{x-4}{2} = \dfrac{x+5}{15} + 2$

## Solving a proportion of the form $\frac{a}{x+b} = \frac{c}{x}$

Learning Page We use the **method of cross products**. State the **method of cross products**.

**EXAMPLE:**
Solve $\dfrac{3}{x+4} = -\dfrac{5}{x-1}$ for $x$.

$$\begin{aligned} \frac{3}{x+4} &= \frac{-5}{x-1} \\ 3(x-1) &= -5(x+4) \\ 3x-3 &= -5x-20 \\ 8x &= -17 \\ x &= -\frac{17}{8} \end{aligned}$$

**YOU TRY IT:**

**19.** Solve $\dfrac{2}{x-1} = \dfrac{1}{x+6}$ for $x$.

## Solving for a variable in terms of other variables in a rational equation: Problem type 2

**EXAMPLE:** Solve for $P$.

$$A = P + Prt$$

Factor out $P$ on right.

$$A = P(1+rt)$$

Divide both sides by $1+rt$.

$$\frac{A}{1+rt} = P$$

**YOU TRY IT:** Solve for $d$.

**20.** $S = \frac{n}{2}(a+d)$

## Solving for a variable in terms of other variables in a linear equation with fractions

**EXAMPLE:** Solve for $F$.

$$C = \tfrac{5}{9}(F-32)$$

Multiply both sides by $\frac{9}{5}$

$$\begin{aligned} \tfrac{9}{5}C &= \tfrac{9}{5}\cdot\tfrac{5}{9}(F-32) \\ \tfrac{9}{5}C &= F-32 \\ \tfrac{9}{5}C + 32 &= F \end{aligned}$$

**YOU TRY IT:** Solve for $c$.

**21.** $A = \frac{1}{3}(a-b+c)$

## Solving a rational equation that simplifies to linear: Denominators $a$, $x$ or $ax$

Open the e-book to complete the following.

Read EXAMPLE 5: **Solving a Rational Equation** to complete the following steps.

Solve the equation and check the solution. ____________________

**Solution:**

$$\frac{12}{x} = \frac{6}{2x} + 3$$

__________ $x$ so that __________

$$\underline{\qquad\qquad}\left(\frac{12}{x}\right) = \underline{\qquad\qquad}\left(\frac{6}{2x} + 3\right)$$

Clear __________ by __________ both sides by the __________. Since $x \neq 0$, this will produce an equivalent equation.

$$\frac{2x}{1}\left(\frac{12}{x}\right) = \frac{2x}{1}\left(\frac{6}{2x}\right) + \frac{2x}{1}\left(\frac{3}{1}\right)$$

Apply the __________ property.

$$24 = \underline{\qquad\qquad\qquad}$$

Simplify.

$$\underline{\qquad\qquad} = 6x$$

Subtract 6 from __________ sides.

$$\underline{\qquad\qquad} = x$$

Check: $\frac{12}{3} \stackrel{?}{=} \frac{6}{2(3)} + \frac{3}{1}$

$4 \stackrel{?}{=} 1 + 3 \checkmark$ true

**YOU TRY IT:**

**22.** Solve $\dfrac{2}{3y} + \dfrac{1}{4} = \dfrac{11}{6y} - \dfrac{1}{3}$.

## Solving a rational equation that simplifies to linear: Denominators $ax$ and $bx$

**EXAMPLE:** Solve the equation.

$$\frac{1}{15} + \frac{4}{3y} = \frac{11}{5y}$$

We first note that $y$ cannot be 0.

Multiply both sides of the equation by the LCD.

$$15y \cdot \left(\frac{1}{15} + \frac{4}{3y}\right) = 15y \cdot \frac{11}{5y}$$

$$15y \cdot \frac{1}{15} + 15y \cdot \frac{4}{3y} = 3 \cdot 11$$

$$y + 20 = 33$$

$$y = 13$$

The only restricted value is $y = 0$ so our solution is $y = 13$.

**YOU TRY IT:**

Solve the equation.

**23.** $\frac{1}{3} - \frac{4}{3w} = \frac{7}{w}$

## Solving a rational equation that simplifies to linear: Unlike binomial denominators

Watch the video *Solving a Rational Equation* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Solve.

**EXAMPLE:** Solve the equation.

$$\frac{x}{x-3} = \frac{3}{x-3} - \frac{3}{4}$$

We first note that $x$ cannot be 3.

Multiply both sides of equation by the LCD.

$$4(x-3) \cdot \frac{x}{x-3} = 4(x-3) \cdot \frac{3}{x-3} - 4(x-3) \cdot \frac{3}{4}$$

Simplify.

$$4\cancel{(x-3)} \cdot \frac{x}{\cancel{x-3}} = 4\cancel{(x-3)} \cdot \frac{3}{\cancel{x-3}} - \cancel{4}(x-3) \cdot \frac{3}{\cancel{4}}$$

$$4x = 12 - 3(x-3)$$

$$4x = 12 - 3x + 9$$

$$7x = 21$$

$$x = 3$$

$x = 3$ is a restricted value so there is no solution.

**YOU TRY IT:** Solve the equation.

**24.** $\dfrac{3}{4t+4} + 1 = \dfrac{2t-5}{t+1}$

## Notes from Focus Group:

# Notes from Focus Group:

# Module 2

## Contents



## Weekly Checklist

☐ Complete MALL time.

☐ Work in ALEKS and Notebook at least 3 days a week.

☐ Complete the weekly Module and Notebook pages by the due date.

☐ Attend Focus Group.

☐ Actively participate in Focus Group.

☐ Earn extra credit: Complete 10 topics by ____________________.

## Using $i$ to rewrite square roots of negative numbers

Open the e-book to complete the following.

**The Imaginary Number $i$**

- $i =$__________ and $i^2 =$__________
- If $b$ is a positive real number, then $\sqrt{-b} =$__________.

**EXAMPLE:** Simplify the following.

a) $\sqrt{-36}$

$\sqrt{-36} = i\sqrt{36} = 6i$

b) $\sqrt{-28}$

$\sqrt{-28} = i\sqrt{28} = i\sqrt{2^2 \cdot 7} = 2i\sqrt{7}$

**YOU TRY IT:** Simplify the following.

**25.** $\sqrt{-49}$

**26.** $\sqrt{-48}$

## Solving an equation written in factored form

Open the e-book to complete the following.

**Zero Product Property**

If ______________, then __________ or __________.

To solve a quadratic equation using the zero product property, set one ______________ of the __________________ equal to ______________ and __________________ the other side.

**EXAMPLE:** Solve for $x$.

$$(x-4)(2x+5) = 0$$

Use the **Zero Product Property**

Set each factor equal to 0.

$$\begin{aligned} x - 4 &= 0 \qquad & 2x + 5 &= 0 \\ x &= 4 & 2x &= -5 \\ & & x &= -\tfrac{5}{2} \end{aligned}$$

$$x = 4, -\tfrac{5}{2}$$

**YOU TRY IT:** Solve for $x$.

**27.** $(3x-2)(x+7) = 0$

## Finding the roots of a quadratic equation of the from $ax^2 + bx = 0$

**EXAMPLE:** Solve for $x$.

$$2x^2 + 16x = 0$$

Factor out a $2x$.

$$2x(x+8) = 0$$

Set each factor equal to 0.

$$2x = 0 \qquad x + 8 = 0$$

Solve each equation.

$$x = 0 \qquad x = -8$$

$$x = 0, -8$$

**YOU TRY IT:** Solve for $y$.

**28.** $3y^2 - 27y = 0$

## Finding the roots of a quadratic equation with leading coefficient 1

Watch the video *Introduction to Quadratic Equations and the Zero Product Property* to complete the following.

**Definition of a Quadratic Equation**

Let $a$, $b$, and $c$ represent real numbers where $a \neq 0$. A **quadratic equation** in the variable $x$ is an equation of the form

__________________________

**Zero Product Property**

If ________________, then __________ or __________.

Solve by applying the zero product property.

**EXAMPLE:** Solve for $y$.

$$\begin{aligned} y^2+4y-21&=0 \\ (y+7)(y-3)&=0 \\ y&=-7,3 \end{aligned}$$

**YOU TRY IT:** Solve for $x$.

**29.** $x^2-8x+15=0$

## Solving a quadratic equation needing simplification

**EXAMPLE:** Solve for $x$.

$$2x^2-x-3=(x+1)^2$$

First rewrite so one side is 0.

$$2x^2-x-3-(x+1)^2=0$$

Simplify the $(x+1)^2$.

$$2x^2-x-3-(x^2+2x+1)=0$$

Distribute negative.

$$2x^2-x-3-x^2-2x-1=0$$

Combine like terms.

$$x^2-3x-4=0$$

Factor.

$$(x-4)(x+1)=0$$

$$x=4,-1$$

**YOU TRY IT:** Solve for $x$.

**30.** $2x^2+x=(x-2)^2-10$

## Finding the roots of a quadratic equation with leading coefficient greater than 1

Watch the video *Summary of Techniques to Solve a Quadratic Equation* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

1. Factor and use the zero product rule.

   Example:

   Example:

2. Use the square root property. Complete the square if necessary.

   - Good choice if the equation is in the form $x^2 + bx + c$ where $b$ is even.

   Example:

   - Good choice if the equation is in the form $ax^2 + c = 0$ (middle term is zero).

   Example:

3. Apply the quadratic formula.
   Example:

**EXAMPLE:** Solve for $x$.

$$8x^2 + 22x = -5$$

First rewrite so one side is 0.

$$8x^2 + 22x + 5 = 0$$

Now factor.

$$(4x+1)(2x+5) = 0$$

Set each factor equal to 0.

$$x = -\frac{1}{4}, -\frac{5}{2}$$

**YOU TRY IT:** Solve for $y$.

**31.** $10y^2 + y = 21$

## Writing a quadratic equation given the roots and the leading coefficient

Learning Page We use the ____________________, which states that if ________ is a root of the polynomial $P(x) = 0$, then ________________ is a factor of the polynomial $P(x)$.

**EXAMPLE:** Write the quadratic equation whose roots are $-2$ and 3, and whose leading coefficient is 7.
$-2$ is a root so $x+2$ is a factor and 3 is a root so $x-3$ is a factor.

$$7(x+2)(x-3) = 0$$
$$7(x^2 - 3x + 2x - 6) = 0$$
$$7(x^2 - x - 6) = 0$$
$$7x^2 - 7x - 42 = 0$$

**YOU TRY IT:**

**32.** Write the quadratic equation whose roots are 5 and $-2$, and whose leading coefficient is 3.

## Restriction on a variable in a denominator: Quadratic

Learning Page Division by ________ is not ________________. So the expression is undefined when its ________________________ is 0.

**EXAMPLE:** Find all excluded values for $\dfrac{y+2}{y^2-9}$.

We must exclude values when the denominator is 0. That is when $y^2 - 9 = 0$.

$$y^2 - 9 = 0$$
$$y^2 = 9$$
$$y = 3, -3$$

$\dfrac{y+2}{y^2-9}$ is undefined when $y = 3$ or $y = -3$.

**YOU TRY IT:** Find all excluded values.

**33.** $\dfrac{u+7}{u^2 - 4u + 4}$.

## Solving a quadratic equation using the square root property: Exact answers, advanced

Watch the video *Introduction to the Square Root Property* to complete the following.

**Square Root Property**

If $x^2 = k$, then ____________________.

The solution set is ____________________ or more concisely ____________________.

Solve by applying the square root property.

a.

b.

c.

**EXAMPLE:** Solve for $x$.

$$2(x+1)^2 = 16$$

Solve for the squared term.

$$(x+1)^2 = 8$$

Apply the square root property.

$$x + 1 = \pm\sqrt{8}$$

$$x = -1 \pm 2\sqrt{2}$$

**YOU TRY IT:**

**34.** Solve: $\frac{1}{2}(x-2)^2 - 5 = 0$

## Applying the quadratic formula: Exact answers

Watch the video *Introduction to the Quadratic Formula* to complete the following.

1. Factor and apply the zero product rule.

   This method works if the ____________ expression is ____________.

2. Complete the square and apply the square root property.

   This method works in ______________.

3. Apply the quadratic formula.

   This method works in ______________. **State the quadratic formula.**

   Solve.

**EXAMPLE:** Solve $2x^2 + 6x - 3 = 0$ using the quadratic formula.

$2x^2 + 6x - 3 = 0$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{-6 \pm \sqrt{36 + 24}}{4}$$

$$x = \frac{-6 \pm \sqrt{60}}{4}$$

$$x = \frac{-6 \pm 2\sqrt{15}}{4}$$

$$x = \frac{-3 \pm \sqrt{15}}{2}$$

**YOU TRY IT:** Solve using the quadratic formula.

**35.** $-4x^2 - 12x + 5 = 0$

## Solving a quadratic equation with complex roots

Open the e-book to complete EXAMPLE 7: **Using the Quadratic Formula**.

Solve the equation by applying the quadratic formula. ______________

**Solution:**

$\frac{3}{10}x^2 - \frac{2}{5}x + \frac{7}{10} = 0$ The equation is in the form ______________.

$\_\_\_\_\_\_\left(\frac{3}{10}x^2 - \frac{2}{5}x + \frac{7}{10}\right) = 0\_\_\_\_\_\_$ Multiply by ________ to clear ____________.

$3x^2 - 4x + 7 = 0$

$a = \_\_\_\_, b = \_\_\_\_, c = \_\_\_\_$ Identify the ____________ of $a$, $b$, and $c$.

$x =$ Apply the quadratic formula.

$x =$

$x =$ Simplify.

$x =$ Simplify the ____________.

$x =$ Factor the ______________ and the ______________.

$x =$ Simplify the ____________.

$x = \frac{2}{3} \pm \frac{\sqrt{17}}{3}i$ Write the solutions in standard form, $a + bi$.

**EXAMPLE:**
Solve $5x^2 - 4x + 1 = 0$ using the quadratic formula.

$$5x^2 - 4x + 1 = 0$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(1)}}{2(5)}$$

$$x = \frac{4 \pm \sqrt{-4}}{10}$$

$$x = \frac{4 \pm 2i}{10}$$

$$x = \frac{2}{5} \pm \frac{1}{5}i$$

**YOU TRY IT:** Solving using the quadratic formula.

**36.** $3x^2 + 2x + 1 = 0$

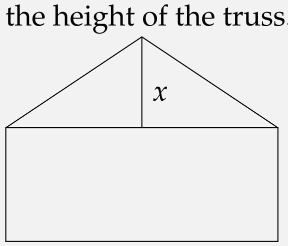
## Solving a word problem using a quadratic equation with rational roots

Watch the video *Using a Quadratic Equation in an Application Involving Area* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Sketch the graph from the video and show all work.

**YOU TRY IT:**

**37.** The front face of a shed is in the shape shown below. The length of the rectangular region is 3 times the height of the truss. The height of the rectangle is 2 ft more than the height of the truss. If the total area of the front face of the shed is 336 $ft^2$, determine the length and width of the rectangular region. Let $x$ be the height of the truss.



## Solving a word problem using a quadratic equation with irrational roots

Open the Instructor Added Resource which will direct you to a video to complete the following.

The population $P$ of a culture of bacteria is given by ____________________, where $t$ is the time in hours since the culture was started. Determine the time(s) at which the population was __________. Round to the nearest hour.

**EXAMPLE:**

If football is kicked straight up with an initial velocity of 128 ft/sec from a height of 5 ft, then its height, $h$, above the earth is a given by $h = -16t^2 + 128t + 5$. When will the football hit the ground?

The football hits the ground when the height is 0, so we set $h = 0$ and solve for $t$.

$-16t^2 + 128t + 5 = 0$

Multiply each by $-1$.

$16t^2 - 128t - 5 = 0$

Use the quadratic formula.

$$x = \frac{128 \pm \sqrt{128^2 - 4(16)(-5)}}{2(16)}$$

$$x = \frac{128 + \sqrt{16704}}{32}, \cancel{\frac{128 - \sqrt{16704}}{32}}$$

There will only be one solution

because cannot have a negative time.

$$x = \frac{128 + \sqrt{16704}}{32} \approx 8.04 \text{ sec}$$

**YOU TRY IT:**

**38.** If football is kicked straight up with an initial velocity of 128 ft/sec from a height of 5 ft, then its height, $h$, above the earth is a given by $h = -16t^2 + 128t + 5$. When will the football be at 37 feet?

## Solving a rational equation that simplifies to quadratic: Binomial denominators, constant numerators

Open the Instructor Added Resource which will direct you to a video to complete the following.

Solve for $x$. $x \neq$ ____________

**YOU TRY IT:** Solve for $x$.

**39.** $\frac{1}{x} + \frac{1}{x-1} = \frac{3}{2}$

## Solving a rational equation that simplifies to quadratic: Binomial denominators and numerators

Watch the video *Solving a Rational Equation that Reduces to a Quadratic* and complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Solve the equation.

**EXAMPLE:** Solve for $x$.

$$\frac{x-3}{x-1} = \frac{x-2}{x-4} - 1$$

$x = 1$ and $x = 4$ are excluded from the solution.

Multiply both sides by the LCD.

$$(x-1)(x-4)\frac{x-3}{x-1} = \left(\frac{x-2}{x-4} - 1\right)(x-1)(x-4)$$

Simplify.

$$\cancel{(x-1)}(x-4)\frac{x-3}{\cancel{x-1}} = \frac{(x-2)(x-1)\cancel{(x-4)}}{\cancel{x-4}} - 1(x-1)(x-4)$$

$$\begin{aligned}
(x-3)(x-4) &= (x-2)(x-1) - (x^2 - 5x + 4) \\
x^2 - 7x + 12 &= x^2 - 3x + 2 - x^2 + 5x - 4 \\
x^2 - 9x + 14 &= 0 \\
(x-7)(x-2) &= 0 \\
x &= 2, 7
\end{aligned}$$

**YOU TRY IT:** Solve for $x$.

**40.** $\dfrac{3x+1}{x+5} = \dfrac{x-1}{x+1} + 2$

## Solving a rational equation that simplifies to quadratic: Proportional form, advanced

Open the Instructor Added Resource which will direct you to a video to complete the following.

Solve for $x$.

**EXAMPLE:** Solve for $x$.

$$\frac{18}{x^2-8x+12} = \frac{-2x}{x-2}$$

Factor the denominator.

$$\frac{18}{(x-2)(x-6)} = \frac{-2x}{x-2}$$

$x = 2$ and $x = 6$ are excluded from the solution.

Multiply both sides by the LCD.

$$(x-2)(x-6)\frac{18}{(x-2)(x-6)} = \frac{-2x}{x-2}(x-2)(x-6)$$

Simplify.

$$\cancel{(x-2)(x-6)}\frac{18}{\cancel{(x-2)(x-6)}} = \frac{-2x}{\cancel{x-2}}\cancel{(x-2)}(x-6)$$

$$\begin{aligned} 18 &= -2x(x-6) \\ 18 &= -2x^2+12x \\ 2x^2-12x+18 &= 0 \\ 2(x^2-6x+9) &= 0 \\ 2(x-3)^2 &= 0 \\ x &= 3 \end{aligned}$$

**YOU TRY IT:** Solve for $y$.

**41.** $\dfrac{2y}{y-6} = \dfrac{12}{y^2-7y+6}$

## Finding a solution to a linear equation in two variables

Learning Page There are ______________________ ordered pairs ______ that are solutions to $Ax + By = C$. To find one, we can choose a value for __________ of the variables and ____________ for the __________ variable.

**EXAMPLE:**
Find an ordered pair that is a solution to $3x + 4y = 8$.

There are infinitely many solutions. We choose a value for either $x$ or $y$, then solve for the other. Several examples are:

- $(0, 2)$
- $(\frac{8}{3}, 0)$
- $(4, -1)$

**YOU TRY IT:**

**42.** Find an ordered pair that is a solution to $6x - 3y = 15$.

## Completing the square

To complete the square of a quadratic expression $x^2 + bx$:

1. Find $\frac{1}{2}$ of the coefficient of $x$.
   $\frac{1}{2} \cdot b$
2. Square the result from 1.
   $\left(\frac{b}{2}\right)^2$
3. Add the result from 2. to the expression and factor.
   $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

*Note*: To complete the square, the leading coefficient must be equal to one.

Watch the video *Introduction to Completing the Square* and complete the following.

Determine the value of $n$ that makes the polynomial a perfect square trinomial. Then factor as the square of a binomial.

a.

b.

c.

## Solving a quadratic equation by completing the square: Exact answers

Watch the video *Solving a Quadratic Equation With Leading Coefficient 1 by Completing the Square* and complete the following.

Solve by completing the square and applying the square root property.

**EXAMPLE:**
Solve $x^2 - 12x + 33 = 0$ by completing the square.

$$x^2 - 12x + 33 = 0$$
$$x^2 - 12x = -33$$
Add $\left(\frac{12}{2}\right)^2$ to each side
$$x^2 - 12x + 36 = -33 + 36$$
Factor the left side.
$$(x-6)^2 = 3$$
$$x - 6 = \pm\sqrt{3}$$
Apply the square root property.
$$x = 6 \pm \sqrt{3}$$

**YOU TRY IT:**

**43.** Solve $x^2 + 2x + 5 = 0$ by completing the square.

# Notes from Focus Group:

## Notes from Focus Group:

# Module 3

## Contents



## Weekly Checklist

☐ Complete MALL time.

☐ Work in ALEKS and Notebook at least 3 days a week.

☐ Complete the weekly Module and Notebook pages by the due date.

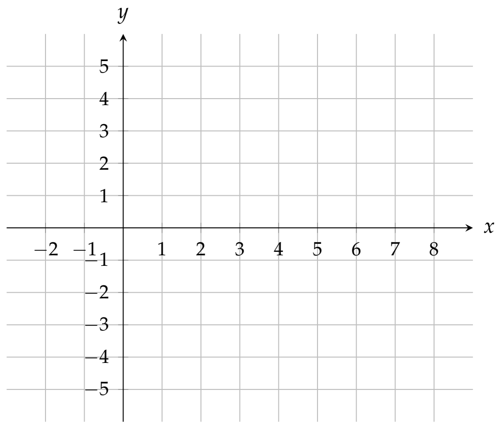
☐ Attend Focus Group.

☐ Actively participate in Focus Group.

☐ Earn extra credit: Complete 10 topics by ____________________.

## Finding the $x$ and $y$ intercepts of the graph of a nonlinear equation

Watch the video *Identifying $x$- and $y$-intercepts* to complete the following.



$x$-intercept(s):

$y$-intercept(s):

**Determining $x$- and $y$-intercepts from an Equation**

Given an equation in $x$ and $y$,

- Find the ____________________ by substituting _____ for _____ in the equation and solving for _____.
- Find the ____________________ by substituting _____ for _____ in the equation and solving for _____.

Determine the $x$- and $y$-intercepts of the graph of the equation. ________________

$x$-intercept(s):

$y$-intercept(s):

**EXAMPLE:**
Find the $x$ and $y$-intercepts of $16x^2 + 25y^2 = 400$.

- Find the $x$-intercepts.

$$\begin{aligned} 16x^2 + 25 \cdot 0^2 &= 400 \\ 16x^2 &= 400 \\ x^2 &= 25 \\ x &= 5, -5 \end{aligned}$$

The $x$-intercepts are $(5, 0)$ and $(-5, 0)$.

- Find the $y$-intercepts.

$$\begin{aligned} 16 \cdot 0^2 + 25y^2 &= 400 \\ 25y^2 &= 400 \\ y^2 &= 16 \\ y &= 4, -4 \end{aligned}$$

The $y$-intercepts are $(0, 4)$ and $(0, -4)$.

**YOU TRY IT:**

**44.** Find the $x$ and $y$-intercepts of $7x^2 + 5y^2 = 35$.

## Finding the $x$ and $y$ intercepts of a line given the equation: Advanced

Learning Page To find the $x$-intercept of a line, ________________________________________.

If the $x$-intercept is $a$, this means the point __________ lies on the line.

To find the $y$-intercept of a line, ________________________________________.

If the $y$-intercept is $b$, this means the point __________ lies on the line.

**EXAMPLE:**
Find the $x$ and $y$-intercept of $4x + 3y = -8$.

a) Find the $x$-intercept. Let $y = 0$.

$$\begin{aligned} 4x + 3(0) &= -8 \\ 4x &= -8 \\ x &= -2 \end{aligned}$$

$(-2, 0)$ is the $x$-intercept.

b) Find the $y$-intercept. Let $x = 0$.

$$\begin{aligned} 4(0) + 3y &= -8 \\ 3y &= -8 \\ y &= -\tfrac{8}{3} \end{aligned}$$

$(0, -\frac{8}{3})$ is the $y$-intercept.

**YOU TRY IT:**

**45.** Find the $x$ and $y$-intercept of $7x - 5y = 3$.

## Finding slope given two points on the line

Watch the video *Determining the Slope of a Line* to complete the following.

slope = ______________________________ = ____________________

$m =$ ____________________

Determine the slope of the line containing the points ____________________ and ____________________

**EXAMPLE:**
Find the slope of the line through $(-3, 5)$ and $(5, -7)$.

$$\begin{aligned} m &= \frac{5-(-7)}{-3-5} \\ &= \frac{5+7}{-8} \\ &= \frac{12}{-8} \\ &= -\tfrac{3}{2} \end{aligned}$$

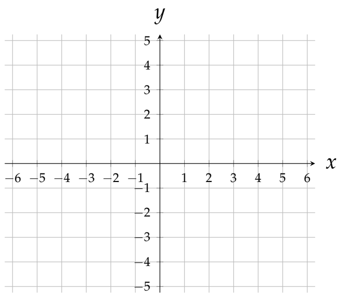
**YOU TRY IT:**

**46.** Find the slope of the line through $(-3, 5)$ and $(6, -1)$.

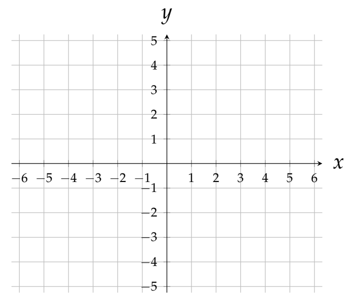
## Finding the slope of horizontal and vertical lines

Watch the video *Investigating Slopes of Horizontal and Vertical Lines* to complete the following.

Sketch in the graphs of the two lines shown in the video.



$m =$ ________________ $=$ ________



$m =$ ________________ $=$ ________________

**EXAMPLE:**

a) Find the slope of the line through $(3, -5)$ and $(3, 1)$.

$$\text{slope} = \frac{-5-1}{3-3} = -\frac{6}{0}$$

The slope is undefined.

b) Find the slope of the line through $(3, 1)$ and $(-2, 1)$.

$$\text{slope} = \frac{1-1}{3-(-2)} = \frac{0}{5} = 0$$

The slope is 0.

**YOU TRY IT:**

**47.** Find the slope of the line through $(4, -7)$ and $(2, -7)$.

## Writing an equation in slope-intercept form given the slope and a point

Watch the video *Using Slope-Intercept Form to Write an Equation of a Line* and complete the following.

1. Use the slope-intercept form to write an equation of the line that passes through ________________ with slope $m =$ ____________.

2. Write the equation using function notation where $y = f(x)$.

**YOU TRY IT:**

**48.** Write the equation of the line with slope $m = \frac{3}{4}$ that passes through $(2, -3)$.

## Writing the equation of the line through two given points

Watch the video *Writing an Equation of the Line Passing Through Two Given Points* and complete the following.

Write an equation of the line that passes through the points ____________ and ____________. Write the answer in slope-intercept form.

**YOU TRY IT:**

**49.** Write the equation of the line through $(2, -4)$ and $(-1, 3)$.

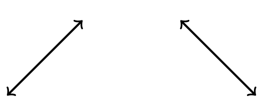
## Writing the equations of vertical and horizontal lines through a given point

Open the e-book to complete the following.

**Linear Equations and Slopes of Lines**

$Ax + By = C$

Slanted line



________ slope ________ slope

$y = k$

Horizontal line



________ slope

$x = k$

Vertical line

________ slope

**YOU TRY IT:**

**50.** Write the equation of the vertical line through $(-4, 3)$

**51.** Write the equation of the horizontal line through $(7, -12)$

## Identifying parallel and perpendicular lines from equations

Learning Page Here are some facts about parallel and perpendicular lines.

**Parallel Lines:**

- Two ____________ lines are parallel if and only if they have the ________________.
- All ____________ lines are parallel to ________________.

  Vertical lines are parallel only to other ________________.

**Perpendicular Lines:**

- Two nonvertical lines are perpendicular if and only if the ________________ is ______.
- All vertical lines are perpendicular to all __________ lines and vice versa.

  Vertical lines are ________________ to horizontal lines and vice versa.

**EXAMPLE:**
Determine if the lines below are parallel, perpendicular, or neither.

$$5y = 2x + 3$$
$$-5y = 3x + 2$$

We first write the lines in slope-intercept form.

$$y = \frac{2}{5}x + \frac{3}{5}$$
$$y = -\frac{3}{5}x + \frac{2}{5}$$

The slope of the first line is $\frac{2}{5}$ and the slope of the second line is $-\frac{3}{5}$. They are not equal so the lines are NOT parallel. $\frac{2}{5} \cdot -\frac{3}{5} \neq -1$ so the lines are NOT perpendicular.

**YOU TRY IT:**

**52.** Determine if the lines below are parallel, perpendicular, or neither.

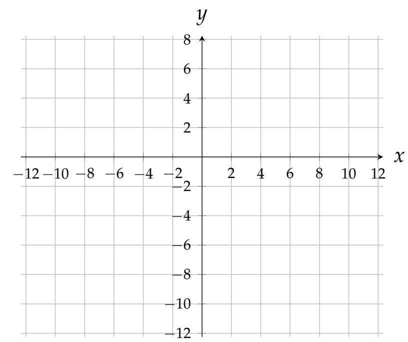
$$6y = 2x + 3$$
$$-2y = 6x + 2$$

## Writing equations of lines parallel and perpendicular to a given line through a point

Watch the video *Writing an Equation of a Line Parallel to Another Line* and complete the following.

Write an equation of the line passing through ________ and parallel to the line ____________.

Pause the video and try graphing the given line and the parallel line yourself.

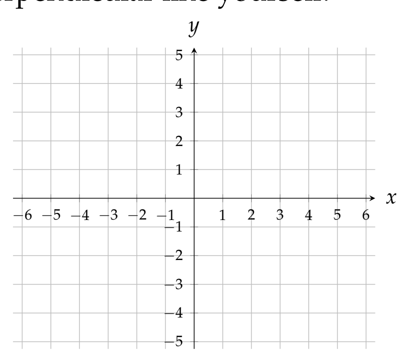



Play the video and check your answers.

Watch the video *Writing an Equation of a Line Perpendicular to Another Line* and complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Write an equation of the line passing through ________ and perpendicular to the line ____________.

Pause the video and try graphing the given line and the perpendicular line yourself.




Play the video and check your answers.

**YOU TRY IT:** Consider the line $4x + 3y = -6$. Find the equation of a line that is:

**53.** perpendicular to $4x + 3y = -6$ and contains $(4, -2)$.

**54.** parallel to $4x + 3y = -6$ and contains $(4, -2)$.

## Graphing a line given its equation in slope-intercept form: Fractional slope

Watch the video *Introduction to Linear Equation in Two Variables* and complete the following.

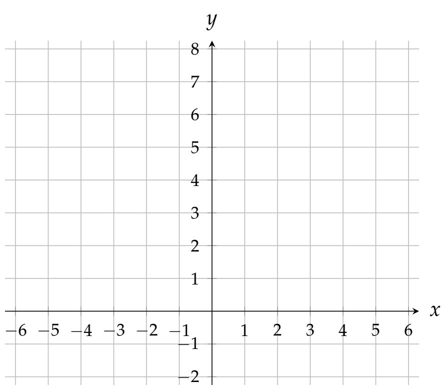
**Linear Equation in Two Variables**

Let $A$, $B$, and $C$ represent real numbers such that $A$ and $B$ are not both zero. A ________________

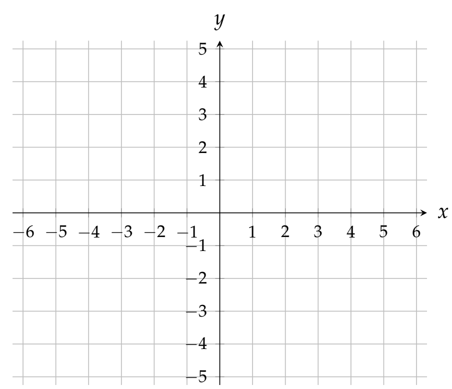
in the variables $x$ and $y$ is an ____________ that can be written in the form:

Graph the equation.

a. ________________

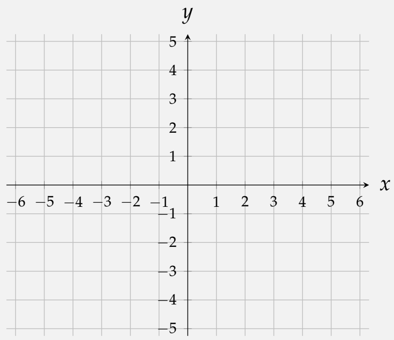



b. ____________



**YOU TRY IT:** Sketch the graph of $y = \frac{2}{3}x + 4$.

**55.**




## Graphing a line given its equation in standard form

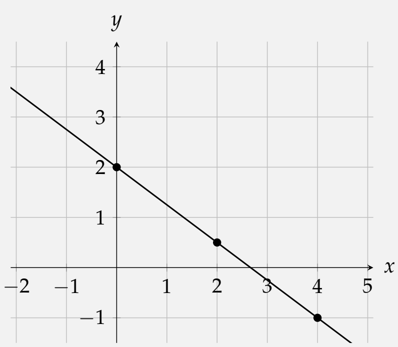
First, solve the equation for ______. Then, choose some ______ values and evaluate.

