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Learning calculus concepts through interactive real-life examples

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Abstract

Calculus concepts are considered the foundation for many theories in our life. Yet, calculus classes internationally face high drop-out rates, failure and negative attitude. This paper presents an action research on a calculus class at one private university in the capital city of the Kingdom of Saudi Arabia. The study aims to show how cooperation, technology and interactive learning (using real-life examples) support students' understanding of calculus concepts and improve their level of enjoyment and appreciation to the subject. Data was collected through questionnaire and focus-group interviews. The results showed that students were enthusiastic about the course. Moreover, students were able to connect calculus concepts to their major and apply them in real-life scenarios. The level of satisfaction was reported to have improved using this approach.

Keywords: calculus; abstract concepts; cooperative learning; motivation

Introduction

Mathematics is known to be the foundation of other subjects. Specifically, calculus is considered the structure of many nowadays concepts or theories. In fact, the discovery of calculus in the seventeenth century is a major shift in many fields including mathematics of course (Lo'pez-Gay, Mart'ınez Sa'ez, Mart'ınez Torregrosa, 2015). For some, its power as a theoretical foundation is one of the most powerful intervention of the mankind. Accordingly, calculus concepts are introduced to students as early as high school in many parts of the worlds. At the college level, calculus is a major requirement for most science, technology, medical and even business programs. To comprehend calculus concepts, students are expected to complete several levels of calculus courses. Depending on the academic program and the mathematical demand of such program, students will be required to complete one or two and up to four levels of calculus.

The Problem

Early levels of calculus introduce the basic concepts behind the differentiation and integration techniques. Students will learn how to solve routine differentiation and integration tasks by using the original definition of the concepts and by using certain algorithms. Applications of calculus concepts or word problems will mostly follow on the subsequent levels or even in other courses. This means that the level of abstraction in the early sequences of calculus courses is high. Abstract concepts are most of the time hard for student to acquire (Kaminski, Sloutsky, & Heckler, 2008). Consequently, some main calculus concepts are hard to comprehend. For example, the basic

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concept of the limit (Rodi, 1986) or the Rate of A change (Herbert, 2013) are not easy for students. Although such concepts have been introduced many many years ago, still many students found them challenging (Baker, Cooley, Trigueros, 2000). Even those who were introduced to calculus in secondary education felt that the level of abstraction at the tertiary level is challenging (Bressoud & Rasmussen, 2015). In fact, Rodi admitted that

> Sitting around in the afternoon with a group of 4 or 5 colleagues discussing subtle points of algebraic topology feels better, is more personally rewarding, and maybe even easier, than worrying and struggling about exposing 18 year olds for the first time to some of history's great ideas like limits and derivative. (1986, p. 118)

Hence, early level of calculus internationally faces high drop-out rates and failures (Bressoud & Rasmussen, 2015; Nobre, 2016). Many students might even switch major or abandon higher education because of calculus classes (Bressoud & Rasmussen, 2015; Cui, Wang, Yang, Nave, & Harris, 2011). In fact, Calculus is known internationally as the Filter. The blame will be on the subject itself and on the teacher who fails to make the rigid concepts interesting for students (Nobre, 2016). Some researchers argued that secondary education is to be blamed for not training students on this level of abstraction (Nobre, 2016). Bressoud and Rasmussen (2015) sadly stated that "Calculus I, as taught in our colleges and universities, is extremely efficient at lowering student confidence, enjoyment of mathematics, and desire to continue in a field that requires further mathematics" (p. 144).

