DEVELOPED PROBABILISTIC REDUCTION FACTORS FOR *LOPHIRA ALATA* (EKKI) TIMBER JOISTS SUBJECTED TO CREEP-RUPTURE

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ABSTRACT
Wood experiences a significant loss of strength and stiffness when loaded over period of time. This phenomenon is known as creep-rupture. Several models were developed for the estimation of the reduction of load carrying capacity of timber with time. In this paper, the results of time dependent structural reliability analysis of timber joist produced with Lophiraalata (Ekki) timber specie was presented. Three load duration models were considered in the study, namely: The Model proposed by Wood, Gerhards model, and Nielsen. The timber joist was designed in accordance with the Eurocode 5. The uncertainties in all the basic design variables were fully accommodated in the time dependent reliability analysis. The entire process was implemented using a developed MATLAB program employing First Order Reliability Method (FORM). Time dependent mathematical models for modification of safety index to account for the effect of load duration were proposed. The use of both Gerhards and Nielsen model, for the design of Lophiraalata timber members was recommended.

Keywords: Creep-rupture, Lophiraalata, reliability, uncertainties, Gerhards

1. INTRODUCTION
1.1 Preamble
Wood experiences a significant loss of strength and stiffness when loaded over a period of time. This phenomenon is known as creep-rupture – often called the duration of load (DOL) effect[1, 2, 3]. The issue has been a subject of particular interest for everyone in the timber engineering community concerned with safe and efficient engineering design. Research on both clear wood [4] and structural timber [5, 6] sustaining long-term constant load is well documented in the literature.

The load duration factors presented in the [7], like all other coefficients in the code were established based on the long term experience of building tradition based on semi-probabilistic approach [8]. In reality, the material properties of timber, such as density, bending strength, modulus of elasticity, tension and compression strength parallel and perpendicular to grain are highly random. The load duration effect is also a random variable [2]. The best way to model effects that are random (uncertain) is to use full probabilistic method [9]. With this in mind, probabilistic analysis was used to evaluate the effect of the load duration on timber considering timber floor joist made with *Lophiraalata* (Ekki) timber specie. The material properties of the timber specie were generated from the laboratory. First order reliability method was used through a developed MATLAB program to implement the structural reliability analysis. The reason for the selection of *Lophiraalata* timber specie is for its wide structural usage. Many structures like timber frame houses, timber Bridges and Railway sleepers were made of *Lophiraalata*.

Modification of the reliability indices rather than the timber material properties is proposed in this study. This is because safety indices are established after consideration of the uncertainties in the design variables and loading. The current recommended load duration factors (k_{mod}) in the Eurocode 5 [7], only affects the material properties, being silent of the possible influence of load effect on the load duration.

Reliability-based design is performed to take into account, the effects of uncertainties and achieve predefined target safety indices. The effects of load duration on timber structures generally lead to non realisation of the target values of the safety indices, because, capacity of a timber structural member for a given failure mode is reduced with age. There is therefore the need to accommodate the loss of structural safety due to load duration effect in design.

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The overall aim of the present study is to develop time dependent mathematical models for modification of safety indices to account for load duration on *Lophiraalata* timber members, considering uncertainties in both material properties of the timber and the load action effects. The aim was achieved through the following objectives:

i. Generated statistical models (mean, coefficient of variation and statistical distribution models) of material properties of *Lophiraalata* timber specie based on experimental data.

ii. Generated statistical models of loading and geometrical properties for timber floor joist from data reported in the literature.

iii. Developed performance (limit state) functions for creep-rupture of simply supported floor joist, considering various load duration models reported in the literature (Woods (Woods, 1951), Gerhards (Gerhards, 1979) and Nielson models (Nielson, 1979).

iv. Developed MATLAB based First Order Reliability Analysis (FORM) program for the evaluation of the performance function to determine the safety indices at varying load duration for each of the considered load duration model.

v. Developed non-linear regression models for prediction of safety indices reduction due to creep rupture for timber floor joist.