**EXAMPLE:** Sketch the graph of $3x + 4y = 8$.
Solve for $y$.

$$4y = 8 - 3x$$
$$y = 2 - \frac{3}{4}x$$

Find points that lie on the graph.

| $x$ | $y$ |
|---|---|
| 0 | 2 |
| 2 | $\frac{1}{2}$ |
| 4 | -1 |




**YOU TRY IT:**

**56.** Sketch the graph of $2x - 3y = 6$.

## Graphing a line by first finding its slope and $y$-intercept

Learning Page State the **slope-intercept** equation of a line. ____________________.

The slope is __________.

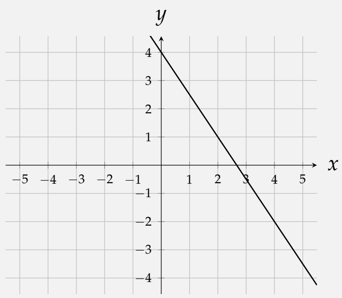
The $y$-intercept is __________.

**EXAMPLE:** Find the slope and $y$-intercept of $3x + 2y = 8$ and sketch the graph.

First write the equation in slope-intercept form.

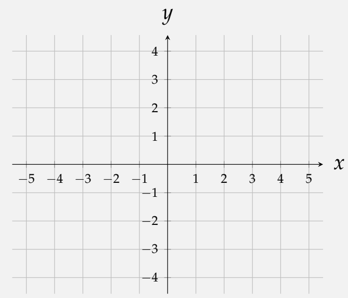
$$\begin{aligned} 3x + 2y &= 8 \\ 2y &= -3x + 8 \\ y &= -\tfrac{3}{2}x + 4 \end{aligned}$$

The slope is $-\frac{3}{2}$ and the $y$-intercept is $(0, 4)$.




**YOU TRY IT:** Find the slope and the $y$-intercept of $2x - 3y = 9$ and sketch the graph.

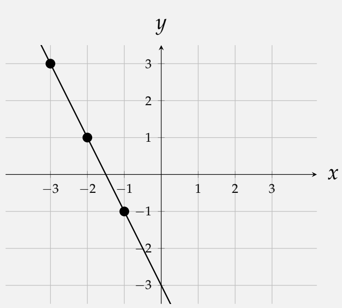
**57.**




## Graphing a line through a given point with a given slope

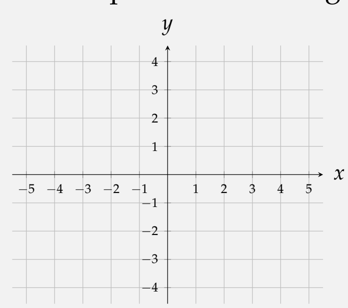
**EXAMPLE:** Graph the line through $(-2, 1)$ with slope $-2$.

- First plot the point $(-2, 1)$.
- The slope gives us the **change in** $y$ over the **change in** $x$ so we
  - plot the point down 2 and right 1 from $(-2, 1)$.
  - Note that we could have also plotted the point up two and left 1 from $(-2, 1)$.
- Connect the dots to graph the line.




**YOU TRY IT:**

**58.** Graph the line through $(1, 2)$ with slope $-\frac{1}{3}$.



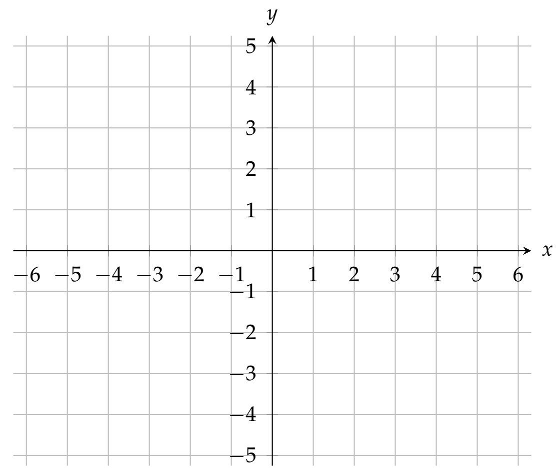
## Graphing a line by first finding its $x$ and $y$-intercepts

Open the Instructor Added Resource which will direct you to a video to complete the following.

Sketch the graph of ____________________ by first finding the $x$ and $y$-intercepts.

$x$-intercept:

$y$-intercept:

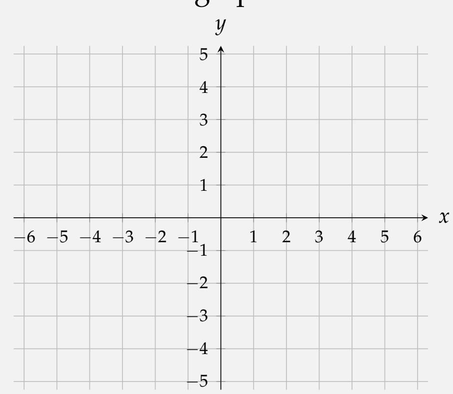



**YOU TRY IT:** Consider the line $3x - 4y = 12$.

**59.** Find the $x$-intercept.

**60.** Find the $y$-intercept.

**61.** Sketch the graph.



## Writing and evaluating a function that models a real-world situation: Advanced

Watch the video *Writing Linear Cost, Revenue, and Profit Functions* and complete the following.

A lawn service company charges __________ for each lawn maintenance call. The fixed monthly cost of __________ includes telephone service and depreciation of equipment. The variable costs include labor, gasoline, and taxes. These amount to __________ per lawn.

a. Write a linear cost function representing the monthly cost $C(x)$ for $x$ maintenance calls.

$$\left(\qquad\right) = \left(\qquad\right) + \left(\qquad\right)$$

b. Write a linear revenue function representing the monthly revenue $R(x)$ for $x$ maintenance calls.

c. Write a linear profit function representing the monthly profit $P(x)$ for $x$ maintenance calls.

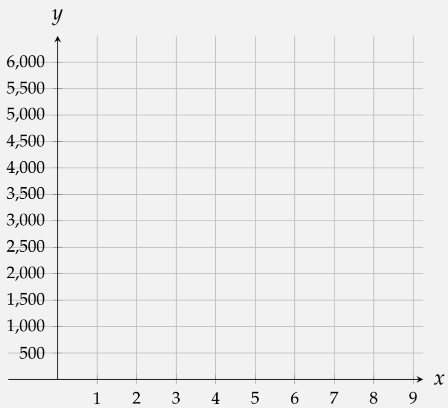
d. Determine the number of calls needed per month for the company to make money.

e. If 42 calls are made for a given month, how much money will the lawn service earn or lose?

## Writing an equation and drawing its graph to model a real-world situation: Advanced

**YOU TRY IT:**

**62.** American Crystal Sugar is going to transport its sugar to market. It will cost \$4350 to rent trucks, and it will cost an additional \$150 for each ton of sugar transported. Let $C$ represent the total cost (in dollars), and let $S$ represent the amount of sugar (in tons) transported. Write an equation relating $C$ to $S$, and then graph the equation.




## Interpreting the parameters of a linear function that models a real-world situation

Open the Instructor Added Resource which will direct you to a video to complete the following.

Jose is driving to Chicago. Let $y$ represent his distance from Chicago (in miles). Let $x$ represent the time he has been driving (in hours). Suppose that $x$ and $y$ are related by the equation ____________________.

a. How far was Jose from Chicago when he began his drive?

b. What is the change in Jose's distance from Chicago for each hour he drives?

This is given by the _______________ of the _______________.

The slope of _________________________ is __________.

This means that for ______________________________ that Jose drives, his ____________________ to Chicago will ____________________ by __________ miles.

**YOU TRY IT:**
Let $y$ represent the total cost of producing a toy. Let $x$ represent the number of toys produced. Suppose that $x$ and $y$ are related by the equation $1100 + 15x = y$.

**63.** What is the change in the total cost for each toy made?

**64.** What is the cost to get started before any toys are made?

## Solving a system of linear equations using substitution

Watch the video *Solving a System of Equations by the Substitution Method* to complete the following.

Solve the system of equations by using the substitution method.

**EXAMPLE:**
Solve the system of equations using substitution.

$$\begin{aligned} 3x - y &= 6 \\ 6x + 5y &= -23 \end{aligned}$$

In the first equation, solve for $y$.

$$\begin{aligned} -y &= -3x + 6 \\ y &= 3x - 6 \end{aligned}$$

Substitute this expression for $y$ into the other equation.

$$\begin{aligned} 6x + 5(3x - 6) &= -23 \\ 6x + 15x - 30 &= -23 \\ 21x &= 7 \\ x &= \frac{1}{3} \end{aligned}$$

We must now find the $y$ value. Either equation may be used.

$$y = 3(\frac{1}{3}) - 6) = 1 - 6 = -5$$

The solution is the ordered pair $\left(\frac{1}{3}, -5\right)$.

**YOU TRY IT:**

**65.** Solve the system of equations using substitution.

$$\begin{aligned} 2x - 3y &= -4 \\ 2x + y &= 4 \end{aligned}$$

## Solving a system of linear equations using elimination with multiplication and addition

Watch the video *Solving a System of Equations Using the Addition Method* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Solve the system by using the addition method.

**EXAMPLE:**
Solve the system of equations using elimination.

$$2x - 3y = -2$$
$$3x - 2y = 12$$

Multiply the first equation by -3 and the second equation by 2.

$$-3(2x - 3y) = -2(-3)$$
$$2(3x - 2y) = 12(2)$$

Simplify the equations. Note that we have a $6x$ in one equation and a $-6x$ in the other.

$$-6x + 9y = 6$$
$$6x - 4y = 24$$

Add the two equations together and solve for $y$.

$$\begin{array}{r} -6x + 9y = 6 \\ 6x - 4y = 24 \\ \hline 5y = 30 \\ y = 6 \end{array}$$

Use one of the equations to solve for $x$.

$$2x - 3(6) = -2$$
$$2x = 16$$
$$x = 8$$

The solution is the ordered pair $(8, 6)$.

**YOU TRY IT:**

**66.** Solve the system of equations using elimination.

$$-2x + 5y = 14$$
$$7x + 6y = -2$$

## Solving a word problem using a system of linear equations of the form $Ax + By = C$

**EXAMPLE:**
Lisa and Tara each get ice cream. Lisa gets 2 scoops of cherry and 1 scoop of mint for a total of 43 grams of fat. Tara has 1 scoop of cherry and 2 scoops of mint for a total of 47 grams of fat. How many grams of fat does 1 scoop of each type of ice cream have?
Let $c$ = grams of fat in cherry and $m$ = grams of fat in mint.

$$2c + m = 43$$
$$c + 2m = 47$$

Multiply the top equation by $-2$.

$$-4c - 2m = -86$$
$$c + 2m = 47$$

Add the two equations together

$$-3c = -42$$
$$c = 14$$

Use $c$ and one of the equations to find $m$.

$$2(14) + m = 43$$
$$m = 43 - 28$$
$$m = 15$$

A scoop of cherry ice cream has 14 grams of fat and a scoop of mint has 15 grams of fat.

**YOU TRY IT:**

**67.** John and Alycia bought school supplies. John spent \$10.65 on 4 notebooks and 5 pens. Alycia spent \$7.50 on 3 notebooks and 3 pens. What is the cost of 1 notebook and what is the cost of 1 pen?

## Notes from Focus Group:

# Notes from Focus Group:

# Module 4-Review

To help you review for your upcoming exam, this module contains all of the topics from the modules since the last exam. Topics that you have already mastered will not appear in your carousel.

☐ Complete this module before you take the ALEKS exam.

☐ Each exam has two parts.

- The ALEKS exam (100 pts)
  - The ALEKS exam must be taken in the MALL.
  - The ALEKS exam is a Comprehensive Knowledge Check.
    - Your score is the number of topics you have mastered out of the number of topics you should have mastered by this point.
  - If you lose topics on your ALEKS exam, your Review Module completion grade will not change.
  - Your scratch work for the ALEKS exam must be numbered and turned in through Blackboard.
- The Written exam (25 pts)
  - Take your written exam in class the day of your focus group.
  - To study for the written exam:
    - Rework your old Focus Group assignments.
    - Rework any topics in ALEKS you may have lost on the ALEKS exam.

| | Score |
|---|---|
| ALEKS Exam | |
| Written Exam | |

# Module 5

## Contents



## Weekly Checklist

☐ Complete MALL time.

☐ Work in ALEKS and Notebook at least 3 days a week.

☐ Complete the weekly Module and Notebook pages by the due date.

☐ Attend Focus Group.

☐ Actively participate in Focus Group.

☐ Earn extra credit: Complete 10 topics by ____________________.

## Solving a linear inequality with multiple occurrences of the variable: Type 1

Open the e-book to complete the following.

**Properties of Inequality**

Let $a$, $b$, and $c$ represent real numbers.

1.

2.

3.

4.

5.

6.

These statements are also true expressed with the symbols ______, ______, and ______.

## Solving a linear inequality with multiple occurrences of the variable: Type 3

Watch the video *Solving a Linear Inequality Involving Fractions* to complete the following.

Solve.

**EXAMPLE:** Solve for $y$.

$$\frac{4}{3}y - \frac{1}{6} \geq \frac{1}{2}y + 3$$

Multiply by the LCD.

$$6(\frac{4}{3}y - \frac{1}{6}) \geq 6(\frac{1}{2}y + 3)$$

$$8y - 1 \geq 3y + 18$$

$$5y \geq 19$$

$$y \geq \frac{19}{5}$$

**YOU TRY IT:** Solve for $x$.

**68.** $\frac{1}{2}x + 2 - \frac{1}{5}x - \frac{3}{5} < -\frac{1}{10}x$

## Introduction to solving an absolute value equation

Watch the video *Introduction to Absolute Value Equations* to complete the following.

$|u| =$ __________ $\longrightarrow$

$|u| =$ __________ $\longrightarrow$

$|u| =$ __________ $\longrightarrow$

$|x + 1| = 3$ Let $k$ represent a real number.

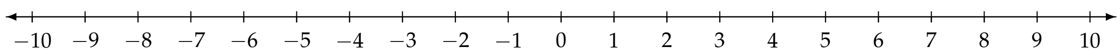
1. If $k > 0$, $|u| = k$ is equivalent to ____________ or ____________.
2. If $k = 0$, $|u| = k$ is equivalent to ____________.
3. If $k < 0$, $|u| = k$ has ________________.

**Continued on the next page**

Open the Instructor Added Resource which will direct you to a video to complete the following.

- How does absolute value relate to distance? ________________________________

- Is the ________________ between $-2$ and 6 ________________ the distance between 6 and $-2$? ________ Sketch the graph from the video.




- The distance between $-2$ and 6 can be written as ________________.

  The distance between 6 and $-2$ can be written as ________________.

- These are both equal to ________ so we can say ____________________

- In general, ____________________.

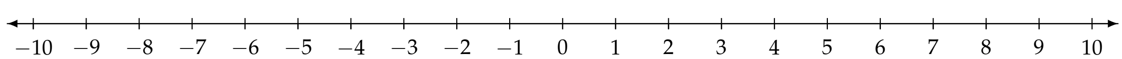
**Absolute Value as Distance**

$$|x - a| = b$$

means that the ________________ between ____ and ____ is ____.

Solve ____________. $\Longrightarrow$ ________________.

The ________________ between ____ and ____ is equal to ____. So $x =$ ________




**YOU TRY IT:**

**69.** Solve $|x| = 7$

## Solving an absolute value equation: Problem type 2

Watch the video *Solving Absolute Value Equations* to complete the following.

Solve the equations.

a.

b.

c.

Let $k$ represent a real number.

1. If __________, $|u| = k$ is equivalent to __________ or __________.
2. If __________, $|u| = k$ is equivalent to __________.
3. If __________, $|u| = k$ has ____________________.

**YOU TRY IT:** Solve for $x$.

**70.** $|x + 7| = 3$

## Solving an absolute value equation: Problem type 4

Watch the video *Solving an Absolute Value Equation* to complete the following.

Solve the equation.

**EXAMPLE:**
Solve the following equations.

a) $2|x+5|-10=0$
First isolate $|x+5|$.

$$\begin{aligned} 2|x+5|-10 &= 0 \\ 2|x+5| &= 10 \\ |x+5| &= 5 \end{aligned}$$

Write the equivalent statements without absolute value.

$$\begin{array}{rcr} x+5=5 & \text{or} & x+5=-5 \\ x=0 & \text{or} & x=-10 \end{array}$$

So $x=0,-10$

b) $6+4|x+3|=2$

$$\begin{aligned} 4|x+3| &= -4 \\ |x+3| &= -1 \end{aligned}$$

No solution.

**YOU TRY IT:**
Solve the following equations.

**71.** $-7|x-5|+4=9$

**72.** $-3|x-7|+5=-1$

## Writing an inequality for a real-world situation

Learning Page Here is how some English sentences can be written as inequalities.

| English sentence | Inequality |
|---|---|
| $A$ is less than $B$ | |
| $A$ is less than or equal to $B$<br>$A$ is at most $B$<br>$A$ is no more than $B$ | |
| $A$ is more than $B$ | |
| $A$ is more than or equal to $B$<br>$A$ is at least $B$<br>$A$ is no less than $B$ | |

**EXAMPLE:** Write an inequality to represent the situation.

The distance to the nearest bathroom is less than 25 yards.

We will use $d$ to represent distance (in yards). The words "less than" indicate we should use the $<$ symbol.

$$d < 25$$

**YOU TRY IT:** Write an inequality to represent the situation.

**73.** The maximum capacity of the scale is no more than 500 pounds.

## Set builder and interval notation

Learning Page The set $\{x \mid$ ______________ $\}$ is ______________________________.

This set is an ____________. It is written using ____________.

We can specify an interval using ______________, a ____________, or ______________

as shown below. Complete the chart.

| Set Builder Notation | Graph | Interval Notation |
|---|---|---|
| $\{x \mid a \leq x \leq b\}$ | | |
| | | $(a, b)$ |
| $\{x \mid a < x \leq b\}$ | | |
| $\{x \mid a \leq x < b\}$ | | |
| | | $[a, \infty)$ |
| $\{x \mid x > a\}$ | | |
| | | $(-\infty, a]$ |
| $\{x \mid x < a\}$ | | |

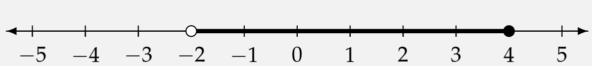
A solid dot shows an endpoint that ____________________.

In interval notation, this is shown using ____________.

A hollow dot shows an endpoint that ____________________.

In interval notation, this is shown using ____________.

**EXAMPLE:**
Given the set $\{x \mid -2 < x \leq 4\}$, graph the set and write the interval notation.




$(-2, 4]$

**YOU TRY IT:**

**74.** Given the set $\{x \mid x \geq -3\}$, graph the set and write the interval notation.

## Union and intersection of intervals

Open the Instructor Added Resource which will direct you to a video to complete the following.

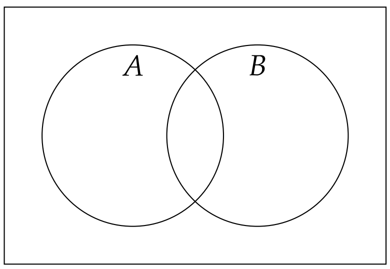
### Union and Intersection of Sets

The **union** of sets $A$ and $B$, denoted __________,

is the set of elements that belong to set $A$ ______

to set $B$ ____________________________.

Shade in $A \cup B$.

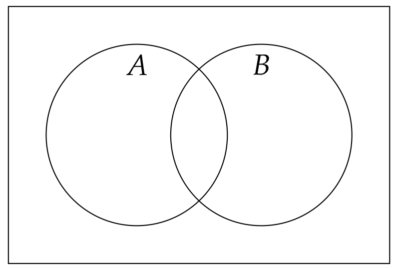



The **intersection** of sets $A$ and $B$,

denoted __________, is the set of elements

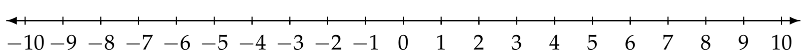
____________________________________.

Shade in $A \cap B$.

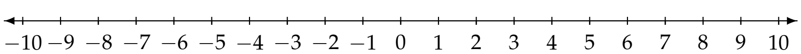



Given the sets $A = \{x \mid x > -6\}$, $B = \{x \mid \ \leq 3\}$, and $C = \{x \mid x \geq 7\}$ find the following. Write your answer in interval notation.

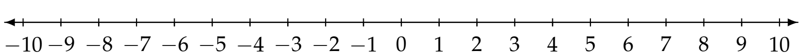
Set A:




Set B:




Set C:




a. $A \cup B =$

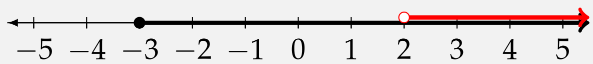
b. $A \cap B =$

c. $B \cap C =$

d. $B \cup C =$

**EXAMPLE:**
Given $A = \{x \mid x > 2\}$ and
$B = \{x \mid x \geq -3\}$.
Find the following.

a) $A \cup B$
We begin by sketching both graphs.




We want values in either of the two intervals.

$A \cup B = [-3, \infty)$

b) $A \cap B = (2, \infty)$
We want the overlap of the two intervals.

**YOU TRY IT:**
Given $D = \{x \mid x \leq 2\}$ and
$E = \{x \mid x > 5\}$. Find the following.

**75.** $D \cap E$

**76.** $D \cup E$

## Solving a radical equation that simplifies to a linear equation: One radical, advanced

Open the e-book to complete the following.

**Solving a Radical Equation**

**Step 1**

**Step 2**

**Step 3**

**Step 4**

In solving radical equations, ______________________________ potentially arise when both sides of the equation are raised to an even power. Therefore, an equation with only __________________ roots will not have extraneous solutions. However, it is still recommended that all potential solutions ____________________.

**EXAMPLE:** Solve for $y$.

$$\sqrt{y+8}+2=4$$

Isolate the radical.

$$\sqrt{y+8}=2$$

Square both sides.

$$(\sqrt{y+8})^2=(2)^2$$

Simplify.

$$y+8=4$$

$$y=-4$$

Check the solution.

$$\sqrt{-4+8}+2\stackrel{?}{=}4$$

$$\sqrt{4}+2\stackrel{?}{=}4$$

$$4=4$$

$y=-4$ is a solution.

**YOU TRY IT:** Solve for $x$.

**77.** $\sqrt{2x+29}+3=1$

## Solving a radical equation that simplifies to a quadratic equation: One radical, advanced

Watch the video *Solving a Radical Equation in which Squaring a Binomial is Required* to complete the following.

Solve the equation.

**EXAMPLE:** Solve for $y$.

$$\begin{aligned}\sqrt{y+18}+2&=y\\ \sqrt{y+18}&=y-2\\ (\sqrt{y+18})^2&=(y-2)^2\\ y+18&=y^2-4y+4\\ 0&=y^2-5y-14\\ 0&=(y-7)(y+2)\\ y&=-2,7\end{aligned}$$

Check the solutions.

$$\begin{aligned}\sqrt{-2+18}+2&\stackrel{?}{=}-2\\ \sqrt{16}+2&\stackrel{?}{=}-2\\ 4+2&\stackrel{?}{=}-2\\ 6&\neq-2\end{aligned} \qquad \begin{aligned}\sqrt{7+18}+2&\stackrel{?}{=}7\\ \sqrt{25}+2&\stackrel{?}{=}7\\ 5+2&\stackrel{?}{=}7\\ 7&=7\end{aligned}$$

$y=7$ is a solution.

**YOU TRY IT:** Solve for $x$.

**78.** $\sqrt{2x+29}+3=x$

## Word problem involving radical equations: Advanced

**EXAMPLE:**
The distance $d$ (in miles) that an observer can see on a clear day is approximated by $d=\frac{49}{40}\sqrt{h}$, where $h$ is the height of the observer in feet. If Rita can see 24.5 mi, how far above ground is her eye level?

$d=24.5$ which can also be written as $d=\frac{49}{2}$. We substitute this into the given equation and solve for $h$.

$$\frac{49}{2}=\frac{49}{40}\sqrt{h}$$

Multiply both sides by $\dfrac{40}{49}$

$$\frac{40}{49}\cdot\frac{49}{2}=\frac{40}{49}\cdot\frac{49}{40}\sqrt{h}$$

$$20=\sqrt{h}$$

Square both sides.

$$400\text{ feet}=h$$

**YOU TRY IT:**

**79.** If an object is dropped from a height of $h$ meters, the velocity $v$ (in m/sec) at impact is given by $v=\sqrt{19.6h}$. Determine the impact velocity for an object dropped from a height of 10 m.

## Solving an equation with exponent $\frac{1}{a}$: Problem type 1

Open the Instructor Added Resource which will direct you to a video to complete the following.

a. $(x+5)^{1/3} = -4$

Check the solution.

b. $(x-1)^{1/4} = 3$

Check the solution.

c. $(x-2)^{1/4} = -5$

Check the solution.

**EXAMPLE:** Solve for $x$.

$$\sqrt[3]{2x-5} = -3$$

Cube both sides.

$$(\sqrt[3]{2x-5})^3 = (-3)^3$$

Simplify.

$$2x - 5 = -27$$
$$2x = -22$$
$$x = -11$$

Check the solution.

$$\sqrt[3]{2(-11)-5} \stackrel{?}{=} -3$$
$$\sqrt[3]{-27} \stackrel{?}{=} -3$$
$$-3 = -3$$

**YOU TRY IT:** Solve for $x$.

**80.** $\sqrt[5]{4x+8} = 2$

## Solving an equation using the odd-root property: Problem type 2

Open the Instructor Added Resource which will direct you to a video to complete the following.

Solve for $x$.

**YOU TRY IT:** Solve for $x$.

**81.** $\frac{1}{2}(x+5)^3 - 64 = 0$

## Identifying functions from relations

Watch the video *Determining Whether a Relation Defines $y$ as a Function of $x$* to complete the following.

**Definition of a Function**

Given a ____________ in $x$ and $y$, we say that ______________________ if for each ________________ in the domain, there is ________________ value of $y$ in the ____________.

Determine whether the relation defines $y$ as a function of $x$.

a.

b.

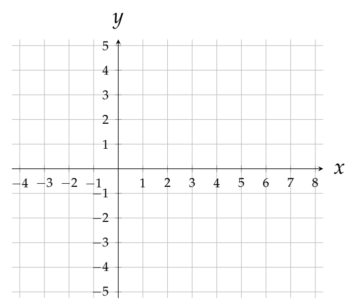
**YOU TRY IT:**
For each relation, determine whether or not it is a function.

**82.** $\{(2,3),(-5,1),(0,3),(5,-4)\}$.

**83.** $\{(1,-2),(-7,3),(1,5),(0,8)\}$.

## Vertical line test

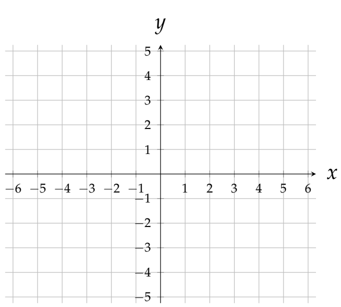
Watch the video *Introduction to the Vertical Line Test* to complete the following.

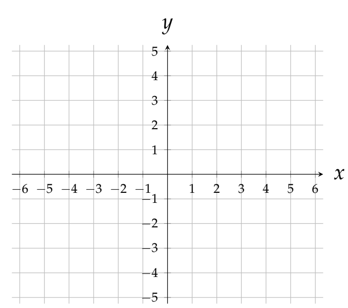



**Using the Vertical Line Test**

Consider a relation defined by a set of points $(x, y)$ graphed on a rectangular coordinate system. The graph defines $y$ as a function of $x$ if ____________ vertical line intersects the graph in ______________________________.

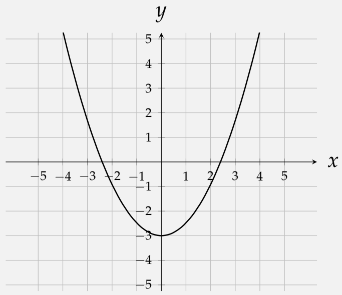
Sketch the graphs from the video below and state if the graph defines $y$ as a function of $x$.



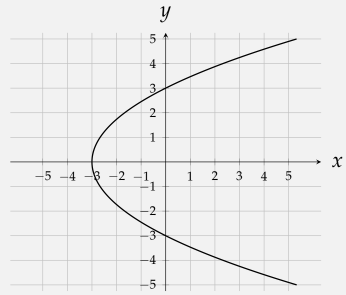



**YOU TRY IT:** For each relation, determine whether or not it is a function.

**84.**




**85.**




## Evaluating a rational function: Problem type 2

Watch the video *Introduction to Function Notation* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

$y =$ ________________

__________ $=$ ________________

Evaluate the function for the given values of $x$.

a. $f(-2)$

b. $f(-1)$

c. $f(0)$

d. $f(1)$

e. $f(-1)$

**EXAMPLE:**
Given $f(x) = \dfrac{x-5}{x^2+x-12}$, evaluate $f(4)$.

We will substitute 4 into our expression for $x$.

$$\begin{aligned} f(4) &= \frac{4-5}{4^2+4-12} \\ &= \frac{-1}{16-8} \\ &= -\frac{1}{8} \end{aligned}$$

**YOU TRY IT:**

**86.** Given $f(x) = \dfrac{x-2}{x^2-2x-15}$, evaluate $f(-4)$.

## Evaluating a function: Absolute value, rational, radical

**EXAMPLE:**
Given $f(x) = 3x^2 - 4x + 7$ and
$g(x) = |1-4x|$, find the following.

a) $f(-2)$

$$\begin{aligned} f(-2) &= 3(-2)^2 - 4(-2) + 7) \\ &= 3(4) + 8 + 7 = 27 \end{aligned}$$

b) $g(6)$

$$\begin{aligned} g(6) &= |1-4(6)| \\ &= |1-24| = |-23| = 23 \end{aligned}$$

**YOU TRY IT:**
Given $f(x) = 3x^2 - 4x + 7$ and
$g(x) = |1-4x|$, find the following.

**87.** $g(-4)$

**88.** $f(3)$

## Evaluating a piecewise-defined function

Watch the video *Interpreting a Piecewise-Defined Function* to complete the following.

Evaluate the function for the given values of $x$.

$$g(x) = \begin{cases} \underline{\qquad\qquad\qquad} & \text{for } x \le -2 \\ \underline{\qquad\qquad\qquad} & \text{for } -2 < x < 3 \\ \underline{\qquad\qquad\qquad} & \text{for } x \ge 3 \end{cases}$$

Pause the video and try these yourself.

a. $g(-3) =$

b. $g(3) =$

c. $g(-2) =$

d. $g(0) =$

e. $g(4) =$

Play the video and check your answers.

## Evaluating a cube root function

Complete the chart below of perfect cubes.

| $x$ | $x^3$ | $x$ | $x^3$ |
|---|---|---|---|
| 1 | | 6 | |
| 2 | | 7 | |
| 3 | | 8 | |
| 4 | | 9 | |
| 5 | | 10 | |

**EXAMPLE:**
Given $f(x) = \sqrt[3]{4x+7}$ find $f(-2)$.

$$\begin{aligned} f(-2) &= \sqrt[3]{4(-2)+7} \\ &= \sqrt[3]{-1} = -1 \end{aligned}$$

**YOU TRY IT:**

**89.** Given $f(x) = \sqrt[3]{4x+7}$ find $f(5)$.

## Table for a square root function

Learning Page

The table gives ____________ $x$ and asks that we find the corresponding ____________ __________.

Compete the table below from the Learning Page.

| $x$ | **Evaluate** $f(x) =$ ________ | $f(x)$ |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

## Finding the total cost including tax or markup

Open the Instructor Added Resource which will direct you to a video to complete the following.

