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RESEARCH PAPER

THROUGHPUT ANALYSIS OF EXTENDED ARQ SCHEMES

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ABSTRACT

Various Automatic Repeat Request (ARQ) schemes have been used to combat errors that befall information transmitted in digital communication systems. Such schemes include simple ARQ, mixed mode ARQ and Hybrid ARQ (HARQ). In this study we introduce extended ARQ schemes and derive expressions for their throughput performance. We then compare their performance with mixed mode ARQ schemes where it is shown that extended ARQ schemes outperform the mixed mode ARQ schemes. We also show how the parameters for the schemes can be chosen to maximize performance.

Keywords: *ARQ, extended ARQ, HARQ, mixed mode ARQ, throughput*

INTRODUCTION

Automatic Repeat reQuest (ARQ) techniques have been widely used to combat the errors suffered in digital communication systems. The techniques come into play when the receiver requests for the retransmission of received information if that information has been received with errors. Such information is then retransmitted until it is received without errors, or until the transmitter times out. When the channel is very noisy or unstable, the performance of the communication systems tends to be very poor. In order to improve the performance, Hybrid ARQ (HARQ) is most often employed. In this case, when the receiver encounters errors in the received information, it attempts to correct them rather than requesting for retransmissions. The receiver therefore only requests for retransmissions if the number of errors in the received information exceeds its capacity to correct. The advantage of ARQ schemes is their simplicity, while their disadvantage is the amount of time it needs to retransmit the information in case of a noisy channel. The advantage of HARQ schemes is that no retransmissions are needed in moderate error conditions, while their disadvantage lies in their complexity. In choosing between ARQ or HARQ therefore, one is forced to weigh their advantages and disadvantages and the noise performance of the channel. Comparisons of ARQ and HARQ schemes is given in various reports (Lin and Costello, 2003, Le Martret *et al.* 2012, Kotuliakova *et al.* 2013).

Various structures of ARQ schemes have been developed in order to reduce the amount of time needed to retransmit information, while capitalizing on simplicity. The structure of ARQ schemes has therefore evolved as technology has advanced and the demand for more robust communication has increased. At the lower end there are the simple ARQ schemes, which include the Stop and Wait (SW), Selective Repeat (SR), Stutter (ST) and Go-Back-N (GBN) (Lin and Costello, 2003). Combinations of these schemes lead to mixed mode schemes which include the SR-GBN, SR-ST1 and SR-ST2. In the mixed mode schemes, when a prescribed number of failures occur in the SR mode, the GBN or ST retransmission mode is applied until the failed packet is received without errors. Another class of ARQ schemes is what we term as *extended*. These schemes can be implemented in the ST, ST-GBN, SR-ST and SR-ST-GBN configurations. The operation of extended ARQ schemes is such that when a prescribed number of failures occur in the SR mode, a number of ST retransmissions is applied in subsequent frames, with these ST retransmissions being repeated in a prescribed number of frames, before the full ST or full GBN mode is applied.

Various methods have been employed to characterise, analyze and adapt ARQ and HARQ schemes in communication. Jin *et al.* (2009) employed queuing models to evaluate the performance of massive-scale wireless multimedia service networks. On their part, Alonzo-Zarate *et. al.* (2012) employed computer simulations to analyze the performance of communication networks employing cooperative ARQ in the face of hidden and exposed terminals. Computer simulations were also used by Vien *at al.* (2012) to determine the ARQ scheme giving best performance when network coding (NC) is employed in two-way relay communication networks. In their study on the selection of a suitable ARQ scheme to yield best performance, Kotuliakova *et al.* (2013) employed throughput and complexity as the criteria. In this report, we apply the approach of representing the operation of the communication systems by transition diagrams, where the z-transform plays a major role (Kundaeli, 2010). We then derive analytical expressions for the throughput of the schemes, and use it to compare their performance.

SYSTEM MODEL AND ANALYSIS

In this analysis, as in previous ones (Kundaeli, 2010, 2013), we assume that information is transmitted in packets, with the round-trip delay between transmitter and receiver occupying N packets. Transmitted information is therefore assumed to be grouped in frames of N packets, and the packets in the frame are numbered 1 to N. All the extended ARQ schemes can be thought of as starting in the SR mode, then ST before ending in either ST or GBN. Therefore, the number of retransmissions in the SR mode before the transmitter switches to the ST mode is v, after which the transmitter may transmit up to k frames in the ST mode. In these k frames, consecutive u ($u < N$) replicas of the failed packet may be transmitted in each of the frames with u being different for each frame. If all k retransmissions end up in failure, the transmitter enters full ST or full GBN retransmission. A snapshot of the timing diagram for the ST (k,u) scheme is shown in Fig. 1, in which $v = 0$, $N = 6$, $k = 3$, $u_1 = u_2 = 3$ and $u_3 = 2$. Note that the scheme starts with ST retransmissions because $v = 0$. Note also that in this diagram, and in subsequent ones, the transmission of a packet of interest is represented by * while the decision on the received packet is indicated by an up arrow. Therefore, in Fig. 1 the packet of interest is first transmitted in slot 1 of frame 0, where its failure at the end of this frame leads to the transmission of 3 replicas of it in frame 1. If these also fail as seen in frame 2, 3 replicas of it are again transmitted. If these also fail, then 2 replicas of it are transmitted in frame 3, and if these also fail, the system enters full ST mode at the end of frame 4.