1.2. Modeling of the Duration of Load Effect

Several models of load effect for timber and other materials can be found in the literature [2, 4-6, 10-18]. Some of these have a conceptual framework from some physical mechanisms leading to failure over time as a background; others are just empirically derived to represent load duration test data. One of the earliest creep rupture model was the one proposed by Wood in 1951 [4]. This model was formulated on the basis of results from laboratory tests on clear wood bending specimens, exposed to loads of constant intensity. Medison curve was then developed based on the following empirical model:

$$sl = 90.4 - 6.3 \log t_f \qquad (1)$$

In (1), sl is the stress level and t_f is the time to failure in hours. What were popularly known as 'damage accumulation models' for creep rupture were proposed by some notable researchers. The most prominent damage models are those proposed by Gerhaard's [13] and Foschi [12].The mathematical expression of Gerhard's model for calculating damage accumulation of a timber member with initial capacity, R_0 sustaining a load S(t), is given by the following equation:

$$\frac{d\alpha}{dt} = e^{(-a+b)\varsigma} \text{ for } 0 \le \alpha \le 12 \qquad (2)$$

In (2), $\varsigma = \frac{S(t)}{R_0}$, α is the degree of damage ($\alpha = 0$ denoting no damage and $\alpha = 1$ denoting failure) a and b are model parameters. Equation 3 shows the mathematical expression of Foschi and Yao's model for calculating damage accumulation of a timber member with initial capacity, R_0 sustaining a load S(t).

$$\frac{d\alpha}{dt} = A\left(\frac{S(t)}{R_0} - \eta\right)^B + C\left(\frac{S(t)}{R_0} - \eta\right)^D \alpha(t) \text{for } S(t) > \eta R3 \quad (3)$$

$$\frac{d\alpha}{dt} = 0 \text{ for } S(t) \le \eta R_0 \qquad (4)$$

In (3), A, B, C and D are model parameters. Other creep rupture models were developed based on fracture mechanic**s.** Typical is the one proposed by Nielsen in 1979 [16]. The idea behind the Nielsen model is that structural timber may be seen as an initially damaged viscoelastic material, with the load-carrying capacity R_0, where the damage is represented by cracks along the fibres. The time-dependent behaviour of timber under load is modeled by a single crack under stress perpendicular to the crack plane. For the case of a constant load level, S a damage accumulation law can be formulated from the DVM-model as:

$$\frac{d\alpha}{dt} = \frac{(\pi FL)^2}{R_0} \alpha_k \frac{(S)^2}{R_0} \left[\alpha_k\left(\frac{(S)^2}{R_0}\right)^{-1} - 1\right]^{(-1/b)} \qquad (5)$$

In (5) FL is the strength level defined as the ratio σ_{cr}/σ_i between the short term strength, σ_{cr} measured in a very fast ramp test and the intrinsic strength of the (hypothetical) non-cracked material σ_i. The damage, α_k is defined as the ratio between the actual crack length, l, and the initial crack length l_0. $\alpha_k = 1$ corresponds to no damage and $\alpha_k\left(\frac{(S)^2}{R_0}\right)^{-1}$ to full damage. τ and b are creep material parameters. q is given as a function of creep exponent $q = \left(0.5(b+1)(b+2)\right)^{(a/b)}$ and considers a parabolic increasing crack propagation.

2. MATERIALS AND METHODS
2.1 Material
The programme of tests was designed to determine the bending strength, modulus, density and moisture content of *Lophiraalata* (Ekki) timber species. Four point bending test was conducted in accordance with [19]. The materials were obtained from Sabongari Zaria timber sheds, Kaduna State.

2.2 Experimentation

Fourty samples were used for both the density and the bending tests based on the EN 384 [20] specification on the required number test specimens. The density and the moisture content tests was conducted according to EN 13153-1 [21], while the bending tests were conducted according to EN 408 [20]. All the tests were done at the heavy duty structures laboratory of the Department of Civil Engineering, Ahmadu Bello University, Zaria. The generated data were analyzed based on EN 384 [20], to determine the mean values and coefficients of variation of the material properties and density. Kolmogorov Smirnov (KS)test was conducted using Easyfit [22] statistical analysis software to fit the most appropriate distribution model to the material properties and the density.

