jfewr ©2019 - jfewr Publications
E-mail:jfewr@yahoo.com
ISBN: 2141 – 1778
Ogana et al., 2019

DETERMINATION OF SELF-THINNING LINE FOR *GMELINA ARBOREA* ROXB STANDS IN NIGERIA

Ogana F.N.[1]*, Ekpa N.E.[2] and Ogana T.E.[1]

[1]Department of Social and Environmental Forestry, University of Ibadan, Ibadan, Nigeria
[2]Department of Forestry and Natural Environmental Management, University of Uyo, Uyo, Nigeria
***Corresponding E-mail:** *ogana_fry@yahoo.com;* +2347037679328

ABSTRACT

Self-thinning line defined the site occupation of species and it is an important tool for manipulation of stand density and simulating thinning regime. Few studies have defined the self-thinning line of Gmelina arborea especially in Nigeria. Therefore, in this study, the self-thinning line of G. arborea in Oluwa Forest Reserve was determined. Data were obtained from twenty-four temporary sample plots (TSPs) of 20 x 20 m size established in the G. arborea stands. Three methodologies were used to define the self-thinning line including quantile regression (QR), stochastic frontier function with half-normal and stochastic frontier function with truncated-normal. Root means squared error (RMSE), Akaike information criterion (AIC) and Bayesian information criterion were used to assess the methods. The results showed that the three methods performed relatively well in describing the self-thinning line of the stand. Stochastic frontier with half-normal with minima AIC and BIC of -5.321 and -0.609, respectively was more suitable. It predicted maximum density of 2630, 1537 and 1079 N/ha at quadratic mean diameter of 15, 25 and 35 cm, respectively. This information would help in the manipulation of the growth condition and determination of thinning schedule of the G. arborea stands.

Keywords: Quantile regression, stand density, stochastic frontier regression, self-thinning,

INTRODUCTION

Self-thinning is an equilibrium relationship that exist between plant growth and plant mortality. This relationship is based on the "-3/2 power law" or "self-thinning rule" (Yoda *et al*., 1963). The self-thinning rule states that the dynamic relationship between the logarithmic of the density and average plant/tree size is a straight line (Zhang *et al*., 2005). This line has been considered as the self-thinning line or maximum size-stand density relationship (Zhang *et al*., 2005; Vospernik and Sterba, 2015; Camacho-Montoya *et al*., 2018; Kara, 2018). It is a measure of maximum stockability of a given stand (Reyes-Hernandez *et al*., 2013). It is also an important tool for developing stand density management diagram from which thinning and harvesting regimes can be prescribed (Solomon and Zhang, 2002).

Zeide (2005) identified two causes of self-thinning in forest stand – decrease in self-tolerance and increase of tree diameter. The number of trees per unit area decreases with increase average tree size (i.e. quadratic mean diameter). Thus, stand density is influenced by competition (Pretzch and Biber, 2005). The Reineke's Stand density index (Reineke, 1933) gives an indicator of the degree to which forest stands are attaining complete site occupancy based on stand density (N tree per ha) and quadratic mean diameter (Dq). The relationship between stand density (N tree per ha) and quadratic mean diameter has been explored to establish self-thinning line for different species.

The intercept of the self-thinning line varies with species, site, region etc. (Jack and Long, 1996; Bi, 2001; Weiskittel *et al*., 2009; Zhang *et al*., 2013; Kara 2018). Vospernik and Sterba, (2015) asserted that a slight change in the intercept can result to a considerable change in stand density of a species. The universal slope of the self-thinning line proposed by Reineke (1933) as 1.605 has been heavily criticised. Studies have shown that the slope of the self-thinning line varies with species, i.e., species-specific (Pretzch and Biber, 2005). The acceptance of a constant slope may lead to formulation of inappropriate thinning regimes (Kara, 2018). Thus, necessitates the need for the establishment of self-thinning line for different species across the globe.