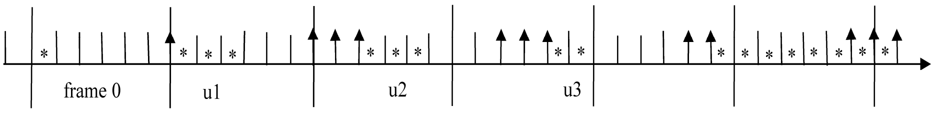

Fig. 1: Timing diagram of the ST (k,u) scheme based on N = 6, k = 3, $u_1 = u_2$ = 3 and u_3 = 2

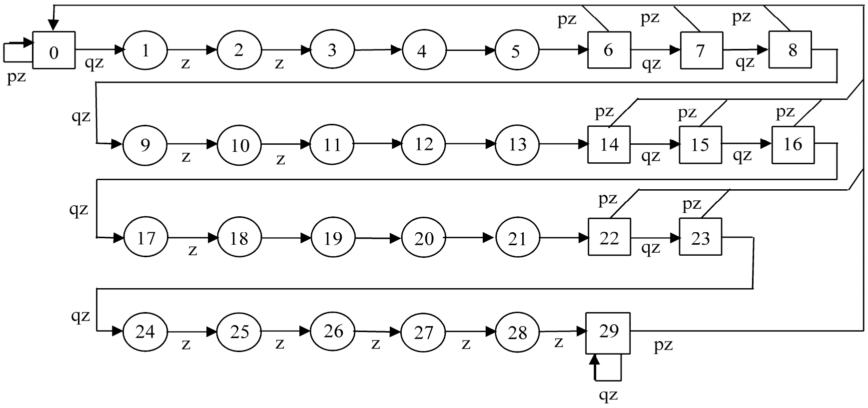

Fig. 2: Transition diagram of the ST (k,u) scheme based on N = 6, k = 3, $u_1 = u_2$ = 3 and u_3 =2

The system represented by Fig. 1 can also be represented by the transition diagram of Fig. 2, where p and q (with $q = 1 - p$) represent the probability of a transmitted packet being received without and with errors respectively. With this diagram, the system remains in state 0 as long as there are no errors. If an error occurs, however, the system transits to state 1, and it may transit through states 2 to 29 before returning to state 0. Note that in this diagram the parameter z appears only in the transitions associated with the packet of interest. Note also that decisions on the received packets are made in states 6, 7, 8, 14, 15, 16, 22, 23 and 29.

The transfer functions for general values of N, k and u can therefore be obtained from Fig. 2 as:

$$\Phi_f(z) = \Phi_{01}(z) = \frac{qz}{1 - pz}$$

$$\Phi_r(z) = \Phi_{10}(z) = z^{u_1 - 1}\sum_{s=0}^{u_1 - 1}(qz)^s pz + \sum_{j=2}^{k}\prod_{i=1}^{j-1}\left(z^{u_i - 1}(qz)^{u_i}\right)\left\{z^{u_j - 1}\sum_{s=0}^{u_j - 1}(qz)^s pz\right\} + \prod_{i=1}^{k}\left(z^{u_i - 1}(qz)^{u_i}\right)\frac{pz^{N}}{1 - qz} \tag{1}$$

from which we obtain the mean transition times for the failure and recovery processes as

$$L_f = \frac{d}{dz}\left(\Phi_f(z)\right)\Big|_{z=1} = \frac{1}{q}$$

$$L_r = \frac{d}{dz}\left(\Phi_r(z)\right)\Big|_{z=1} = \sum_{s=0}^{u_1-1}(u_1 + s)q^s p + \sum_{j=2}^{k}\prod_{i=1}^{j-1} q^{u_i}\sum_{s=0}^{u_j-1} q^s p\left(u_j + \sum_{i=1}^{j-1} 2u_i - j + s + 1\right) + \frac{\prod_{i=1}^{k} q^{u_i} p\left[\left(N + \sum_{i=1}^{k} 2u_i - k\right)(1 - q) - (-q)\right]}{(1 - q)^2} \tag{2}$$