Some students might succeed in completing calculus courses but with limited level of comprehension. This limited level of comprehension affects their success of other related fields (Tokgoz, & Gualpa, 2015). In addition, such limitation translated into the limited ability to only perform manipulations and algorithms and failing to demonstrate the knowledge (Baker et al., 2000) and applying it in real world situations such as physics phenomena (Herbert, 2013). Actually, students themselves assume that the use of differential calculus is limited to the use of algorithms (Lo'pez-Gay et al., 2015). It is greed among many researchers that the values of calculus appear when we solve real-world problem (like physics scenario). However, students even if they are using calculus to solve real-world phenomena tend not to explain why they are using it (Lo'pez-Gay et al., 2015). For example, when solving acceleration or disposition tasks, students cannot sometimes explain the answer or how the results will address the problem they are facing. Moreover, it has been also argued that preformatting calculus manipulations is the easiest part of solving a calculus task. In fact, performing calculus symbolic manipulation is easier than understanding the concept behind it (Doorman & Van maanen, 2008). To make things worse, in the assessment process of early-level calculus courses, most tasks focus on procedural, algorithmic tasks with minimal focus on real-world problems.

Previous Solutions

One of the challenges in calculus classes is that the course is not motivational because students cannot see the value of it. As suggested in the literature—in all classes and not specifically calculus—cooperation and interactive learning might address this issue (Ronfeldt, Farmer, McQueen & Grissom, 2015). In such environment, students depend on each other for help and support more than depending on the teacher. In many cases, such environment will be implemented by placing students in groups to complete certain tasks. This approach helps students improve their teamwork skills (Cui et al., 2011). It was also noted that students tend to learn from each other more than the teacher. They are not afraid or ashamed to talk about their weaknesses. They have the courage to ask even basic or trivial questions because they are not worried to be judged by their classmates (Sankar & Karri, 2016). Collaborative learning has been also found to have positive impact not only on students but also on teachers (Ronfeldt et al., 2015). In other cases, the power of technology is used. Digital collaboration is a new approach that is used in many classrooms (not only mathematics) and at all levels (not only higher education). Scalise (2016) indicated that through digital collaboration, students can gain life-time skills. Embracing such approach supports social network for students not to mention that it empowers their emotional growth. Moreover, the use of software and technology has been supported as a motivational teaching tools to students (Sankar & Karri, 2016; Tay, & Mensah-Wonkyi, 2018). It also has been argued that the use of software foster understanding of mathematical concepts (Cui et al., 2011; Tay & Mensah-Wonkyi, 2018)

Project-based learning is one approach used by instructors to build an effective collaborative learning environment and to foster understanding of abstract concepts (Cui et al., 2011). Through this approach, students build their own knowledge (Sankar & Karri, 2016) and connect the knowledge they are learning to their field (Cui et al., 2011). Adding the presentation approach supports students' reflection of their knowledge. Chang, Cromley, and Tran (2016) suggested that presentations of mathematical abstract concepts help. In fact, multiple representation of concepts and connecting ideas is what has been encouraged in calculus research. Relating to students' life and representing the concepts with familiar examples is another approach to make it interesting, tangible, and enjoyable (Kaminski et al., 2008). It was recognized in the literature that students learn better when they build their own knowledge and are given the opportunity to reflect and apply the knowledge they learned (Bressoud & Rasmussen, 2015).

Especially when we are considering non-math major students, the effect of the level of abstraction in early levels of calculus affect not only the level of comprehension to calculus concepts but also their level of appreciation to the subject (Cui et al., 2011). It is important to realize here that such students lack the basic foundation or theoretical background. Students even if they score good grades, do not see calculus as a meaningful course. Such students are not interested in proofs or theories for example, but more on the technical part. Hence, it is important to show students the beauty of the abstract concepts and connect the theory part to real-world applications.

Study Description

At one private university in the capital city of Saudi Arabia, the teacher of calculus II used a non-traditional approach to engage students. The teacher has been teaching the calculus series for the past seven years to non-math major students. The passing rate over the years has been average and students' grades were normally distributed. However, the teacher was always not happy with the level of satisfaction, engagement and motivation of the students. Even those who completed the course earning good grades, showed in several ways that they really do not appreciate the subject. Hence, during second semester of academic year 2017-2018, the teacher used a different pedagogical approach during the 2nd half of the course. This study aimed to report on how cooperation and technology along with the interactive learning approach helped in relating calculus concepts to students' fields, and how this helped in improving their level of motivation and enjoyment of the course. The study looked also at the effect of this approach on enhancing students' understanding of the subject.