There are different fitting methodologies that have been used to develop self-thinning line for several species. These include: the randomly hand fitting method, ordinary least squares regression (OLS), principal component analysis (PCA), reduce major axis regression, quantile regression, deterministic frontier, stochastic frontier regression, linear mixed model etc. (Drew and Flewllin, 1977; Solomon and Zhang, 2002; Zhang *et al*., 2005; VanderSchaaf and Burkhart, 2007; Weiskittel *et al*., 2009; Camacho-Montoya et al., 2018; Salas-Eljatib and Weiskittel, 2018). The first three methods are rather subjective and do not utilize all available data points in the model-fitting process of self-thinning line (Solomon and Zhang, 2002). In addition, Zhang *et al*. (2005) stated that the "OLS and PCA describe average maximum-size density line rather than the biological maximum size-density". This line should indicate the upper limit of the chosen data points (Camacho-Montoya et al., 2018). Detailed of these methods are well documented in Solomon and Zhang (2002), Zhang *et al*. (2005) and Salas-Eljatib and Weiskittel (2018).

Gmelina arborea Roxb is an important exotic species which occupies vast expanse of land in Nigeria (Ogana *et al*. 2017). It is tolerant to drought and light demanding (Duke, 1983). *G. arborea* is popularly grown for timber and serves as raw materials for pulp and paper industries (Ajayi *et al*., 2004). Despite the importance of self-thinning line to forest management and the period over which the methodology has existed, there is still dearth of study on the self-thinning line of *G. arborea* species, especially in Nigeria. Knowledge of the self-thinning line of *G arborea* would help in the manipulation of the forest stand for optimal growth and production. Therefore, the main purpose of this study is to develop self-thinning line for *G. arborea* stands in Nigeria using different methods.

MATERIALS AND METHODS

Study area

The data used for this study were obtained from the *G. arborea* stands in Oluwa Forest Reserve In the humid tropical zone of Southwestern Nigeria. Oluwa forest reserve is in Odigbo Local Government Area, Ondo State, Nigeria. It is situated between latitude 6º55´ and 7º20´N and longitude 3º45´ and 4º32´E with an area of 87,816 ha (Onyekwelu, 2001). Annual rainfall ranges from 1700 to 2200 mm. Annual temperature in Oluwa is 26 °C, and mean elevation of 123m above sea level (Onyekwelu *et al*., 2006). The data were collected from 1,052 trees on 24 temporary sample plots (TSPs) of 0.04 ha size. Diameter and height were measured to accuracy of 0.1 cm and 0.1 m with diameter tape and hypsometer, respectively. These were used to compute stand variables including quadratic mean diameter, basal area per ha (G) and number of trees per ha (N). The descriptive statistics of the variables are presented in Table 1.

Table 1: Descriptive statistics of the stand variables

Variables	Statistics			
	Mean	Max	Min	SD
Age (yr)	29.0	39.0	19.0	7.91
Dq (cm)	25.5	31.2	18.9	3.47
N (tree/ha)	1078	1525	625	252.84
G (m^2/ha)	54.14	77.41	32.88	12.48

Modelling approach

The maximum density relationship as proposed by Reineke (1933) is based on the number of trees per ha and quadratic mean diameter (in logarithm scale); expressed as:

$lnN = \beta_0 + \beta_1 lnDq$ .... (1)

Where N = number of trees per ha; Dq = quadratic mean diameter; β_0 and β_1 = intercept and slope of the regression model.

To date, different methods have been used for fitting self-thinning line for many species. However, due to the paucity of data, only methods that utilize full data range were adopted for this study. These are quantile regression (QR) and stochastic frontier regression with half-normal and stochastic frontier regression with truncated-normal.