which simplifies to

$$L_r = u_1\left(1 - q^{u_1}\right) + \frac{q - q^{u_1} - pq^{u_1}(u_1 - 1)}{p} + \sum_{j=2}^{k}\prod_{i=1}^{j-1}\frac{q^{u_i}}{p}\left[p\left(1 - q^{u_j}\right)\left(u_j + \sum_{i=1}^{j-1} 2u_i - j + 1\right) + \left(q - q^{u_j} - pq^{u_j}(u_j - 1)\right)\right] + \prod_{i=1}^{k}\frac{q^{u_i}}{p}\left[\left(N + \sum_{i=1}^{k} 2u_i - k\right)p + q\right]. \tag{3}$$

The throughput efficiency is then given by

$$\eta = \frac{L_f}{L_f + L_r} \tag{4}$$

In the ST-GBN (*k*,*u*) scheme, the operation is the same as that of the ST (*k*,*u*) scheme except that when persistent failures occur, the transmitter ends up with GBN retransmissions instead of ST retransmissions. The timing diagram for such a system with $N = 6$, $k = 3$, $u_1 = u_2 = 3$ and $u_3 = 2$ is given in Fig. 3 and the corresponding transition diagram in Fig. 4.

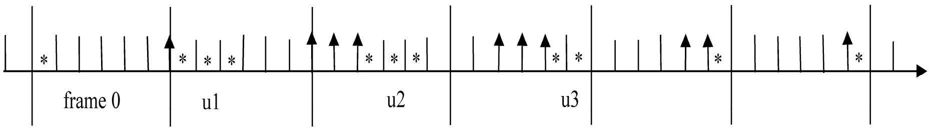

Fig. 3: Timing diagram of the ST-GBN (*k*,*u*) scheme based on *N* = 6, *k* = 3, $u_1 = u_2 = 3$ and $u_3 = 2$

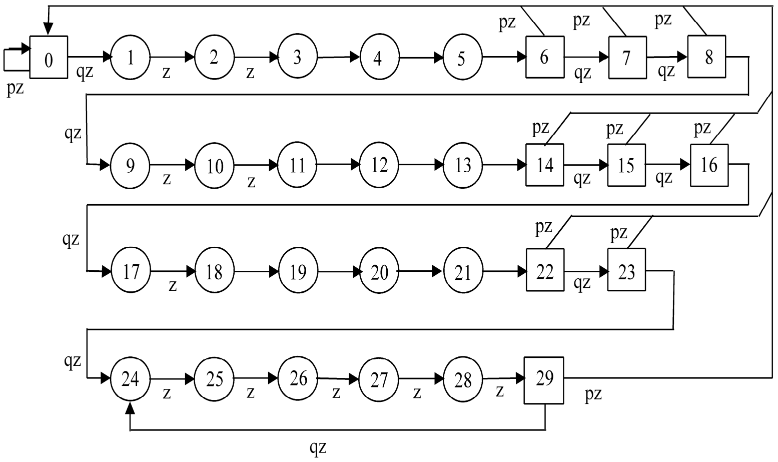

Fig. 4: Transition diagram of the ST-GBN (*k*,*u*) scheme based on *N* = 6, *k* = 3, $u_1 = u_2 = 3$ and $u_3 = 2$

The general transfer functions can be derived from the transition diagram as

$$\Phi_f(z) = \frac{qz}{1-pz}$$

$$\Phi_r(z) = z^{u_1-1}\sum_{s=0}^{u_1-1}(qz)^s pz + \sum_{j=2}^{k}\prod_{i=1}^{j-1}\left(z^{u_i-1}(qz)^{u_i}\right)\left\{z^{u_j-1}\sum_{s=0}^{u_j-1}(qz)^s pz\right\} + \prod_{i=1}^{k}\left(z^{u_i-1}(qz)^{u_i}\right)\frac{pz^N}{1-qz^N} \qquad (5)$$

from which we obtain

$$L_f = \frac{1}{q}$$

$$L_r = \sum_{s=0}^{u_1-1} (u_1 + s) q^s p + \sum_{j=2}^{k} \prod_{i=1}^{j-1} q^{u_i} \sum_{s=0}^{u_j-1} q^s p \left(u_j + \sum_{i=1}^{j-1} 2u_i - j + s + 1 \right) \tag{6}$$

$$+ \frac{\prod_{i=1}^{k} q^{u_i} p \left[\left(N + \sum_{i=1}^{k} 2u_i - k \right) (1-q) - (-Nq) \right]}{(1-q)^2}$$

