COMPARATIVE INVESTIGATION OF METHODS FOR DETERMINING THE LATERAL STIFFNESS OF COUPLED RC SHEARS WALLS

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ABSTRACT

In this study, the lateral stiffness of coupled RC shear walls is studied using the continuum method, equivalent frame and finite element methods. For this purpose, asix-story coupled shear walls with typical dimensions are considered and the lateral displacements of system are calculated under a variety of lateral loads such as: uniform, triangular distributed and concentrated loads, then the results are compared with together. The results show that under the rectangular and concentrated loadings, equivalent frame and continuum indicate more displacements compared finite element approach; therefore, these methods approximate less lateral stiffness for coupled RC shear walls. In addition, equivalent frame technique in most cases, except triangular loading, compared with continuous medium method determines more soft behavior for the structure.

Keywords 1-Coupled RC shear wall 2-Lateral stiffness 3-Equivalent frame 4-Continuum method 5-finite element

1. INTRODUCTION

Horizontal loads applied on structures, such as wind and earthquake effects, should be tolerated by the lateral resisting system[1]. According to the regulations for the design of buildings, these systems are categorized as: load-bearing walls, braced frame, moment resisting frames, dual systems and cantilever systems. Each of these systems has its own advantages and disadvantages and they are used in certain situations in practice. For example, in designing the buildings upper ten floors, generally frame performance with flat slab-beam system together with columns is not sufficient [2]. In this situations, shear walls as one of the most important structural elements in supplement of lateral strength, with high in-planar strength and stiffness, are suitable for bracing the structures for up to 35 floors[3]. In general, in structures with shear walls ,with increasing the structural stiffness and reducing the extreme displacements, the risk of structural failure will be reduced [4]. The advantages of a beamless flat roof in high-rise structures, also can be provided by the strategy of using the shear walls as lateral load resisting elements[5].

In general, shear walls are categorized as concrete and steel (see[6]), and in selecting an appropriate place to locate shear walls two principal points should be considered:

1- To increase the torsional resistance as much as possible, walls should be in the perimeter of plan. (Refer to Figure1)

2- Wall location should be chosen such that tension under lateral load as much as possible should be decreased by gravity load[4].
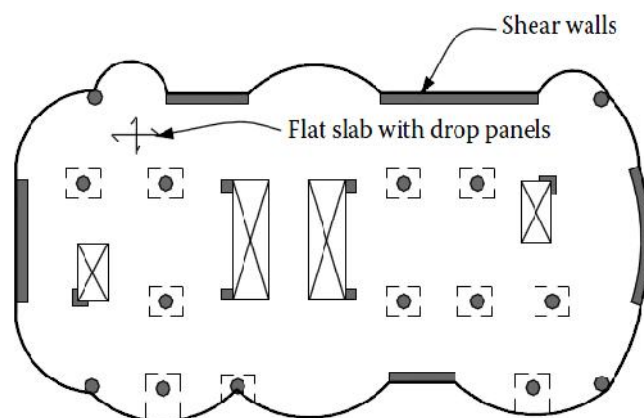

Fig.1. The suitable place shear walls to increase the torsional resistance [7]

In terms of structural behavior, as shown in Figure 2, shear walls can be categorized as proportionate and disproportionate systems [8]. As their name implies in a proportionate system, the flexural rigidity of the wall is constant at height. For example, a system that the height of walls is unchanged at height, and the variations of wall thickness are the same at any level, is proportionate. It should be noted that proportionate systems are determined, so they can be analyzed using the equilibrium equations and distribution of external moments and shears in terms of flexural rigidity of the walls. But in a disproportionate undetermined systems, analyzing is more difficult than proportionate system[9].
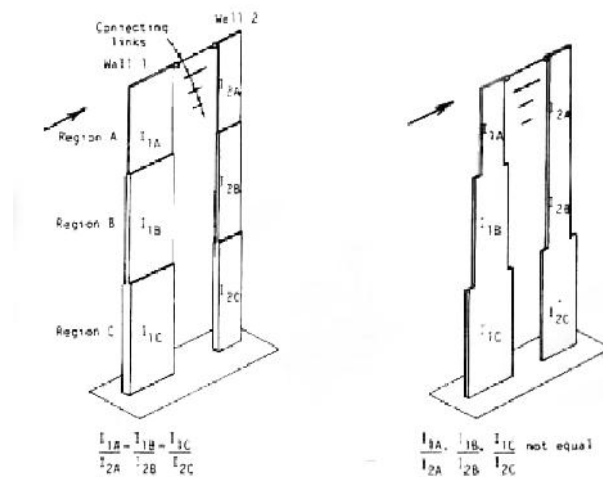
**Fig.2.** proportionate and disproportionate systems[3].

