EXTENDING ENTROPY STABILITY MEASURE TO EXTERNAL DEBT STRUCTURE

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ABSTRACT
This study focuses on the measure of stability of external debt using entropy. This is achieved by modifying the conglomerate of Shannon and Boltzmann entropy. This modification rectifies the limitations of these models. Practical illustration of the modified model is also given to justify its use.

Keyword: Conglomerate, entropy, tenure classes and stability

INTRODUCTION
Generally, entropy is a measure of the degree of disorder, uncertainty or randomness in a system. It is also defined as the thermodynamics variables of the system under consideration (McClean and Abodunde, 1978). Some authors use entropy to measure the stability of experience existing in a manpower system, Vissiliou (1984), McClean (1986), Omosigho and Osagiede (1999), while others apply it to combat degradation, Rodrigues (1989), queueing systems, Guiasu (1986), Kouvatsos (1988).

In some of these works, the basic entropy model is either applied as it is or modified to capture the variables and the scope of study as in McClean (1986), Rodrigues (1989), etc.

This study attempts to measure the stability of external debt structure of Nigeria using entropy models. Our uphill task here is to modify the basic entropy model of McClean and Abodunde (1978) to capture the stages of economic growth and to remove the problem of large values of n associated with the thermodynamic model of Tyler (1989).

THE ENTROPY MODELS
The basic entropy model of manpower system is as given in McClean and Abodunde (1978) as

$$H = \frac{-\sum_{i=1}^{k} P_i \log P_i}{\log k} \qquad (1)$$

where i is the tenure class, p_i is the probability that a member of staff is in tenure class i and k is the number of tenure classes. It is obvious from equation (1) that $0 \leq H \leq 1$. H = 0 implies no stability and therefore, the absence of experience in the manpower system. But H = 1 means that the system is highly stable and a high degree of experience in the system. For intermediate values of 0<H<1, entropy is a good measure of experience in the system (Omosigho and Osagiede, 1999).

The system in equation (1) is referred to as Shannon entropy.

The thermodynamic model for manpower system for Tyler (1989) is given by

$$H_B = \log\left(\frac{N!}{N_1!\,N_2!\,..N_C}\right) \Big/ \log C \qquad (2)$$

where
C is the number of tenure classes,
N_i – number of staff in tenure class i
N – number of staff in the manpower system.

Equation (2) is known as Boltzmann entropy. The assumption of a constant workforce size before the system in equation (1) can be used poses a limitation to its use. Also the system in equation (2) can fail with very large N. Hence, the need to work out a modification for these two models that undermines the limitations of these two models before applying it to the nature of problem and data in this study.

MODEL CONSTRUCTION

To measure the entropy of external debt stock, we write equation (1) as

$$H = \frac{-\sum_{i=1}^{k}\left(\frac{n_i}{\sum_{i=1}^{k} n_i}\right)\log_e\left(\frac{n_i}{\sum_{i=1}^{k} n_i}\right)}{\log_e k} \qquad (3)$$

where n_i denotes the external debt stock owed by the economy in stage i, and i refers to the stage of economic growth.

Further simplification of the formula in equation (3) yields:

$$H = \frac{1}{\left(\sum_{i=1}^{k} n_i\right)\ln k}\ \ln\frac{\left(\sum_{i=1}^{k} n_i\right)^{\sum_{i=1}^{k} n_i}}{\prod_{i=1}^{k} n_i^{\,n_i}} \qquad (4)$$

By Stirling's formula given in Stephenson (1973) and Gupta (1993)

$$n! \approx \sqrt{(2\pi)}\ \ e^{n}\ \ n^{n+\frac{1}{2}} \qquad (5)$$

for large n.

