THE EFFECTS OF OFF TAKE ANGLE ON THE VELOCITY DISTRIBUTION AND RATE OF SILTATION OF CANALS

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ABSTRACT

The problem of excessive siltation in canals (navigation, irrigation, water supply, etc) was tackled by the Schwarz-Christoffel transformation, neglecting gravity and assuming a constant depth of flow. This implies that large off take angles will encourage more intake of sediments by the canal. In addition, it was also observed that large off take angles exhibit higher and lower (wider range) velocities. That is, near the stagnation point, a large off take angle will posses lower velocities than small off take angles thus encouraging siltation, while near the point of infinite velocity, a large off take angle will posses higher velocities thereby increasing sediment intake by canal. It is therefore recommended that canals off take angles should be as small as possible but not too small. If the off take angle is too small, the bank between the branch canal and the main canal will be eroded gradually leading to flooding and eventual destruction of the canal. The results obtained can be applied to navigation, irrigation and water supply canals. The results obtained show that the larger the off take angle, the higher will be the off take discharge as well as the off take entrance velocity distribution. The results were found to agree with both laboratory data obtained using a model and field data, giving correlation coefficients of 0.76, 0.77 and 0.62.

Notations: v = complex velocity, U = upstream velocity, U_1 = downstream velocity, U_2, downstream velocity in branch canal, h = main channel width, h_2 = branch channel width.

INTRODUCTION

Canals have been put to beneficial uses in the areas of navigation, irrigation and water supply. In the Niger Delta region of Nigeria, there is a proliferation of navigation canals dug by oil companies to enable them have access to their off shore facilities. These canals can be classified as tidal and non-tidal canals. Tidal canals are those in the coastal areas located directly adjacent to the ocean or that connect the sea to a water body where facilities are located. Non-tidal canals are those connecting natural river channels.

Around the world, canals are being threatened by serious siltation problems. Siltation reduces the capacity of canals, thereby defeating design purposes. Siltation also promotes flooding which has a devastating effect on human activities and aquatic lives. Excessive siltation also hinders navigation. Sediment transport in irrigation canals is also an important issue in the design and operation of irrigation systems. Frequently observed problems in irrigation systems are, for example, clogging of turnouts and reduction of the conveyance capacity of canals by siltation, and instability of side slopes and of structures due to erosion. Each year large investments are required to maintain or to rehabilitate these systems and to keep them in an acceptable condition for irrigation purposes (Depeweg and Méndez, 2002). However, results obtained by Mingzhou et al (2006) indicate that siltation can have a beneficial effect on agricultural activities by improving the quality of soil. Some canals may be suffering from excessive siltation because they were dug without due consideration of the environmental, hydrodynamic, and geotechnical factors that govern sediment transport and deposition. Sediments could either originate in the canals themselves through bank and bed erosion or they could be transported from the surroundings by flood and run off, or they could originate from the main river channel that feeds the canal. Mostly, dredging has been the commonest resort when it comes to tackling siltation problems, however, it is a very costly operation. Other disadvantages of dredging are: turbidity resulting from improper disposal of dredged materials; increased mortality of aquatic lives due to choking; hindered ship traffic during dredging (Barneveld, 2008); and accelerated bank erosion and failure.

Siltation is a phenomenon that is inextricably woven into the process of erosion because the one hardly occurs without the other. In fact, an idealized fluvial system assumes that degradation (erosion) takes place in the upper reaches of the canal while aggadation (siltation) takes place in the lower reaches of the canal (Valero-Garces et al, 1999). The particle is first detached and then transported by the erosive agent and then finally deposited when the transporting energy is no longer sufficient to drag the particle along canal bed slope (Schum and Khan, 1971), Reynolds number (D'Souza and Morgan, 1976), velocity of flow (Meyer, 1965; Meyer and Wischmeier, 1969) and discharge and particle size (Carson and Kirby, 1972) are factors that influence sediment transport.

Since dredging appears to be a curative rather than a preventive measure, this research was aimed at finding a way to reduce (prevent) excessive siltation by controlling the off take angle which is one of the main factors that govern sediment intake by canals. The reason for choosing this parameter is because it is a sort of sediment exclusion parameter. It will be difficult to control the behaviour of sediments once they enter the canal. The study showed how different off take angles influence velocity distribution around the canal entrance which will in turn influence the quantity of sediments deposited along the canal bed. It also showed how various off take angles will influence velocity downstream of the canal. Velocity distribution around the canal entrance were compared for different off take angles.