To facilitate the grading of the *Lophiraalata* timber specie in accordance with EN 338 [23], the density, bending strength and modulus of elasticity were adjusted to 12% moisture content using Equations (6), (7) and respectively [24, 25, 26]:

$$f_{m,12\%} = \frac{f_{measured}}{1 + 0.0295(12 - u)} \quad (6)$$

$$E_{12\%} = \frac{E_{measured}}{1 + 0.0143(12 - u)} \quad (7)$$

$$\rho_{12\%} = \rho_u \left(1 - \frac{0.5(u - 12)}{100}\right) \quad (8)$$

In (6), (7) and (8), $f_{m,12\%}$ is the bending strength at 12% moisture content. $f_{measured}$ is the bending strength at the measured moisture content. u is the measured moisture content in %, E_{12} is the 12% moisture content adjusted modulus of elasticity, $E_{measured}$ is the measured modulus of elasticity, ρ_{12} is 12% moisture content adjusted density, ρ_u is the measured density.

2.3 Structural Reliability Analysis

The most probable point (MPP) method is employed for the probabilistic analysis in this study. Based the data on material properties and loading defining the performance function of a given mode of failure, suitable distribution laws (normal, lognormal, uniform, Weibull, Gumbel) are selected. Then the random variables were transformed into a standard normalized space. For the transformation, the mean value and the standard deviation of the random variables are necessary. This information is presented in Table 1.

In the standard normalized space, a normalized vector *U*, can be calculated by: $U = T(y)$, where $T(y)$ is the probabilistic transformation function. For a given scenario, the reliability index β, introduced by Hasofer and Lind in 1974 is evaluated by solving a constrained minimization problem:

$$Min \qquad\qquad : d(U) \qquad (9)$$

$$Subject\ to \qquad H(H) \leq 0 \qquad (10)$$

In (9), U is the vector modulus in the normalized space, measured from the origin. Figures 1 and 2 shows the physical and the normalised space respectively
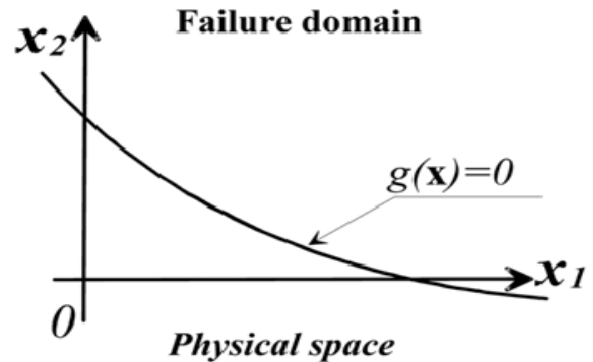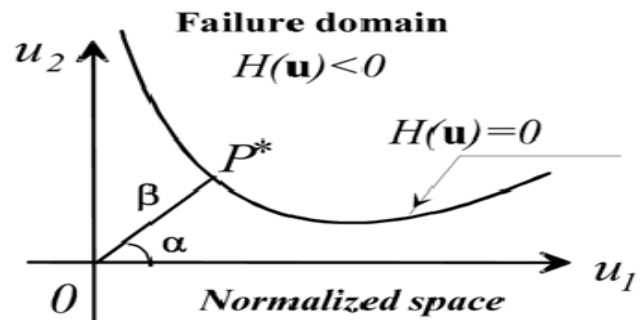

Figure 1: Physical space

Figure 2: Normalised apace

The minimum distance *d(U)* is given by :

$$d = \sqrt{\sum U_i^2} \quad i = 1, \ldots, n \qquad (11)$$

where n is the variable number.

The solution of the constrained optimisation problem (Equation 10), define the design point *P**. The resulting minimum distance between the limit state function H (U) and the origin is called the reliability index, which in general can be obtained in terms of the probability of failure P_f given by:

$$\beta = \phi^1(P_f) \qquad (12)$$

The flow chart for the MATLAB implementation of the MPP method is presented in Figure 3.