which then simplifies to

$$L_r = u_1 \left(1 - q^{u_1}\right) + \frac{q - q^{u_1} - pq^{u_1}(u_1 - 1)}{p} \tag{7}$$

$$+ \sum_{j=2}^{k} \prod_{i=1}^{j-1} \frac{q^{u_i}}{p} \left[p\left(1 - q^{u_j}\right)\left(u_j + \sum_{i=1}^{j-1} 2u_i - j + 1 \right) + \left(q - q^{u_j} - pq^{u_j}(u_j - 1)\right) \right]$$

$$+ \prod_{i=1}^{k} \frac{q^{u_i}}{p} \left[N + \left(\sum_{i=1}^{k} 2u_i - k \right) p \right].$$

The scheme proposed by Weldon (1982) resembles the ST (*k*,*u*) and ST-GBN (*k*,*u*) schemes except that when *k* retransmissions have been attempted, the ST mode is applied with the packet being transmitted u_k times in the *k*th ST retransmission. The general transfer functions for this system are then given by

$$\Phi_f(z) = \frac{qz}{1 - pz}$$

$$\Phi_r(z) = z^{u_1-1} \sum_{s=0}^{u_1-1} (qz)^s pz + \sum_{j=2}^{k} \prod_{i=1}^{j-1} \left(z^{u_i-1} (qz)^{u_i} \right) \left\{ z^{u_j-1} \sum_{s=0}^{u_j-1} (qz)^s pz \right\} \tag{8}$$

$$+ \prod_{i=1}^{k} \left(z^{u_i-1} (qz)^{u_i} \right) \frac{z^{u_k-1} \sum_{s=0}^{u_k-1} (qz)^s pz}{1 - z^{u_k-1} (qz)^{u_k}}$$

from which we obtain

$$L_f = \frac{1}{q}$$

$$L_r = \sum_{s=0}^{u_1-1} (u_1 + s) q^s p + \sum_{j=2}^{k} \prod_{i=1}^{j-1} q^{u_i} \sum_{s=0}^{u_j-1} q^s p \left(u_j - 1 + \sum_{i=1}^{j-1} (2u_i - 1) + s + 1 \right) \tag{9}$$

$$+ \frac{\left[\prod_{i=1}^{k} q^{u_i} \sum_{s=0}^{u_k-1} q^s p \left(u_k - 1 + \sum_{i=1}^{k} (2u_i - 1) + s + 1 \right) \left(1 - q^{u_k}\right) - \prod_{i=1}^{k} q^{u_i} \sum_{s=0}^{u_k-1} q^s p \left(-(2u_k - 1) q^{u_k} \right) \right]}{\left(1 - q^{u_k}\right)^2}$$

which simplifies to

$$
\begin{aligned}
L_r = {} & \frac{(q+pu_1)-(q+2pu_1)q^{u_1}}{p} \\
& + \sum_{j=2}^{k}\prod_{i=1}^{j-1}\frac{q^{u_i}}{p}\left[\left(1+pu_j+p\sum_{i=1}^{j-1}2u_i-jp\right)-\left(1+2pu_j+p\sum_{i=1}^{j-1}2u_i-jp\right)q^{u_j}\right] \\
& + \prod_{i=1}^{k}\frac{q^{u_i}}{p\left(1-q^{u_k}\right)}\left[\left(1-p+pu_k+p\sum_{i=1}^{k}2u_i-kp\right)-\left(1+p\sum_{i=1}^{k}2u_i-kp\right)q^{u_k}\right].
\end{aligned} \tag{10}
$$

The SR-ST (v,k,u) scheme is similar to the ST (k,u) scheme except that $v > 0$. The transition diagram for the system for $N = 6$, $v = 2$, $k = 2$, $u_1 = 3$ and $u_2 = 2$ is given in Fig. 5.

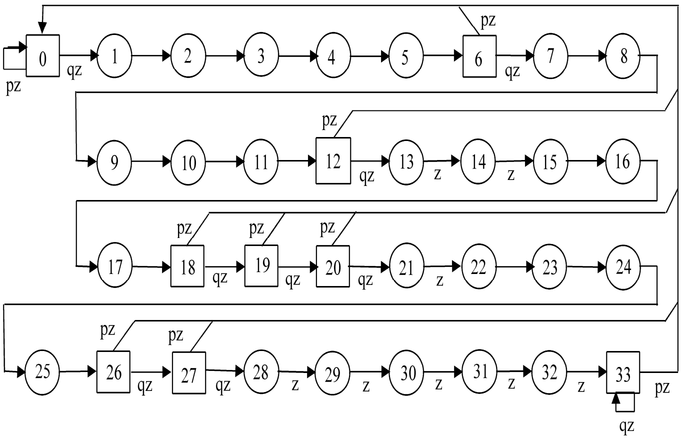

Fig. 5: Transition diagram of the SR-ST (*v,k,u*) scheme based on *N* = 6, *v* = 2, *k* = 2, *u*$_1$ = 3 and *u*$_2$ = 2