Substituting the formula in equation (5) into equation (4), we obtain

$$H \approx \frac{1}{\left(\sum_{i=1}^{k} n_i\right)\ln k}\ \left\{\frac{1}{2}\ln\frac{(2\pi)^{(k-1)}\prod_{i=1}^{k} n_i}{\sum_{i=1}^{k} n_i} + \ln\frac{\left(\sum_{i=1}^{k} n_i\right)!}{\prod_{i=1}^{k}(n_i)!}\right\} \qquad (6)$$

The second part of equation (6) is the Boltzmann's formula given in equation (2) reduced by the scaling factor

$$\left(\sum_{i=1}^{k} n_i\right)\ln k$$

Thus, we have in gamma notation:

$$H_e = \frac{1}{\left(\sum_{i=1}^{k} n_i\right)\ln k}\ \left\{\frac{1}{2}\ln\frac{(2\pi)^{(k-1)}\prod_{i=1}^{k} n_i}{\sum_{i=1}^{k} n_i} + \ln\frac{\Gamma\left(\sum_{i=1}^{k} n_i + 1\right)}{\prod_{i=1}^{k}\Gamma(n_i+1)}\right\} \qquad (7)$$

H_e is the extended entropy measure (EEM).

The values of the gamma function can be read from gamma table. Gamma function is defined as

$$\Gamma(n) = \int_0^{\infty} e^{-x} x^{n-1}\, dx,\ n > 0 \qquad (8)$$

[see Gupta (1993) and Stephenson (1973)]

Moreover, EEM has some advantages over other entropy measures in the literature. This encapsulates:

i) It is a short computational formula as we need not compute the probability P_i (as in McClean and Abodunde, 1978).

ii) It can be used for data having decimal fractions, which is the case of external debt stock, unlike the Boltzmann's entropy formula in equation (2) as given by Tyler (1989).

iii) It solves the problem associated with large values of n_i, which is associated with the Boltzmann's formula in equation (2).

However, for large values of n_i, we reduce n_i by a factor f so that

$$\left(\frac{n_i}{f}\right) \in [1,\ 2]$$

Notwithstanding, if

$$\left(\frac{n_i}{f}\right) \in (0,\ 1],\ then\ \Gamma\left(\frac{n_i}{f}\right) = \frac{\Gamma\left(\frac{n_i}{f}+1\right)}{\left(\frac{n_i}{f}\right)} \tag{9}$$

The reason for this transformation is that the gamma table has values for n such that $1 \leq n \leq 2$. In this regard, a correction factor, C, is included in the formula in equation (7) so as to ensure that

$$H_e \in [0,\ 1]$$

From simplification of equation (7) it is easy to see that

$$H_e^* = \frac{1}{f\left(\sum_{i=1}^{k} n_i^*\right)\ln k}\left\{\frac{1}{2}\ln\frac{(2\pi)^{(k-1)}\prod_{i=1}^{k} n_i^*}{\sum_{i=1}^{k} n_i^*} + \ln\frac{\Gamma\left(\sum_{i=1}^{k} n_i^* + 1\right)}{\left(\prod_{i=1}^{k}\Gamma\left(n_i^*+1\right)\right)} + C\right\} \tag{10}$$

where $n_i^* = \frac{n_i}{f}$

$$C = (f-1)\left\{\left(\sum_{i=1}^{k} n_i^*\right)\ln\left(\sum_{i=1}^{k} n_i^*\right) - \sum_{i=1}^{k} n_i^* \ln n_i^*\right\} \tag{11}$$

and H_e^*

is the corrected extended entropy measure (EEM).

NUMERICAL APPLICATION OF THE MODEL TO NIGERIA'S DEBT STRUCTURE

To estimate the entropy value of external debt stock in Nigeria, data were drawn from Iyoha (2005) for the period 1971 to 2003 (see Table I in List of Tables).

The stages of development in Nigeria can be divided into three, viz: the pre – SAP period (1960 – 1985), the SAP period (1986 – 1988) and the post – SAP period (1989 – till date), see Iyoha (2005), Okojie (2002), and Oyesusi and Mogbolu (2002). Thus, we obtain the results given in Table 2 (see List of Tables).

Note that the acronym SAP is used to denote Structural Adjustment Programme

We illustrate the system in equation (10) using the results in Table 2. The term C in equation (10) is calculated as follows taking $f = 10^6$;

$$C = (10^6 - 1)\{0.61\ln 0.61 - (0.08\ln 0.08 + 0.08\ln 0.08 + 0.45\ln 0.45)\}$$

$$= 4.62 \times 10^5$$

Hence

$$H_e^* = \frac{1}{10^6}\left\{\frac{1}{2}\ln\frac{4\pi^2(0.08)^2(0.45)}{0.61} + \ln\frac{\Gamma(1.61)}{[\Gamma(1.08)]^2\Gamma(1.45)} + 4.62\times 10^5\right\}$$

$$\Rightarrow H_e^* = 0.68928 \approx 0.69$$

Note: The gamma functions $\Gamma(.)$ are read from gamma table.

