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USING GRAPHIC CALCULATOR IN TRIGONOMETRIC LEARNING: A DIDACTICAL DESIGN FOR TEACHER

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Abstract:
This study explored the activities using a graphing calculator of 10th-grade high school students on the trigonometry topic. A total of 65 students were divided into two classes, the first class consisted of 32 students, and the second class consisted of 33 students. The main objective of this study is to present classroom activities enriched with graphing calculators and observe responses about teaching with this approach. This study is research on developing learning designs within the Didactical Design Research (DDR) framework presented in an exploratory narrative. Data were collected by using two trigonometry worksheets and lesson observations. The worksheets were constructed based on the following focus areas: behaviour of graphs, constructing related-angle formula, and overall performance of the learner. The result of this research is a learning design that has been tested on 65 high school students. The research results on how to analyze the results of the implementation of structured learning designs provide information that educators can use, including the use of graphing calculators to help and make it easier for students to identify graphs more quickly and make students accustomed to using trigonometric function symbols. Another finding from structured instructional designs is that working in groups allows students to share experiences more effectively.

Keywords: Classroom Activity, Graphic Calculator, Trigonometric Functions

KALKULATOR GRAFIK DALAM PEMBELAJARAN TRIGONOMETRI: DESAIN DIDAKTIS UNTUK GURU

Abstrak:
Penelitian ini mempelajari aktivitas kelas 10 siswa SMA dalam topik trigonometri dengan menggunakan kalkulator grafik. Sebanyak 65 siswa yang terbagi menjadi dua kelas, yaitu 32 siswa di kelas pertama dan 33 siswa di kelas kedua. Tujuan utama dari penelitian ini adalah untuk menyajikan aktivitas kelas yang diperkaya dengan kalkulator grafik dan mengamati respons tentang pengajaran dengan pendekatan ini. Penelitian ini merupakan penelitian pengembangan dengan desain pembelajaran dalam bingkai Didactical Design Research (DDR) yang disajikan secara naratif exploratory. Hasil dari penelitian ini merupakan rancangan desain pembelajaran yang

telah diuji coba kepada 65 siswa SMA. Data diperoleh dari dua lembar kerja siswa dan observasi kelas. Lembar kerja menitikberatkan pada: bentuk grafik, konstruksi rumus sudut berelasi, dan bagaimana siswa membuat grafik tersebut. Hasil penelitian mengenai bagaimana analisis hasil implementasi dari desain pembelajaran yang disusun memberikan informasi yang bisa dimanfaatkan oleh para pendidik diantaranya adalah penggunaan kalkulator grafik membantu dan mempermudah siswa mengidentifikasi grafik secara lebih cepat dan membuat siswa terbiasa menggunakan simbol fungsi trigonometri. Temuan lain dari desain pembelajaran yang disusun adalah bahwa bekerja secara berkelompok dapat membuat siswa membagikan pengalaman dengan cara yang lebih efektif.

Kata Kunci: Desain Pembelajaran, Kalkulator Grafik, Fungsi Trigonometri

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INTRODUCTION

Numerous approaches were conducted to encourage students in trigonometry (Challenger, 2009; Maknun, Rosjanuardi, & Jupri 2018, 2019). Each approach has different aims and materials, particularly trigonometric function, which many students have more difficulties than trigonometry as a ratio (Kamber & Takaci, 2017; Tuna, 2013). Teachers need to apply specific teaching methods that encouraged students' active participation in trigonometry learning (Kagenyi, 2016). One of the trigonometry topics is the related angles. The related angles in trigonometry mean for some different angles, it has equal trigonometric value. For instance, $\sin 30^o = \sin 120^o$ or in radian measure, $\sin\left(\frac{1}{6}\pi\right) = \sin\left(\frac{5}{6}\pi\right)$. This is not the only equality of trigonometric function which later summarized into some patterns and formulas. However, the mathematics textbook used in class does not explain how the relationship of trigonometric values at the related angles was obtained and only wrote the formulas. Based on the preliminary observation, the teacher introduced this concept in a cartesian plane through the transformation concept of an angle's ray. Yet, understanding the transformation concept has its challenges because not only time-consuming but also the students should have not learned this concept since they are supposed to learn transformation at twelfth grade as what describes in the mathematics school curriculum. This approach means that the students constructing the related angles of trigonometry through a concept that they have not learned yet.