The transfer functions are then given by

$$
\begin{aligned}
\Phi_f(z) &= \frac{qz}{1-pz} \\
\Phi_r(z) &= \sum_{m=0}^{v-1}(qz)^m pz + (qz)^v z^{u_1-1}\sum_{s=0}^{u_1-1}(qz)^s pz \\
&\quad + (qz)^v \sum_{j=2}^{k}\prod_{i=1}^{j-1}\left(z^{u_i-1}(qz)^{u_i}\right)\left\{z^{u_j-1}\sum_{s=0}^{u_j-1}(qz)^s pz\right\} \\
&\quad + (qz)^v \prod_{i=1}^{k}\left(z^{u_i-1}(qz)^{u_i}\right)\frac{pz^N}{1-qz}
\end{aligned} \tag{11}
$$

which gives

$$\begin{aligned} L_f &= \frac{1}{q} \\ L_r &= \sum_{m=0}^{v-1}(m+1)q^m p + q^v \sum_{s=0}^{v-1} q^s p(v+u_1+s) \\ &\quad + q^v \sum_{j=2}^{k} \prod_{i=1}^{j-1} q^{u_i} \left\{ \sum_{s=0}^{u_j-1} q^s p \left(v + \sum_{i=1}^{j-1}(2u_i-1) + u_j + s \right) \right\} \\ &\quad + \frac{q^v \prod_{i=1}^{k} q^{u_i} p \left\{ \left(v + N + \sum_{i=1}^{k}(2u_i-1) \right)(1-q) - (-q) \right\}}{(1-q)^2} \end{aligned} \tag{12}$$

and which simplifies to

$$\begin{aligned} L_r &= \frac{(1-q^v - vpq^v)}{p} + (v+u)(1-q^{u_1})q^v + \frac{q^v\left(q - q^{u_1} - pq^{u_1}(u_1-1)\right)}{p} \\ &\quad + q^v \sum_{j=2}^{k} \prod_{i=1}^{j-1} \frac{q^{u_i}}{p} \left\{ \left(v + \sum_{i=1}^{j-1} 2u_i + u_j + 1 - j \right)(1-q^{u_j})p + \left(q - q^{u_j} - pq^{u_j}(u_j-1)\right) \right\} \\ &\quad + q^v \prod_{i=1}^{k} \frac{q^{u_i}}{p} \left\{ \left(v + N + \sum_{i=1}^{k} 2u_i - k \right)p + q \right\}. \end{aligned} \tag{13}$$

The SR-ST-GBN (*v*,*k*,*u*) scheme is similar to the SR-ST (*v*,*k*,*u*) schemes in which the final operation after persistent failures is full GBN instead of full ST retransmissions. By comparing Figs. 4 and 5, the general transfer functions for this scheme can be obtained as

$$\begin{aligned} \Phi_f(z) &= \frac{qz}{1-pz} \\ \Phi_r(z) &= \sum_{m=0}^{v-1}(qz)^m pz + (qz)^v z^{u_1-1} \sum_{s=0}^{u_1-1}(qz)^s pz \\ &\quad + (qz)^v \sum_{j=2}^{k} \prod_{i=1}^{j-1} \left(z^{u_i-1}(qz)^{u_i} \right) \left\{ z^{u_j-1} \sum_{s=0}^{u_j-1} (qz)^s pz \right\} \\ &\quad + (qz)^v \prod_{i=1}^{k} \left(z^{u_i-1}(qz)^{u_i} \right) \frac{pz^N}{1-qz^N} \end{aligned} \tag{14}$$

which give

$$
\begin{aligned}
L_f &= \frac{1}{q} \\
L_r &= \sum_{m=0}^{v-1} (m+1) q^m p + q^v \sum_{s=0}^{v-1} q^s p (v + u_1 + s) \\
&\quad + q^v \sum_{j=2}^{k} \prod_{i=1}^{j-1} q^{u_i} \left\{ \sum_{s=0}^{u_j - 1} q^s p \left(v + \sum_{i=1}^{j-1} (2u_i - 1) + u_j + s \right) \right\} \\
&\quad + \frac{q^v \prod_{i=1}^{k} q^{u_i} p \left\{ \left(v + N + \sum_{i=1}^{k} (2u_i - 1) \right) (1-q) - (-qN) \right\}}{(1-q)^2}
\end{aligned}
\tag{15}
$$

