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Explicit Pre A*-algebra

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ABSTRACT
This manuscript is a study on Birkhoff centre of a Pre-A*-algebra. In fact, it is proved that Birkhoff centre of a Pre A*-algebra is also a Pre A*-algebra and identified that the centre of Birkhoff centre of a Pre A*-algebra is a Boolean algebra.

Keywords: Pre A*-algebra, Centre, Birkoff centre, Boolean algebra.

AMS Mathematics subject classification (2000): 03G25(03G05, 08G05).

1. INTRODUCTION

The notions of lattice concerned aspects were detailed conferred by Birkhoff (1948). In an outline script Manes (1993) initiated the perception of Ada, based on C-algebras by Fernando and Craig (1990).

Chandrasekhararao et al. (2007) bring in the impression Pre A*-algebra $(A, \wedge, \vee, (-)^{\sim})$ akin to C-algebra as a reduct of A*- algebra. Venkateswararao et al. (2009) added the structural compatibility of Pre A*-Algebra with Boolean algebra. Further, Satyanarayana and Venkateswararao (2011) ascertained the thought of ideals of Pre A*-algebras. Boolean algebra depends on two element logic. C-algebra, Ada, A*- algebra and our Pre A*-algebra are standard expansions of Boolean logic to 3 truth values, anywhere the third truth value indicates an undefined one.

We recognize the Birkhoff Centre of a Pre A*-algebra and attest various associated results as well. Swamy and Murti (1981) initiated the perception of centre of a C-algebra and bear out that it is a Boolean algebra through induced operations. Furthermore, Swamy and Pragathi (2003) initiated the observation of Birkhoff's centre of a semigroup and extended the above concept for a general semigroup and proved that the Birkhoff's centre of any semigroup is a relatively complemented distributive lattice.

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Let us summon up with the objective of, if S is a semigroup and there exists 0, 1 such that x 0 = 0 x = 0 and 1 x = x for all x belongs to S, then S is named a semigroup with 1. An element a in S is referred as a Birkhoff central element of S if there exit semigroups say S_1 and S_2 with 0 and 1 an isomorphism S onto $S_1 \times S_2$ which maps a onto (0, 1). The set with Birkhoff central elements of S is referred to be Birkhoff centre of S. This conception is extended to a Pre A*-algebra with 1 and attested that the set of all central elements of a Pre A*algebra with 1 is a Pre A*-algebra.

1. Preliminaries

1.1. Definition (Chandrasekhararao et al., 2007**):**

An algebra $(A, \wedge, \vee, (-)^{\sim})$ where A is a non-empty set with 1; $\wedge, \vee$ are binary operations and $(-)^{\sim}$ is a unary operation on A satisfying:

(a) $x^{\sim\sim} = x$ for all x in A

(b) $x \wedge x = x$ for all x in A

(c) $x \wedge y = y \wedge x$ for all x in A

(d) $(x \wedge y)^{\sim} = x^{\sim} \vee y^{\sim}$ for all x, y in A

(e) $x \wedge (y \wedge z) = (x \wedge y) \wedge z$ for all x, y, z in A

(f) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ for all x, y, z in A

(g) $x \wedge y = x \wedge (x^{\sim} \vee y)$ for all x, y in A is called a Pre A*-algebra.

1.1. Example (Chandrasekhararao et al., 2007**):**

The set **3** = {0, 1, 2} by means of operations $\wedge, \vee, (-)^{\sim}$ defined below is a Pre A*-algebra.

$\wedge$	0	1	2	$\vee$	0	1	2	x	$x^{\sim}$
0	0	0	2	0	0	1	2	0	1
1	0	1	2	1	1	1	2	1	0
2	2	2	2	2	2	2	2	2	2

1.1. Note (Chandrasekhararao et al., 2007**):** The above example make sense the following:

(a) $2^{\sim} = 2$. (The only element in the set **3** with this property)

(b) $1 \wedge x = x$ for all $x \in$ **3.** (1 is the meet ($\wedge$) identity in **3**).

(c) $0 \vee x = x$ for all $x \in$ **3.** (0 is the join ($\vee$) identity in **3**).

(d) $2 \wedge x = 2 \vee x = 2$ for all $x \in$ **3**.