METHODOLOGY

Schwarz and Christoffel independently published the same theorem for the mapping of the interior of any simple closed polygon onto the upper half plane. A simple closed polygon is one with the following properties:

(i) The boundaries are made up of straight lines

(ii) One vertex or more could be at infinity

(iii) It is possible to go from any assigned point of the boundary to any other point of the boundary by following a path which never leaves the boundary.

(iv) The boundary divides the points of the plane into two regions the points of which may be called the *interior* and *exterior*.

In most problems relating to hydrodynamics as the problem at hand, the boundaries of the polygon extend to infinity. Thus we can safely state that a dividing flow in a open channel is the interior of a simple closed polygon with some vertices extending to infinity and the sides of the channel forming the boundaries of the polygon. If a, b, c ……..n are points on the real axis on the *t-plane* such that a < b < c < ……….< n and α, β, γ are interiors of a simple closed polygon of n vertices, so that α + β + γ……+ n = (n – 2)π; then the transformation of the simple closed polygon to the upper half plane as given by Schwarz and Christoffel is

$$\frac{dz}{dt} = K(t-a)^{\frac{\alpha}{\pi}-1}(t-b)^{\frac{\beta}{\pi}-1}(t-c)^{\frac{\gamma}{\pi}-1} \tag{1}$$

The usual procedure is to open up the polygon at one of its vertices. In opening up, the vertex in question describes a very large semi circle on the *t-plane* such that it coincides with the points -∞ and +∞ while the other vertices describe very small semi circles $re^{i\theta}$ which are indentations on the real axis of the *t-plane* such that *r→0 as z → a, b or c* on the *z-plane.*

DERIVATION

The channel was mapped onto the upper half plane and subsequently to a plane of uniform flow ie an infinite strip. The polygonal figure was then opened up at the extreme points of the canal ie point EF thus placing the point at infinity on the *t-plane* The point HA was made the origin on the *t-plane* for convenience. This therefore makes point HA a source at origin while point BC becomes a sink at t = 1 and FE a sink at infinity.

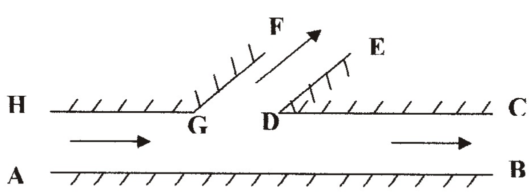


Fig 1: Line Representation of Canal

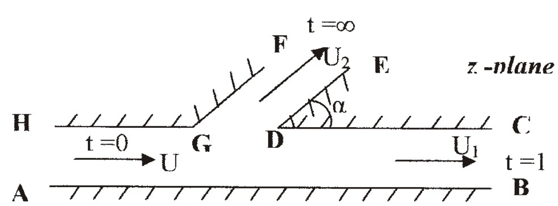


Fig 2: Channel with a Branch Canal Taking of at Angle α

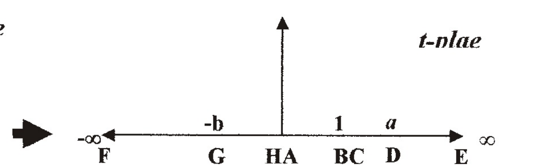


Fig 3: Mapping of the System onto the Upper Half Plane

Knowing th

at the interior angles are $\pi + \alpha$ for point G, $2\pi - \alpha$ for point D and π for points HA and BC respectively, the channel (*z-plane*) can be mapped onto the upper half plane using equation one so that we have

$$dz = A(t+b)^{\frac{\alpha}{\pi}} t^{-1} (t-1)^{-1} (t-a)^{1-\frac{\alpha}{\pi}} dt$$

Where A is the constant of transformation The next step was to map from an infinite strip back to the same upper half plane. The canal was simply rotated in a clockwise sense to become parallel with the main river as shown below and then afterwards mapped onto the upper half plane. The *w-plane* is a plane of uniform flow with boundary CED as streamline. If we let AB(lower boundary) be the horizontal axis, then HF(upper boundary) (2) is $\Psi = U_1h1$.