The wholesale price for a paperback book is ______________. A store marks up the wholesale price by ______________. Find the price of the book in the store.

Sonia went to Target and bought a shirt that cost ______________ including ______________ sales tax. What was the price of the shirt?

**YOU TRY IT:**

**90.** A laptop has a listed price of $499 before tax. If the sales tax rate is 6.5%, find the total cost of the laptop with sales tax included.

## Finding the original price given the sale price and percent discount

Open the Instructor Added Resource which will direct you to a video to complete the following.

Today only, a table is being sold for ______________. This is ______________ of its regular price. What was the price yesterday?

**EXAMPLE:** A chair is on sale this week for \$217. The sign says this is a 38% discount from the original price. What was the original price?

Solution: Let $x =$ the original price.
Original price $-$ Discount amount $=$ Sale price

$$\begin{aligned} x - 0.38x &= 217 \\ 0.62x &= 217 \\ x &= 350 \end{aligned}$$

The original price was \$350.

**YOU TRY IT:**

**91.** Today only, a phone is being sold at a 76% discount. The sale price is \$158.40. What was the price yesterday?

Notes from Focus Group:

## Notes from Focus Group:

# Module 6

## Contents



## Weekly Checklist

☐ Complete MALL time.

☐ Work in ALEKS and Notebook at least 3 days a week.

☐ Complete the weekly Module and Notebook pages by the due date.

☐ Attend Focus Group.

☐ Actively participate in Focus Group.

☐ Earn extra credit: Complete 10 topics by ____________.

## Determining whether an equation defines a function: Basic

Watch the video *Determining if a Relation Defines y as a Function of x* to complete the following.

Determine if the equation defines $y$ as a function of $x$.

a.

b.

c.

## Variable expressions as inputs of functions: Problem type 1

Watch the video *Evaluating a Function* to complete the following.

Given ____________________, evaluate ____________________.

**EXAMPLE:**
Given $g(x) = \sqrt{1-4x^2}$, find $g(3x)$.

$$\begin{aligned} g(3x) &= \sqrt{1-4(3x)^2} \\ &= \sqrt{1-4(9x^2)} \\ &= \sqrt{1-36x^2} \end{aligned}$$

**YOU TRY IT:**

**92.** Given $f(x) = 3x^2 - 4x + 7$, find $f(5x)$.

## Variable expressions as inputs of functions: Problem type 2

If you have not already done so, watch the video *Evaluating a Function* and take notes in the video box for the previous topic **Variable expressions as inputs of functions: Problem type 1**.

**EXAMPLE:**
Given $f(x) = 3x^2 - 4x + 7$, find $f(x-2)$.

We substitute $x - 2$ into the expression for $x$.

$f(x-2) = 3(x-2)^2 - 4(x-2) + 7$

FOIL and distribute.

$= 3(x^2 - 4x + 4) - 4x + 8 + 7$

Distribute and simplify.

$= 3x^2 - 12x + 12 - 4x + 15$

$= 3x^2 - 16x + 27$

**YOU TRY IT:**

**93.** Given $g(x) = \sqrt{1-4x}$, find $g(x^2-4)$.

## Domain and range from ordered pairs

Learning Page

The ______________ of a relation is the set of all ____________________________ in the ordered pairs.

The ______________ of a relation is the set of all ____________________________ in the ordered pairs.

**YOU TRY IT:**

**94.** Find the domain and range of the relation $S = \{(2,3), (-5,1), (0,3), (5,-4)\}$.

## Domain of a rational function: Excluded values

Learning Page The fraction ________ have a ____________ of ______.

**YOU TRY IT:**

**95.** Find all values of $x$ that are NOT in the domain of $f(x) = \dfrac{x+4}{x^2-9}$

## Domain of a rational function: Interval notation

Learning Page

The domain of any rational function is the set of $x$ for which the ________________.

There are ______________________ on the domain of a rational function.

**EXAMPLE:**
Find the domain of $f(x) = \dfrac{x-5}{x^2+x-12}$.

We must determine where the denominator is zero. $x^2+x-12 = (x+4)(x-3) = 0$. So $x = 3, -4$. These are the values we want to exclude from the domain.

Domain: $(-\infty, -4) \cup (-4, 3) \cup (3, \infty)$

**YOU TRY IT:**

**96.** Find the domain of $f(x) = \dfrac{x-2}{x^2-2x-15}$.

## Domain of a square root function: Advanced

Open the e-book to complete the following.

**Guidelines to Find Domain of a Function**

To determine the implied domain of a function defined by $y = f(x)$,

- Exclude values of $x$ that make the ________ of a ____________________.
- Exclude values of $x$ that make the ____________________ within an even-indexed root.

Read **EXAMPLE 9 c.** to complete the following.

$h(t) =$ __________ The ____________ is restricted to the ________ numbers that

__________ $\geq 0$ make the radicand ______________ or ________ to _____.

$-t \geq$ _______ Divide by _______ and __________ the inequality sign.

$t$__________

Domain: __________

**EXAMPLE:**
Find the domain of $g(x) = \sqrt{5x-8}$.

We must determine where $5x - 8$ is greater than or equal to zero.

$$5x - 8 \geq 0$$
$$5x \geq 8$$
$$x \geq \frac{8}{5}$$

So the domain is $[\frac{8}{5}, \infty)$

**YOU TRY IT:**

**97.** Find the domain of $h(x) = \sqrt{4 - 7x}$.

## Finding the domain of a fractional function involving radicals

Watch the video *Determining Domain and Range of a Function from its Equation* to complete the following.

Write the domain of the function in interval notation.

a.

b.

c.

**EXAMPLE:** Find the domain of the function.

$$f(x) = \frac{\sqrt{3-x}}{x-1}$$

We must consider two parts.

- We may not have a zero in the denominator, so

$$x - 1 \neq 0$$
$$x \neq 1$$

- We also must have 0 or a positive value under the square root.

$$3 - x \geq 0$$
$$-x \geq -3$$
$$x \leq 3$$

The domain is the intersection of these two sets. In interval notation: $(-\infty, 1) \cup (1, 3]$

**YOU TRY IT:** Find the domain of the function.

**98.** $g(x) = \dfrac{4-2x}{\sqrt{9-7x}}$

## Domain and range of a linear function that models a real-world situation

Learning Page

- **Description of values for the domain:**

The domain of a function is the ______________________________.

- **Description of values for the range:**

The range of a function is the ______________________________.

To find the range, let's look at the ____________________ for some values of the ____________________.

**EXAMPLE:**
The Perfect Pickle delivers pickles to its customers. Let $C$ be the total cost to transport the pickles, in dollars. Let $P$ be the amount of pickles transported in pounds. The company can transport up to 30 pounds of pickles. Suppose that $C$=130P+1500 gives $C$ as a function of $P$. Describe the domain and range in words and determine the domain and range.

**Domain:** The domain will be the amount of pickles transported in pounds.
The domain is $[0, 30]$.

- The amount of pickles cannot be negative so the domain must be greater than or equal to 0.
- The company cannot transport more than 30 pounds of pickles so the domain must be less than or equal to 30.
- The amount of pickles could be any amount between 0 and 30.

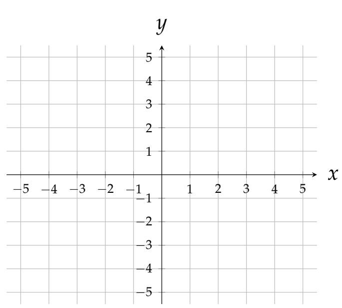
**Range:** The range will be the cost to transport the pickles in dollars.
The range is $[1500, 5400]$.

- What would the cost be if 0 pounds of pickles were transported? $C = 1500$.
- What would the cost be if 30 pound of pickles were transported? $C = 130(30) + 1500 = 5400$
- The cost to transport any other amount of pickles will be in between \$1500 and \$5400.

## Domain and range from the graph of a continuous function

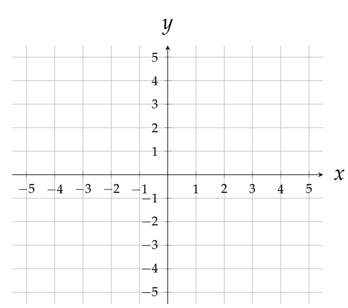
Watch the video *Determining Domain and Range of a Function from its Graph* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Determine the domain and range.



Domain:

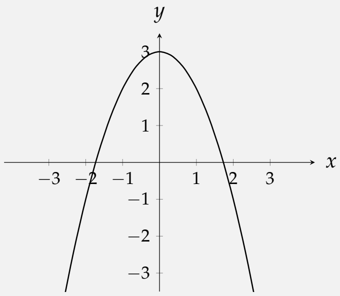
Range:



Domain:

Range:

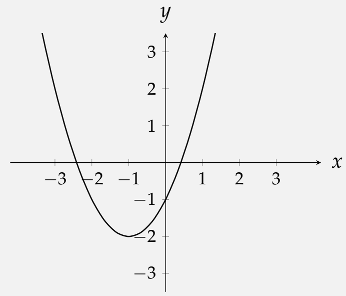
**EXAMPLE:**
Find the domain and range of the function from the graph.



Domain: $(-\infty, \infty)$
Range: $(-\infty, 3]$

**YOU TRY IT:**

**99.** Find the domain and range of the function from the graph.



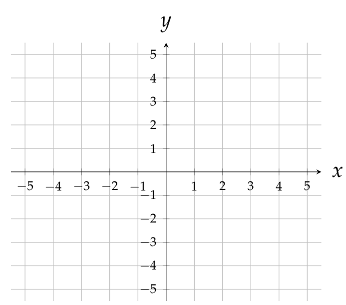
## Domain and range from the graph of a piecewise function

Watch the video *Interpreting Function Values from the Graph* to complete the following.

a. Determine $f(-2)$

b. Determine $f(3)$

c. Find $x$ for which $f(x) = -1$.



d. Find $x$ for which $f(x) = -4$.

e. Determine the $x$-intercept(s).

f. Determine the $y$-intercept.

g. Determine the domain.

h. Determine the range.

## Finding domain and range from a linear graph in context

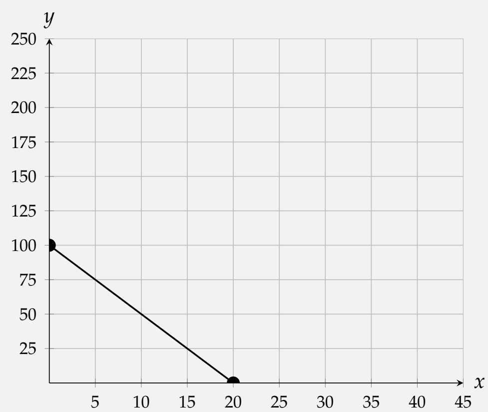
Learning Page

The __________ is the set of all the numbers that appear as ____________________ of __________ on the graph.

The __________ is the set of all the numbers that appear as ____________________ of __________ on the graph.

**YOU TRY IT:**

**100.** Amir drained an aquarium. He took 20 minutes. The graph shows the amount of water (in liters) in the aquarium versus time (in minutes). Find the domain and the range of the function shown.




## Finding inputs and outputs of a function from its graph

Learning Page

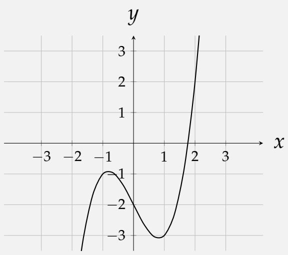
Each point on the graph of a function $f$ can be written as an ______________________________.

For each point $(x, y)$ on the __________________, the $x$ coordinate gives an __________________ of the function.

The $y$ coordinate gives the corresponding __________________. That is ____________________

The video *Interpreting Function Values from the Graph* may also be helpful. You may find space to take notes under the topic **Domain and range from the graph of a piecewise function**.

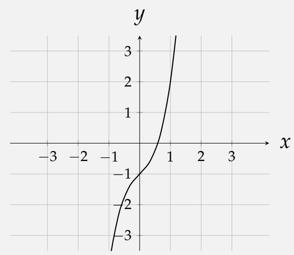
**EXAMPLE:**
Use the graph to find the following.




a) $f(2)$
We see the point $(2,2)$ on the graph, so $f(2) = 2$.

b) One value of $x$ for which $f(x) = -2$
From the graph we see that $f(0) = -2$ so $x = 0$. There are also two other values of $x$ where $f(x) = -2$.

**YOU TRY IT:**
Use the graph to find the following.




**101.** $g(1)$

**102.** One value of $x$ for which $g(x) = -1$

## Finding inputs and outputs of a two-step function that models a real-world situation: Function notation

**EXAMPLE:**
A crew can lay 5 miles of track each day. They need to lay 175 miles of track. The length, $L$, in miles, that is left to lay after $d$ days is given by the function $L(d) = 175 - 5d$.

a. How many miles of track does the crew have left to lay after 12 days?

We want to substitute 12 in for $d$ to find $L(12)$.

$$\begin{aligned} L(12) &= 175 - 5(12) \\ &= 175 - 60 \\ &= 115 \text{ miles} \end{aligned}$$

b. How many days will it take the crew to lay all of the track?

We want to know when $L(d) = 0$.

$$\begin{aligned} 175 - 5d &= 0 \\ -5d &= -175 \\ d &= 35 \text{ days} \end{aligned}$$

**YOU TRY IT:**
Steve wants to save \$700 to buy a computer. He saves \$18 each week. The amount $A$, in dollars he still needs after $w$ weeks is given by the function $A(w) = 700 - 18w$.

**103.** How much money does Steve still need after 5 weeks?

**104.** If Steve still needs \$394, how many weeks has he been saving?

## Finding the average rate of change of a function

Watch the video *Determining Average Rate of Change* to complete the following.

Determine the average rate of change of the function on the given interval.

$f(x) =$ ____________ $m =$ ____________

a. ____________

b. ____________

c. ____________

**EXAMPLE:**
Find the average rate of change of $f(x) = x^2 + x - 4$ from $x = 1$ to $x = 3$.

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(3) - f(1)}{3 - 1}$$
$$= \frac{(3^2 + 3 - 4) - (1^2 + 1 - 4)}{2}$$
$$= \frac{8 - (-2)}{2} = 5$$

**YOU TRY IT:**

**105.** Find the average rate of change of $f(x) = 3 - 2x - x^2$ from $x = -1$ to $x = 2$.

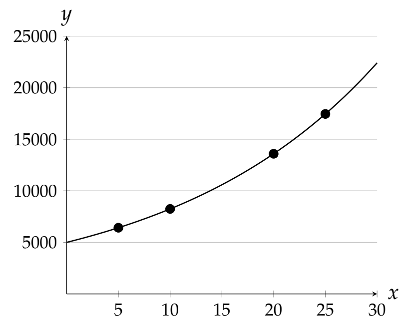
## Finding the average rate of change of a function given its graph

Watch the video *Determining Average Rate of Change 1* to complete the following.

The function given by $y = f(x)$ shows the value of __________ invested at __________ interested compounded continuously after $x$ years.

a. Find the average amount earned per year between the __________ year and __________ year.

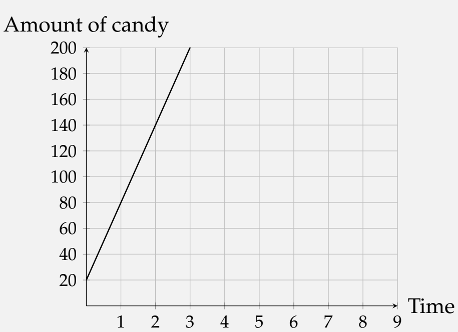
$m =$ ________________________




b. Find the average amount earned per year between the __________ year and __________ year.

## Finding the initial amount and rate of change given a graph of a linear function

**YOU TRY IT:**
At a candy factory, a machine is putting candy into a container. The graph shows the amount of candy, in pounds, in the container versus time in minutes.




**106.** What is the amount of candy in the container at 0 minutes?

**107.** Describe how the time and amount of candy are related.

## Finding the initial amount and rate of change given a table for a linear function

Open the Instructor Added Resource which will direct you to a video to complete the following.

Sergio is adding water to a swimming pool at a constant rate. The table below shows the amount of water in the pool after different amounts of time.

| Time (minutes) | 6 | 9 | 12 | 15 |
|---|---|---|---|---|
| Water (gallons) | 118 | 142 | 166 | 190 |

1. How much water was already in the pool when Sergio started adding water?

   From the table we have the points ________________ and ________________.

   Slope:

   To find how much water was in the pool when Sergio started adding water, we substitute

   ____________ into our equation.

   $y =$ ________________

   There were ________ gallons of water in the pool when Sergio started adding water.

2. As time increases is the amount of water in the pool increasing or decreasing? At what rate?

## Word problem involving average rate of change

Learning Page The average rate of change is the ______________ of the line passing through

____________________ and ____________________.

**EXAMPLE:**
Travis is cooking a beef roast. The table below gives the temperature $R(t)$ of the roast in degrees Celsius, at a few times $t$ in minutes after he removed it from the oven. Find the average rate of change for the temperature from 10 to 50 minutes.

| Time $t$ | Temperature $R(t)$ |
|---|---|
| 0 | 226.6 |
| 10 | 205.6 |
| 30 | 157.6 |
| 50 | 119.6 |
| 70 | 61.6 |

The average rate of change over $[x_1, x_2]$ is given by the formula below. In this problem $x_2 = 50$ and $x_1 = 10$. We find the values of the function from the table above.

$$\begin{aligned}\frac{f(x_2) - f(x_1)}{x_2 - x_1} &= \frac{f(50) - f(10)}{50 - 10} \\ &= \frac{119.6 - 205.6}{40} \\ &= \frac{-86}{40} = -2.15^{\circ}C \text{ per minute}\end{aligned}$$

## Notes from Focus Group:

## Notes from Focus Group:

# Module 7

## Contents



## Weekly Checklist

☐ Complete MALL time.

☐ Work in ALEKS and Notebook at least 3 days a week.

☐ Complete the weekly Module and Notebook pages by the due date.

☐ Attend Focus Group.

☐ Actively participate in Focus Group.

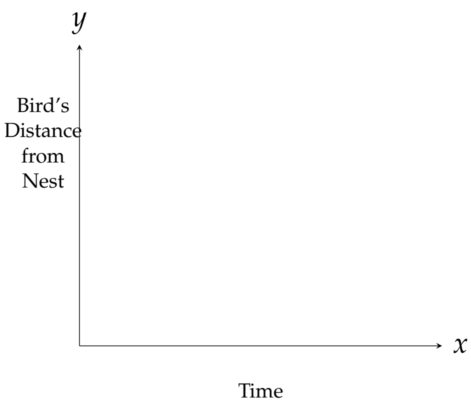
☐ Earn extra credit: Complete 10 topics by ____________________.

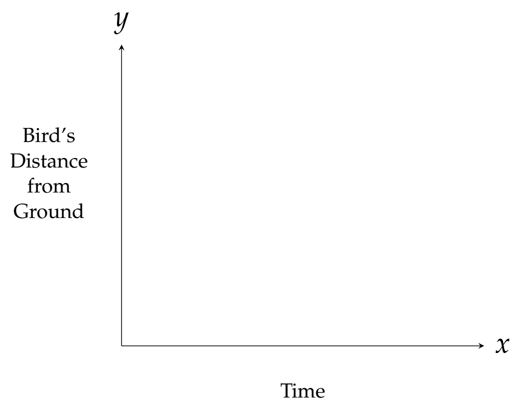
## Choosing a graph to fit a narrative: Basic

Open the Instructor Added Resource which will direct you to a video to complete the following.

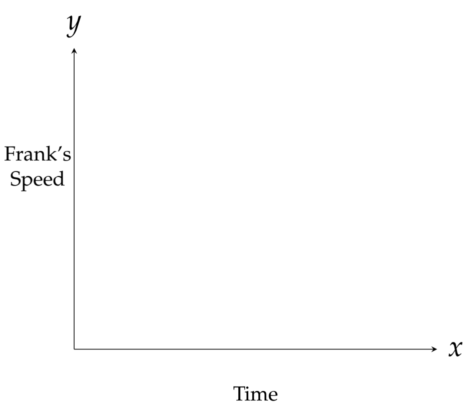
Sketch the graph that best describes the scenario below.

(a) A ______________ flies ______________ from its nest to go hunting.




(b) Frank drives at a ______________ speed for a while.

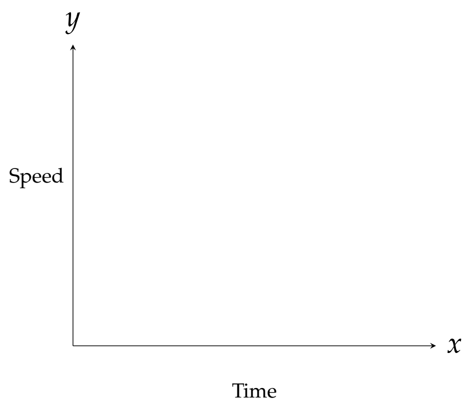


## Choosing a graph to fit a narrative: Advanced

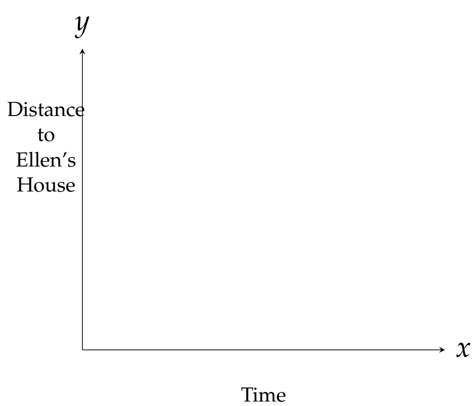
Open the Instructor Added Resource which will direct you to a video to complete the following.

Sketch the graph that best describes the scenario below.

(a) Hector begins his jogging workout by running ______________ for about a minute. Once he hits a comfortable pace, he runs at that pace for ______________ minutes. Then he gradually ______________ to a stop over the next few minutes.

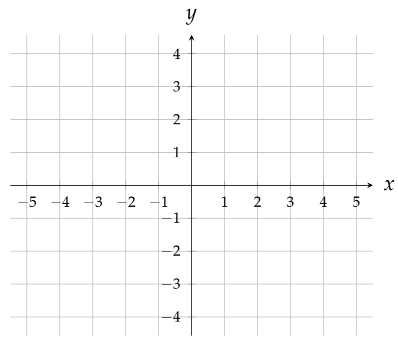



(b) Tina is delivering a pizza to Ellen's house. She drives at a ______________ speed toward the house until she hits a traffic jam and has to ______________ for several minutes. After, she starts up again and drives at a ______________ speed than before.



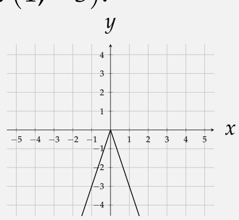
## Graphing an absolute value equation of the form $y = A|x|$

Learning Page Sketch the graph of $y = |x|$.



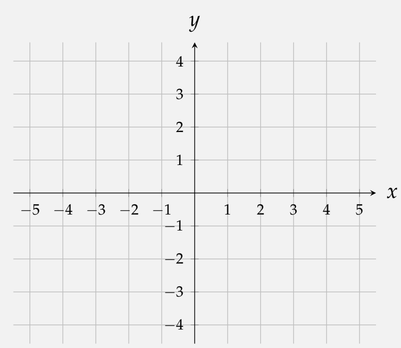

**EXAMPLE:** Sketch the graph of $y = -3|x|$.

- We first plot the vertex at $(0, 0)$.
- Next we plot a point on either side of the vertex, use $x = -1, 1$.
  - If $x = -1$, then $y = -3|-1| = -3$. Plot $(-1, -3)$.
  - If $x = 1$, then $y = -3|1| = -3$. Plot $(1, -3)$.




**YOU TRY IT:**

**108.** Sketch the graph of $y = 2|x|$.




## Graphing an absolute value equation in the plane: Advanced

Learning Page We will be graphing equations of the form $y = a|x - b| + c$.

The graphs of these equations will always have a ____________________ shape.

The vertex of the "V shape" occurs at the _______________ that makes ____________________.

To graph these equations, we first plot the _______________ and ________________________________________.

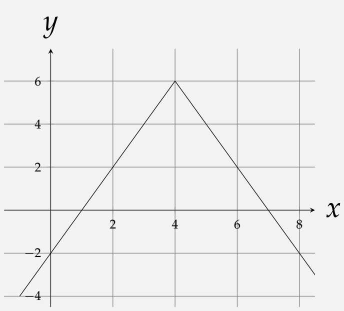
We then draw rays starting from the _______________ that pass _______________ these points.

**EXAMPLE:** Sketch the graph of $g(x) = -2|x-4| + 6$.

- This is the graph of $g(x) = |x|$ shifted right 4, up 6, reflected across the $x$-axis and stretched by a factor of 2.
- The vertex will be $(4, 6)$ and it will open down because 2 is negative.
- We also find the $x$ and $y$ intercepts to obtain the graph.
  - Let $x = 0$ to find the $y$-intercept: $y = -2|-4| + 6 = -2(4) + 6 = -2.$
  - Let $y = 0$ to find the $x$-intercept(s):

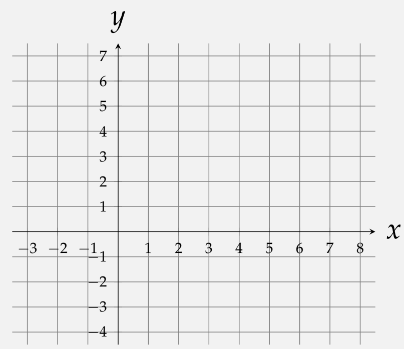
$$-2|x-4| + 6 = 0$$
$$|x-4| = 3$$

$$x - 4 = 3 \quad \text{or} \quad x - 4 = -3$$
$$x = 7 \quad \text{or} \quad x = 1$$




**YOU TRY IT:**

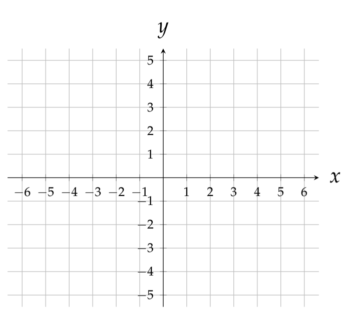
**109.** Sketch the graph of $f(x) = \frac{1}{2}|x-4| - 1$




## Graphing a square root function: Problem type 1

Watch the video *Graphing a Function with a Horizontal Shift* to complete the following.

Graph ________________________.



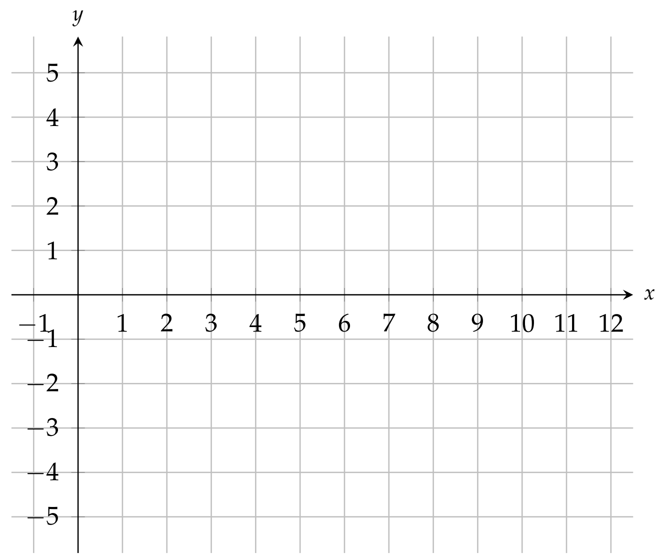
## Graphing a square root function: Problem type 2

Open the Instructor Added Resource which will direct you to a video to complete the following.

Sketch the graph of $f(x) =$ ____________________.

$f$ is the graph of _______________ shifted _____ units _______________ and _____ units _______________.

$x$-intercept:

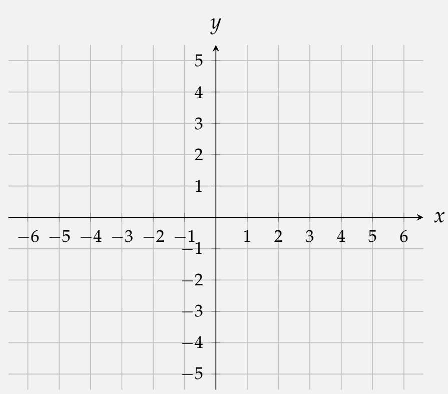



Choose __________

Choose __________

**YOU TRY IT:**

**110.** Sketch the graph of $g(x) = \sqrt{x+1} + 2$.



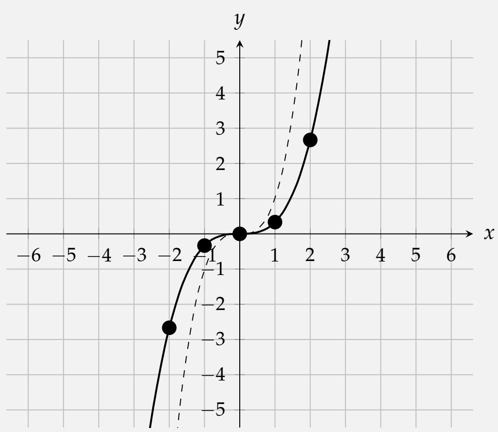
## Graphing a cubic function of the form $y = ax^3$

**EXAMPLE:**
Sketch the graph of $y = \frac{1}{3}x^3$.

We will complete the chart below to obtain the points to graph.

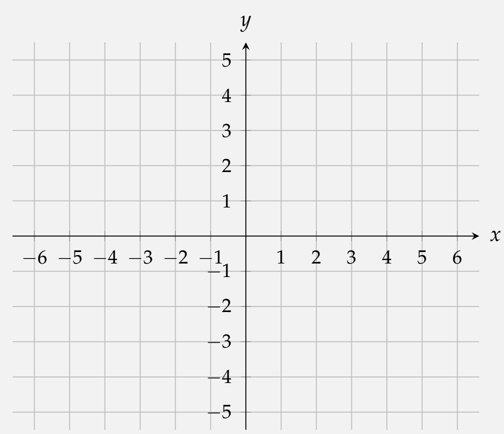
| $x$ | $y = \frac{1}{3}x^3$ | $(x, y)$ |
|---|---|---|
| -2 | $y = \frac{1}{3}(-8) = -\frac{8}{3}$ | $(-2, -\frac{8}{3})$ |
| -1 | $y = \frac{1}{3}(-1) = -\frac{1}{3}$ | $(-1, -\frac{1}{3})$ |
| 0 | $y = \frac{1}{3}(0) = 0$ | $(0, 0)$ |
| 1 | $y = \frac{1}{3}(1) = \frac{1}{3}$ | $(1, \frac{1}{3})$ |
| 2 | $y = \frac{1}{3}(8) = \frac{8}{3}$ | $(2, \frac{8}{3})$ |

The graph of $y = x^3$ is drawn below as a dashed line so you can see how the value of $a$ changes the graph.



**YOU TRY IT:**

**111.** Sketch the graph of $y = -\frac{3}{2}x^3$.

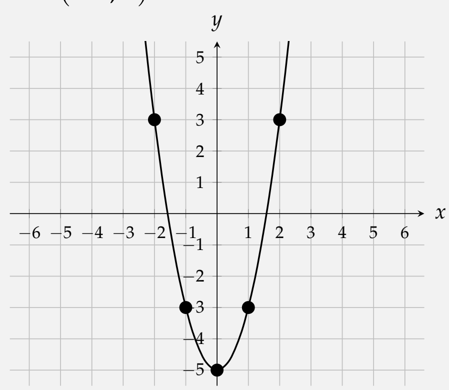


## Graphing a parabola of the form $y = ax^2 + c$

Learning Page A parabola with equation $y = ax^2 + c$ has its vertex at ____________.

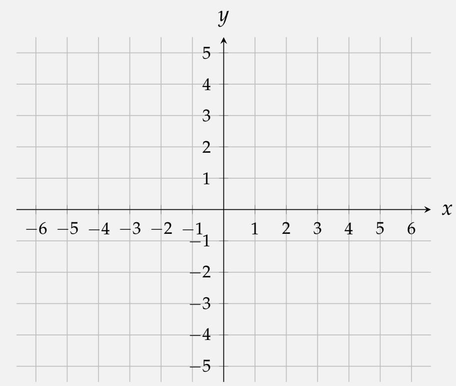
**EXAMPLE:** Sketch the graph of $y = 2x^2 - 5$.