Maknun, Rosjanuardi, & Jupri (2020) reveal that analyzing trigonometric graphs can help students in understanding trigonometric functions. This matter is in line with Dundar (2015), shows that different representations can enhance the awareness of the questions presented and enhance the understanding of the mistakes better. The different representations also changed students' perspectives, provided them with multiple possibilities for problem-solving, and allowed them to express their cognitive and visualization processes. In other words, the efficiency of the representations is apparent. Associated with understanding trigonometric functions, Blackett and Tall (1991) show the advantages of a computer approach compared to traditional approaches, which allow students to manipulate images and link their dynamically changing status to the appropriate numerical concepts. This approach has the potential to increase student understanding.

Morobe (2000) recorded some positive changes in which the graphing calculator was an essential component. More significant changes in student's conceptual understanding of functions can be attained if students are exposed to at least three types of learning experiences: (a) lecture, (b) pencil-and-paper tutorial, and (c) tutorial using either a computer or graphing calculator as a learning resource. The use of technology in learning mathematics is widely used and can improve students' motivation, understanding, and problem-solving skills. Further, the massive use of technology, especially mobile apps among adolescents, can be a positive signal to acknowledge them using technology as an interactive learning resource that easily reaches and operates. Mosese (2017) shows that ICT had a significant positive effect on students' ability to make connections between different representations of trigonometry functions, on the analysis and interpretations of the given tasks of various contexts of trigonometry functions, and transformations of trigonometry functions. For instance, in his study, GeoGebra was shown to be more effective in the teaching and learning of trigonometry functions. The use of ICT is effective in the students' ability to make connections between different representations and contexts of trigonometry functions. The use of ICT is effective in improving the students' ability to interpret and analyze trigonometry functions.

However, Barnes, Lawson, Geer, Sweeney, and White (2011) argue that in the use of technology, such as graphing calculators, in studying trigonometric charts, it is found that students simply copy the graphical representation that is displayed in the graphing calculator without understanding how the process is. Although the application of technology in mathematics learning also does not guarantee positive results in learning (Maknun, 2018; Trouche, 2005). Besides, how students access graphing calculators needs to be considered before integrating technology into learning (Barnes, Lawson, Geer, Sweeney, & White, 2011). Thus, some considerations need to be made to ensure that all students have access to the graphing calculator via the student's smartphone. Students are familiar with the sine and cosine function graphs manually (drawing directly without using a graphing calculator). It is the teachers' responsibility to make sure whatever content being taught is at the level of the learners strictly following the syllabus but not the class textbook. Thus, considering applying technology not to vanish the mathematics epistemology is as important as designing the didactical process.

In the light of this information, the main purpose of this study is to present a classroom activity in related angle in the trigonometric function that incorporates with the technology used graphing calculator particularly. This purpose means showing what learning sequence can be proposed and what students experience about this activity, and whether they acquire something from the teaching. It aims to help them see the stages of mathematical concepts based on what they have learned in an effective way as possible, provide students with a different perspective, and bring technology in lessons using graphing calculators. As Constructionist acknowledges the fact that; students come to class with an established world view formed by years of prior experience and learning. Students learn from each other as well as from the teacher. Students learn better by doing. Allowing and creating opportunities for all to have a voice promotes the construction of new ideas. These facts were essential to improve the teaching and learning interactions in a trigonometry lesson (Kagenyi, 2016). In this study, a particular concern in how didactical design and classroom environment when conducting the design tasks is the main aim. The research problem proposed is: "How is the didactical design in learning trigonometry using a graphing calculator?"

METHOD

1. Research Method

This study is a research and design study. Therefore, the process in constructing the lesson design follows, which has three main stages, i.e., a prospective analysis, a metapedadidactics procedures of the didactic design research (DDR), which is proposed by Suryadi (2010), analysis and retrospective analysis (Suryadi, 2010).