2.4 Structural Configuration of the *Lophiraalata* Floor Joists

Timber floor joists for a domestic dwelling (Figure 4) produced with *Lophiraalata* (Ekki) timber specie were considered for the time-dependent reliability-based determination of the safety index modification factor. The width of the floor b, is 3.6 m and the floor span, l, is 3.4 m.

Table 1: Stochastic models of the basic design variables

| S/No	Variable	Unit	Variable Type	Coefficient of Variation	Distribution Model	Reference
1	Resistance Model Uncertainty, ϕ_R	-	Random	0.1	Lognormal	[9]
2	Creep Rupture Model, λ_d	-	Random	0.15	Lognormal	[9]
3	Bending Strength, f_m	N/mm²	Random	0.16	Lognormal	[9]
4	Load action model uncertainty, ϕ_s	-	Random	0.1	Lognormal	Test reslts
5	Variable load, Q	kN/m²	Random	0.2-0.4	Gumbel	[27]
6	Dead to live load ration, α	-	Deterministic	-	-	-
7	Joist span, L	M	Deterministic	-	-	-
8	Joist width, b	Mm	Random	0.05	Normal	[28]
9	Joist Depth, d	Mm	Random	0.05	Normal	[28]
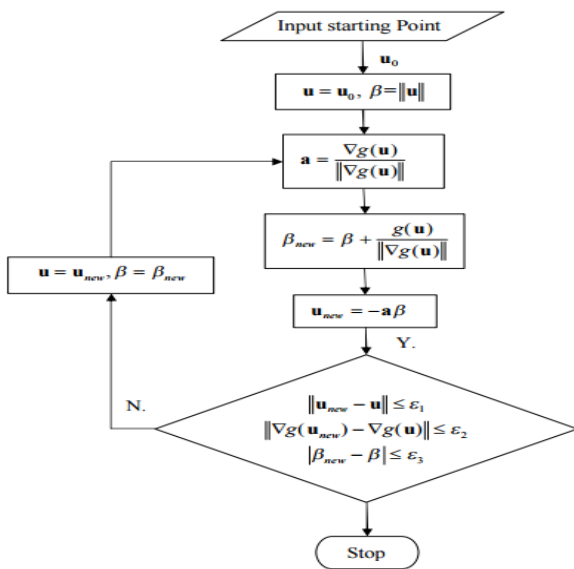
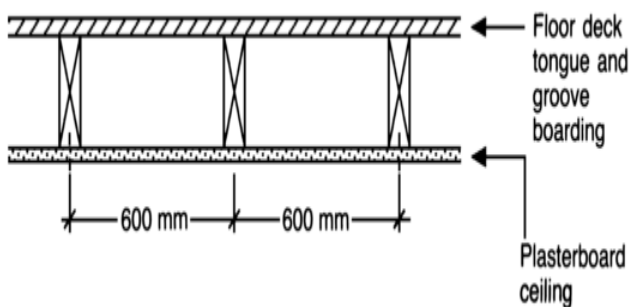*Figure 3: Flow chart for the MATLAB implementation of the MPP method*

Figure 4: Structural Model of the Flooring System Fracture Mechanics Model

The joists are spaced at 600 mm centres. It was subjected to characteristic variable action of 1.50 kN/m²with permanent to variable load ratio defined by alpha (α). The design of the joist according to Eurocode 5 revealed that timber joists with width b equal to 50mm, and depth h, equal 150mm spaced 600mm would be adequate.

2.5 The Flexural Limit State Function

The structural reliability analysis requires the formulation of the performance function for the failure mode of interest. In this study, only flexural limit state function is considered. The limit state function is given by Equation (13)

$$G(x) = \phi_R \lambda_d f_m - \frac{2.25e^4 Q(\alpha + 1)L^2}{bd^2} \qquad (13)$$

In (13), ϕ_R is the model uncertainty for bending resistance, λ_d is the creep-rupture model, f_m is the bending strength of the *Lophiraalata* timber joist, ϕ_s is the model uncertainty for the load action, Q is the variable load action, α is the dead to live load ratio, L is the joist span in m, b and d are joist width and depth respectively in mm. The *Lophiraalata* timber species was classified into timber strength class D60, according to EN 338 [23].

