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Full Length Research Article

DYNAMIC RESPONSE ANALYSIS OF A UNIFORM CONVEYING FLUID PIPE ON TWO – PARAMETER ELASTIC FOUNDATION

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ABSTRACT

In this study, the Dynamic Response of a Fluid Conveying Pipe on a Pasternak Foundation is investigated using a Galerkin finite element method. Several factors affecting the vibration of the conveying pipe such as foundation stiffness, length of pipe, fluid density have been studied extensively. From results obtained, it shows that foundation and length of the pipe affects the mechanical behaviour of fluid conveying pipe.

Keywords: Vibration; Deflection; Elastic; Pipe; Mass Matrix

INTRODUCTION

The technology of conveying fluid, such as petroleum liquids and waters through long or slender pipelines, which cover different types of foundation, has evolved over the years. The speed of this liquid or fluid in the pipeline has impart energy to the pipeline making it to vibrate. The vibration of the conveying pipe can put pressures on the walls of the pipe resulting on the pipe to deflect. Therefore, having understanding knowledge of conveying pipe structure on an elastic foundation, have enable conveying pipes on some type of foundation that can handle certain type of displacement and load so as to increase the structure life span. There are a lot of literatures problem of conveying fluid and numerical method of solving them as Mohamed et al (2016) used finite element method to examine the stability of a fluid conveying pipe. Effect of spring location and linear spring was examined on the pipe; Kesimli et al (2016) used multiple scale method to analyze linear vibration of a fluid conveying pipe with clamped end support. Hamilton principle was used to obtain the equation of motion of the system. The velocity of the fluid was assumed to change harmonically. The results were simulated graphically to show the effect of certain parameters on the vibration of the pipe. In a Similar manner Ghaitani et al (2014) studied the Effect of Elastic Foundation on bending behaviour of oil pipeline. The governing equation considered was derived based on Stress – strain and Strain – displacement relation coupled with Hamilton method. The foundation type considered in this study is Pasternak foundation subjected to lateral load on the oil pipeline. Olayiwola (2016) investigate static and Dynamic response of a fluid conveying pipe that is harmonically excited resting on a viscoelastic foundation. Method of integral transform and complex variable function is employed to obtain the analytical solution of the model equation. Effects of damping force, Coriolis force and elastic medium are investigated, Wang & Ni (2009) Review some works on Dynamic of Slender structures subjected to axial flow or towed on quiescent fluid. They show that fundamental understanding and methodology used in the studies of Dynamical model of fluid – conveying pipe has proved to be very useful in the study of other dynamical models such as shells conveying or immersed on axial flow, nanotube conveying tube etc. Others include Chellapilla & Simha (2008), Kulper & Melnkine (2004), Liu & Xuan (2010), Mostapha (2014), Inuwa (2017), Haos (2012) etc.

MATERIALS AND METHODS

The problem to be considered is the Vibration Analysis of Conveying Pipe carrying Fluid on a Pasternak Elastic foundation. The derivation of the equation is based on Bernoulli – Euler elementary beam theory. The physical model of a fluid conveying pipe system is shown in Figure 1; Figure 2 shows forces on fluid element while Figure 3 shows forces and moment of pipe element.

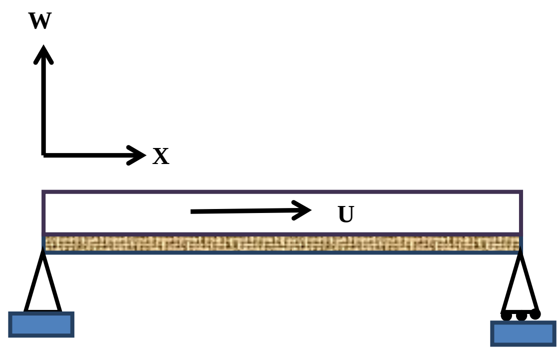


Figure 1: showing a simply supported pipe conveying fluid on a Pasternak foundation

