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Performance Evaluation of the Earth Gravitational Model 2008 (EGM2008) – A Case Study

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Abstract

Ghana's local geodetic reference network which is based on the War Office 1926 ellipsoid was established using astro-geodetic observations during the British Colonial era with data in latitude, longitude and orthometric height $(\varphi,\ \lambda,\ H)$ *without the existence of ellipsoidal height. The Global Positioning System (GPS) observations which is an example of the Global Navigation Satellite System (GNSS) is being referenced to the World Geodetic System 1984 (WGS84) ellipsoid with data in latitude, longitude and ellipsoidal heights* $(\varphi,\ \lambda,\ h)$*. This prevalent situation makes it difficult to apply standard forward transformation equation for direct conversion of ellipsoidal heights (h) which is global to a practical height (H) within Ghana local geodetic reference network. In order to overcome such a challenge, many researchers resort to various methods of determining the geoidal undulations for a local and national geodetic network and improving the recent New Earth Gravitational Model accuracies and its performances. This present study therefore seeks to evaluate such method of estimating geoidal heights using the Earth Gravitational Model 2008 (EGM08) in a part of the University of Mines and Technology, UMaT, Tarkwa. The estimated geoid heights obtained by the EGM08 model were compared with 328 discrete geometrical heights from co-located GPS and Total station orthometric heights of the University Primary Levelling Networks. The methods applied include estimating the geoidal heights using the EGM08 model, a geometric method and a polynomial mathematical model for improving the estimated EGM08 geoid heights values. The statistics of the differences between derived geoid heights by the geometric approach and corresponding geoid heights* *obtained from the geoid model (EGM08) suggests that, the EGM08 model is most suitable at this moment. The RMSE, Mean Error, and the Standard deviation of their geoidal height differences are 0.120825 m, 2.18823 m, and 3.47678 m, which is better in the area of interest. The study concluded that, the recent geoid model can be applied in UMaT and the polynomial mathematical model is the best model for modelling EGM08 geoid heights values for a local geoid model.*

Keywords: GPS/levelling, Earth Gravitational Model, Polynomial mathematical model, Geoid undulations

1 Introduction

Over the years, one of the most interesting and challenging tasks in the field of geodetic surveying is the accurate determination of orthometric heights from GNSS, in particular GPS measurements (Al-Ghamdi and Dawod, 2013, Lee *et al*., 2012) for a local geodetic datum. This poses a challenge for high order engineering works such as engineering surveys or 3D coordinate transformation and mapping (Featherstone *et al*., 2001; Fotopoulos, 2003). This has therefore drawn the attention of many researchers in the area of orthometric height determination (Ulotu, 2009). Moreover, converting the GPS ellipsoidal height to a physical meaning require the determination of orthometric height and the geoid undulation of the area (Shen and Han, 2013; Dumrongchai *et al*., 2012). The EGM96 and EGM08 are some of the models used to calculate the geoid undulation of an area to determine the orthometric height from GPS measurements (Do, 2011). The original technique that was used to compute the geoid undulation was the Stokes' integral (Heiskanen and Moritz, 1967).

In line of the above, the EGM08 method has been the most widely used for height conversion in some countries and to a relatively high degree of accuracy. The EGM08 is good enough for geodetic applications like determining the topographic heights of points on the globe that require the geoid which approximates Mean Sea Level (MSL) as the datum/reference surface (Yilmaz *et al*., 2010; Abeho *et al*., 2014). The EGM08 was preceded by EGM96 which had a lower degree of accuracy (Pavlis *et al*., 2008). In this modern era, EGM08 is capable of obtaining a sufficiently accurate model of the gravity field over the surface of the earth (Kotsakis *et al*., 2009; Kotsakis and Sideris, 1999). This is a great achievement in the fields of geodesy and geophysics since we can obtain heights with physical meaning without necessarily carrying out the tedious and time-consuming procedures of obtaining these heights by geometric or trigonometric levelling (Hirt, *et al*., 2011, Gruber *et al*., 2011). The EGM08 derived geoid heights can reach the accuracy of regional or local geoid models after modelling the differences between the GPS/levelling geoid heights and EGM08 derived geoid heights at identified control points (Dawod, 2008; Dawod *et al*., 2010; Soycan, 2014).

In view of the above, several researchers were motivated to come up with both empirical and geometric approach for the improvement of the earth gravitational model and also to develop their own regional and national geoid (Chandler and Merry, 2010; Kuroishi *et al*., 2002; Roman *et al*., 2009; Toth *et al*., 2000). Some of the global geoid model that have been used for geoid modelling include OSU91A (Rapp *et al*., 1991), EGM96 (Lemoine *et al*., 1998), GGM02C (Tapley *et al*., 2005, EGM08 (Pavlis *et al*., 2008), AGP2003 (Merry *et al*., 2005), AGP2006 (Parker *et al*., 2007), TZG07 (Olliver, 2007), GEM-T3 (Lerch *et al*., 1994), GRIM4-C2 (Rapp and Wang, 1993), and TG09 (Erol and Erol, 2012). Several mathematical models have been proposed over the decades to improve the working efficiency of the EGM08. Some of the mathematical models proposed include Stokes's formula (Featherstone, 2012), Least Squares Collocation (Lee *et al*., 2013), and polynomial methods (Soycan, 2014; Erol, 2011). This is because the global geopotential models (GGMs) prompt the long wavelength components of the Earth's gravity field very well (Daho *et al*., 2008; Krynski and Lyszkowicz, 2006). They do not only provide a basis for the gravity field when emergent high-precision geoid models, but they are also momentous as reference surfaces for conniving local geoids (Bae *et al*., 2011; Dawod *et al*., 2010). Countries that are yet to develop geoid models have been using GGMs for the calculation of geoid heights and gravity anomalies through spherical harmonic analysis (Lee *et al*., 2008).