Quantile Regression (QR)

QR (Knoeker and Bassett, 1978) is a robust method and insensitive to outlier and makes full use of data set in the modelling process. It is expressed as:

$\hat{y}_\tau(lnN) = \beta_0 + \beta_1 lnDq$ ....(2)

Where $\hat{y}_\tau(lnN)$ is the estimated value of the τth quantile of the number of trees per ha at quadratic mean diameter (*Dq*), the intercept (β_0) and slope (β_1) from the quantile regression were obtained by minimizing the sum of absolute residual expressed as:

$$\hat{\beta}_{(\tau)} = \text{argmin}_{\beta \in R^2} \sum_{i=1}^{n} \rho_\tau(y_i - x_i'\beta)$$ .... (3)

Where $\hat{\beta}$ = parameters β_0 and β_1; τ = quantile (0.95).

Stochastic Frontier Regression (SFR)

Stochastic frontier is a production function that specifies the maximum output for a given input. In other word, it gives the maximum achievable output from a given data. It was introduced by Aigner *et al.* (1977) and since then, it has been applied to other fields of research including forestry. The SFF consists of three components: "the efficient production function, technical inefficiency and random variation of the data" (Camacho-Montoya *et al.*, 2018). This approach relaxes the assumption of heteroscedasticity while testing the effect of covariates (Bi et al., 2000; Zhang *et al.* 2005; Weiskittel *et al.*, 2009). The SFR is given by:

$y_i = f(x_i;\beta) + v_i - u_i$ .... (4)

Applying this function to the self-thinning line would be:

$lnN = \beta_0 + \beta_1 lnDq + v_i - u_i$ ....(5)

Where y_i is the output (production term); x_i is the vector of the input (k x q); β is the vector of the parameters to be estimated; v_i and u_i account for the compound error. v_i is usually assumed to be a symmetrical distribution like the normal with a zero mean and constant variance. However, the u_i term contains the asymmetric part which account for the technical inefficiency in the observation. In this study, the distribution of the asymmetric term was assumed to be half-normal and truncated normal. Hereafter referred to as stochastic frontier regression with half-normal (SFR-HN) and stochastic frontier regression with truncated-normal (SFR-TN). Details of these methods are well documented in Solomon and Zhang (2002) and Zhang *et al*. (2005).

All methods were fitted in R (R Core Team, 2017). The methods were assessed based on root mean square error (RMSE), Akaike information criterion (AIC) and Bayesian information criterion (BIC). The graphical performance of the self-thinning lines from the different methods was assessed by overlaying them on the field inventory data.

$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n-p}}$ .... (6)

$AIC = -2loglik + 2p$ **.................** (7)

$BIC = -2loglik + pln(n)$ **...........** (8)

Where: n = sample size, p = number of parameters; Y_i is the observed value and $\hat{Y}_i$ is the theoretical value predicted by the model.

One main application of Reineke's equation is the construction of density management diagrams (DMD). DMD is a graphical model that can be used to determine thinning schedules (Camacho-Montoya *et al.*, 2018). The best modelling approach was used to construct DMD based on stand density index (SDI), number of tree per ha, quadratic mean diameter. The SDI for the stand was obtained with this expression:

$$SDI = N\left(\frac{Ref_{Dq}}{Dq}\right)^{\beta_1} \quad ....(9)$$

Thus, for a given SDI, the number of trees per ha was estimated from:

$$N = SDI\left(\frac{Dq}{Ref_{Dq}}\right)^{\beta_1} \quad ....(10)$$

Where Ref_{Dq} is the reference diameter which was taken as the average of the quadratic mean diameter (25.2 cm) for the G. arborea stand. Other variables are previously defined in equation 1.

RESULT

Determination of the self-thinning line

The results of the estimated self-thinning lines for the model were three: quantile regression (QR), stochastic frontier regression with half-normal (SFR-HN) and truncated-normal (SFR-TN) as shown in table 2. The results showed that the three methods had negative slope for the self-thinning line which ranged from -1.051 to -0.951. The estimated intercept and slope for the different methods were also significant at 5% level. The SFR-HN had the smallest AIC and BIC of -5.321 and -0.609, respectively with lowest standard errors for the slope parameter and the error variances. This was followed by SFR-TN, and lastly by QR method.