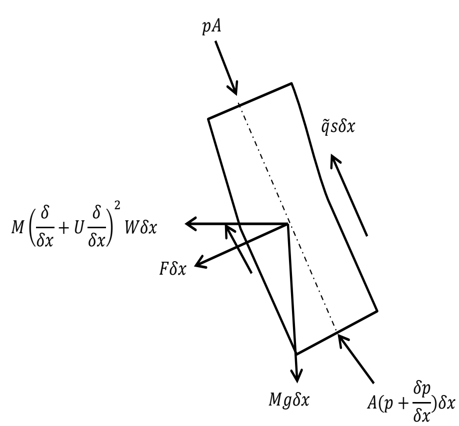


Figure 2: Forces on the Fluid Element

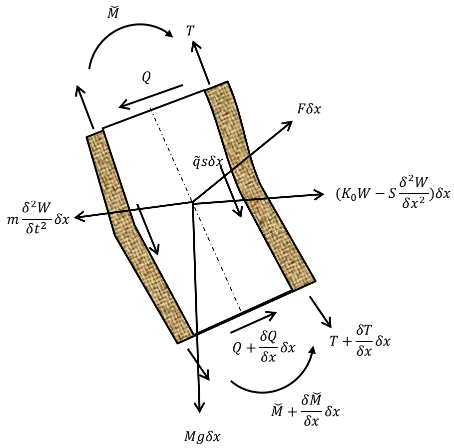


Figure 3: Forces and Moment on Pipe Element

The equation for Conveying Pipe carrying Fluid on a Pasternak Elastic foundation is given as:

$$EI\frac{\partial^4 w}{\partial x^4} + k_o w - s\frac{\partial^2 w}{\partial x^2} + MU^2\frac{\partial^2 w}{\partial x^2} + 2MU\frac{\partial^2 w}{\partial x \partial t} + (m+M)\frac{\partial^2 w}{\partial t^2} = \mathrm{F(x,t)} \quad (1)$$

Initial condition

$$\left.\begin{array}{l} w(x,0) = 0 \\ \dfrac{\partial w(x,0)}{\partial t} = 0 \end{array}\right\} \quad (2)$$

And Boundary condition

$$\left.\begin{array}{l} w(0,t) = 0 \\ w(L,t) = 0 \\ EI\dfrac{\partial^2 w(x,t)}{\partial x^2} - M_p = 0 \\ EI\dfrac{\partial^3 w(x,t)}{\partial x^3} - V_P = 0 \end{array}\right\} \text{at } x = 0 \;\&\; x = L \quad (3)$$

Where M_p and V_P are the prescribed moment and shear forces respectively.

Let L be the length of the conveying pipe. Through the discretization of the conveying pipe of length L into n elements of length ℓ, the displacement function $W(x,t)$ can be express as:

$$W(x,t) = \sum_{i=1}^{n} N_i(x) q_i(t) \quad (4)$$

Where;

$N_i(x)$ represent the shape function of the bending

$q_i(x)$ is the function which represents the shape of the displacements and rotations at nodes.

n is the number of degree of freedom. In this work the degree of freedom is 4

Therefore, equation (4) becomes

$$W(x,t) = \sum_{i=1}^{4} N_i(x) q_i(t) \;\; ,\mathrm{i} = 1,2,3,4 \quad (5)$$

$$= N_1(x)q_1(t) + N_2(x)q_2(t) + N_3(x)q_3(t) + N_{41}(x)q_4(t) \quad (6)$$

$$= \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} \quad (7)$$

Applying finite element method to the governing partial differential equation obtained in (1) and *F(x, t) =0* we have

$$\int_0 [\mathrm{N_i}]^{\mathrm{T}} \begin{pmatrix} \mathrm{EI}\dfrac{\partial^4 \mathrm{w}}{\partial \mathrm{x}^4} + \mathrm{k_o w} - \mathrm{s}\dfrac{\partial^2 \mathrm{w}}{\partial \mathrm{x}^2} + \mathrm{MU}^2\dfrac{\partial^2 \mathrm{w}}{\partial \mathrm{x}^2} + 2\mathrm{MU}\dfrac{\partial^2 \mathrm{w}}{\partial \mathrm{x}\partial \mathrm{t}} + \\ (\mathrm{m}+\mathrm{M})\dfrac{\partial^2 \mathrm{w}}{\partial \mathrm{t}^2} \end{pmatrix} \mathrm{dx} = 0 \quad (8)$$