In practice, most shear walls (as in this study is intended) are planar. But in some cases, for better compatibility with the plan and creating more stiffness for the structure L,T, I, and U sections are also use. Figure 3 shows the cross-sectional shape of the shear walls.
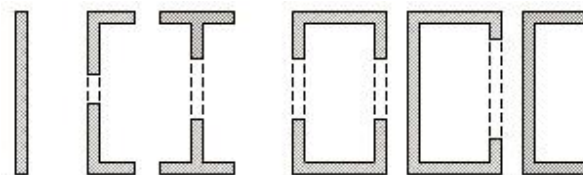

Fig.3. sections of shear walls[10]

Shear wallsdue to their importance in improving the performance of structures have always been considered by researchers from different aspects. For example Sarvghad et al. have worked on pattern of the reinforcements on the seismic performance of the short shear walls in nuclear power stations. Short shear walls, height to width ratio of less than 2, and mostly shear behaviors dominant on them. In that study, using finite element software ATENA 3D, Failure modes and residual and ultimate strength of these walls by changing some parameters such as transverse and longitudinal bars at the edges, change the horizontal and vertical bars as well as the use of diagonal bars, have been examined. They also concluded that, contrary to long shear walls, focus of bars on the edges of the wall will not increase the ductility of these structures[11].Doran under article titled "Elastic-plastic analysis of R/C coupled shear walls: The equivalent stiffness ratio of the tie elements"considering stiffness of elements of link beam and using Drucker-Prager and Von-Mises criteria presented a model for nonlinear analysis of shear walls[12]. Carillo and et al. also examined the effect of lightweight and low-strength concrete on seismic performance of thin lightly-reinforced shear walls. They used quasi-static test and shaking-table to investigate the behavior of twelve shear walls. Test results show that the shear strength, the drift and damped energy will increase compared with shear walls made of non-lightweight concrete. In another part of this article, reduction coefficient in also ACI-318-11 is criticized [13].

Another aspect that has been considered in the literature is study of the behavior of coupled shear walls. In general, shear walls are divided into two categories: separate and linked walls. In the first, lateral load on the structure will fund by the independent behavior of each walls. But in many cases, due to architectural considerations and wall Openings, coupled shear walls composed of two walls connected by connecting beams is used (Figure 4).
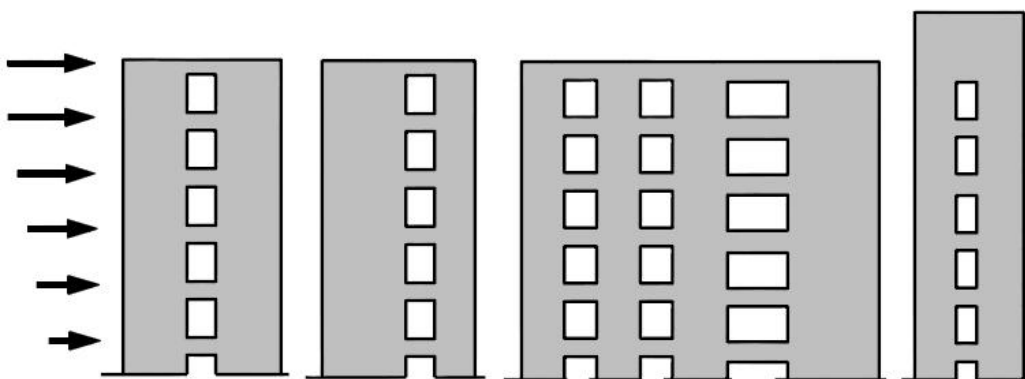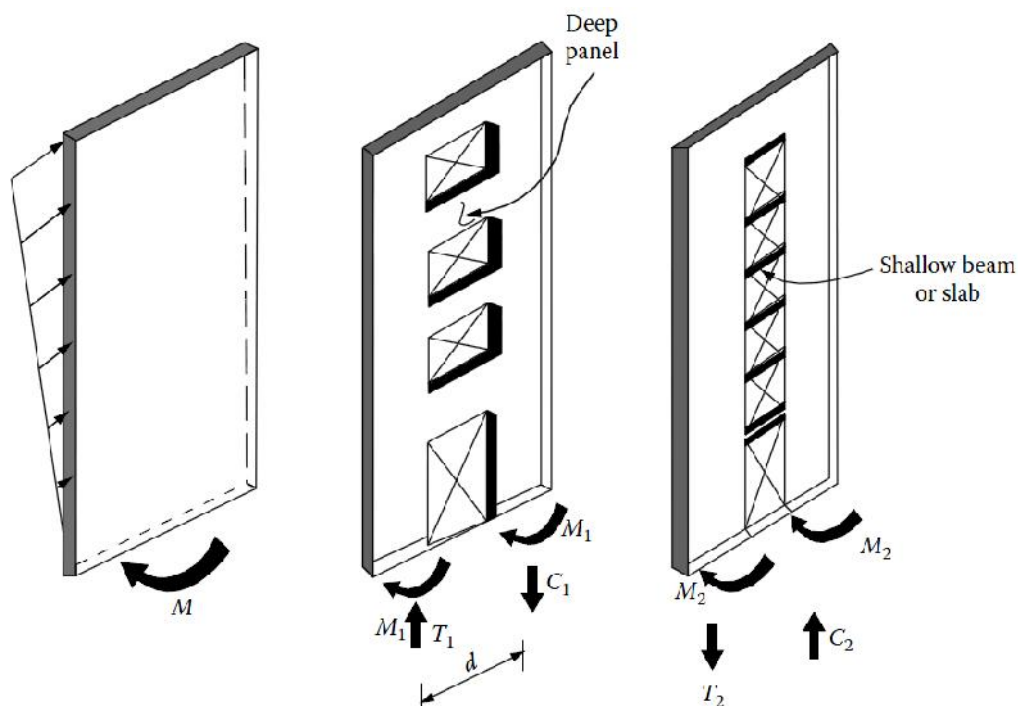