An accurate geoid model is essential for determining orthometric heights using the GNSS technology, which is being accepted globally for geodetic purposes (Fotopoulus *et al*., 1999). Many researchers have improved the working efficiency of EGM08 with the polynomial mathematical model. Although several mathematical models exist for the modelling of the EGM08 geoid heights, the application of polynomial approach was adopted in this present study to improve the working efficiency of the EGM08 in the study area. The motivation for using the polynomial model was based on its simplicity in application and promising results reported in literature (Erol, 2011; Al-Kragy *et al*., 2014; Dawod *et al*., 2010; Dawod, 2008; Soycan, 2014). In addition, it can be used for GPS/levelling at the local scale and significant for local geoid modelling (Dawod, 2008). Hence, the polynomial approach provides a provides a promising evidence for its future use in various geodetic applications (Erol, 2011).

The present study considered the University of Mines and Technology (UMaT) campus as a case of application of the EGM08. This is because the local geodetic datum of the University of Mines and Technology (UMaT) is non-geocentric with data in latitude, longitude and orthometric height with geoid model. Therefore, in order to convert the GPS ellipsoidal height for practical engineering applications, there is the need to determine the geoidal undulation of the co-located ellipsoidal and orthometric heights in the local War Office 1926 ellipsoid and World Geodetic System (WGS84) datums. In so doing, the accuracy of the obtained geoidal undulation from the EGM08 can be accessed with those attained from GPS/levelling measurement. Moreover, to the best of our knowledge, the applicability and performance assessment of EGM08 for height conversion in Ghana has not been evaluated. Therefore, this study evaluated and tested the accuracy of using the EGM08 as a method for height conversion within the University of Mines and Technology (UMaT) campus and improving its accuracies by a polynomial mathematical model. Therefore, this study constitutes a good foundation for future research into EGM08 in Ghana.

2 Study Area and Data Source

The study area (Figure 1) is located in the mining town of Tarkwa which happens to be the administrative capital of the Tarkwa Nsuaem Municipal Assembly. It is located in the southwest of Ghana with geographical coordinates between longitudes 1° 59′ 00″ W and latitude 5° 18′ 00″ N and is 78 m above mean sea level. It is about 85 km from Takoradi, which is the regional capital, 233 km from Kumasi and about 317 km from Accra (Ziggah, 2012). The topography is generally described as remarkable series of ridges and valleys. The ridges are formed by the Banket and Tarkwa Phyllites whereas upper quartzite and Huni Sandstone are present in the valleys. Surface gradients of the ridges are generally very close to the Banket and Tarkwa Phyllites. The University and its environs generally lie within the mountain ranges covered by thick forest interjected by undulating terrain with few scarps. The type of coordinate system used in the study area is the Ghana projected grid derived from the Transverse Mercator 1 ° NW and the (WGS84) (UTM Zone 30N). The datum of the University of Mines and Technology are the World Geodetic System 1984 (WGS84); War Office Ellipsoid; Leigon; and the geoid which approximates the Mean Sea Level (MSL). The projection of the University of Mines and Technology is the Transverse Mercator (Ziggah, 2014; (Seidu, 2004)). The study area has a South-western Equatorial climate with seasons influenced by the moist South-West Monsoon winds from the Atlantic Ocean and the North-East Trade Winds. The mean rainfall is approximately 1500 mm with peaks of more than 1700 mm in June and October. Between November and February, the rainfall pattern decreases to between 20 mm to 90 mm (Forson, 2006). The mean annual temperature is approximately 25 °C with small daily temperature variations. Relative humidity varies from 61 % in January to a maximum of 80 % in August and September (Ziggah, 2012).

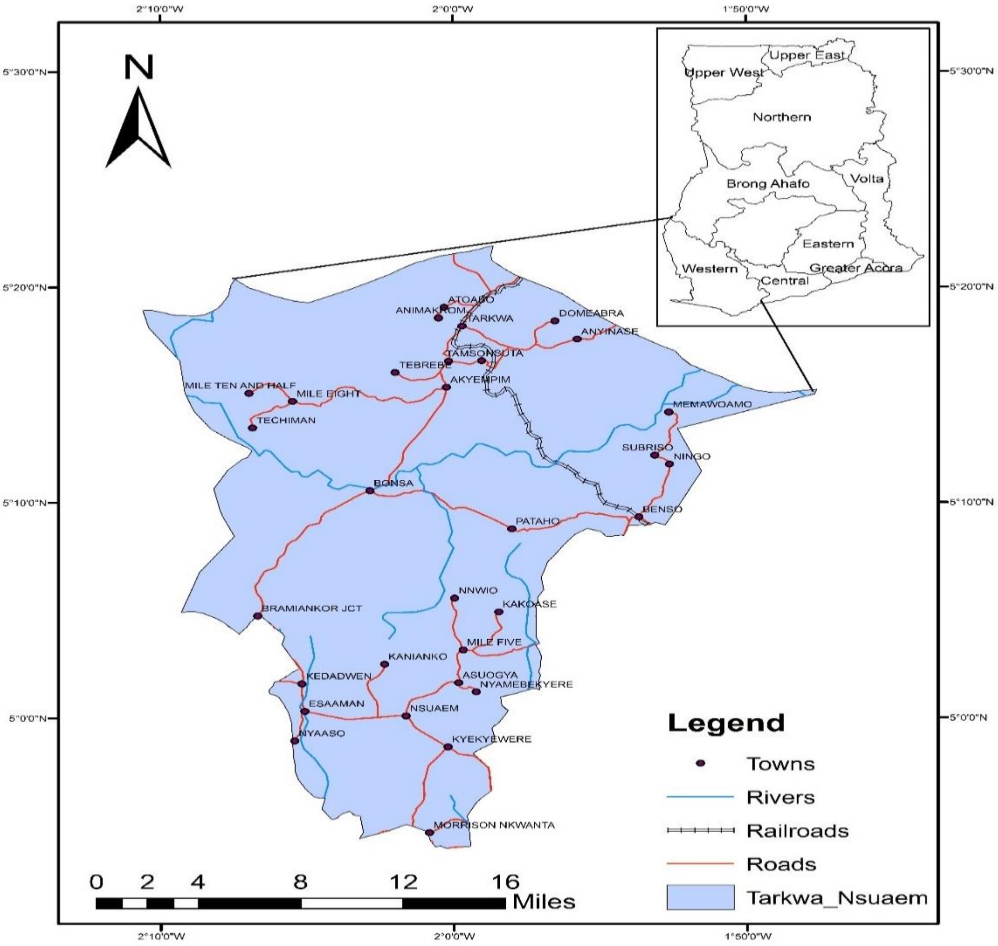

Figure 1 The Study Area

In this present study, a total of 28 Total station and GPS data was used for this present study. This data was obtained directly from field measurement and it covers some part of the University of Mines and Technology, UMaT campus. Total station and the GPS receivers' instruments were used for the collection of data. The Total station data was recorded in Eastings, Northings and Orthometric heights (E, N, H) whilst the GPS data was recorded in Latitude, Longitude and Ellipsoidal height (ɸ, λ, h). Table 1 is a sample of the data used to embark this present study.

