

## UNIT COMMITMENT: A NEW TRUNCATED METHOD OF UNIT COMBINATION

By

I. N. IWUAGWU and N. A. ANWAH  
Department of Electrical Engineering  
University of Nigeria, Nsukka

### Abstract

The utility industry can reduce its fuel cost by proper commitment of schedulable generating unit. In this paper, a new truncation of unit combinations is proposed which will greatly reduce the number of unit combinations to be considered for large systems. A dynamic programming optimization based digital computer program has been developed for this new approach.

Data obtained from the National Electric Power Authority (NEPA) has been used to test the efficacy of this approach. The results indicate a significant reduction in cost over the method of just considering the first N possible unit combinations generated by binary powers.

### Introduction

The objective of unit commitment is to decide which generating units should be on-line and their level of operation during an approximately 24 hour commitment period while satisfying various technical and energy constraints. Unit commitment is an economic problem and its solution has the Potential to save thousands of Naira in a day. Much work has been done in this area [1-11].

Unit commitment is a complex problem both in nature, and: size. The local operating constraints (both technical and energy) for every unit in the system must be respected. Some of these constraints include unit power limits, minimum up/down times. Other constraints on the system such as spinning reserve requirements, transmission constraints, system load, etc. must also be observed. The total cost to be minimised includes the total unit production cost, start-up and shut-down costs. The energy expended in bringing a unit on-line from an on-line state gives rise to the start-up cost because this energy does not result in any MW generation from the unit. Since the total cost should be minimized, the system production cost per hour must also be minimized. This brings in the problem of economic dispatch which is only a sub-problem of the broad problem of unit commitment. These have been recently considered in literature[5].

Many mathematical techniques have been developed for analysing the unit commitment problem [2 – 10]

However, the results obtainable currently with these procedures may be classified as “near optimum” for practical sized systems. The purpose of this presentation is to show an improvement over all existing technique. It

must be pointed out that although the solution with our proposal may not be the optimum, it has been shown to be closer to the optimum than an existing technique.

### 2 Unit Commitment Methods

It is instructive to briefly review the various unit commitment methods. Unit commitment methods can be divided into heuristic and mathematical optimization techniques. Heuristically, units are committed according to a priority based on unit full load average production cost. It is also known as the complete priority order method [4]. Mathematically, unit commitment is a discrete decision problem. A number of sophisticated mathematical techniques have been proposed to minimize the total production cost [2 – 10], although the electric utility industry has essentially employed empirically based unit commitment. Some of these techniques are now discussed.

(i) Mixed Integer Linear Programming Method [2] Linear programming (LP) is a means of minimizing some linear function subject to linear inequality and equality constraints. In mixed integer LP some of the variables must necessarily be integer number (e.g. the number of units). The objective function to be minimized is the total fuel cost. This may be expressed, as

$$FC = \sum_{i=1}^{NG} \sum_{k=1}^{NH} [\gamma_{ik}^{\lambda} P_i(k) + \alpha_{ik} SUC_{ik} \beta_{ik} SUC_{ik}] \quad (1)$$

FC = total fuel cost

NG = number of generators

NH = number of hours in the study period

$\lambda_i$  = incremental heat rate in ₦/hour for unit i  
 $P_i(k)$  = MW production from unit i in hour k (MW)  
 $SDC_{ik}$  = shut-down cost of unit IS in hour  
 $SUC_{ik}$  = start up cost of unit in hour k  
 For hour k,  $\gamma = 1$  if unit i is in operation else it is 0  
 $\alpha_{ik} = 1$  if unit i is started up else it is 0  
 $\beta_{ik} = 1$  if unit i is shut-down else it is 0

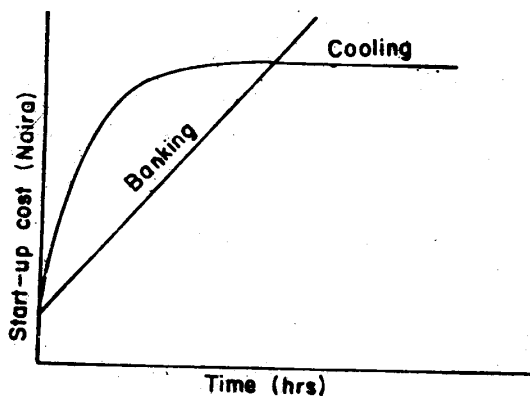


Figure 1: Starting - up cost curves

Equation (1) is minimized subject to equality and inequality constraints. Some of these constraints include:

(a) Unit Power limits which may be modelled as follows

$$P_{g,i}^{\min} \leq P_{g,i} \leq P_{g,i}^{\max} \quad (2)$$

where  $P_{g,i}$  = MW output from unit i

$P_{g,i}^{\min}$   $P_{g,i}^{\max}$  denote lower and upper MW levels respectively for unit i. This constraint means that each committed unit can only operate within its power limits.

