ON THE ESTIMATION OF PARAMETERS OF THICK CURRENT SHELL MODEL OF EQUATORIAL ELECTROJET USING OPTIMISATION METHOD

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ABSTRACT
Equatorial electrojet, an intense current flowing eastward in the low latitude ionosphere within the narrow region flanking the dip equator, is a major phenomenon of interest in geomagnetic field studies. For the first time the five parameters required to fully describe the Onwumechili's composite thick current shell model format of equatorial electrojet have been evaluated from a single autonomous set of ground data at solar minimum. The non-linear model was applied to four data points, each with a pair of simultaneously measured horizontal H and vertical Z variation field components. The resultant system of eight non-linear equations with five unknown model parameters were subjected to non-linear least square optimisation method taking advantage of the robust Levenberg-Madquart optimisation subroutine of licensed MATLAB 6.0 version. The thick current shell format model parameters estimated for Indian sector are shown to be within the appropriate limits and in excellent agreement with literature and physical expectation.

Keywords: *Equatorial electrojet; Numerical models; Optimisation*

INTRODUCTION

Modeling the ionosphere and its current systems has continued to gain attention (Jadhav *et al.*, 2002; Doumoya *et al.*, 2003; Holme *et al.*, 2004) due to its increasing significance in the earth-satellite communication, applications in space weather studies and source field problems in magnetotellurics. The daytime dynamo E region of the ionosphere in the neighbourhood of magnetic dip equator have been identified to consist of two layers of currents responsible for the quiet solar daily variations; one known as the worldwide Sq, flowing at an altitude of (118 ± 7) km, the other one is the intense non-uniform east west current named as equatorial electrojet, EEJ, by Chapman (1951), which flows at a lower altitude of (106 ± 2) km (Chapman, 1951; Richmond, 1973; Rastogi, 1975; Onwumechili, 1997).

Since Chapman (1951) presented the first model of equatorial electrojet, EEJ, a number of models have been proposed (Onwumechili, 1966a, b, c;

Untiedt, 1967; Richmond, 1973; Suzuki, 1973; Fambitakoye and Mayaud, 1976). Onwumechili (1966a, b, c; 1967) presented a two dimensional model of the continuous current distribution responsible for EEJ as:

$$j = jo\ a^2(a^2 + ax^2)b^2(b^2 + bz^2)\,/\,(a^2 + x^2)^2\,(b^2 + z^2)^2 \quad (1)$$

Where j (μA m^{-2}) is the eastward current density at the point (x, z). The origin is at the centre of the current, x is northwards, and z is downwards. The model is extensible to three dimension by introducing the coordinate y or longitude $\emptyset$ or eastwards local time t. j_0 is the current density at the centre, a and b are constant latitudinal and vertical scale lengths respectively, a and b are dimensionless parameters controlling the current distribution latitudinally and vertically respectively. It is a meridional plane model, which in this simple form has to be applied to specific longitudes or local times. Once the five (5) parameters j_o, a, a, b, b are determined by fitting observational data, a number of physical parameters of the current and its magnetic field can be calculated from them. The model is a realistic model having both width and thickness.

Onwumechili (1966c) used the Biot-Savart law to obtain the northwards X and vertical Z components of the magnetic field variation with latitude on the horizontal plane *(v = constant)* as a result of the current distribution in (1) as follows:

$$(sg.\ z)\ P^4 X = \tfrac{1}{2} k\ [(1+b)(v + av + 2aa)(u + b)^2$$
$$+\ 2(1- b)(v + av + 4a - 2aa)(u + b)$$
$$+\ (1+ b)(v + av + 2a)(v + a)^2] \quad\quad (2)$$

$$-(sg.x)\ P^4 Z = \tfrac{1}{2} k\ [(1+ a)(1+ b)(u + b)^3 + ((1+a)(1+ b)(u + b)^2$$
$$+\ (1+ b)(v + av + 3a - aa)\,(v + a)\,(u + b)$$
$$-\ (1- b)\,b\,(v + av + 3a - aa)\,(v + a)] \quad (3)$$

Where $P^2 = (u + b)^2 + (v + a)^2$ (4)
    $k = 0.1\pi^2 abj_0$ (5)
    $u = /x/$ and $v = /z/$ (6)
    $sg.x =$ sign of (x/u) and is ± 1 when $x = 0$ (7)
    $sg.z =$ sign of (z/v) and is ± 1 when $z = 0$ (8)

Equations 2 and 3 give the horizontal and vertical magnetic field variations respectively, due to thick current shell format.