Table1 Sample of Data collected from the field

POINT ID	LATITUDE	LONGITUDE	h	H
BM1	5.299648	2.001587	105.0657	79.2000
BM2	5.299471	2.001814	106.1565	80.3900
BM3	5.299346	2.001813	93.3399	68.4770
BM4	5.299505	2.001931	106.3422	80.5390
BM5	5.299301	2.002049	109.1203	82.4000
BM6	5.299204	2.002202	107.6099	81.7900
BM7	5.299211	2.002393	105.6151	79.7320
BM8	5.299563	2.001292	106.3142	80.4690
BM9	5.298738	2.001984	103.9401	78.1920
BM10	5.298880	2.002044	106.0406	80.4210

3 Methods

3.1 GPS/Levelling Derived Geoidal Heights

The computation of geoidal heights from GPS observations and orthometric heights was done according to Equation 1. The geoidal heights from GPS-derived ellipsoidal heights and the Total station orthometric heights are referred to as GPS/Levelling (Dawod, 2008)).

$$N_{GPS/Levelling} = h - H \tag{1}$$

Where $N_{GPS/Levelling}$ is the estimated geoidal heights, h is the ellipsoidal height from GPS measurements and H is the orthometric height obtained from levelling procedure.

3.2 Geoidal Heights from EGM08 Geoid Model

The geoidal heights obtained from this model was done using the EGM08 calculator. The inputs were the geodetic coordinates and the outputs were the geodetic coordinates with their geoidal heights. The geoidal undulations computed from the EGM08 geopotential model coefficients refer to the tide-free system as far as the permanent tide is concerned (Pavlis *et al.*, 2008). The calculated geoid heights obtained from the EGM08 model was computed based on the spherical harmonic equation (Lu *et al.*, 2014) as denoted by Equation 2:

$$V_{(p,\phi,\lambda)} = \sum_{n=0}^{\infty} \frac{1}{p^{n+1}} \sum_{k=0}^{n} (a_{nk} \cos k\lambda + b_{nk} \sin k\lambda) p_{nk}(\cos\phi) \tag{2}$$

Where; (p,ϕ,λ) = are the spherical coordinates a_{nk} and b_{nk} = are the coefficients of the Earth's gravity field $p_{nk}(\cos\phi)$ = represents the associated Legendre polynomials n is degree, and k is order (Pavlis *et al.,* 2008; Pavlis *et al*., 2012; Lu *et al*., 2014).

3.3 Models Performance Evaluation

In order to evaluate the adequacy of the EGM08 and the polynomial model, several statistical indicators were utilised. These include mean error (ME), root mean square error (RMSE) and standard deviation (SD). Their mathematical expressions are given in Equation 3 to 5 respectively. In computing the differences between the geoid models, the assumption made here is that, the geoidal undulations obtained from the GPS/Levelling are standard to which the geoidal heights provided by the EGM08 was compared. The geoidal undulation difference, ΔN between the GPS/Levelling geoidal undulations, $N_{GPS/Levelling}$ and the computed geoidal undulations referred to EGM08, N_{EGM08} is given as denoted by Equation 3:

$$\Delta N = N_{GPS/Levelling} - N_{EGM\,2008} \tag{3}$$

Where ΔN is the geoidal height difference between geoidal heights obtained from geometrical techniques ($N_{GPS/Levelling}$) and EGM approach ($N_{EGM2008}$)

The mean difference, ΔN_{mean} is the average of the geoidal height differences, ΔNj for the EGM08 model. The mean is computed as denoted by Equation (4):

$$\Delta N_{mean} = \frac{1}{n}\sum_{j=1}^{n} \Delta N_j \tag{4}$$

Where $j = 1,2,3,....,n$ and $i = 1,2,3,....,n.$

The root mean square (RMSE) value of the differences in the model is computed from as denoted by Equation 5:

$$RMSE = \sqrt{\frac{\sum_{j=1}^{n} \Delta N_j{}^2}{n}} \tag{5}$$

The RMSE gives a sense of the typical size of the value.

The standard deviation from the mean of the differences (error) in geoidal undulations in the model was computed using Equation 6:

$$SD = \sqrt{\sum_{j=1}^{n} \frac{\left(\Delta N_j - \Delta N_{mean}\right)^2}{n-1}} \tag{6}$$

Where $n-1$ is the degree of freedom.

The standard deviation measures how closely the data are clustered about the mean.

3.4 Hypothesis of the Problem

A hypothesis is a statement about the parameters of a distribution. A test of a hypothesis is a rule that, based on the sample values, leads to a decision to accept or reject the null hypothesis. Normally, a test statistic is computed from the sample values (observations) and from the specification of the null hypothesis. If the test statistic falls within a critical region, the null hypothesis is rejected otherwise it is accepted.