- We first plot the vertex at $(0, -5)$.
- Next we plot 2 points on either side of the vertex.
  *All parabolas have symmetry so we can use this when finding points.
  - If $x = 1$, then $y = 2(1)^2 - 5 = -3$. Plot $(1, -3)$.
  - If $x = -1$, then $y = 2(-1)^2 - 5 = -3$. Plot $(-1, -3)$.
    We could also have used symmetry. Because the points $x$ values are the same distance from the $x$ value of the vertex, they must have the same $y$-coordinate.
  - If $x = 2$, then $y = 2(2^2) - 5 = 3$. Plot $(2, 3)$ and using symmetry plot $(-2, 3)$.




**YOU TRY IT:**

**112.** Sketch the graph of $y = -\frac{1}{2}x^2 + 3$.



## Graphing a parabola of the form $y = (x - h)^2 + k$

Watch the video *Graphing a Parabola Given an Equation in Vertex Form* to complete the following.

Given ____________________.

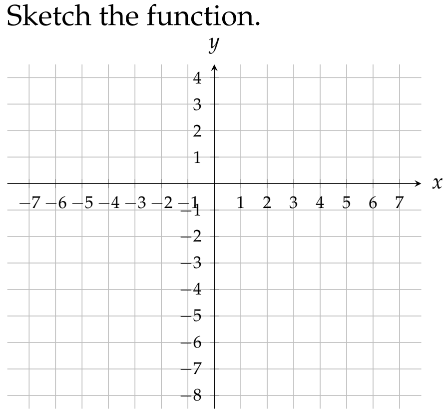
a. Determine whether the graph of the parabola opens upward or downward.

b. Identify the vertex.

c. Determine the $x$-intercept(s).

d. Determine the $y$-intercept.

e. Sketch the function.

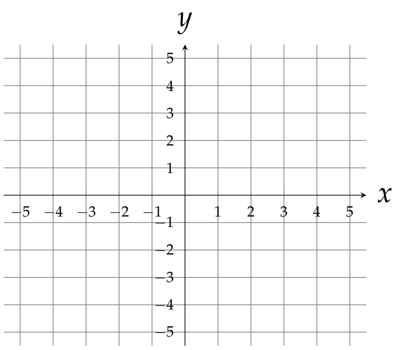



f. Determine the axis of symmetry.

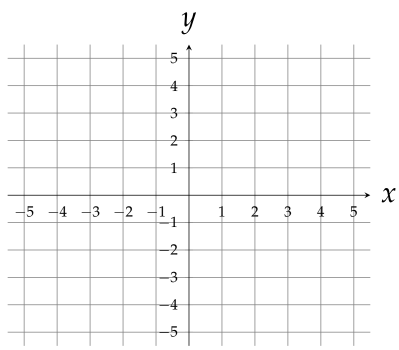
# Matching parent graphs with their equations

**Basic functions and Their Graphs**

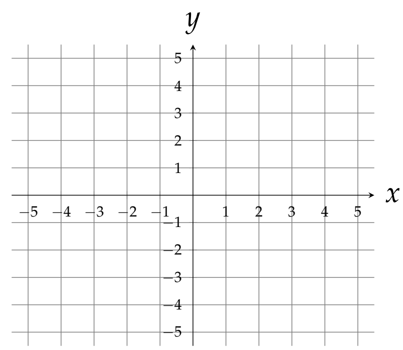
1. Linear functions
$f(x) = mx + b$



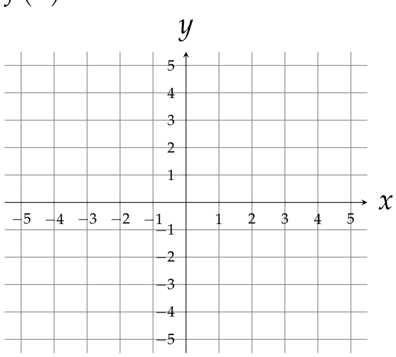

Constant functions
$f(x) = b$



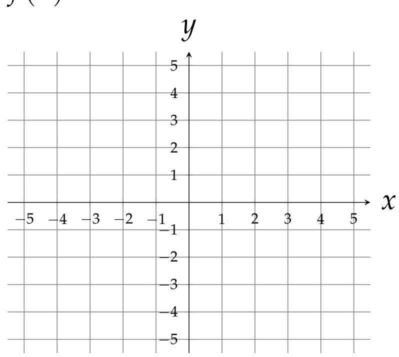

2. Identity function
$f(x) = x$



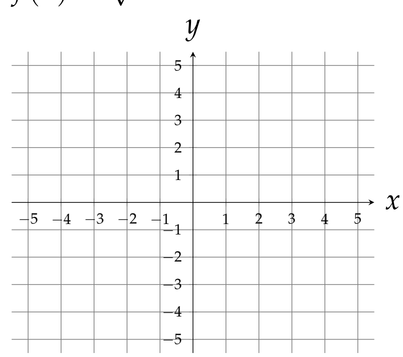

3. Quadratic function
$f(x) = x^2$



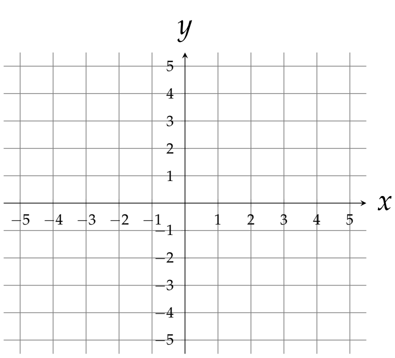

4. Cube function
$f(x) = x^3$



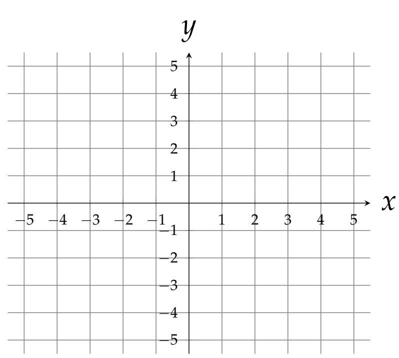

5. Square root function
$f(x) = \sqrt{x}$



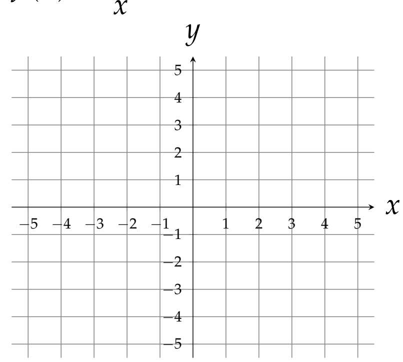

6. Cube root function
$f(x) = \sqrt[3]{x}$




7. Absolute value function
$f(x) = |x|$




8. Reciprocal function
$f(x) = \dfrac{1}{x}$



## How the leading coefficient affects the graph of a parabola

Learning Page A equation of the form ________________ ($a \neq 0$) describes a ______________________ whose ________________ is at the ________________.

The value of the leading ___________________________ $a$ tells us how the parabola looks.

(a) A ________________ leading coefficient, ___________, gives a parabola that opens ______________________.

A ________________ leading coefficient ___________, gives a parabola that opens ______________________.

(b) A ______________________ parabola has a leading coefficient $a$ ______________________ to ________.

A ______________________ parabola has a leading coefficient $a$ ______________________ from ________.

## Translating the graph of a function: One step

Open the e-book to complete the following.

**Vertical Translations of Graphs**

Consider a function defined by $y = f(x)$. Let $k$ represent a positive real number.

- The graph of ________________ is the graph of $y = f(x)$ shifted ___________________________.
- The graph of ________________ is the graph of $y = f(x)$ shifted ___________________________.

**Horizontal Translations of Graphs**

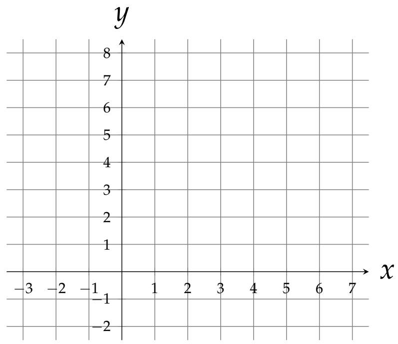
Consider a function defined by $y = f(x)$. Let $h$ represent a positive real number.

- The graph of ________________ is the graph of $y = f(x)$ shifted ___________________________.
- The graph of ________________ is the graph of $y = f(x)$ shifted ___________________________.

## Translating the graph of a function: Two steps

Watch the video *Using Rigid Transformations to Graph a Function* to complete the following.

Graph ____________________. Sketch the parent function using a dashed line and $c(x)$ using a solid line.

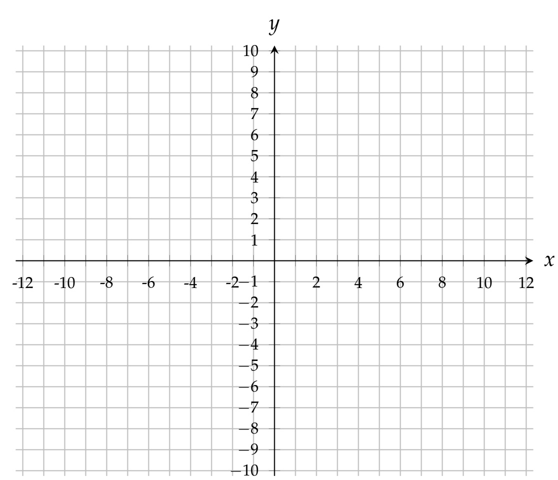



## Translating the graph of an absolute value function: Two steps

Open the Instructor Added Resource which will direct you to a video to complete the following.

Sketch the graph of ______________________.

Translations:




$x$-intercept(s):

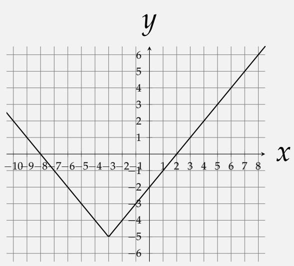
$y$-intercept:

**EXAMPLE:**
Sketch the graph of $g(x) = |x+3| - 5$.

- This is the graph of $g(x) = |x|$ shifted left 3 and down 5.
- We also find the $x$ and $y$ intercepts to obtain the graph.
    - Let $x = 0$ to find the $y$-intercept: $y = |0+3| - 5 = 3 - 5 = -2$.
    - Let $y = 0$ to find the $x$-intercept(s):

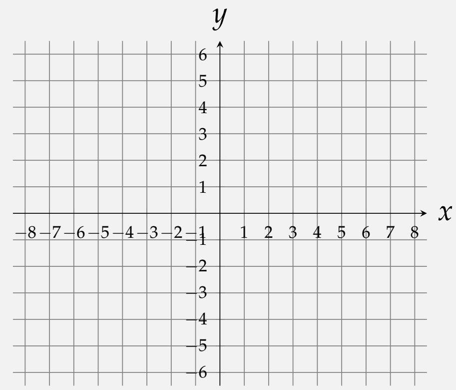
$$|x+3| - 5 = 0$$
$$|x+3| = 5$$

$$x + 3 = 5 \quad \text{or} \quad x + 3 = -5$$
$$x = 2 \quad \text{or} \quad x = -8$$




**YOU TRY IT:**

**113.** Sketch the graph of $f(x) = |x+2| - 3$.




## Transforming the graph of a function using more than one transformation

Open the e-book to complete the following.

**Steps for Graphing Multiple Transformations of Functions**

To graph a function requiring multiple transformations, use the following order.

1.

2.

3.

4.

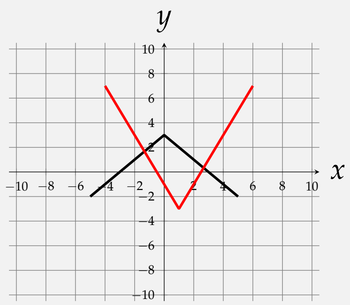
**EXAMPLE:** The graph of $y = f(x)$ is shown. Draw the graph of $y = -2f(x-1)+3$.

This is the graph of $y = f(x)$ that is

- stretched vertically by a factor of 2
- reflected across the $x$-axis
- shifted right 1 and up 3.

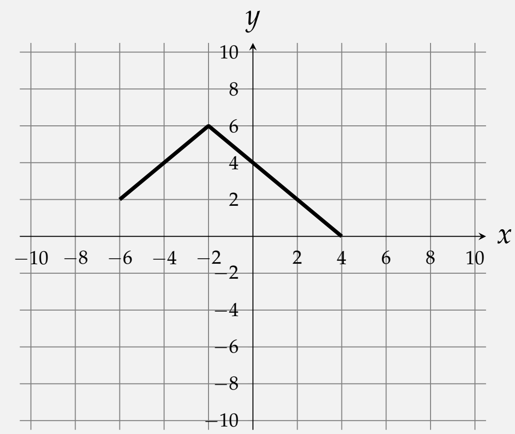
Consider the following points:

| Original | Stretch | Reflect | Shift |
|---|---|---|---|
| $(-5,-2)$ | $(-5,-4)$ | $(-5,4)$ | $(-4,7)$ |
| $(0,3)$ | $(0,6)$ | $(0,-6)$ | $(1,-3)$ |
| $(5,-2)$ | $(5,-4)$ | $(5,4)$ | $(6,7)$ |



**YOU TRY IT:**

**114.** The graph of $y = g(x)$ is shown. Draw the graph of $y = \frac{1}{2}f(x+2) - 3$.



## Transforming the graph of a function by shrinking or stretching

Watch the video *Investigating Horizontal Shrinking and Stretching* to complete the following.

**Horizontal Shrinking and Stretching of Graphs**

Consider a function defined by $y = f(x)$. Let _____ represent a __________ real number.

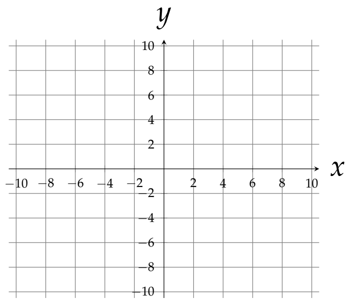
- If __________, then the graph of __________ is the graph of $y = f(x)$ _______________ _______________ by a _______________ of $a$.
- If _______________, then the graph of __________ is the graph of $y = f(x)$ ______________________________ by a factor of __________.

*Note:* for any point __________ on the graph of $y = f(x)$, the point __________ is on the graph of $y = f(ax)$.

**Continued on the next page**

Sketch the graph of $y = f(x)$ using a dashed line and the transformed graph using a solid line.

Given $y = f(x)$, Graph $y = f(3x)$.



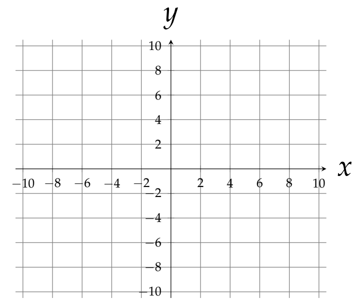
Points on $y = f(x)$:

________, ________, ________

Points on $y = f(3x)$:

________, ________, ________

Given $y = f(x)$, Graph $y = f(\frac{1}{3}x)$.



Points on $y = f(x)$:

________, ________, ________

Points on $y = f(\frac{1}{3}x)$:

________, ________, ________

Open the e-book to find the following definition.

**Vertical Shrinking and Stretching of Graphs**

Consider a function defined by $y = f(x)$. Let $a$ represent a positive real number.

- If ________, then the graph of ________ is the graph of $y = f(x)$ ____________ ____________ by a factor of $a$.
- If ____________, then the graph of ________ is the graph of $y = f(x)$ ________________________ by a factor of $a$.

*Note:* for any point ________ on the graph of $y = f(x)$, the point ________ is on the graph of $y = af(x)$.

## Transforming the graph of a function by reflecting over an axis

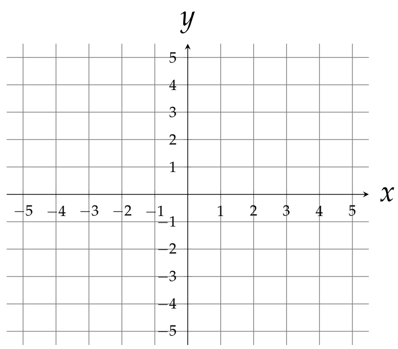
Watch the video *Investigating Reflections Across the x and y-Axes* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

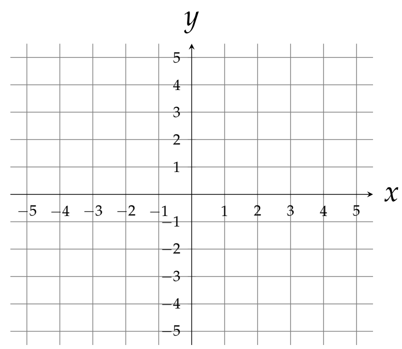
**Reflections Across the $x$ and $y$-Axes**

Consider a function defined by $y = f(x)$.

- The graph of ____________ is the graph of $y = f(x)$ reflected across the ____________.
- The graph of ____________ is the graph of $y = f(x)$ reflected across the ____________.

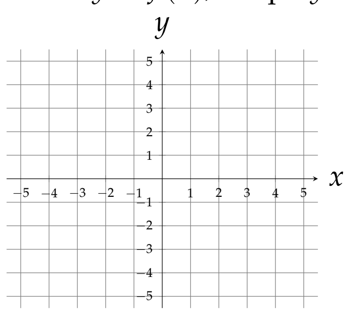
Sketch the blue graph from the video using a dashed line and the red graph using a solid line.





Sketch the graph of $y = f(x)$ using a dashed line and the transformed graph using a solid line.

Given $y = f(x)$, Graph $y = f(-x)$.



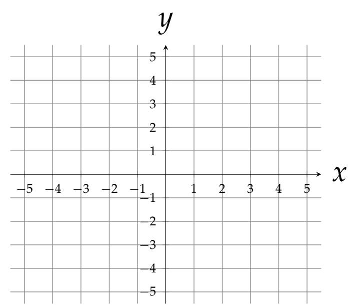
Points on $y = f(x)$:

________, ________, ________, ________

Points on $y = f(-x)$:

________, ________, ________, ________

Given $y = f(x)$, Graph $y = -f(x)$.



Points on $y = f(x)$:

________, ________, ________, ________

Points on $y = -f(x)$:

________, ________, ________, ________

## Transforming the graph of a quadratic, cubic, square root, or absolute value function

Possible transformations on a graph are reflecting about an axis, shifting, stretching, and shrinking. The chart below summarizes all the possible transformations of parent functions.

**Transformations of functions**

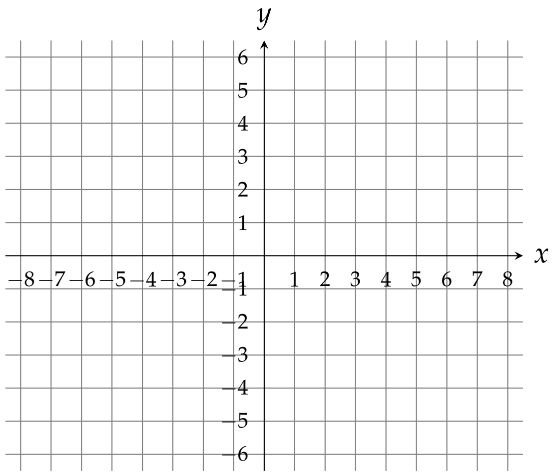
Consider a function defined by $y = f(x)$. If $h$, $k$, and $a$ represent positive real numbers, then the graphs of the following functions are related to $y = f(x)$ as follows.

| Transformation | Effect on the Graph of $f$ | Changes to Points on $f$ |
|---|---|---|
| **Vertical Translations of Graphs** | | |
| $y = f(x) + k$ | Shift ______ units | Replace $(x, y)$ by ______ |
| $y = f(x) - k$ | Shift ______ units | Replace $(x, y)$ by ______ |
| **Horizontal translations** | | |
| $y = f(x - h)$ | Shift ______ units | Replace $(x, y)$ by ______ |
| $y = f(x + h)$ | Shift ______ units | Replace $(x, y)$ by ______ |
| **Vertical stretch/shrink** | Vertical ______ if $a > 1$ | |
| $y = af(x)$ | Vertical ______ if $0 < a < 1$ | Replace $(x, y)$ by ______ |
| | Graph is stretched/shrunk vertical by a factor of ____ | |
| **Horizontal stretch/shrink** | Horizontal ______ if $a > 1$ | |
| $y = f(ax)$ | Horizontal ______ if $0 < a < 1$ | Replace $(x, y)$ by ______ |
| | Graph is shrunk/stretched horizontally by a factor of ____ | |
| **Reflection** | | |
| $y = -f(x)$ | Reflection across the ______ | Replace $(x, y)$ by ______ |
| $y = f(-x)$ | Reflection across the ______ | Replace $(x, y)$ by ______ |

## Writing an equation for a function after a vertical and horizontal translation

Open the Instructor Added Resource which will direct you to a video to complete the following.

Using translations of the base graph $y = |x|$, write the equation of the graph shown below.

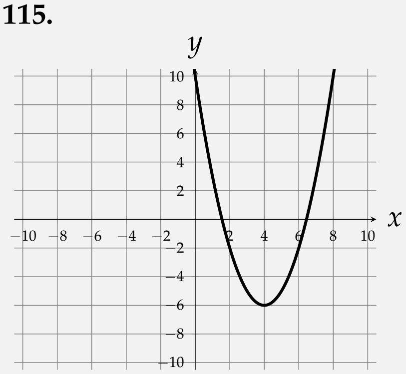


The base graph has been moved _____ units to the ______________ and _____ units ______________.

The equation of the graph is ____________________.

**YOU TRY IT:** Write the equation of the graph given below.

**115.**



## Domain and range from the graph of a quadratic function

Learning Page It is possible to determine the domain and range of a function from its graph.

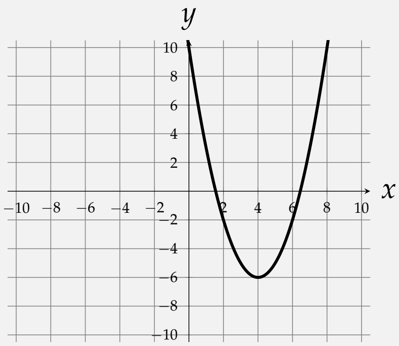
The ______________ is the set of all the numbers that appear as ________________ of points on the graph.

The ______________ is the set of all the numbers that appear as ________________ of points on the graph.

The graph of a ______________ function is a ______________.

**YOU TRY IT:** Find the domain and range of the quadratic given below.

**116.**




## Notes from Focus Group:

# Notes from Focus Group:

# Module 8-Review

To help you review for your upcoming exam, this module contains all of the topics from the modules since the last exam. Topics that you have already mastered will not appear in your carousel.

☐ Complete this module before you take the ALEKS exam.

☐ Each exam has two parts.

- The ALEKS exam (100 pts)
  - The ALEKS exam must be taken in the MALL.
  - The ALEKS exam is a Comprehensive Knowledge Check.
    - Your score is the number of topics you have mastered out of the number of topics you should have mastered by this point.
  - If you lose topics on your ALEKS exam, your Review Module completion grade will not change.
  - Your scratch work for the ALEKS exam must be numbered and turned in through Blackboard.
- The Written exam (25 pts)
  - Take your written exam in class the day of your focus group.
  - To study for the written exam:
    - Rework your old Focus Group assignments.
    - Rework any topics in ALEKS you may have lost on the ALEKS exam.

| | Score |
|---|---|
| ALEKS Exam | |
| Written Exam | |

# Module 9

## Contents



## Weekly Checklist

☐ Complete MALL time.

☐ Work in ALEKS and Notebook at least 3 days a week.

☐ Complete the weekly Module and Notebook pages by the due date.

☐ Attend Focus Group.

☐ Actively participate in Focus Group.

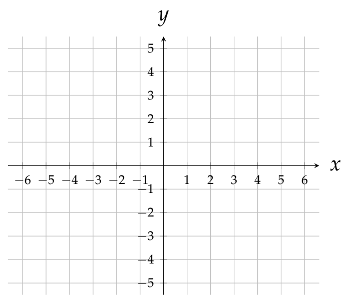
☐ Earn extra credit: Complete 10 topics by ____________________.

## Determining if graphs have symmetry with respect to the $x$-axis, $y$-axis, or origin

Watch the video *Introduction to Symmetry* to complete the following.

On each axis below, sketch in the blue graph with a solid line and the black graph with a dashed line.

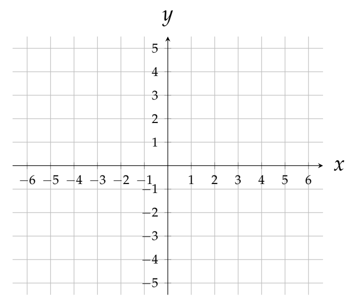
Symmetry with respect to the $x$-axis



Every point $(x, y)$ has a

mirror image ________________.

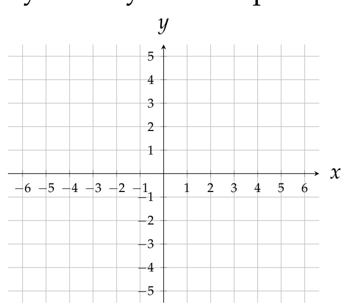
Symmetry with respect to the $y$-axis



Every point $(x, y)$ has a

mirror image ________________.

Symmetry with respect to the origin

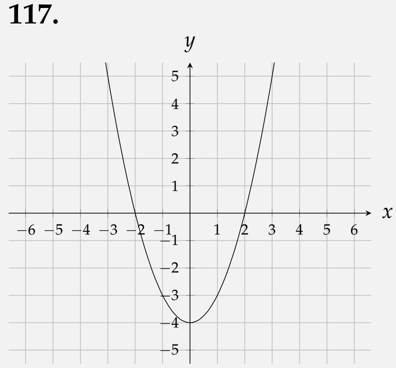


Every point $(x, y)$ has a

mirror image ________________.

**YOU TRY IT:** Determine what kind of symmetry (if any) applies to the graph.

**117.**



## Testing an equation for symmetry about the axes and origin

Watch the video *Testing for Symmetry* to complete the following.

**Tests for Symmetry**

Consider an equation in the variables $x$ and $y$.

- The graph of the equation is **symmetric with respect to the** ____________ if substituting ____________________ in the equation results in an ______________ equation.
- The graph of the equation is **symmetric with respect to the** ____________ if substituting ____________________ in the equation results in an equivalent equation.
- The graph of the equation is **symmetric with respect to the** ____________ if substituting ____________________ and ____________________ in the equation results in an equivalent equation.

Determine whether the graph of the equation is symmetric with respect to the $x$-axis, $y$-axis, origin, or none of these.

a.

b.

**EXAMPLE:**
Determine whether the graph of the equation is symmetric with respect to the $x$-axis, the $y$-axis, or the origin.

$$x^2y^2 + xy = 4$$

- Replace $y$ with $-y$.
$$x^2(-y)^2 + x(-y) = 4$$
$$x^2y^2 - xy = 4$$
This is not equivalent to $x^2y^2 + xy = 4$ so it is not symmetric to the $x$-axis.

- Replace $x$ with $-x$.
$$(-x)^2y^2 + (-x)(y) = 4$$
$$x^2y^2 - xy = 4$$
This is not equivalent to $x^2y^2 + xy = 4$ so it is not symmetric to the $y$-axis.

- Replace $x$ with $-x$ and $y$ with $-y$.
$$(-x)^2(-y)^2 + (-x)(-y) = 4$$
$$x^2y^2 + xy = 4$$
This is equivalent to $x^2y^2 + xy = 4$ so it is symmetric to the origin.

**YOU TRY IT:**

**118.** Determine whether the graph of the equation is symmetric with respect to the $x$-axis, the $y$-axis, or the origin.

$$5x^2 + 8y^2 = 14$$

## Finding local maxima and minima of a function given the graph

Watch the video *Introduction to Relative Maxima and Minima* to complete the following.

**Relative Minimum and Relative Maximum Values**

- $f(a)$ is a **relative maximum** of $f$ if there exists an open interval containing $a$ such that

  ____________________ for all $x$ in the interval.

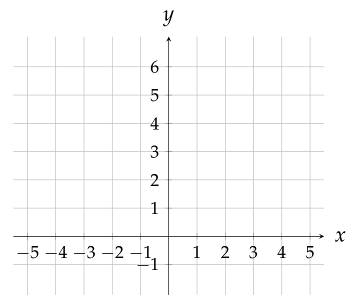
- $f(a)$ is a **relative minimum** of $f$ if there exists an open interval containing $a$ such that

  ____________________ for all $x$ in the interval.

*Note:* An __________ interval is an interval in which the endpoints are _________________________.

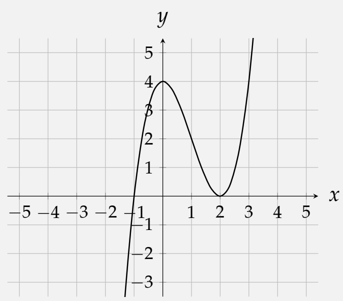
**Continued on the next page**

a. Determine the relative maxima.




b. Determine the relative minima.

**EXAMPLE:**
Use the graph of the function $f$ below to find:



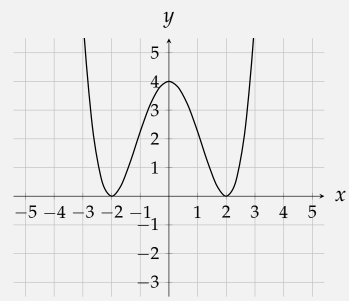

a) All local maximum and minimum values of $f$

- Local maximum value: 4
- Local minimum value: 0

b) All values at which $f$ has a local maximum and minimum

- Local maximum at $x = 0$
- Local minimum at at $x = 2$

**YOU TRY IT:**
Use the graph of the function $f$ below to find:




**119.** All local maximum and minimum values of $f$

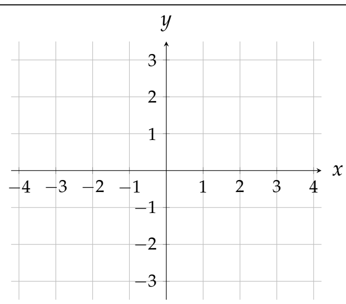
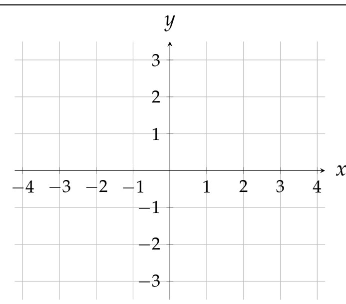
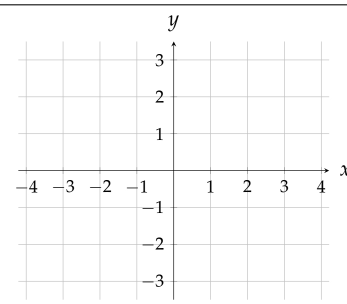
**120.** All values at which $f$ has a local maximum and minimum

## Finding where a function is increasing, decreasing, or constant given the graph: Interval notation

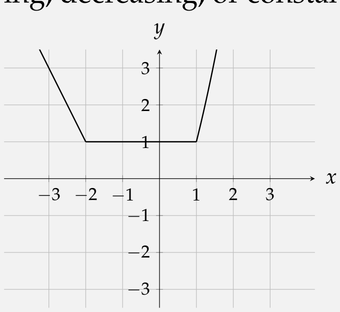
Learning Page

- A function $f$ is (strictly) **increasing** on an interval if, for all $a$ and $b$ in that interval, $a < b$ implies ________________________.
- A function $f$ is (strictly) **decreasing** on an interval if, for all $a$ and $b$ in that interval, $a < b$ implies ________________________.
- A function $f$ is **constant** on an interval if, for all $a$ and $b$ in that interval, $a < b$ implies ________________________.

Sketch a graph of each type of function on the axes below.

| **Increasing** | **Decreasing** | **Constant** |
|---|---|---|
|  |  |  |

**EXAMPLE:**
Determine where the function below is increasing, decreasing, or constant.

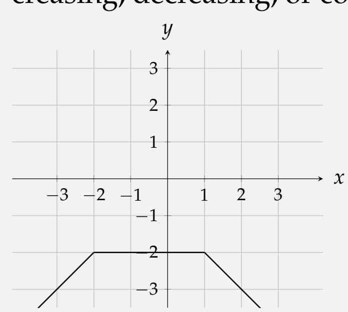


The function is

- Increasing on $(1, \infty)$
- Decreasing on $(-\infty, -2)$
- Constant on $(-2, 1)$

**YOU TRY IT:**

**121.** Determine where the function below is increasing, decreasing, or constant.



## Finding the absolute maximum and minimum of a function given the graph

Learning Page We will use the following information about absolute maximums and minimums, vertical asymptotes, and "holes".

Suppose the domain of a function $f$ is an interval.

- Absolute maximums and minimums:

  The absolute ____________ of $f$ is the ____________ ____________ of any point on the graph of $f$.

  The absolute ____________ of $f$ is the ____________ ____________ of any point on the graph of $f$.

- Vertical asymptotes:

  Suppose the graph of $f$ has a vertical asymptote, ________.