(b) Minimum Up-time: This constraint demands that an on line unit must run for at least a certain amount of time before it is shut down.

(c) Minimum Down-time: Engineering considerations do not allow plants to be switched in and out frequently. A unit must be off-line for at least a certain amount of time before it is re-committed. There are two approaches to treating thermal unit during its down time. The first allows the unit boiler to cool down and the second, called banking, requires that sufficient energy be input to the boiler to just maintain its operating temperature. The approach to adopt depends on

the length of time the unit is to be off-line. This is illustrated in figure 1.

(d) Load Balance: During each interval of time, k, the power balance between demand and generation may be expressed as

$$\sum_{i=1}^{NG} P_i(k) = P_D(k) + P_L(k) \quad (3)$$

where  $P_i(k)$  = generation by unit i in hour k  
 $P_D(k)$  = total system demand less total hydroelectric generation  
 $P_L(k)$  = system loss  
 $NG$  = number of units

(e) Spinning Reserve Requirements: Capacity in excess of load must be committed if load interruption is to be minimal.

(ii) Multiple Area Representation [4].

In the multiple area representation, the system is divided into a number of interconnected areas which greatly reduces the dimensionality of the unit commitment problem. Each area has its own generating units, load demand and reserve requirements. The unit commitment problem is then solved for each area using any optimization method. Load demand can be satisfied through the import of power from other areas.

(iii) Dynamic Programming (DP) Methods (4)

As unit commitment is mathematically a discrete decision problem, the only basic mathematical optimization approaches available for this type of problem are DP and mixed integer LP. However, LP is best suited for linear objective function subject to linear inequality and equality constraints.

Linearization could help but this implies forcing a solution method to the problem. Of the two, DP is more attractive than LP because of the generality of the problem formulation to which it can be applied. DP is a fast recursive means of finding an optimum solution. No requirements as to the form of the objective function are imposed because nonlinearities in the system equations can easily be handled. DP divides a given problem into stages or sub-problems and solves the sub-problems until the initial problem is solved. The objective function of Equation (1) can be reduced to a form that is convenient for DP. This may be expressed for hour k as

$$TC(k,I) = \{F\}_{\min} [TC(k-1, L) + SC(k-I, k:k, I) + PC(k, I)] \quad (4)$$

where

$TC(k,I)$  = least total cost to arrive at STATE(k,I)

STATE(k,I) = I<sup>th</sup> unit combination at hour k  
 SC(k-1, L:k, I) = transitional cost from STATE  
 (k-1, L) to STATE(k,I)

PC(k,I) = production cost for STATE(k,I)

{F} = set of admissible states at interval k

Description of equation (4) has been presented in [3, 12] and will not be repeated here.

Equation(4) is a mathematical statement of Bellmans principle of optimality [13]. TC(k-1, L) is the minimum cost in going to the beginning (initial stage or hour) from state(L) at stage (or hour) k. The transitional cost SC(.) to go from state (L) to state(k) and the production cost PC(.) at hour k are to be minimized at hour k. The major limitation of the standard DP algorithm is the computational requirement associated with it. This is called the Bellmans "curse of dimensionality" [13]. This "curse" is seen In:

(i) Amount of high speed storage required to store TC(k-1, L) during the computation of

SC(.) and PC(.).

(ii) The number of storage locations required is one for each state (possible unit combination), a quantity that increases exponentially (2<sup>n</sup>) with the number of units.

(iii) Large amount of computer time is required to carry out the calculations.

(iv) Large amount of off-line storage is required to store the results.