1.2. Example (Chandrasekhararao et al., 2007)**:**

2 = {0, 1} with operations $\wedge$, $\vee$, $(-)^{\sim}$ defined below is a Pre A*-algebra.

$\wedge$	0	1	$\vee$	0	1	x	$x^{\sim}$
0	0	0	0	0	1	0	1
1	0	1	1	1	1	1	0

1.2. Definition (Satyanarayana and Venkateswararao, 2011**):**

Let A be a Pre A*-algebra. An element $x \in A$ is described as a central element of A if $x \vee x^{\sim}$ and the set $\{x \in A / x \vee x^{\sim}=1\}$ of all central elements of A is referred the centre of A, denoted B (A).

1.1. Theorem (Satyanarayana and Venkateswararao, 2011):

Let A be a Pre A*-algebra with 1. Subsequently, B (A) is a Boolean algebra in the midst of the operations $\wedge$, $\vee$, $(-)^{\sim}$.

1.1. Lemma (Satyanarayana and Venkateswararao, 2011**)**:

Every Pre A*-algebra with 1 satisfies the following:

(a) $x \vee 1 = x \vee x^{\sim}$. (b) $x \wedge 0 = x \wedge x^{\sim}$.

1.2. Lemma (Satyanarayana and Venkateswararao, 2011):

Every Pre A*-algebra by means of 1 satisfies the following:

(a) $x \wedge (x^{\sim} \vee x) = x \vee (x^{\sim} \wedge x) = x$ (b) $(x \vee x^{\sim}) \wedge y = (x \wedge y) \vee (x^{\sim} \wedge y)$

(c) $(x \vee y) \wedge z = (x \wedge z) \vee (x^{\sim} \wedge y \wedge z)$

1.3. Definition (Satyanarayana and Venkateswararao, 2011**):**

Let A be a Pre A*-algebra. An element x in A is a central element of A if $x \vee x^{\sim}=1$ and the set $\{x \in A / x \vee x^{\sim}=1\}$ of all central elements of A is referred the centre of A denoted B (A).

1.4. Note (Venkateswararao et al., 2009): If A is a Pre A*- algebra with 1, then 1, 0 are in B (A). If the centre of a Pre A*- algebra concurs with {0, 1}, then we declare that A has trivial centre.

1.2. Theorem (Venkateswararao et al., 2009)**:**

Let A be a Pre A*-algebra with 1. Then B(A) is a Boolean algebra by means of the operations $\wedge$, $\vee$, $(-)^{\sim}$.

2. BIRKHOFF'S CENTRE

In this segment, we describe Birkhoff centre of a Pre A*- algebra, in addition we shall bear out assorted properties.

2.1. Definition:

Let A be a Pre A*- algebra with meet identity. An element $a \in A$ is assumed to be Birkoff central element of a Pre A*- algebra A if there exist Pre A*- algebras A_1 and A_2 with 1 (meet ($\wedge$) identity) and an isomorphism $f: A \rightarrow A_1 \times A_2$ such that $f(a) = (1_1, 0_2)$. (Where 1_1, is the meet identity in A_1 and 0_2 is the join ($\vee$) identity in A_2 in that order).

2.2. Definition:

The set of all Birkhoff central elements of a Pre A*- algebra A is described Birkoff centre of A and denoted BC (A).

2.1. Lemma:

Let A be a Pre A*- algebra with meet identity. Then for each element $a \in BC(A)$ entails $a^{\sim} \in BC(A)$.

Proof: Let a be an element in BC (A). Then there subsist Pre A*- algebras A_1 and A_2; and an isomorphism $f: A \rightarrow A_1 \times A_2$ such that $f(a) = (1_1, 0_2)$.

Now, define $g: A \rightarrow A_2 \times A_1$ such that $g(x) = (x_2, x_1)$ whenever $f(x) = (x_1, x_2)$.

Let $x, y \in A$ such that $f(x) = (x_1, x_2)$ and $f(y) = (y_1, y_2)$.

At that moment, $f(x \wedge y) = (x_1 \wedge y_1, x_2 \wedge y_2)$, as f is a homomorphism.