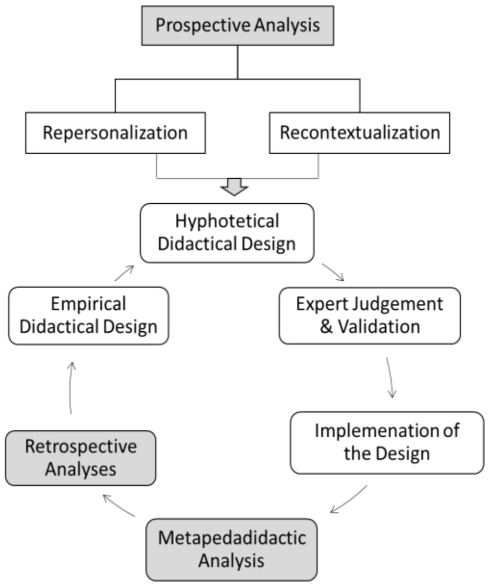


Figure 1. Research Flow

The description of the procedure in conducting the didactical design research is shown as follows:

a) A prospective analysis, an analysis carried out before learning is done such as literature review about the topic and looking at the syllabus to decide how the learning sequences and consider the learning trajectory about the topic.
b) A metapedadidactics analysis, is an activity to investigate the learning process experienced by students as a whole, look at the situation comprehensively, identify and analyze essential things that occur, and take appropriate actions so that the learning stages run smoothly and as a result, students learn optimally (Suryadi, 2010).
c) A retrospective analysis is aimed to reflect the learning process and analyze the possibility of learning obstacles that might be happening in the implementation process.

This research study aimed to propose a didactical design using an implementation of technology in helping the student understands the trigonometric formula, especially related angles properties. The design was constructed through prospective analysis and was validated by the expert. In this study, there were two implementations, the first implementation for the initial lesson design and the second implementation for the revised lesson design. The result of the second implementation would be the empirical design proposed in this study. To avoid misinterpretation, let us stress that the purpose of the study was not to demonstrate that the learning instruction better than the instruction used in a typical trigonometry classroom. In particular, the research method used in this study should not use to make such a claim. Also, there is no single or simple solution to effective technology implementation in teaching and learning. Thus, the analysis in this study was the students' response to the tasks. Hence, the results from the study are meant to be suggestive and illustrative.

2. Participant

This study was carried out to the 10th high school students in Bandung, Indonesia public school in 2018. A total of 65 students were divided into two classes, the first class consisted of 32 students, and the second class consisted of 33 students. The first class was used for the first implementation for further revision of the initial hypothetical didactical design. At the same time, the second class was used to confirm the didactical design after the first implementation. Students' mathematical ability ranges from low to middle-skill in mathematics, which means they can follow mathematics learning. However, they are not skilled enough in analyzing and exploring mathematical questions. For example, before the research was conducted, the students had learned about the trigonometry definition both as a ratio and as a function, drawing the trigonometric functions manually and analyzing the trigonometric graphs' properties. However, they had not learned advanced analyses in related angles formulas yet.

3. Collecting Data and Validity

The data in this study were obtained through four ways, i.e., the interviews, the class observations, the documentation, and the students' work result, which were collected in the video recordings and the photographs. Students' worksheets had been designed to fill the learning sequences that the students had learned before. Since the students had learned about the trigonometric functions' properties and their general graphical representation, the worksheets proposed started by recalling the students' basic knowledge of the trigonometric graph. The validity of this research was evaluated based on two criteria, and they are Credibility (Internal validity) and Transferability (External validity) (Sugiyono, 2016). The credibility was conducted into four ways as follows:

1) Increased research persistence through double-check the data found
2) Source and technique triangulation. Source triangulation is the process by which the data obtained from different sources using the same technique. Meanwhile, technical triangulation is a process of collecting data from the same subject using different techniques
3) Reference Material using the camera to prove the obtained data.