With some simplifying assumptions, the thick current shell format represented by equations 2 and 3 were approximated to give the horizontal and vertical magnetic field variations respectively, due to thin current shell format as:

$(sg.\ z)\ P^4 X = \tfrac{1}{2} Ka\ [(v + av + 2aa)(u + b)^2 + (v + av + 2a)(v + a)^2]$ (9)
$-(sg.x)\ P^4 Z = \tfrac{1}{2} Ka(u + b)\ [(1+ a)(u + b)^2 + (v + av + 3a - aa)\,(v + a)]$ (10)

where;
      $K = 0.2\ \pi J_0$ (11)
Or $Ka = k(1+ b)$ (12)

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K is the magnetic field, being the magnetic field of an infinite plane current sheet with constant intensity J_0 A km^{-1}. It should be noted that *b,* the parameter that controls the vertical scale of the current is conspicuously missing in the thin shell format field equations 9 and 10.

In the neigbourhood of dip equator northwards component X and horizontal component H are approximately equal as the inclination is very small, therefore $\Delta X \approx \Delta H$. ΔZ and ΔH are measurable field components at observatories, where *v* is the current altitude taken to be 106 km (0.96°) as determined by rocket and satellite measurements (Onwumechili, 1997), *u* is the dip latitude of the point of observation.

The advantages of Onwumechili's model over others have been identified in Onwumechili (1997). The current distribution of Onwumechili (1966a, b, c) model, henceforth designated as OM66, has been used by several authors to model equatorial electrojet, for examples Onwumechili and Agu (1989), Onwumechili and Ezema (1992), Oko *et al.*, (1966), Rigoti *et al.*, (1999), Jadhav *et al.*, (2002). The simplification of OM66 into measurable magnetic field components necessary for evaluation of model parameters resulted in two current shell formats, one known as the thick current shell format and the other known as thin current shell format. The latter is the approximation of the former. With the exception of Onwumechili and Ezema (1992) who applied the OM66 in its thick current shell format to POGO satellite data, all others have used the approximation known as the thin shell format. Onwumechili and Ogbuehi (1967) and Onwumechili *et al.*, (1989) used satellite data to evaluate three parameters *K, a,* and *a* from the thin shell format; Onwumechili and Ezema (1992) employed the POGO satellite data to evaluate the five parameters using the thick current shell format; Oko *et al.*, (1996) evaluated only three parameters j_o (obtainable from *K* or *k*), *a, a,* using the thin current shell format; Rigoti *et al.*, (1999) evaluated three model parameters j_o, *a, a,* for Brazilian sector using the thin current shell format with a set of ground

data; Jadhav *et al.*, (2002) also employed the thin current shell format using Oersted satellite data between April 1999 and March 2000 to evaluate three parameters *K, a*, and *a*. Electrojet models computed under the thin-shell approximation have been shown to be inadequate by Untiedt (1967), Sugiura and Poros (1969), and Richmond (1973) due to the neglect of vertical currents.

It is observed that the thick current shell of OM66 employed by Onwumechili and Ezema (1992) was evaluated at high solar activity period, 1967 to 1969, and results were published only for 1100 and 1200 hours local time. Even then it is interesting to see the cumbersome nature of their long analytical process of evaluation, of the parameters using satellite data, which necessitated some approximations. Onwumechili (1997) highlighted the many successes of his continuous current distribution over the years and noted that the five model parameters have never been obtained from a single autonomous set of ground data using the thick current shell format. The thick current shell format of the model, which takes into account both the width and the thickness of the jet, contains all the five parameters in a non-linear form that makes it complicated for trivial attempt. It is a composite format capable of describing in detail the latitudinal and vertical flow of EEJ. The primary objective of this work is to evaluate the five (5) parameters that completely define the continuous current distribution of Equatorial electrojet from a single set of ground data during low solar activity for the first time, and generate the daytime hourly profiles of the parameters.