  As the $x$-coordinatesof the graph of $f$ approach $a$, the $y$-coordinates approach ________ or ________.

  If the $y$-coordinates approach ________, then the function will ____________ have an absolute ____________.

  If the $y$-coordinates approach ________, then the function will ____________ have an absolute ____________.

- "Holes":

  A "hole" in the graph of $f$ is show as a ____________ ____________.

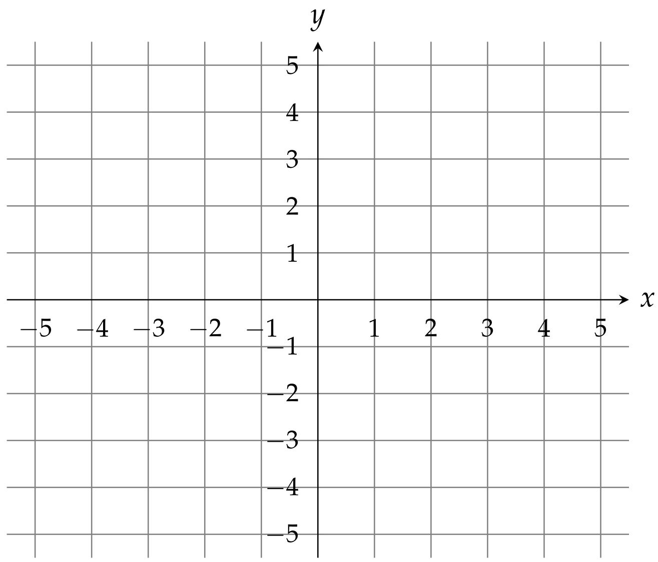
  A "hole" is a point that is ____________ on the graph of $f$.

  If a "hole" in the graph of $f$ has a ____________ $y$-coordinate than any point on the graph of $f$, then the function does ____________ have an absolute ____________.

  If a "hole" in the graph of $f$ has a ____________ $y$-coordinate than any point on the graph of $f$, then the function does ____________ have an absolute ____________.

## Finding values and intervals where the graph of a function is zero, positive, or negative

Open the Instructor Added Resource which will direct you to a video to complete the following.




a. Is $f(-2)$ negative?

b. For which value(s) of $x$ is $f(x) < 0$?

c. For which value(s) of $x$ is $f(x) = 0$?

d. For which value(s) of $x$ is $f(x) > 0$

## Finding a difference quotient for a linear or quadratic function

Watch the video *Finding a Difference Quotient for a Nonlinear Function* to complete the following.

*NOTE:* This may not be the first video that pops up. Select the appropriate video in the video box.

Given ____________________, find the difference quotient.

**EXAMPLE:**
Find the difference quotient for
$f(x) = 3x^2 - 4x + 5$.
First, find $f(x+h)$.

$$\begin{aligned} f(x+h) &= 3(x+h)^2 - 4(x+h) + 5 \\ &= 3(x^2 + 2xh + h^2) - 4x - 4h + 5 \\ &= 3x^2 + 6xh + 3h^2 - 4x - 4h + 5 \end{aligned}$$

Now find $\dfrac{f(x+h) - f(x)}{h}$.

$$\begin{aligned} &\frac{f(x+h) - f(x)}{h} \\ &= \frac{3x^2 + 6xh + 3h^2 - 4x - 4h + 5 - (3x^2 - 4x + 5)}{h} \\ &= \frac{3x^2 + 6xh + 3h^2 - 4x - 4h + 5 - 3x^2 + 4x - 5}{h} \\ &= \frac{6xh + 3h^2 - 4h}{h} \\ &= \frac{h(6x + 3h - 4)}{h} \\ &= 6x + 3h - 4 \end{aligned}$$

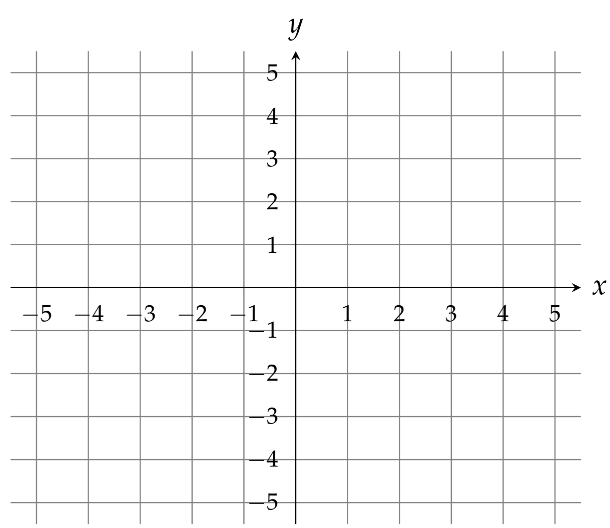
**YOU TRY IT:**

**122.** Find the difference quotient for
$f(x) = -4x^2 + 5x - 3$.

## Graphing a piecewise-defined function: Problem type 1

Open the Instructor Added Resource which will direct you to a video to complete the following.

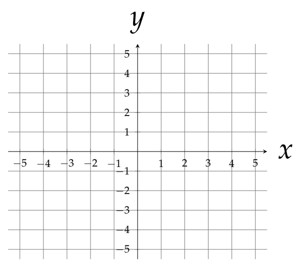
$$g(x) = \begin{cases} \underline{\hspace{1.5cm}} & \text{if } -2 \leq x < -1 \\ \underline{\hspace{1.5cm}} & \text{if } -1 \leq x < 0 \\ \underline{\hspace{1.5cm}} & \text{if } 0 \leq x < 1 \\ \underline{\hspace{1.5cm}} & \text{if } 1 \leq x < 2 \end{cases}$$

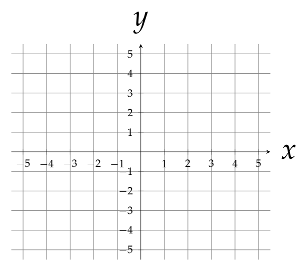


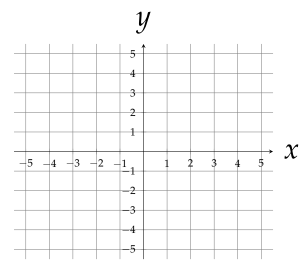
## Graphing a piecewise-defined function: Problem type 2

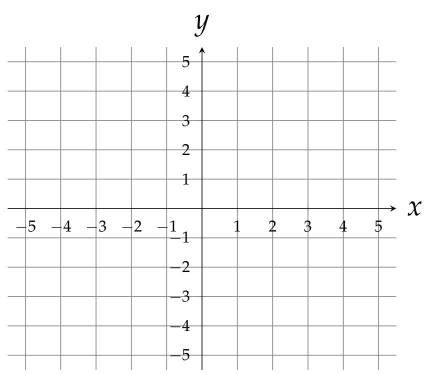
Watch the video *Graphing a Piecewise-Defined Function* to complete the following.

Graph. $m(x) = \begin{cases} \underline{\hspace{2.5cm}} & \text{if } -4 < x < -1 \\ \underline{\hspace{2.5cm}} & \text{if } -1 \leq x < 3 \\ \underline{\hspace{2.5cm}} & \text{if } x \geq 3 \end{cases}$



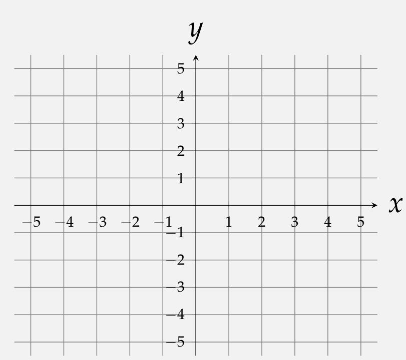

The graph of $f(x)$ is **continuous** if there are no "holes" or "jumps" in the graph. In other words, you can draw the graph without lifting your pencil.

## Graphing a piecewise-defined function: Problem type 3

If you did not complete the video *Graphing a Piecewise-Defined Function* under the topic **Graphing a piecewise-defined function: Problem type 2**, click the video link now and complete the work.

**YOU TRY IT:**

**123.** Sketch the graph of $f(x) = \begin{cases} -2 & \text{if } x < -3 \\ x+1 & \text{if } -3 \leq x \leq 2 \\ 4 & \text{if } x > 2 \end{cases}$



## Sum, difference, and product of two functions

Watch the video *Introduction to Operations on Functions* to complete the following.

**Sum, Difference, Product, and Quotient of Functions**

Given the functions $f$ and $g$, the functions $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$ are defined by:

$$(f+g)(x) = \underline{\hspace{5cm}}$$

$$(f-g)(x) = \underline{\hspace{5cm}}$$

$$(f \cdot g)(x) = \underline{\hspace{5cm}}$$

$$\left(\frac{f}{g}\right)(x) = \underline{\hspace{5cm}}$$

The domains of the functions $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$ are all real numbers in the

______________________________ of the individual functions $f$ and $g$.

For $\frac{f}{g}$ we further restrict the domain to ______________________________.

Given $f(x) =$ __________ and $g(x) =$ __________, find $(f+g)(x)$.

**EXAMPLE:**
Given $f(x) = x^2 - 3x$ and $g(x) = \sqrt{4x-1}$, find the function and its domain.

$$\begin{aligned}(f \cdot g)(x) &= f(x) \cdot g(x) \\ &= (x^2 - 3x)\sqrt{4x-1}\end{aligned}$$

The domain of $f$ is $(-\infty, \infty)$ and the domain of $g$ is $[\frac{1}{4}, \infty)$ so the domain of $f \cdot g$ is the intersection of the two domains. Interval notation: $[\frac{1}{4}, \infty)$.

**YOU TRY IT:**
Given $f(x) = 3x^2 + 2x$ and $g(x) = 1 - \frac{1}{x}$, find the function and its domain.

**124.** $(g \cdot f)(x)$

## Quotient of two functions: Basic

Watch the video *Evaluating Functions for a Given Value of x* to complete the following.

Evaluate the functions for the given values of $x$.

$f(x) =$ ______________ $g(x) =$ ______________ $h(x) =$ ______________

a.

b.

**YOU TRY IT:** Given $f(x) = x^2 - 3x$ and $g(x) = \sqrt{4x+1}$, find the following.

**125.** $\left(\frac{f}{g}\right)(2)$

**126.** $\left(\frac{f}{g}\right)(-4)$

## Combining functions: Advanced

Watch the video *Combining Functions and Finding Domain* to complete the following.

Given ______________ and ______________, evaluate the given function and write the domain in interval notation.

a.

b.

## Combining functions to write a new function that models a real-world situation

**EXAMPLE:**
A website designer creates videos on how to create websites. He sells the video packages for \$40 each. His one-time initial cost to produce a package is \$5000. The cost to ship each video is \$2.80.

a. Write a function that represents the cost $C(x)$ to produce and ship $x$ video packages.

$C(x) = 2.8x + 5000$

b. Write a function that represents the revenue $R(x)$ for selling $x$ video packages.

$R(x) = 40x$

c. Evaluate $(R - C)(x)$ and interpret its meaning in the context of this problem.

$(R - C)(x) = 40x - (2.8x + 5000) = 37.2x - 5000$
This represents the profit for selling $x$ video packages.

**YOU TRY IT:**
An artist makes jewelry from polished stones. The rent for her studio and utilities comes to \$640 per month. It also costs her \$3.50 for supplies to make one necklace. She sells the necklaces for \$25 each.

**127.** Write a function $C(x)$ that represents the cost to produce $x$ necklaces during a one month period.

**128.** Write a function $R(x)$ that represents the revenue for selling $x$ necklaces.

**129.** Evaluate $(R - C)(x)$ and interpret its meaning in the context of this problem.

## Introduction to the composition of two functions

Watch the video *Composing Functions* to complete the following.

**Composition of Functions**

The **composition of** $f$ **and** $g$ denoted __________ is defined by __________________________.

The domain of __________ is the set of real numbers $x$ in the __________________________

such that __________ is in the domain of ____.

Evaluate the given functions for

$f(x) =$ _______________ $g(x) =$ _______________ $h(x) =$ _______________

a.

b.

## Composition of two functions: Basic

Open the e-book to find and watch the Animation: *Introduction to the composition of functions* to complete the following. The Animation is found right after Figure 2-41.

Given ____________________ and _______________, evaluate,

a. $(f \circ g)(-2)$

b. $(f \circ g)(x)$

## Composition of two functions: Advanced

Watch the video *Composing Functions and Determining Domain 1* to complete the following.

For the given functions, evaluate $(q \circ m)(x)$ and write the domain in interval notation.

**EXAMPLE:**
Given $f(x) = \frac{x}{x+2}$ and $g(x) = \frac{1}{x-4}$, find the following function and its domain. $(f \circ g)(x)$

$$\begin{aligned} f(g(x)) &= f\left(\frac{1}{x-4}\right) \\ &= \frac{\frac{1}{x-4}}{\frac{1}{x-4}+2} \\ &= \frac{\frac{1}{x-4}}{\frac{1}{x-4}+2} \cdot \frac{x-4}{x-4} \\ &= \frac{1}{1+2(x-4)} \\ &= \frac{1}{2x-7} \end{aligned}$$

We must exclude 4 from the domain and we must also exclude values of $x$ where $\frac{1}{x-4} + 2 = 0$. We solve this equation for $x$.

$$\begin{aligned} \frac{1}{x-4} + 2 &= 0 \\ 1 + 2(x-4) &= 0(x-4) \\ 1 + 2x - 8 &= 0 \\ 2x &= 7 \\ x &= \frac{7}{2} \end{aligned}$$

The domain of $f \circ g$ is $(-\infty, \frac{7}{2}) \cup (\frac{7}{2}, 4) \cup (4, \infty)$.

**YOU TRY IT:**
Given $f(x) = \frac{3}{x}$ and $g(x) = \frac{x-1}{x-4}$, find the following functions and their domains.

**130.** $(g \circ f)(x)$

## Composition of a function with itself

Open the Instructor Added Resource which will direct you to a video to complete the following.

Given ____________________, find and simplify $(f \circ f)(x)$.

**EXAMPLE:**
Given $f(x) = x^2 + 2$ and $g(x) = \frac{1}{x-4}$, find the following.

a) $(f \circ f)(x)$

$$f(f(x)) = f(x^2+2)$$
$$(x^2+2)^2+2$$
$$= (x^4 + 2x^2 + 2x^2 + 4) + 2$$
$$= x^4 + 4x^2 + 6$$

b) $(g \circ g)(x)$

$$g(g(x)) = g\left(\frac{1}{x-4}\right)$$
$$= \frac{1}{\frac{1}{x-4}-4}$$
$$= \frac{1}{\frac{1}{x-4}-4} \cdot \frac{x-4}{x-4}$$
$$= \frac{x-4}{1-4(x-4)}$$
$$= \frac{x-4}{1-4x+16}$$
$$= \frac{x-4}{17-4x}$$

**YOU TRY IT:**
Given $f(x) = \frac{3}{x}$ and $g(x) = x^2 - 5$, find the following functions and their domains.

**131.** $(f \circ f)(x)$

**132.** $(g \circ g)(x)$

## Expressing a function as a composition of two functions

Watch the video *Decomposing a Function* to complete the following.

Find two functions $f$ and $g$ such that $h(x) = (f \circ g)(x)$.

## Word problem involving composition of two functions

Open the e-book and read **EXAMPLE 10** to complete the following.

At a popular website the cost to download individual songs is ______________ per song. In addition, a first time visitor to the website has a one-time coupon for ______________ off.

a. Write a function to represent the cost $C(x)$ (in \$) for a first-time visitor to purchase $x$ songs.

$C(x) =$ ______________

The cost function is a ______________________________________.

b. The sales tax for online purcahses depends on the location of the business and customer. If the sales tax rate on a purchase is __________, write a function to represent the total cost $T(a)$ for a first-time visitor who buys $a$ dollars in songs.

$T(a) =$ ______________ $=$ __________

The total cost is the ______________________________________.

c. Find $(T \circ C)(x)$ and interpret the meaning in context.

$(T \circ C)(x) = T(C(x)) =$ __________ $=$ ____________________ $=$ ____________________

$(T \circ C)(x)$ represents the ______________________________________ for a first-time visitor to the website.

d. Evaluate $(T \circ C)(10)$ and interpret the meaning in context.

$(T \circ C)(10) =$ ____________________ $=$ __________

The ______________ for a first-time visitor to ______________________________.

## Notes from Focus Group:

## Notes from Focus Group:

# Module 10

## Contents



## Weekly Checklist

☐ Complete MALL time.

☐ Work in ALEKS and Notebook at least 3 days a week.

☐ Complete the weekly Module and Notebook pages by the due date.

☐ Attend Focus Group.

☐ Actively participate in Focus Group.

☐ Earn extra credit: Complete 10 topics by ____________________.

## Constructing a scatter plot

Open the dictionary to complete the following.

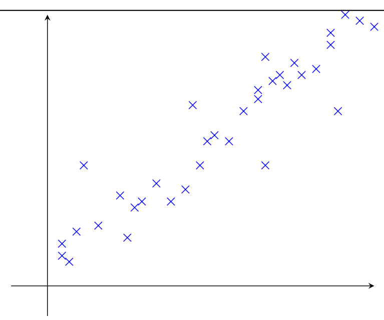
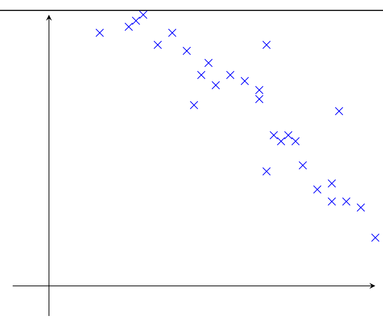
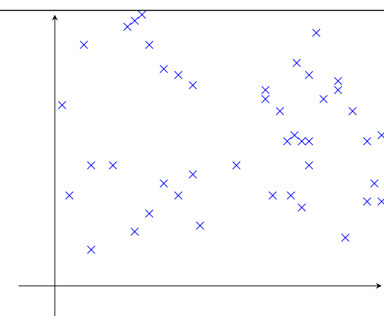
A *scatter plot* is a ____________________ representation of values of ________________ variables.

The paired values are represented as ________________ in the ____________________________.

## Scatter plots and correlation

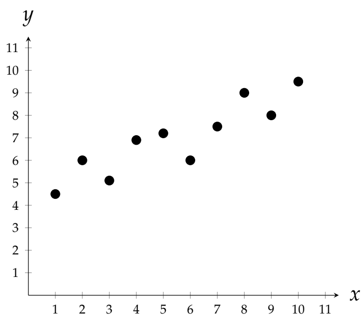
Learning Page The correlations between two ________________ is an ________________ of how the ________________ are related.

The figures below show different types of correlation. Fill in the blanks in the table below.

| ______________ correlation | ______________ correlation | ____ correlation |
|---|---|---|
|  |  |  |
| As $x$ ________________, $y$ tend to ________________ | As $x$ ________________, $y$ tends to ________________ | There is ____ ____________ pattern. |

# Classifying linear and nonlinear relationships from scatter plots

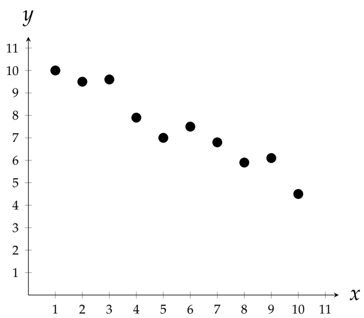
Learning Page Four scatter plots are shown below. From the Learning Page, find the corresponding graph and label it as **positive linear** relationship, **negative linear** relationship, **no** relationship, or **nonlinear** relationship.




______________________________ relationship

The data points appear to follow a *line*.

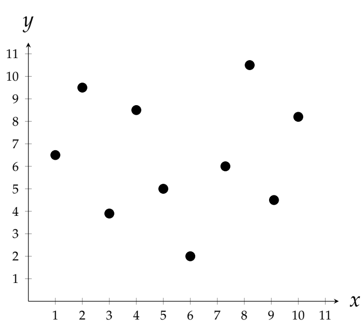
This line goes _____ from left to right.




______________________________ relationship

The data points appear to follow a *line*.

This line goes __________ from left to right.

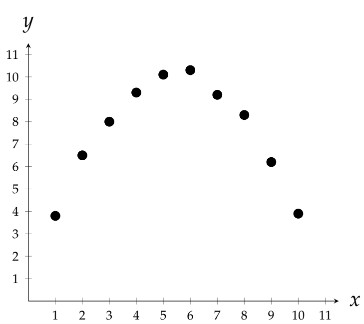



______________________________ relationship

There is no _______________ pattern to the data points.

They _______________ appear to folllow a

_______________ or a simple curve.




______________________________ relationship

The data points appear to follow a simple

_______________.

There does appear to be a _______________ to the data.

But this pattern is __________ in the form of a

_______________.

## Identifying outliers and clustering in scatter plots

Learning Page Data sets can sometimes have **clusters**.

A cluster is a ____________________________________.

An a cluster doesnt have ____________ (or any) data points ________________.

Data sets can sometimes have **outliers**.

An ________________ is a data point that is ________________________ from the

________________________ points.

## Sketching the line of best fit

Learning Page Informally, the line of best fit is a ________________ that lies as ________________ as

possible to all the ________________ points.

It is a line that shows the ________________ of the data points as ________________ as any other.

## Predictions from the line of best fit

Learning Page Suppose we can draw a ____________ that follows the ____________ of the data shown in

a ________________ plot.

Then, we can ________________ the relationship between the ________________ as ____________.

We can use the line to ________________ the corresponding ____ for a given ____.

We can also use the line's ____________ to ________________ how $y$ will ____________ as ____ changes.

For a ________________ increase in ____, the ________________ change in ____ is equal to the

____________ of the line.

## Approximating the equation of a line of best fit and making predictions

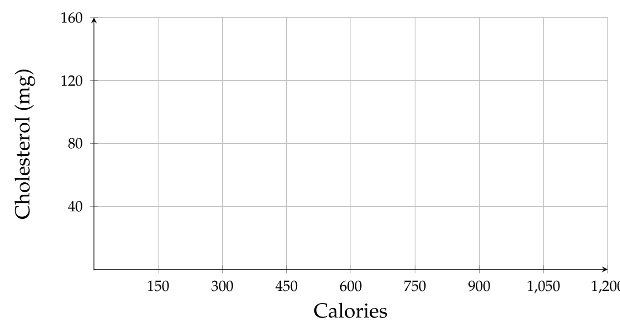
Watch the video *Writing a Linear Model to Relate Two Variables in an Application* to complete the following.

The table gives the number of calories and the amount of cholesterol for selected fast food hamburgers.

a. Graph the data in a scatter diagram using the number of calories as the independent variable $x$ and the amount of cholesterol as the dependent variable $y$.

| Hamburger Calories | Cholesterol (mg) |
|---|---|
| 220 | |
| 420 | |
| 460 | |
| 480 | |
| 560 | |
| 590 | |
| 610 | |
| 680 | |
| 720 | |
| 800 | |
| 1050 | |

**Amount of Cholesterol vs. Number of Calories for Selected Hamburgers**




b. The amount of cholesterol is approximately linearly related to the number of calories. Use the points

__________ and __________ to write a linear function that defines the amount of cholesterol $c(x)$ as a linear function of the number of calories, $x$.

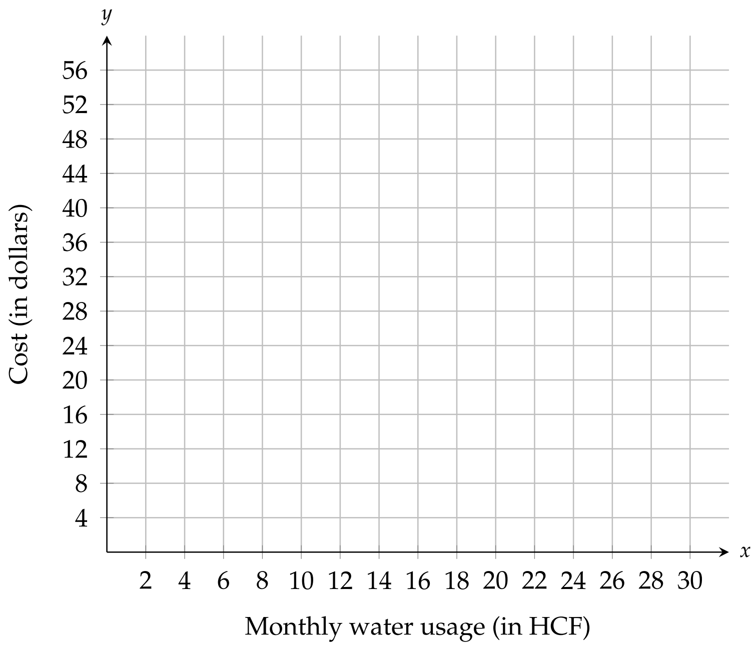
c. Interpret the meaning of the slope in the context of this problem.

d. Use the model from part (b) to predict the amount of cholesterol for a hamburger with 650 calories.

## Interpreting the graphs of two functions

Open the Instructor Added Resource which will direct you to a video to complete the following.

The water company has a different monthly pricing plan for residential customers than for business customers. For each pricing plan, cost (in dollars) depends on water used (in hundreds of cubic feet, HCF). Draw in the Residential Plan graph using a solid line and the Business Plan graph using a dashed line. Answer all questions using complete sentences using the context of the problem.




1. If the monthly water usage is 22 HCF which plan costs less?

   How much less does it cost than the other plan?

2. For what amount of monthly water usage do the plans cost the same?

   If the monthly water usage is less than this amount, which plan costs less?

## Computing residuals

**Learning Page** **Residual**

A residual is a ________ of how far the ________ value is from the ________ value.

In particular, we compute the ________ for a particular data point as follows.

________ = ________ − ________

$y$-value $y$-value

## Interpreting residual plots

Learning Page A residual is a ____________________ of how far a ____________________ value is from an ____________________ value.

We can find residuals ____________________ when looking at a ____________________ plot that has a line of _______________ fit.

A residual ____________________ how far we move __________ or _______________ from a point (the ____________________ $y$-value) to the line (the ____________________ $y$-value).

Points **above** the line have ____________________ residuals, and points **below** the line have ____________________ residuals.

A residual plot can be used to determine how _______________ the line of best fit ____________________ a data set.

- The line is a ____________________ model for the data set if the residuals ______________________________.
- The line is a _______________ (but not perfect) model for the data set is the following are _______________.
  - The points on the residual plot appear to be ______________________________, with no ____________________ pattern.
  - There are about as many ____________________ residuals as ____________________ residuals.
  - The points on the residual plot are ____________________ fairly close to the ____________________.
- The line might __________ be an ____________________ model if there is a _______________ in the residual plot.

## Linear relationship and the correlation coefficient

Learning Page The correclation coefficient, _____, measures the _______________ ____________________

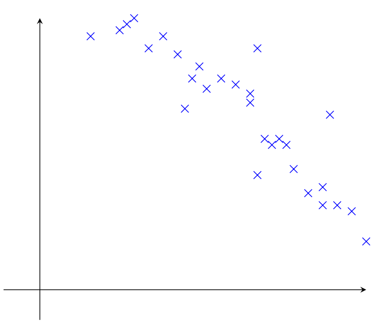
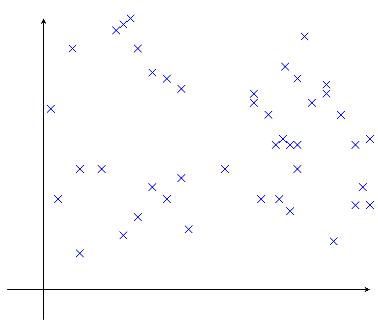
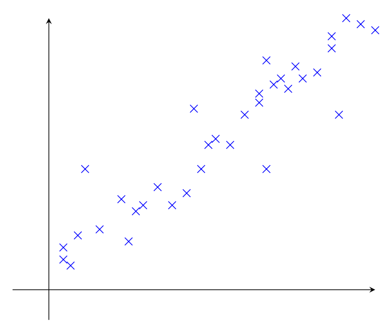
between two variables. The value of $r$ is a ____________________ from _____ to _____.

A ____________________ value of $r$ indicates a ____________________ linear relationship between the two

____________________.

A value of $r$ _______________ to _____ indicates there is _______________ to __________ linear relationship.

A ____________________ value of $r$ indicates a positive ____________________ relationship.

| _______________ linear relationship | __________ linear relationship | _______________ linear relationship |
|---|---|---|
|  |  |  |
| As $x$ increases, $y$ tends to _______________ | There is no ____________________ pattern. | As $x$ increases, $y$ tends to _______________ |

The _______________ the points are to ____________________ on a ____________________ line, the

____________________ the ____________________ relationship.

The ____________________ the linear relationship, the _______________ $r$ is to _____ or _____.

A value of __________ indicates a ____________________ positive ____________________ relationship.

The points lie ____________________ on a straight line that _______________ from _______________ to

_______________.

A value of __________ indicates a perfect ____________________ linear relationship.

The _______________ lie exactly on a ____________________ line that _______________ from left to right.

## Finding outliers in a data set

Learning Page An **outlier** is a data ______________ that is ______________ smaller or larger than most of the ____________________________.

It is a value that is "____________________________" and is very __________________ from most of the other values.

**YOU TRY IT:** Identify all values that are outliers.

**133.** 181, 494, 497, 500, 505, 511, 513, 516, 518, 832

## Choosing a quadratic model and using it to make a prediction

Learning Page Informally, the curve that fits the data best is the curve that lies __________________ to the ______________ points. It shows the ______________ of the data points __________________ than the other curves.

## Finding the zeros of a quadratic function given its equation

Learning Page The zeros of a function are the ______________ that give an _______________________.

So, to find the zeros, we set _______________________ and _______________________.

**EXAMPLE:** Find all zeros of the quadratic.

$$y = x^2 - 14x + 33$$

We set $y = 0$ and solve for $x$

$$0 = x^2 - 14x + 33$$

Factor the quadratic.

$$0 = (x - 11)(x - 3)$$

Set each factor equal to 0.

$$x = 11, 3$$

**YOU TRY IT:** Find all zeros of the quadratic.

**134.** $y = x^2 + 5x - 14$

# Finding the vertex, intercepts, and axis of symmetry from the graph of a parabola

Open the e-book to complete the following.

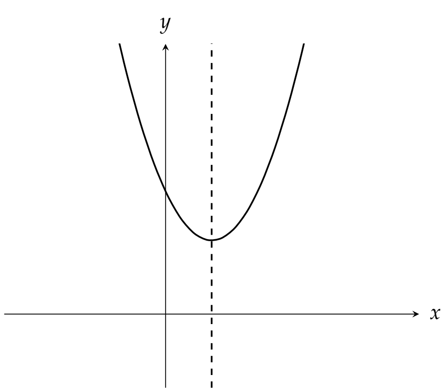
**Quadratic Function**

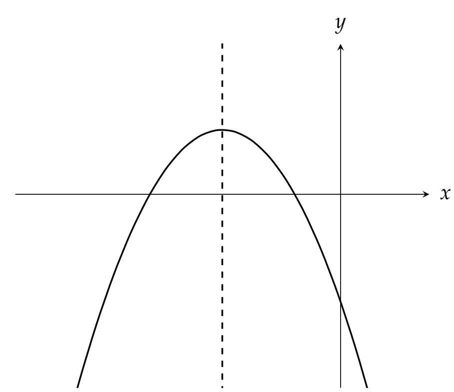
A function defined by ________________ ($a \neq 0$) is called a **quadratic function**. By completing the square $f(x)$ can be expressing in vertex form as $f(x) = a(x-h)^2 + k$.

- The graph of $f$ is a ____________ with vertex __________.
- If __________, the parabola opens ____________, and the ____________ is the ____________ point. The ____________ value of $f$ is _____.
- If __________, the parabola opens ____________, and the ____________ is the ____________ point. The ____________ value of $f$ is _____.
- The ____________________ is __________. This is the ____________ line that passes through the ____________.

On the graphs below, label

- the axis of symmetry with $x = h$
- the vertex with $(h, k)$
- the value of $a$ with $a > 0$ or $a < 0$



## Finding the maximum or minimum of a quadratic function

Watch the video *Applying the Vertex Formula and Graphing a Parabola* to complete the following.

Given $g(x) =$ ____________________.

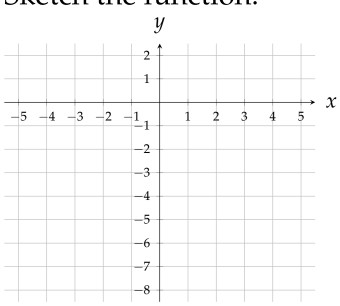
a. Determine whether the graph of the parabola opens upward or downward.

b. Identify the vertex.

c. Determine the $x$-intercept(s).

d. Determine the $y$-intercept.

e. Sketch the function.




f. Determine the axis of symmetry.

g. Determine the minimum or maximum value of the function.

h. Domain: Range:

## Graphing a parabola of the form $y = a(x-h)^2 + k$

Watch the video *Graphing a Parabola Given an Equation in Vertex Form* to complete the following.