The transferability was evaluated by the expert, teacher, and peer-researcher while the study was going on. While the dependability testing (Reliability) to find out whether the approach taken by the researcher is reliable, the procedures used were:

1) Check the results of the transcription to make sure there are no errors in the transcription process
2) Make sure there are no ambiguous meanings
3) Share the analysis with experts and peers.

Further, to support the credibility and dependability, the confirmability was conducting (e.g., providing documentary evidence of how learning activities took place).

4. Data Analysis

In this study, the data were analyzed into several stages: preparing data, reading the entire data, analyzing in more detail, determining the themes, describing the theme, and interpreting the data. In writing the results of the analysis, the researcher used a narrative approach. This approach includes discussing how learning activities are carried out, important events in the field, and the relationship between how previous learning affects students' actions in learning. In addition, pictures of student work and illustrations of how learning activities take place were provided to help the analysis process. Each learning process and situation in the learning process analyzed will be interpreted by data based on the connection with previous studies, the researcher's assumptions, questions, and conjectures that confirm whether the research results will support or deny previously believed information. In this case, the researcher does not rule out new questions that need to be answered in further research.

RESULT AND DISCUSSION

1. Prospective Analysis

The design task proposes in the study focus on the trigonometric function graph, which is presented in table 1 and table 2. The students were asked to complete the output of trigonometric function graphs. This task directly relates to the student's ability to recall the previous material, which is frequently cited as evidence that students are ready to acquire the new materials, making the learning effective (Rohmah, Utomo, & Zukhrufurrohmah, 2019). For instance, the students need to understand the trigonometric value and its angles. Another consideration of why this task was included was to indicate how well the students know about the sine and cosine function graph. They can recall (or derive) specific properties of trigonometric function graphs since the students need to understand the concept first before they grasp the graphing calculator's process to display graphs (Mitchelmore & Cavanagh, 2000).

Table 1. Students' Worksheet 1

1. Complete this graph below by putting the angles and the value of sine and cosine functions, also, identify which one is $y = \sin x$, and $y = \cos x$.
2. Further, using Desmos Graphing Calculator to draw the graphs of $y = -\sin x$, $y = -\cos x$, $y = \cos(-x)$ and $y = \sin(-x)$,

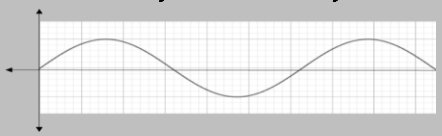
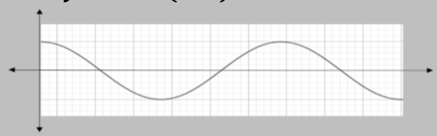



Table 2. Students' worksheet 2

Open the Desmos Graphing Calculator, then figure out these graphs!. Next, identify the same graphs, then classify each graph which similar to:

$\boldsymbol{y = \sin x}$	$\boldsymbol{y = -\sin x}$	$\boldsymbol{y = \cos x}$	$\boldsymbol{y = -\cos x}$
1. $y = \sin\left(x + \frac{\pi}{2}\right)$		9. $y = \cos\left(x + \frac{\pi}{2}\right)$	
2. $y = \sin\left(x - \frac{\pi}{2}\right)$		10. $y = \cos\left(x - \frac{\pi}{2}\right)$	
3. $y = \sin\ (x + \pi)$		11. $y = \cos\ (x + \pi)$	
4. $y = \sin\ (x - \pi)$		12. $y = \cos\ (x - \pi)$	
5. $y = \sin\left(x + \frac{3}{2}\pi\right)$		13. $y = \cos\left(x + \frac{3}{2}\pi\right)$	
6. $y = \sin\left(x - \frac{3}{2}\pi\right)$		14. $y = \cos\left(x - \frac{3}{2}\pi\right)$	
7. $y = \sin\ (x + 2\pi)$		15. $y = \cos\ (x + 2\pi)$	
8. $y = \sin\ (x - 2\pi)$		16. $y = \cos\ (x - 2\pi)$	

In constructing the students' worksheet, a consideration about students' work with a graphing calculator was the main aim. Thus, the students could explore whether they can identify either the similarity and difference among the functions. Also, the students could input the formula in a graphing calculator and provide correct output, whether they can classify each graph in the correct table. In the theoretical perspectives, the ability to work systematically through cases in an exhaustive and the ability to interpret and systematically extend solutions to problems indicate high order thinking ability (Maharaj & Wagh, 2016). This ability, while important, was not central to the main goals of the study reported in this article and is discussed only briefly in the results section. Finally, the students were asked to analyze and conclude from the table they had filled. This matter is the main goal of the learning sequences that the students could analyze the graphs, particularly in related angles.