Fig.4. Coupled shear walls[4]

If the connection beams were pinned, the moment of walls obtains from their stiffness ratios and the maximum stress is created on the edge of the wall. On the other hand, if the beams are rigid structural performance will be similar to a double vertical cantilever and the maximum stress occurs at the edges of the bottom wall. But in practice, the real beams are flexible in structures; behavior of the system is placed between the two modes. In this case, the bendingof link beams will reduce the amount of the moment of walls (coupling effect).Figure 5, describes performance of the link beam (which sometimes also referred as coupled beam).
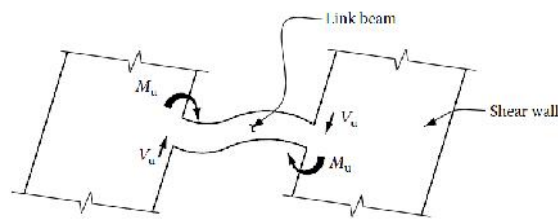
Beam [4]**Fig.5.** Resistance of connection

Figure 6 shows the coupled shear walls with deep and shallow link beam.It is clear that in this case as the beam be weakened, by reducing the coupling effect, the structures tend toward two separate cantilevers.

[4]**Fig 6.** The effect of deep or shallow beam connection

Several researchers have worked on the performance of coupled shear walls.For example, Hisatako and Matano proposed a seismic design method for high-rise buildings with shear walls and the results of their work were evaluated by performing tests[14]. Amar also utilizes finite element method analysis by ANSYS software with solid elements have studied on coupled shear walls in a 10-story commercial building. In this study non-linear response of the structure and pattern of formed cracks in the structure were examined[15]. Subedi et al. associates a simple approach based on the concept of the total moment for the analysis of coupled shear walls with one or two band. Experimental studies were used to estimate the failure mode and ultimate strength in this study[16].Finite element method in the analysis of coupled shear walls, usually using two-dimensional plane stress elements occurs, has high accuracy (see [17]).But due to the complexity of finite element analysis, (especially in the initial design of structures) approximate methods such as continuous medium method are used[18][19]. These methods have its own advantages and disadvantages, for example, Kuang has shown that continuum approach, where the walls are asymmetrical sides of the coupled beam loses his accuracy. Because in this case it is assumed i.p. in the middle of the connection beams, which is one of the basic assumptions of this approach is destroyed[20].It is also when changing the thickness of the wall height there be in trouble.

Continuum method

This method is one of the most common approximated methods in order to analyze the coupled shear walls with taking into account the effect of walls and connecting beams to withstand against lateral force the structure. It is noteworthy that some researchers modified this approach for dynamic analysis of a coupled shear wall structures [21]. To explaining the basis of continuous method, consider a plane (or two-dimensional) coupling shear wall according to Figure 7, which is equivalent with a continuum connecting medium.
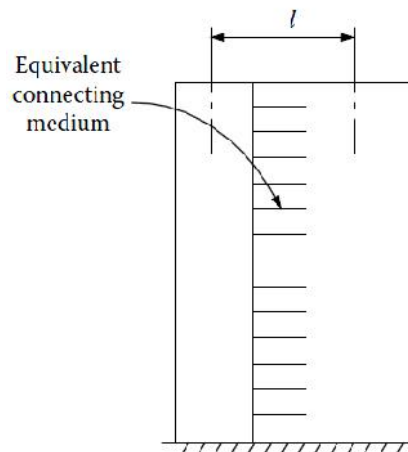
[4]medium **Fig 7.** Equalization of link beam by continues

To obtain the differential equation of the problem, as shown in Figure 8, the following assumptions should be applied here:

- Planar sections, before and after bending remains plane. (Bernoulli hypothesis)
- Story heights are constant (h=cte) and properties of walls and link beam assumed to be constant in height.

