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Full Length Research Article

# MATHEMATICAL MODELING AND SIMULATION OF CLOSING FUNCTION OF WATER HAMMER SYSTEM DURING GAS PRODUCTION

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**ABSTRACT**

Mathematical modelling and simulation of closing function of water hammer system during gas production described the speed variation of gas as valves close automatically. The fluctuation of speed witnessed was as a result of automatic valve closing and always give back pressure and the shape of the pressure curve where a wide variety of closing modes exists, depending on the valve type and their operation is mathematically represented by a function. The pressure wave obtained in this work can be significantly important to those in gas producing industries. It is in this regard that the author aim is to reduce the problem encountered in gas producing industries by developing a one-dimensional system of governing equations. The equation was simplified to generic function which was formulated to accommodate a suitable closing laws by means of a polygonal segmented structure and solve by Laplace Transformation. The boundary conditions used in this work is generated from a special algothrim that described the transitory. It is observed that back pressure wave shape and amplitude depend on the closing function of valves and in unique relationship. The results can prevent premature closure of gas in gas producing industries. This work has presented an information about the over-pressure peak, shape and phase of the pressure wave during the gas production. It has an advantage of helping gas production industries in choosing the best type of closing laws and help in arresting the over increase in pressure which may cause rupture to a pipe or cause damage to equipment.

**Keywords:** Closing Function, generic function, Laplace Transformation and Special Algothrim.

## INTRODUCTION

Modelling and simulation of Closing Function of the Water Hammer system during Gas Production play a very important role in understanding the performance of the well. During production, operating valves causes back pressure that generates changes on the flow condition. This phenomenon has its origin during automatic closure of valves which is always referred to as closing function or water hammer system. During simulation of the closing function, pressure and fluid flow behaviour are observed to be alternate succession of crest and troughs that attenuates with time.

Many authors has been reporting on closing function in the water hammer modelling that it provides information about the shape, over-pressure peak and phase of the pressure wave during the transient development.

Authors like Bergant et al., (2008), investigates parameters that may significantly affect closing function wave attenuation, shape and timing and possible sources that may affect the waveform. They predict the pressure shape by using classical closing function that include unsteady friction and cavitation. Aly et al., (2019), developed a model that can investigate the behaviour of high pressure and reported that produced pressures water hammer effect can rupture the pipeline and its components, while the low negative pressures can collapse the pipeline. Provenzano et al (2011), development a mathematical model that takes into account the closing law, and the formulation of an algorithm that allows to describe a wide range of closing functions. Enbin et al, (2017), reported that closing phenomena can occur easily during operational processes especially at the starting and stopping of a pump station, the adjustment of the speed of a pump unit, and a shutdown due to an incident and sometime failure of a regulating valve. They carry out a theoretical study and a numerical simulation of closing function that occur due to the collapse of flow interruption.

Boran et al., (2018), applied the Lax-Wendroff method to predict the transient pressure caused by a valve closing in a gravitational pipe with continuous air entrainment. The study provides an access by considering the influence of the pipe flow velocity on the wave propagation to simulate transient processes caused by closing function during pipe flow operation. Xaixiao et al., (2019), applied analytical investigation of water hammer pressure variation in a reservoir-pipe-valve system. They used Laplace transform method and obtained analytical solutions for different characteristics of the valve closing using sudden valve closing and piecewise linear closing. Norazlina and Norsarahaida, (2015), investigate the closing function on transient flow of hydrogen-natural gas mixture in a horizontal pipeline with the view of determining the relationship between pressure waves and different modes of closing and opening of valves.

In all the literature none has mention the closing function that occurs during production. The presence work studied the different type of closing laws on the pressure and the behaviour of the flow at the instance of closing during production.

## MODEL FORMULATION

In gas production, the most commonly cause of pressure surge is the valve closing flow perturbation. Usually, the point at which perturbation took place is considered as a frontier of the system. Flow changes either after or before the point are different.

An angle is automatically formed between the upstream and downstream pressure waves that travel across the entire well. It is these characteristics that allow to evaluate the valve closing as a boundary condition. A one-dimensional mathematical model for the closing function during gas production is developed from the principle of conservation laws using a simple system, constituted by a reservoir, a single vertical constant diameter conduction with a production valves that operate automatically.

