THE EFFECT OF WELL-BORE REVERSE FLOW OF FLUID ON PRESSURE INTERPRETATIONS IN OIL WELLS

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ABSTRACT
Well-bore storage may dominate the bottom-hole pressure profile of a well particularly for the short time situation, The dominance may be strongly accentuated in cases where reverse flow into a passive sand or casing leakage down-hole cannot be isolated from the test zone. This analysis shows that reverse flow of fluid in a well-bore has a retarding effect on pure storage phenomenon. Thus in effect, type-curve analysis of bottom-hole pressures as a means of evaluating petro-physical properties of a reservoir sand may be rendered unreliable if reverse flow effect is not properly considered.

NOTATION

C	Well-bore radius storage constant b1/psi
C_D	dimensionless well-bore storage constant
C_t	system compressibility
h	active sand thickness
h_1	passive sand thickness
k	permeability of active sand
k_1	permeability of passive sand
p	pressure
p_i	initial reservoir
Δp	pressures change at well – bore
Δp_D	dimension less pressure change at well-bore
q	surface flux per unit well-bore volume
q_a	storage induced flux
q_e	effective surface flux
r_w	Well – bore radius
r_D	Dimensionless radius
t	Time
t_D	Dimensionless time
Φ	Formation porosity
β	Reverse-flow constant
$\Delta\psi_D$	Dimensionless pressure drawn- down at pressure sand – face
μ	fluid viscosity
τ	Time parameter

INTRODUCTION

The effects of well-bore storage and skin factors on the performance of a producing oil well are very well known. One of the most recent innovations in bottom-hole pressure analysis is the use of type-curves for evaluating short-term pressure data which are otherwise regarded as unsuitable for analysis using the more conventional techniques. Type-curve matching is however only accurate when the storage factor remains unchanged through the duration of a test [1], and when such storage factor is purely due to after-flow or annulus as described by the inner boundary conditions of the system's equation [2]. The case of any other well- bore defect which radically departs from the above conventional storage-type problem is yet to receive any attention in the petroleum literature.

The aim of this paper therefore is to investigate one of these unconventional but common practical oil field problems that may have significant Influence on the pressure performance of an oil well. Specifically, the paper will investigate the problem of reverse-flow from active sand into a passive oil sand as this affects the pressure characteristics of the oil well.

2. THEORETICAL CONSIDERATIONS

Consider a homogeneous, isotropic oil reservoir of infinite extent. The reservoir is penetrated by a well which is perforated for flow within an interval, h. For a well producing with a source straight, q, the bottom-hole pressure change response at the well bore taking account of well-bore storage is given in the form [3]:

$$\Delta_p = \frac{1}{\emptyset c_t} \int_0^t q\, G(r,\tau) d\tau \; - \; \int_0^t q_a\, G(r,\tau) d\tau \quad (1)$$

In equation [1] the first term on the RHS is the pressure change response when there is no storage or after-flow effect. The second term is the pressure change contribution due to the effect of after- flow (storage); and G(r,τ) is the instantaneous Green's function for the source system. The after-flow strength q_a is related to the pressure draw-down $\Delta p(r,t)$, at the well-bore, through the equation [2]:

$$q_a = C \frac{\partial(\Delta p)}{\partial t} \quad (2)$$

Where C is the well - bore storage constant if we define dimensionless variables as follows:

$\Delta p_D = \frac{(p(r,t) - p_i)}{q_e \mu}$; dimensionless pressure draw-down

$t = \left(\frac{kt}{\phi \mu c_t r_w^2}\right)$; dimensionless time

$C_D = \left(\frac{C}{2\pi\phi h c_t r_w^2}\right)$; dimensionless storage constant

$r_D = \left(\frac{r}{r_w}\right)$; dimensionless radius

$q_e = q(\pi r_w^2 h)$; effective well-bore flux (3)

Then equation (1) and (2) yield the dimensionless equation:

$$\Delta p_D (r_D, t_D) = \int_0^{t_D} G(r_D,\tau_D) d\tau_D - C_D \int_0^{t_D} \frac{\partial \Delta p_D(r_D, t_D - \tau_D)}{\partial \tau_D} G(r_D,\tau_D) d\tau_D \quad (4)$$

Equation (4) is perfectly general. It makes no assumption about the natures of the well or source system under consideration.