2.6 Determination of the Safety Index Reduction Factor due to Creep Rupture

The floor joists considered were first designed according to Eurocode 5, [7], with the load duration factor k_{mod} taken as 1. The flexural failure mode of the joists was then subjected to structural reliability based design, and the safety index was found to be 5.92 corresponding to the Eurocode 5[7] criteria for the design load. The computed safety index is above the Eurocode 0 [8] recommended minimum safety index of 3.

Three damage models proposed by Wood [4], Gerhards [13] and Nielson [16] were used to evaluate the time dependant capacity reduction of safety indices due to creep rupture. The structural reliability analysis was conducted considering each of the three models. The time dependent capacity reduction as measure of reduction of joist safety is obtained by Equation (14).

$$CR(t) = \frac{\beta_i(t) - \beta_r(t)}{\beta_i(t)} \qquad (14)$$

The time dependent capacity modification factor is the compliment of the time dependent capacity reduction factor, CR(t), given by Equation (15).

$$\lambda_d(t) = 1 - CR(t) \qquad (15)$$

Here, CR(t) is the time dependent capacity reduction, $\beta_i(t)$ is the safety index of the just constructed timber

joist, $\beta_r(t)$ is the time dependent safety index of the joist, $\lambda_d(t)$ is the time dependent safety index modification factor, t is the age of the joist from erection in years.

Investigation was made for up to 50 years load duration, and the results subjected to non-linear regression analysis. Time dependent mathematical models for the safety index modification were developed for Wood, Gerhards and the Nielsen models.

3. RESULTS AND DISCUSSION

The results of the time dependent structural reliability analysis of the effect of load duration on timber members are presented in Figures 5 to 8.

The relationship between safety index and load duration of timber corresponding to the three considered damage models (Wood model, Gerhards model and Nielsen models) is presented in Figure 5. It is clear from the plot that, the relationship is non-linear. The model proposed by wood, shows very high capacity reduction at all ages. For instance at the 40 years load duration the Wood model gave safety index on 2.5, while the Gerhards and Nielsen models gave safety indices of 3.15 and 3.2 respectively. Structural reliability using the Eurocode 5 recommended load duration factors k_{mod} for long term action, gave safety index of 3.03. Since 40 years is within the long term action category of the Eurocode 5, it can be concluded that, the Gerhards and the Nielsen models closely predicted the Eurocode 5 load duration effect

proposal. The Wood model however, over-estimated the load duration effect.

Figure 6, 7 and 8 presented the non-linear regression plots for the Wood, Gerhards and Nielsen models respectively.

Based on the results presented, the following mathematical models were developed for reduction of safety indices obtained from structural analysis due to load duration for timber member produced with *Lophiraalata* timber specie:

i) based on Wood model:
$$\lambda_d(t) = -0.005 \ln(t) + 0.615 \qquad (16)$$
(ii) based on Gerhard model
$$\lambda_d(t) = -0.04 \ln(t) + 0.70 \qquad (17)$$
(iii) based on Nielsen model
$$\lambda_d(t) = -0.04 \ln(t) + 0.615 \qquad (18)$$

The R^2 values for all the models were found to be 0.998, indicating strong correlation between the safety index reduction factor and the load duration.