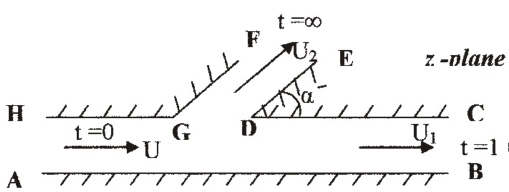


Fig 4: Channel with a Branch Canal Taking of at Angle α

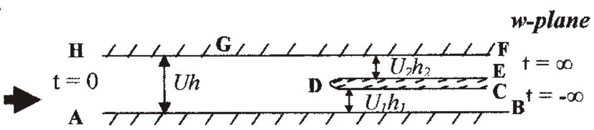


Fig 5: Transformation of Canal to a Plane of Uniform Flow

The interior angles are 2π for point D and π for points HA, BC and EF respectively. Hence using equation (1) to map *w-plane* to *t-plane*(upper half plane), we have

$$dw = B\frac{t-a}{t(t-1)}dt$$

(3)

Next we obtain the expression for complex velocity as follows

$$\frac{dw}{dz} = \frac{dw}{dt} \times \left(\frac{dz}{dt}\right)^{-1}$$

(4)

Combining equations (2) and (3) gives the complex velocity. Hence,

$$\frac{dw}{dz} = \frac{B}{A}\left(\frac{t-a}{t+b}\right)^{\frac{\alpha}{\pi}}$$

(5)

Designating $\frac{B}{A}$ as K then equation (5) becomes

$$\frac{dw}{dz} = K\left(\frac{t-a}{t+b}\right)^{\frac{\alpha}{\pi}}$$

(6)

Determination of constant *K*

(a) Considering the *z* and *t-planes,* at point HA where t = 0, the velocity $\frac{dw}{dz} = -U.$

Substituting this in equation (6) yields

$$K = -U\left(\frac{b}{a}\right)^{\frac{\alpha}{\pi}} \ell^{i\alpha}$$

(b) The point where t = 1 on the *t-plane corresponds to the point BC on the* z-plane with velocity U_1. Substituting this again in (6) yields

$$K = \left(\frac{1+b}{1-a}\right)^{\frac{\alpha}{\pi}} U_1$$

Equating (7) and (8) gives

$$\left(\frac{U}{U_1}\right)^{\frac{\pi}{\alpha}} = \left(\frac{a/b+a}{a-1}\right)$$

(c) As $t \to \infty$ on the *t-plane*, $\dfrac{dw}{dz} \to -U_2 e^{i\alpha}$

Substituting this in (6) and combining with (7) and (9) yields

$$\left(\frac{U}{U_1}\right)^{\frac{\pi}{\alpha}} = \left(\frac{a + \left(\frac{U}{U_2}\right)^{\frac{\pi}{\alpha}}}{a - 1}\right)$$

Determination of the Expression for *a* and *b*

From equation (3), we have

$$w = B\int\left(\frac{t-a}{(t-1)}\right)\frac{dt}{t}$$

$$\therefore w = B[a \ln t + (1-a)\ln(t-1)] + N \qquad (12)$$

Considering the t-plane and the w-plane

(a) At point D on the *w-plane*, $w = 0+iU_2h_2$. On the *t-plane*, this corresponds to the point $t = a$. Hence

$0 + iU_1h_1 = B[a \ln a + (1-a)\ln(a-1)] + N$

Since $a-1 > 0$ and $a>0$, $\ln a$ and $\ln(a-1)$ are real.

$$\Rightarrow \qquad N = iU_1h_1 \qquad (13)$$

(b) Along AB on the *w-plane*, $w = \varphi + i0$. On the *t-plane*, AB corresponds to the range $0 \le t \le 1$. Hence

$\varphi + i0 = B[a \ln t + (1-a)\ln(t-1)] + iU_1h_1$

Since $0 \le t \le 1$, $\ln t$ is real but $\ln(t-1)$ is complex. So breaking into real and imaginary parts, we have

$\varphi + i0 = B[a \ln t + (1-a)\ln|t-1| + i(1-a)\pi] + iU_1h_1$

$\Rightarrow B(1-a)\pi = -U_1h_1$. Hence

$$B = \frac{U_1h_1}{(1-a)\pi} \qquad (14)$$

(c) Along HG on the *w-plane*, $w = \varphi + iUh$. On the *t-plane*, HG corresponds to the range $-b \le t \le 1$ such that $\ln t$ and $\ln(t-i)$ are complex. Hence