However, the null hypothesis is that the differences have a normal distribution with mean, φ and variance σ^2. The sample mean, ΔN_{mean} and sample variance, S^2 were tested to see if they really belong to normal distribution N (φ, σ2). For statistical testing, the assumption made is that the population mean, φ and variance, σ^2 are normally distributed. Thus, in order to see if the sample mean ΔN_{mean} and variance S^2 are within the confidence interval of the population mean, φ and variance σ^2 from which the sample is drawn, the following hypothetical statistical tests was used:

- Let n_j be the geoid undulation differences from the recent geoid models such that $(j = 1,2,3,....,n)$ with estimated statistics, ΔN_{mean} and S. Then, the sample mean N_{mean} has a T-distribution function given by Equation 7:

$$\frac{\Delta N_{mean} - \varphi}{s/\sqrt{n}} t_{n-1} \tag{7}$$

Where (n-1) is the degree of freedom, the equal to sign (=) indicates that the right-hand side is distributed with respect to left hand side. Thus at 95% probability level, the interval of population mean, φ should be given as denoted by Equation 8:

$$\Delta N_{mean} - t_{0.95}^{(n-1)} \frac{s}{\sqrt{n}} \le \varphi \le \Delta N_{mean} + t_{0.95}^{(n-1)} \frac{s}{\sqrt{n}} \tag{8}$$

4 Results and Discussions

4.1 Training Results

The EGM08 calculator was used to compute the various Geoid heights for the study area. Three hundred and twenty-eight GPS data points within the University of Mines and Technology reference to the WGS84 ellipsoid were used for the training. The inputs were the geodetic coordinates (latitude and longitude) and the outputs were the geoid heights. An ordinary geometric mathematical approach was used to derive the Mean, the Root Mean Square and the Standard deviation were further estimated to assess the accuracy of the computed geoid heights. Table 2 is a sample of the estimated geoid heights by the EGM08 model and Table 3 present a summary of the computed geoid heights ($N_{EGM2008}$) by the EGM2008 model with their estimated Mean, Root Mean Square, and their Standard deviation. Figures 2 and 3 shows the mathematical statistics and analysis of the EGM2008 geoid heights.

Table 2 Sample of the estimated heights by the EGM model

POINT ID	**LATITUDE**	**LONGITUDE**	**UNDULATIONS (m)**
BM1	5.299648	-2.001587	25.8208
BM2	5.299471	-2.001814	25.8202
BM3	5.299346	-2.001813	25.8199
BM4	5.299505	-2.001931	25.8202
BM5	5.299301	-2.002049	25.8196
BM6	5.299204	-2.002202	25.8193
BM7	5.299211	-2.002393	25.8192
BM8	5.299563	2.001292	25.828
BM9	5.298738	-2.001984	25.8182
BM10	5.298839	-2.002044	25.8185

Table 3 A summary of the computed geoid heights by the EGM08 model

GEOID MODEL	**MINIMUM (m)**	**MAXIMUM (m)**	**MEAN (m)**	**RMS (m)**	**SD (m)**
EGM08	25.8126	25.8225	25.81895823	1.42561348	0.002519702