To show the graphical performance of the self-thinning lines from the different methods, the lines were overlaid on the field inventory data (Fig 1). The self-thinning line of QR method estimated the upper limiting boundary line correctly compared to SFR-HN and SFR-TN. SFR-HN produced a self-thinning line not distinguishable from that of SFR-TN but lower than the limiting boundary line of the data. The Reineke's universal slope (i.e., theoretical slope) was also included in Fig 1. It was defined as: lnN = β_0 – 1.605lnDq; thus, the intercept (β_0) which varies with species was 12.142 for the *G. arborea* stand. Self-thinning line of the universal slope was higher than the SFF for stand with small quadratic mean diameter, but became lower than stochastic frontier lines at Dq > 24.5 cm. Thus, the Reineke's universal slope was steeper than those of quantile regression and stochastic frontier functions.

Table 2: Estimated parameters and fit indices for the different methods

Methods	Parameters	Estimate	SE	Lower	Upper	RMSE	AIC	BIC
QR	β_0	10.382*	0.948	0.477	12.288	0.398	8.305	11.839
	β_1	-0.951*	0.288	-1.529	-0.373			
SFR-HN	β_0	10.035*	1.038	8.319	12.387	0.183	-5.321	-0.609
	β_1	-1.051*	0.027	-1.586	-0.516			
	σ_v^2	0.034	0.009	0.016	0.052			
	σ_u^2	9.55E-6	0.026	-0.052	0.052			
SFR-TN	β_0	10.345*	0.842	8.695	11.994	0.183	-3.321	2.569
	β_1	-1.047*	0.226	-1.490	-0.604			
	σ_v^2	0.034	0.013	0.008	0.059			
	σ_u^2	0.006	0.319	-0.620	0.631			
	μ	-0.028	0.501	-1.010	0.955			

*OLS = ordinary least square; QR = quantile regression; SFF = stochastic frontier function; SE = standard error; * = significant at 5% level*

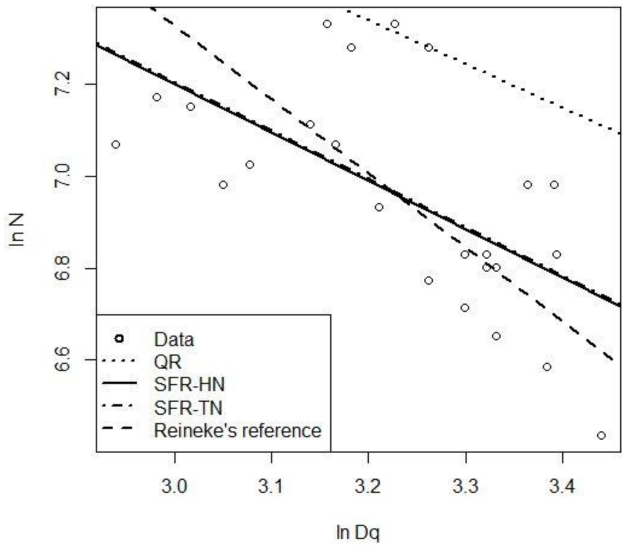

Fig. 1: Self-thinning lines derived from three modelling methods: quantile regression (QR), stochastic frontier regression with half-normal (SFR-HN) and truncated-normal (SFR-TN); and the Reineke's constant slope (-1.605)

Density management diagram (DMD)

Following the methodology of Camacho-Montoya *et al*. (2018), the stand density of four SDI classes was computed. This was done to delimit the zones of competition in the density management diagram. These zones include: the upper and lower limits of the self-thinning lines (i.e., maximum density) which were established at 100% and 55% of the SDI, respectively; and the lower and upper limits of the constant growth and free growth zones were fixed at 35% and 20%, respectively. The competition zones under QR, SFR-HN and SFR-TN are presentable in Table 3. The estimates from SFR-HN were relatively higher below the reference diameter (< 25 cm) and lowest above the reference point. QR predictions were higher above 25 cm diameter. SFR-HN and SFR-TN had the same predictions across the four zones at 25 cm quadratic diameter. A simplified density management diagram constructed under SFR-HN for the *G. arborea* stand is presented in Fig 2. The diagram showed the number of trees per ha, quadratic mean diameter and the delimited zones.