4. CONCLUSION

In this study, the results of reliability-based evaluation of the effect of load duration on timber members produced with *Lophiraalata* timber specie were undertaken. Mathematical models for reduction of computed safety indices from structural analysis were developed considering three damage models reported in the literature (Wood, Gerhards model and Nielsen models).
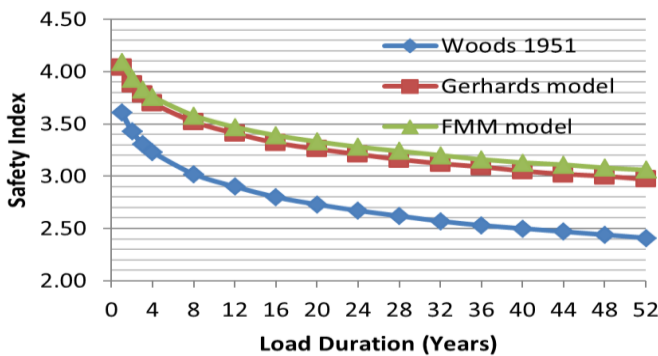

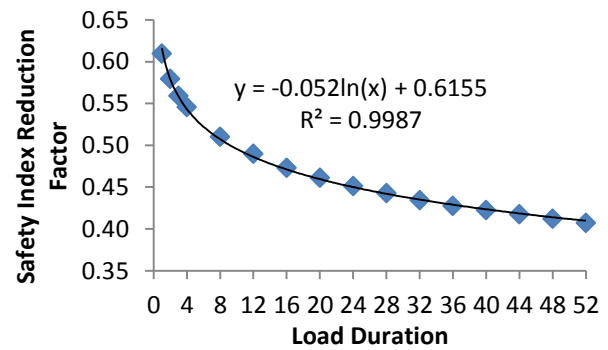Figure. 5: Effect of load duration on safety index for various damage models

Fig. 6: Non-Linear regression plots for Safety index reduction model (Woods, model)
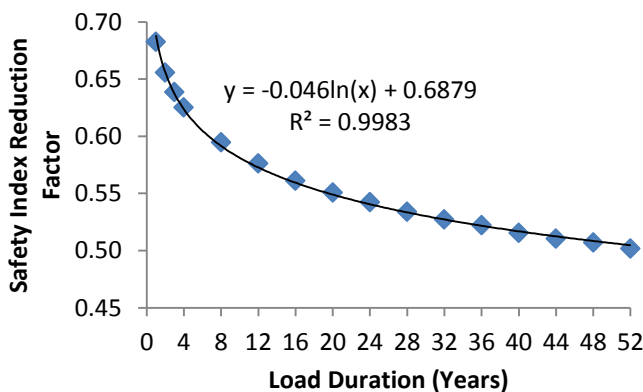

Fig. 7: Non-Linear regression plots for Safety index reduction model (Gerhards model)
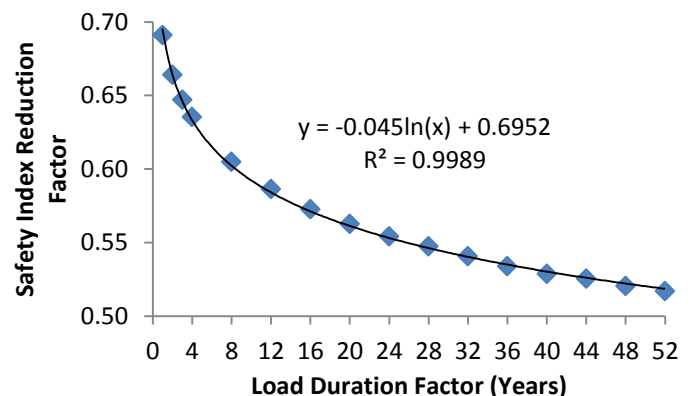

Fig. 8: Non-Linear regression plots for Safety index reduction model (Nielsen model)

It should be noted that, the Woods model is based on test performed on small clear timber specimen (free of defect), while the Gerhardsand Nielsen models were based of test results performed on structural timber. Timber design codes that are based on permissible stress methods, such as the Nigerian code of practice for timber [26], use material properties obtained from tests on small clear timber specimen, while the material properties for limit state design related codes such as EN 338 (2009) are based on test on structural timber. With this in mind, the use of the capacity reduction factors for safety indices derived based on Gerhards and Nielsen models is recommended for design of *Lophiraalata* timber structural members with the Eurocode 5.

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