METHODOLOGY
A look at equations 2 and 3 reveals that each of H and Z, which are measurable quantities at magnetic observatories are expressed in terms of *k, a, a, b,* and *b*, as well as *v* and *u*. The first five parameters (*k, a, a, b,* and *b)* are the model parameters while *u* and *v* are known values at any point of observation. Rewriting equations 2 and 3 such that:

$$(sg.\ z)\ P^4\ X\ -\ \tfrac{1}{2}\ k\ [(1+b)(v\ +\ av\ +2aa)(u\ +\ b)^2\ +\ 2(1-\ b)(v\ +\ av\ +\ 4a\ -2aa)(u\ +\ b)$$
$$+\ (1+\ b)(v\ +\ av\ +\ 2a)(v\ +\ a)^2]\ =\ 0 \qquad (13)$$

$$-\ (sg.x)\ P^4\ Z\ -\ \tfrac{1}{2}\ k\ [(1+\ a)(1+\ b)(u\ +\ b)^3\ +\ ((1+\ a)(1+\ b)(u\ +\ b)^2\ +\ (1+\ b)(v\ +\ av\ +\ 3a\ -\ aa)$$
$$(v\ +\ a)\ (u\ +\ b)\ -\ (1-\ b)\ b\ (v\ +\ av\ +\ 3a\ -\ aa)\ (v\ +\ a)]\ =\ 0 \qquad (14)$$

Implies that a non linear function F (k, a, a, b, b) of magnetic field variations in each of the components X and Z can be written such that
$$F(k, a, a, b, b) = 0 \qquad (15)$$

For each hour, we applied equations 13 and 14 to a set of simultaneously derived electrojet index pairs H_i and Z_i for a set of four data stations each at dip latitude u_i (i =1-4), so we have a set of eight (8) non linear simultaneous equations. Hence the model parameters are overdetermined.

Data preparation/smoothing

Simultaneously recorded hourly horizontal H and vertical Z field values were obtained from 5 stations whose coordinates are shown in Table 1. These hourly horizontal and vertical field values were treated for hourly departures, non-cyclic and storm-time disturbance, Dst, variations to ensure absolute quiet condition as required. The electrojet index was obtained by subtracting the hourly values of worldwide solar quiet daily variation, Sq, as obtained at Hyderabad, a station just outside of electrojet, from other four stations that fall within the electrojet influence.

The current distribution described an external field and so it became necessary to separate the internal field from the external field. It is known that the observed values of H and Z are algebraic sum of the external ionospheric current and internal effects, such that:
$$\Delta H_e + \Delta H_i = \Delta H \qquad (16)$$
$$\Delta Z_e\ -\ \Delta Z_i = \Delta Z \qquad (17)$$

Onwumechili (1997, page 128) reported the ratios of 0.28 ± 0.08 and -0.17 ± 0.02 for $\Delta H_i/\Delta H_e$ and $\Delta Z_i/\Delta Z_e$ respectively found in excellent agreement with Davis *et al.,* (1967) result. We used these ratios to filter out the internal field from the observed values such that ΔH and ΔZ reflect the variation field due to external source of interest, the ionosphere. We therefore generated a hourly profiles of EEJ index in H and Z due to external current system of equation 1 for selected 60 quiet days of the solar minimum year 1986 (Sunspot number R = 13.4).

Levenberg-Marquardt Method

The Levenberg-Marquardt (LM) method (Levenberg, 1944; Marquardt, 1963; Shepherd, 1997; Press *et. al.,* 1992) is an advanced, and the most widely used, non-linear optimisation algorithm which significantly outperforms simple gradient descent and other conjugate gradient methods in a wide variety of problems. It works very well in practice and has become the standard of non-linear least squares routines (Press *et al.*, 1992). LM method is widely presented in

Table 1: Coordinates of the geomagnetic observatories

| Station | Code | Geog. | | Dip latitude |
		Lat. N°	long °E	(°N)
Trivandrum	TRD	8.29	76.57	0.20
Ettaiyapuram	ETT	9.10	78.00	0.50
Kodaikanal	KOD	10.23	77.47	2.14
Annamalainagar	ANN	11.4	79.7	3.28
Hyderabad	HYB	17.42	78.55	9.33

More (1977), Bates and Watts (1981), Gill *et al.*, (1981) and Bishop (1995).

LM finds the minimum of a function F(x) which is a sum of squares of non-linear functions:

$$F(x) = \frac{1}{2} \sum_i [f_i(x)]^2 \qquad (18)$$

Where x = $(x_1, x_2, …..x_n)$ is a vector, in our case n = 8.

The LM method employs a search direction that is a solution of the linear set of equations:

$$[J(x_k)^T J(x_k) + \lambda_k] d_k = -J(x_k)F(x_k) \qquad (19)$$

Where $J_i(x_k)$ is the Jacobian of $f_i(x)$; λ_k are non-negative scalars that control the magnitude and direction d_k, and I is the identity matrix. When λ_k is zero, the direction d_k is identical to that of the Gauss-Newton method. As λ_k tends to infinity, d_k tends toward a vector of zeros and a steepest descent direction. LM method is a blend of gradient descent and Gauss-Newton iteration. LM also demonstrates an increased robustness over Gauss-Newton method.