Consequently we have,
$$g(x \wedge y) = (x_2 \wedge y_2, x_1 \wedge y_1) = (x_2, x_1) \wedge (y_2, y_1) = g(x) \wedge g(y).$$

In the same way, we can provide evidence that, $g(x \vee y) = (x) \vee g(y)$.

To substantiate that g (x˜) = [g(x)]˜.

Let f (x) = (x_1, x_2). Then g (x) = (x_2, x_1).

Regard as, f (x) = (x_1, x_2). This entails, (f (x))˜ = $(x_1, x_2)^{\sim}$ = ($x_1^{\sim}$, $x_2^{\sim}$).

Hence we have, f ($x^{\sim}$) = ($x_1^{\sim}$, $x_2^{\sim}$). (Since f is a homomorphism).

As a result, g(x˜) = ($x_2^{\sim}$, $x_1^{\sim}$) = $(x_2, x_1)^{\sim}$ = (g(x))˜. Therefore, g is a homomorphism.

Besides reflect on, f (a˜) = (f(a))˜ = $(1_1, 0_2)^{\sim}$ = ($1_1^{\sim}$, $0_2^{\sim}$) = (0_1, 1_2).

Subsequently we be obliged to have that g(x˜) = (1_2, 0_1).

In view of the fact that g is defined as so is f and as f is a bijection, then clearly so is g.

Therefore, g is an isomorphism.

So we categorize x˜is a Birkhoff's central element. Hence, x˜ ∈ BC (A).

2.2. Lemma:

Let t be an element in a Pre A*- algebra A. Then t A = {t ∧ α / α ∈ A} is a Pre A*- algebra by the induced operations ∧ and ∨ of A and the unary operation defined by $(t \wedge \alpha)^{*} = t \wedge \alpha^{\sim}$.

Proof: Given that t A= { t ∧ a / $a \in$ A}.

Let us choose the elements, t ∧ x, t ∧ y, t ∧ z from the set t A, where x, y, z are in A.

(1) Reflect on, (t ∧ x) ** = (t ∧ x˜)* = t ∧ x ˜˜ = t ∧ x.

As a result, (t ∧ x) ** = t ∧ x.

(2) Regard as, (t ∧ x) ∧ (t ∧ x) = $t \wedge$(x ∧ x) = t ∧ x.

Therefore, (t ∧ x) ∧ (t ∧ x) = t ∧ x.

(3) Mull over, (t ∧ x) ∧ (t ∧ y) = t ∧ (t ∧ y) = t ∧ (y ∧ x) = (t ∧ y) ∧ (t ∧ x).

Therefore, (t ∧ x) ∧ (t ∧ y) = (t ∧ y) ∧ (t ∧ x).

(4) Consider, ((t ∧ x) ∧ (t ∧ y))* = (t ∧ (x ∧ y))* = t ∧ (x ∧ y) ˜ = t ∧ (x ˜ ∨ y ˜)

= (t ∧ x ˜) ∨ (t ∧ y ˜) = (t ∧ x)* ∨ (t ∧ y)*.

Therefore, ((t ∧ x) ∧ (t ∧ y))* = (t ∧ x)* ∨ (t ∧ y)*.

(5) Consider, (t ∧ x ∧((t ∧ y) ∧ (t ∧ z)) = $t \wedge (x \wedge (y \wedge z)) = t \wedge ((x \wedge y) \wedge z)$

= ((t ∧ x) ∧ (t ∧ y)) ∧ (t ∧ z).

Therefore, (t ∧ x) ∧ ((t ∧ y) ∧ (t ∧ z)) = ((t ∧ x) ∧ (t ∧ y)) ∧ (t ∧ z).

(6) Consider, (t ∧ x) ∧((t ∧ y) ∨ (t ∧ z)) = (t ∧ x) ∧(t ∧ (y ∨ z))

$= t \wedge (x \wedge (y \vee z)) = t \wedge ((x \wedge y) \vee (x \wedge z))$

$= (t \wedge (x \wedge y)) \vee (t \wedge (x \wedge z))$

$= ((t \wedge x) \wedge (t \wedge y)) \vee ((t \wedge x) \wedge (t \wedge z))$.