We can now consider a situation where the same storage-influenced well is also open to another sand of thickness h_1 which is situated below the objective sand and is passive. This kind of situation can occur in cases where is leakage in the well casing or a plug- there back job is not well done in the neighborhood of the passive sand, or n open hole completions. In any such case t can be assumed that both the active and the passive sands are initially in equilibrium with the liquid column within the well-bore. However, when the well is actuated, the state of static equilibrium previously maintained will be disturbed as a result of pressure variations within the well-bore. The consequent pressure 'build-up' within the well-bore may force a condition of reverse flow into the passive sand particularly if the permeability of the passive sand is much higher than that of the active sand. This form of indirect pressure depletion of the reservoir is expected to influence the ultimate pressure performance of the reservoir. The level of pressure depletion due to this phenomenon alone at the sand face of the passive sand can be represented by $\Delta\psi_D$ (r_D, t_D). This level of pressure draw-down can also be expressed in terms of the relative flow capacities of the two sands viz:

$$\Delta\psi_D(r_D, t_D) = \Delta p_D(r_D, t_D)\beta \qquad (5)$$

where, $\beta = \frac{k_1 h_1}{kh}$

and kh and kh, are the flow capacities of the active and the passive sands respectively.

From equations (4) and (5), the complete pressure response profile, taking account of depletion due to reverse flow at the well-bore, becomes,

$$\Delta p_D(r_D, t_D) = (rD, tD)\int_0^{t_D} G(r_D, \tau_D)\,d\tau_D - C_D\int_0^{t_D} \frac{\partial \Delta p_D(r_D, \tau_D)}{\partial \tau_D} G(r_D, t_D - \tau_D)d\tau_D - \Delta p_D(r_D, t_D)\beta \qquad (6)$$

Rearranging Equation (6) yields –

$$\Delta p(r_D, t_D).(1+\beta) = \int_o^{t_D} G(r_D, \tau_D)d\tau_D - C_D\int_0^{t_D} \frac{\partial \Delta p(r_D, \tau_D)}{\partial \tau_D}.G(r_D, \tau_D - \tau_D)d\tau_D \qquad (7)$$

Taking the Laplace transform of equation (7) gives –

$$\Delta p_D(r_D, s).(1+\beta) = \frac{1}{s}G(r_D, s) - C_D\, s\, \Delta p_D(r_D, s)G(r_D, s)$$

Or,

$$\Delta p_D(r_D, s) = \frac{\frac{1}{s}G(r_D, s)}{(1+\beta) + s\, C_D\, G(r_D, s)} \qquad (8)$$

where s is the Laplace parameter.

Equation (8) is a general equation which describes the pressure response function for any from of source system, the pressure response function and hence the instantaneous Green's function $G(r_D, t_D)$ is very well known. The Laplace transform of the instantaneous Green's function due to a continuous cylindrical source system can therefore be written as [4]:

$$G(r_D, s) = \frac{k_0(r_D\sqrt{s})}{s^{\frac{1}{2}}\, k_1(r_D\sqrt{s})} \qquad (9)$$

Where $k_0(x)$ and $k_1(x)$ are the modified Bessel functions of the second kind, of orders zero and one respectively. Using equation (9) in (8) yields-

$$\Delta p_D(r_D, s) = \frac{1}{(1+\beta)}\,\frac{1}{s}\left(\frac{K_0(r_D\sqrt{s})}{\sqrt{s}K_1(r_D\sqrt{s}) + \frac{C_D\ s}{(1+\beta)}K_0(r_D\sqrt{s})}\right) \qquad (10)$$

One interesting feature of equation (10) is the fact that the reverse flow parameter β can be seen as having a retarding influence on the pure storage parameter C_D. It is in the light of this that one feels that an operative storage parameter can be defined for the system under study.

If we define an effective dimensionless well-bore storage constant as:

$$C_{D_e} = \frac{C_D}{(1+\beta)} \qquad (11)$$

$$\Delta p_D(r_D, s) = \frac{1}{(1+\beta)}\frac{1}{s}\left(\frac{K_0(r_D\sqrt{s}\ )}{\sqrt{s}\ \ K_1(r_D\sqrt{s}) + C_{D_e}\ s\ K_0(r_D\sqrt{s})}\right) \qquad (12)$$

since Δp_D is measured at the sand- face, r_D=1 hence,

$$\Delta p_D(1, s) = \frac{1}{(1+\beta)}\frac{1}{s}\frac{5 \qquad K_0(\sqrt{s})}{\sqrt{s}\ \ K_1(\sqrt{s})\ + C_{D_e}\ s\, K_0\ (\sqrt{s}\ )} \qquad (13)$$

Equation (13) which expresses the pressure response function of a cylindrical source system in the Laplace space is very similar to the equation previously obtained by other investigators [2], [4] when these equations are evaluated for the special case of zero skin factor considered in this study. It is however different in that the concept of an effective well- bore storage which takes into account the possibility of revere flow into a passive sand or casing leakage is introduced. It could also be observed that the effective well-bore storage parameter C_D and the dimensionless well-bore storage C_D become identical when β, the reverse flow parameter of the passive sand is zero. For this special case, equation (13) degenerates to the equation of Agarwal et al [2] for zero skin-factor.