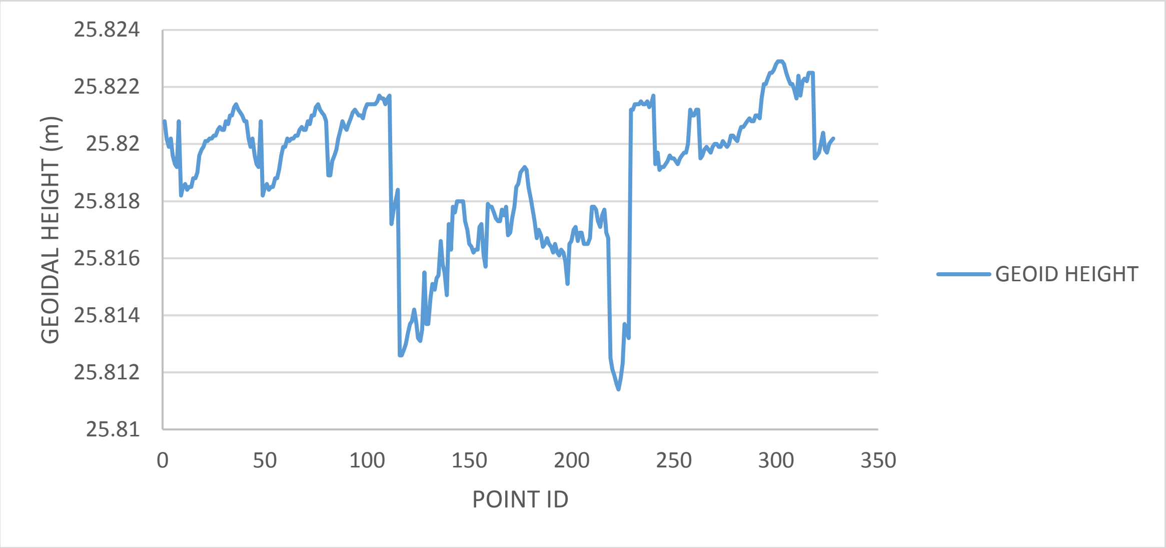

Figure 2 A graph of the EGM2008 geoidal undulations points

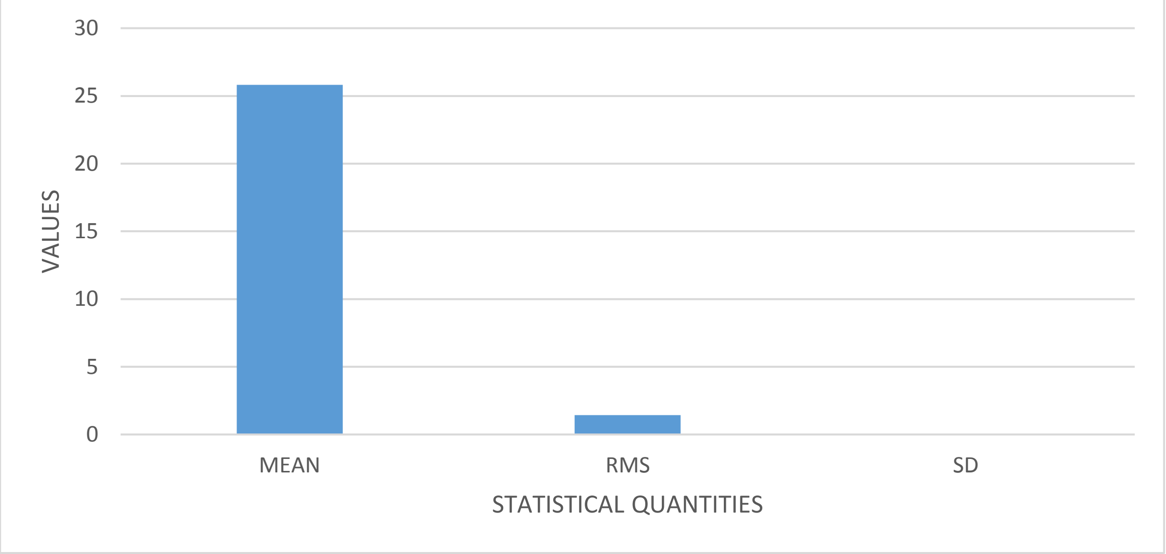

Figure 3 A graph depicting the mathematical statistics of the EGM2008 geoid height

4.2 Testing Results

The testing was carried out using 328 common points from the GPS and the Total station to compute the geoidal undulations of the study area. The geoidal undulations from GPS-derived ellipsoidal heights and the Total station orthometric heights are referred to as GPS/Levelling. A geometric method was used to compute the Mean, the Root Mean Square, and the Standard deviation for further estimate to access the accuracy of the computed geoidal undulations. Table 4 presents a sample of the results of geoidal heights obtained by the geometric method ($N_{GPS/Levelling}$) and Table 5 present a summary of the computed GPS/Levelling geoidal undulations ($N_{GPS/Levelling}$) by an ordinary geometric approach with their estimated Mean, Root Mean Square, and their Standard deviation. Figure 4, and 5 displays the three hundred and twenty points from the Total station and the statistical graphs of the GPS/Levelling geoidal undulations.

Table 5 Sample of the results obtained by the geometric method

POINT ID	**h**	**H**	**UNDULATION (m)**
BM1	105.0657	79.2000	25.8657
BM2	106.1565	80.3900	25.7665
BM3	93.3399	68.4770	24.8629
BM4	106.3422	80.5390	25.8032
BM5	109.1203	82.4000	26.7203
BM6	107.6099	81.7900	25.8199
BM7	105.6151	79.7320	25.8831
BM8	106.3142	80.4690	25.8452
BM9	103.401	78.1920	25.7481
BM10	106.0406	80.4210	25.6196

Table 5 A summary of the computed GPS/Levelling geoid height

MODEL	**MIN (m)**	**MAXI (m)**	**MEAN (m)**	**RMS (m)**	**SD (m)**
GPS/LEVELLING	11.1536	50.9063	27.92206474	1.54173811	3.478905399

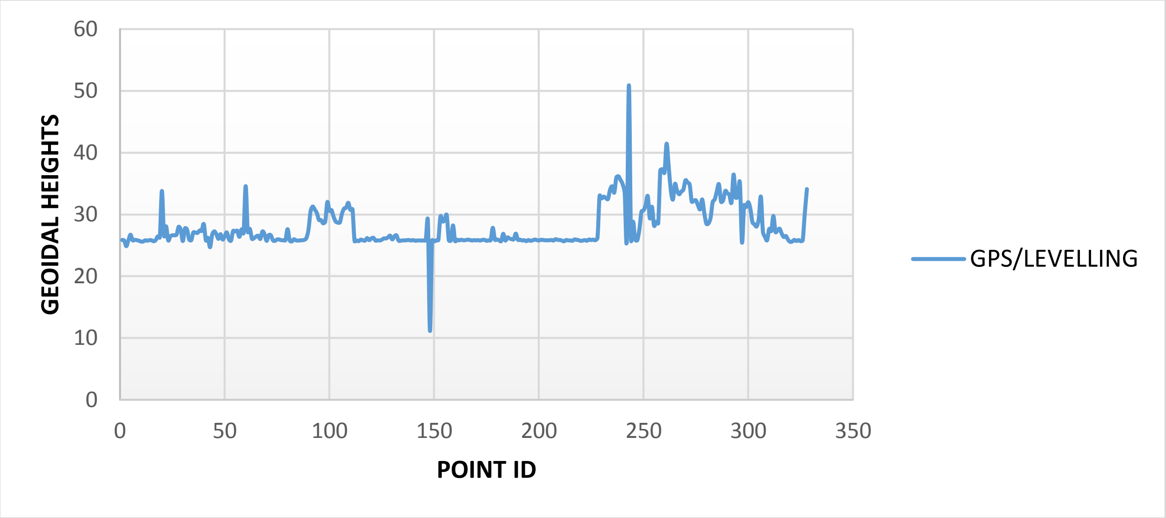

Figure 4 A graph of the GPS/levelling geoid heights

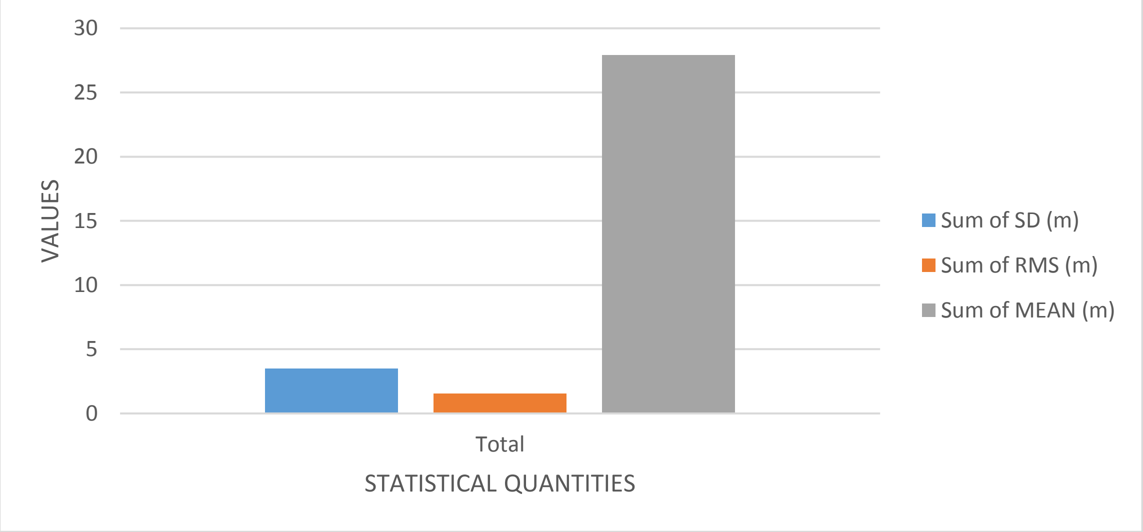

Figure 5 A graph depicting the mathematical statistics of the GPS/levelling geoid heights