- Flexural rigidity of beams (EI_b), is replaceable with the distributed flexural rigidity of the continues medium(EI_b / h).
- In the deformation of the structure, i.p. are formed in the middle of link beam and the curvature of the walls at the height of the structure is constant and therefore the bending moment is proportional to the flexural rigidity of each wall.
- Axial force, bending moments, and shear of connecting beams respectively can be substituted by continuous distribution and intensity of n, q and m in unit height of the structure. (Figure 9)
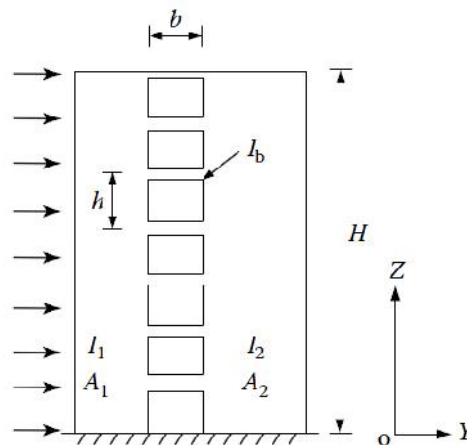
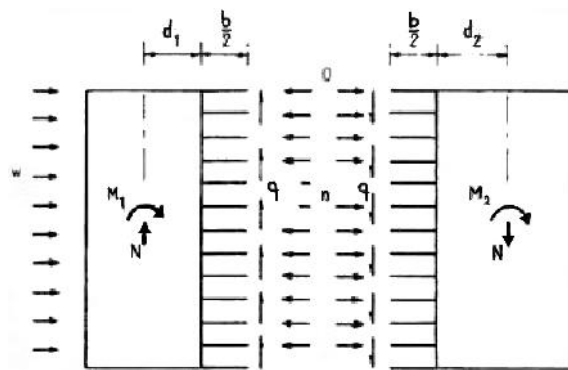**Fig.8.** parameters in coupled shear wall [4]

Fig.9. Coupled shear wall properties [3]

By applying the above assumptions, and also assume a solid foundation (which is common scenarios in practical uses),compatibility equation states that on the path of inflection points of bending of beams connection there should be no vertical displacement

$$\left(L \frac{dy}{dz} + \frac{b^3 h}{12 EI_e} \frac{dN}{dz} - \frac{1}{E}\left(\frac{1}{A_1} + \frac{1}{A_1}\right)\int_0^z N dz = 0 \right) 1$$

Considering the moment-curvature relationship in the walls can write

$$\left(EI_1 \frac{d^2 y}{dz^2} = M_1 = M - \left(\frac{b}{2} + d_1\right)\int_z^H q dz - M_a \right) 2$$

And,

$$3)\ \left(EI_2 \frac{d^2y}{dz^2} = M_2 = M - \left(\frac{b}{2} + d_2 \right) \int_z^H qdz - M_a \right.$$

Where, M_a is the moment of connecting axial forces.

From the two equations (2) and (3), the overall moment-curvature relationship is obtained for coupled shear walls

$$4)\ \left(E(I_1 + I_2) \frac{d^2y}{dz^2} = M - L \int_z^H qdz = M - NL \right.$$

By combining the above equations, differential equations governing the behavior of coupled shear walls, in terms of lateral displacement y is

$$5)\ \left(\frac{d^4y}{dz^4} - (k\alpha)^2 \frac{d^2y}{dz^2} = \frac{1}{EI} \left(\frac{d^2M}{dz^2} - \frac{(k\alpha)^2(k^2-1)}{k^2} M \right) \right.$$

Where,

$$\alpha^2 = \frac{12I_e L^2}{b^3 hI}$$

$$k^2 = 1 + \frac{AI}{A_1 A_2 L^2}$$

$$I = I_1 + I_2$$

$$A = A_1 + A_2$$

In relationships before, E is the modulus of elasticity and all other parameters are defined in the figures 8 and 9.