The real inversion integral solution to equation (14) can be written as (4):

$$\Delta p_D(r_D, t_D) = \frac{}{\pi^{2(1+\beta)}} \int_0^\infty \frac{\left(1 - e^{-U^2 t_D}\right) dU}{U^3\{[UC_{D_e} J_0(U) - J_1(U)]^2 + [\,UC_{D_e} Y_0(U) - Y_1(U)]^2\}} \quad (14)$$

$J_o(U)$ and $J_I(U)$ are the Bessel functions to the first kind of orders zero and one, while Y_0 (U) and $Y_1(U)$ are the Bessel functions of the second kind of the respective orders.

3. DISCUSSION

The integral solution expressed by equation (14) describes the well-bore pressure variation with time. The equation cannot be expressed in close form and hence must be evaluated by numerical integration. The Rhomberg numerical integration technique was used to evaluate equation (14) for various values of β and C_D. Figure 1 shows the graphical representation of the numerical results for C_D value of 10^2, 10^3 and 10^4 and for values of the parameter β equals 0 and 9.0.

It should be pointed out here again that the graph for β=0 corresponds to the case where there is no reverse flow in the welt-bore, whereas the second case of β =9.0 implies the existence of a measure of well-bore reverse flow. It is clear from these graphs that the effect of any reverse flow or casing leakage on bottom-hole pressure measurements can be substantial. The immediate consequence of this is the introduction of significant errors in the calculated values of formation petro physical properties if the analysis for these are based purely on the type-curve matching technique, using Ramey's [5] correlation curves uncorrected. Notwithstanding this however, the unit slope diagnosis for storage controlled short-time data is expected to remain preserved even with a very significant effect of after-flow. This is not surprising in view of the fact that the after-flow parameter β comes into the system's dynamics as a mere modification of the pure storage term. In fact, it is not only the unit slope of the short-time pressure-time data that is preserved; the entire shape of the pressure profile is preserved for all values of β as can easily be observed in fig 1.

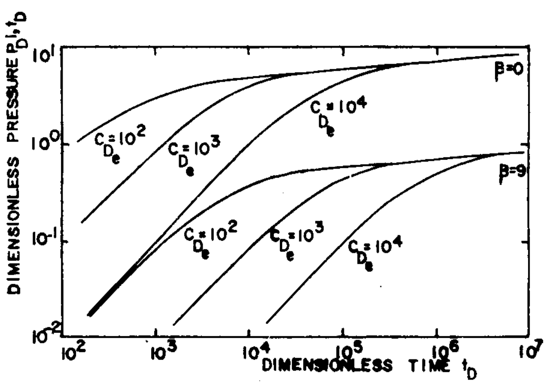

Fig. 1. Type curves for reverse-flow well -bore system.

This unique characteristic of the pressure profile implies that the slope of the pressure-time curve is also independent of the reverse-flow parameter S. The uniqueness of the pressure-time slope has a very important implication for practical application of the above theory to bottom-hole pressure analysis. It ensures that analyses can be based on established type-curve correlations subject to appropriate data interpretations. Specifically, a good correlation within the storage-controlled section of the pressure data would give a value for the storage constant from which the well-bore storage volume could be estimated. Such estimated volume should give a close check with the well- bore volume calculated from well-completion data. However, in cases where there is a measure of reverse flow in the well-bore, the value of well-bore storage volume deduced from type-curve correlation is expected to be in excess of the real well-bore volume calculated from well completion data. This follows from the definitions of the effective storage constant C_{De} in equation (11).

As an illustration, consider a situation where data correlation for well-bore storage gives a storage constant of 10^3. From this the following results:

$$C_{D_e} = \frac{C_D}{(1+\beta)} = 10^3 \quad (15)$$

Or,

$$C_D = (1 + \beta)\, 10^3$$

For the case where there is no reverse-flow β= 0 and C_D is truly 10^3. However, when reverse flow exists, β >0 thus confirming the contention that:

$$C_D = (1 + \beta)\, 10^3 > 10^3 \quad (16)$$

Equation (15) provides a useful avenue for evaluating the reverse flow parameter β in cases where the well-bore volume and hence C_D can be estimated from well completion data. Finally, when sufficient information is available, to carry out a conventional analysis of the pressure data beyond the storage controlled region, the slope measurement technique of Horner [6] provides a very useful check on the value of transmissibility factor(kh/μ) obtained from type -curve matching.

4. CONCLUSION

From the preceding theory and illustrations it is clear that type-curve matching is by no means a replacement for the more conventional method of bottom-hole pressure analysis. The role of type curve matching in pressure analysis is largely a supplementary one, particularly in the storage - dominated region of the pressure profile. In this region large variety of useful information about the intrinsic nature of the well-bore can be deduced if analysed data are properly interpreted.

This paper has discussed an additional interesting possibility of reverse-flow of fluid in a storage-influenced well-bore; and illustrated how this influences the pressure behaviour of such system. It is clear from the illustration that baffling analysis result, which could easily be dismissed as error-prone may in fact be a good pointer to a yet undiscovered feature of the system under consideration. Type curve matching when properly used may provide the necessary insight into some of such apparent well-bore puzzles.

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