Therefore, $(t \wedge x) \wedge ((t \wedge y) \vee (t \wedge z)) = ((t \wedge x) \wedge (t \wedge y)) \vee ((t \wedge x) \wedge (t \wedge z))$.

(7) Consider, $(t \wedge x) \wedge ((t \wedge x)^* \vee (t \wedge y))$

$= (t \wedge x) \wedge ((t \wedge x^{\sim}) \vee (t \wedge y))$

$= (t \wedge x) \wedge (t \wedge (x^{\sim} \vee y))$

$= t \wedge (x \wedge (x^{\sim} \vee y))$

$= t \wedge (x \wedge y)$

$= (t \wedge x) \wedge (t \wedge y)$.

As a result, $(t \wedge x) \wedge ((t \wedge x)^* \vee (t \wedge y)) = (t \wedge x) \wedge (t \wedge y)$.

As a consequence, $(tA, \wedge, \vee, {}^*)$ is a Pre A*-algebra.

2.3. Lemma: BC (A) is a Pre A*- algebra**.**

Proof: Let a, b be any elements from BC (A). Then there exists Pre A*- algebras A_1, A_2 and A_3, A_4 with 1 and isomorphisms f: $A \to A_1 \times A_2$ such that f (a) = $(1_1, 0_2)$ and g: $A \to A_3 \times A_4$ such that g (b) = $(1_3, 0_4)$.

Now, we have to prove that $a \wedge b$ is an element in BC (A). That is, we have to find an isomorphism h: $A \to A_5 \times A_6$ such that h $(a \wedge b) = (1_5, 0_6)$ (where $1_5 \in A_5$ and $0_6 \in A_6$).

Suppose that g (a) = (t_3, t_4), where. $t_3 \in A_3$ and $t_4 \in A_4$.

Define, $A_5 = t_3 A_3$ where t_3 $(=1_3)$ is a meet identity in A_3 and $t_3 \wedge t_3^{\sim}$ $(= 0_3)$ is a join identity and $t_3 A_3 = \{t_3 \wedge \alpha / \alpha \in A_3\}$.

As a result of lemma 2.2, $t_3 A_3$ is a Pre A*- algebra with $1_5 = t_3$ (meet identity in A_5) and join identity 0_5 $(= t_3 \wedge t_3^{\sim} = 0_3)$.

In addition, define, $A_6 = t_4 A_4 \times A_2$. Then A_6 is also a Pre A*- algebra in the company of meet identity $1_6 = (t_4, 1_2)$ $(= 1_4, 1_2)$, and join identity $0_6 = (t_4 \wedge t_4^{\sim}, t_2 \wedge t_2^{\sim})$ $(= (0_4, 0_2)$.

Note that $0_2 = t_2 \wedge t_2^{\sim} = 0_2 \wedge t_2$).

For any x in A, let f (x) = (s_1, s_2) and g (x) = (x_3, x_4) where $s_1 \in A_1$, $s_2 \in A_2$ and $x_3 \in A_3$, $x_4 \in A_4$.

Define h: $A \to A_5 \times A_6$ by h (x) = $(t_3 \wedge x_3, ((t_4 \wedge x_4), s_2))$ for any $x \in A$.

Let f (y) = (r_1, r_2) and g (y) = (y_3, y_4).

Subsequently, $f(x \wedge y) = (s_1 \wedge r_1, s_2 \wedge r_2)$, $g(x \wedge y) = (x_3 \wedge y_3, x_4 \wedge y_4)$, $f(x^{\sim}) = (s_1^{\sim}, s_2^{\sim})$ and $g(x^{\sim}) = (x_3^{\sim}, x_4^{\sim})$ as f and g are isomorphisms.

Consider, $h(x \wedge y) = (t_3 \wedge x_3 \wedge y_3), (t_4 \wedge x_4 \wedge y_4, s_2 \wedge r_2))$

$= (t_3 \wedge x_3 \wedge t_3 \wedge y_3), (t_4 \wedge x_4 \wedge t_4 \wedge y_4, s_2 \wedge r_2))$

$= (t_3 \wedge x_3, (t_4 \wedge x_4, s_2)) \wedge (t_3 \wedge y_3, (t_4 \wedge y_4, r_2))$

$= h(x) \wedge h(y)$.