Methodology

Population and Sample

This is an action research at one private university in the Kingdom of Saudi Arabia. The targeted population are non-mathematics major students who are studying the first or second level of calculus courses. The sample was the group of students from the College of Computer Sciences and Information Systems majoring in computer sciences, information systems, or software engineering who completed Calculus I and Calculus II in two consecutive semesters at the private university by the end of second semester of academic year 2017-2018. Both courses in the two semesters were taught by the same instructor who is the second author of this paper.

The Intervention

The pedagogical style of the calculus series followed the traditional approach of lecturing in Calculus I and during 50% of Calculus II. In the 2nd half of Calculus II, the students were introduced to a new approach. Students were divided in groups of three. Each group was assigned to prepare a presentation of specific sections/ topics that was introduced briefly to students. Students were asked to search other recourses, create activities using digital tools and provide examples linked to their major and to real-world settings. Students were expected to present in class the concepts they learned and link other previously learned concepts using examples from real-life situations. Students were encouraged to show *out of the box examples* that connect to calculus concepts. For instance, activities focused on explaining natural phenomena or creating the waves in the curly hair of the Disney character "Brave" in the movie "Brave". Another group showed how calculus concepts are related to our daily life by simply showing a video of how functions represent activities during a daily life of a young boy. Such activities include eating breakfast, trip to school, studying, and playing basketball. Finally, the instructor applied three levels of evaluation. First, self-evaluation: the students are expected to evaluate their work by taking a video to their presentation and then watch it and write their assessment; second, peer evaluation: each group evaluated the others; third, teacher's evaluation: this approach done solely by the teacher.

Data Collection

Data was collected using several qualitative and quantitative approaches. First, as an institutional approach, all students of the university are asked to complete an anonymous questionnaire for all courses they registered for in a semester. This is an institutional approach to evaluate the delivery of each course by the end of each semester. The questionnaire items address the teacher, course syllabi, used recourses, assessment methods, and environment and pedagogy. Teachers are provided with the aggregated results for their courses upon the end of the semester. Hence, a total of 43 students who completed Calculus I and 31 students who completed Calculus II in academic year 2017-2018 in two consecutive semesters completed the questionnaire. For this study, the teacher (second author of this paper) shared students' feedback about the two courses they completed with her (Calculus I and Calculus II) for academic year 2017-2018. For the purpose of this study, only items concerning the goal of the study were considered (Appendix A). It is important to mention here that the results of Calculus I questionnaire were completed before students start Calculus II and students were not aware at that time that the pedagogical style might be modified.

Second, upon completion of Calculus II, the instructor asked the students to reflect on the new approach and give their feedback. The instructor created a short survey concerning the teaching style used in the course (Appendix B). All 31 registered students in Calculus II completed the survey. Finally, selected students were invited through email to participate in a focus-group interview. The selection of students was based on willingness to participate and academic levels. Students from various academic levels were invited. The invitational email explained the purpose of the study and provided privacy and anonymity data collection assurance. All interviewees signed the consent form. A total of three focus-group interviews were conducted with a total of nine participants. Each interview lasted for about 30 minutes. The goal was to get in-depth feedback about the effectiveness of the approach and to hear their opinion directly from them. Both authors of this paper administered the interviews during the period of 26 April 2018 through 9 May 2018. The meeting took place in the first author's office.

The interview protocol was semi-structured. The researchers developed six open-ended questions and used follow up questions when appropriate (Appendix C). The questions focused on students' experience, opinion and suggestions. The participants were asked to honestly reflect on negative or positive experiences. It is important to mention here that data was collected only upon the Institutional Review Board (IRB) approval. Upon completing the interviews, the data was transcribed by the first author. The data was letter-number coded. For example, letter A indicted the first focus group, and Participant B2 indicated the 2nd participant in the second focus group interview. Then, the researchers read through the transcribed data, organized it and categorized it. The researchers then identified themes and perspectives that helped them in writing their interpretations of the data.