4.3 Comparison of Geoid Heights from GPS/Levelling and the EGM2008

The differences of geoidal heights from the GPS/Levelling derived geoidal heights and those from the geoid models at co-located benchmarks provide discrete geometric control in validation purposes. In this section, the geoidal heights differences between the geoid models against the GPS/Levelling derived geoidal heights at the 328 benchmarks were obtained. The computed geoidal heights from the GPS/levelling and the corresponding computed and predicted geoidal heights from the EGM08 are illustrated in Figure 6.

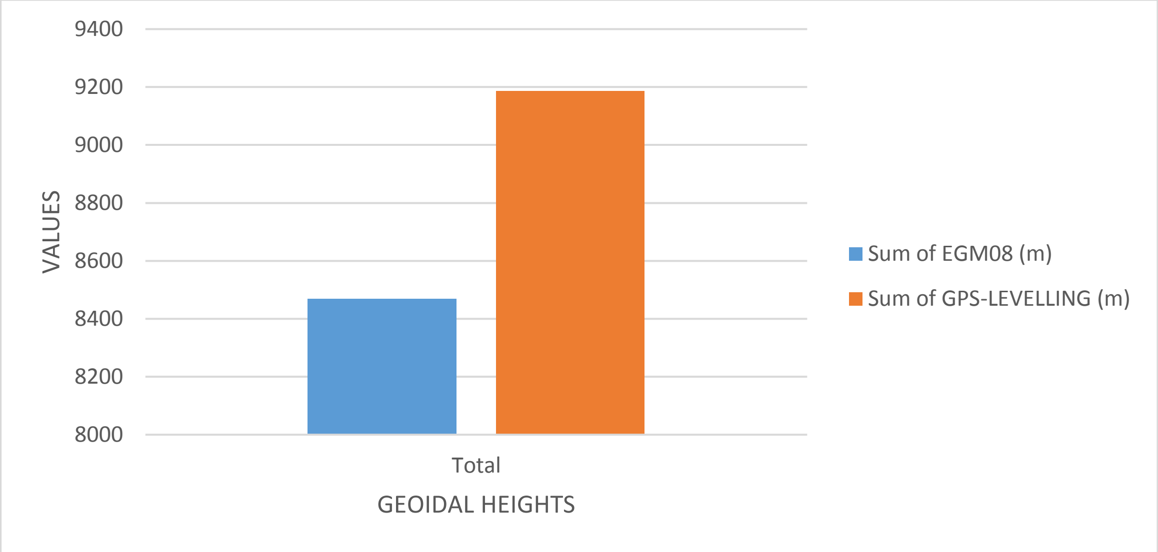

Figure 6 A graph depicting the Geoid height of EGM2008 and GPS/levelling

4.3.1 Results from the comparison of Geoid heights from GPS/levelling and EGM2008

Sample of the results of the geoid heights differences are tabulated in Table 6. The summary of the results obtained from the differences between the two geoidal undulations are shown in Table 7 for the recent geoid models in the area of interest. The statistics of the differences are also shown with respect to minimum differences, maximum differences, mean of the differences, root mean square of the differences and the standard deviation from the mean of the difference. Figure 7 depicts the graphs of the geoid undulation differences of the geoid model EGM2008 and the GPS/levelling.

Table 6 Sample of the geoidal heights differences results

POINT ID	**GPS/LEVELING (m)**	**EGM2008 (m)**	**dN (m)**
BM1	25.8657	25.8208	0.04490
BM2	25.7665	25.8202	-0.0537
BM3	24.8629	25.8199	-0.9570
BM4	25.8032	25.8202	-0.0170
BM5	26.7203	25.8196	0.90070
BM6	25.8199	25.8193	0.00060
BM7	25.8831	25.8192	0.06390
BM8	25.8452	25.8208	0.02440
BM9	25.7481	25.8182	-0.0701
BM10	25.6196	25.8185	-0.1989

Table 7 A summary of the geoid height differences

HEIGHT	**MINIMUM (m)**	**MAXIMUM (m)**	**MEAN (m)**	**RMS (m)**	**SD (m)**
dN	-0.0012	25.0872	2.188234756	0.120825052	3.476781863