Simplifying further, we have

$$Aq + Bq + Cq + Dq + E\,\dot{q} + F\,\ddot{q} = 0 \quad (9)$$

The individual matrices can be represented as follows:

$$A = \frac{2EI}{l^3}\begin{bmatrix} 6 & 3l & -6 & 3l \\ 3l & 2l^2 & -3l & l^2 \\ -6 & -3l & 6 & -3l \\ 3l & l^2 & -3l & 2l^2 \end{bmatrix},$$

$$B = \frac{k_o l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix},$$

$$C = \frac{s}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ 36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix},$$

$$D=\frac{MU^2}{30l}\begin{bmatrix}36 & 3l & -36 & 3l\\ 3l & 4l^2 & -3l & -l^2\\ 36 & -3l & 36 & -3l\\ 3l & -l^2 & -3l & 4l^2\end{bmatrix},$$

$$E=\frac{MU}{30}\begin{bmatrix}-30 & -6l & 30 & 6l\\ 6l & 0 & -6l & l^2\\ 30 & 6l & 30 & -6l\\ -6l & -l^2 & 6l & 0\end{bmatrix},$$

$$F=\frac{(m+M)l}{420}\begin{bmatrix}156 & 22l & 54 & -13l\\ 22l & 4l^2 & 13l & -3l^2\\ 54 & 13l & 156 & -22l\\ -13l & -3l^2 & -22l & 4l^2\end{bmatrix}$$

Then, we may write

$$F\ddot{q}+E\dot{q}+(A+B+C+D)q=0 \qquad (10)$$

This is a second order differential equation with respect to time (t) representing one element.

For k number of elements, we have

$$\sum_{i=1}^{k}F\,\ddot{q}+E\,\dot{q}+(A+B+C+D)\,q=0 \qquad (11)$$

The finite difference formula for the Newmark Beta Scheme is

$$\{\ddot{q}_{t+\Delta t}\}=\frac{1}{\beta\Delta t^2}(q_{t+\Delta t}-q_t)-\frac{1}{\beta\Delta t}\dot{q}_t-(\frac{1}{2\beta}-1)\ddot{q}_t \qquad (12)$$

$$\{\dot{q}_{t+\Delta t}\}=\frac{\alpha}{\beta\Delta t}(q_{t+\Delta t}-q_t)-\left(\frac{\alpha}{\beta}-1\right)\dot{q}_t-\Delta t(\frac{\alpha}{2\beta}-1)\ddot{q}_t \qquad (13)$$

The equation of motion (11) can be solved to obtain the displacement, velocity and acceleration at time $t+\Delta t$

By substituting (12) and (13) into (11), we have

$$[\overline{\mathrm{M}}]\{C_{t+\Delta t}\}=[\overline{\mathrm{F}_{t+\Delta t}}] \qquad (14)$$

Where the Effective mass matrix is

$$[\overline{\mathrm{M}}]=\frac{1}{\beta\Delta t^2}[\mathrm{M}]+\frac{\alpha}{\beta\Delta t}[\mathrm{D}]+[\mathrm{K}] \qquad (15)$$

$$[\overline{\mathrm{F}_{t+\Delta t}}]=[\mathrm{F}_{t+\Delta t}]+\left[\left(\frac{1}{2\beta}-1\right)[\mathrm{M}]+\Delta t\left(\frac{\alpha}{2\beta}-1\right)[\mathrm{q}]\right]\{\ddot{q}\}_t+\left[\frac{1}{\beta\Delta t}[\mathrm{M}]+\left(\frac{\alpha}{\beta}-1\right)[\mathrm{q}]\right]\{\dot{q}_t\}+\left[\frac{1}{\beta\Delta t^2}[\mathrm{M}]+\frac{\alpha}{\beta\Delta t}[\mathrm{q}]\right]\{q_t\}$$

