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ON THE FUZZY NATURE OF CONSTRUCTED ALGEBRAIC STRUCTURE

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ABSTRACT
In this paper, some fuzzy nature of a newly constructed an algebraic structure G_p $(p \geq 5\ and\ p\ is\ always\ prime)$ were been investigated by construction of a modified fuzzy membership function on G_p and it was used to investigate the $\propto - cut\ level$ of G_p and it was established that the $\propto - cut\ level$ of G_p is the domain $G_p|w_{p-1}$, the $support(Supp)$ of G_p was also been investigated and we arrived at a conclusion that, the support of the G_p structure is the entire domain of the structure (G_p).
Keywords: fuzzy set, ∝- cut level, support of a fuzzy set, permutation pattern, cycles, successors and membership function.

1.0 INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh(1965), by defining them in terms of mappings from a set into a unit interval on the real line. Fuzzy sets were introduced to provide means to describe situations mathematically which gives rise to ill-defined classes, i.e. collection of objects for which there is no precise criteria for membership, collections of this type have a vague boundaries (Fuzzy), there are objects for which it is impossible to determine whether or not they belong to the collection. The classical mathematical theories, by which certain types of certainty can be expressed, are the classical set theory and probability theory, in terms of set theory, uncertainty is expressed by any given set of possible alternatives in situations where only one of the alternatives may actually happen. Uncertainty expressed in terms of sets of alternatives results from the non-specificity inherent in each set. Probability theory expresses uncertainty in terms of a classical measure of subsets of a given set of alternatives. The set theory introduced by Zadeh, presents the notion of membership in a given subset as a matter of degree rather than of totally in or totally out. With a fuzzy set theory, one obtains a logic in which statements may be true or false to different degrees rather than the bivalent situations (on or off) of being true or false.

Permutation pattern have been used in the past decades to study mathematical structures Audu(1986), Ibrahim(2005)studied the concept of permutation pattern using some elaborate scheme to determine the order of precedence and the position of each of the elements in a finite set of prime size have also been established in Ibrahim (2007), Garba (2018) and also an idea of embedment as an algebraic structure has yielded some interesting results by Ibrahim (2005), Garba (2018),they studied the structure and developed a scheme for the range of such cycles and use it to investigate further number theoretic and algebraic properties of $\mathbf{G_p}$, and furthermore a group theoretic properties was also investigated by Garba (2018) and the concept of Fuzzy nature of $\mathbf{G_p}$ has also been studied by Aremu (2017) and investigated the alpha-level cut of $\mathbf{G_p}$. Ibrahim (2007)studied the concept of permutation pattern using some elaborate scheme to determine the order of precedence and the position for each of the elements in a finite set of prime size, and establish a scheme the scheme for generating each element in the permutation. Garba (2009) studied the $G_p$structure using number theoretic properties of Catalan numbers, and also developed a scheme for range of such cycles defined to be $|\Delta_f^l|$ where l is the last element in the cycle and f is the first element in the cycle, and established that for all cycles in $G_p$the range exist, they also use it to investigate further number theoretic properties of G_p.Usman (2011)investigated the group theoretic properties of $G_p$using composition of functions, by investigating the properties of a group and established that the structure is an Abelian group, using additive group of integers modulo n, where n is necessarily a prime.

Garba (2015) extended the $\boldsymbol{G_p}$ structure to the $\boldsymbol{G_P}'$ where a special cycle (w_p) is introduced and the special cycle is been used as an identity of permutation and established the closure, commutativity and associativity properties from the structure. Aremu (2017)studied the $\boldsymbol{G_P}'$structure and investigated the $\propto -cut$ level ($\propto$ is a fixed numerical value $\propto \in R^+$) of $\boldsymbol{G_P}'$and defined the level to be $\frac{1}{p}$ ($\propto = \frac{1}{p}$, p is a prime p≥ 5) and established that the $\propto -cut$ level of $\boldsymbol{G_P}'$based on$\propto = \frac{1}{p}$ is the entire domain of $\boldsymbol{G_p}$.

2.0 Preliminaries

2.1 Fuzzy Set

If X is a collection of object then the fuzzy set $\tilde{A} \subset X$ is a set of ordered pairs, $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$,where $\mu_{\tilde{A}}(x) : X \to [0,1]$ is called degree of membership of x in A**(**Zadeh 1965).