In general, the general solution of the differential equation (5) is

$$6)\ \left(y = C_1 + C_2 z + C_3 \cosh(k\alpha z) + C_4 \sinh(k\alpha z) + \frac{-1}{EI(k\alpha)^2} \left[\frac{1}{(k\alpha)^2} + \frac{D^2}{(k\alpha)^4} + ... \right] \left(\frac{d^2M}{dz^2} - \frac{(k^2-1)}{k^2} M \right) \right.$$

Where, D is the differentiation operator and coefficients C_1 to C_4 are integration constants obtained applying appropriate boundary conditions.

In practice, by assuming fixed base for the wall

)7
$$(y(0) = 0 \quad \& \quad y'(0) = 0$$

Also in the basethe variation of axial forces respect to height, is zero

) 8
$$(z = 0 \quad \rightarrow \quad \frac{dN}{dz} = 0$$

At the top of the structure (z = H) as well as bending moments and axial forces do not exist

)9
$$(z = H \quad \rightarrow \quad \frac{d^2y}{dz^2} = 0 \quad \& \quad N = 0$$

In this section, relations for axial force of walls N, shear ofconnected elements q, moments of walls M1 and M2, and lateral displacement y, in the case of rectangular loadingare given

)10
$$(N = \frac{wH^2}{k^2L} \left[\frac{1}{2} \left(1 - \frac{z}{H} \right)^2 + \frac{1}{(k\alpha H)^2} \left[1 - \frac{\cosh(k\alpha z) + k\alpha H \sinh(k\alpha(H - z))}{\cosh(k\alpha H)} \right] \right]$$
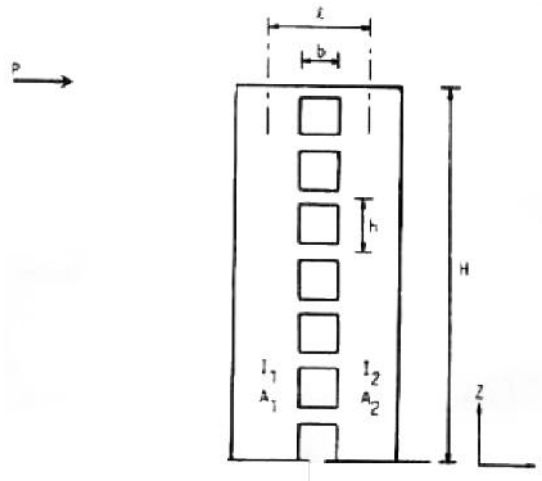
)11
$$(q = \frac{wH^2}{k^2L} \left[\left(1 - \frac{z}{H} \right) + \frac{1}{k\alpha H} \left[1 - \frac{\sinh(k\alpha z) - k\alpha H \cosh(k\alpha(H - z))}{\cosh(k\alpha H)} \right] \right]$$

)12
$$(M_1 = \frac{I_1}{I} \left[\frac{1}{2} wH^2 (L - \frac{z}{H})^2 - NL \right]$$

)13
$$(M_2 = \frac{I_2}{I} \left[\frac{1}{2} wH^2 (L - \frac{z}{H})^2 - NL \right]$$

)14

$$y = \frac{wH^4}{EI}\left[\frac{1}{24}\left[\left(1-\frac{z}{H}\right)^4 + 4\frac{z}{H} - 1\right] + \frac{1}{k^2}\left\{\left(\frac{1}{2(k\alpha H)^2}\right)\left(\frac{2z}{H} - \left(\frac{z}{H}\right)^2\right)\right.\right.$$
$$\left.\left. - \frac{1}{24}\left(\left(1-\frac{z}{H}\right)^4 + 4\frac{z}{H} - 1\right) - \frac{1 + k\alpha H\sinh(k\alpha H) - \cosh(k\alpha z) - k\alpha H\sinh(k\alpha(H-z))}{(k\alpha H)^4\cosh(k\alpha H)}\right]\right.$$

If the lateral load is a concentrated load **p** at the top of the structure, relations are

Fig10. Shear wall under concentrated load[3]

)15

$$N = \frac{pH}{k^2L}\left[\left(1-\frac{z}{H}\right) - \frac{\sinh(k\alpha(H-z))}{k\alpha H\cosh(k\alpha H)}\right]$$

)16

$$q = \frac{p}{k^2L}\left[1 - \frac{\cosh(k\alpha(H-z))}{\cosh(k\alpha H)}\right]$$

)17

$$y = \frac{pH^3}{3EI}\left[\frac{1}{2}\left[\left(\frac{k^2-1}{k^2}\right)\left[3\left(\frac{z}{H}\right)^2 - \left(\frac{z}{H}\right)^3\right]\right] + \frac{3}{k^2}\left(\frac{1}{(k\alpha H)^2}\frac{z}{H} - \frac{\sinh(k\alpha H) - \sinh(k\alpha(H-z))}{(k\alpha H)^3\cosh(k\alpha H)}\right)\right]$$