The MATLAB algorithm for LM implementation is described in detail in the Optimisation toolbox user guides by Mathworks (2000). LM algorithm is the default method used by LSQNONLIN of MATLAB for medium scale algorithm and the Jacobian is computed by default using finite differencing. By default LSQNONLIN chooses the large-scale algorithm which is a subspace trust region method based on the interior-reflective Newton method. LSQNONLIN with 'options.LargeScale' set to "off" uses the medium scale algorithm with the LM method with line search by default.

The optimisation algorithm of MATLAB 6.0 employs a pair of files in same directory. The first step entailed the development of an M-file that computes the values of objective function (equation 15). The second step called the non-

linear least squares routine of MATLAB with an appropriate starting guess x_0 and option.

The statement:
Option = optimset ('LargeScale', 'off')   *(20)*

Suppresses the default Large Scale subroutine and favours LM algorithm

The statement:
x = LSQNONLIN(@modelfun, x_0, option)   *(21)*

invokes the optimixer which makes use of the LM as subroutine. LM computes Jacobian matrices by finite difference technique for default estimation. LSQNONLIN solves problems of the form:

$$\min \sum_i [f_i(x)]^2 \qquad (22)$$

where x and the values returned by $f_i(x)$ can be vectors or matrices.

LSQNONLIN(@modelfun, x_0, option) starts at the matrix x0 and finds a minimum x to the sum of squares of the functions in modelfun. It tends to optimize f(x) and obtain the vector x, a matrix whose elements are the five model parameters that satisfies f(x) as represented in equation 15. Modelfun is the name of M-File that accepts input x and returns a vector (or matrix) of function values F evaluated at x.

We found it necessary to normalize the equations such that the output of the program gives the results in order of unity, this is emphasized by Press *et al.*, (1992). One outstanding performance we observed is that with appropriate starting values of iteration, which we simply chose to be within the range of values available in the literature, the program does not need any constraint. Rather it optimized to give the result we obtained.

We estimated the model parameters at every hour and take the seasonal means according to

Llyod's classification: E-season (March, April, September, October), J-season (May, June, July, August) and D-season (November, December, January, February).

Table 2: Daytime Seasonal and annual means of the evaluated model parameters with their standard deviations (SD)

Parameters	Seasons			Annual
	E	J	D	
k (100A)	101.8027	97.4532	98.5170	99.2576
SD	9.8734	6.2797	6.7001	7.0749
a (°)	3.6879	3.6828	3.6833	3.6874
SD	0.0081	0.0077	0.0074	0.0073
a	-1.8171	-1.9620	-1.8588	-1.8793
SD	0.3827	0.2303	0.2543	0.2628
b (°)	0.0778	0.0796	0.0790	0.0788
SD	0.0047	0.0030	0.0032	0.0034
b	0.4874	0.5019	0.4970	0.4954
SD	0.0375	0.0240	0.0256	0.0270

Table 3: Annual mean hourly values of the parameters using thick current shell model

Local time (hrs)	k(100A)	a (°)	a	b (°)	b
0700	90.1789	3.6731	-2.1531	0.0827	0.5267
0800	94.2770	3.6795	-2.0014	0.0808	0.5113
0900	101.7332	3.6867	-1.7078	0.0771	0.4818
1000	107.5295	3.6905	-1.5168	0.0747	0.4627
1100	109.3809	3.6943	-1.5206	0.0739	0.4564
1200	107.3849	3.6924	-1.6123	0.0750	0.4655
1300	103.7775	3.6907	-1.7614	0.0770	0.4804
1400	99.0073	3.6874	-1.9540	0.0794	0.4998
1500	94.8184	3.6830	-2.1069	0.0813	0.5152
1600	92.5686	3.6793	-2.1859	0.0823	0.5232
1700	91.1776	3.6743	-2.1522	0.0827	0.5266
Mean	99.2576	3.6847	-1.8793	0.0788	0.4954
SD	7.0749	0.0073	0.2628	0.0034	0.0270

Table 4: Comparison of noontime means of evaluated model parameters with available piecewise data from literature for Indian sector