Given $h(x) =$ ____________________

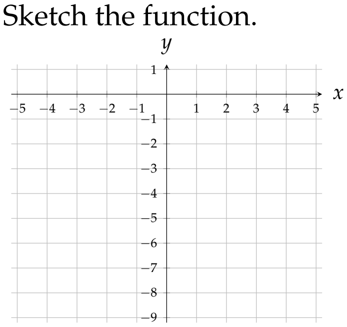
a. Determine whether the graph of the parabola opens upward or downward.

b. Identify the vertex.

c. Determine the $x$-intercepts.

d. Determine the $y$-intercept.

e. Sketch the function.




f. Determine the axis of symmetry.

g. Determine the minimum or maximum value of the function.

h. Domain:

Range:

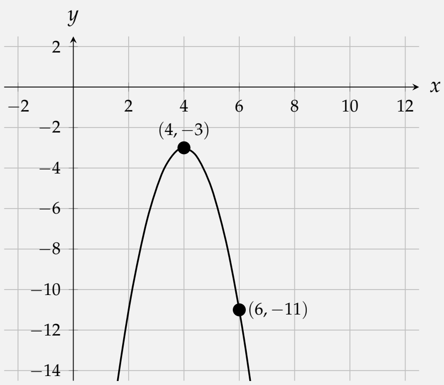
## Writing the equation of a quadratic function given its graph

Learning Page The graph of a quadratic function is a ________________.

Any quadratic function $f$ whose graph has vertex ____________ can be written in the following form.

$$f(x) = \underline{\hspace{4cm}},\text{ where } a \neq 0$$

**EXAMPLE:**
Find the equation of the quadratic function $f$ whose graph is shown below.




A parabola with vertex $(h, k)$ has the form: $y = a(x-h)^2 + k$

The graph has vertex $(4, -3)$ so we have $y = a(x-4)^2 - 3$.

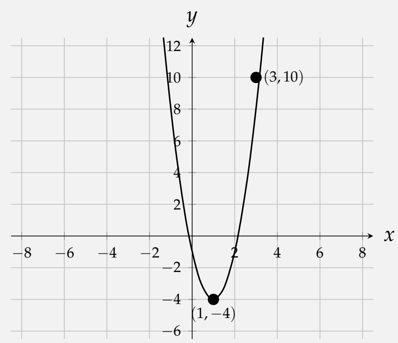
We need to find $a$. We use the other given point: $(6, -11)$, which gives us an $x$ and a $y$ value to substitute and solve then for $a$.

$$\begin{aligned} y &= a(x-4)^2 - 3 \\ -11 &= a(6-4)^2 - 3 \\ -8 &= a(2)^2 \\ -2 &= a \end{aligned}$$

Equation of parabola: $y = -2(x-4)^2 - 3$

**YOU TRY IT:**
Find the equation of the quadratic function $f$ whose graph is shown below.

**135.**



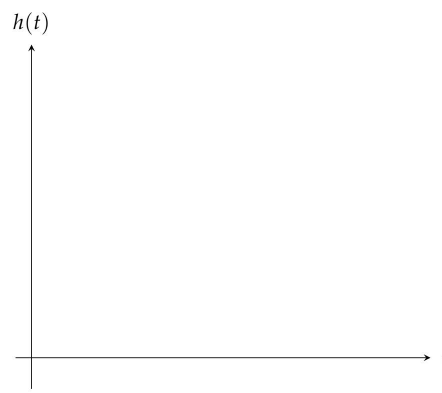
# Word problem involving the maximum or minimum of a quadratic function

Watch the video *Interpreting the Vertex of a Parabola in an Application* to complete the following.

A fireworks mortar is launched straight upward from a pool deck platform 3 m off the ground at an initial velocity of 42 m/sec. The height of the mortar can be modeled by ____________________, where $h(t)$ is the height in ______________ and $t$ is the time in ______________ after launch.

a. Determine the time at which the mortar is at its maximum height. Round to 2 decimal places.

b. What is the maximum height? Round to the nearest meter. Sketch in the graph of the function on the right, labeling the vertex.




**YOU TRY IT:** A ball is thrown vertically upward. After $t$ seconds, its height $h$, in feet, is given by the function $h(t) = 100t - 20t^2$.

**136.** When will the ball reach a maximum height?

**137.** What is the maximum height that the ball will reach?

## Word problem involving optimizing area by using a quadratic function

Watch the video *Applying a Quadratic Function in Geometry* to complete the following.

Suppose that a family wants to fence in an area of their yard for a garden. One side is already fenced from the neighbor's property.

Draw the picture to illustrate this example.

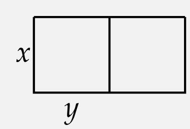
a. If the family has enough money to buy ________ ft of fencing, what dimensions would produce the maximum area for the garden?

Constraint equation: ________________ = ________________

Area equation: ________________

b. What is the maximum area?

**YOU TRY IT:** Two pens are to be built adjacent to one another from 120 ft of fencing.



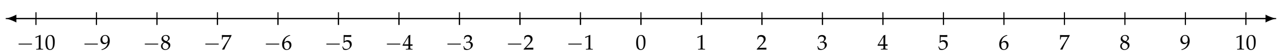

**138.** What dimensions should be used to maximize the area of an individual coop?

**139.** What is the maximum area of an individual coop?

## Solving a quadratic inequality written in factored form

Open the Instructor Added Resource which will direct you to a video to complete the following.

Graph the solution to the inequality ____________________.




$x =$ __________ $x =$ __________ $x =$ __________

## Solving a quadratic inequality

Watch the video *Solving Quadratic Inequalities* to complete the following.

Solve the inequality.

**EXAMPLE:**
Graph the solution to the inequality $x^2 - x < 12$.

We rewrite the inequality, then factor.

$$x^2 - x < 12$$
$$x^2 - x - 12 < 0$$
$$(x-4)(x+3) < 0$$

- We want the values of $x$ that make $(x-4)(x+3)$ less than zero (negative).
- $(x-4)(x+3)$ is equal to zero when $x = 4$ or $x = -3$.

$$\begin{array}{ccccc} & 0 & & 0 & \\ \text{———————} & | & \text{———————} & | & \text{———————} \\ & -3 & & 4 & \end{array}$$

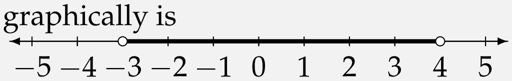
We will test a point in each interval on the number line above.

- For $x = -4$, we have $(-)(-) = +$
- For $x = 0$, we have $(-)(+) = -$
- For $x = 5$, we have $(+)(+) = +$

Note that we do not need the VALUE, just whether it will be positive or negative.

$$\begin{array}{ccccc} +++++++++ & 0 & ------- & 0 & +++++++++ \\ \text{———————} & | & \text{———————} & | & \text{———————} \\ & -3 & & 4 & \end{array}$$

The solution in interval notation is $(-3, 4)$. And graphically is

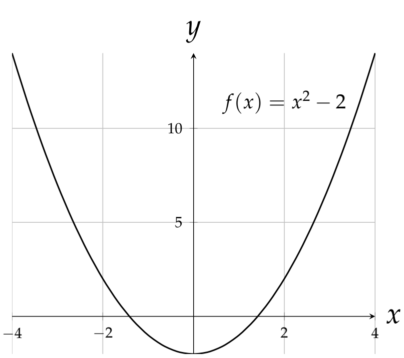



**YOU TRY IT:**

**140.** Graph the solution to the inequality

$$2x^2 - 9x \geq 5.$$

An alternative method to the one shown above is to graph the parabola and determine the answer from the graph. Solve $x^2 - 2 \geq 0$




We can find the $x$-intercepts of the graph $(\sqrt{2}, 0)$ and $(-\sqrt{2},, 0)$. We want the $x$ values where the graph lies on or above the $x$-axis.

The solution is $(-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$.

## Notes from Focus Group:

# Notes from Focus Group:

# Module 11

## Contents



## Weekly Checklist

☐ Complete MALL time.

☐ Work in ALEKS and Notebook at least 3 days a week.

☐ Complete the weekly Module and Notebook pages by the due date.

☐ Attend Focus Group.

☐ Actively participate in Focus Group.

☐ Earn extra credit: Complete 10 topics by ______________.

## Finding the zeros of a quadrtic function given its equation

Learning Page The zeros of a function are the ______________ that give an ______________ of ____.

In our function ________________, they are the ______________ of ____ that make __________.

So, to find the ______________, we set ____ equal to ____ and __________ for ____.

$$0 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$0 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

The product of __________ and __________ must __________ 0.

This will be true if and only if at __________ one of the expressions equals 0.

So we have the following.

$$\_\_\_\_\_\_\_\_\_\_ = 0 \quad \text{or} \quad \_\_\_\_\_\_\_\_\_\_ = 0$$

We solve these two equations for ____.

$$\_\_\_\_\_\_\_\_\_\_ \quad \text{or} \quad \_\_\_\_\_\_\_\_\_\_$$

## Finding a polynomial of a given degree with given zeros: Real zeros

Learning Page The **Factor Theorem** tells us the following.

A number $c$ is a zero of a polynomial $f(x)$ if and only if ____________________.

We also get that, if $c$ is a zero of ______________, then ____________________.

**YOU TRY IT:**

**141.** Find a polynomial $p(x)$ of degree 5 that has zeros $-2, 0, 1$ (multiplicity 2), 7.

# Identifying polynomial functions

Watch the video *Introduction to Polynomial Functions* to complete the following.

Polynomial Function

Not a Polynomial Function

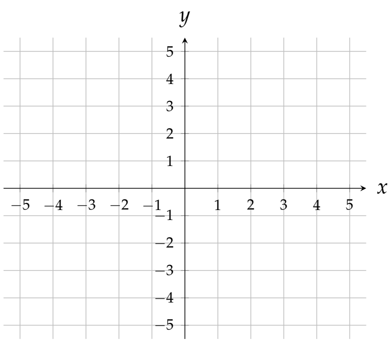
**Definition of a Polynomial Function**

Let $n$ be a whole number and $a_n, a_{n-1}, a_{n-2}, \ldots, a_1, a_0$ be ______________________, where $a_n \neq 0$. Then a function defined by

$$f(x) = \underline{\hspace{6cm}}$$

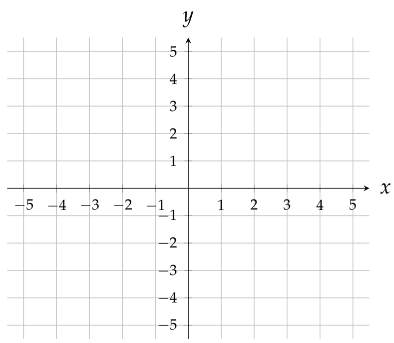
is called a **polynomial function of degree** _____.

$f(x) =$ _____



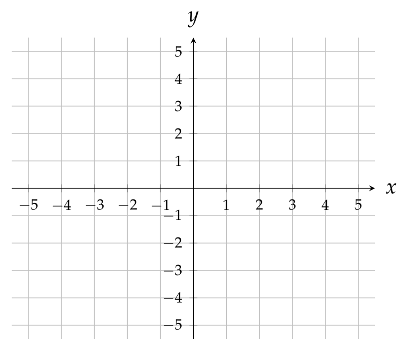

degree = _____

$f(x) =$ __________



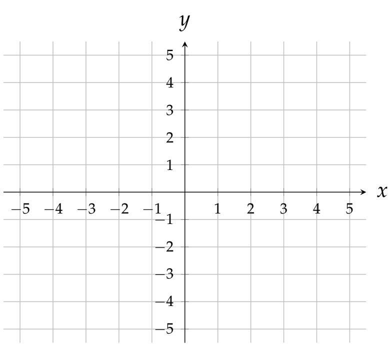

degree = _____

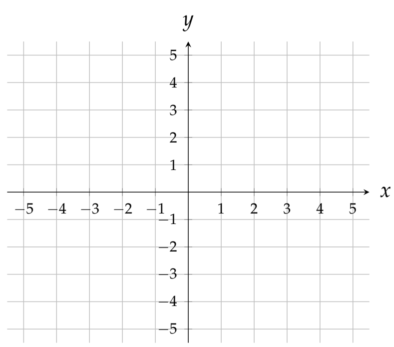
$f(x) =$ _______________

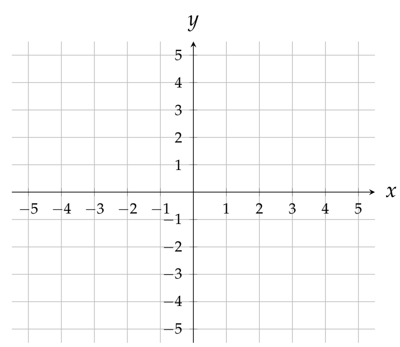



degree = _____

Graph three functions that are NOT polynomials.



**EXAMPLE:**
Identify which of the following are polynomials.

a) $A(x) = 3x^5 - 2x^3 + 5x^{-4}$
This is not a polynomial because the exponent on the term $5x^{-4} = \frac{5}{x^4}$ is not a whole number.

b) $B(x) = x^3 + \sqrt{5}x^2 - 3x + \sqrt{7}$
This is a polynomial. All coefficients are real numbers and all exponents are whole numbers.

c) $C(x) = \dfrac{3-x}{7}$
This is a polynomial, it can be rewritten as $C(x) = \frac{3}{7} - \frac{1}{7}x$. All coefficients are real numbers and all exponents are whole numbers.

d) $D(x) = \dfrac{4-x^2}{x-1}$
This is not a polynomial. It is a ratio of polynomials so is a rational function.

**YOU TRY IT:**
Identify which of the following are polynomials.

**142.** $a(x) = 3x^5 - 2\sqrt{x} + 5x^2$

**143.** $b(x) = \dfrac{5x^4 - 2x^2 + x}{3}$

**144.** $c(x) = -6$

**145.** $d(x) = 2x(x+4)(x-7)(x+1)^4$

## Finding zeros of a polynomial function written in factored form

Learning Page The ______________ of $f$ are the real numbers $x$ for which ______________.

So we set ______________ and ______________.

For a product to ____________, at least one of the ____________ must ____________ 0.

**YOU TRY IT:**

**146.** Find the zeros of $f(x) = 3x^2(x^2-9)(x+4)$

## Finding zeros and their multiplicities given a polynomial function written in factored form

Watch the video *Determining Zeros and Multiplicities* to complete the following.

Determine the zeros of the function and state their multiplicities.

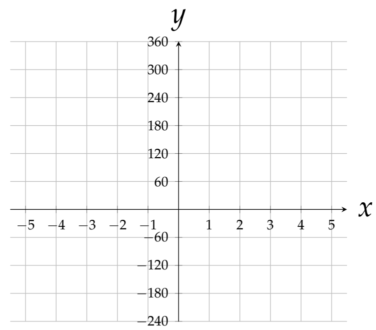
$f(x) =$ ____________________

Zero: ________ Multiplicity: ________

Zero: ________ Multiplicity: ________

Zero: ________ Multiplicity: ________

Zero: ________ Multiplicity: ________




**EXAMPLE:**
Consider the polynomial

$$p(x) = -4x(x-3)^2(x+7)^3(x-1).$$

List each zero and its multiplicity.

Zeros of multiplicity one: 0, 1
Zero of multiplicity two: 3
Zero of multiplicity three: -7

**YOU TRY IT:**

**147.** Consider the polynomial

$$q(x) = 5x^2(x-1)^4(x+5)^2(x+6).$$

List each zero and its multiplicity.

## Finding $x$ and $y$ intercepts given a polynomial function

Learning Page A ***y*-intercept** is the ____________ of a point where the graph ____________ ____________.

A function's graph has ________ $y$-intercept.

To find it, we find the ____________________.

**Continued on the next page**

Watch the video *Identifying Zeros and Multiplicities* to complete the following.

Given a polynomial function defined by $y = f(x)$:

The values of $x$ in the ____________ of $f$ for which ____________ are called the ____________ of the function. These are also called the ________________ of the equation ____________.

Determine the zeros of the function and state their multiplicities.

**EXAMPLE:**
Find all intercepts of $p(x) = 3x^3 + x^2 - 2x$.

a) $y$-intercept
$p(0) = 3(0)^3 + 0^2 - 2(0) = 0$
$(0, 0)$ is the $y$-intercept.

b) $x$-intercept

$$\begin{aligned} 3x^3 + x^2 - 2x &= 0 \\ x(3x^2 + x - 2) &= 0 \\ x(3x-2)(x+1) &= 0 \\ x &= 0, \tfrac{2}{3}, -1 \end{aligned}$$

$(0, 0)$, $\left(\frac{2}{3}, 0\right)$, and $(-1, 0)$ are $x$-intercepts.

**YOU TRY IT:**

**148.** Find all intercepts of $q(x) = 2x^4 - 2x^3 - 24x^2$.

# Determining the end behavior of the graph of a polynomial function

Open the e-book to complete the following.

## Notation for Infinite Behavior of $y = f(x)$

| | |
|---|---|
| $x \to \infty$ | is read as ____________________<br>This means that $x$ becomes infinitely large in the __________ direction |
| $x \to -\infty$ | is read as ____________________<br>This means that $x$ becomes infinitely large in the __________ direction |
| $f(x) \to \infty$ | is read as ____________________<br>This means that the $y$ value becomes infinitely large in the __________ direction |
| $f(x) \to -\infty$ | is read as ____________________<br>This means that the $y$ value becomes infinitely large in the __________ direction |

## The Leading Term Test

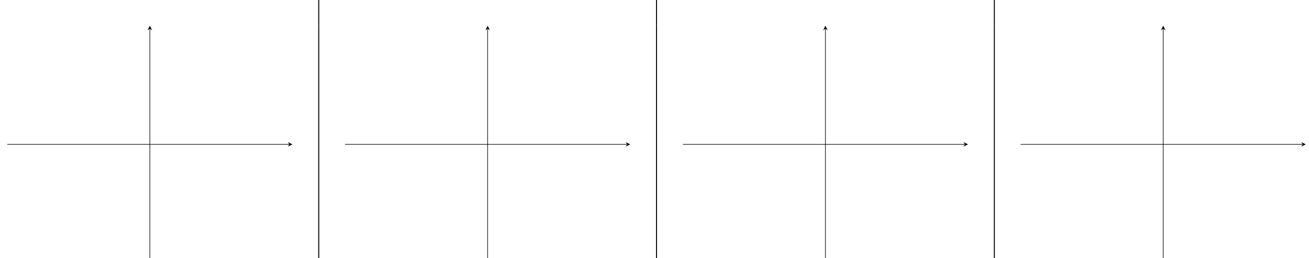
Consider a polynomial function given by

$$f(x) = \underline{\hspace{6cm}}$$

As $x \to \infty$ or as $x \to -\infty$, $f$ eventually becomes forever increasing or forever decreasing and will follow the general behavior of __________.

Compete the chart below, then sketch a graph in each box that represents the correct end behavior.

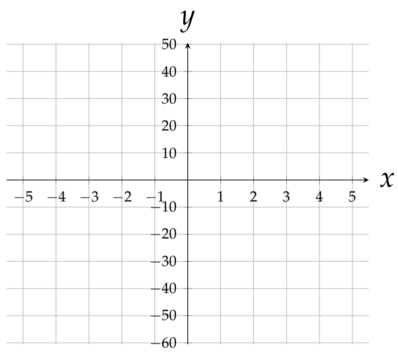
| $n$ is even | | | | $n$ is odd | | | |
|---|---|---|---|---|---|---|---|
| $a_n$ positive | | $a_n$ negative | | $a_n$ positive | | $a_n$ negative | |
| As $x \to -\infty$,<br>$f(x) \to$ ___ | As $x \to \infty$,<br>$f(x) \to$ ___ | As $x \to -\infty$,<br>$f(x) \to$ ___ | As $x \to \infty$,<br>$f(x) \to$ ___ | As $x \to -\infty$,<br>$f(x) \to$ ___ | As $x \to \infty$,<br>$f(x) \to$ ___ | As $x \to -\infty$,<br>$f(x) \to$ ___ | As $x \to \infty$,<br>$f(x) \to$ ___ |



## Determining end behavior and intercepts to graph a polynomial function

Watch the video *Graphing a Polynomial Function* to complete the following.

Graph $k(x) =$ ______________________




leading term:

$y$-intercept:

Zeros:

## Matching graphs with polynomial functions

Open the e-book to complete the following.

**Touch Points and Cross Points**

Let $f$ be a polynomial function and let $c$ be a real zero of $f$. The point __________ is an $x$-intercept of the graph of $f$. Furthermore,

- If $c$ is a zero of __________ multiplicity, the graph ______________ the $x$-axis at $c$.

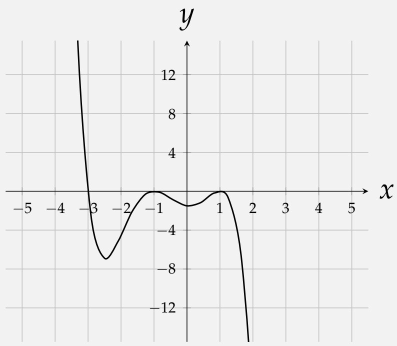
  The point $(c, 0)$ is called a ____________________.

- If $c$ is a zero of __________ multiplicity, the graph ______________ the $x$-axis at $c$.

  The point $(c, 0)$ is called a ____________________.

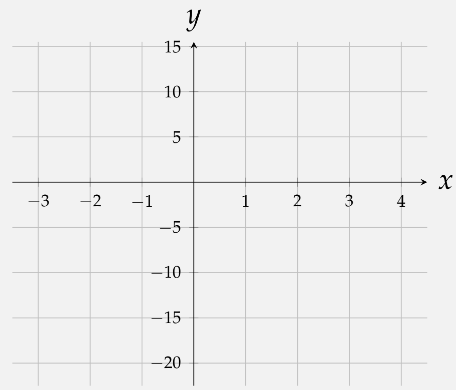
**EXAMPLE:**
Sketch the graph of
$$f(x) = -\tfrac{1}{2}(x-1)^2(x+3)(x+1)^2.$$

- The touch points of $f$ are $(1,0)$ and $(-1,0)$.
- The cross point of $f$ is $(-3,0)$.
- The $y$-intercept of $f$ is $\left(0,-\frac{3}{2}\right)$.
- The degree of $f$ is 5 and $a_n$ is negative so as $x \to \infty$, $f(x) \to -\infty$ and as $x \to -\infty$, $f(x) \to \infty$.




**YOU TRY IT:**

**149.** Sketch the graph of
$$g(x) = x^2(x+1)^2(x-3)(x-2).$$




## Inferring properties of a polynomial function from its graph

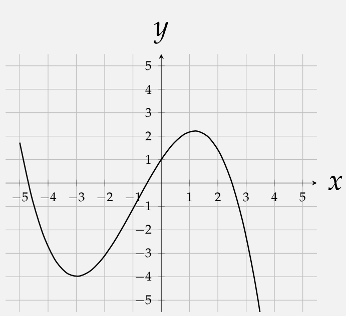
Watch the video *Turning Points of a Graph of a Polynomial Function* to complete the following.

**Number of Turning Points of a Polynomial Function**

Let $f$ represent a polynomial function of ____________________. Then the graph of $f$ has at most

__________ turning points.

**YOU TRY IT:** Below is the graph of a polynomial function $f$ with real coefficients. Use the graph to answer the following questions.




**150.** At what $x$-values does $f$ have local minima?

**151.** What is the sign of the leading coefficient of $f$?

**152.** What is the lowest possibility for the degree of $f$?

## Polynomial long division: Problem type 2

Watch the video *Long Division of Polynomials with a Linear Divisor* to complete the following.

**EXAMPLE:**
Use polynomial long division to evaluate:
$(x^4 + 3x^3 + x - 5) \div (x^2 - 3)$

$$
\begin{array}{r|rrrrr}
 & x^2 & +3x & +3 & & \\
\cline{2-6}
x^2 - 3 & x^4 + 3x^3 & & & +x & -5 \\
 & -x^4 & & +3x^2 & & \\
\cline{2-4}
 & & 3x^3 & +3x^2 & +x & \\
 & & -3x^3 & & +9x & \\
\cline{3-5}
 & & & 3x^2 & +10x & -5 \\
 & & & -3x^2 & & +9 \\
\cline{4-6}
 & & & & 10x & +4
\end{array}
$$

So $(x^4 + 3x^3 + x - 5) \div (x^2 - 3)$

$$= x^2 + 3x + 3 + \frac{10x + 4}{x^2 - 3}$$

**YOU TRY IT:**
Use polynomial long division to evaluate:

**153.** $(2x^5 + x^4 - x^3 - x - 1) \div (x^2 - 2x + 1)$

## The Factor Theorem

Watch the video *Introduction to the Factor Theorem* to complete the following.

**Factor Theorem**

Let $f(x)$ be a polynomial.

1. If $f(c) = 0$, then ______________ is a ______________ of $f(x)$.

2. If ______________ is a factor of $f(x)$, then ______________.

Use the Factor Theorem to determine if the given binomial is a factor of $f(x)$.

$$f(x) = x^4 + 11x^3 + 41x^2 + 61x + 30$$

a. ______________

b. ______________

**EXAMPLE:**
Use the Factor Theorem to determine whether $x + 1$ is a factor of $p(x) = -3x^3 + 4x^2 - 2x - 6$.

$$\begin{aligned} p(-1) &= -3(-1)^3 + 4(-1)^2 - 2(-1) - 6 \\ &= 3 + 4 + 2 - 6 \\ &= 3 \end{aligned}$$

$p(-1) \neq 0$ so $x + 1$ is not a factor of $p(x)$.

**YOU TRY IT:**

**154.** Use the Factor theorem to determine whether $x + 4$ is a factor of $q(x) = x^3 - 13x + 12$.

## Synthetic division

Watch the video *Introduction to Synthetic Division* to complete the following.

Divide.

**EXAMPLE:**
Use synthetic division to evaluate:
$(x^4 - 14x^2 + 5x - 9) \div (x + 4)$

$$\begin{array}{r|rrrrr} -4 & 1 & 0 & -14 & 5 & -9 \\ & & -4 & 16 & -8 & 12 \\ \hline & 1 & -4 & 2 & -3 & 3 \end{array}$$

So $(x^4 - 14x^2 + 5x - 9) \div (x + 4)$

$$= x^3 - 4x^2 + 2x - 3 + \frac{3}{x+4}$$

**YOU TRY IT:**
Use synthetic division to evaluate:

**155.** $(2x^4 - x^3 - 3x - 1) \div (x - 2)$

## Using a given zero to write a polynomial as a product of linear factors: Real zeros

Watch the video *Factoring a Polynomial Given a Zero of the Polynomial* to complete the following.

a. Factor $f(x) =$ ______________________ given that $\frac{1}{4}$ is a zero.

b. Solve ____________________________

## Finding the intercepts, asymptotes, domain, and range from the graph of a rational function

Watch the video *Introduction to Rational Functions* to complete the following.

a. As $x \to -\infty$, $f(x) \to$ __________

b. As $x \to 4^{-}$, $f(x) \to$ __________

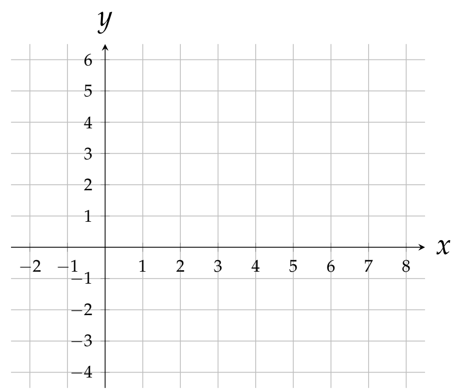
c. As $x \to 4^{+}$, $f(x) \to$ __________

d. As $x \to \infty$, $f(x) \to$ __________

e. Increasing:

f. Decreasing:

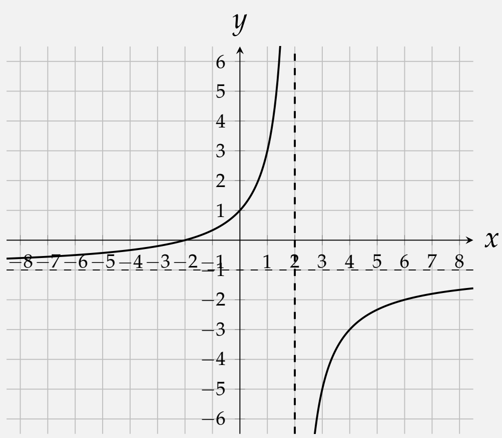
g. Domain:



h. Range:

In mathematics, $\to$ means the word ________________.

**YOU TRY IT:** Use the graph to answer the following questions about $f(x)$.



**156.** Find the domain of $f(x)$.

**157.** Find the range of $f(x)$.

**158.** Find all asymptotes of $f(x)$.

## Finding the asymptotes of a rational function: Constant over linear

Learning Page

**Vertical asymptote(s):**

A rational function in simplest form has vertical asymptotes at the ________________ of the __________________________.

**Horizontal asymptote(s):**

A rational function can have _______________________________ horizontal asymptote.

To find the horizontal asymptotes (if any), we compare the __________________ $n$ of the _______________________ with the _______________________ of the __________________________.

- If ________________, the horizontal asymptote is ________________.
- If ________________, the horizontal asymptote is given by _______________________________.
- If ________________, there is __________ horizontal asymptote.

**YOU TRY IT:**

**159.** Find all vertical and horizontal asymptotes of the function $f(x) = \dfrac{7}{3x-2}$.

## Finding the asymptotes of a rational function: Linear over linear

Open the e-book to complete the following.

**Definition of a Vertical Asymptote**

The line ______________ is a **vertical asymptote** of the graph of a function $f$ if $f(x)$ approaches

______________ or ________________________ as $x$ approaches _________ from either side.

**Identifying Vertical Asymptotes of a Rational Function**

Consider a rational function $f$ defined by ___________________, where $p(x)$ and $q(x)$ have

____________________________ other than 1.

If $c$ is a ____________________________, then ______________ is a ______________ asymptote of the graph of $f$.

**Definition of a Horizontal Asymptote**

The line ______________ is a **horizontal asymptote** of the graph of a function $f$ if

____________________________ infinity or negative infinity.

**Identifying Horizontal Asymptotes of a Rational Function**

Let $f$ be a rational function defined by

$$f(x) = \frac{a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + b_{m-2}x^{m-2} + ... + b_1 x + b_0}$$

The definition of $f(x)$ indicates that _____ is the ______________ of the ______________ and _____ is the ______________ of the ______________.

1.

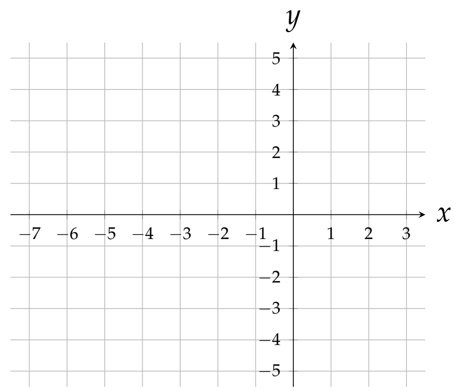
2.

3.

## Finding horizontal and vertical asymptotes of a rational function: Quadratic numerator or denominator

Watch the video *Identifying Vertical Asymptotes Algebraically 1* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

$f(x) =$ ________________ $=$ ________________




Denominator is zero when: __________________

Numerator is zero when: ______________

Vertical asymptote(s): ______________

**EXAMPLE:**
Find all asymptotes of

$$f(x) = \frac{x+1}{(x-2)(x+3)}.$$

- The numerator and denominator share no common factors other than 1.
  - To find the vertical asymptotes we consider the zeros of the denominator which are 2 and -3.
- Vertical asymptotes:
  - $x = 2$
  - $x = -3$
- Horizontal asymptote:
  - We look at the degree of the top compared to the degree of the bottom.
  - As $x$ gets large, $y$ will get close to zero so the horizontal asymptote is $y = 0$.

**YOU TRY IT:**

**160.** Find all asymptotes of

$$f(x) = \frac{x^2}{x^2 - 9}.$$

## Graphing a rational function: Constant over linear

Open the e-book to complete the following.

**Graphing a Rational Function**

Consider a rational function $f$ defined by $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials with no common factors.

1. Determine the ____________________ by evaluating __________.
2. Determine the ____________________ by finding the __________ solutions of _______________.