And also in the case of triangular load,

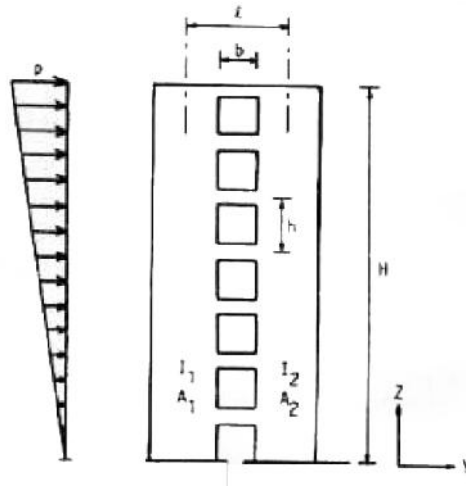**Fig.11.** Shear wall under triangular load[3]

$$
(18) \quad N = \frac{pH^3}{k^2 L} \Bigg[\frac{\big[\cosh(k\alpha H) - (k\alpha H/2) + (1/k\alpha H)\big]}{(k\alpha H)^2 \cosh(k\alpha H)} \sinh(k\alpha(H-z)) - \frac{1}{(k\alpha H)^2} \cosh(k\alpha(H-z)) + \\
\frac{1}{2}\left(1 - \frac{z}{H}\right)^2 - \frac{1}{6}\left(1 - \frac{z}{H}\right)^3 + \frac{1}{(k\alpha H)^2}\left(\frac{z}{H}\right) \Bigg]
$$

$$
(19) \quad q = \frac{pH}{k^2 L} \Bigg[\frac{\big[\sinh(k\alpha H) - (k\alpha H/2) + (1/k\alpha H)\big]}{(k\alpha H) \cosh(k\alpha H)} \cosh(k\alpha(H-z)) - \frac{1}{(k\alpha H)} \sinh(k\alpha(H-z)) + \\
\left(1 - \frac{z}{H}\right) - \frac{1}{2}\left(1 - \frac{z}{H}\right)^2 + \frac{1}{(k\alpha H)^2} \Bigg]
$$

$$
y = \frac{pH^4}{2EI} \Bigg[\frac{1}{60}\left(\frac{k^2 - 1}{k^2}\right)\left[20\left(\frac{z}{H}\right)^2 - 10\left(\frac{z}{H}\right)^3 + \left(\frac{z}{H}\right)^5\right] + \\
(20) \quad \frac{1}{k^2(k\alpha H)^2}\Big[(1 - \frac{2}{(k\alpha H)^2})\left(\frac{z}{H}\right) - \frac{1}{3}\left(\frac{z}{H}\right)^3 + \\
\frac{2}{(k\alpha H)^2 \cosh(k\alpha H)}[\cosh(k\alpha z) - 1 + \left(\frac{1}{(k\alpha H)} - \frac{(k\alpha H)}{2}\right)[\sinh(k\alpha H) - \sinh(k\alpha(H-z))]
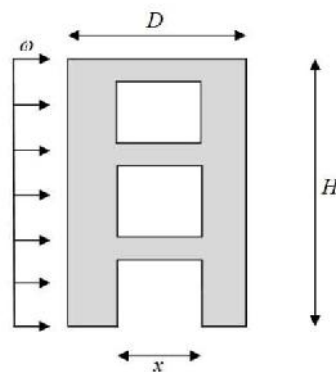$$

Equivalent frame approach

In this way, by defining a frame of beams and columns that their stiffness are obtained by opening dimensions, the coupled shear walls are analyzed.
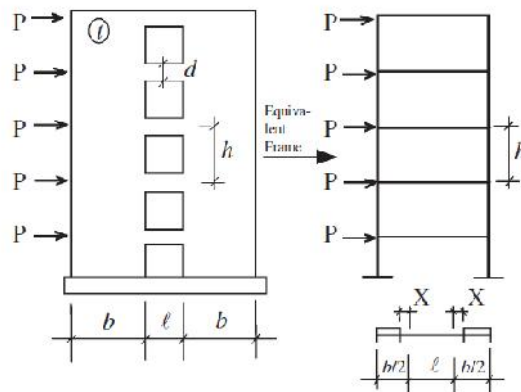
Fig.12. equivalent frame for analyzing the Coupled shear walls [12]

To better understand this method, consider a three-story shear wall as shown in Figure 13.