Results

Students' satisfaction with the intervention with Calculus I

For Calculus I, 43 students completed the institutional evaluation. Students were asked to reflect on their level of satisfaction with the recourses of the course, cooperation with their classmates and the course in general. The results indicated that almost 70% are satisfied with the recourses of the course. More than 50% enjoyed the level of cooperation with their classmates and more than 60% are happy with the course in general. See Table 1 for more details.

Table 1 Students feedback on selected items of the institutional questionnaire for Calculus I

Statement	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
In general, I am satisfied with the learning resources used in this course	16	14	5	3	5
I enjoyed the level of cooperation with my classmates	13	9	10	4	7
In general, I am satisfied with the course	19	7	7	2	8

Students' satisfaction with the intervention with Calculus II

In the 2nd level of calculus, 31 students answered the survey. The results indicated that more than 83% are satisfied with the recourses and enjoyed working with their classmates. Around 87% enjoyed the class in general. Such results are displayed in Table 2.

Table 2 Students feedback on selected items of the institutional questionnaire for Calculus II

Statement	Strongly agree	Agree	Neutral	disagree	Strongly disagree
In general, I am satisfied with the learning resources used in this course	21	5	2	2	1
I enjoyed the level of cooperation with my classmates	21	5	2	1	1
In general, I am satisfied with the course	22	5	3	1	0

Second: Course Survey and Interviews

The students were asked to reflect on the new approach and give their feedback. All 31 students completed the survey. The results showed quite an agreement among students in favor of the interactive approach. Moreover, 80% if the students recommended such approach for other mathematics courses. Table 3 below shows more details.

Table 3 Students' Feedback on the Pedagogical Approach Used in Calculus II

Question	Strongly agree	Agree	I don't know	Disagree	Strongly Disagree
I learned a new quick technique during the activity	20	11			
I got a better understanding of the integration technique	19	8	4		
I had an opportunity to search and find out a new application can help me in my studies	23	8			
I had an opportunity to search and find out a new application can help me in my studies	23	8			
I was well involved and motivated in the classes	18	8		5	
I recommended this type of activities for all the math courses	16	8	7		

Following the survey, selected students were invited to participate in focus-group interviews. Analysis of the interview data revealed that students focused on certain key words, even without being directly asked about it. For example, in the interviews, students talked about the benefit of the *cooperation* and interaction with each other. Participant C1 said "Calculus I was rigid, it was only me and the paper". Moreover, participant B3 indicted that "the presentation helped in improving between me and other students and the teacher so the feedback I got from them was helpful", while another student from the same group said "I learned to work with everyone, some people I never worked with".

On the other hand, the power of the *presentation* style also was clear to students. Participant A1 said "Everyone learned something new in the presentation". The student also talked about the soft skills learned by using this style. "The confidence level improved… talking in front of people, so communication skills improved a lot also" (Participant A1). However, group C agreed that the presentation approach also influences understanding of the contents. "Whatever you present you will understand more….so the learning is deeper and we learned new things" (Participant C2, & C3). The third participant in group C argued that it is not only the power of presentation that helped in comprehending the concepts. "We searched very well, so we learned more; we searched for new tricks, techniques, new ways, so it deepened our understanding" (Participant C1).

The depth of understanding was also argued by group A. Participant A1 honestly said "You know, I feel in five years I would still remember the materials in Calculus II", while another one commented that "I really learned! Because I have to understand to be able to present". Interestingly, this group of students talked about Calculus I and compared it with Calculus II without being asked to do so. Participant A3 openly said "to be honest in Calculus I, I was only a *copy machine* and I did not understand. But in this one, it is different". In addition, one participant from group B summarized it all as: "Now I feel I am ready for anything"!

The participants also indicted that because they solved tasks using different approaches and because they ***connected the concepts to their major***, their understanding is fostered and empowered. These comments came from all three groups: "I am happy that I know where to use it"….."I can see how it is related to my major"…."It also connected my major with calculus"…. "In Calculus I, it was only lecturing, but in Calculus II we learned how to apply it in real life"…"It helped in our major, we searched for more useful of these applications related to our major" (Participants A2, A3, B1, C1, C3). Moreover, connecting calculus concepts to students' majors means connecting concepts together. Participant B3 said "Mind map is very important, so you connect the concepts. Also you can show real life examples. For example, you know…Disney and how to make artificial waves through integration".