Source (model format)	Model parameters				
	`k	a	a	b	b
	SD(Amp)	SD (°)		SD (°)	
Our result	Ground data (thick)	10738	3.692	-1.61	0.075	0.466
		313	0.003	0.12	0.001	0.009
Onwumechili and Ogbuehi (1967)	Satellite data (thin)		4.270	-1.59		
			1.04	0.08		
Onwumechili *et al.* 1989	Satellite data (thin)		2.937	-1.86		
			0.171	0.06		
Onwumechili and Ezema (1992)	Satellite data (thick)	17282	3.342	-1.53	0.079	0.526
		1319	0.081	0.08	0.001	0.018
Oko *et al.* 1996	Ground data (thin)		3.570	-1.56		
			0.040	0.02		

RESULTS AND DISCUSSION

Seasonal and annual means of the evaluated model parameters *k, a, a, b, b* for the Indian sector are displayed in Table 2 with their standard deviations. These seasonal values have no basis for comparison in literatures as only noontime values of *a* (-1.58 ± 0.01) and *a* (3.53 ± 0.04) were reported for equinox noontime by Oko *et al.*, 1996. Rather most literatures including Onwumechili (1997) discussed the seasonal variations of the derived parameters, from this model, which of course are of interest to the geophysical society. Table 3 presents the mean annual hourly values of the model parameters for daytime (hours 0700 to 1700).

Table 4 compares noontime model parameters from our work with the available ones in the literature, indicating the data source (ground or satellite) and the type of current shell format employed. Obviously the thin current shell format permits direct evaluation of only the latitudinal parameters *a, a*, and an implicit function of current density j_0 in terms of the magnetic constant *K* which is not shown in Table 4. The values of *a, a, b,* and *b* compare so well with the existing values as they all fall within the appropriate limits of standard deviations. The value of *b* as measured by satellite and adopted for so long as (8.5 ± 0.5) km (Onwumechili, 1997) equivalent of (0.0767 ± 0.0045°) is in excellent agreement with our value (0.0778 ± 0.0047°). The significant variation in our own parameter *k* (10738 ± 313) Amp and the only available basis for comparison, that of Onwumechili and Ezema (1992), (172383 ± 1319) Amp can be accounted for by the solar activity difference in the periods of data acquisition. Model parameter *k* by definition is directly proportional to the current density (see equation 5). We estimated the parameter for low solar activity period (sunspot number = 13.4), while Onwumechili and Ezema, (1992) considered high solar activity period (average sunspot number = 101.7) using available satellite data and analytical procedure. Oko *et al.*, (1996) estimated thin current shell parameters for low solar activity period (R=13.4) and obtained

lower current density than Onwumechili and Ezema, (1992).

We estimated the magnetic constant, *K*, expressed by definition (Onwumechili, 1997) as:

$$k = \frac{k(1+\beta)}{\alpha} \qquad (22)$$

And obtained the correlation coefficient and the probability of significance, between our estimated magnetic constant K_m from our thick current shell and that of Oko *et al.*, (1996) K_o obtained directly from thin current shell format, using the syntax:

$$[R,P]=CORRCOEF(K_m, K_o) \qquad (23)$$

Where R is the correlation coefficient and P returns the probability of getting a correlation as large as the observed value by random chance, when the true correlation is zero. If P is small, say less than 0.05, then the correlation R is significant.

We obtained a high positive correlation coefficient of 0.9903 and p-value of 4.9×10^{-9} (<<0.05). This demonstrates a high level of significance and gives credence to our estimation using the approach we have taken. It is sufficient to mention that the other landmark and current parameters of EEJ derived from these 5 model parameters are quite in agreement with literature and physical expectation.

CONCLUSION

The five parameters that fully described the current distribution of EEJ have been evaluated from an autonomous set of ground data. This was accomplished by a computer program based on the Levenberg-Marquardt least square method. The program consisted of a pair of M-file and the execution file that invoke the state of the art optimisation of licensed MATLAB 6.0 version. The results obtained compare well with earlier piecewise obtained parameters. The mean noontime values of the model parameters *k, a, a, b,* and *b*, are (10738 ± 313) A, (3.6924 ±

0.0023)°, (-1.6123 ± 0.1169), (0.0750 ± 0.0012)°, and (0.4655 ± 0.0091) respectively. The highly significant correlation of 0.99 and estimated p value of 4.9×10^{-9} which is much less than 0.05 between the values of our derived magnetic constant *K* and those obtained directly from thin shell format by Oko *et al.*, (1996) demonstrated excellent agreement between our parameters and their data.

Most importantly we have demonstrated that with a simple algorithm and appropriate computer code, a more suitable model can be applied to anomalous equatorial geomagnetic phenomena in a bid to explain its detailed feature and thus produce a more reliable and excellent composite model description.

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