Now consider, $h(x^{\sim}) = (t_3 \wedge x_3^{\sim}, (t_4 \wedge x_4^{\sim}, s_2^{\sim}))$ (since $(t_3 \wedge x_3)^* = t_3 \wedge x_3^{\sim}$)

$= (x_3^*, (x_4*, s_2^{\sim}))$

$= (h(x))^{\sim}$.

Consider, $h(x \vee y) = (t_3 \wedge (x_3 \vee y_3), (t_4 \wedge (x_4 \vee y_4, s_2 \vee r_2))$

$= ((t_3 \wedge x_3) \vee (t_3 \wedge y_3), ((t_4 \wedge x_4) \vee (t_4 \wedge y_4), s_2 \vee r_2))$

$= (t_3 \wedge x_3, (t_4 \wedge x_4, s_2)) \vee (t_3 \wedge y_3, (t_4 \wedge y_4, r_2))$

$= h(x) \vee h(y)$.

In view of that, h is a homomorphism.

To show h is injective, first we prove $h(a \wedge b) = (1_5, 0_6)$. $(\in A_5 \times A_6 = t_3 A_3 \times (t_4 A_4 \times A_2)))$.

Note that $1_5 \in A_5 = t_3 A_3 = \{t_3 \wedge \alpha / \alpha \in A_3\}$ and $0_6 \in A_6 = t_4 A_4 \times A_2$ $= \{t_4 \wedge \alpha / \alpha \in A_4\} \times A_2$, where 1_5 is the meet identity in $t_3 A_3$ and 0_6 is the join identity in $t_4 A_4 \times A_2$.

We have, $f(a) = (1_1, 0_2)$, $g(a) = (t_3, t_4)$ $(= (1_3, t_4))$, $g(b) = (1_3, 0_4)$, $f(b) = (t_1, t_2)$.

Now consider, $h(a \wedge b) = h(a) \wedge h(b)$ (since h is a homomorphism)

(as $h(a), h(b) \in A_5 \times A_6 = t_3 A_3 \times t_4 A_4 \times A_2$ and a, b are Birkoff central elements of A)

$= (t_3 \wedge t_3, (t_4 \wedge t_4, 0_2)) \wedge (t_3 \wedge 1_3, (t_4 \wedge 0_4, t_2))$

$= (t_3, (t_4 \wedge 0_4, 0_2 \wedge t_2))$

$= (t_3, (t_4 \wedge 0_4, t_2 \wedge t_2^{\sim}))$

$= (t_3, (t_4 \wedge t_4^{\sim}, 0_2))$

(by lemma 1.1(b), we have, $x \wedge 0 = x \wedge x^{\sim}$ and $t_2 \wedge t_2^{\sim}$ defined as 0_2)

$= (1_5, 0_6)$.

Let $x, y \in A$ such that $h(x) = h(y)$. To prove that $x = y$.

Then $t_3 \wedge x_3 = t_3 \wedge y_3$ and $t_4 \wedge x_4 = t_4 \wedge y_4$ and $s_2 = r_2$. In order to prove $x = y$ we require to prove $s_1 = r_1$ and $s_2 = r_2$. So it suffixes to prove $s_1 = r_1$ as already we have $s_2 = r_2$.

Now consider $g(a) \wedge g(x) = (t_3, t_4) \wedge (x_3, x_4)$

$= (t_3 \wedge x_3, t_4 \wedge x_4)$

$= (t_3 \wedge y_3, t_4 \wedge y_4)$

$= g(a) \wedge g(y)$.

Since g is a homomorphism, $g(a \wedge x) = g(a \wedge y)$.

This implies, $a \wedge x = a \wedge y$ (since g is one-one).

Subsequently, $f(a \wedge x) = f(a \wedge y)$ (since f is well defined).

Hence, $f(a) \wedge f(x) = f(a) \wedge f(y)$ (since f is a homomorphism).

This leads to, $(1_1, 0_2) \wedge (s_1, s_2) = (1_1, 0_2) \wedge (r_1, r_2)$.

Hence, $(1_1 \wedge s_1, 0_2 \wedge s_2) = (1_1 \wedge r_1, 0_2 \wedge r_2)$.