One of the most amazing comments came from students is about how much they enjoyed the course. Students simply indicted that it was exciting! Participant A1 said "I learned that calculus is meaningful", while Participant B2 argued that "This approach was better, it makes calculus more interesting". Remarkably, even if the students did not get a straight A score, they still enjoyed the course. Examples of such comments: "Calculus I was boring and easy, Calculus II is more challenging but the project made it easier"…."I did better in Calculus I as a grade but Calculus II was more fun"….."to be honest, I thought of changing my major after Calculus I, I also thought or dropping Calculus II at the beginning. I am glad I didn't. In fact, my sister changed her major just because of Calculus I" (Participants C3, B1, B3). This last comments showed that being demotivated in a course could lead to serious decisions. To summarize, Participant A2 gave the final advice: "I think we should use this presentation approach in all calculus and math classes. We should always be able to see the effect of math in our life."

Discussion and Conclusion

This action research looked at the effect of an interactive approach used in a calculus course. The interview and survey results showed that students were excited about this new approach. It is very promising to see that students are actually excited and that they thought that a calculus class was fun! Which might change the perception about calculus as a de-motivational subject (Bressoud & Rasmussen, 2015). Moreover, students were able to connect to their major and apply in real-life scenarios, which is what have been argued in the literature to lack in calculus classes (Lo'pez-Gay et al., 2015).

When the traditional and the interactive approach were compared, it was also clear which approach is favorable by students. Although the level of students' satisfaction in Calculus I (Table 1) is good, it is clearly much better in the 2nd course (Table 2) when the new approach is used. The teacher of both courses (2nd author of this paper) observed that the students' participation level increased in the classroom. Their enthusiastic about the subject was also clear as they were eager to show out of the box examples that connect to calculus concepts. Their interest in the course expanded when they started their project and as they were thinking how to utilize the concepts in their field. It was also noticed that this collaborative project helped students to develop their interpersonal and communication skills.

Limitations and Recommendations

As indicated by the teacher of the course, the project began in the mid of the semester and lasted for five weeks. Students might have benefited more from the approach if it lasted longer. On the other hand, this approach took time from students to search for the activities and prepare the presentations. In addition, the presentations were scheduled during the class time, which is a concern for the time constrain. It is also important to mention here that the sample size used in this study is small and the presented results are all descriptive. Hence, generalization of the findings is not suggested. However, the findings coincide with many other previous studies that argue for collaborative, interactive, digital approach where students can connect rigid concepts to real-life scenarios, which is exactly what we encourage for all calculus classes.

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Appendix A: Selected Items from the Institutional Questionnaire

- In general, I am satisfied with the learning resources used in this course
- I enjoyed the level of cooperation with my classmates
- In general, I am satisfied with the course

Appendix B: The Survey Items

Dear student: please answer the following questions based on your latest experience in solving application problems for the Surface Area and the Arc Length

- I had an opportunity to search and find out new applications that can help me in my studies
  strongly agreed agree disagree strongly disagree
- I learned a new quick technique during the activity strongly agreed
  strongly agreed agree disagree strongly disagree
- I got a better understanding of the integration technique
  strongly agreed agree disagree strongly disagree
- I was well involved and motivated in the classes
  strongly agreed agree disagree strongly disagree
- I recommend this type of activities for all the math courses
  strongly agreed agree disagree strongly disagree

Appendix C: Interview Semi Structured Items

1. Up to what extent the new approach you used in class was similar or different than the usual approach used for other applications
2. How was your understanding of the integration techniques supported with the new approach?
3. What in your opinion some of the skills that you gained in the new approach that benefited you?
4. If you are given the choice to choose the approach to learn about application problems, would you choose the new approach for all applications? Why?
5. What suggestions would you give to improve the process?
6. Overall, how would you rate your experience?