Fig.13. Defining the parameters of the equivalent frame method

If the walls in this case are considered as a frame that has a series of beams and columns, flexural stiffness of beams is

$$k_{beams} = \left(\frac{EI}{L}\right)_{beams} \quad \rightarrow \quad k_{beams} = \frac{Eth_b^3}{12x} \qquad (21)$$

Where, t is the wall thickness and h_b is the height of the link beam. Also, the flexural stiffness ofcolumnsis

$$)22(\qquad k_{columns} = \left(\frac{EI}{L}\right)_{columns} \qquad \rightarrow \qquad k_{columns} = Et\left(\frac{D-x}{2}\right)^3 / 12h$$

Where, h is the length of column.

As the equivalent structures is indetermine, internal forces are dependent on relative values of stiffness (not absolute values), so the stiffness of beams is assumed to be unit and the relative stiffness of columns in this case is

$$)23(\; k_{beams} = 1 \quad \rightarrow \quad k_{columns} = \frac{Et\left(\frac{D-x}{2}\right)^3}{\dfrac{12h}{\dfrac{Eth_b^3}{12x}}} = \frac{x}{8h}\left(\frac{D-x}{h_b}\right)^3$$

If we use the dimensionless numbers

$$)24(\; x/D = r$$

$$)25(\; h_b / H = s$$

The following equation is obtained

$$k_{beams} = 1 \; , \quad k_{columns} = \left(\frac{D}{h}\right)^4 \frac{r\,(1-r)^3}{8s^3} \tag{26}$$

After the calculation of relative stiffness analysis can be performed using a suitable analytical method by using a computer program.

Numerical modeling and results

A Proportionateshear wall accordance with specifications shown in Table1.is supposed.

Table1. Considered properties for coupled shear wall

Value	index	parameter
6	N	No. of stories
3m	h	Story height
30cm	t	Wall thickness
40cm	h_b	Depth of link beams

2m	$b\,(or\ x)$	length of link beams
8m	D	Total width of wall
23MPa	E	Modulus of elasticity
0.15	ϵ	Poisson's ratio

By substituting the values of parameters, the relative stiffness of columns in this case is:

$$k_{columns} = \left(\frac{8}{3}\right)^4 \frac{0.25\,(1-0.25)^3}{8(0.133)^3} \simeq 281 \qquad (27)$$

The span of beams in this case, is assumed to be 5m (center to center the walls).

The following figure shows the equivalent frames under three different loading conditions.
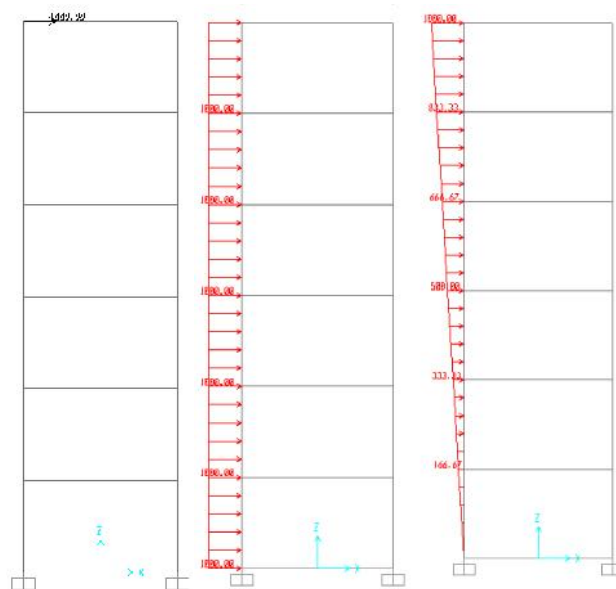
**Fig.14.** Equivalent frames under different loads

Table 2 shows the lateral displacement calculated in different ways under the rectangular, triangular and point loads.

Table 2. Results of analysis

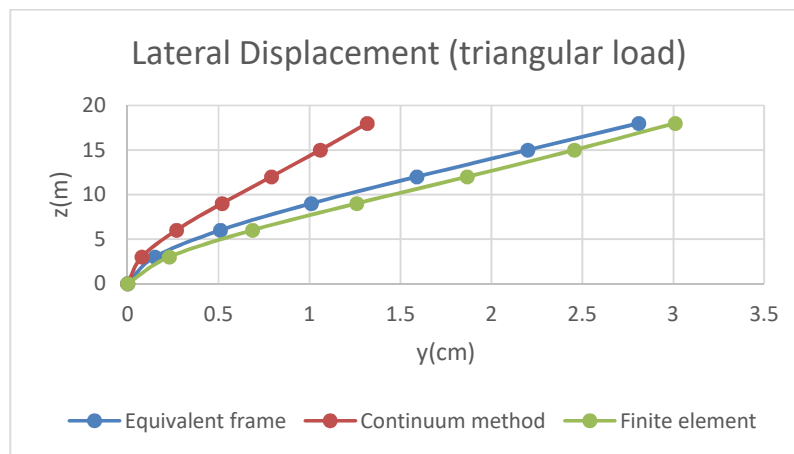| Lateral displacements of shear wall y(cm)		
Concentrated loading	Triangular loading	Rectangular loading