The dimension of the problem can be reduced by considering only some of the combinations of schedulable units. This can be achieved by employing Dynamic Programming - Sequential Combination (DP-SC) [4] or Dynamic Programming - Truncated Continuations (DP-TC) [3]. In these DP methods, the unit commitment process is exactly the same except for the way in which the various unit combinations are generated. This is illustrated in Table 1 (or a system with 4 units, wherein the

**Table 1: Methods of Generating Unit Combination**

DP-CE	DP-SC	DP-TC
4, 3, 2, 1	4, 3, 2, 1	3, 2, 1
0 0 0 0	0 0 0 0	0 0 0
0 0 0 1	0 0 0 1	0 0 1
0 0 1 0	0 0 1 1	0 1 0
0 0 1 1	0 1 1 1	0 1 1
0 1 0 0	1 1 1 1	1 0 0
0 1 0 1		1 0 1
0 1 1 0		1 1 0
0 1 1 1		1 1 1
1 0 0 0		
1 0 0 1		
1 0 1 0		
1 0 1 1		
1 1 0 0		
1 1 0 1		
1 1 1 0		
1 1 1 1		

where (0 0 0 0) implies all units off-line  
 (1 1 1 1) implies all units on-line

column DP- CE stands for Dynamic Programming - Complete Enumeration, when all combinations are considered. (Only this exhaustive enumeration can guarantee an optimum). In DP- TC, unit 4 is given a must-run status. It is clear that how close to optimum our solution is depends on the number of states considered.

In this paper, we are proposing that the unit-list be ordered according to the full load average production costs of the units. This means that most efficient units should be placed on top of the list

and less efficient units placed on the bottom. This procedure will yield more economic schedules (because of the way the unit combinations are generated using binary powers). The unit commitment problem should then be solved first as DP-SC, and then using the feasible states to replace some of the initial combinations in our first in states (from DP-CE). The reason for this proposal is that some of the initial configurations in DP-CE are infeasible due to inadequate available capacity to meet demand and reserve requirements.

### 3. Implementation of the Proposal

This section describes the implementation of the proposal. A realistic test system, NEPAs, has been used to carry out an economic study using this new approach on an IBM 4361 mainframe computer using FORTRAN IV programming language.

#### i. Preparation of Input Data:

Various system data are required to run the computer program [3, 11]. The system is divided into a convenient number of areas with each area having a different number of plants and each plant has a number of generating units. The thermal units are classified into one of the unit types depending on the startup costs. Other unit characteristics required for each unit are the maximum and minimum MW levels, minimum up and down times in hours, incremental heat rate in MBtu/MWh, no load cost, fuel cost, shutdown cost and initial status.

The system losses can be incorporated as a percentage of total generation. The load on the thermal subsystem is the difference between total demand and total hydrogeneration. The load should be given on hourly basis and by implication, the study period is divided into hours. Some of the requirements which ensure adequate and proper distribution of spinning reserve are included in the program. These are minimum total system spinning reserve in MW and maximum p.u. unit capacity for spinning reserve. The outputs from the program include the fuel cost for the study period, the optimal (or near optimal) path etc.

#### ii. Numerical Example

The thermal portion of the National Electric Power Authority (NEPA) as at May 1981 [14] has been used to provide meaningful results for the proposed method. The system is divided into three areas as follows:

Area 1 - Sapele

Area 2 - Afam

Area 3 - Delta,

Each of the areas has one thermal plant. In order to reduce the number of units at a plant, some smaller units are combined to form larger units. Typical unit parameters are then assumed for the larger units [15]. The resulting 15 thermal units are classified into one of four unit types and constant incremental cost curves are assumed. The unit information and initial conditions are shown in Table 2 and Table 3 respectively with minimum up/down times in hours. Zero unit shutdown costs have also been assumed. The time dependent startup costs for the four unit types are presented in Table 4. In Table 3, negative hours in the second column denote the number of hours the corresponding unit has been out of service at the start of the study period while positive hours show the number of hours the unit has been running. The priority order shown in Table 3 is in accordance with the incremental heat rate.

The results of two separate runs are shown in Tables 5 and 6 where a 24-hour study period is used. Although 100 strategies were saved per hour in each of the two studies, the major difference between them is that the second has been solved using the newly proposed method. The results indicate that a savings of ₦3,755/day is obtainable using the proposed method.

### 4. Conclusions

A new approach to the unit commitment problem which can reduce the number of combinations to be considered has been presented in this paper. The implementation of the proposed method is described. Details of the preparation of the input data are also given. A sample realistic test system (NEPAs thermal system as at May 1981) has been used to demonstrate the efficacy of the new proposed method of unit commitment and the results indicate a significant reduction in cost (₦3,755/day) with this proposal over just choosing the first  $n$  states from DP-CE. In addition, this proposed new method leads to savings in computer memory size and CPU time so that DP can be applied to existing power systems.