(16)

RESULTS AND DISCUSSION

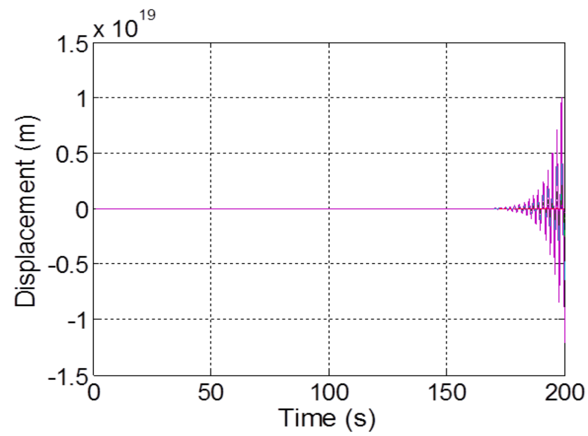
MATLAB software was used to obtain the Global system mass matrix, Global system stiffness matrices and Global system damping matrix. To investigate Dynamic Response of a Fluid Conveying Pipe on a Pasternak Foundation the following material data are adopted. The computation is carried out by using 50 elements.

$L=2m$,

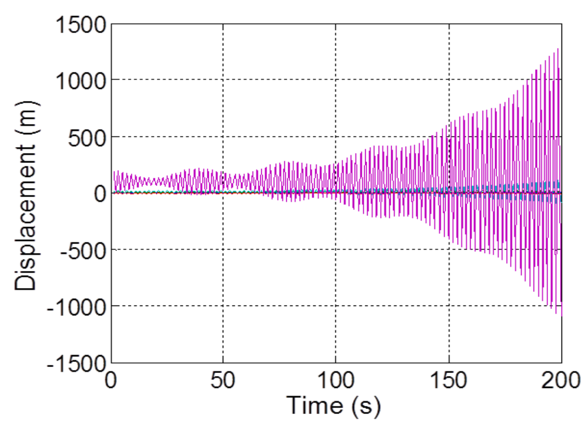
$l=0.04m$

$n=L/l=50\ elements$,

$E=2\times10^5\ N/m^2$,



(a)

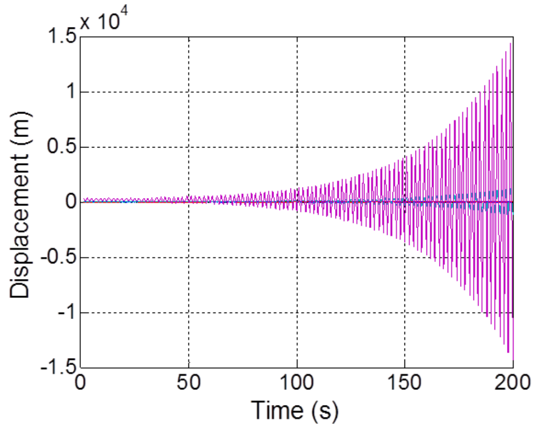


(b)

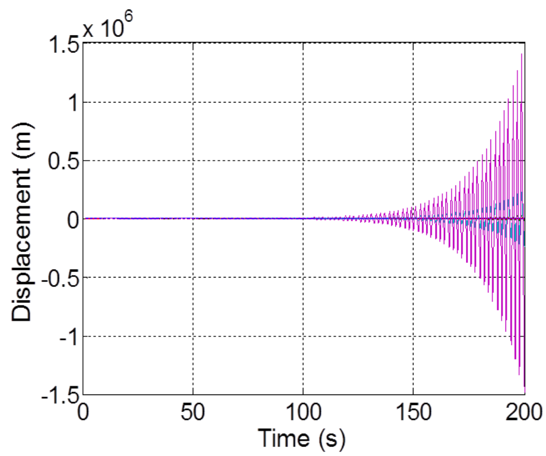
Figure 4: Effect of Velocity on Deflection of the Pipe at