In carried out the learning process, the researcher chose the Desmos Graphing Calculator application version 3.0.0.2. This application was chosen because of its simple appearance. In addition, it is easier for students to get used to this application since too long in adjusting to the use of technology can become a limitation in learning (Curri, 2012). Further, numerous research and learning activities had used Desmos as a tool to engage students innovatively in mathematics (Ebert, 2014; King, 2017; Liang, 2016). Desmos Graphing Calculator is a graphic processing application from Desmos Inc. This application can be downloaded for free via the play store. The features offered by this application include graphing, sliders for creating interactive animations of variables, creating function tables, enlarging images, statistics, determining the coordinates of points on curves, scientific calculators, and creating graphs. Also, this application can be open both online and offline mode.

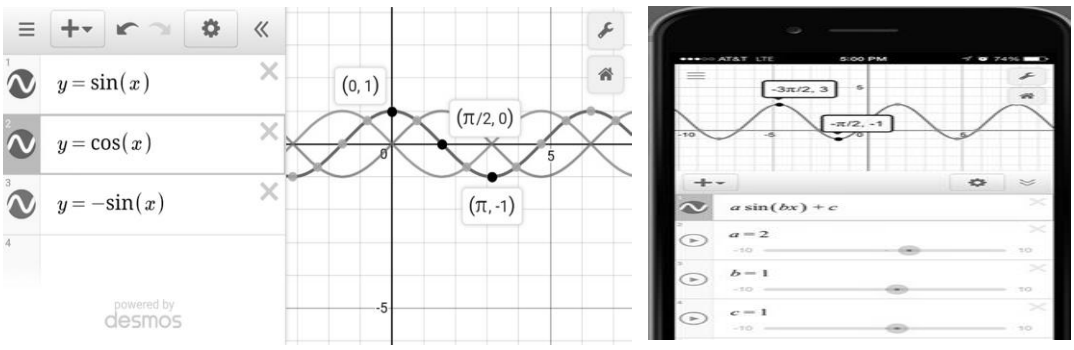


Figure 2. The Appearance of Desmos Graphic Calculator

2. Didactical Design

The sequences of the learning activities begin by checking students' access to the graphing calculator on their phones and arrange the students into groups with five to six students for each group. Then, they were given worksheets in groups related to the trigonometric function graphs. Through conducting, evaluating, and analyzing by comparing the graphs of sine and cosine functions graphs to the other graphs using graphic calculator. Then, the students made a concluding remark on the graph. The teacher is role as a facilitator in the learning process and institutionalisation.

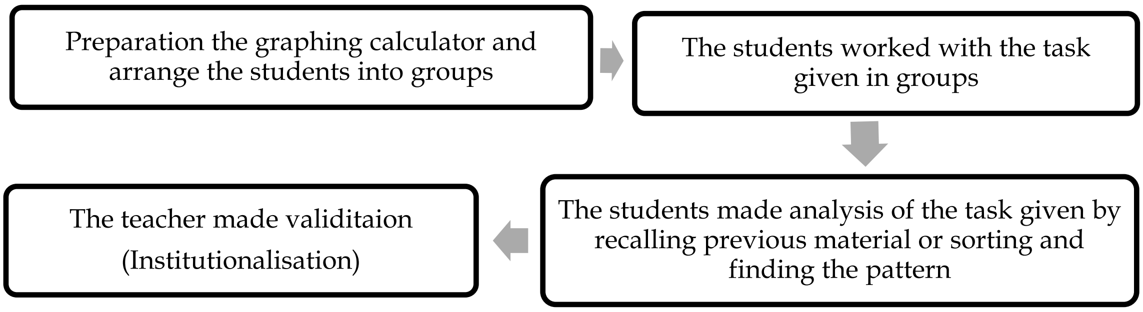


Figure 3. Flow Chart of Learning Sequences

The didactical situation was arranged in two cases which represented the aims of learning as follows:

1) Understanding the functions of $\sin(-x) = -\sin(x)$ and $\cos(-x) = \cos x$ based on the graph.
2) Understanding related angles in sine and cosine function based on graph. In conducting the didactical design, the students did two worksheets about graphical representation in trigonometric function, which is provided in the appendix.

3. Metapedadidactics Analysis

The initial stage of the learning process was students had been asked to fill the first students' worksheet in group work. Although each of the students had access to the Desmos, the group activities could make them share their opinion and results in the most effective way. Before the students started to use the graphing calculator, they need to identify the graphs given in the students' worksheet, i.e., deciding which graph is either sine or cosine graphs. All of the students could identify the graphs. It meant that the students could recall previous material, thus they were ready to learn new material. However, the students wrote informally, i.e., "sine graph" instead wrote it formally as $y = \sin x$. Thus, it is important to use various kinds of representations, including symbolic. Egan (2005) argues that teaching is to help learners engage with the symbolic representations of knowledge that remain hidden without the acquisition by learners of the cultural and social tools needed to translate them. Also, the graphing calculator contains symbolic representation to compute the graph. Thus, the students were allowed to proceed with the task by implementing the graphing calculator to get used in symbolic representation.

The result of this study was in line with what Man and Poon (2014) find that providing opportunities to the students with the aid of calculator and allowing them to use different representations (e.g., graphical, numerical, or symbolic representation) success to present their understanding of the limit concepts of trigonometric functions in the explanation parts. The use of the graphing calculator could make them used to the symbolic representation of the trigonometric function.

Figure 4. Students' Activities in Using Desmos Graphing Calculator

The students were asked in their group work about the analysis and the conclusion of the $y = -\sin x$ and $y = \sin\ (-x)$ Graph.

Researcher : What do you get when compared to the sine graph?

Student : The opposite

Based on the conversation above, it showed that the students found the fact of the equality between the sine graph in negative angle and the negative of the sine graph. The activities continued to analyze and conclude the relationship between the cosine graph on the negative angle and the negative of the cosine graph. One of the student's summaries is presented in figure 5.

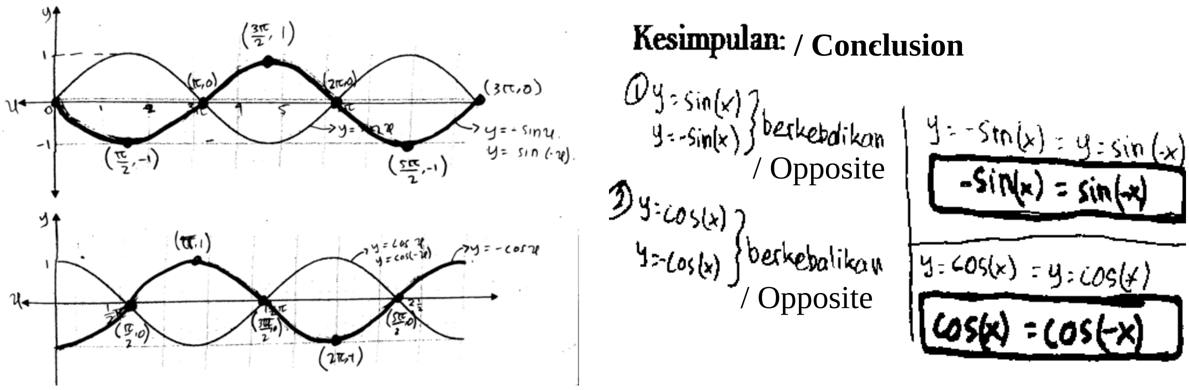
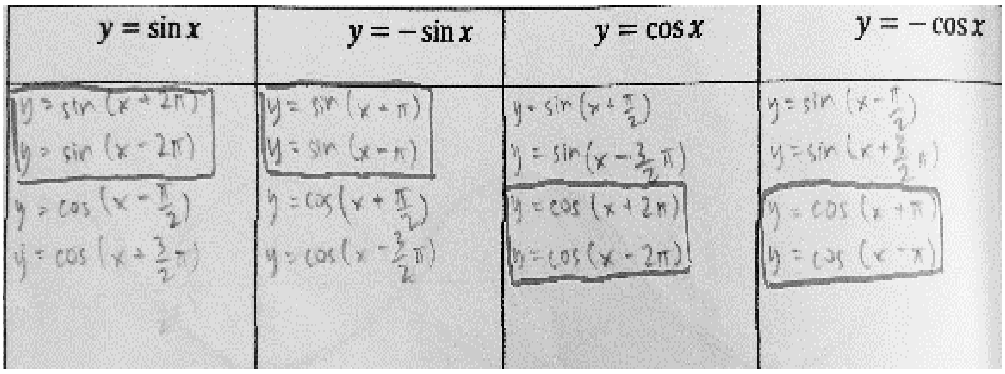