$\varphi + iUh = B[a \ln|t| + a\pi i + (1-a)\ln|t-1| + (1-a)\pi i] + iU_1h_1$

Equating real and imaginary parts gives

$$B = \frac{Uh - U_1h_1}{\pi} \qquad (15)$$

Combining (14) and (15) we have

$$a = \frac{U_1h_1}{Uh - U_1h_1} + 1 \qquad (16)$$

From continuity conditions in flow systems, we have $Uh = U_1h_1 + U_2h_2$. Substituting this and (16) in (10) gives the expression for U_1 and U_2 as written below.

$$U_1 = U\left[\left[\frac{1 + \left[\frac{h}{h_2} - \frac{U_1h_1}{Uh_2}\right]^{-\frac{\pi}{\alpha}}}{\left[\frac{Uh}{U_1h_1} - 1\right]^{-1}}\right] + 1\right]^{-\frac{\alpha}{\pi}}$$

If we set $\dfrac{Uh}{h_1} = m$; $\dfrac{h}{h_2} = n$; $\dfrac{h_1}{Uh_2} = p$;

equation (17)

$$U_1 = U\left[\frac{1 + [n - U_1p]^{-\frac{\pi}{\alpha}}}{\left[\frac{m}{U_1} - 1\right]^{-1}} + 1\right]^{-\frac{\alpha}{\pi}} \qquad (18)$$

The value of U_1 can be obtained from (18) by iteration while the value of U_2 can subsequently be obtained by substituting the value of U_1 in the continuity equation.

VELOCITY DISTRIBUTION AT CANAL ENTRANCE

From (6) and (7), the expression for complex velocity v is

$$v = -\frac{dw}{dz} = U\left(\frac{b}{a}\right)^{\frac{\alpha}{\pi}} e^{i\alpha}\left(\frac{t-a}{t+b}\right)^{\frac{\alpha}{\pi}} \qquad (19)$$

Since we obtained velocity as a function of t and not a function of z, it is necessary to define the appropriate path that represents the canal entrance on the *t-plane*. Following the reasoning of Milne-Thomson(1968), it is safe to assume that the canal entrance is a semi circle of diameter $a+b$ (see figures below).

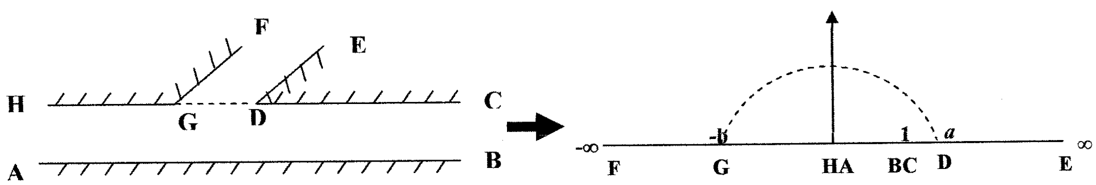


Fig 6: Representing the Canal Entrance (DG) on the Upper Half Plane

In this research, two categories of cases were examined

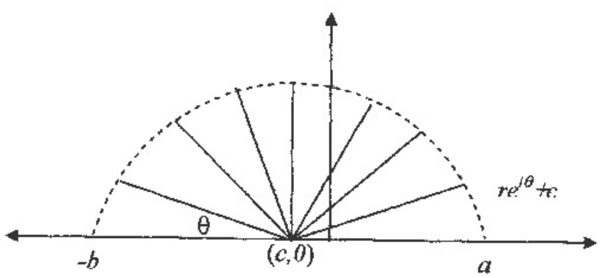
(a) Constant off take angle (α) with variable main channel (h) / branch canal widths (h_2)
  - (i) $h/h_1 = 0.5$ (ie channel width < canal width)
  - (ii) $h/h_1 = 1$ (ie channel width = canal width)
  - (iii) $h/h_1 = 2$
  - (iv) $h/h_1 = 3$

(b) Variable off take angle(α) with constant main channel (h) / branch canal widths (h_2). Here cases where $\alpha = \pi/3$, $\pi/2$ and $2\pi/3$ were examined.