**TABLE 2: Unit Information.**

UN	PUL	TY	MU	MD	PMIN	PMAX	IHR	UNLC
1	2	4	1	1	14.00	55.00	14.46	30.00
2	2	4	1	1	24.00	96.00	12.97	30.00
3	3	4	4	4	18.00	72.00	11.30	80.00
4	2	4	2	2	25.00	100.00	9.57	34.00
5	3	4	4	4	30.00	120.00	9.56	160.00
6	1	2	8	8	30.00	120.00	9.55	160.00
7	1	2	8	8	30.00	120.00	9.55	160.00
8	1	2	8	8	30.00	120.00	9.55	160.00
9	1	2	8	8	30.00	120.00	9.55	160.00
10	1	2	8	8	30.00	120.00	9.55	160.00
11	1	2	8	8	30.00	120.00	9.55	160.00
12	3	4	4	4	32.00	126.00	9.55	180.00
13	1	3	6	7	36.00	143.00	9.55	200.00
14	1	3	6	7	36.00	143.00	9.55	200.00
15	2	1	10	15	108.00	427.00	8.69	220.00

where:

UN = Unit number

PUL = Plant where unit is located

TY = Unit type

MU = Minimum up time

MD = Minimum down time

PMIN = Economic low limit (MW)

PMAX = Economic high limit (MW)

IHR = Incremental heat rate (MBtu/MWh)

UNLC = No load cost (N/h)

Spinning reserve requirements:

Min. total system sp. res. in MW = 150.000

Max. P.U. unit Capacity for Sp. res. = 0.100p.u.

**Table 3 Initial Status and Fuel Costs:**

Unit	Hour	N/ MBtu	N/ MWh	Priority
1	-6	6.00	86.76	15
2	-4	6.00	77.82	14
3	8	6.00	67.80	13
4	8	6.00	57.42	12
5	-6	6.00	57.36	11
6	10	6.00	57.36	10
7	-9	6.00	57.30	10
8	-8	6.00	57.30	9
9	15	6.00	57.30	7
10	-7	6.00	57.30	6
11	10	6.00	57.30	5
12	8	6.00	56.70	4
13	10	6.00	56.40	3
14	10	6.00	56.40	2
15	11	6.00	52.14	1

**Table 4:** (Time Dependent) Start-up Cost in Naira

Hour	Unit Type			
	1	2	3	4
1	240.00	120.00	60.00	18.00
2	320.00	168.00	92.00	37.20
3	400.00	216.00	124.00	56.40
4	480.00	264.00	156.00	75.60
5	560.00	312.00	188.00	94.80
6	640.00	360.00	220.00	105.26
7	720.00	408.00	252.00	111.09
8	800.00	456.00	284.00	115.28
9	880.00	405.00	305.33	118.29
10	960.00	547.27	320.61	120.46
11	1040.00	575.18	333.85	122.02
12	1102.10	600.38	348.33	123.14
13	1148.13	623.14	355.28	123.94
14	1190.49	643.78	363.90	124.52
15	1229.48	662.45	371.38	124.94
16	1265.37	679.35	377.86	125.24
17	1298.39	694.64	383.47	125.45
18	1328.79	798.47	388.34	125.61
19	1366.76	720.99	392.66	125.72
20	1382.51	732.31	396.21	125.80
21	1406.20	742.56	399.38	125.85
22	1428.08	751.83	402.13	125.89
23	1448.08	760.22	404.31	125.92
24	1466.55	767.81	406.58	125.95

**Table 5**

Number of strategies to be saved = 100

Number of search states in the present hour = 100

Initial state not found in priority list, adding state to list

**OPTIMUM COMMITMENT SCHEDULE**

Total cost = ₦ 1011540.75

Hour	State	Unit - status (1 → unit on - line      0 → unit off - line)														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
24	40	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1
23	48	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1
22	64	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
21	64	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
20	64	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
19	64	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
18	64	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
17	64	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
16	101	0	0	1	1	0	1	0	0	0	0	1	1	1	1	1
15	101	0	0	1	1	0	1	0	0	0	0	1	1	1	1	1
14	101	0	0	1	1	0	1	0	0	0	0	1	1	1	1	1
13	101	0	0	1	1	0	1	0	0	0	0	1	1	1	1	1
12	101	0	0	1	1	0	1	0	0	0	0	1	1	1	1	1
11	101	0	0	1	1	0	1	0	0	0	0	1	1	1	1	1
10	101	0	0	1	1	0	1	0	0	0	0	1	1	1	1	1
9	101	0	0	1	1	0	1	0	0	0	0	1	1	1	1	1
8	32	0	0	0	0	0	1	0	0	0	0	1	1	1	1	1
7	28	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1
6	20	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1
5	20	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1
4	20	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1
3	20	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1
2	20	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
1	28	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1