The equations governing the closing function law are the conservation of mass and momentum derived as follows

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}=0 \quad (1)$$

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho u^2)}{\partial x}=-\frac{\partial p}{\partial x}-\rho g\sin\theta-\frac{f\rho u^2}{2d} \quad (2)$$

In dealing with the closing function, $\theta=0$ and the friction is assumed to be zero that is $(f=0)$, and simplifying equations (1) and (2), the equation for the closing function is obtained as in (3).

$$\frac{\partial p^2}{\partial x^2}+\delta\frac{\partial^2 u}{\partial t\partial x}=0 \quad (3)$$

Equation (3), therefore, can be transform into generic equation using Laplace Transformation.

$$\overline{P}=A\cosh\left(\frac{sx}{a}\right)+B\sinh\left(\frac{sx}{a}\right)+\frac{p_0}{s} \quad (4)$$

The Constant $A$ and $B$ are obtained through integration, to determine the particular solution for (4), the boundary condition for the closing law are as follows:

For $t>\iota,\ t=0,\ u=u_0$, For $t\geq 0$

$x=0$, $p=p_0$, For $0<t<z$, $x=L$

$u=u(t)\quad (clo\sin g\ Law)$

Transient fluid speed during production can be modified in many different ways depending on the action of the closing laws (valves action). Some of the closing function can be classified according to how the operation reduces flowing fluid pressure and are described as follows:

I. Convex closing law: this causes decrease in the speed flow during the early closing time $(\tau)$.
II. Concave closing law: this causes high decrease of the speed flow at the initial closing time $(\tau)$ that increases with time as production goes.
III. Linear Closing law: This type of closing law is where speed flow reduction is uniformly distributed throughout the closing time.
IV. Instantaneous closing time: this is the speed changes instantly from the initial valve to its final limit or zero.

A further equation can be developed from (2) that can accommodate all the closing functions. The polygonal closing function approached which allow an ordered succession of segments. Putting the closing time $(\tau)$ staged in equal parts will yield the closing function.

The local velocity $u$ has been proposed as in Provenzano et al., (2011)

$$u(t)=(u_0-u_\tau)\left[1-\left(\frac{t}{\tau}\right)^m\right]+u_\tau$$

$$u(t)=(u_0-u_\tau)\left[1-\left(\frac{i\beta}{\tau}\right)^m\right]+u_\tau \quad (5a, b)$$

where $0\leq i\leq N$, $\tau=N\beta$, $0\leq m<\infty$ and $u_\tau$ is the gas speed at the closing operation, $m$ determines the closing curves law depending on its valve. If $m=0$ the closing is instantaneous, $0\leq m<1$ it is concave closing, $m=1$ it is linear closing and if $1\leq m<\infty$ it is convex closing.

Solution for the closing function during production can be obtain by applying polygonal approach procedure and the Laplace transformations as in equa.2 with the boundary condition. Similarly the time solution before the closing end $0<t\leq\tau$ is obtain as follows:

$$p(x,t)=p_0+$$

$$+\frac{\rho}{g}a\sum_{i=1}^{j<k-1}\frac{(u_i-u_{i-1})}{\beta}\frac{8L}{\pi^2 a}\left\{\sum_{n=1}^{\infty}\frac{(-1)^n\cos(2n-1)\pi\frac{a}{2L}(t-(i-1)\beta)\sin(2n-1)\pi\frac{x}{2L}}{(2n-1)^2}\right\}$$

$$\left\{-\sum_{n=1}^{\infty}\frac{(-1)^n\cos(2n-1)\pi\frac{a}{2L}(t-i\beta)\sin(2n-1)\pi\frac{x}{2L}}{(2n-1)^2}\right\}-$$

$$-\frac{\rho}{g}a\frac{(u_{j+2}-u_{j+1})}{\beta}\left\{\frac{x}{a}-\frac{8L}{a\pi^2}\frac{\sum_{n=1}^{\infty}(-1)^n\cos(2n-1)\pi\frac{a}{2L}(t-(j-1)\beta)\sin(2n-1)\pi\frac{x}{2L}}{(2n-1)^2}\right\}$$

(6)