Knowing the velocity (U) upstream of the main channel, the first step in the analysis is to obtain U_1 from (18) by iteration and subsequently U_2 from continuity equation. Having observed the range of velocity in the canals studied by Dike (2002), we assume an upstream velocity (U) of 0.2m/s. We recall that the canal entrance describes a semi circle when transformed onto the upper half plane. Equations (9) and (16) enable us to obtain a and b which are the two extreme points of the semi circle. Hence we can obtain the radius of the semi circle which represents the canal entrance. We represent equal intervals along the canal entrance on the *z-plane* by equal intervals along the circumference of the semi circle (not the real axis) on the *t-plane* (see figure below). In order to have enough points to plot, the semi circle was divided into ten equal sectors each having an included angle of 18^0. The co-ordinates of t are obtained by $re^{i\theta}+c$ if the centre of the circle lies on the right of the origin and $re^{i\theta}-c$ if the centre lies on the left of the origin. θ is the angle between any point on the circumference of the semi circle and the positive real axis. The co-ordinates of t thus obtained were used to generate the velocity distribution for each set of co-ordinates chosen. Next, with a view to investigating average velocity along the canal, somewhere far away from the canal entrance and comparing it with downstream velocity of the main channel somewhere far away from the branch point, we plotted graphs of U_1 and U_2 against various off take angles.

Fig. 7: Equal Intervals across Canal Entrance Represented on the Upper Half Plane

Critical Points



On the *t-plane*, the points G and D have the values $-b$ and a.

$$v = -\frac{dw}{dz} = U\left(\frac{b}{a}\right)^{\frac{\alpha}{\pi}} e^{i\alpha}\left(\frac{t-a}{t+b}\right)^{\frac{\alpha}{\pi}}$$

Substituting a in the expression for velocity, we obtain $v = 0$ which implies that D is a ***stagnation point.*** Substituting $-b$ in the expression for velocity, we obtain $v = \infty$ which implies that G is a point of ***theoretically infinite velocity.***

DISCUSSION

The complex velocity expression shows that the upstream end (point G0 of the canal entrance is a point of infinite velocity while

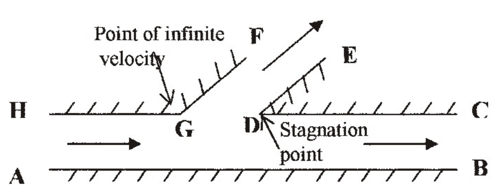


Fig 8: Channel with Branch Canal Showing Critical Points

the downstream end (point D) is a stagnation point regardless of off take angle. For any given off take angle, the velocity at the entrance will increase as the ratio of the main channel width to the canal width increases. Furthermore, for a constant canal width, a larger off take angle exhibits higher velocity near the upstream end of the canal entrance than smaller off take angles and a lower velocity near the downstream end of the canal entrance. As the off take angle increases, the velocity in the branch canal increases, while velocity of flow in the downstream end of the main canal decreases. Between 30^0 and 180^0 off take angles, 60^0 exhibits about the lowest velocity in the branch canal.

From the foregoing, it can be deduced that the upstream end of the canal entrance (point D) will always be prone to siltation. This is because there is little or no tractive force there, thus making it to act as a sedimentation basin. In addition, large off take angles will encourage both deposition of silt at the canal entrance at the upstream end and intake of sediment into the canal from the downstream end of the canal entrance. Between angles 30^0 and 60^0, 60^0 has the lowest velocity in the branch canal and will therefore encourage deposition of sediments along the canal bed. Angles higher that 60^0 will normally generate very high velocity near the upstream junction of the canal entrance thereby leading to higher possibility of scouring. In order to avoid both siltation and scouring, angles less than 60^0 should always be used for canals that are usually prone to siltation. However, for canals where siltation is not a problem, 60^0 should be used because it is less prone to scouring. Very small off take angles should be avoided as much as possible since they could lead to flooding and erosion of banks between canal and main channel due to proximity of flow. Furthermore, the canal width should just large enough to allow for navigation where applicable. This width should be used as the critical upper limit for canal design. This is because for a fixed off take angle, speed of flow at the canal entrance is higher for smaller widths thus preventing the deposition of sediments at the canal entrance.