TESTING THE STABILITY OF THE EXTENDED ENTROPY MEASURE (EEM)

The stability of the measure is demonstrated using bootstrap re-sampling technique applied to the data in Table 1. Generally, there is no yardstick for determining the number of bootstrap samples to be generated from a given data. However, it is advocated that the number of 'bootstrap samples' and hence the number of replicates, B, be taken so large so that bootstrap standard deviation

Tables 1: Trends in External Debt Burden Indicators, 1971 – 2003

Year	External Debt US $ Million	Debt Per Capital US $	Debt / GDP Ratio Percentage	Debt Service Ratio Percentage
1971	309	6.4	2.7	2.3
1972	401	8.0	3.7	1.8
1973	421	8.2	2.5	1.8
1974	523	9.8	1.8	1.3
1975	559	10.2	1.6	0.5
1976	594	10.5	1.4	0.7
1977	763	13.1	1.1	0.3
1978	2164	36.1	3.5	2.7
1979	2825	45.6	3.7	1.7
1980	3444	53.2	3.7	0.8
1981	3668	54.9	4.6	5.0
1982	13124	191.9	17.1	8.9
1983	17765	251.3	18.5	17.8
1984	17347	237.6	23.3	29.1
1985	18904	250.4	23.9	31.7
1986	25574	328.3	56.7	28.0
1987	28316	351.8	92.6	11.9
1988	30693	368.9	92.2	24.2
1989	31589	372.1	106.9	25.3
1990	33099	381.8	114.6	23.9
1991	33730	381.1	101.4	25.7
1992	27565	302.9	99.0	18.2
1993	28718	305.5	90.8	16.2
1994	29429	304.0	71.1	18.8
1995	32585	327.5	36.7	15.2
1996	28060	274.6	22.5	13.4
1997	27087	258.0	21.0	10.9
1998	28773	265.4	22.9	16.1
1999	28039	179.3	77.2	9.0
2000	28274	245.4	86.4	9.0
2001	28347	236.6	57.9	11.9
2002	30990	253.7	62.3	7.5
2003*	32917	261.3	64.4	39.8

***Estimated**

Source: Iyoha (2005) citing from Debt Management Office Nigeria, 2004. Annual Report and State of Account for the year ended December 31st, 2003.

would exactly equal standard deviation of the original distribution. Notwithstanding, Efron (1982) stated that "the values of B were deliberately taken large for investigating quantities other than standard deviation and that the smallest value of B performed almost as well in all three situations". Efron (1982) also stated that "In most cases there is no point in taking B so large that the bootstrap standard deviation agrees very closely with the standard deviation of the original distribution, since the latter itself will be highly

Table 2: External Debt Stock for the Stages of Development, 1971 – 2003

Stage I	Classification	External Debt Stock (US $ million)	External Debt Stock (US $ trillion)
1	Pre – SAP Period	82811	0.08
2	SAP Period	84583	0.08
3	Post – SAP Period	449202	0.45
	Total		**0.61**

Source: Computed from Table 1

variable for estimating the true standard deviation".

In our study, bootstrap samples of 100 (i.e. 100 bootstrap replicates) is considered adequate to determine the standard deviation and the bias of the EEM. Hence the stability of the EEM can be established.

Bootstrap samples of 100 replicates were generated using computer. The resultant entropy values for the 100 bootstrap samples using equation (10) are presented in Table 3.

Bootstrap standard deviation is given by

$$\hat{SD} = \left\{ \frac{1}{B-1} \sum_{b=1}^{B} \left[H_{eb}^{*} - \hat{H}_{\bullet}^{*} \right]^{2} \right\}^{1/2} \quad (12)$$

b =1,2,…,100, B is the number of bootstrap samples,

$$\hat{H}_{\bullet}^{*} = \frac{1}{B} \sum_{b=1}^{B} H_{eb}^{*}$$

and,

H_{eb}^{*} is the estimated entropy values for the bootstrap sample b using equation (10). In our case B is 100.