And for time after closing $(t>\tau)$

$$p(x,t)=p_0+$$

$$+\frac{\rho}{g\tau}a\sum_{i=1}^{k}\frac{(u_i-u_{i-1})}{\beta}\frac{8L}{a\pi^2}\left\{\sum_{n=1}^{\infty}\frac{(-1)^n\cos(2n-1)\pi\frac{a}{2L}(t-(i-1)\beta)\sin(2n-1)\pi\frac{x}{2L}}{(2n-1)^2}\right\}-$$

$$-\left\{\sum_{n=1}^{\infty}\frac{(-1)^n\cos(2n-1)\pi\frac{a}{2L}(t-i\beta)\sin(2n-1)\pi\frac{x}{2L}}{(2n-1)^2}\right\}$$

(7)

## RESULTS AND DISCUSSION

The result of the closing function was carried out for different closing functions using equation (4) and equation (5). From the analysis, the shape of transient back pressure due to closing valve was obtained and compared to an existing work of Provenzono et al., (2011). The result shows good agreement. At $m=0.05$, the shape of the pressure wave evolves from square towards trapezoidal shapes as shown in fig.2. When pressure wave increases from $0.05<m<0.3$, it shows triangular wave

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evolving toward trapezoidal shapes with right bias fig. 3. If we increase the valve of $m$ to $0.3 < m < 8$, the pressure wave shape becomes triangular and moving toward trapezoidal shapes with left bias fig. 4. The situation is different when $m = 1$, there is a Strick triangular pressure wave shape with much fluctuation as shown in fig. 5. If $m$ is increased to 10 the pressure wave decreases at upstream and increases at downstream as shown in fig.6 and 7.