leading to

$$
\begin{aligned}
L_r = {} & \frac{\left(1 - q^v - vpq^v\right)}{p} + (v+u)\left(1 - q^{u_1}\right) q^v + \frac{q^v \left(q - q^{u_1} - pq^{u_1}(u_1 - 1)\right)}{p} \\
& + q^v \sum_{j=2}^{k} \prod_{i=1}^{j-1} \frac{q^{u_i}}{p} \left\{ \left(v + \sum_{i=1}^{j-1} 2u_i + u_j + 1 - j \right) \left(1 - q^{u_j}\right) p + \left(q - q^{u_j} - pq^{u_j}(u_j - 1)\right) \right\} \\
& + q^v \prod_{i=1}^{k} \frac{q^{u_i}}{p} \left\{ N + \left(v + \sum_{i=1}^{k} 2u_i - k \right) p \right\}.
\end{aligned}
\tag{16}
$$

RESULTS AND DISCUSSION

The results of the analysis are given in Figs. 6 to 12. In these results, unless otherwise indicated, the parameters have been fixed at number of packets in a frame (*N*) = 100, and the number of SR retransmissions (*v*) = 2. Fig. 6 shows that the throughput efficiency decreases with increase in the packet failure rate as expected. It is also seen that schemes employing SR retransmissions have higher throughput efficiency than those not employing such retransmissions. We can deduce from these results that use of ST retransmissions lowers the efficiency and thus use of SR retransmissions reduces the probability of the schemes entering ST modes. The results also reveal that use of GBN retransmissions further decreases the throughput efficiency. These results therefore show that the SR-ST (*v*,*k*,*u*) scheme gives the highest throughput efficiency.

In Fig. 7 it is seen that higher values of the repeat frames *k* lead to higher throughput efficiency. Using the same reasoning as in Fig. 6, it is also seen that SR retransmissions lead to increased throughput efficiency while GBN retransmissions lead to decreased throughput efficiency This trend is also seen in Fig. 8. The increase in throughput efficiency with repeat frames in the two figures can be deduced by noting that repeat frames are special SR retransmissions in which *u* > 1. The results also indicate that higher values of ST retransmissions lead to higher throughput efficiency at lower repeat frames but degrade the performance in schemes not employing SR retransmissions.

The effects of ST retransmissions are seen in Figs. 9 and 10, where the effects of SR retransmissions are also evident. In both cases it is seen that higher ST retransmissions lead to lower throughput efficiency. We can deduce that since after a packet fails *u* copies of it are transmitted

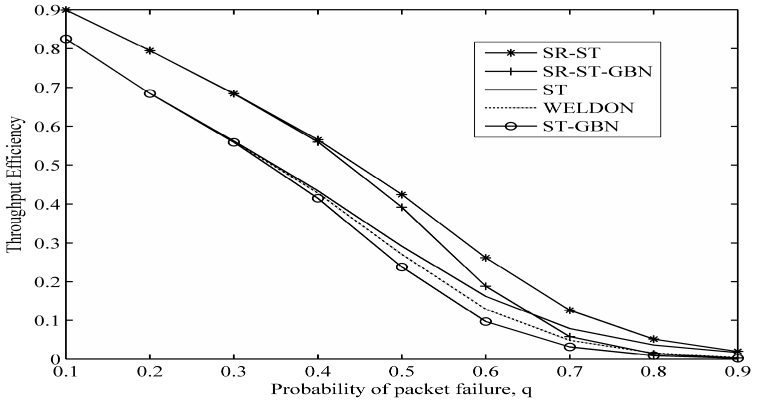

Fig. 6: Throughput efficiency vs probability of packet failure, *q* with *v* = 2, *k* = 3 and *u* = 2

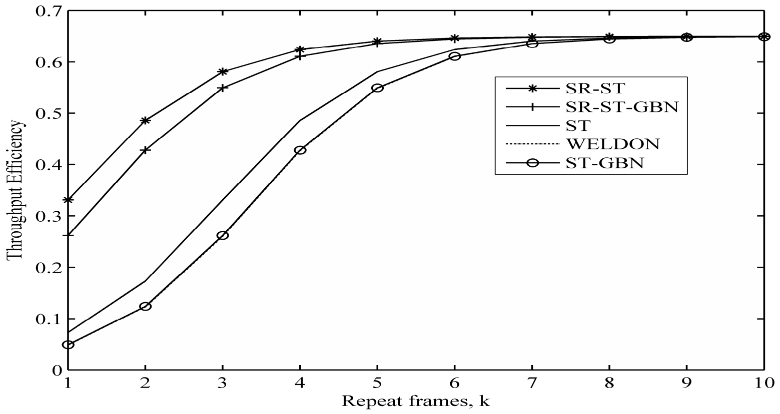

Fig. 7: Throughput efficiency vs number of repeat frames, *k* with *q* = 0.35, *v* = 2 and *u* = 1