Table 3: Stand density by SDI class derived with the Reineke equation using quantile regression (QR), stochastic frontier regression with half-normal (SFR-HN) and truncated normal (SFR-TN)

Dq (cm)	QR				SFR-HN				SFR-TN			
	100%	55%	35%	20%	100%	55%	35%	20%	100%	55%	35%	20%
15	2496	1674	1612	1473	2630	1752	1714	1536	2625	1749	1710	1534
17	2216	1486	1431	1308	2306	1536	1502	1347	2302	1534	1500	1345
19	1993	1337	1287	1176	2051	1367	1336	1198	2049	1365	1335	1197
21	1812	1216	1170	1070	1846	1230	1203	1078	1845	1229	1202	1078
23	1662	1115	1073	981	1678	1118	1093	980	1677	1117	1093	980
25	1535	1030	991	906	1537	1024	1001	898	1537	1024	1001	898
27	1427	957	921	842	1418	944	924	828	1418	945	924	829
29	1333	894	861	787	1315	876	857	768	1316	877	857	769
31	1251	839	808	738	1226	817	799	716	1227	817	799	717
33	1179	791	761	696	1148	765	748	670	1149	766	749	671
35	1115	748	720	658	1079	719	703	630	1080	720	704	631

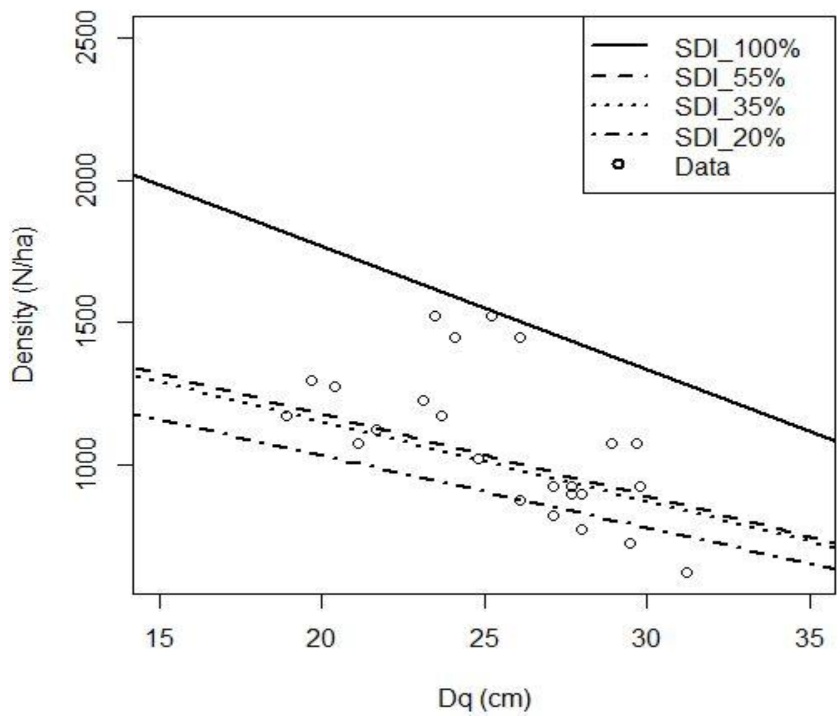

Fig. 2: Density management graph for Gmelina arborea stands derived from stochastic frontier regression with half-normal (SFR-HN).