(a) $V=0m/s$, $L=2m$ and $k=1000$

(b) $V=30m/s$, $L=2m$ and $k=1000$

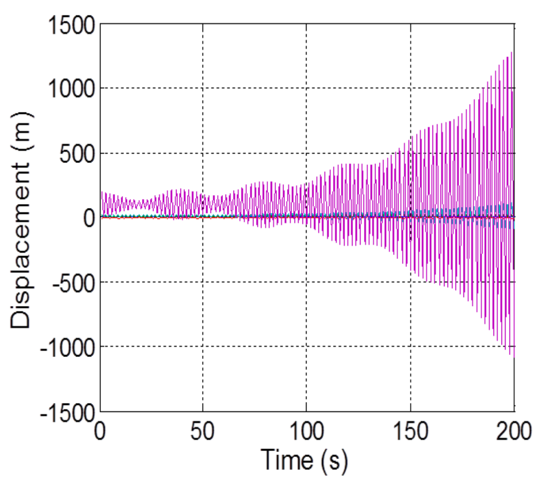


(a)

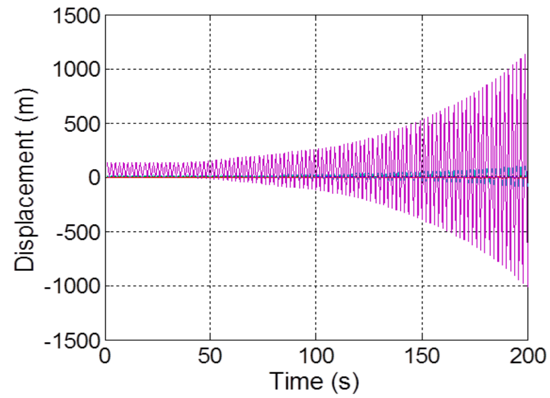


(b)

Figure 5: Effect of Length of Pipe on Deflection of the Pipe at
(a) $V = 20m/s$, $L = 2m$ and $k = 1000$
(b) $V = 20m/s$, $L = 4m$ and $k = 1000$



(a)



(b)

Figure 6 Effect of Stiffness of Foundation on Deflection of the Pipe at
(a) $V = 30m/s$, $L = 2m$ and $k = 1000$
(b) $V = 30m/s$, $L = 2m$ and $k = 3000$

In Figure 4 (a) when the velocity of the fluid is at $V = 0m/s$, the deflection was small at zero second and gradually increases at around 160s, (b) $V = 30\,m/s$, the deflection at around zero second and increase in both direction. In Figure 5 show the effect of Length of pipe on the deflection of the pipe (a) when the length of the pipe is $L = 2m$ the vibration was higher from the earlier seconds and gradually increasing while in (b) $L = 4m$, the vibration was smaller compare to when it was $L = 2m$. Figure 6 show the effect of foundation stiffness of the deflection of the pipe. Figure 6 (a) $k = 1000$ show a higher unstable vibration of the pipe as compare to (b) when $k = 3000$. The shows that foundation stiffness affects the deflection of the pipe are the stiffness increases.

Conclusion

This study is performed to evaluate the effect of the following parameters on the dynamic behavior of the pipe: Velocity of the fluid; Length of pipe; Stiffness of foundation.
From the study made in this research work, we conclude that:

1. The Velocity of the Fluid affect the deflection of the pipe
2. As the Length of the pipe increased, the deflection of the pipe is reduced at a constant velocity
3. The Coefficient of stiffness of foundation affects greatly the vibration of the fluid conveying pipe.

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