This implies, $(s_1, 0_2 \wedge s_2) = (r_1, 0_2 \wedge r_2)$.

Thus, $s_1 = r_1$ and $s_2 = r_2$ (already in the above we have, $s_2 = r_2$)

Therefore, $(s_1, s_2) = (r_1, r_2)$.

So, $f(x) = f(y)$.

This implies, $x = y$ (since f is one-one).

Therefore, h is one – one.

Let $(x, y) \in A_5 \times A_6$. Then $(x, y) = (t_3 \wedge x_3, (t_4 \wedge x_4, s_2))$ for some $x_3 \in A_3$, $x_4 \in A_4$ and $s_2 \in A_2$.

Since $t_3 \wedge x_3 \in t_3 A_3 \subseteq A_3$, $(t_3 \wedge x_3, t_4 \wedge x_4) \in A_3 \times A_4$ and g is onto, there exist $t \in A$ such that $g(t) = (t_3 \wedge x_3, t_4 \wedge x_4)$.

Now $g(a \wedge t) = g(a) \wedge g(t)$

$= (t_3, t_4) \wedge (t_3 \wedge x_3, t_4 \wedge x_4)$

$= (t_3 \wedge t_3 \wedge x_3, t_4 \wedge t_4 \wedge x_4)$

$= (t_3 \wedge x_3, t_4 \wedge x_4)$

$= g(t)$

Therefore, $g(a \wedge t) = g(t)$ --- 1

This implies. $a \wedge t = t$ (since g is one-one).

Hence, $f(a \wedge t) = f(t)$ (since f is well defined).

Then we have, $f(a) \wedge f(t) = f(t)$ (since f is a homomorphism).

This leads to $(1_1, 0_2) \wedge (y_1, y_2) = (y_1, y_2)$ (since $t \in A$),

(Here, $f(t) = (y_1, y_2)$, where, $y_1 \in A_1$, $y_2 \in A_2$).

As a result, $(1_1 \wedge y_1, 0_2 \wedge y_2) = (y_1, y_2)$.

Consequently, $(y_1, 0_2 \wedge y_2) = (y_1, y_2)$. -- 2

Therefore, $y_2 = 0_2 \wedge y_2$

Now, by above we observe that, $y_1 \in A_1$, $y_2 \in A_2$. Subsequently, $(y_1, y_2) \in A_1 \times A_2$.

Since f is onto, there exists, say n ∈ A such that $f(n) = (y_1, y_2)$.

Now consider, $f(a \wedge n) = f(a) \wedge f(n)$

$= (1_1, 0_2) \wedge (y_1, y_2)$

$= (1_1 \wedge y_1, 0_2 \wedge y_2)$

$= (y_1, 0_2 \wedge y_2)$

$= (y_1, y_2)$. (by (2))

$= f(t)$.

Since f is one-one, $a \wedge n = t$ and since g is well defined $g(a \wedge n) = g(t)$ ---------------------------- 3

Also, since, n ∈ A we have, $g(n) = (z_1, z_2)$.

This implies, $(t_3 \wedge t_3 \wedge x_3, t_4 \wedge t_4 \wedge x_4) = g(a \wedge t)$

$= g(t)$ (by (1)

$= g(a \wedge n)$ (by (3)

$= g(a) \wedge g(n)$ (since g is a homomorphism)

$= (t_3, x_4) \wedge (z_1, z_2)$

$= (t_3 \wedge z_1, t_4 \wedge z_2)$.

Therefore, $t_3 \wedge t_3 \wedge x_3 = t_3 \wedge z_1$ and $t_4 \wedge t_4 \wedge x_4 = t_4 \wedge z_2$ -- 4

Now consider, $h(n) = (t_3 \wedge z_1, t_4 \wedge z_2, s_2)$

$= (t_3 \wedge t_3 \wedge x_3, (t_4 \wedge t_4 \wedge x_4, s_2))$

$= (t_3 \wedge x_3, (t_4 \wedge x_4, s_2))$ (by (4))

$= (x, y)$.

Therefore, h is onto. Since, $a, b \in BC(A)$ implies $a \wedge b \in BC(A)$ and by lemma 2.1, $a \in BC(A)$ implies $a^{\sim} \in BC(A)$. In addition to this, $a \vee b \in BC(A)$.