**Table 6**

Number of strategies to be saved = 100

Number of search states in the present hour = 100

Initial state not found in priority list, adding state to list

**OPTIMUM COMMITMENT SCHEDULE:**

Total cost = N 1007784.88

Hour	State	Unit - status (1 → unit on - line      0 → unit off - line)														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
24	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1
23	3	0	0	0	1	0	0	0	0	0	0	0	1	1	1	1
22	3	0	0	0	1	0	0	0	0	0	0	0	1	1	1	1
21	3	0	0	0	1	0	0	0	0	0	0	0	1	1	1	1
20	3	0	0	0	1	0	0	0	0	0	0	0	1	1	1	1
19	3	0	0	0	1	0	0	0	0	0	0	0	1	1	1	1
18	64	0	0	0	1	0	0	0	0	0	0	0	1	1	1	1
17	64	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
16	64	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
15	64	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
14	4	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
13	4	0	0	0	1	0	0	0	0	0	0	1	1	1	1	1
12	4	0	0	0	1	0	0	0	0	0	0	1	1	1	1	1
11	4	0	0	0	1	0	0	0	0	0	0	1	1	1	1	1
10	3	0	0	0	1	0	0	0	0	0	0	0	1	1	1	1
9	3	0	0	0	1	0	0	0	0	0	0	0	1	1	1	1
8	2	0	0	0	1	0	0	0	0	0	0	0	1	1	1	1
7	1	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1
6	8	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1
5	8	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
4	8	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
3	8	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
2	8	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
1	8	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1



**References**

1. Engles, L.; Larson, R.E.; et. al.; 1975 "Dynamic, programming Applied to Hydro and Thermal Generation Scheduling" in IEEE Tutorial Course on Probabilistic Analysis of Power System Reliability.
2. Edwin K. W, Machat, R.D.; Tans, R.J.; 1980, "Comparative Evaluation of Unit Commitment Approaches", IEEE International Conference on Power System Monitoring and Control; London.
3. Pank, ex, Chen, H.C.; 1976 1976. "Optimal Short-Term Thermal Unit Commitment", *IEEE Trans. on Power App. and Syst.*, Vol. 90, pp. 1336 - 1346.
4. Park C.Y.; Sheble, G.B.; Albuyeh, F.; 1981, "Evaluation of Dynamic Programming Based Methods and Multiple area Representation for Thermal Unit commitment", *IEEE Trans. on Power App. and Syst.*, Vol. 90, pp. 1212 - 1218.
5. Sasson, A.M.; Merrill, H.M.; 1974, "Some Applications of Optimization Techniques to Power System Problems", Proc. of the IEEE, Vol. 62, pp. 959 - 972.
6. Happ, H.H.; Johnson, R.C.; wright, W.J. 1974, "Large Scale Hydrothermal Unit Commitment Method and Results"; IEEE Trans, on Power App. and Syst.; Vol. 90, pp. 1373 - 1384.
7. Muckstadt, J .A.; Wilson, R.C.; 1968, "An Application of Mixed-Integer programming Duality to Scheduling Thermal Generating Systems", IEEE Trans, on Power App. and Syst., Vol. 87, pp. 1968 - 1977.
8. Henuk, K.; 1972, "Optimal Scheduling of Thermal Power Stations" Proc. 4th PSCC (Grenoble, France). Paper 2.5.
9. Guy, J.D.; 1971, "Security Constrained Unit Commitment", *IEEE Trans. on power App. and Syst.*, Vol. 90, pp. 1385 - 1390.
10. Ayoub, A.K.-; Patton, A.D.; 1971, "Optimal Thermal Generating Unit Commitment", *IEEE Trans. on Power App; and Syst.*, Vol. 90, pp. 1952 - 171.
11. Iwuagwu, J.N.; 1988, "Economic Short-Term Scheduling of Thermal-Generating Units Using Dynamic Programming", M.Eng. Thesis, University of Nigeria, Nsukka.
12. Wood, A. J.; Wollenberg B. F.; 1987, "Power Generation, Operation, and control", Water, New York.
13. Bellman, R.; 1957, "Dynamic programming" Princeton University Press. Princeton, New Jersey.
14. National Electric Power Authority; "Power System Development Plan (1982-1990)"; 1982 Lagos.
15. Electric Power Research Institute "(EPRI) Report; "Synthetic Electric Utility System for Evaluating Advanced Technologies".