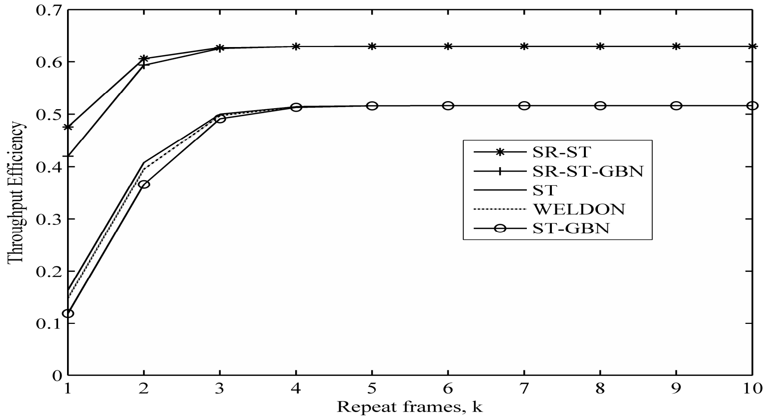

Fig. 8: Throughput efficiency vs number of repeat frames, *k* with *q* = 0.35, *v* = 2, and *u* = 2

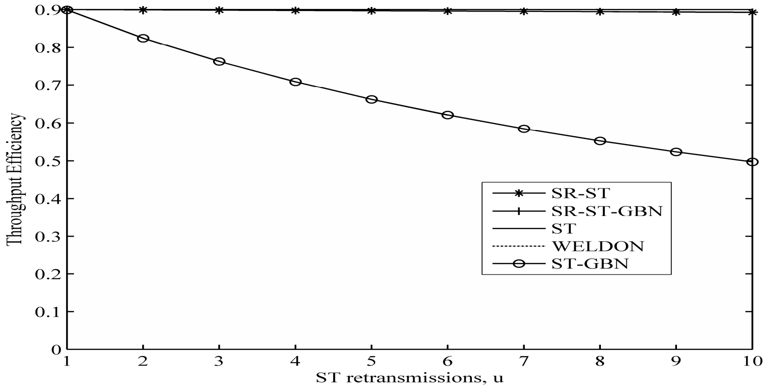

Fig. 9: Throughput efficiency vs number of ST retransmissions, *u* with *q* = 0.1, *v* = 2 and *k* = 4

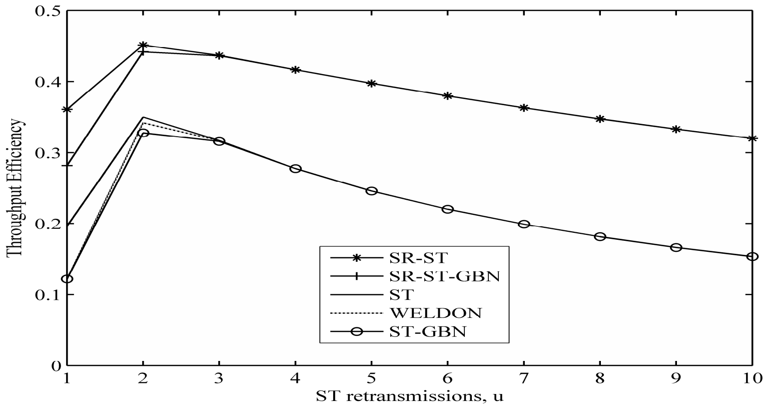

Fig.10: Throughput efficiency vs number of ST retransmissions, *u* with *q* = 0.5, *v* = 2 and *k* = 4

consecutively, the transmission of many copies of the failed packet leads to wastage of bandwidth where only a few copies could fulfil the required verification. This is revealed in Fig. 10 where the transmission of two copies of the failed packet leads to highest throughput efficiency and the transmission of a different number from this leads to lower efficiency. The results in Figs. 9 and 10 also reveal the superior effects of SR retransmissions and inferior effects of GBN retransmissions. It can also be deduced from the two results that higher values of *q* require higher values of *u*.