As a result, BC(A) is a sub-algebra of a Pre A*- algebra A and for this reason BC(A) is a Pre A*-algebra.

2.1. Note: Let us bring to mind the designation of centre of a Pre A*- algebra [6]. Let BC(A) be a Pre A*- algebra with meet identity 1. Then the centre of BC(A) is defined as the set $B(BC(A)) = \{a \in BC(A) / a \vee a^{\sim} = 1\}$.

One can see that B (BC(A)) is a Boolean algebra under the operations induced by those on BC(A).

2.4. Lemma:

Let $a \in B$ (BC(A)). Then for all x in BC(A), $a \wedge x = a$ if and only if, $a \vee x = x$.

Proof: Suppose that $a \wedge x = a$.

Consider, $a \vee x$

$= (a \wedge x) \vee x$ (since $a \wedge x = a$)

$= [a \wedge (a^{\sim} \vee x)] \vee x$ (by axiom (vii) of definition [1.1], we have, $x \wedge y = x \wedge (x^{\sim} \vee y)$)

$= (a \vee x) \wedge [(a^{\sim} \vee x) \vee x]$

$= (a \vee x) \wedge (a^{\sim} \vee x)$

$= (a \wedge a^{\sim}) \vee x = 0 \vee x = 0.$

Consequently, $a \vee x = x$.

On the other hand presume that $a \vee x = x$.

Consider, $a \wedge x = a \wedge (a \vee x) = a$ (since a in B (BC(A))).

Hence, $a \wedge x = a$.

2.5. Lemma:

Let BC (A) be a Pre A*- algebra and a be an element in B (BC (A)). In case, the set $S_a = \{ x \in$ BC(A) / $a \wedge x = a\}$, then S_a is closed under the operations $\wedge$ and $\vee$. Also for any x in the set S_a, define, $x^* = a \vee x^{\sim}$. Then $(S_a, \wedge, \vee, {}^*)$ is a Pre A*-algebra.

Proof: Let x, y, z be from the set S_a. Then, $a \wedge x = a$ and $a \wedge y = a$, $a \wedge z = a$.

This entails, $a \vee x = x$ and $a \vee y = y$, $a \vee z = z$. (By above lemma [2.4])

Now reflect on, $a \wedge (x \wedge y) = (a \wedge x) \wedge y = a \wedge y = a$.

Hence, $x \wedge y$ belongs to the set S_a.

Also consider, $a \wedge (x \vee y) = (a \wedge x) \vee (a \wedge y) = a \vee a = a$.

This implies, $x \vee y$ is an element in the set S_a.

Consequently, S_a is closed under the operation $\wedge$ and $\vee$.

Reflect on, $a \wedge x^* = a \wedge (a \vee x^{\sim}) = a$ (since a is in B(A)).

This involves, x^* belongs to S_a.

In consequence S_a is closed under $*$.

Now we have the following:

(1) Regard as, $x^{**} = (a \vee x^{\sim})^{*}$

$= a \vee (a \vee x^{\sim})^{\sim} = a \vee (x^{\sim} \wedge x)$

$= (a \vee a^{\sim}) \wedge (a \vee x) = a \vee x$ (as a is a Boolean element, $a \vee a^{\sim} = 1$) $= x$.

For that reason, $x^{**} = x$.

(2) Reflect on, $x \wedge x = (a \vee x) \wedge (a \vee x) = a \vee (x \wedge x) = a \vee x = x$.

As a result, $x \wedge x = x$.

(3) By Considering, $x \wedge y = (a \vee x) \wedge (a \vee y) = (a \vee y) \wedge (a \vee x) = y \wedge x$.

Accordingly, $x \wedge y = y \wedge x$.

(4) Regard as, $(x \wedge y)^{*} = a \vee (x \wedge y)^{\sim}$

$= a \vee (x^{\sim} \vee y^{\sim}) = (a \vee x^{\sim}) \vee (a \vee y^{\sim}) = x^{*} \vee y^{*}$.

Consequently, $(x \wedge y)^{*} = x^{*} \vee y^{*}$.