DISCUSSION

The quantile regression (QR), stochastic frontier regression with half-normal (SFR-HN) and stochastic frontier regression with truncated-normal (SFR-TN) methods have been used to determine the self-thinning line of *G. arborea* stands. The parameter estimates from the three methods were negative (downward slope from left to right) and as such, they are biologically reasonable. This is because the number of tree per ha is expected to decrease with increasing average tree size (Zeide, 2005). Though the three methods performed relatively well in defining the self-thinning line, the SFR-HN with the smallest fit indices values and lowest standard errors was the most suitable for modelling the self-thinning line of the *G. arborea* stand. Parallel result was reported in Camacho-Montoya *et al*. (2018) who found the SFR-HN to be more suitable than ordinary least square and SFR-TN for modelling the self-thinning line of *Pinus patula*. The ordinary least square (OLS) was not used in this study because studies have shown that it produces inappropriate slope for the self-thinning line and only represent central trend line (e.g., Solomon and Zhang, 2002; Zhang *et al*., 2005).

The self-thinning lines obtained in this study seems to be smoother than the universal slope of -1.605 proposed by Reineke (1933). This further confirms that different species exhibit different slope values for the self-thinning line due different growth rates. Site variation is another factor that could affect the slope parameter (Weiskittel *et al*., 2009; Kara, 2018; Salas-Eljatib and Weiskittel, 2018). For example, Weiskittel *et al*. (2009) reported that aspect, site index and stand origin affect the slope parameter of the self-thinning line. Furthermore, Pretzsch and Biber (2005) asserted that the use of Reineke's universal slope value may provide unrealistic thinning regimes.

The QR method seems to represent the upper limit of the data compared SFR-HN and SFR-TN. However, a major setback with the QR method is that "statistical inference" is relatively problematic (Zhang *et al*., 2005). Similar observation was reported by Zhang *et al*. (2005) for *Pinus strobus* L. stand. One important advantage of stochastic frontier is that it relaxes the assumption of heteroscedasticity while testing the effect of covariates (Bi *et al*., 2000; Weiskittel *et al*., 2009). In addition, with the stochastic frontier regression method, it is possible to estimate the maximum feasible density through the frontier that limits the estimates of the parameters (Camacho-Montoya *et al*., 2018). The density management diagram (DMD) constructed under SFR-HN for the *G. arborea* stand can be used to manipulate different conditions of the stand. For example, if the average tree size is 25 cm (Dq), the expected density for the different delimited zones would be 1537, 1024, 1001 and 898 N/ha. This information is required for the routine management of the *G. arborea* stand.

CONCLUSION

Developing self-thinning line requires efficient and effective method so that realistic thinning regimes can be prescribed. In this, we found the stochastic frontier regression especially, with half-normal to be the most suitable method for the *G. arborea* stand in Nigeria. The information provided in this study would help in the manipulation of the growth condition and determination of thinning schedule of the *G. arborea* stands.

REFERENCES

Aigner, D.J., Lovell, C.A. and Schmidt, P.J. (1977). Formulation and estimation of stochastic frontier production functions models. *Journal of Econometrics*, 6: 21-37

Ajayi, S., Ogar, N.E. and Anyaorah, C.N. (2004). A mathematical programming approach to sustainable management of *Gmelina arborea* (Roxb) plantations in a Nigerian rain forest. *International Journal of Education*, 2(1-2): 146-157.

Bi, H. 2001. The self-thinning surface. *Forest Science*, 47: 361-370

Bi, H., Wan, G. and Turvey, N.D. (2000). Estimating the self-thinning boundary line as a density-dependent stochastic biomass frontier. *Ecology*, 81: 1477-1483.

Camacho-Montoya J, Santiago-Garcia W, Rodriguez-Ortiz G, Antunez P, Santiago-Garcia E and Suarez-Mota ME (2018): Self-thinning and density management in even-aged Pinus patula Schiede ex Schlechtdl. & Cham. stands. In *Revista Mexicana de Ciencias Forestales* 9 (49): 1–24.

Comeau P.G., White M., Kerr G and Hale S.E. 2010. Maximum density–size relationships for Sitka spruce and Douglas-fir Britain and Canada. *Forestry* 83 (5), pp. 461–468.