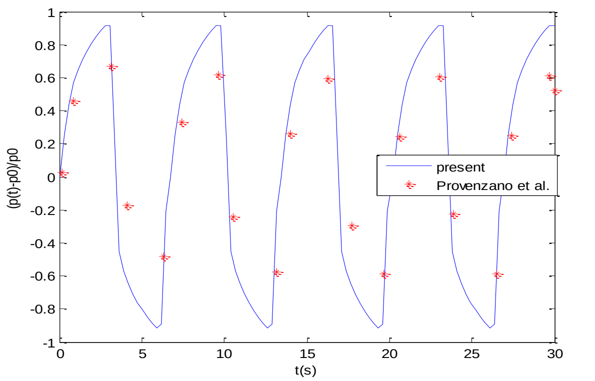


**Figure 1:** Provenzomo et.al (2011) - Pressure Due to Closing Valve

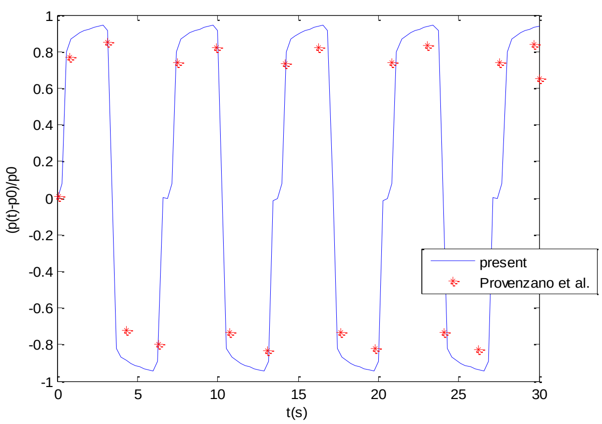


**Figure 2:** Pressure wave at m=0.05

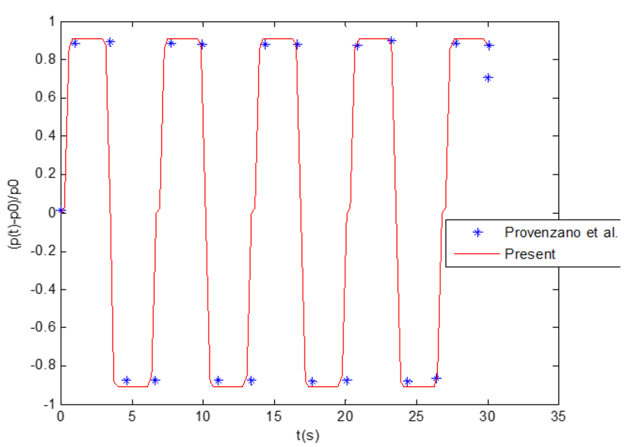


**Fig. 3**: pressure wave at $0.05 < m < 0.3$

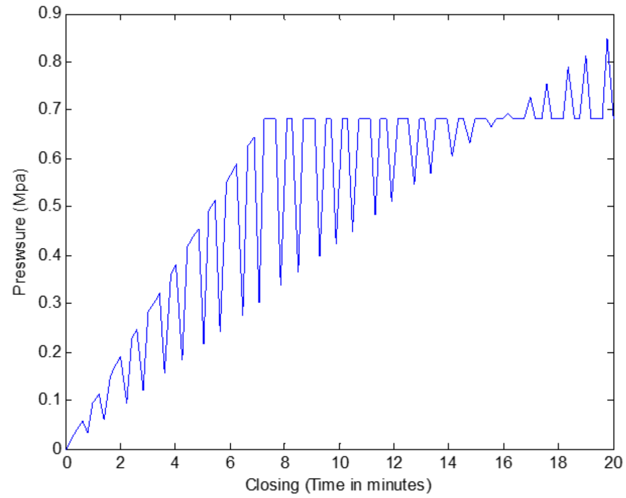


**Fig. 4**: pressure wave at $0.3 < m < 8$

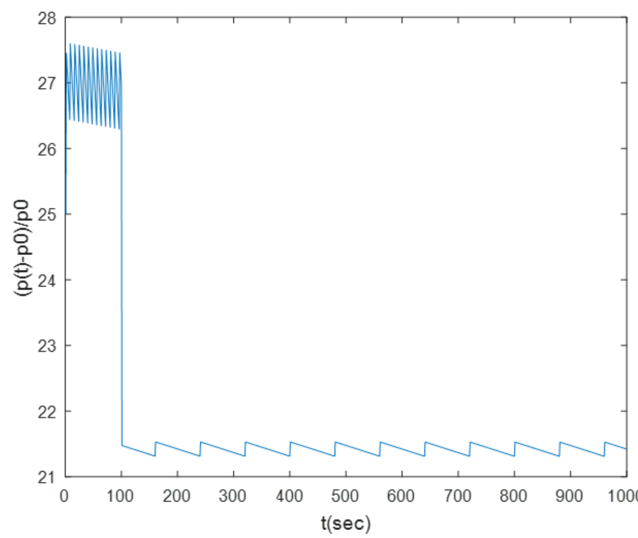


**Fig. 5**: Strick pressure wave at $m = 1$

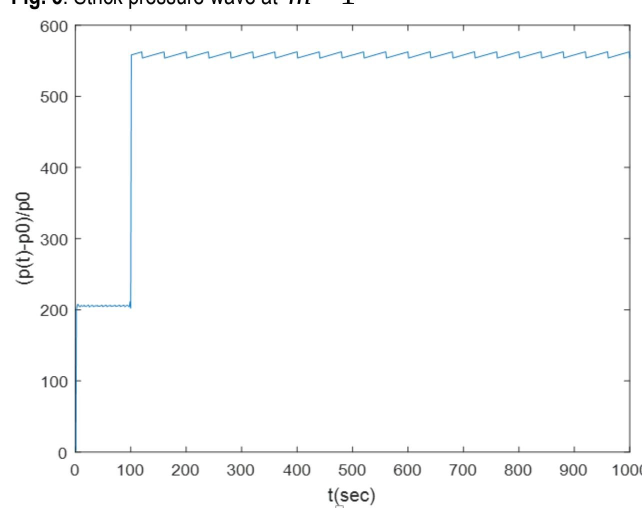


**Fig. 6**: pressure wave at $m = 10$ upstream

**Conclusion**

Equations for the closing function during gas production has been formulated and presented. Different closing functions has been discussed at the upstream toward the well head and at downstream. The inclusion of the closing function in gas production is to give a mathematical solution that provides information about the shape, over-pressure peak and phase of the pressure wave. Modelling of the closing law has given the analysis of a wide variety of closing shapes that appears in the operation during production period. It has been observed that the linear law (m = 1), triangular type waves prediction gives a lower over-pressure peak than other closing laws. The result obtained has also allowed to examine the behavior of the pressure during the closing period, extending the work to the beginning of the valve operation.

The pressure wave at $0.3 < m < 8$ (convex functions) show very moderate oscillations during closing and the pressure reaches the first peak value at t = τ

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