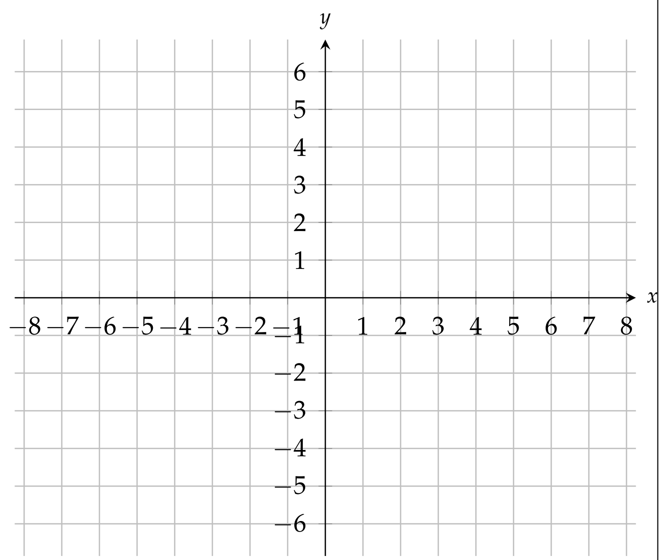
   The value $f(x)$ equals __________ when ____________________.
3. Identify any _________________________ and graph them as _______________ lines.
4. Determine whether the function has a ______________________________ or a slant asymptote (or neither), and graph the asymptote as a _______________ line.
5. Determine where the function crosses the _______________ or slant asymptote (if applicable).
6. If a test for ____________________ is easy to apply, use ____________________ to plot additional points. Recall:
   - $f$ is an even function (symmetric to the _______________) if ____________________.
   - $f$ is an odd function (symmetric to the _______________) if ____________________.
7. Plot at least one point on the intervals defined by the $x$-intercepts, vertical asymptotes, and any points where the function crosses a horizontal or slant asymptote.
8. Sketch the function based on the information found in steps 1-7.

## Graphing a rational function: Linear over linear

Open the Instructor Added Resource which will direct you to a video to complete the following.

Sketch the graph of $f(x) =$ ____________________

Vertical asymptote:



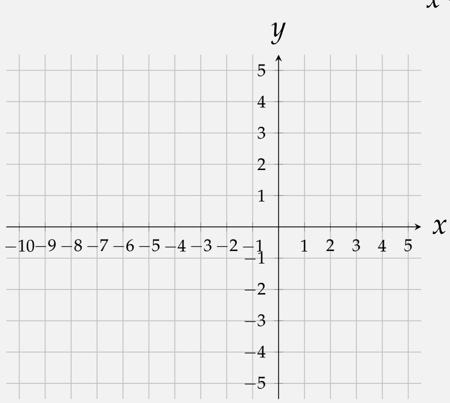
Horizontal asymptote:

$x$-intercept(s):

$y$-intercept:

**YOU TRY IT:**

**161.** Sketch the graph of $f(x) = \dfrac{2x-2}{x+3}$.



## Matching graphs with rational functions: Two vertical asymptotes

Watch the video *Graphing a Rational Function* to complete the following.

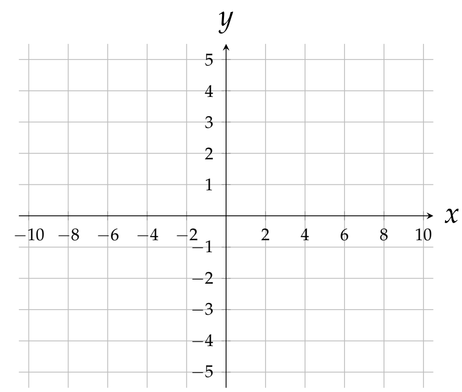
Graph $r(x) =$ ____________________ $=$ ____________________

$y$-intercept:

$x$-intercept(s):

Vertical asymptote(s):

Horizontal or slant asymptote:




## Notes from Focus Group:

# Notes from Focus Group:

# Module 12-Review

To help you review for your upcoming exam, this module contains all of the topics from the modules since the last exam. Topics that you have already mastered will not appear in your carousel.

- ☐ Complete this module before you take the ALEKS exam.
- ☐ Each exam has two parts.
  - The ALEKS exam (100 pts)
    - The ALEKS exam must be taken in the MALL.
    - The ALEKS exam is a Comprehensive Knowledge Check.
      - Your score is the number of topics you have mastered out of the number of topics you should have mastered by this point.
    - If you lose topics on your ALEKS exam, your Review Module completion grade will not change.
    - Your scratch work for the ALEKS exam must be numbered and turned in through Blackboard.
  - The Written exam (25 pts)
    - Take your written exam in class the day of your focus group.
    - To study for the written exam:
      - Rework your old Focus Group assignments.
      - Rework any topics in ALEKS you may have lost on the ALEKS exam.

| | Score |
|---|---|
| ALEKS Exam | |
| Written Exam | |

# Module 13

## Contents



## Weekly Checklist

- ☐ Complete MALL time.
- ☐ Work in ALEKS and Notebook at least 3 days a week.
- ☐ Complete the weekly Module and Notebook pages by the due date.
- ☐ Attend Focus Group.
- ☐ Actively participate in Focus Group.
- ☐ Earn extra credit: Complete 10 topics by ____________________.

## Horizontal line test

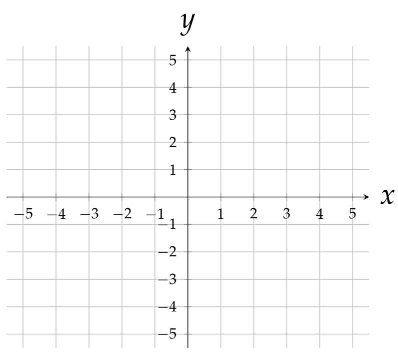
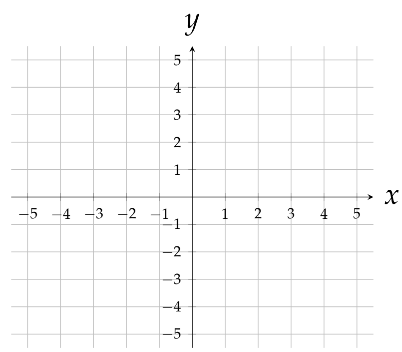
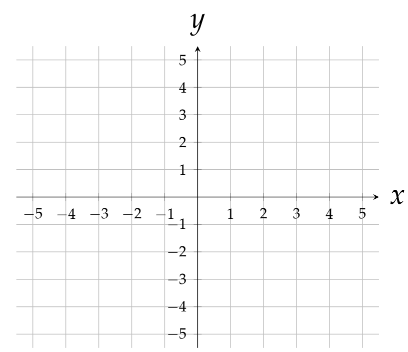
Watch the video *Applying the Horizontal Line Test* to complete the following.

**Horizontal Line Test**

A function defined by $y = f(x)$ is ________________ if ____________________

intersects the graph in ________________________.

Sketch the graph and determine if the relation defines $y$ as a one-to-one function of $x$.

## Graphing the inverse of a function given its graph

Learning Page To get the graph of ________, we take the ________________________ of ____ and

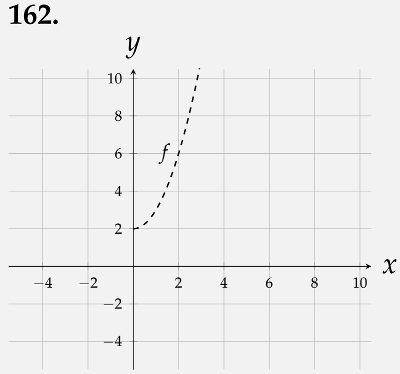
____________ them.

That is, we ________________________________________________.

We see that the graph of $f^{-1}$ is the ______________ of the ____________ of ____ over the line ________.

**YOU TRY IT:** The graph of $f(x)$ is given below. Sketch the graph of $f^{-1}(x)$ on the same axes.

**162.**



## Determining whether two functions are inverses of each other

Watch the video *Determining Whether Two Functions are Inverses* to complete the following.

Determine whether the two functions are inverses.

$f(x) =$ ______________ and $g(x) =$ ______________

Let $f$ be a ________________ function. Then $g$ is the inverse of $f$ if the following conditions are both true.

1.

2.

**YOU TRY IT:**

**163.** Determine if $f(x) = 3x + 7$ and $g(x) = \frac{x-3}{7}$ are inverses.

## Inverse functions: Linear, discrete

Learning Page For a given ______________ function $f$, there is a related function, _________, which is the ____________________________.

The function $f$ maps __________________ if and only if $f^{-1}$ maps __________________.

So, the ____________________________ are the ____________________________ and vice versa.

More precisely,

__________________ if and only if __________________.

There is a general method to find the inverse of a function that is defined by an equation.

**Step 1:**

**Step 2:**

**Step 3:**

**Step 4:** .

The composition of a function with its inverse always gives an ______________ ______________ to the ______________.

Watch the video *Introduction to Inverse Functions* to complete the following.

$f=$ _________, _________, _________, _________

Domain:

Range:

$f^{-1}=$ _________, _________, _________, _________

Domain:

Range:

**EXAMPLE:**
Given $f = \{(1,3),(2,4),(5,7)\}$, find the following

a) $f^{-1}$
The inverse function $f^{-1}$ reverses the ordered pairs of $f$.
$f^{-1} = \{(3,1),(4,2),(7,5)\}$.

b) $f^{-1}(7)$
From part a) we see $f^{-1}(7) = 5$.

c) $(f^{-1} \circ f)(1)$

$$(f^{-1} \circ f)(1) = f^{-1}(f(1)) = f^{-1}(3) = 1$$

**YOU TRY IT:**
Given $g = \{(3,0),(2,5),(4,6),(7,9)\}$, find the following.

**164.** $g^{-1}$

**165.** $g^{-1}(5)$

**166.** $(g^{-1} \circ g)(7)$

**EXAMPLE:**
Given $g(x) = 3x - 7$, find the following

a) $g^{-1}(x)$

$$\begin{aligned} g(x) &= 3x - 7 \\ y &= 3x - 7 \\ x &= 3y - 7 \\ x + 7 &= 3y \\ \frac{x+7}{3} &= y \\ g^{-1}(x) &= \frac{x+7}{3} \end{aligned}$$

b) $(g \circ g^{-1})(4)$
From the definition of inverse function we know $(g \circ g^{-1})(x) = x$ for all $x$ in the domain. So $(g \circ g^{-1})(4) = 4$.

**YOU TRY IT:**
Given $f(x) = \frac{1}{7}x + 5$.

**167.** Find $f^{-1}(x)$

**168.** Find $(f \circ f^{-1})(-3)$.

## Inverse functions: Quadratic, square root

Watch the video *Finding the Inverse of Function with a Restricted Domain* to complete the following.

a. Graph $f(x) =$________________; ____________.

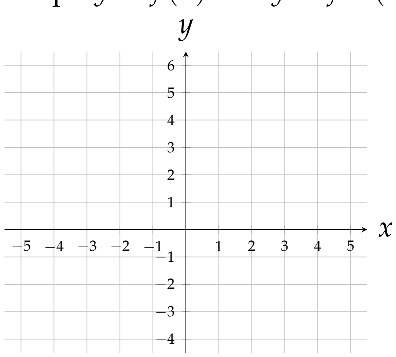
b. From the graph of $f$, is $f$ a one-to-one function?

c. Write the domain of $f$ in interval notation.

d. Write the range of $f$ in interval notation.

e. Write an equation for $f^{-1}(x)$.

f. Graph $y = f(x)$ and $y = f^{-1}(x)$ on the same coordinate system.




g. Write the domain of $f^{-1}(x)$ in interval notation.

h. Write the range of $f^{-1}(x)$ in interval notation.

**EXAMPLE:**

a) Find the inverse of $f(x) = \sqrt{x-4} + 3$.

$$
\begin{aligned}
f(x) &= \sqrt{x-4} + 3 \\
y &= \sqrt{x-4} + 3 \\
x &= \sqrt{y-4} + 3 \\
x - 3 &= \sqrt{y-4} \\
(x-3)^2 &= y - 4 \\
x^2 - 6x + 9 + 4 &= y \\
f^{-1}(x) &= x^2 - 6x + 13 \quad \text{for } \; x \geq 3
\end{aligned}
$$

We need the extra condition $x \geq 3$ because otherwise $f^{-1}(x)$ is NOT one-to-one.

b) Find the inverse of $g(x) = x^2 + 2x - 4$ where $x \geq -1$.

$$
\begin{aligned}
g(x) &= x^2 + 2x - 4 \\
y &= x^2 + 2x - 4 \\
x &= y^2 + 2y - 4 \\
x &= y^2 + 2y + 1 - 1 - 4 \\
x &= (y+1)^2 - 5 \\
x + 5 &= (y+1)^2 \\
\sqrt{x+5} &= y + 1 \\
\sqrt{x+5} - 1 &= f^{-1}(x)
\end{aligned}
$$

**YOU TRY IT:**

**169.** Find the inverse of $f(x) = \sqrt{3x-1} + 2$.

**170.** Find the inverse of $g(x) = x^2 - 6x - 4$ where $x \geq 3$.

## Inverse functions: Cubic, cube root

**EXAMPLE:**

Find the inverse of $f(x) = \sqrt[3]{x-7} + 4$.

$$y = \sqrt[3]{x-7} + 4$$

Switch $x$ and $y$.

$$x = \sqrt[3]{y-7} + 4$$

Subtract 4.

$$x - 4 = \sqrt[3]{y-7}$$

Cube both sides.

$$(x-4)^3 = (\sqrt[3]{y-7})^3$$

Simplify.

$$(x-4)^3 = y - 7$$

Add 7.

$$(x-4)^3 + 7 = y$$

$$f^{-1}(x) = (x-4)^3 + 7$$

**YOU TRY IT:**

**171.** Find the inverse of $f(x) = (x+4)^3$.

## Finding, evaluating, and interpreting an inverse function for a given linear relationship

**EXAMPLE:** Steve is walking and his distance $D$ in miles from Fargo after $x$ hours of walking is given by $D(x) = 11.6 - 4x$.

a. Describe in words what $D^{-1}(x)$ means.

   With a function and its inverse we are "switching" the domain and range.
   The input for $D^{-1}(x)$ will be a distance and the output will be a time.

   $D^{-1}(x)$ represents the amount of time in hours that Steve has walked when he is $x$ miles from Fargo.

b. Find $D^{-1}(x)$.

$$\begin{aligned} y &= 11.6 - 4x \\ x &= 11.6 - 4y \\ x - 11.6 &= -4y \\ \tfrac{x-11.6}{-4} &= y \\ D^{-1}(x) &= \tfrac{11.6-x}{4} \end{aligned}$$

## Table for an exponential function

Learning Page The table gives ______________ $x$ and their corresponding ______________ $h(x)$.

We use the rule $\left(\frac{a}{b}\right)^{-n}$ = ______________

**YOU TRY IT:** Complete the tables below.

**172.**

| $x$ | $g(x) = 5^x$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| $-1$ | |
| $-2$ | |
| $-3$ | |

**173.**

| $x$ | $f(x) = (\frac{1}{3})^x$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| $-1$ | |
| $-2$ | |
| $-3$ | |

## Graphing an exponential function and its asymptote: $f(x) = b^x$

Watch the video *Graphing Exponential Functions* to complete the following.

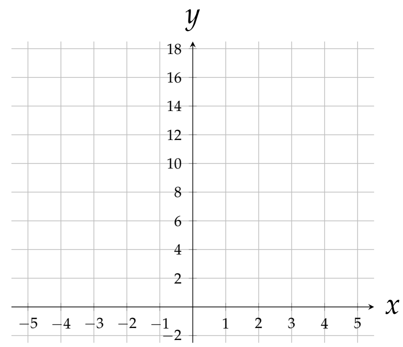
Graph the functions. Sketch $g(x)$ with a solid line and $k(x)$ with a dashed line.

a. $g(x) =$ ___________

| $x$ | $g(x)$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| $-1$ | |
| $-2$ | |
| $-3$ | |

b. $k(x) =$ ___________

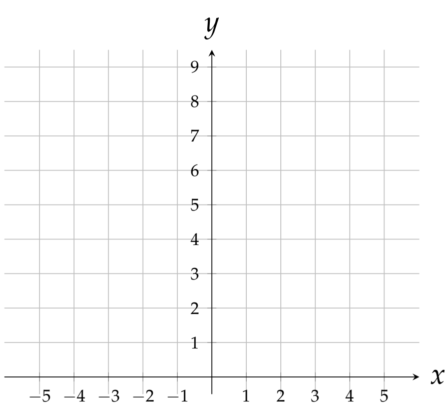
| $x$ | $k(x)$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| $-1$ | |
| $-2$ | |
| $-3$ | |

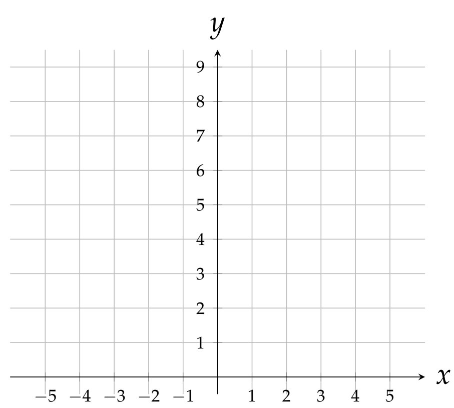


## Translating the graph of an exponential function

Watch the video *Graphing an Exponential Function Using Transformations* to complete the following.

Graph $g(x) =$ ________________

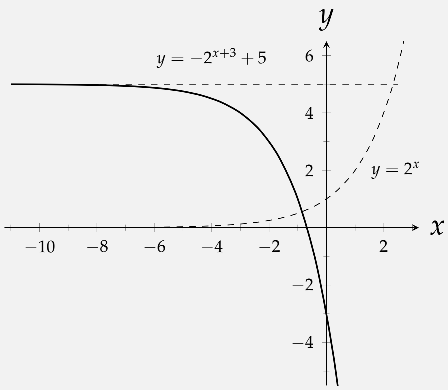




**EXAMPLE:** Sketch the graph of $y = -2^{x+3} + 5$.

This is the graph of $y = 2^x$ transformed by

- Shifting left 3 units
- Reflecting across the $x$-axis
- Shifting up 5 units




**YOU TRY IT.**

**174.** Sketch the graph of $y = 3^{x-2} - 4$

## The graph, domain, and range of an exponential function

Open the e-book to complete the following.

**Graphs of $f(x) = b^x$**

The graph of an exponential function defined by $f(x) = b^x$ where $b > 0$ and $b \neq 1$ has the following properties.

1. If $b > 1$, $f$ is an ________________ exponential function, sometimes called an exponential ________________ function.

   If $0 < b < 1$, $f$ is a ________________ exponential function, sometimes called an exponential ________________ function.

2. The domain is ________________________________.

3. The range is ________________.

4. The line ________ is a ________________.

5. The function passes through the point ________ (this is the $y$-intercept) because $f(0) = b^0 = 1$.

## Transforming the graph of a natural exponential function

Learning Page

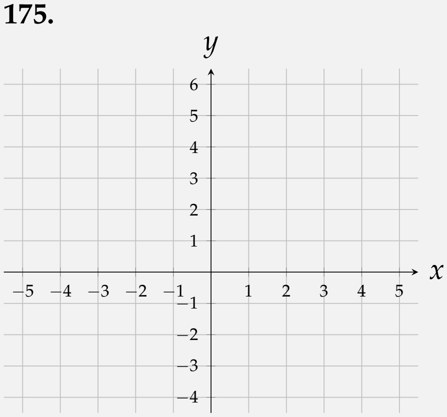
Some ways to transform the graph of a function.

1.
2.
3.

In what order is it a good idea to perform the transformations? ____________________

**YOU TRY IT:** Sketch the graph of $y = e^{x-1} - 3$

**175.**



## Evaluating an exponential function with base $e$ that models a real-world situation

Watch the video *Applying an Exponential Function to Newton's Law of Cooling* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

The temperature $T(t)$ of an object set to cool is modeled by

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

where $T_a$ is the ____________ of the surrounding ______

$T_0$ is the _________ temperature of the object

$t$ is the _________ since the hot object was set to cool

$k$ is a _________ related to the physical ____________ of the object

A cake comes out of the oven at ______ and is placed on a cooling rack in a ______ kitchen.

After checking the temperature several minutes later, it is determined that the cooling rate $k$ is ______.

a. Write a function that models the temperature $T(t)$ (in °F) of the cake $t$ minutes after being removed from the oven.

b. What is the temperature of the cake 10 min after coming out of the oven? Round to the nearest degree.

c. It is recommended that the cake should not be frosted until it has cooled to under 100°F. If Jessica waits 1 hr to frost the cake, will the cake be cool enough to frost?

**EXAMPLE:**
A bacteria population size increases according to $P(t) = 1700e^{0.18t}$ where $t$ is measured in hours. Find the initial number in the population and the number after 7 hours.

- Initial number
  We want the number of bacteria after 0 hours so we compute $P(0)$.
  $P(0) = 1700e^{0.18(0)} = 1700$
- Number after 7 hours $P(7) = 1700e^{0.18(7)} \approx 5993$

**YOU TRY IT:**
The velocity $v(t)$ in m/s of an object falling near Earth's surface is given by $v(t) = 49(1 - e^{-0.22t})$ where $t$ is measured in seconds.

**176.** Find the velocity of the object after 4 seconds.

## Evaluating an exponential function that models a real-world situation

**EXAMPLE:**
The dollar value $c(t)$ of a car that is $t$ years old is given by $c(t) = 19,900(0.86)^t$. Find the value initial value of the car and the value of the car after 11 years.

- Initial value
  The initial value will be the value of the car at 0 years so we compute $c(0)$.
  $c(0) = 19,900(0.86)^0 = \$19,900$
- Value after 11 years
  We are computing $c(11)$.
  $c(11) = 19,900(0.86)^{11} \approx \$3787$

**YOU TRY IT:**
A radioactive substance has a half-life of 14 hours. The amount $a(t)$ in grams of a sample remaining after $t$ hours is given by

$$a(t) = 2800\left(\frac{1}{2}\right)^{\frac{t}{14}}$$

**177.** Find the initial amount in the sample.

**178.** Find the amount remaining after 30 hours.

## Converting between logarithmic and exponential equations

Watch the video *Converting from Logarithmic Form to Exponential Form* to complete the following.

Write each equation in exponential form.

a.

b.

________________ is the same as ________________

c.

Watch the video *Converting from Exponential Form to Logarithmic Form* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

Write each equation in logarithmic form.

a.

b.

________________ is the same as ________________

c.

**EXAMPLE:**

a) Write $\log_5 x = y$ as an exponential equation.

$$\log_5 x = y$$
$$5^y = x$$

b) Write $c^6 = 3$ as a logarithmic equation.

$$c^6 = 3$$
$$\log_c 3 = 6$$

**YOU TRY IT:**

**179.** Write $\log_4 5 = x$ as an exponential equation.

**180.** Write $7^y = 9$ as a logarithmic equation.

## Converting between natural logarithmic and exponential equations

Learning Page For any numbers $a$, $b$, and $c$, with $a$ and $c$ positive ($a \neq 1$), we have the following equivalence.

$\log_a c = b$ if and only if ______________

The first is a ____________________ equation, and the second is an ____________________ equation.

However, when the base is __________, we do __________ write ____________________.

Instead, we write __________, which is read as ______________________________

$e$ is a special ____________________ number. Its value is $e =$______________

So, when the base of the logarithm is $e$, we write the relationship as follows.

____________________ if and only if ____________________

**EXAMPLE:**

a) Write $\ln 8 = x$ as an exponential equation.

$$\ln 8 = x$$
$$e^x = 8$$

b) Write $e^y = 2$ as a logarithmic equation.

$$e^y = 2$$
$$\ln 2 = y$$

**YOU TRY IT:**

**181.** Write $\ln x = 5$ as an exponential equation.

**182.** Write $e^r = t$ as a logarithmic equation.

## Evaluating logarithmic expressions

Watch the video *Evaluating Common and Natural Logarithms* to complete the following.

Simplify the expressions.

a.

b.

**YOU TRY IT:** Simplify the expressions.

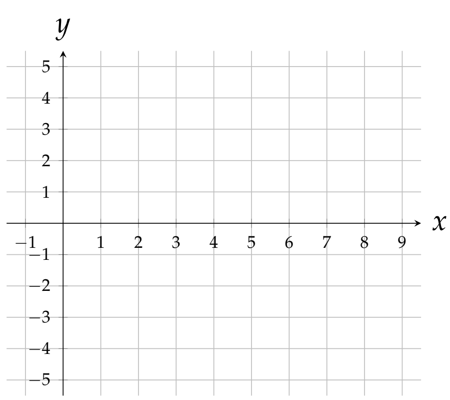
**183.** $\log_5 \frac{1}{125}$

**184.** $\ln e^5$

## Graphing a logarithmic function: Basic

Watch the video *Graphing a Logarithmic Function* to complete the following.

Graph ____________________.



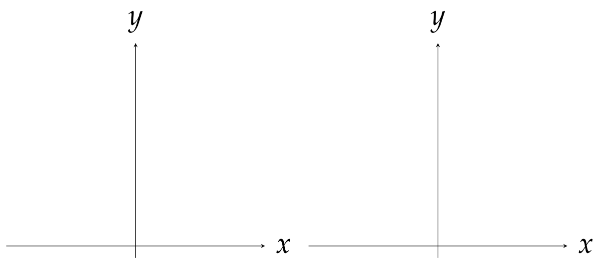

| | $y$ |
|---|---|
| | 0 |
| | |
| | 2 |
| | |
| | $-2$ |

## The graph, domain, and range of a logarithmic function

Open the e-book to complete the following.

**Graphs of Exponential and Logarithmic Functions**

**Exponential Functions**




$b > 1$ $\quad$ $0 < b < 1$

Domain: ____________

Range: ____________

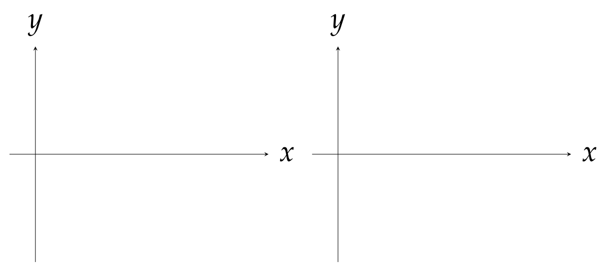
Horizontal asymptote: ____________

Passes through: ____________

If $b > 1$, the function is ____________.

If $0 < b < 1$, the function is ____________.

**Logarithmic Functions**




$b > 1$ $\quad$ $0 < b < 1$

Domain: ____________

Range: ____________

Vertical asymptote: ____________

Passes through: ____________

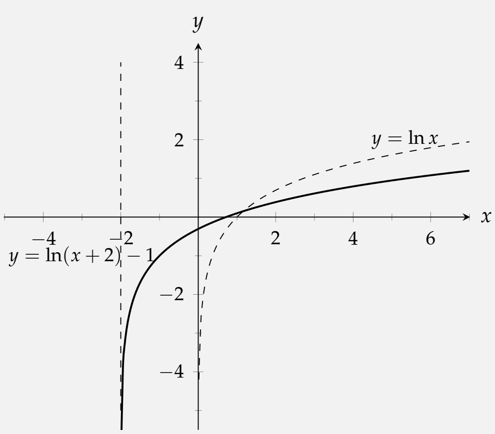
If $b > 1$, the function is ____________.

If $0 < b < 1$, the function is ____________.

**EXAMPLE:**
Sketch the graph of $y = \ln(x+2) - 1$.

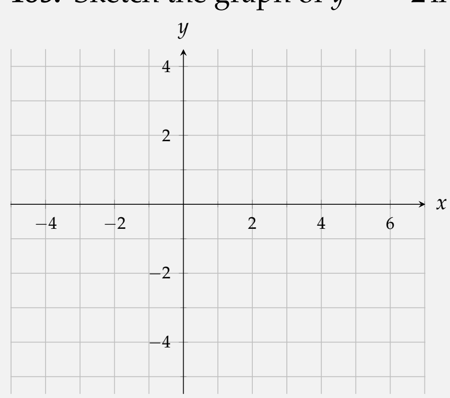
This is the graph of $y = \ln x$ shifted

- left 2 units
- down 1 unit




**YOU TRY IT:**

**185.** Sketch the graph of $y = -2\ln(x+3) + 1$.



## Domain of a logarithmic function: Advanced

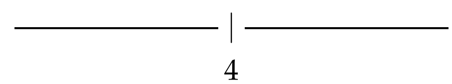
Watch the video *Identifying the Domain of a Logarithmic Function* to complete the following.

The domain of ____________________ is restricted to _______________

Write the domain in interval notation.

a. $f(x) =$ _______________

b. $r(x) =$ _______________




**YOU TRY IT:** Find the domain of the function. Write your answer in interval notation.

**186.** $g(x) = \log(x+7)$

## Notes from Focus Group:

## Notes from Focus Group:

# Module 14

## Contents



## Weekly Checklist

☐ Complete MALL time.

☐ Work in ALEKS and Notebook at least 3 days a week.

☐ Complete the weekly Module and Notebook pages by the due date.

☐ Attend Focus Group.

☐ Actively participate in Focus Group.

☐ Earn extra credit: Complete 10 topics by ____________________.

## Basic properties of logarithms

Open the e-book to complete the following.

**Product Property of Logarithms**

Let $b$, $x$, and $y$ be positive real numbers where $b \neq 1$. Then

$$\log_b(xy) = \underline{\hspace{4cm}}$$

The logarithm of a product equals the ______________________________________.

**Quotient Property of Logarithms**

Let $b$, $x$, and $y$ be positive real numbers where $b \neq 1$. Then

$$\log_b\left(\frac{x}{y}\right) = \underline{\hspace{4cm}}$$

The logarithm of a quotient equals the ______________ of the logarithm of the ______________ and the ______________ of the ______________.

**Power Property of Logarithms**

Let $b$, $x$, and $y$ be positive real numbers where $b \neq 1$. Let $p$ be any real number. Then

$$\log_b x^p = \underline{\hspace{4cm}}.$$

**Properties of Logarithms**

Let $b$, $x$, and $y$ be positive real numbers where $b \neq 1$, and let $p$ be any real number. Then the following properties of logarithms are true.

1. $\log_b 1 =$ _____
2. $\log_b b =$ _____
3. $\log_b b^p =$ _____
4. $b^{\log_b x} =$ _____

## Using properties of logarithms to evaluate expressions

Learning Page Let $a$, $b$, and $c$ be any real numbers, with $a$ and $c$ positive, and $a \neq 1$.

We have the following definition of the **logarithm**.

____________________ if and only if ____________________

From this definition, we get the following fact.

_________________________

However, when the base is __________, we do __________ write _______________.

Instead we write __________, which is read as _________________________.

____________________ if and only if ____________________

From this definition, we get the following fact.

_________________________

We also have the following properties of logarithms.

| | |
|---|---|
| Logarithm of a product: | $\log_a M + \log_a N =$ _______________ |
| Logarithm of a quotient: | $\log_a M - \log_a N =$ _______________ |
| Logarithm of a power: | $p \log_a M =$ _______________ |

For these properties, $a$, $M$, and $N$ are ____________________ numbers, with __________, and _____ is any

____________________.

**YOU TRY IT:** Use the properties of logarithms to evaluate the expression.

**187.** $6 \ln e^4 - \ln e^3$

## Expanding a logarithmic expression: Problem type 1

Watch the video *Applying the Product Property of Logarithms* to complete the following.

**Product Property of Logarithms**

Let $b$, $x$, and $y$ be positive real numbers where __________. Then,

_______________ = ____________________

Example:

Write the logarithm as a sum and simplify if possible.

**EXAMPLE:** Expand $\log\left(\dfrac{x^3y^5}{z}\right)$.

Use the Quotient Property

$$\log\left(\frac{x^3y^5}{z}\right) = \log(x^3y^5) - \log z$$

Use the Product Property

$$= \log x^3 + \log y^5 - \log z$$

Use the Power Property

$$= 3\log x + 5\log y - \log z$$

**YOU TRY IT:**

**188.** Expand $\ln\left(\dfrac{xz^2}{y^5}\right)$.

## Expanding a logarithmic expression: Problem type 2

Watch the video *Writing a Logarithmic Expression in Expanded Form* to complete the following.

Write the expression as the sum or difference of logarithms.

## Writing an expression as a single logarithm

Watch the video *Writing the Sum or Difference of Logarithms as a Single Logarithm 2* to complete the following.

Write the logarithmic expression as a single logarithm with coefficient 1, and simplify if possible.