(5) Consider, $x \wedge (y \wedge z) = (a \vee x) \wedge \{(a \vee y) \wedge (a \vee z)\}$

$= a \vee \{x \wedge (y \wedge z)\}$

$= a \vee \{(x \wedge y) \wedge z\}$ (since x, y, z are in A)

$= (x \wedge y) \wedge z$.

Thus, $x \wedge (y \wedge z) = (x \wedge y) \wedge z$.

(6) Consider, $x \wedge (y \vee z) = (a \vee x) \wedge \{(a \vee y) \vee (a \vee z)\}$

$= \{(a \vee x) \wedge (a \vee y)\} \vee \{(a \vee x) \wedge (a \vee z)\}$

$= \{a \vee (x \wedge y)\} \vee \{(a \vee (x \wedge z)\}$

$= (x \wedge y) \vee (x \wedge z)$.

Therefore, $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$.

(7) Consider, $x \wedge (x^{*} \vee y) = x \wedge \{ (a \vee x^{\sim}) \vee y\}$

$= \{x \wedge (a \vee x^{\sim}))\} \vee (x \wedge y)$

$= (x \wedge x^{\sim}) \vee (x \wedge y)$ (since $a \vee x = x$)

$= x \wedge (x^{\sim} \vee y)$

$= x \wedge y$.

Therefore, $x \wedge (x^{*} \vee y) = x \wedge y$.

Thus, $(S_a, \wedge, \vee, ^{*})$ is a Pre A*-algebra.

2.6. Lemma:

Let BC (A) be a Pre A*- algebra and $a \in B(BC(A))$. Then, $f_a : BC(A) \rightarrow S_a$ is an anti-homomorphism.

Proof: Let $f_a : BC(A) \rightarrow S_a$ be a mapping defined by $f_a(x) = a \vee x^{\sim}$.

Consider, $f_a(x \wedge y) = a \vee (x \wedge y)^{\sim}$

$= a \vee (x^{\sim} \vee y^{\sim})$

$= (a \vee x^{\sim}) \vee (a \vee y^{\sim})$

$= f_a(x) \vee f_a(y)$.

Therefore, $f_a(x \wedge y) = f_a(x) \vee f_a(y)$.

Again consider, $f_a(x \vee y) = a \vee (x^{\sim} \wedge y^{\sim})$

$= (a \vee x^{\sim}) \wedge (a \vee y^{\sim})$

$= f_a(x) \wedge f_a(y)$.

Therefore, $f_a(x \vee y) = f_a(x) \wedge f_a(y)$.

Finally, consider, $[f_a(x)]^* = (a \vee x^{\sim})^*$

$= a \vee (a \vee x^{\sim})^{\sim}$

$= a \vee (a^{\sim} \wedge x)$

$= a \vee x$

Similarly on the other hand consider, $f_a(x^*) = a \vee (x^*)^{\sim}$

$= a \vee (a \vee x^{\sim})^{\sim}$

$= a \vee (a^{\sim} \wedge x)$

$= a \vee x$

Therefore we must have, $[f_a(x)]^* = f_a(x^*)$.

Therefore, $f_a: BC(A) \rightarrow S_a$ is an anti-homomorphism.

3. CONCLUSION

This study has been endow with the notion of Birkhoff's centre of a Pre A*-algebra and concerned results as well. In fact, it is pragmatic that the set of all Birkhoff's central elements of a Pre A*-algebra, that is; Birkhoff centre of Pre A*-algebra, structures yet again a Pre A*-algebra. Auxiliary, it is acknowledged that the set of all central elements of a Birkhoff centre of Pre A*-algebra shapes again a Boolean algebra. It is identified that centre of the Birkhoff centre of a Pre A*-algebra is a Boolean algebra and any element a of such algebra satisfies $a \wedge x = a$ if and only if $a \vee x = x$ for all x in Birkhoff centre of a Pre A*-algebra. A crucial set $S_a = \{x \in BC(A) / a \wedge x = a\}$, was defined by taking an element a from the Pre A*algebra (the Birkhoff's centre of a Pre A*algebra) and proved that it is also again a Pre A*-algebra. Finally, it is obtained an anti-homorphism between those algebras.

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