Figs. 11 and 12 show the variation of the throughput efficiency with the SR retransmissions. It is seen that not only does SR retransmissions lead to higher throughput efficiency, but also that the efficiency increases with number of SR retransmissions. We can deduce that both the ST and GBN retransmissions lead to lower throughput efficiency, and that systems employing SR retransmissions have higher throughput efficiency because SR retransmissions prevent the systems from entering the ST and GBN retransmissions. It is also seen that the SR-ST scheme has the best performance. We also note the effects of packet failure *q* on the performance

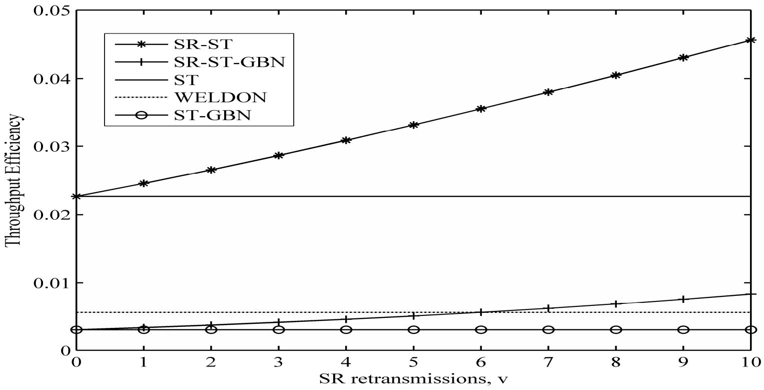

Fig.11: Throughput efficiency vs number of SR retransmissions, *v* with *q* = 0.9, *k* = 5 and *u* = 2

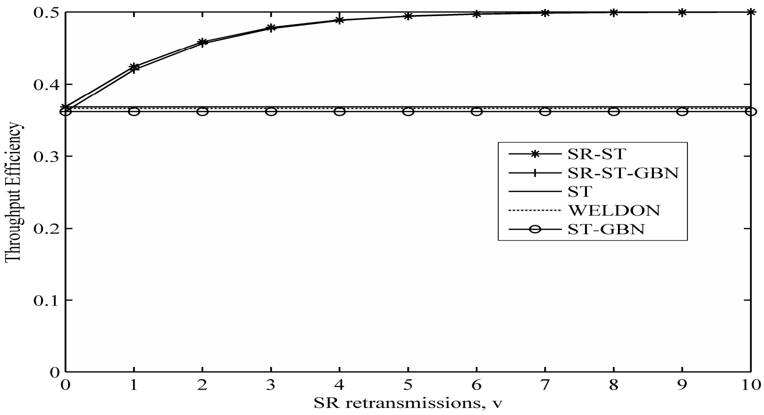

Fig.12: Throughput efficiency vs number of SR retransmissions, *v* with *q* = 0.5, *k* = 5 and *u* = 2

of the systems.

In comparing the extended and non-extended schemes, it was shown that the SR-ST and SR-ST-GBN schemes outperform the other schemes in earlier reports (Kundaeli, 2010, 2013). In this report, Figs. 6 to 10 also show that the SR-ST and SR-ST-GBN schemes outperform the other schemes. We also note that the effects of SR retransmissions are the same in both extended and non-extended schemes, in that performance increases with SR retransmissions. We next note that when repeat frames are not applied (k = 1), the extended schemes revert to the non-extended schemes analysed in the earlier reports. We then note from Figs. 7 and 8 that the performance of the extended schemes increases with number of repeat frames. This therefore shows that the extended schemes outperform the non-extended schemes. We next note that at high error rates, ST retransmissions should exceed 1 in both the extended and non-extended schemes in order to increase throughput efficiency. As also pointed out in the earlier reports, the results of the analysis can be used to choose the design parameters for the ARQ schemes. This is also the case for the extended schemes, in which the number of ST retransmissions that gives highest throughput efficiency can be obtained from Fig. 10 as 2. The comparisons in this analysis therefore show that the extended schemes have better throughput performance over the non-extended schemes. The analysis also shows that in both cases, the SR-ST schemes have best performance over all the other schemes.

CONCLUSION

The analysis of extended ARQ schemes has been undertaken using transition diagrams and the z-transform. This work is an extension of similar work on simple and mixed mode ARQ schemes. The analysis shows that the extended ARQ schemes outperform the mixed mode ARQ schemes in throughput efficiency. The analysis also shows that the SR-ST scheme yields the best throughout efficiency. The parameters that give best performance to match the noise in the channel have also been derived. Although the comparison on throughput efficiency often suffices to rank performance, a combined throughput-delay comparison gives a better picture, especially for delay sensitive communication. Moreover, the structures of extended ARQ schemes appear to be more complex than those of simple or mixed mode ARQ schemes. This tends to suggest inherent higher delays in the extended ARQ schemes. The inclusion of delay analysis will therefore give a better comparison with the simple and mixed mode schemes. Future work on extended ARQ schemes will therefore be on delay analysis.

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