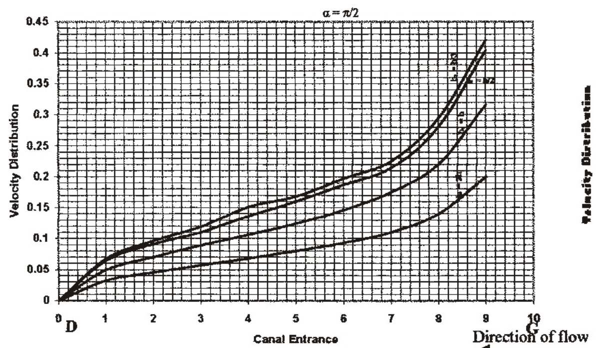


Fig 8: Comparison of Velocity Distribution Across Canal Entrance for Different Ratios of Canal and Channel Widths

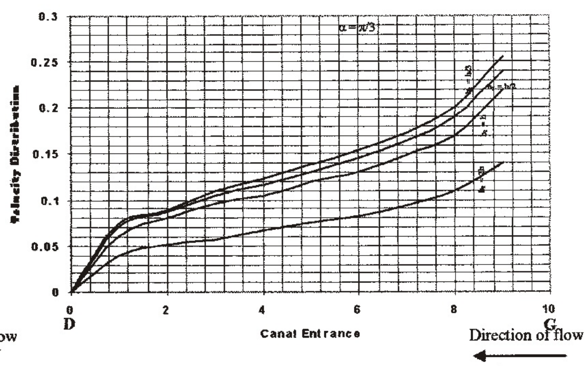


Fig 9: Comparison of Velocity Distribution Across Canal Entrance for Different Ratios of Canal and Channel Widths

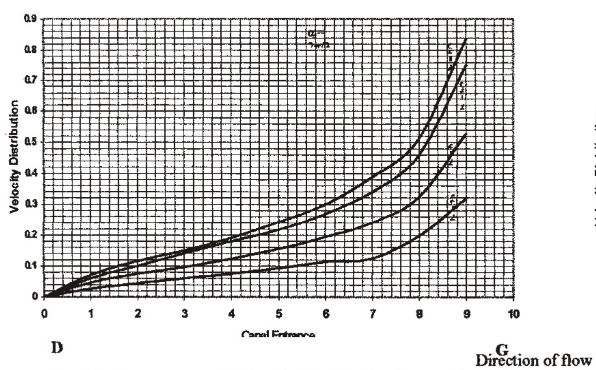


Fig 10: Comparison of Velocity Distribution Across Canal Entrance for Different Ratios of Canal and Channel Widths

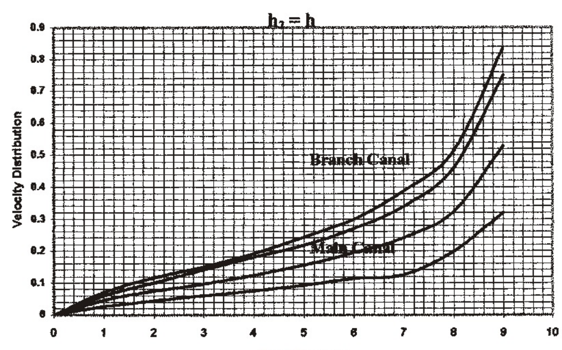


Fig 11: Comparison of Downstream Velocities of Canal and Channel for Different Off Take Angles ($h_2 = h$)

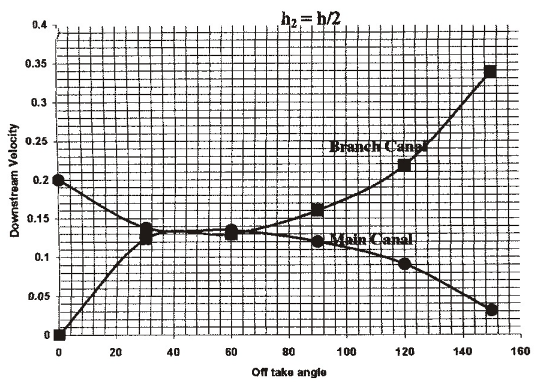


Fig 12: Comparison of Downstream Velocities of Canal and Channel for Different Off Take Angles ($h_2 = h/2$)