Drew, J. and Flewelling, J. (1977). Some recent Japanese theories of yield-density relationships and their application to Monterey pine plantations. *Forest Science*, 23: 517-534

Duke, J.A. (1983). *Handbook of energy crops*: Gmelina arborea Roxb. Retrieved from https://www.hort.purdue.edu/newcrop/duke-energy/Gmelina_arborea.html

Jack, S.B. and Long, J.N. 1996. Linkages between silviculture and ecology: an analysis of density management diagrams. *Forest Ecology and Management*, 86: 205-220

Kara, F. 2018. The growing space utilization of main tree species in northern Turkey. In *CERNE* 24 (2), pp. 133–139.

Koenker R., Bassett G. (1978). Regression quantiles. *Econometrica* 46: 33-50.

Ogana, F.N., Itam, E.S. and Osho, J.S.A. 2017. Modelling diameter distributions of *Gmelina arborea* plantation in Omo Forest Reserve, Nigeria with Johnson's S_B. *Journal of Sustainable Forestry*, 36: 121-133.

Onyekwelu, J. C. (2001). Growth Characteristic and management scenarios for plantation grown *Gmelina arborea* and *Nauclea diderichii* in south-western Nigeria. Ph.D. Thesis; Hieronymus Publishers, Munich; ISBN 3-89791-235-X

Onyekwelu, J. C. Mosandl and Stimm, B. (2006). Productivity, site evaluation and state of nutrition of *Gmelina arborea* plantation in tropical rainforest zone in south-western Nigeria. Forest Ecology and management 229:214-227.

Pretzsch, H. and Biber, P. (2005). A re-evaluation of Reineke's rule and stand density index. *Forest Science*, 51:304-320.

R Core Team (2017). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL http://www.R-project.org/ Accessed 30 June 2017

Reineke, L.H. 1933. Perfecting a stand-density index for even-aged forests. *Journal of Agricultural Research*, 46(7): 627-638

Reyes-Hernandez V; Comeau P.G and Bokalo M. (2013). Static and dynamic maximum size density relationships for mixed trembling aspen and white spruce stands in western Canada. In *Forest Ecology and Management* 289, pp. 300–311.

Salas-Eljatib C and Weiskittel A.R. (2018). Evaluation of modelling strategies for assessing self-thinning behaviour and carry capacity. *Ecol Evol*. 00:1-12, https://doi.org/10.1002/ece3.4525

Shaw; J.D (Eds.) (2005). Reineke's Stand Density Index: Where are we and where do we go from here? Proceedings: Society of American Foresters 2005 National. Bethesda, October 19-23. Society of American Foresters.

Solomon D.S. and Zhang L (2002). Maximum size_density relationships for mixed softwoods in the northeastern USA. In *Forest Ecology and Management* 155, pp. 163–170.

Vanderschaaf, C.L. and Burkhart, H.E. (2007). Comparison of methods to estimate Reineke's maximum size-density relationship species boundary line slope. *Forest Science*, 53: 435- 442.

Vospernik S; Sterba H. 2015. Do competition-density rule and self-thinning rule agree? In *Annals of Forest Science* 72, pp. 379–390.

Weiskittel, A., Gould, P. and Temesgen, H. (2009). Sources of variation in the self-thinning boundary line for three species with varying levels of shade tolerance. *Forest Science*, 55: 84-93.

Yoda, K., Ogawa, H. and Hozumi, K. (1963). Self-thinning in overcrowded pure stands under cultivated and natural conditions (Intraspecific competition among higher plant XI). Japan Polytechnic Institute, Osaka, Japan. 14(Series D). 107-129.

Zeide B (2005): How to measure stand density. *Trees* 19, pp. 1–14.

Zhang J., Oliver W.W and Powers R.F. (2013). Re-evaluating the self-thinning boundary line for ponderosa pine (Pinus ponderosa) forests. *Canadian Journal of Forest Research*, 43: 963-971.

Zhang L, Bi H, Jeffrey H.G. and Heath L.S. (2005). A comparison of alternative methods for estimating the self-thinning boundary line. *Canadian Journal of Forest Research*, 35: 1507–1514.