2.2 Crisp Set

The crisp set is a set defined using characteristics functions that assign to each element of the universe a Boolean state of obedience.

2.3 The $\propto -\boldsymbol{Cut\ Level\ Set}$

The $\propto -level$ of a fuzzy set $\tilde{\boldsymbol{A}}$ is a crisp set **A**thatcontains the elements that have membership functions in $\tilde{\boldsymbol{A}}$greater than or equal to $\propto$. and its represented as

$$A_\propto = \{ \propto \mid \mu_{\tilde{A}}(x) \geq \propto \}$$

2.4 The Support of a Fuzzy Set

The support of a fuzzy set (denoted Supp) is the crisp set of all $x \in X\ for\ which\ \mu_{\tilde{A}}(x) > 0$**(**Zadeh 1965).

2.5 Cycle and Successor

Let Ω be a non-empty, totally ordered and finite subset of$\mathbb{N}$. Let$\boldsymbol{G_p} = \{w_1, w_2, \dots, w_{p-1}\}$ be a structure such that each w_i is generated from the arbitrary set Ω for any prime $p \geq 5$, using the scheme

$$w_i = \big((1)(1+i)_{mp}(1+2i)_{mp} \dots (1 + (p-1)i)_{mp}\big)$$

Wheremp is modulop

Then each w_i is called a cycle and the elements in each w_i are distinct and called successors**(**Ibrahim 2004).

2.5.1 $\boldsymbol{n^{th}\ successor}$

The$n^{th}\ successor$ of a cycle w_i is given by $a_n = (1 + (n-1)i)_{mp}$

where $1 \leq n \leq p, and\ 1 \leq i \leq p-1$. the number of distinct successors in a cycle is called the length of the cycle**(**Ibrahim 2004).

2.5.2 Range of Cycle

The range of a cycle $w_i \in \boldsymbol{G_p}$ is defined as$\pi(w) := |\Delta_f^l(w)|$, where $\Delta_f^l(w)$ is the difference between the first and last successor in a cycle w**(**Garba, 2009).

2.5.3 Definition of G_p'

Let $\boldsymbol{G_p} = \{w_1, w_2, \dots w_{p-1}\}$ be as defined as above then$\boldsymbol{G_p'} := \boldsymbol{G_p} \cup \{w_p\}$, where $w_p := \{pp \dots p\}$. that is$\boldsymbol{G_p'} = \{w_1, w_2, \dots w_{p-1}, w_p\}$.

Using the above setting, if p=5, then we have the following set of permutations$w_1 = (12345), w_2 = (13524)$, $w_3 = (14253)$, $w_4 = (15432)$ and this shows

that$G_5 = \{(12345), (13524), (14253), (15432)\}$

Note that 0 and 5 are equivalent in modulo 5, thus instead of using 0 in modulo p we will be using p**(**Garba, 2009).

3.0 RESULT AND DISCUSSION

In this section, the discussion of the result is carried out by figures, tables and proofs.

3.1 Fuzzy Nature of G_p

Let $\boldsymbol{G_p'} := \boldsymbol{G_p} \cup \{w_p\}$and $\boldsymbol{G_p} \subseteq \boldsymbol{G_p'}$, then $\boldsymbol{G_p}$is a fuzzy set defined by

$$\hat{G}_p = \{(\mu_{\hat{G}_p}(w_i) : w_i \in \boldsymbol{G_p'}\}$$

Where$\mu_{\hat{G}_p}(w_i) = \left(i, \frac{\pi(w_i)}{p+2}\right), i < p$

$$\pi(w_i) = |\Delta_f^l(w_i)|$$

$\pi(w_i) = |\Delta_f^l(w_i)|$where$l$ is the last successor and f is the first successor

Illustration: consider $\boldsymbol{G_p'}$ where p=5, $\boldsymbol{G_5'} = \{w_1, w_2, w_3, w_4, w_5\}$**,** $let\boldsymbol{G_5} \subseteq \boldsymbol{G_5'}$

Then $\boldsymbol{G_5} = \{w_1, w_2, w_3, w_4\}$**,** Defined $\mu_{\hat{G}_5}(w_i) = \left(i, \frac{\pi(w_i)}{7}\right), i < 5$

$$\pi(w_i) = |\Delta_f^l(w_i)|$$

$$\mu_{\hat{G}_5}(w_1) = (1, 0.6)$$

$$\mu_{\hat{G}_5}(w_2) = (2, 0.4)$$

$$\mu_{\hat{G}_5}(w_3) = (3, 0.3)$$

$$\mu_{\hat{G}_5}(4) = (4, 0.1)$$

$$\hat{G}_p = \{(\mu_{\hat{G}_p}(w_i)) : w_i \in \boldsymbol{G_p'}\}$$

$\hat{G}_5 = \{(1,0.6), (2,0.4), (3,0.3), (4,0.1)\}$, then $\hat{G}_5$ is a fuzzy set.