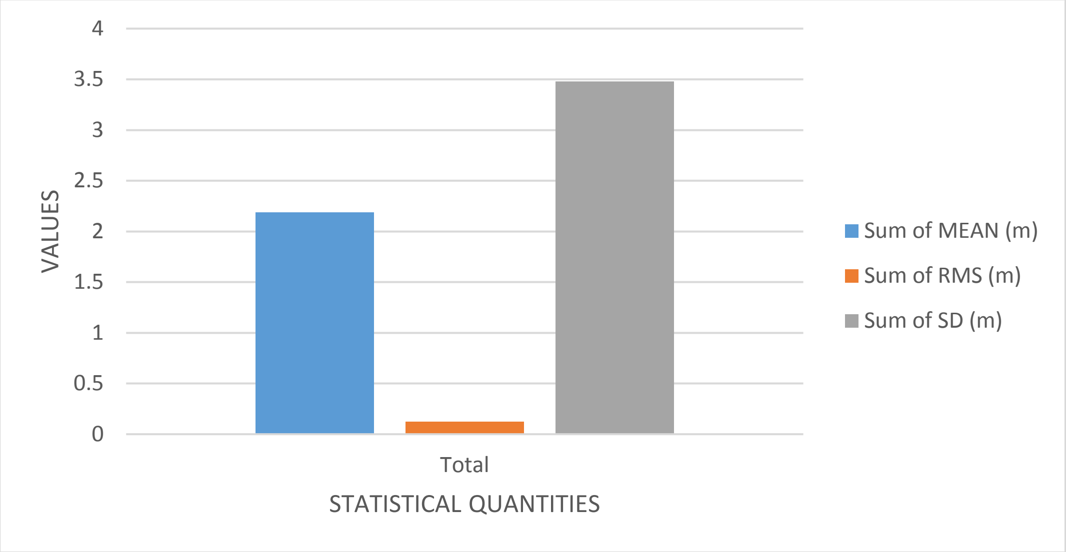

Figure 7 A graph depicting the statistics of the geoid height differences

The results obtained in this study show that there are differences between GPS/Levelling derived geoidal heights and those obtained from the New Earth Gravitational Model 2008. With reference to Table 3 and Figure 5 above, it was observed that the geoidal height differences from the GPS/levelling geoidal heights and those from the EGM2008 model ranges from -0.0012 m to 25.0872 m. The results showing the geoidal height differences are also presented graphically above. It was also seen that; the residuals range from the positive to the negative and depicting the trend of the geoid height differences from the recent geoid model based on the standard GPS/Levelling derived geoidal heights. The minimum differences are -0.0012m, the maximum differences are 25.0872 m, the mean differences are 2.18823 m, the Root Mean Square differences are 0.12082 m and the standard deviation differences are 3.47678 m. The mean differences, the Root Mean Square value and the sample Standard deviation are also represented graphically as shown in Figure 7 above.

5 Polynomial Models Developed

5.1 Polynomial mathematical model

In this project, a polynomial mathematical model was applied on the EGM2008 derived geoidal heights within the University of Mines and Technology geodetic reference network to improved its performance, mean square errors, root mean errors and standard deviation respectively. The general polynomial equation is given by Equation 9:

$$Z = a_0 + a_1 + a_2 + a_3 + a_n \tag{9}$$

The process of determining the geoidal heights and improving its performance with the polynomial mathematical model requires a lot of computational tasks which will be practically be a difficult task without the use of a computer programming language. For this task, a Microsoft Excel and Matlab 2014 software computer programming language was written to handle the geoidal heights computed and the polynomial models. The input data consist of the heights obtained from the EGM2008 calculator. The output for the polynomial mathematical model formed after determining its value were used to improve the EGM2008 geoidal heights. Three polynomial models were used for modelling the EGM08 geoid height values, the simple planar surface, the bi-linear saddle, and the quadratic surface. Their polynomial equation is given by equation (10), (11), and (12) (Soycan, 2014; Dawod *et al.*, 2010; Dawod, 2008; Al-Kragy *et al.*, 2014; Erol, 2011).

$$N = a_0 + a_1\lambda + a_2\phi \tag{10}$$

$$N = a_0 + a_1\lambda + a_2\phi + a_3\lambda\phi \tag{11}$$

$$N = a_0 + a_1\lambda + a_2\phi + a_3\lambda^2 + a_4\lambda\phi + a_5\phi^2 \tag{12}$$

A Least Squares approach according to Equation 13 was used to compute the unknown parameters as denoted by:

$$AX = L + V \tag{13}$$

$$A = \begin{bmatrix} 1 & \lambda_1 & \phi_1 \\ \vdots & \vdots & \vdots \\ 1 & \lambda_n & \phi_n \end{bmatrix} X = \begin{bmatrix} a_0 \\ \vdots \\ a_n \end{bmatrix} L = \begin{bmatrix} dN_1 \\ \vdots \\ dN_n \end{bmatrix}$$

Where, A= Matrix of coefficients for the unknown parameters, X= Matrix of unknown parameters, L= Observation matrix; and V= Matrix of the residuals.

Therefore, the residuals (V) matrix is given by Equation 14:

$$V = AX - L \tag{14}$$

5.2 Testing of the Improved EGM2008 Geoidal Heights

The accuracies of the improved EGM2008 geoid height values were tested. In all, 328 points from each data sets were used, that is $N_{GPS/Levelling}$ and N_{EGM08}. The testing also provided the platform to know the accuracies of the two data sets when the polynomial model was applied on the EGM2008 geoid height values. This was achieved by comparing the two N heights. Equation 15, 16, and 17 was used to improve the new earth gravitational model.

Firstly, a trend surface is fitted to the application of method by using control points. Trend surface may be fitted by the polynomial mathematical method. According to this study, usage of the simple planar surface, bi-linear saddle, and quadratic surface polynomial functions can be sufficient in practice. Determination of the improved model was carried out through 3, 4, 6 parameter trend solution according to the polynomial order of degree.

$$T_i = a_0 + a_1\lambda + a_2\phi \tag{15}$$

$$T_i = a_0 + a_1\lambda + a_2\phi + a_3\lambda\phi \tag{16}$$

$$T_i = a_0 + a_1\lambda + a_2\phi + a_3\lambda^2 + a_4\lambda\phi + a_5\phi^2 \tag{17}$$

The trend values calculated for each point the dN values were calculated by subtracting the geoid height differences as denoted by Equation 18 (Soycan, 2014; Dawod *et al*., 2010):

$$dN_i = \Delta N_i - T_i \tag{18}$$

Subsequently, dN values were modelled by least squares fitting with a suitable surface. For this purpose, the Matlab 2014 software algorithm was used. Thus, dN values were calculated for each model (Soycan, 2014).

Finally, the Improved EGM08 geoid height values at each point can be calculated by adding the trend value (Ti), and the difference value (dN) to known EGM2008 geoid height as denoted by Equation 19:

$$N_i^{\mathrm{Im}\,proved} = N_i^{EGM\,2008} + T_i + dN_i \tag{19}$$

As a result, the polynomial mathematical model had a best agreement with the GPS/levelling derived geoid heights. Looking at the Figure 8 and Table 8, it is obvious that the three models improved the EGM2008 geoid heights values to equally match the GPS/Levelling geoidal height values. Thus, the results of the polynomial mathematical model have revealed that the EGM08 geoid model performs exceedingly over the study area.