**EXAMPLE:** Write $\frac{1}{2}\ln y - \frac{1}{3}\ln x + \ln 2$ as a single log.

$$\begin{aligned}\frac{1}{2}\ln y - \frac{1}{3}\ln x + \ln 2 &= \ln y^{1/2} - \ln x^{1/3} + \ln 2 \\ &= \ln\sqrt{y} - \ln\sqrt[3]{x} + \ln 2 \\ &= \ln\left(\frac{\sqrt{y}}{\sqrt[3]{x}}\right) + \ln 2 \\ &= \ln\left(\frac{2\sqrt{y}}{\sqrt[3]{x}}\right)\end{aligned}$$

**YOU TRY IT:**

**189.** Write $\log(x-1) + \log 3 - 3\log x$ as a single log.

## Solving an equation of the form $\log_b a = c$

Learning Page

For any numbers $a$, $b$, and $c$, with $a$ and $c$ positive ($a \neq 1$), we have the following relationship.

____________________ if and only if _______________

**EXAMPLE:** Solve.

$\log_2 x = -3$

Use the relationship above.

$2^{-3} = x$

$\frac{1}{8} = x$

**YOU TRY IT:** Solve.

**190.** $\log_x 2 = \frac{1}{3}$

## Solving a multi-step equation involving a single logarithm: Problem type 1

Read EXAMPLE 8: **Solving a Logarithmic Equation** to complete the following steps.

Solve. ______________________

**Solution:**

$4\log_3(2t-7) = 8$

| | |
|---|---|
| $\log_3(2t-7) = 2$ | Isolate the _____________ by _____________ both sides by 4. |
| | The equation is in the form _________________ where _________________ |
| _________________ | Write the equation in _____________ form. |
| $2t - 7 = 9$ <br> $t = 8$ | Check: $4\log_3(2t-7) = 8$ <br> $4\log_3[2(8)-7] \stackrel{?}{=} 8$ <br> $4\log_3 9 \stackrel{?}{=} 8$ <br> $4 \cdot 2 \stackrel{?}{=} 8\checkmark$ |

**YOU TRY IT:** Solve.

**191.** $5\log_6(7x+1) = 10$

## Solving a multi-step equation involving a single logarithm: Problem type 2

Watch the video *Solving a Logarithmic Equation by Writing Exponential Form* to complete the following.

### Solving Logarithmic Equations by Using Exponential Form

**Step 1** Given a logarithmic equation, ________________________________.

**Step 2** Use the ________________________ to write the equation in the form ________________ where $k$ is a constant.

**Step 3** Write the equation in ____________________.

**Step 4** ____________ the equation from ________________.

**Step 5** ____________ the potential solution(s) in the ____________________.

Solve.

Check:

**EXAMPLE:** Solve.

$$\log_3(x-1) - \log_3 4 = 2$$

Use Quotient Property of Logs.

$$\log_3 \frac{x-1}{4} = 2$$

Use Def of Log.

$$\frac{x-1}{4} = 3^2$$

$$x - 1 = 36$$

$$x = 37$$

**YOU TRY IT:** Solve.

**192.** $-7 + \log_4(x+3) = -5$

## Solving a multi-step equation involving natural logarithms

Open the Instructor Added Resource which will direct you to a video to complete the following.

Solve for $x$.

**YOU TRY IT:** Solve for $x$.

**193.** $\ln(x+2) = 4$

## Solving an equation involving logarithms on both sides: Problem type 1

Watch the video *Solving a Logarithmic Equation 2* to complete the following.

Solve ______________________________

**YOU TRY IT:** Solve the equation.

**194.** $\log_3 x + \log_3(x+6) = 3$

## Solving an equation involving logarithms on both sides: Problem type 2

Watch the video *Solving a Logarithmic Equation by Using the Equivalence Property* to complete the following.

**Equivalence Property of Logarithmic Expressions**

Let $b$, $x$, and $y$ be positive real numbers with $b \neq 1$. Then,

____________________ implies that ________________

Solve ____________________.

**EXAMPLE:**
Solve.

$$\begin{aligned} \log_5(x+18) + \log_5(x-6) &= 2\log_5 x \\ \log_5((x+18)(x-6)) &= \log_5 x^2 \\ (x+18)(x-6) &= x^2 \\ x^2 + 12x - 108 &= x^2 \\ 12x - 108 &= 0 \\ 12x &= 108 \\ x &= 9 \end{aligned}$$

**YOU TRY IT:**
Solve.

**195.** $\log_2 x + \log_2(x-4) = \log_2(x+24)$

## Solving an exponential equation by finding common bases: Linear exponents

Watch the video *Solving an Exponential Equation by Using the Equivalence Property* to complete the following. NOTE: This may not be the first video that pops up. Select this video from the list of videos on the left of the video box.

**Equivalence Property of Exponential Expressions**

Let $b$, $x$, and $y$ be real numbers with $b > 0$ and $b \neq 1$. Then,

$b^x = b^y$ implies that ________________

Solve.

Check:

**Continued on the next page**

Learning Page For any positive number $A$ such that $A \neq 1$, we have the following.

____________________ if and only if _______________

We can write each side of our equation with the ____________________ and then apply this property.

**EXAMPLE:**
Solve.

$$32^{x-4} = 64$$

Rewrite each side with base 2.

$$(2^5)^{x-4} = 2^6$$

Simplify exponent on left.

$$2^{5x-20} = 2^6$$

Use property from above.

$$5x - 20 = 6$$
$$5x = 26$$
$$x = \frac{26}{5}$$

**YOU TRY IT:**
Solve.

**196.** $4^{x+2} = \frac{1}{2^x}$

## Solving an exponential equation by using logarithms: Exact answers in logarithmic form

Watch the video *Solving an Exponential Equation by Using Logarithms 3* to complete the following.

Solve _________________________

**EXAMPLE:**
Solve.

$$\begin{aligned} 4^{x+2} &= 7^x \\ \ln 4^{x+2} &= \ln 7^x \\ (x+2)\ln 4 &= x \ln 7 \\ x\ln 4 + 2\ln 4 &= x\ln 7 \\ x\ln 4 - x\ln 7 &= -2\ln 4 \\ x(\ln 4 - \ln 7) &= -2\ln 4 \\ x &= \frac{2\ln 4}{\ln 4 - \ln 7} \end{aligned}$$

**YOU TRY IT:**
Solve.

**197.** $e^{x-2} = 9$

## Finding the time given an exponential function with base $e$ that models a real-world situation

Read EXAMPLE 5 Part b: **Creating a Model for Exponential Decay** to complete the following steps.

An archeologist uncovers human remains at an ancient Roman burial site and finds that ________ of the carbon-14 still remains in the bone. How old is the bone? Round to the nearest hundred years.

**Solution:**

| | |
|---|---|
| ________________ $= Q_0 e^{-0.000121t}$ | The quantity $Q(t)$ of carbon-14 in the bone is ________ of _____ |
| ________________ $= e^{-0.000121t}$ | Divide by _____ on both sides. |
| $\ln 0.7666 =$ ____________ | Take the ____________ ________________ of both sides. |
| $t =$ ________________ $\approx$ ________ | Divide by ________ to isolate _____. |

The bone is ________________ ________ years old.

## Finding the initial amount and rate of change given an exponential function

Learning Page

A function in the following form models ______________________________.

______________________ (where $a > 0, b > 0,$ and __________)

Here, $y$ is an ______________ and $t$ is the ______________________________. Note the following.

- The constant __________ is the _________________________, that is, the value of

  __________________________.

- The constant __________ tells whether the functions models ______________________________.
  - If _______________, then the function models _______________.
  - If _______________, then the function models _______________.

- From the value of __________, we can also ge the _______________ of growth or decay.
  - If _______________, then $b$ equals _______________, where $r$ is the _________________________.

    That is __________ is the _________________________ (expressed as a decimal) for each

    ______________________________.
  - If _______________, then $b$ equals _______________, where $r$ is the _________________________.

    That is __________ is the _________________________ (expressed as a decimal) for each

    ______________________________.

## Notes from Focus Group:

## Notes from Focus Group:

# Module 15-Final Review

To help you review for your upcoming final exam, this module contains all of the topics from the course. Topics that you have already mastered will not appear in your carousel.

- ALEKS final exam
  - The ALEKS final exam must be taken in the MALL.
  - The ALEKS final exam is a Comprehensive Knowledge Check.
  - The ALEKS final exam must be completed by ______________________.
- To study for the final exams:
  - Complete this ALEKS Final Review Module.
  - Rework the problems on your old exams.
  - Review your old Focus Group assignments.

# Solutions

**Module 1**

**1.** $-\frac{8}{125}$

**2.** $\frac{1}{25}$

**3.** $\frac{4}{x^5}$

**4.** $3x^7$

**5.** $\frac{x^{12}}{27}$

**6.** $\sqrt{x^7}$

**7.** $x^{4/3}$

**8.** 2

**9.** 2

**10.** 4

**11.** 27

**12.** 64

**13.** $\frac{1}{9}$

**14.** $7\sqrt{6}$

**15.** $x \neq \frac{1}{3}$

**16.** $x = -\frac{39}{11}$

**17.** $y = -7$

**18.** $x = -2$

**19.** $x = -13$

**20.** $d = \frac{2S-an}{n}$

**21.** $c = 3A - a + b$

**22.** $y = 2$

**23.** $w = 25$

**24.** $t = \frac{27}{4}$

**Module 2**

**25.** $7i$

**26.** $4i\sqrt{3}$

**27.** $x = \frac{2}{3}, -7$

**28.** $y = 0, 9$

**29.** $x = 5, 3$

**30.** $x = -3, -2$

**31.** $y = -\frac{3}{2}, \frac{7}{5}$

**32.** $3x^2 - 9x - 30 = 0$

**33.** $u = 2$

**34.** $x = 2 \pm \sqrt{10}$

**35.** $x = \frac{-3\pm\sqrt{14}}{2}$

**36.** $x = -\frac{1}{3} \pm i\frac{\sqrt{2}}{3}$

**37.** length: 24 ft
height: 10 ft

**38.** $x = 4 \pm \sqrt{14}$ sec
$x \approx .26$ sec, $7.74$ sec

**39.** $x = \frac{1}{3}, 2$

**40.** $x = -\frac{1}{3}$

**41.** $y = 3, -2$

**42.** $(0, -5)$, Answers may vary

**43.** $x = -1 \pm 2i$

**Module 3**

**44.** $x$-intercepts: $(\sqrt{5}, 0), (-\sqrt{5}, 0)$
$y$-intercepts: $(0, \sqrt{7}), (0, -\sqrt{7})$

**45.** $x$-intercept: $\frac{3}{7}$
$y$-intercept: $-\frac{3}{5}$

**46.** $-\frac{2}{3}$

**47.** 0

**48.** $y = \frac{3}{4}x - \frac{9}{2}$

**49.** $y = -\frac{7}{3}x + \frac{2}{3}$

**50.** $x = -4$

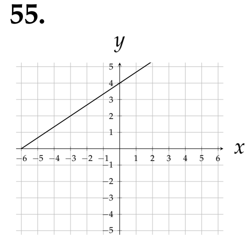
**51.** $y = -12$

**52.** perpendicular

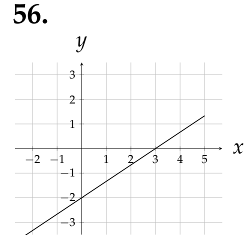
**53.** $y = \frac{3}{4}x - 5$

**54.** $y = -\frac{4}{3}x + \frac{10}{3}$

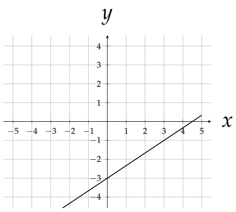
**55.**



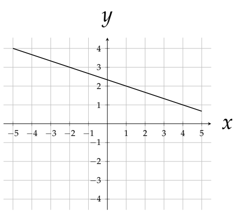

**56.**



**57.** Slope is $\frac{2}{3}$
$y$-intercept $(0, -3)$



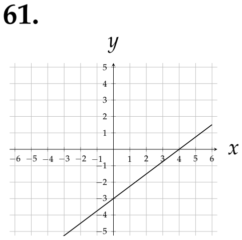

**58.**



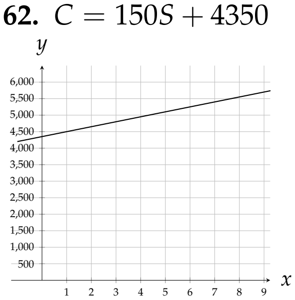

**59.** 4

**60.** $-3$

**61.**




**62.** $C = 150S + 4350$




**63.** \$15 per toy produced

**64.** \$1100

**65.** $(1, 2)$

**66.** $(-2, 2)$

**67.** A notebook is \$1.85 and a pen is \$0.65.

**Module 5**

**68.** $x < -\frac{7}{2}$

**69.** $x = 7, -7$

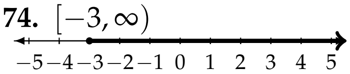
**70.** $-4, -10$

**71.** No solution

**72.** $x = 5, 9$

**73.** $c \leq 500$

**74.** $[-3, \infty)$




**75.** $\emptyset$

**76.** $(-\infty, 2] \cup (5, \infty)$

**77.** No Solution

**78.** $x = 10$

**79.** 14 m/sec

**80.** $x = 6$

**81.** $x = 4\sqrt[3]{2} - 5$

**82.** Function

**83.** Not a Function

**84.** Function

**85.** Not a Function

**86.** $f(-4) = -\frac{2}{3}$

**87.** 17

**88.** 22

**89.** 3

**90.** \$531.44

**91.** \$660

**Module 6**

**92.** $75x^2 - 20x + 7$

**93.** $\sqrt{17 - 4x^2}$

**94.** domain: $\{2, -5, 0, 5\}$
range: $\{3, 1, -4\}$

**95.** $3, -3$

**96.** $(-\infty, -3) \cup (-3, 5) \cup (5, \infty)$

**97.** $(-\infty, \frac{4}{7}]$

**98.** $(-\infty, \frac{9}{7})$

**99.** Domain: $(-\infty, \infty)$
Range: $[-2, \infty)$

**100.** domain: $0 \leq x \leq 20$
range: $0 \leq y \leq 100$

**101.** 2

**102.** 0

**103.** \$610

**104.** 17 weeks

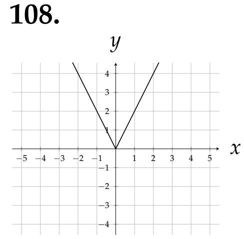
**105.** $-3$

**106.** 20

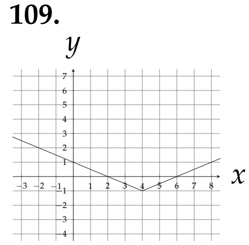
**107.** As time increases, the amount of candy in the container increases by 60 pounds per minute.

**Module 7**

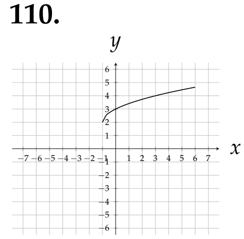
**108.**



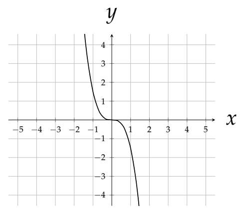

**109.**



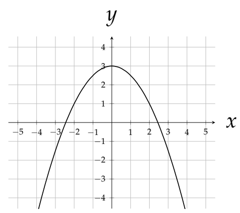

**110.**



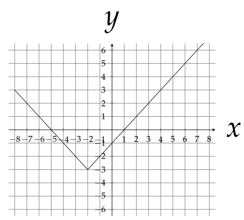
**111.**



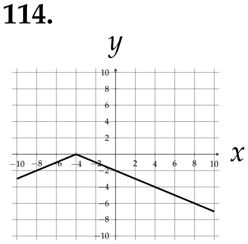
**112.**



**113.**



**114.**



**115.** $y = (x-4)^2 - 6$

**116.** Domain: $(-\infty, \infty)$
Range: $[-6, \infty)$

**Module 9**

**117.** $y$-axis

**118.** symmetric to the $x$-axis, $y$-axis and the origin

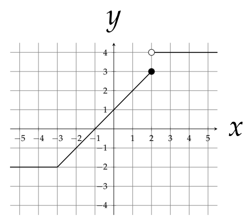
**119.** local min value: 0
local max value: 4

**120.** max at $x = 0$
min at $x = -2, 2$

**121.** Increasing on $(-\infty, -2)$
Decreasing on $(1, \infty)$
Constant on $(-2, 1)$

**122.** $-8x - 4h + 5$

**123.**



**124.** $3x^2 - x - 2$
D: $(-\infty, 0) \cup (0, \infty)$

**125.** $-\frac{2}{3}$

**126.** Not defined

**127.** $C(x) = 3.5x + 640$

**128.** $R(x) = 25x$

**129.** $(R-C)(x) = 21.5x - 640$
Represents the monthly profit for selling $x$ necklaces.

**130.** $(g \circ f)(x) = \frac{3-x}{3-4x}$
D: $(-\infty, 0) \cup (0, \frac{3}{4}) \cup (\frac{3}{4}, \infty)$

**131.** $(f \circ f)(x) = x$

**132.** $(g \circ g)(x) = x^4 - 10x^2 + 20$

**Module 10**

**133.** $181,832$

**134.** $x = 2, -7$

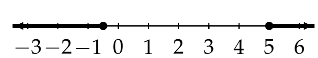
**135.** $y = \frac{7}{2}(x-1)^2 - 4$

**136.** 2.5 sec

**137.** 125 feet

**138.** 20 ft by 15 ft

**139.** 300 ft$^2$

**140.** 

**Module 11**

**141.** $p(x) = x(x+2)(x-1)^2(x-7)$

**142.** Not a polynomial

**143.** polynomial

**144.** polynomial

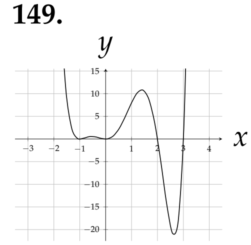
**145.** polynomial

**146.** $0, 3, -3, -4$

**147.** Zero of multiplicity one: -6
Zeros of multiplicity two: 0,-5
Zero of multiplicity four: 1

**148.** $x$-intercepts: $(0,0)$, $(-3,0)$, $(4,0)$
$y$-intercept: $(0,0)$

**149.**



**150.** $x = -3$

**151.** negative

**152.** 3

**153.** $2x^3 + 5x^2 + 7x + 9 + \frac{10x-10}{x^2-2x+1}$

**154.** $q(-4) = 0$ so $x+4$ is a factor.

**155.** $2x^3 + 3x^2 + 6x + 9 + \frac{17}{x-2}$

**156.** $(-\infty, 2) \cup (2, \infty)$

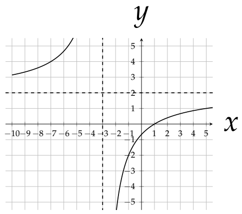
**157.** $(-\infty, -1) \cup (-1, \infty)$

**158.** Vertical asymptote: $x = 2$
Horizontal asymptote: $y = -1$

**159.** Vertical: $x = \frac{2}{3}$
Horizontal: $y = 0$

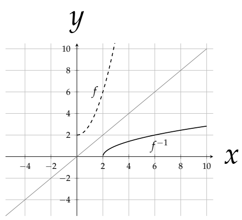
**160.** $x = 3, x = -3, y = 1$

**161.**




## Module 13

**162.**




**163.** $(f \circ g)(x) = \frac{3x+40}{7}$ so $f$ and $g$ are NOT inverses.

**164.** $g^{-1} = \{(0,3), (5,2), (6,4), (9,7)\}$

**165.** 2

**166.** 7

**167.** $f^{-1}(x) = 7x - 35$

**168.** $-3$

**169.** $f^{-1}(x) = \frac{1}{3}x^2 - \frac{4}{3}x + \frac{5}{3}$ for $x \geq 2$

**170.** $g^{-1}(x) = \sqrt{x+13} + 3$

**171.** $f^{-1}(x) = \sqrt[3]{x} - 4$

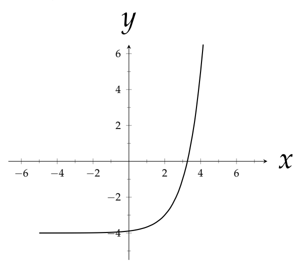
**172.**

| $x$ | $g(x) = 5^x$ |
|---|---|
| 0 | 1 |
| 1 | 5 |
| 2 | 25 |
| 3 | 125 |
| $-1$ | $\frac{1}{5}$ |
| $-2$ | $\frac{1}{25}$ |
| $-3$ | $\frac{1}{125}$ |

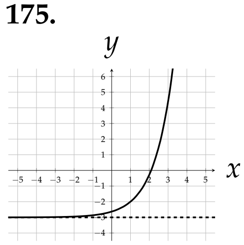
**173.**

| $x$ | $f(x) = (\frac{1}{3})^x$ |
|---|---|
| 0 | 1 |
| 1 | $\frac{1}{3}$ |
| 2 | $\frac{1}{9}$ |
| 3 | $\frac{1}{27}$ |
| $-1$ | 3 |
| $-2$ | 9 |
| $-3$ | 27 |

**174.**




**175.**




**176.** 29 m/s

**177.** 2800 grams

**178.** 634 grams

**179.** $4^x = 5$

**180.** $\log_7 9 = y$

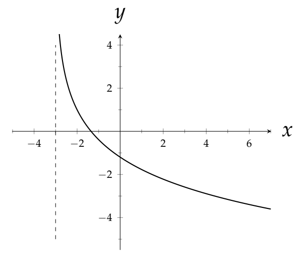
**181.** $e^5 = x$

**182.** $\ln t = r$

**183.** $-3$

**184.** 5

**185.**




**186.** $(-7, \infty)$

## Module 14

**187.** 21

**188.** $\ln x + 2\ln z - 5\ln y$

**189.** $\log\left(\frac{3x-3}{x^3}\right)$

**190.** $x = 8$

**191.** 5

**192.** $x = 13$

**193.** $x = e^4 - 2$

**194.** 3

**195.** $x = 8$

**196.** $x = -\frac{4}{3}$

**197.** $x = \ln 9 + 2$

# Index

| ARITHMETIC PROPERTIES | | | |
|---|---|---|---|
| **Associative:** | addition: $a+(b+c)=(a+b)+c$<br>multiplication: $a(bc)=(ab)c$ | **Identity:** | addition: $0+a=a$<br>multiplication: $1 \cdot a = a$ |
| **Commutative:** | addition: $a+b=b+a$<br>multiplication: $ab=ba$ | **Inverse:** | addition: $a+(-a)=0$<br>multiplication: $a \cdot \frac{1}{a}=1,\ a \neq 0$ |
| **Distributive:** | $a(b+c)=ab+ac$ | | |

| FRACTIONS | | | |
|---|---|---|---|
| **Adding:** | $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$ | **Multiplying:** | $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$ |
| **Subtracting:** | $\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$ | **Dividing:** | $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{ad}{bc}$ |

| FACTORING | |
|---|---|
| **Difference of Two Squares**<br>$a^2-b^2=(a-b)(a+b)$<br>$a^2+b^2=$Does not factor | **Sum and Difference of Two Cubes**<br>$a^3+b^3=(a+b)(a^2-ab+b^2)$<br>$a^3-b^3=(a-b)(a^2+ab+b^2)$ |
| **Perfect Square Trinomials**<br>$a^2-2ab+b^2=(a-b)^2$<br>$a^2+2ab+b^2=(a+b)^2$ | |

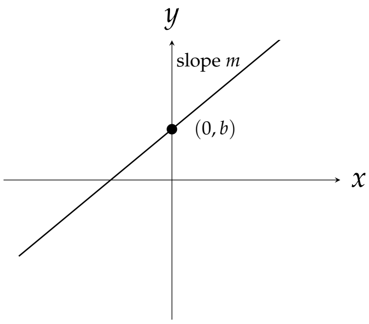
| DISTANCE AND MIDPOINT FORMULAS | |
|---|---|
| Distance between $(x_1,y_1)$ and $(x_2,y_2)$<br>$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ | Midpoint between $(x_1,y_1)$ and $(x_2,y_2)$<br>$m=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ |

| ABSOLUTE VALUE | | | |
|---|---|---|---|
| Statement | Equivalent Statement | Statement | Equivalent Statement |
| $\|x\|=a$ | $x=a$ or $x=-a$ | $\|x\|\leq a$ | $-a\leq x\leq a$ |
| $\|x\|=\|y\|$ | $x=y$ or $x=-y$ | $\|x\|\geq a$ | $x\leq -a$ or $x\geq a$ |

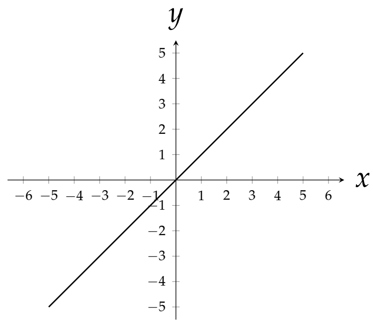
| CIRCLE |
|---|
| Standard Form of a Circle with center $(h,k)$ and radius $r$: $(x-h)^2+(y-k)^2=r^2$ |

## COMMON GRAPHS

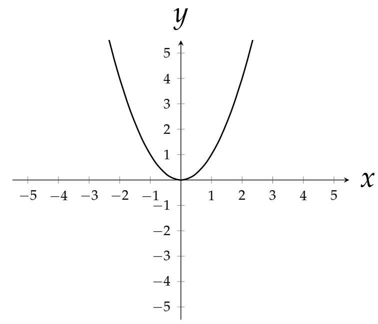
$f(x) = mx + b$



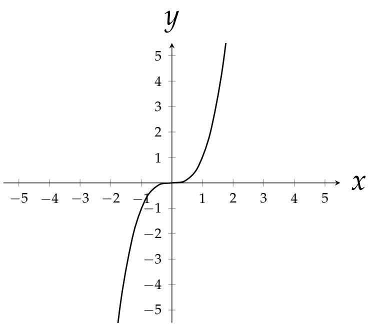

$f(x) = x$



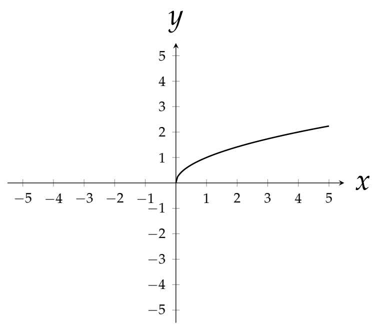
$f(x) = x^2$



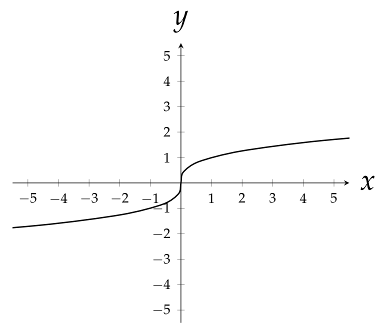
$f(x) = x^3$



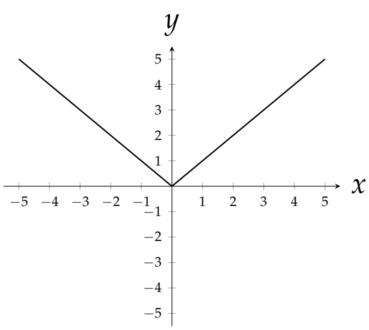
$f(x) = \sqrt{x}$



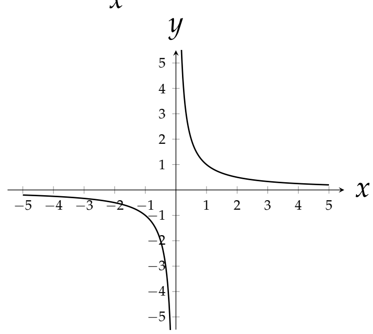
$f(x) = \sqrt[3]{x}$



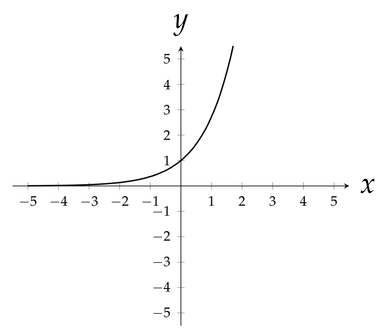
$f(x) = |x|$



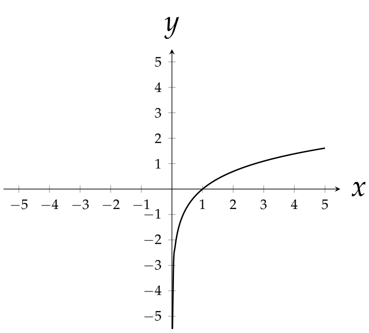
$f(x) = \dfrac{1}{x}$



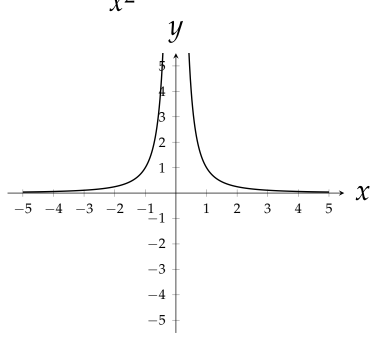
$f(x) = e^x$



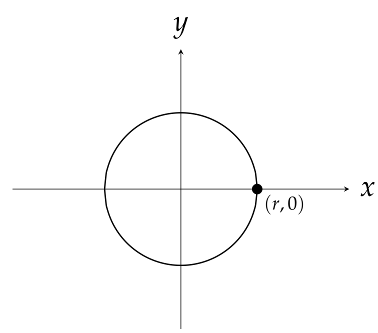
$f(x) = \ln x$

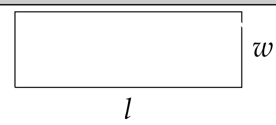
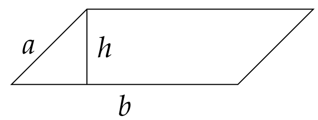
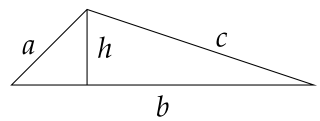
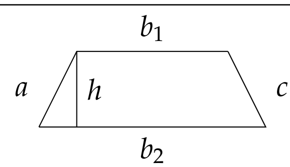
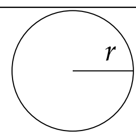
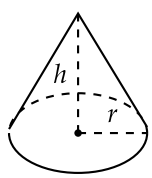
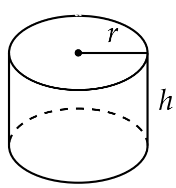
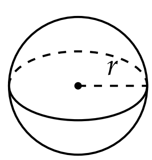
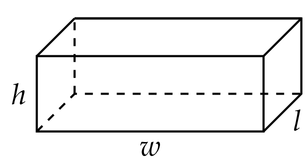


$f(x) = \dfrac{1}{x^2}$



$x^2 + y^2 = r^2$



| GEOMETRY | | | |
|---|---|---|---|
| Rectangle |   | Perimeter<br>$P = 2l + 2w$ | Area<br>$A = lw$ |
| Parallelogram |   | Perimeter<br>$P = 2a + 2b$ | Area<br>$A = bh$ |
| Triangle |   | Perimeter<br>$P = a + b + c$ | Area<br>$A = \frac{1}{2}bh$ |
| Trapezoid |   | $\mathrm{P} = a + b_1 + b_2 + c$ | Area<br>$A = \left(\frac{b_1 + b_2}{2}\right)h$ |
| Circle |   | Circumference<br>$C = 2\pi r$ | Area<br>$A = \pi r^2$ |
| Right Circular Cone |   | Volume<br>$V = \frac{1}{3}\pi r^2 h$ | Surface Area<br>$A = \pi r\sqrt{r^2 + h^2}$ |
| Right Circular Cylinder |   | Volume<br>$V = \pi r^2 h$ | Surface Area<br>$A = 2\pi rh$ |
| Sphere |   | Volume<br>$V = \frac{4}{3}\pi r^3$ | Surface Area<br>$A = 4\pi r^2$ |
| Parallelepiped |   | Volume<br>$V = lwh$ | Surface Area<br>$A = 2(lw + lh + wh)$ |

| PROPERTIES OF EXPONENTS | | | |
|---|---|---|---|
| $a^m \cdot a^n = a^{m+n}$ | $\frac{a^m}{a^n} = a^{m-n}$ | $(a^n)^m = a^{nm}$ | $(ab)^m = a^m b^m$ |
| $a^0 = 1, a \neq 0$ | $a^{-n} = \frac{1}{a^n}$ | $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ | |

| DEFINITION OF LOGARITHM | |
|---|---|
| $log_a x = y \iff a^y = x$ | $\ln x = y \iff e^y = x$ |

| LAWS OF LOGARITHMS | |
|---|---|
| $\log_a m + \log_a n = \log_a mn$ | $\ln m + \ln n = \ln mn$ |
| $\log_a m - \log_a n = \log_a \frac{m}{n}$ | $\ln m - \ln n = \ln \frac{m}{n}$ |
| $\log_a m^n = n \log_a m$ | $\ln m^n = n \ln m$ |