3.2 Proposition: The $\propto -cut$ level of any $G_p$is $G_p|w_{p-1}$

Proof

An $\propto -cut$ level is a set that contains values from the membership functions greater than or equal to $\propto$.$\propto$ is an arbitrary value with the range of fuzzy $[0,1]$;$let\ \boldsymbol{G_p}$be a fuzzy set and $\boldsymbol{G_p} \subseteq \boldsymbol{G_p'}$, then the $\propto -cut$ level is a set.$G_{p_\propto} = \{w_i : \mu_{G_p}(w_i) \geq \propto\} \quad for \ \propto = \frac{1}{p}$,where(p$\geq 5$)

Since$G_{p_\propto} = \{\mu_{G_p}(w_i) : \mu_{G_p}(w_i) \geq \propto, i < p\}$

$$\mu_{\hat{G}_p}(w_i) = \left(i, \frac{\pi(w_i)}{p+2}\right), i < p$$

Without loss of generality, $\mu_{\hat{G}_p}(w_{p-1}) = \left(i, \frac{\pi(w_{p-1})}{p+2}\right)$,

$$\frac{\pi(w_{p-1})}{p+2} < \frac{1}{p}, \text{ for any } \boldsymbol{G_p}.$$

Where$\boldsymbol{G_p} = \{w_1, w_2, \dots w_{p-1}\}$

$$\mu_{\hat{G}_p} = \{w_1, w_2, \dots w_{p-2}\} \geq \frac{1}{p}, \text{but } \mu_{\hat{G}_{p-1}} < \frac{1}{p}.$$

=> $G_p|w_{p-1}$is the domain of the alpha-cut level of the set $\hat{G}_p$.

Illustration consider when p=5,

$$\boldsymbol{G_5} = \{w_1, w_2, w_3, w_4\},$$
$$\mu_{\hat{G}_5}(w_1) = (1, 0.6)$$
$$\mu_{\hat{G}_5}(w_2) = (2, 0.4)$$
$$\mu_{\hat{G}_5}(w_3) = (3, 0.3)$$
$$\mu_{\hat{G}_5}(4) = (4, 0.1)$$
$$if \ \propto = \frac{1}{p}, then \ \propto = \frac{1}{5} = 0.2,$$
$$=> \ w_{p-1} < \propto.$$

From the illustration above it implies that, the $\propto -cutlevel$ is the domain$G_p|w_{p-1}$. The table below gives a complete description of the alpha-cut-level of the constructed algebraic structure, the alpha level of each **Gp** exist, and is unique.

Table 3.1: Membership Functions of w_i and $\propto -Cut\ Level$

s/n	w_i	$\mu_{\hat{G}_p}(w_i)$
1	w_1	0.6
2	w_2	0.4
3	w_3	0.3
4	w_4	0.1
5	$\propto$-cut level	0.2

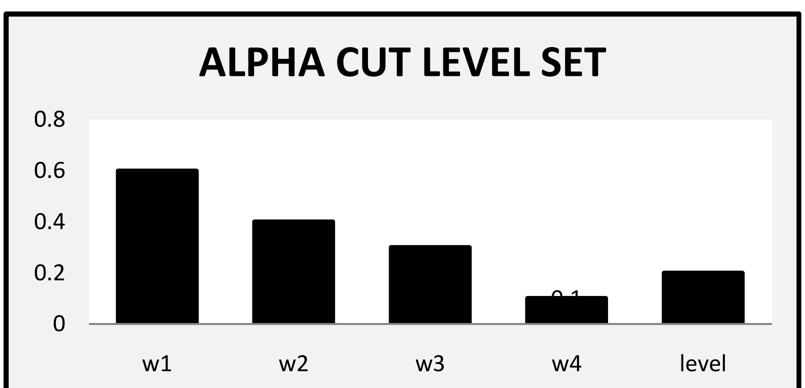

Figure 3.1: Alpha Cut Level Set

Figure 3.1 illustrate the $\propto -cut\ level$ of the $\mathbf{G_p}$ .the vertical axis represents the membership functions while the horizontal axis represents the permutations w_i .