Table 8 Sample of the improved EGM geoid heights values

POINT ID	**GPS/LEVELLING (m)**	**MODEL A (m)**	**MODEL B (m)**	**MODEL C (m)**
BM1	25.8657	25.8657	25.8653	25.5000
BM2	25.7665	25.7665	25.7669	25.6250
BM3	24.8629	24.8629	24.8632	25.1250
BM4	25.8032	25.8032	25.8034	25.8750
BM5	26.7203	26.7203	26.7203	26.7500
BM6	25.8199	25.8199	25.8202	25.6250
BM7	25.8831	25.8831	25.8827	26.1250
BM8	25.8452	25.8452	25.8457	25.5000
BM9	25.7481	25.7481	25.7485	25.7500
BM10	25.6169	25.6169	25.6191	25.3750

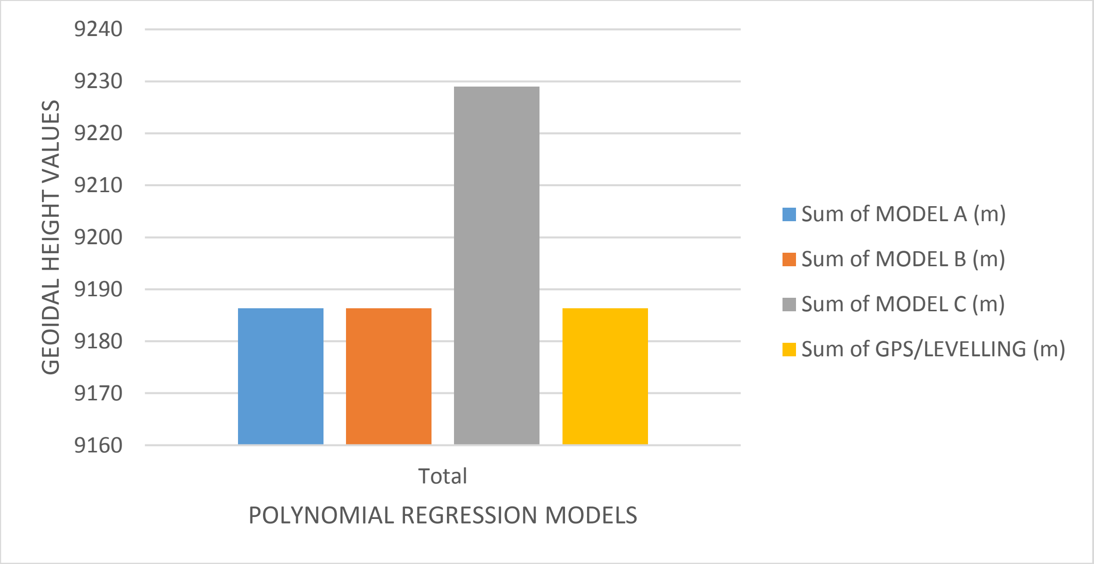

Figure 8 A graph of the various polynomial models

6. Hypothesis testing

These tests were performed in order to quantify if the sample mean, ΔN_{mean} and variance, S^2 are within the confidence interval of the estimate population mean, φ and variance σ^2 from which the sample is drawn from the geoid model. The information which is used to find the truthiness in validating the geoid models are those found in Table 3.

Using Equation (20) the population mean, φ can be tested as follows;

Let the null hypothesis $H_0 : \varphi_{EGM\,2008} = 2.1882$ (20)

From the statistical Tables $t_{0.95}^{(n-1)} = 1.960, \Delta N = 2.1882, \sqrt{n} = 18.1108, S = 3.4768$ and degree of freedom $(n-1) = 327$, then the interval becomes as denoted by Equation 21:

$$1.8119 \leq \varphi_{EGM\,2008} \leq 2.5645 \tag{21}$$

Equation (21) entails the acceptance of the null hypothesis in equation (20).

7 Conclusion

To conclude, it can be fairly stated that EGM08 approach have not been applied and tested within the Ghana local geodetic reference network for estimating geoidal heights (N) in order to convert GPS heights (h) to a practical height (H). Moreover, it is well known that the accuracy of the determined local geoidal heights has an influence on the transformed GPS ellipsoidal heights to orthometric heights. Hence there is the need to investigate the efficiency and performance of the EGM08 in estimating local geoid heights within Ghana’s local geodetic network. This study evaluated, compared and improved the recent geoid model (EGM08) derived geoid heights in the University of Mines and Technology, UMaT, Tarkwa using GPS/Levelling (geometric approach) derived geoidal heights as an independent tool for validation of results obtained from the geoid model (EGM08). In order to ascertain the efficiency of the EGM08, the longitude and latitude obtained from the GPS measurements were applied in the EGM08 model to estimate geoid heights to be able to convert GPS ellipsoidal heights from the World Geodetic System 1984 (WGS84) to local geodetic system (Accra datum). According to the results and the objectives of this project, the polynomial mathematical model best agree with the geometric estimated geoid heights

(GPS/Levelling). The difference between the EGM and geometric estimated heights with mean differences of 2.18823 m and the RMSE of 0.1208 m lead to the conclusion that the EGM2008 geoid model is a better model for GPS/levelling in the study area at the moment. The accuracies of the geoid model for a local geoid scale have been assessed, it was realized from the results obtained that each polynomial model has varying degree of accuracies. Based on the results, the polynomial regression is the proposed model for a local geoid modelling in the study area. This study will therefore create the opportunity for geospatial practitioners in developing countries like Ghana to arrive at a consensus on the most appropriate alternative technique applicable for estimating local geoid heights within the local geodetic reference network. This study will also create the opportunity to know the efficiency and performance of applying EGM as a plausible practical alternative technology to the traditional geometric method in estimating geoidal heights. It could also improve the general accuracy method in topographic mapping and three-dimensional (3D) coordinate transformation.

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