Finite element	Continuum method	Equivalent frame	Finite element	Continuum method	Equivalent frame	Finite element	Continuum method	Equivalent frame	elevation
0.275566	0.2598	0.56	3.010582	1.3171	2.81	1.484901	1.8113	3.85	18
0.206376	0.1984	0.42	2.456132	1.0592	2.2	1.222826	1.4731	3.04	15
0.146211	0.1396	0.29	1.867076	0.7907	1.59	0.940863	1.1179	2.23	12
0.091835	0.0865	0.18	1.259805	0.5192	1.01	0.644225	0.7505	1.44	9
0.04633	0.0425	0.08	0.686494	0.2688	0.51	0.357185	0.3992	0.75	6
0.013995	0.0117	0.02	0.229178	0.0780	0.15	0.121878	0.1196	0.23	3

For better comparison between different methods, the results in Table 2, respectively, is depicted in Figures 15-17.
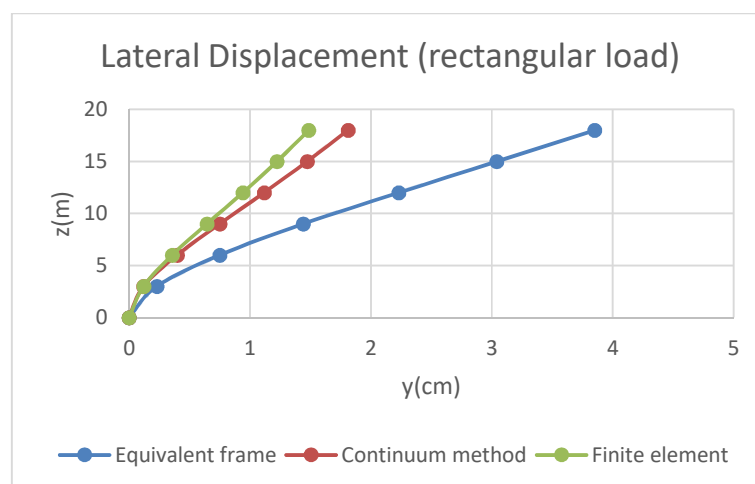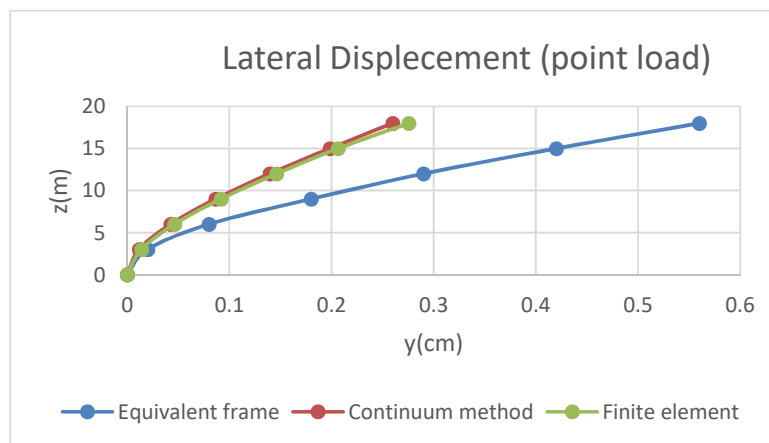

Fig.15. Comparing the lateral displacement obtained from different methods under rectangular load

Fig.16. Comparing the lateral displacement obtained from different methods under triangular load

Fig.17. Comparing the lateral displacement obtained from different methods under concentrated load

CONCLUSION

The lateral stiffness of coupled RC shear walls is investigated in this research by using the continuum method, equivalent frame and finite element methods under different types of loading. The results show that the continuum method and equivalent frame approach have larger displacement compared finite element method; so, the use of these methods will be considered the coupled shear wall structures softer. Continuum methods in all load conditions, except for the triangular case, is better than equivalent frame method compared to the finite element method.,

and it seems that at this state of the loadings, this approximate analysis have better results to the initial design of coupled shear walls. Also, continuum method under triangular loading, has shown minimum displacement and thus the most stiffness and in this case, the equivalent frame method has better agreement with finite element method.

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