For $\boldsymbol{G_5}$the$\propto - cut$ level is $G_5|w_4$.

For $\boldsymbol{G_7}$the$\propto - cut$ level is $G_7|w_6$.

For $\boldsymbol{G_{11}}$the$\propto - cut$ level is $G_{11}|w_{10}$.

For $\boldsymbol{G_{13}}$the$\propto - cut$ level is $G_{13}|w_{12}$, e.t.c

This generalize the proof.

3.3 Proposition: The Support of the fuzzy set $\hat{G}_p$ of any $\boldsymbol{G_p}$is the entire domain.

Proof

The support of a fuzzy set (denoted by Supp) are those members of the set in which their membership degree is > 0,

$$Supp(\hat{G}_p) = \left\{\mu_{G_p(w_i)} : \mu_{G_p}(w_i) > 0\right\}$$

And$\pi(w_i)$ is never zero, then, the result follows.

$Supp(\hat{G}_p)$is the entire domain

Table 3.2:Membership Functions of $\boldsymbol{w_i}$

s/n	w_i	$\mu_{\hat{G}_p}(w_i)$
1	w_1	0.6
2	w_2	0.4
3	w_3	0.3
4	w_4	0.1

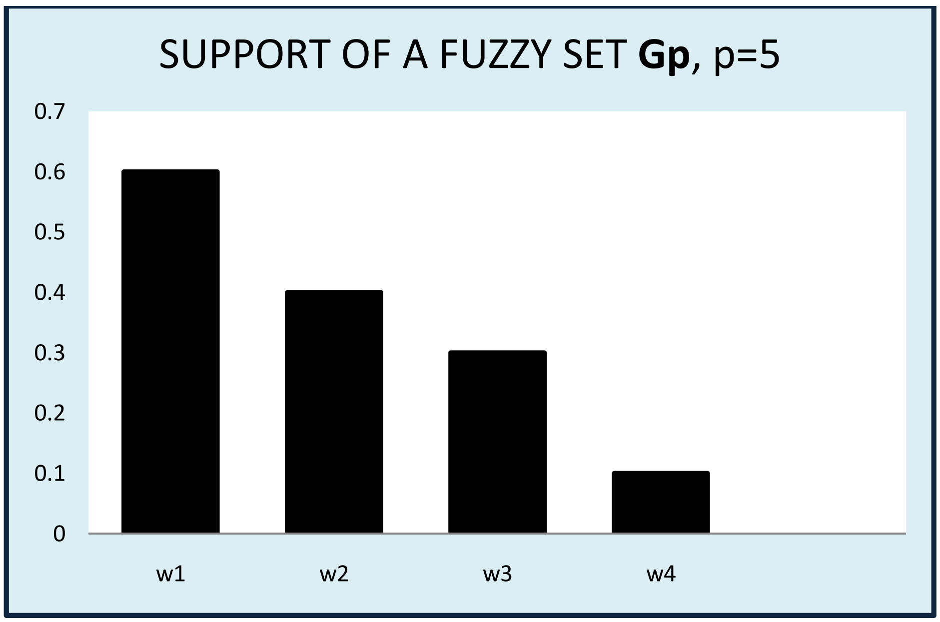

Figure 3.2: Support of a Fuzzy SetGp

Figure 3.2 gives the description of the **Supp(Gp),**and it can be seen that all the values are >0, the vertical axis represents the membership functions while the horizontal axis represents the permutations $w_i$and is true for any **Gp,** then the support of any fuzzy set in **Gp** is the entire domain.

3.5 CONCLUSION

The construction of an algebraic structures and investigating their algebraic properties cannot be over emphasized as it has a lot of applications in different field of mathematics, in this paper we investigated some fuzzy nature of an algebraic structure G_p that was constructed earlier, where we discovered that if $\hat{G}_p$ is a fuzzy set, then the $\propto -$ cut level set of any G_p is a set $G_p|w_{p-1}$, and the $support$ of $\hat{G}_p$ is the entire domain, In the above constructed algebraic structure the first element of the permutation is always fixed.

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