Applying equation (12) to the results in Table 3, we have

$\hat{SD}$ =0.1220; $\hat{H}_{\bullet}^{*}$ =0.7020.

Therefore,

Bias = $\hat{H}_{\bullet}^{*}$ - H_{e}^{*} =0.7020 -0.69

$\Rightarrow$ Bias =0.012

Table 3: Entropy Values for 100 Bootstrap Samples

0.6647	0.4716	0.5439	0.6440	0.4249
0.6749	0.8006	0.5755	0.8047	0.7442
0.7807	0.6952	0.9319	0.6223	0.7232
0.7836	0.6575	0.6681	0.6457	0.6053
0.6856	0.8205	0.7150	0.8384	0.8326
0.7301	0.7777	0.5875	0.6557	0.8895
0.7760	0.7088	0.7149	0.9707	0.8742
0.6159	0.8028	0.7264	0.9297	0.4722
0.7115	0.6237	0.7844	0.6839	0.8257
0.8472	0.8380	0.7917	0.4911	0.7103
0.7103	0.9340	0.6949	0.8293	0.8932
0.8410	0.7323	0.6979	0.5420	0.6324
0.6626	0.6257	0.6336	0.6449	0.5367
0.7474	0.4733	0.6005	0.6596	0.8683
0.8255	0.7656	0.7246	0.8149	0.4753
0.7656	0.7052	0.6717	0.8780	0.6954
0.8037	0.4197	0.7736	0.8807	0.6937
0.9096	0.5004	0.9212	0.6189	0.7928
0.6345	0.8019	0.6350	0.4315	0.7558
0.8277	0.5013	0.7580	0.8693	0.5845

Bootstrap standard deviation of $\hat{SD}$ =0.1220 is low. The bootstrap estimate of bias is 0.012 which is approximately 1 percent. This value is also low. Hence the stability of our measure is guaranteed.

The value H_{e}^{*} =0.69 is reliable.

DISCUSSION

In the analysis of debt structure, high entropy value indicates stability of debt stock. A stable debt stock implies a continuous increase in debt stock and low debt servicing capacity. Low entropy value shows instability of debt stock. The debt stock is unstable when loans contracted are accompanied by high debt servicing capacity.

From the computation in this section, $H_e^*= 0.69$. This is a high entropy value and a good measure of the distribution of the system, since $0< H_e^*<1$ (see Omosigho and Osagiede, 1999). The data used for the estimation, from 1971 to 2003, are sufficient as Nigeria's external debt stock grew in leaps and bounds since the late 1970s. (Obadan, 2002; Iyoha; 2002; 2005).

In the light of the foregoing, the Nigerian economy is faced with a stable external debt built – up. The reasons for this in literature are:

- Contracting of project – tied loans without consideration for economic viability;
- Continued decline in the terms of trade;
- Misalignment of exchange rates;
- Uncontrolled fluctuations in export earnings;
- Indiscriminate and uncontrolled borrowing abroad; and
- Poor debt management policies
  (Obadan; 2002; Iyoha, 2005)

Furthermore, the high value of H_e^* may also be attributed to accumulated interest arising from the debt (Jhingan, 2003), and low debt service ratio, below 30% in the period under study (Obadan, 2002).

The Structural Adjustment Programme (SAP) is used as a base to categorise the stages of development in Nigeria since the Nigerian economic crisis was a manifestation of deep – rooted structural imbalances in the country (Oyefusi and Mogbolu, 2002).

However, devaluation which was a feature in SAP led to an increase in total debt (see Iyoha, 2004), hence it is a factor for the high value of H_e^*.

The high value of H_e^*, which indicates a stable external debt built – up, has adverse effect on the economy as high external debt is deleterious to economic growth and development (Iyoha; 2002; 2005). In order to lower the value of H_e^*, Nigeria's external debt should be carefully managed as stipulated in Anyanwu (1997), Iyoha (2005) and Obadan (2002).

CONCLUSION

The method developed in this study has been applied to practical situation and the result obtained reflects reality. Interestingly, the method can be applied to any value of n, no matter how large thereby removing the limitations associated with Shannon and Boltzmann entropy.

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