Figure 5. Students' Work and Conclusion in Worksheet 1

Even though the students learned with negative angles measures during the trigonometry unit in their previous learning, including negative angles in the unit circle representation, the students revealed it was easier to find out the trigonometric values by seeing the graphs. Thus, it can clear up a misconception with negative angles on the unit circle demonstrated by the students. This result was in line with Marchi's (2012) study, which shows that the students' understanding of negative angle measures is best recalled in the graphical representation.



$y = \sin x$	$y = -\sin x$	$y = \cos x$	$y = -\cos x$
$y = \sin(x + 2\pi)$ $y = \sin(x - 2\pi)$ $y = \cos(x - \frac{\pi}{2})$ $y = \cos(x + \frac{3}{2}\pi)$	$y = \sin(x + \pi)$ $y = \sin(x - \pi)$ $y = \cos(x + \frac{\pi}{2})$ $y = \cos(x - \frac{3}{2}\pi)$	$y = \sin(x + \frac{\pi}{2})$ $y = \sin(x - \frac{3}{2}\pi)$ $y = \cos(x + 2\pi)$ $y = \cos(x - 2\pi)$	$y = \sin(x - \frac{\pi}{2})$ $y = \sin(x + \frac{3}{2}\pi)$ $y = \cos(x + \pi)$ $y = \cos(x - \pi)$

Figure 6. Students' Work in Worksheet 2

This stage aims to determine the trigonometric value at an angle that has a relation with another angle. The sample of students' works demonstrates in Figure 6. The students were able to classify and categorize the graph based on their similarities. Students marked the square sign in figure 4 to easily figure out a relevant and specific conclusion. For instance, in the first column, they gathered $y = \sin(x + 2\pi)$ and $y = \sin(x - 2\pi)$ in a group, since they could simplify it as $y = \sin(x \pm 2\pi)$ and so do the other columns. Hence, they concluded it in a primary conclusion as shown in figure 5. What they had done in this stage was a process of inductive reasoning. Inductive reasoning refers to the detection of regularities and irregularities to form rules and make generalizations. The relationship between inductive reasoning and mathematics academic achievement is especially salient, as both inductive reasoning and mathematics performance require individuals to find patterns and relations among numbers and figures. The Desmos Graphing Calculator allowed each trigonometric graph to be visualized and decreased complicated calculations, thus allowing the students to focus on essential and critical ideas such as the relationship among the graph (Naidoo, Govender, Africa, Africa, Naidoo, & Africa, 2014).