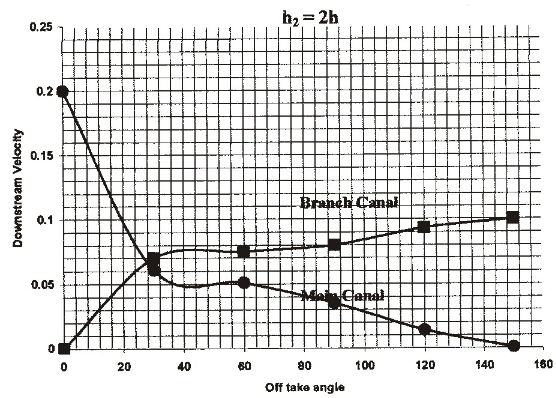


Fig 13: Comparison of Downstream Velocities of Canal and Channel for Different Off Take Angles ($h_2 = 2h$)

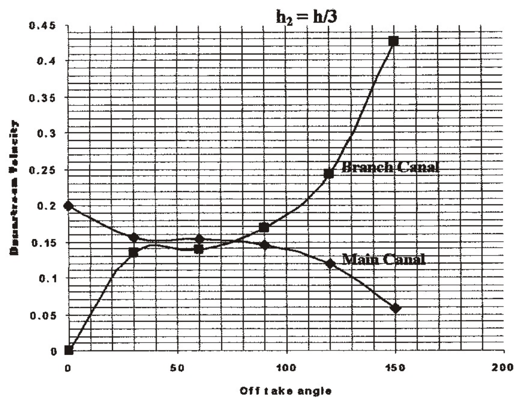


Fig 14: Comparison of Downstream Velocities of Canal and Channel for Different off Take Angles ($h_2 = h/3$)

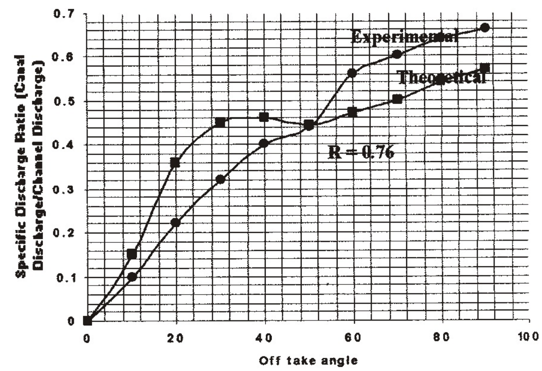


Fig 15: Specific Discharge Ratio Versus Off Take Angle for Q = $0.196 m^3/s$

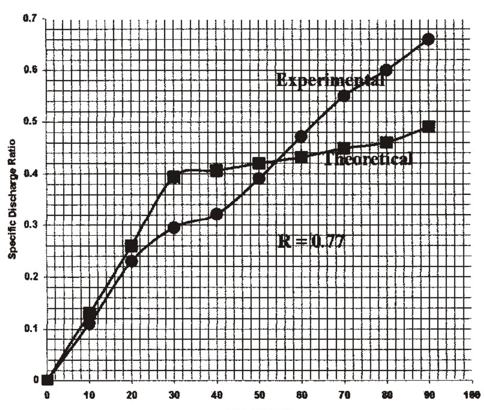


Fig 16: Specific Discharge Ratio Versus Off Take Angle for Q = $0.126 m^3/s$

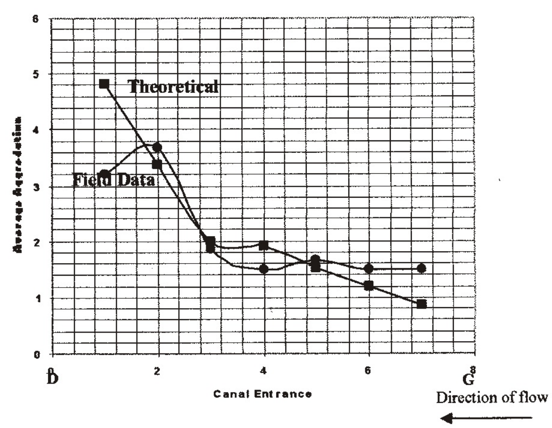


Fig 17: Average Aggradation across Canal Entrance for $\alpha = 90$

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