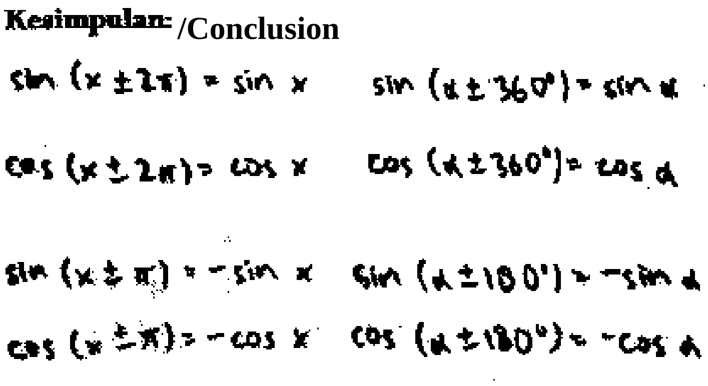


Figure 7. Students' Conclusion in Worksheet 2

An inductive approach, in a concrete way, derives from the experience gained. Nevertheless, in many cases, the inductive inferences are valid and provide an essential basis for understanding regularities in mathematics (Papageorgiou & Christou, 2007). Taking into consideration that inductive reasoning ability improves the learning of mathematics.

Moreover, a condition in which the students were working in a group which consisted of various range of mathematics skill among the group members, it can be seen the interaction of the students in solving the task since class interaction pattern was very important in teaching and learning activities of trigonometry (Kagenyi, 2016). Further, Kagenyi's notes that the main challenges of the methods, such as discussion groups were time-consuming. Also, Maknun (2018) aware that technology can be quite challenging for illiterate students in technology. However, those were not found in this study. Therefore, we assume that the simple features provided in graphing calculator made a significant contribution.

4. Retrospective Analysis

At first implementation, the researcher planned that students work individually, but due to conditions where not all students had installed the application. Thus, at the second implementation, they work in groups. Further, working in a group situation make students exchange information on how to use the application so that the learning process runs well. This result contrasts with the first implementation learning design at the first meeting, where the results of the observations at the meeting showed a lack of effectiveness. DeJarnette (2014) argues that technology can increase student motivation in learning. However, whether technology also increases motivation in group work needs further research. The process of analyzing the results of the activities carried out by students went well. At the end of the lesson, the practice questions apply the formulas that students have just learned. However, it would be better if the students were given more practice questions to students and provided opportunities for students.

Based on the description above, it can be concluded that several important things for the improvement of this didactical design which mention as follows:

a) The study finds that the students need time to get used to the Desmos applications at the beginning of the learning process. Thus, they are focused on how to operate the application.
b) The tasks given to the students were limited and did not vary. This could make an epistemological obstacle in the future because of the limitation of the tasks.

CONCLUSION

This study shows that the graphing calculator helps students identify and analyze numerous trigonometry graphs, so the time spent in learning can be reduced without decreasing the content itself. The use of the graphing calculator could make them used to the symbolic representation of the trigonometric function. Also, the students found the equality between the sine graph in negative angle and the negative of sine graph, and they revealed that it was easier to find out the trigonometric values by seeing the graphs. Further, a group activity could make them share their opinion and results in the most effective way. Based on the result of the study, the didactical design and task design proposed in this study can be an alternative way to implement technology and content knowledge in classroom activity. However, some consideration needs to be taken into accounts, such as the access to graphic calculators and the task design for students. We realize that it takes time to develop the skills necessary to use graphing calculators effectively in teaching. Although there is no single or simple solution to the effective use of graphing calculators in teaching and learning, teachers need support to develop both new technical and new pedagogical skills. To sum up, the curriculum and its assessment need the flexibility to accommodate technological change. When available, the data generated from this study not only contribute to theory-building on the development of functional concepts but it should also serve as a basis for developing appropriate instructional materials, including books and manuals. More importantly, it enables curriculum developers to review the school mathematics curriculum so that the idea of function becomes a unifying theme.

ETHICS

Regarding ethical issues in this study, such as ethical issues in research objectives, collecting data, writing, and publishing research results, we explain that we presented the subject's weaknesses and strengths objectively. Moreover, both the school and the teacher knew the purpose of this study. During and after the research process, there are no activities that endanger the participants. The publication of the results of this study is intended for academic purposes and is limited to the academic environment to prevent data misuse.

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